Truth, Signification and Paradox*

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Abstract

Thomas Bradwardine's solution to the semantic paradoxes, presented in his *Insolubilia* written in Oxford in the early 1320s, turns on two main principles: that a proposition is true only if things are wholly as it signifies; and that signification is closed under consequence. After exploring the background in Walter Burley's account of the signification of propositions, I consider the extent to which Bradwardine's theory is compatible with the compositional principles of the distribution of truth over conjunction, disjunction, negation and the conditional.

Keywords: truth, meaning, paradox, sophism, bivalence, closure principles, conjunction, disjunction, negation, conditional, compositionality, distributivity; Bradwardine, Burley, Geulincx, Ricardus Sophista.

1 A Sophism about Saying That

We find an intriguing sophism about signification, or saying that, in an oration delivered at the University of Cambridge in 1660:

> "I don't doubt that the proverb 'The ass and the lyre' was coined for you alone. Yet surely you are not worthy to be proctor in the schools of the sophists. For a sophist attacks the Proctor like this: 'Whoever says you are an animal says something true; and whoever says you are an ass says you are an animal; so whoever says you are an ass says something true.' 'I grant the whole thing,' says the Proctor: 'I wouldn't dare deny it on account of my ears.' See, then, the Proctor confesses himself to be an ass by auricular confession."[1]

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[1]Raine (1843), p. xii: "Dubito certe annon in te solum cudatur proverbium Asinus ad lyram. At profecto tu non dignus es qui esses procurator in sophistarum scholis:

The oration is a typical, if unusually rich, example of academic humour, with its allusion to Aesop, its elaborate pun on auricular confession, and its citation of a sophismatic argument dating back to the thirteenth century. The sophism, in fact, appeared shortly afterwards in Geulincx (1662, IV.ii.16.8) (Land, 1891, I p. 451-2), called by him *sophisma splendida*, the splendid or brilliant sophism. Perhaps the anonymous author of the oration had heard Geulincx's lectures in Louvain or Leiden, possibly as a royalist exile during the Commonwealth.[2] The sophism also appeared in the Port-Royal Logic of Arnauld and Nicole (1662, III 12, p. 279), with a goose (oison, literally 'gosling') taking the place of the ass, and Geulincx himself comments that any falsehood could take the place of 'You are an ass', e.g., 'A white thing is black'.

Apart from this flurry of popularity in the mid-seventeenth century, the sophism appears to survive in few medieval treatises: we find it in the *Abstractiones* of around 1240 of Ricardus Sophista (sometimes therefore referred to as the 'Magister Abstractionum', known only through this work);[3] in the Shorter Treatise of Walter Burley's *De Puritate Artis Logicae* ('On the Essence of the Art of Logic'), written in Paris around 1323;[4] and in John Buridan's *Sophismata* (Buridan (2004, p. 51), (2001, p. 864)), written in Paris about a generation later. In the first two cases, it is given as a putative counter-example to what has more recently been entitled 'Suffixing': that whatever follows from the consequent follows from the antecedent.[5]

Burley's response is to distinguish two senses of 'saying that', depending on whether what is said, the *dictum* (e.g., 'that you are an animal'), supposits for an utterance or for a thing. In other words, the *dictum* can be taken in material supposition or in personal supposition, where supposition was, along with signification, one of the medievals' main semantic concepts by which to articulate their theories of meaning, truth and consequence. For example, if I say, 'I am looking at Burley', this is true taking 'Burley' in

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sic enim insurgit quidam sophista contra Procuratorem; 'Qui dicit te esse animal dicit verum; at qui dicit te esse asinum dicit te esse animal ergo, qui dicit te esse asinum dicit verum.' 'Concedo totum,' inquit Procurator: 'non ausus sum negare pro auribus.' Videtis, itaque, Procurator fatetur se esse asinum per confessionem auricularem." I am grateful to my colleague, Professor Sarah Broadie, for her help in trying to capture the subtlety of the original Latin in English. The proverb comes from Aesop. 'Auricular confession' apparently means confession made vocally by the penitent to a priest, as distinct from silently addressed to God. Presumably there's a supposed threat that if he says he's not a donkey then they'll make it true that he's not a donkey by cutting off his (donkey's) ears.

[2] Geulincx (1662) actually uses the example 'Charles is now king of England': see Nuchelmans (1988, p. 269).

