Free Trade vs. Autarky under Asymmetric Cournot Oligopoly

Rabah Amir*

University of Iowa

Jim Y. Jin**

University of St Andrews

Michael Tröge***

ESCP-Europe

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ABSTRACT: The paper compares free trade with autarky in an asymmetric multi-country world with Cournot competition, constant returns and linear demand. We first derive conditions for free trade to hurt a country's consumers, to benefit its firms, to induce it to export, to increase its output, and to raise its welfare. We further show these conditions are linked in a clear order, with each one implying the next. We then demonstrate that with different reservation prices trade can reduce world output and total consumer surplus as well as world welfare and correct oversights in earlier findings by Dong and Yuan (2010).

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* Department of Economics, University of Iowa, Iowa City, IA 52242, U.S.A. E-mail: rabah-amir@uiowa.edu.

** School of Economics and Finance, University of St Andrews, St Andrews KY16 9AL, UK, e-mail: jyj@st-andrews.ac.uk.

*** Correspondant author: ESCP Europe, 97 av. de la République, 75543 Paris Cedex, France, e-mail: troege@escpeurope.eu

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1. Introduction

This paper re-examines the effects of free trade compared to autarky in the classic partial equilibrium Cournot model with linear demand and cost functions, a fixed number of firms and integrated markets without trading costs. Helpman and Krugman (1985, p. 88) observe that in this setting, *"the direction of trade cannot as in a purely competitive model, be determined simply by a comparison of costs or of pretrade prices. There are three sets of variables here - costs, market sizes, and numbers of firms - and all must be taken into account."*

Starting with Helpman and Krugman, (1985), the literature has identified conditions under which trade leads to certain effects on a country, such as increasing consumer surplus, increasing profits, inducing the country to export or import, increasing production or welfare. A number of papers have also demonstrated that these effects are connected. For example, Markusen (1981), Cordella (1993) and Dong and Yuan (2010) (D&Y) use two-country models to show that a country's welfare can only fall under free trade if the country is an importer and consumers can only be worse off if firms are better off. However, the literature has not identified general conditions in an asymmetric world with more than two countries.

The first objective of this paper is to extend the earlier findings and provide simple conditions for these effects and their relations. In particular, we show that there is a clear ranking among these conditions: If free trade hurts a country's consumers, it must benefit its firms. If trade benefits firms, the country must be an exporter and this in turn implies that its output rises. Finally, if a country's output rises, so does its welfare.

The second goal of this paper is to investigate the possibility for free trade to lower world welfare, total consumer surplus and output in a simple linear Cournot model. It is well known that there exist special situations where trade has undesirable effects[1] and D&Y recently derived a necessary and sufficient condition for free trade to lower total welfare with two countries. Their analysis, however, contains several oversights. We clarify these errors and demonstrate that in a more general model trade not only can reduce world welfare but also world output and consumer surplus.

2. Model

There are m ($\geq$ 2) countries. Every Country i's has a representative consumer with a quadratic utility function $z_i + a_i y_i - 0.5 b_i y_i^2$, where z_i is the numeraire good and y_i is the consumption of the oligopoly good. Utility maximization implies an inverse demand function $p_i = a_i - b_i y_i$. There are n_i ($\geq$ 1) firms in each country which have identical and constant marginal cost $c_i < a_i$. In equilibrium, every firm chooses its output q_i to maximize its profit $(p_i - c_i) q_i$. Under autarky, the first-order condition $p_i - c_i - b_i q_i = 0$ yields the Cournot equilibrium price: $p_i^A = \dfrac{a_i + n_i c_i}{n_i + 1}$. A country's autarky output must be equal to its consumption, so we have $n_i q_i^A = y_i^A = \dfrac{n_i (a_i - c_i)}{b_i (n_i + 1)}$.

[1] The insight that trade can harm individual countries goes back to Bhagwati (1971) and Johnson (1965). Situations where trade can reduce world welfare include: inefficient specialization (Krugman 1979, Markusen 1981, Eckel 2008), segmented markets and transportation costs (Brander and Krugman 1983), the absence of insurance markets (Newbery and Stiglitz 1984), strongly increasing returns and non-linear demand (Markusen and Melvin 1988), firms' location choice (Eaton and Kierzkowski 1984), increasing dispersion of markups (Epifani and Gancia 2011). None of these effects are present in our model.