[3] The *Abstractiones* are as yet unpublished; a preliminary edition can be found online at Ricardus (nd).

[4] Burley's treatise was edited in Burley (1955), and translated into English in Burley (2000). The counter-example also appears in the treatise on 'Consequences' of 1302 attributed to Burley and edited in Green-Pedersen (1980, p. 113), but the later passage of the Shorter Treatise is little more than a verbatim repetition of the earlier text. The example is discussed, rather inconsequentially, in Jacquette (2007a) (which is a shorter version of Jacquette (2007b)).

[5] Burley (1955, p. 200 ff.): "Quidquid sequitur ad consequens, sequitur ad antecedens."

material supposition, for I am looking at the word 'Burley' as I write it, but it is false in personal supposition, for Burley is long dead and no images of him remain.

Applying the distinction to the sophism, we note that, from the fact that I say, that is, utter the words, 'You are an ass', it does not follow that I utter the words 'You are an animal', so in material supposition the major premise of the argument is false. On the other hand, in personal supposition, just because I say you are an animal (which I might do by saying you are an ass) it does not follow that I say something true:

> "But if [the act of saying] takes the *dictum* as its object with reference to things, then the inference 'I say that you are an animal; therefore, I say something true' is not valid, because the antecedent can be true without the consequent. For if I say that you are an ass, I say that you are an animal, insofar as the act of saying takes the *dictum* as its object with reference to things. And yet in saying that you are an ass I am not saying something true."[6]

Setting aside the material interpretation where the *dictum* supposits for an utterance, and concentrating on the case where the *dictum* refers to things (as Burley puts it), it is clear that Burley accepts the inference 'If I say you are an ass, I say you are an animal', and rejects the subsequent inference, 'If I say you are an animal, I say something true'. The principle of Suffixing has been saved, but to understand the response, we need shortly to look more closely at Burley's theories of signification and of truth.

We find a similar diagnosis in Geulincx, in the Magister Abstractionum and in Buridan. Geulincx distinguishes two senses of 'saying that', *dicere formaliter vel expresse*, corresponding to direct speech, where the exact words are used; and *dicere consequenter vel implicite*, roughly indirect speech, committing oneself to every proposition entailed by the original.[7] Then the major premise, 'If I say you are an animal I say something true', is only true if 'say' is taken in direct speech (*formaliter*), in which case the minor premise, 'If I say you an ass I say you are an animal', is false, true only *dicens consequenter*.[8]

Similarly, the Magister Abstractionum defends suffixing as a necessary maxim, insisting that the inference 'If someone says you are an animal he says something true, so if someone says you are an ass he says something true' is necessary, "since it is superior to say you are an animal than to say you are an ass."[9] But he denies that everyone saying you are an animal says something true, "for this is not valid: that you are an animal is true, so everyone saying you are an animal says something true. It is a fallacy of figure of speech, for there is a change from one mode of supposition into

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[6]Burley (1955, p. 205), my own translation.
[7]Geulincx (1662, II.i.3.2) (Land, 1891, I p. 238). Cf. Nuchelmans (1994, p. 94).
[8]Geulincx (1662, IV.ii.16.9-10) (Land, 1891, I pp. 452-3).
[9]"Cum superius sit dicere te esse animal quam dicere te esse asinum."

another."[10] This particular fallacy was noted by Aristotle in ch. 24 of his *De Sophisticis Elenchis*, a very broad class of fallacy. Peter of Spain (De Rijk, 1972, pp. 144-5), writing around the same time as the Magister Abstractionum, distinguishes three modes of the fallacy of figure of speech, the third where there is a change of supposition. The Magister Abstractionum concedes, as does Burley, that if someone says you are an ass, he says you are an animal. But if he says you are an ass, he says something false. So sometimes if someone says you are an animal, he does not say something true.

To explain this phenomenon, Burley distinguishes subjective truth from objective truth: "for I say that truth in as much as it is subjectively in the mind is none other than some equating (*adaequatio*) of the mind to a true proposition which only has objective being in the mind." (Brown, 1973, §1.27) These terms 'subjective' and 'objective' need to be approached with care when used in medieval texts. As Sir William Hamilton (1863, pp. 806 ff.) noted, the terms underwent a nearly complete reversal of sense during the eighteenth century. For the medievals, something is subjectively in the mind when it is in the mind as a (real) quality of the subject. In contrast, something is objectively in the mind, or has objective being, when it is an object of thought—indeed, that is its etymology, as something "thrown in the way of" thought (*ob-iacere*), as, e.g., the moon is literally thrown in the way of the sun in a solar eclipse. So for Burley, a thought (a mental proposition, existing as a quality subjectively in the mind) is true if it corresponds to a real proposition, a *propositio in re*, existing only objectively in the mind. Indeed, for him, the notion of proposition was four-fold: there is the written proposition, itself a sign of a spoken proposition (writing is a way of recording speech); the spoken proposition is a conventional sign of a mental proposition, from which it derives its signification; but the ultimate significate of the spoken proposition is the real proposition.[11] Whereas the mental proposition is a compounding of concepts (if affirmative, or dividing them if negative), and the spoken proposition is a compounding of spoken terms, the real proposition is a compounding or dividing of real things. Burley cites Averroes with approval when he wrote: "Things are made true by the mind when it divides them from one another or compounds them with one another."[12] For example, consider 'A man is an animal' (*Homo est animal*). There are numerous subjective mental propositions compounding the concepts of man and animal, all of which are true by their correspondence to the one true real proposition which identifies man and animal. It is this real proposition which is signified by the spoken proposition 'A man is an animal', just as the spoken term 'man' signifies man (the animal) and 'animal' signifies animal (the universal):

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[10] "Et non ualet: te esse animal est uerum, ergo omnis dicens te esse animal dicit uerum; sed est fallacia figurae dictionis; commutatur enim unus modus supponendi in alium."

[11] See, e.g., Conti (2011, §4); Cesalli (2007, pp. 190 ff.).

[12] Brown (1973, §1.03): "Oratio in mente componitur ex rebus patet per Commentatorem, VI *Metaphysicae*, in fine, qui dicit quod entia vera, cuiusmodi sunt propositiones, facta sunt ab intellectu quando dividit ea ab invicem vel componit ea ad invicem."

"Hence I say that the thing signified by 'A man is an animal' does not depend on the mind nor does the truth of this thing, for it would be true even if no mind thought about it ... I say that to the truth 'A man is an animal' having being outside the mind there correspond many truths having subjective being in the mind, for many thoughts can correspond to the same thing." (Brown, 1973, §1.27)

One may well be reminded here of Bertrand Russell's early theory of propositions. Russell (1903, p. 47) wrote: "A proposition, unless it happens to be linguistic (i.e., to be about words) does not contain words: it contains the entities indicated by words." At that time, Russell held an identity theory of truth. Such a theory rejects any correspondence between thought and reality, exemplified by Frege's remark (1997, p. 327) that if facts and thoughts "did correspond perfectly ... they would coincide"; "a fact," he says, "is a thought that is true" (p. 342). In his rejection of idealism, Russell proclaimed that in thought we directly apprehend the fact containing the objects in question.

However, in both Frege and Russell, what we apprehend is in the modern sense objective and mind-independent. Burley's account of the real proposition is closer to that articulated recently by Jeffrey King (2007, ch. 2). King locates the proposition in the relation we create between objects by constructing a sentence in some language whose terms refer to them. Thus for King, as for Burley, the proposition results from a semantic act of ours, compounding or dividing objects with or from one another:

"The facts that are propositions are facts of there being a context and there being *some words* in *some language L* whose semantic values relative to the context are so-and-so occurring in such-and-such way in so-and-so sentential relation that in L encodes such-and-such." (King, 2007, p. 45)

On Burley's account, what makes a subjective proposition (a thought) true is its correspondence to a true objective, that is, real proposition; and the truth of the real proposition itself is nothing other than things' being as the mind considers them to be. Burley elaborates on Averroes' remark, cited above:

"The mind makes things true by compounding those with one another which are in reality united or dividing those from one another which are in reality divided. For if the mind asserts some things to be the same, then it compounds them with one another; but if it asserts them to be divided then it divides them from one another ... For when the mind compounds correctly or divides correctly, then there is truth in the mind, and when the mind does not compound correctly or does not divide correctly, as when it compounds those which are in reality divided or divides from one another those which are in reality the same, then there is falsehood in the mind." (Brown, 1973, §1.24)

To return to our sophism: Burley accepts the inference 'If I say you are an ass, I say you are an animal' (talking of things, not of words), but rejects the inference 'If I say you are an animal, I say something true'. We can now understand why he says this. What 'You are an ass' signifies is the real proposition (*propositio in re*) which compounds you and being an ass together. But being an ass necessitates being an animal, as part of its form. Being an animal is a formal consequence of being an ass. Burley (1955, p. 86) writes in *De Puritate*:

> "Formal consequence is of two kinds: one kind holds by reason of the form of the whole structure (*complexio*), ... Another kind of formal consequence holds by reason of the form of the constituent terms (*incomplexa*), e.g., an affirmative consequence from an inferior to its superior is formal, but holds by reason of the terms."