Under free trade, there is a single world price of p^{T}. Country i's demand will be $y_i = (a_i - p^{\mathrm{T}})/b_i$ and the world total demand $y^{\mathrm{T}} \equiv \sum_{i=1}^{m} \frac{a_i - p^T}{b_i}$. Using $\beta \equiv \sum_{i=1}^{m} 1/b_i$ to indicate the size of the world market, we can write the equilibrium price with free trade as $p^{\mathrm{T}} = \frac{1}{\beta}(\sum_{i=1}^{n} \frac{a_i}{b_i} - y^{\mathrm{T}})$. When every firm in Country i maximizes its profit $(p^{\mathrm{T}} - c_i)q_i$, the first-order condition is $p^{\mathrm{T}} - c_i - q_i/\beta = 0$. A firm's output is $q_i^T = \beta(p^{\mathrm{T}} - c_i)$ and the world total output and consumption $y^{\mathrm{T}} = \sum_{i=1}^{m} n_i q_i^T$. The total number of firms is denoted by $N \equiv \sum_{i=1}^{m} n_i$. The equilibrium price can be solved as:

$$p^{\mathrm{T}} = \frac{1}{N+1} \sum_{i=1}^{m} (\frac{a_i}{\beta b_i} + n_i c_i) \tag{1}$$

We assume $\min\{a_i\} \geq p^{\mathrm{T}} \geq \max\{c_i\}$, so that consumers and firms in all countries are active in the market under free trade. Country i's consumer surplus CS_i = $a_i y_i - 0.5 b_i y_i^2 - p_i y_i = 0.5(a_i - p_i)^2/b_i$, total profit $\pi_i = n_i q_i (p_i - c_i)$ and social welfare $SW_i = 0.5(a_i - p_i)^2/b_i + n_i q_i (p_i - c_i)$. The respective values can be found by substituting p_i and q_i under free trade and autarky, as solved above. Next we will investigate the effect of trade on individual countries.

3. The Effects of Trade

In this section we consider the effects of trade on a country's welfare, consumer surplus, profits, trade position and output. A key variable that will help us simplify mathematical expressions is the ratio of the price margin under autarky over the margin under free trade, $(p_i^A - c_i)/(p^{\mathrm{T}} - c_i)$. We denote this ratio by d_i.

The value of d_i depends in an interesting way on the model's primitives, and more specifically on the average cost and reservation price. Using (1) we can express p^{T} c_i as the sum of $\sum_{j=1}^{m} \frac{a_j}{\beta b_j} - c_i$ and $\sum_{j=1}^{m} n_j (c_j - c_i)$ divided by N + 1. The price margin under autarky $p_i^A -$ c_i can be viewed as a special case of this expression with only one country *i*. If c_i is equal to the average cost $\sum_{j=1}^{m} n_j c_j$ /*N*, the second term of p^{T} – c_i disappears. Similarly, the first term reduces to a_i – c_i if a_i is equal to $\sum_{j=1}^{m} \frac{a_j}{\beta b_j}$, which can be interpreted as the world's (size weighted) average reservation price as $\sum_{j=1}^{m} \frac{1}{\beta b_j}$ = 1. Hence, if a country's reservation price and costs are equal to the world averages, we have d_i = (N + 1)/(n_i + 1) > 1, indicating that the change of price caused by trade only depends on the number of firms under autarky and free trade, but is independent of how costs and reservation prices are distributed. If c_i is lower than the average cost, or if a_i is lower than the average reservation price, d_i will be smaller than this value, as less efficient foreign firms and higher foreign demand weaken competition. When c_i and a_i are sufficiently low, d_i can be lower than 1, implying a higher price under free trade. Next we will use d_i to express various conditions for certain effects due to free trade.

(i) ***Consumer surplus*****:** Consumers gain or lose if and only if free trade decreases or increases the price, i.e., whether $p^{\mathrm{T}} < p_i^A$, or d_i > 1. The value of d_i depends on every parameter in this model through p^{T}. However, we can obtain a sufficient condition for d_i

> 1, which only depends on a country's reservation price a_i, and $\sum_{j=1}^{m} \frac{a_j}{\beta b_j}$. So we have a sufficient condition for free trade to benefit a country's consumers (see Appendix I).

Proposition 1: *Free trade benefits a country's consumers if its reservation price is not lower than the world average reservation price.*

The literature (e.g. D&Y) often assumes $a_i = a$, for all i, which guarantees the condition. Hence, if all countries have the same reservation price, all consumers are better off under free trade compared to autarky. Usually consumers in rich countries tend to have higher reservation prices and are therefore more likely to benefit from trade than their counterparts from poor countries.