Hence Burley accepts that signification is closed under consequence, at least, formal consequence. If the mind is compounding (incorrectly) you and being an ass, it is inevitably compounding you and being an animal, since animal is superior to ass, and being an animal is formally included in being an ass.

It follows, as Burley notes, that it is incorrect to infer from my saying you are an animal that what I am saying is true and that I say something true. For my saying you are an animal may be merely consequent on my incorrectly compounding things which are not united, as when I say you are an ass. Not everyone saying you are an animal says something true. I must compound and divide correctly if I am to say something true. In other words, everything I say must be that way in reality for my (subjective) proposition to be true.

2   Truth and the Liar

We find these two claims, that truth requires that things are only or wholly as signified and that signification is closed under consequence, utilized in Thomas Bradwardine's proposed solution to the semantic paradoxes in his *Insolubilia*, composed in Oxford a year or two after the Shorter Treatise of Burley's *De Puritate* was composed in Paris. Bradwardine's main object of attack in this work is Burley's restrictivist solution to the paradoxes, but in other respects, Bradwardine seems to share similar views to Burley's.[13] He sets out two definitions, six postulates and two theorems, among which we read:

> "First Definition (D1): A true proposition is an utterance signifying only as things are.
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> Second Definition (D2): A false proposition is an utterance signifying other than things are.

[13]See Bradwardine (2010, 'Introduction' §4).

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Second Postulate (P2): Every proposition signifies or means ... everything which follows from it ...

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Second Theorem (T2): If a proposition signifies itself not to be true or itself to be false, it signifies itself to be true and is false."
(Bradwardine, 2010, ¶¶6.2 - 6.4)

Theorem (T2) depends on the fact that propositions can signify (conjunctively) several things; e.g., if a proposition signifies that it itself is not true, it will also signify itself to be true. If a proposition signifies that you are an ass, by (P2) it will also signify that you are an animal. But by (D1), a proposition is true only if things are altogether as it signifies, that is, everything it signifies obtains. Though a proposition might signify that you are an animal, it does not follow that, since you are an animal, it is true. It may also signify that you are an ass, and so not everything it signifies obtains. Similarly, an insoluble such as the Liar, e.g., 'This proposition is false', or Bradwardine's favourite example, Socrates' utterance (only) of 'Socrates says something false', is false. But it does not follow that it is true, since although that is what it signifies, it is not all it signifies (by T2), for it also signifies that it is true, and things cannot be altogether as it signifies, since no proposition is both true and false (by P1: 'Every proposition is true or false,' and not both).

Bradwardine's argument for (T2) runs as follows: suppose some proposition signifies itself not to be true. Then either that is all it signifies, or not. First, suppose that is all it signifies; and suppose that it is not true. Then by (D1), things are not as it signifies, namely, not true, so it is true. That is, its being true follows from its not being true. But it signifies that it is not true, so by (P2) it follows that it signifies that it is true.

So its not being true is not all it signifies: suppose it also signifies that q, say. (There may be many other things it signifies. Let q be their conjunction.) Again, suppose it is not true. Then by (D1), things are not wholly as it signifies, that is, by a De Morgan conversion (P4), either it is true or not-q. So again by (P2), it signifies that either it is true or not-q. But we supposed that it signified that q, and from its either being true or not-q, and q, it follows by Disjunctive Syllogism (P5) that it is true. So once again, by (P2), it follows that it signifies that it is true.

Next, suppose some proposition signifies that it itself is false. If it is false, it's not true, by (P1). So by (P2), it follows that it signifies that it is not true, and so by the above argument, it signifies that it is true.

Finally, we have already noted that things cannot be wholly as any proposition signifies which signifies both that it is not true and that it is true. So by (D2), any such proposition is false. Thus any paradoxical proposition, such as the Liar, is false and not true. So too with Socrates' utterance of 'Socrates says something false'. For if that is his only utterance,

then by (P2) it signifies that his only utterance is false, that is, that it itself is false. So by (T2), it also signifies that it is true, and is false.

The use of Disjunctive Syllogism (P5) in Bradwardine's argument might make one think it depends on taking the disjunction truth-functionally. Not so; suppose s signifies that s is not true, and perhaps other things too, call them q. It follows by (D2) that if s is false, then if it's not q that fails to obtain, it must be the falsity of s that fails, that is, if s is false and q holds, s must be true. If the conditionals here are strict or relevant conditionals, for example, the inference follows.[14] So since s signifies that s is false and q, it follows that s signifies that s is true, by (P2).

Bradwardine has made great play here with his second postulate, (P2). I've interpreted it as a closure postulate, that signification is closed under consequence, and that is certainly how Bradwardine repeatedly uses it. But it is not quite how he states it. He says "a proposition signifies everything which follows from it," not "from what it signifies". Paul Spade (1981, p. 120) took him more literally, but found that Bradwardine's own reasoning did not then go through. He attributed to Bradwardine the principle (BP), in the form of a schema:

> (BP) If p only if q, then P signifies that q,

where what replaces 'P' names the proposition which replaces 'p'. With a naming function $\ulcorner \quad \urcorner$, we could express this as

> If p only if q, then $\ulcorner p \urcorner$ signifies that q,

Suppose that 'signifies' is closed under consequence, that is,

> (P2) If p only if q, then if s signifies that p then s signifies that q.

As an instance we have:

> If p only if q, then if $\ulcorner p \urcorner$ signifies that p then $\ulcorner p \urcorner$ signifies that q.

Then (BP) follows from (P2) together with the plausible assumption that $\ulcorner p \urcorner$ signifies that p. On that assumption, (P2) entails (BP), but is stronger than (BP) since it can be used, and was used, in Bradwardine's proof of (T2), while (BP) cannot. On the other hand, (BP) entails that $\ulcorner p \urcorner$ signifies that p, which (P2) does not. Bradwardine nowhere states this in general, though his practice certainly assumes it.

Bradwardine restricts (T2) to claiming only that every proposition which signifies that it is not true signifies its own truth. John Buridan (in his early writings) and Albert of Saxony, some twenty or thirty years later, claimed that every proposition whatever ("*omnis propositio mundi*") signifies its own

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[14]$p \to (q \to r)$ entails $(p \wedge q) \to r$ (but not vice versa) in the logics of strict implication **S2** (and stronger) and of relevant implication, **R**: on the latter, see, e.g., Anderson and Belnap (1975, §29.3.1, R30 and R31).

truth.[15] Spade (1981, p. 120 n.17) observes that Burley does so too, not only in the Longer Treatise of his *De Puritate*, written shortly after Bradwardine's *Insolubilia*, but also in his own treatise on insolubles, written in 1302.[16] Spade also finds the claim in John Duns Scotus in his *Quaestiones in duos libros Perihermeneias*, written thirty years earlier, around 1295,[17] and even earlier in Bonaventure:[18]

> "An affirmative proposition makes a double assertion: one in which the predicate is affirmed of the subject and the other in which the proposition is asserted to be true. By virtue of the first assertion an affirmative proposition is differentiated from a negative one, which denies the predicate of the subject. So far as the second assertion is concerned, however, affirmative and negative statements agree since they both assert something to be true. Contradiction is concerned not with the second type of assertion but with the first. For when it is stated that no truth exists, insofar as it negates the predicate of the subject this proposition does not imply its opposite, *viz* that some truth exists. But to the extent that it asserts this to be true, it does entail that some truth exists."

None of them provides much, if anything, in the way of an argument for this claim. Geulincx gives this argument:[19] any proposition says things to be (*dicit esse*), indeed, it says them to be what it says them to be. But things being as it says them to be is for it to be true. So it says itself to be true. Nuchelmans (1988, pp. 280-1) comments that the sense of 'say' here must be 'dicere consequenter' (see above, §1).