(ii) ***Profit***: Intuitively, the interest of firms and consumers regarding trade are not necessarily aligned. When producers suffer from imported goods, consumers usually benefit. Likewise, if high export demand increases prices, consumers will suffer, but firms generate high profits. To find conditions under which firms benefit from trade, we need to compare their profit under free trade, $(p^{\mathrm{T}} - c_i)q_i^T = \beta\,(p^{\mathrm{T}} - c_i)^2$ with that under autarky, i.e. $(p_i^A - c_i)q_i^A = (a_i - c_i)^2/(n_i + 1)^2 b_i$. Clearly, the former is larger than the latter if and only if $\sqrt{\beta b_i}\,(p^{\mathrm{T}} - c_i) > (a_i - c_i)/(n_i + 1)$, or $d_i < \sqrt{\beta b_i}$. Hence, we have:

Proposition 2: *Free trade benefits a country's firms if and only if* $d_i < \sqrt{\beta b_i}$.

Since $\sqrt{\beta b_i} > 1$, it is impossible to have $d_i < 1$ and $d_i > \sqrt{\beta b_i}$ simultaneously. Hence, consumers and firms cannot both be worse off. Furthermore, as $d_i < 1$ implies $d_i < \sqrt{\beta b_i}$, a reduction in consumer surplus implies an increase in profits. Likewise, as d_i

$> \sqrt{\beta b_i}$ implies $d_i > 1$, lower profits means higher consumer surplus. Note, that for firms' profits to increase, p^{T} does not need to be higher than p_i^A. As the price under free trade is less sensitive to a firm's output than under autarky ($\beta > 1/b_i$), a firm produces more even if prices do not change. So it is better off as long as p^{T} is not too much lower than p_i^A. The larger the relative increase in market size indicated by βb_i, the lower p^{T} can be without making the firms worse off.

(iii) *Export/Import*: While D&Y and others show that an exporting country must be better off under free trade in a two-country model, this relation may change in a multi-country setting as now one country's export does not any more correspond to the other country's import. To find out if a country is exporting under free trade, we need to compare its output $n_i q_i^T = \beta n_i (p^{\mathrm{T}} - c_i)$ to its consumption, $y_i^T = (a_i - p^{\mathrm{T}})/b_i$, i.e., whether $\beta n_i (p^{\mathrm{T}} - c_i) > (a_i - p^{\mathrm{T}})/b_i$. Hence, we have:

***Proposition 3*:** *A country exports if and only if* $d_i < \dfrac{1 + \beta b_i n_i}{1 + n_i}$.

It is easy to see that $d_i < \sqrt{\beta b_i}$ implies $d_i < (1 + \beta b_i n_i)/(1 + n_i)$. So, if a country has higher profits under free trade, it must be exporting, but the reserve is not necessarily true. Common sense seems to suggest the opposite: an exporting country should generate higher profits under free trade. In fact, a country is more likely to export than to earn higher profits, because free trade generally depresses prices even if it increases demand.

(iv) *Output*: Free trade generally stimulates production, because firms know that an increase in their output has less impact on the price and will consequently produce more, given the same price. However, as D&Y have shown, it is possible that a low cost country ends up producing less, due to an excessive output expansion by a high cost country. A rise in Country *i*'s production requires that every firm's output under free trade $q_i^T = \beta\,(p^{\mathrm{T}} - c_i)$ exceeds that under autarky, $(a_i - c_i)/b_i(n_i + 1)$. Comparing these two terms we obtain:

Proposition 4**:** *A country's output rises under free trade if and only if* $d_i < \beta b_i$.

Note, that $\beta b_i > (1 + \beta b_i\, n_i)/(1 + n_i)$ always holds. So if a country exports, i.e. $d_i < (1 + \beta b_i\, n_i)/(1 + n_i)$, we have $d_i < \beta b_i$, i.e. its output must rise.

(v) *Social Welfare*: Since firms and consumers cannot both lose under free trade, a country's welfare may fall either if its firms' loss in profits exceeds its consumers' gain, or if its' firms gains are lower that it's decrease in consumers surplus. Interestingly, we will show that the latter is impossible: A welfare loss can only occur when firms lose and consumers gain. We find that free trade always benefits a country as a whole if $\Delta_i \equiv n_i - 2(\beta b_i - 1) < 0$. If the country has few firms, consumers are likely to benefit from trade. If the world market is much larger than the home market (high βb_i), firms are likely to gain. So firms' loss cannot exceed consumers' gain. When this condition fails, welfare will fall when d_i is sufficiently close to $1 + n_i$. We can obtain a necessary and sufficient condition for a country's welfare loss (see Appendix II):

Proposition 5: *Free trade will reduce a country's social welfare if and only if* $\Delta_i > 0$ *and* $\left| 1 + n_i - d_i \right| < \sqrt{n_i \Delta_i}$.

This result indicates the trade-off between consumers' gain and firms' loss. If $d_i < 1$, we have $p^{\mathrm{T}} > p_i^A$, hence consumers lose, firms gain, and welfare always rises. When

$d_i > 1$, consumers gain but firms may not lose as we explained earlier. Since the market is less sensitive to a firm's output ($\beta > 1/b_i$), profit still rises as long as $d_i < \sqrt{\beta b_i}$. When $d_i > \sqrt{\beta b_i}$, free trade benefits consumers but hurts firms. As d_i rises further to satisfy the condition in Proposition 5, consumers' gain is equal to firms' loss, and welfare does not change. For an even higher d_i total welfare falls under free trade until the condition is met again. Beyond this point a higher d_i will guarantee that consumers' gain dominates firms' loss, implying a higher welfare under free trade. Hence, a welfare loss cannot happen if d_i is either too high or too low. If d_i is very high, p^{T} is relatively low and the consumers' gain will dominate any profit loss. If d_i is very low, p^{T} is relatively high and firms cannot lose enough to offset consumers' gain.

Finally, we can show that an increase in output always ensures a welfare gain. A welfare loss requires $n_i > 2(\beta b_i - 1)$, so $n_i + 1 > 2\beta b_i - 1 > \beta b_i$. An output increase implies $d_i < \beta b_i$, so we have $|1 + n_i - d_i| > |1 + n_i - \beta b_i|$. Since $(n_i + 1 - \beta b_i)^2 > n_i \Delta_i$ always, we have $|1 + n_i - d_i| > \sqrt{n_i \Delta_i}$, i.e. welfare must increase.

(vi) *Relations between Conditions*: In the discussion above we have already characterized relations between pairs of conditions. Simply by linking these pairwise connections we can establish a clear ordering:

***Proposition 6*:** *If a country's consumers are worse off under free trade, its firms must be better off. If profits increase, the country must export, which implies that its output increases. Finally a higher output guarantees a welfare gain for this country.*

Conversely, we easily see that, if a country's social welfare falls under free trade, its output must fall, which implies that the country is an importer. This in turn means that firms' profits fall, which implies that consumer surplus must increase.

To get some intuition for these relationships, Figure 1 shows how a low cost Country 1 is affected when trading with a high cost Country 2. The indifference curves show for which combinations of Country 1's reservation price a_1 and the number of firms n_1, its welfare, production, trade position, profits and consumer surplus remain unaffected. We also add the indifference curve for the sum of both countries' welfare which indicates the possibility of a world welfare loss.