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[15]Buridan famously rejected this claim in his later writings, saying not that every proposition signifies its own truth, but that it virtually implies it. His main reason for doing so was his rejection of the notion of the complexly signifiable (*complexe significabile*), what is signifiable only by a *complexum*, that is, a proposition. Given the similarity between this doctrine and Burley's notion of the real proposition, Bradwardine would not share Buridan's worries, if he did indeed accept Burley's semantic account. See, e.g. Klima (2009, chs. 9-10, esp. §10.2).

[16]See Burley 1955, p. 25; 2000, p. 108: "Quaelibet propositio asserit seipsam esse veram," and Roure (1970, p. 272): "Quilibet dicens asserit suum dictum esse verum".

[17]See Lib. I Questiones 7-9 §10 (Andrews et al., 2004, I p. 181): "Quaelibet propositio significat se esse veram, ergo ista 'tu eris albus cras' significat se esse veram. Antecedens patet, quia ad omnem propositionem veram sequitur suum dictum fore verum. Similiter contradictorium affirmativae ut ista 'tu non eris albus' infert hanc '"te non fore album" est verum'. Utrumque igitur contradictoriorum in illis de futuro significat se esse determinate veram." This occurs as part of an objection, but in his response Scotus does not question the basic principle.

[18]*Quaestiones disputatae de mysterio Trinitatis*, q1 a1, translated in Wippel and Wolter (1969, pp. 310-11).

[19]Geulincx (1663, ch. 1) (Land, 1891, II p. 25): "Sit enunciatio quaecunque, nempe A. Dico quod A dicat se esse veram. Quia A dicit esse, et dicit esse quod dicit esse, sed esse id, quod A esse dicit, est A esse veram. Cum igitur A prius dicat, dicit etiam posterius, seu A dicet A, id est seipsam, veram esse." Cf. Geulincx (1662, II.i.1.4) (Land, 1891, I p. 234) and Nuchelmans (1988, p. 280). Similar proofs are found in Albert of Saxony and John Buridan. See, e.g., Read (2002, §3).

In fact, that every proposition signifies its own truth follows straightforwardly (though unremarked by Bradwardine) from his postulate (P2). Suppose some proposition s signifies that q_1, q_2 and so on. Let q be their conjunction—everything s signifies. Then by (D1), s is true if and only if everything s signifies obtains, that is, iff q. In particular, if q then s is true. But s signifies that q. So by (P2), s signifies that s is true.

One might now worry about a circularity. To show that s is true, we need to check that everything it signifies obtains. One of the things it signifies is that s is true. So to check that s is true we need first to check that s is true. That threatens to open up a vicious regress. The objection is ill-founded, however. (D1) tells us that s is true iff everything s signifies obtains. So to check that s is true we need to check that everything it signifies obtains, and of course, that condition is equivalent to s's being true. So we need to check that s is true. But that is no more than we are doing. There is no regress here, just a repetition of the task we are set.

More worrying, perhaps, is the open-ended nature of the condition: to check that s is true, check that everything s signifies obtains, where what s signifies is everything entailed by what it signifies (by P2). Of course, having checked that one thing that s signifies obtains, one can be sure that everything entailed by that also obtains. But there may well be other things s signifies that are not entailed by what has been checked. For example, suppose s is 'You are an ass'. That signifies that you are an ass, that you are an animal, that you are alive and many other consequences. Checking that you are an animal confirms that you are alive and so on. But it does not confirm that s is true. One has further to check that you can bray, that you have long ears, in short, that you are an ass. As soon as one of these significates is found not to hold, we know by (D2) that s is false. But if s is true, this check could in principle, and perhaps in practice, never be completed. Does Bradwardine's account of truth mean that no proposition is ever true?

First, the objection confuses the ontological criterion for s's being true with the epistemological condition for knowing that s is true. There is nothing problematic about the first being indefinitely, even infinitely, complex. But even the epistemology is confused. We can know, and be certain, that Brownie is a donkey, even if we have not checked implausible subterfuges, that Brownie is not a heavily disguised CIA spy, or a Martian robot or whatever. That Brownie is a donkey entails that he is not a robot or a disguised human being. Knowledge is fallible. If Brownie turns out to be a robot, then we were badly misled, did not know he was a donkey, and he wasn't. Fortunately, we are not faced with such tricks too often, and we do know (defeasibly) that Brownie is a donkey.