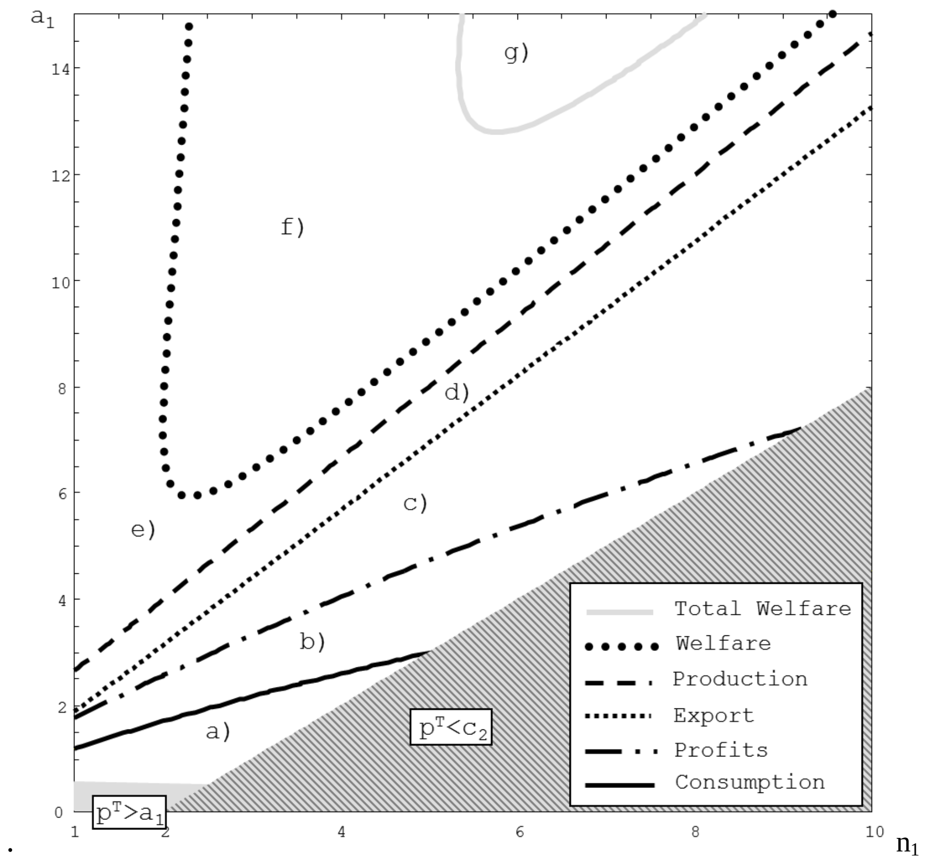

Figure 1: Indifference curves for Country 1's consumers, firms, export, production and welfare given $a_2 = 3,\ b_1 = b_2 = 1,\ n_2 = 9,\ c_1 = 0,\ c_2 = 0.5$

In the grey areas at the bottom of this graph, under free trade some firms or consumers exit the market, which violates our assumptions. In region a), with very low a_1, p_1^A is lower than p^T, implying a loss of consumer surplus. In region b), with higher a_1, trade reduces prices so consumers are better off. A further increase of a_1 leads to region c), where firms are worse off, as they lose the high profits they would made under autarky. As a_1 continues to rise, we enter region d), where the large domestic market starts to attract foreign goods and turns Country 1 to an importer. The next region e) has a higher a_1. Now imports force domestic firms to reduce production, despite having lower costs. We notice that all indifference curves are upward sloping. This is because with higher n_1, the market is more competitive, and less affected by Country 2's high cost producers. We therefore need more increase in a_1 to move from one region to another. Finally, if $n_1 > 2$ and a_1 further rises we enter region f), where Country 1's welfare decreases.

For most parameter values we still find that trade increases consumer surplus, decreases profits and raise welfare (regions (c) + (d) + (e)). However, the range of parameter constellations for welfare decrease is surprisingly large (region (f)).

4. Inefficient Trade

The above analysis provides some clues for understanding how trade can lead to a reduction of world welfare. In the example presented in Figure 1, the inefficient Country 2 will always benefit from trade. However, if Country 1 is sufficiently large and has sufficient but not too many firms, its welfare loss exceeds Country 2's welfare gain. This will lead to the decrease of world welfare in region g). Essentially, in this case, opening trade is equivalent to allowing the entry of Country 2's inefficient firms in Country 1's market. We know from Lahiri and Ono (1988) that under Cournot competition this can reduce welfare. Unfortunately, it is algebraically very cumbersome to provide the precise conditions for this to happen. To our knowledge D&Y are the first to have carried out this analysis. However, their result suffers from a few small errors which need clarification.

D&Y's model is a special case of our setup with two countries and identical reservation prices. They assume demand functions $y_1 = a - bp_1$, and $y_2 = \gamma(a - bp_2)$, where $\gamma \geq 0$ is the relative size of the two countries' demand. To simplify their formulae and avoid confusion with our parameters b_i, we set their parameter $b = 1$, without loss of generality. We can then write the inverse demand functions as $p_1 = a - y_1$, and $p_2 = a - y_2/\gamma$. D&Y assume the marginal cost in Country 1 ("Southern") to be higher than in Country 2 ("Northern"), i.e. $c_1 > c_2$. They argue that a necessary condition for a decrease in world welfare is *"the displacement of production of the Northern country by that of the southern Country"* (p. 826). Their Proposition 1 gives a sufficient condition for Country 2's output reduction:

$$\frac{a - c_2}{c_1 - c_2} > \frac{n_2(n_2 + 1)(1 + \gamma)}{\gamma n_1 - n_2 - 1} \tag{2}$$

This condition is incorrect. If the two countries have the same size ($\gamma = 1$) and the same number of firms ($n_1 = n_2$), (2) holds, but in this case the low cost Country 2's output should increase. The correct condition can be derived from our Proposition 4, which states that Country 2's output falls if and only if $\beta b_2 < d_2$. Using D&Y's

parameter γ, which corresponds to b_1/b_2 in our notation, we have $\beta b_2 = 1 + 1/\gamma$. Then as $p_2^A - c_2 = \dfrac{a_2 - c_2}{n_2 + 1}$ and $p^T - c_2 = \dfrac{a - c_2 + n_1(c_1 - c_2)}{n_1 + n_2 + 1}$, $\beta b_2 < d_2$ holds if and only if

$$(\gamma n_1 - 1 - n_2)(a - c_2) > (1 + \gamma)(1 + n_2) n_1 (c_1 - c_2) \qquad (3)$$

When dividing (3) by $c_1 - c_2$ and $\gamma n_1 - 1 - n_2$, D&Y apparently ignore the possibility of a negative sign which will reverse the direction of the inequality. In addition their n_2 on the right hand side should be n_1. This error seems to have resulted in a follow-up mistake. D&Y claim in their Proposition 7 that free trade reduces total welfare if and only if c_2 is higher than a particular threshold c_2^{**} and Country 2's relative size γ is sufficiently small. However, their c_2^{**} is defined, in our notation, as:

$$c_2^{**} = \frac{(2n_1 + n_2 + 2)a + c_1[2(n_1+1)^2(n_1 + n_2 + 1) - (n_1 n_2 + 2n_1 + 2n_2 + 2)]}{[2(n_1+1)^2 + n_2(2n_1+1)](n_1+1)} \qquad (4)$$

This can be simplified to $c_2^{**} = c_1 + \dfrac{(2n_1 + n_2 + 2)(a - c_1)}{(n_1+1))[2(n_1+1)^2 + n_2(2n_1+1)]}$. Since $a > c_1$, we have $c_2^{**} > c_1$. So the condition for total welfare loss, $c_2 > c_2^{**}$ violates their assumption that $c_1 > c_2$. However, if we assume $c_2 > c_1$, D&Y's Proposition 7 will be essentially correct,[2] i.e. the total welfare falls due to free trade if $\dfrac{a + n_1 c_1}{n_1 + 1} > c_2 > c_2^{**}$ and γ is sufficiently small.

In line with the intuition discussed above, it can be shown that the maximum welfare loss in this D&Y's two-country model occurs when $\gamma = 0$, $n_1 = 1$, $n_2 =$ infinity,

[2] There is another small error in the definition of DY's critical $\gamma^{**} = -D/F$. The term $a(2n_1 + n_2 + 2)$ in F should be $a(2n_2 + n_1 + 2)$.

i.e. when a large number of inefficient firms from a country with negligible size enters an efficient monopoly. The maximum welfare loss in this case is 1/9 of the original total welfare, which is quite significant.

Furthermore asymmetric demand intercepts in our model increases the scope for generating a total welfare loss compared to D&Y’s setup. Figure 1 shows a total welfare loss in region g), which does not require extreme parameter combinations and different country sizes. In the Appendix we illustrate this with a simple numerical example with $b_1 = b_2$, (i.e. $\gamma = 1$ in D&Y’s notation).

Asymmetric demand intercepts also lead to another surprising result. In D&Y’s model trade will always increase total output and every country’s consumer surplus. This is a direct consequence of our Proposition 1, as in their model both countries have identical reservation prices. If this restriction is relaxed, we can find parameter constellations for which trade decreases world consumer surplus and total output.

We illustrate this possibility in Figure 2, representing the combinations of a_2 and n_2 for which world consumer surplus and/or world output decrease. In this case consumers in Country 1 lose as its efficient firms export, but the price in Country 2 does not fall significantly, resulting in a decrease in total consumer surplus.