In claiming that the Liar proposition, indeed, perhaps every proposition, signifies its own truth in addition to what it more obviously signifies, Bradwardine is claiming that such propositions are exponible. Exponible propositions were defined by the medievals as propositions which, though

grammatically simple, were logically complex.[20] A standard example was an exclusive proposition such as 'Only a man is running', apparently a simple subject-predicate proposition, but implicitly complex, to be "expounded" or analysed as 'A man is running and nothing other than a man is running'. Burley (1955, p. 134) points out that in consequence, the negation of an exclusive proposition is implictly disjunctive: 'Not only Socrates is running' is equivalent to the disjunction 'Socrates is not running or something other than Socrates is running'. The same is true of the Liar. Since it is implicitly conjunctive, its negation is implicitly disjunctive: to contradict, e.g., 'This proposition is false', which signifies both that it is false and that it is true, we need to assert either that it is true or that it is false.

3   Compositionality

Bradwardine's approach to the Liar belongs to a class of solutions that revise and constrain the theory of truth rather than the underlying logic. Bradwardine's logic is robust and orthodox, endorsing such principles as Bivalence (P1), the De Morgan equivalences (P4) and Disjunctive Syllogism (P5). He is also committed to what Tim Maudlin (2004, p. 112) calls Downward T-Inference and Hartry Field (2008, p. 121) calls T-OUT, embodied in one half of Tarski's T-scheme:

If $\ulcorner p \urcorner$ is true then p.

This is a special case, given that $\ulcorner p \urcorner$ signifies that p, of the more general rule which Bradwardine accepts, namely, that 's is true' implies that anything that s signifies obtains, which follows from (D1). What Bradwardine rejects is Upward T-Inference (Field's T-IN):

If p then $\ulcorner p \urcorner$ is true.

It does not follow that a proposition is true merely from the fact that something that it signifies obtains—though one may defeasibly infer it. I may infer 'Brownie is an ass' is true from the fact that Brownie can bray—though I may have to retract the claim when I find a voice recorder hidden under the cleverly life-like fake skin. I may infer that the Liar is true from the fact that it is false—but again, have to retract my claim when I see Bradwardine's demonstration that the Liar signifies both that it is false and that it is true.

Bradwardine's theory shares with other theories which reject T-IN, such as Kripke's and Maudlin's, a difficulty in justifying the standard compositional, or distributive, principles for conjunction and disjunction. Maudlin (2004, p. 144) notes the problem:

"The absence of the Upward Inferences is a severe constraint. In essence, one loses information when using the Downward Inferences, and has no means of semantic ascent again. For example,

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[20]See, e.g., Yrjönsuuri (1993).

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whenever it is permissible to assert that a conjunction is true, it is permissible to assert that each conjunct is true, but the system as we have it does not allow this inference. From the claim that the conjunction is true one can assert the conjunction itself (by the Downward T-Inference), and hence can assert each conjunct (by & Elimination), but since there is no Upward T-Inference one cannot assert that the conjunct is true."

His response is bold. He simply adds the requisite compositional principles as an axiom. So did Bradwardine. He writes:

"Sixth Postulate (P6): If a conjunction is true each part is true and conversely; and if it is false, one of its parts is false and conversely. And if a disjunction is true, one of its parts is true and conversely; and if it is false, each part is false and conversely." (Bradwardine, 2010, ¶6.3)

But this seems unsatisfactory. The compositional principles should follow from the theory of truth in conjunction with the meaning of the connectives. Indeed, there is a risk of inconsistency in Maudlin's procedure. Consider the corresponding principle for negation:

(Neg) If a negation is true, its negated part is false and conversely; and if it is false, its negated part is true and conversely.

The Liar is a counter-example to this. Let L be $\ulcorner L$ is not true$\urcorner$. L is false, but the negated part 'L is true' is also false. Consequently, if one were to add (Neg) to Bradwardine's theory, as a new postulate, the theory would be inconsistent. As we noted, L is implicitly contradictory, to be analysed or expounded as a conjunction, so its contradictory is a disjunction. 'L is true' no more contradicts 'L is not true' than, e.g., 'Only Socrates is running' contradicts 'Only Socrates is not running', or 'The King of France is bald' contradicts 'The King of France is not bald'.[21]

Similarly, the corresponding principle for conditionals runs into trouble with Curry's paradox.[22] To adapt an example from Jean de Celaya, writing around 1500:[23] let C be the conditional 'If C is true then you are an ass', and suppose we adopted the principle:

(Cond) If a conditional is true then either the antecedent is false or the consequent is true, and conversely; and if it is false, then the antecedent is true and the consequent is false, and conversely.