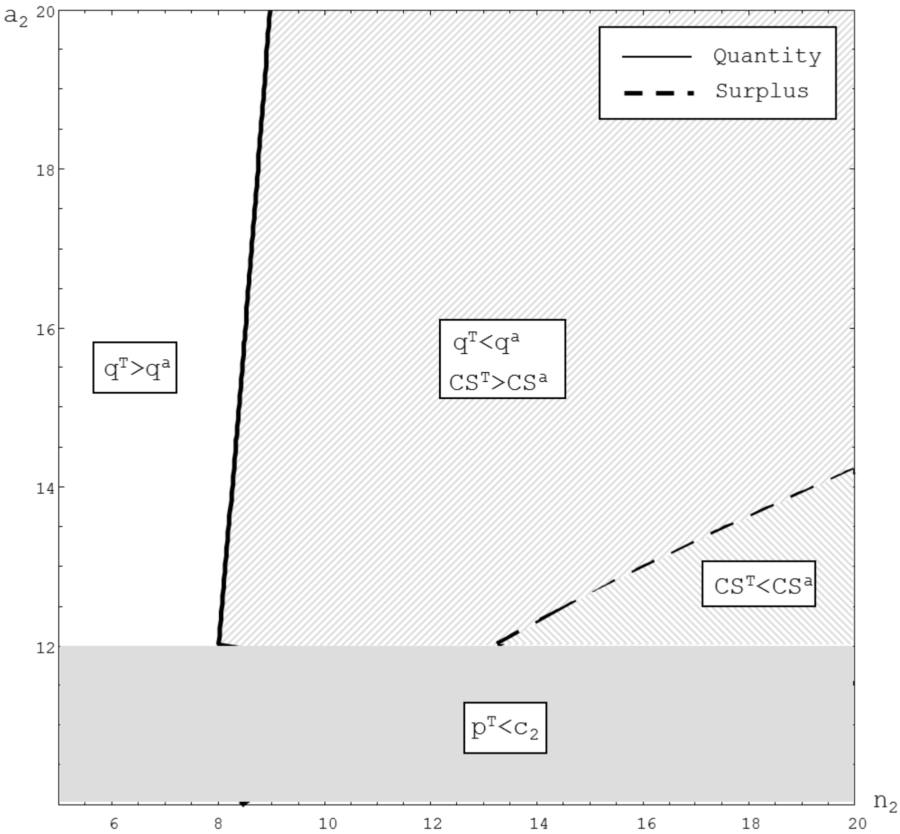

Figure 2: Reduction of world output and consumer surplus
$a_1 = 9,\ b_1 = b_2 = 1,\ n_1 = 1,\ c_1 = 0,\ c_2 = 5.25$

Interestingly, whereas world consumer surplus can fall only if would output falls, there is a large parameter space where a decrease in output does not lead to a decrease in consumer surplus, as trade will allocate the smaller output more efficiently to consumers with high demand. Again we provide in the Appendix a numerical example (with $a_2 =$ 12 and $n_2 = 20$) for a situation where both total output and consumer surplus fall.

5. Concluding remarks

This paper first obtains conditions under which free trade with Cournot competition has a positive or negative impact on a country's consumers, firms, welfare, export/import position and output. We then provide a clear ranking for these conditions.

In the second part of this paper we clarify some oversights in D&Y's analysis of welfare reducing trade, and illustrate that in a more general model trade can also lead to a reduction in world output and total consumer surplus.

While theoretically interesting, we do not think, however that our results should be viewed as a strong argument against free trade. Except in extreme cases, the magnitude of the total welfare loss is very small compared to the potential gains.

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Appendix I, Proof of Proposition 1:

Without loss of generality we assume $a_1 \geq \sum_{i=1}^{m} \frac{a_i}{\beta b_i}$, and show $d_1 > 1$, i.e.

$$\frac{1}{N+1}\left(\sum_{i=1}^{m} \frac{a_i}{\beta b_i} + \sum_{i=1}^{m} n_i c_i\right) < c_1 + \frac{a_1 - c_1}{1+n_1} \tag{A1}$$

(A1) holds if $(n_1 + 1)(a_1 + \sum_{i=1}^{m} n_i c_i) < (N + 1)(n_1 c_1 + a_1)$, i.e.

$$a_1 + n_1 c_1 > \frac{n_1+1}{N-n_1} \sum_{i=2}^{m} n_i c_i\ . \tag{A2}$$

Let $\bar{c} = \max.\{c_i\}$. As $p^{\mathrm{T}} > \bar{c}$ and $a_1 \geq \sum_{i=1}^{m} \frac{a_i}{\beta b_i}$, we have $a_1 + \sum_{i=1}^{m} n_i c_i > (N + 1)\bar{c}$, i.e. $a_1 + n_1 c_1 > (N + 1)\bar{c} - \sum_{i=2}^{m} n_i c_i > (n_1 + 1)\bar{c}$. So (A2) must hold if $(n_1 + 1)\bar{c} \geq \frac{n_1+1}{N-n_1} \sum_{i=2}^{m} n_i c_i$, which is true as $\bar{c} = \max.\{c_i\}$. Hence (A1) must hold.

Appendix II, Proof of Proposition 5:

(i) Country i's welfare under autarky is $0.5(a_i - c_i)^2[1 - 1/(1 + n_i)^2]/b_i$. Under free trade, it is $0.5(a_i - p^{\mathrm{T}})^2/b_i + \beta n_i (p^{\mathrm{T}} - c_i)^2$. The former is larger if and only if

$$L \equiv (a_i - p^{\mathrm{T}})^2 + 2 b_i \beta n_i (p^{\mathrm{T}} - c_i)^2 - (a_i - c_i)^2[1 - \frac{1}{(1+n_i)^2}] < 0.$$

As $\partial^2 L/\partial p^{\mathrm{T}2} = 4 b_i \beta n_i + 2 > 0$, L reaches its minimum when $\partial L/\partial p^{\mathrm{T}} = 0$, i.e. $p^{\mathrm{T}} = c_i + \frac{a_i - c_i}{1+2\beta b_i n_i}$. At this p^{T}, we find the minimum $L^* = (a_i - c_i)^2[\frac{1}{(1+n_i)^2} - \frac{1}{1+2\beta b_i n_i}]$.

It is positive if $1 + 2\beta b_i n_i > (1 + n_i)^2$, or $n_i < 2(\beta b_i - 1)$.

(ii) Let $n_i > 2(\beta b_i - 1)$, we solve $L = 0$. After re-arrangement, we get

$$(1 + 2\beta b_i n_i)(p^{\mathrm{T}} - c_i)^2 - 2(a_i - c_i)(p^{\mathrm{T}} - c_i) + \frac{(a_i - c_i)^2}{(1+n_i)^2} = 0 \tag{A3}$$

So we have $L < 0$ if and only if $p^{\mathrm{T}} - c_i$ lies between the two solutions of (A3), i.e.

$$\frac{a_i - c_i}{1+2\beta b_i n_i}\left(1 - \frac{\sqrt{n_i \Delta_i}}{1+n_i}\right) < p^{\mathrm{T}} - c_i < \frac{a_i - c_i}{1+2\beta b_i n_i}\left(1 + \frac{\sqrt{n_i \Delta_i}}{1+n_i}\right)$$

Given our definition $(a_i - c_i)/(p^{\mathrm{T}} - c_i)(1 + n_i) = d_i$, this inequalities can be written as:

$$d_i\,[1 + n_i - \sqrt{n_i \Delta_i}\,] < 1 + 2\beta b_i\, n_i < d_i\,[1 + n_i + \sqrt{n_i \Delta_i}\,]$$

These inequalities hold if and only if $|\,1 + n_i - d_i\,| < \sqrt{n_i \Delta_i}$.

Appendix III, Examples:

***Example* 1, *Total welfare loss under free trade*:**

Assume $m = 2,\ a_1 = 14,\ a_2 = 3,\ b_1 = b_2 = 1,\ n_1 = 6,\ n_2 = 9,\ c_1 = 0,\ c_2 = 0.5$.

Under autarky, we find $p_1^A = 2 = q_1^A$, $p_2^A = 0.75$, $q_2^A = 0.25$. As $SW_i = 0.5(a_i - p_i)^2/b_i + n_i q_i\,(p_i - c_i)$, we find $SW_1^A = 0.5\times12^2 + 6\times2\times2 = 96$, and $SW_2^A = 0.5(3 - 0.75)^2 + 9/16 = 99/32$. So the total welfare $SW_1^A + SW_2^A = 96 + 99/32 = 99.1$.

Under free trade, we obtain $p^{\mathrm{T}} = 13/16$, $q_1^T = 13/8$, $q_2^T = 5/8$. Thus we have $SW_1^T = 0.5\times(14 - 13/16)^2 + 6\times(13/8)(13/16)$, and $SW_2^T = 0.5(3 - 13/16)^2 + 9\times(5/8)(13/16 - 0.5)$. Then we find $SW_1^T + SW_2^T = 99 < SW_1^A + SW_2^A$.

***Example* 2, Total consumer surplus and output fall under free trade:**

Let $m = 2,\ a_1 = 9,\ a_2 = 12,\ b_1 = b_2 = 1,\ n_1 = 1,\ n_2 = 20,\ c_1 = 0,\ c_2 = 21/4$.

Under autarky, $p_1^A = 4.5$, $p_2^A = 39/7$. As $CS_i = 0.5(a_i - p_i)^2/b_i$, total consumer surplus is $0.5(9 - 4.5)^2 + 0.5(12 - 39/7)^2 = 30.8$. Under free trade, $p^{\mathrm{T}} = 21/4$ and total consumer surplus is equal to $0.5(9 - 21/4)^2 + 0.5(12 - 21/4)^2 = 29.8 < 30.8$.

For total output under autarky we have $n_1 q_1^A + n_2 q_2^A = 4.5 + 45/7 = 10.9$ whereas under free trade $n_1 q_1^T = 10.5$ and $n_2 q_2^T = 0$. So the world output is $10.5 < 10.9$.