If C is true, then by (D1), if C is true you are an ass, so by absorption,[24] if C is true you are an ass. But you are not an ass and could not be (your

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[21]See Read (2008, p. 217).

[22]So called from Curry (1942). Cf. Geach (1955).

[23]See Roure (1962, p. 262).

[24]So called by Geach (1955): in symbols, from $p \rightarrow (p \rightarrow q)$ infer $p \rightarrow q$; nowadays usually referred to as "contraction".

essence is incompatible with that of an ass), so C is necessarily false. Now apply (Cond): given that C is false, it follows that it is true that C is true and false that you are an ass. No harm in the second conjunct, but the first conjunct entails that C is true, by T-OUT. Contradiction. So we cannot endorse the compositional principle (Cond), at least not in the form given.

(Cond) makes the conditional truth-functional, so one might consider adapting it to treat conditionals non-truth-functionally, for example:

> (Cond′) If a conditional is true then the truth of the antecedent is incompatible with the falsity of the consequent, and conversely; and if it is false, then the truth of the antecedent is compatible with the falsity of the consequent, and conversely.

If we now apply (Cond′) to the fact that C is false, it follows that its being true that C is true is compatible with the falsity of your being an ass. But anything compatible with a truth could be true. So it could be true that C is true, so C could be true. But we showed that C cannot be true, so once again, we have a contradiction. The compositional principle (Cond′) cannot be accepted either.

No such contradiction follows, however, from (P6), the compositional principles for conjunction and disjunction, despite the fact that there are similar paradoxes for 'and' and 'or', as Bradwardine (2010, ¶¶8.9.1-2) noted. Let D be the disjunction 'You are an ass or D is false'. Suppose you are an ass or D is false. Since you are not an ass, it follows by (P5) that D is false. But D signifies that you are an ass or D is false, so by (P2), D signifies that D is false. Hence by (T2), D signifies that D is true and so is false. By (P6), given that the disjunction D is false, it follows that it is false that D is false. So D is false and it is false that D is false. But that is not a contradiction, though it may seem surprising. The explanation is that the falsity of D does not suffice to make it true that D is false—to repeat, Upward T-Inference fails in general. 'D is false' entails, by (P6), as we just noted, that it is false that D is false. By (BP), 'D is false' signifies that D is false. So by (P2), 'D is false' signifies that it is false that D is false, whence by (T2), 'D is false' also signifies that 'D is false' is true and so 'D is false' is false.

Similarly, let E be the conjunction 'There is a God and E is false'. Then by a similar argument we show that E signifies its own falsehood and so by (T2), E is false. Hence by (P6), one conjunct is false, and it's not the first, so the second, that is, it's false that E is false. It doesn't follow that E is true, for E is false not because it signifies its own falsehood and it's not false, but because it signifies its own truth and it's false that it's true.

One may still be puzzled why (Neg) and (Cond) lead via L and C to contradiction, whereas (P6) does not produce contradiction through D and E. The explanation is that by (P6), the falsehood of a complex proposition implies only the falsehood of one or both components, whereas by (Neg) and (Cond), the falsehood of a complex proposition implies the truth, or at least the possible truth, of one of its parts, a part that must be false.

Given that they are consistent with Bradwardine's theory, are the compositional principles of the distribution of truth over conjunction and disjunction in fact derivable from the theory together with the standard rules for 'and' and 'or'? First, consider the following diagram:

$$s_1 \quad \text{entails} \quad s_2$$

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$$p \quad \Rightarrow \quad q$$
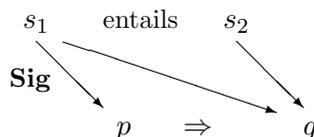
I have so far interpreted (P2) as saying that if s_1 signifies that p, and p entails q (in symbols, $p \Rightarrow q$), then s_1 signifies that q. Paul of Venice interprets it slightly differently, however. He writes:

> "I say that any proposition signifies the significate of any proposition following from it formally ... This is how the common saying, 'Any proposition signifies whatever follows from it', should be understood."[25]

My interpretation follows what we might call the "southern" route in the diagram, Paul's the "northern" route. Arguably, the diagram commutes, and s_1 signifies q whichever route one takes.

Now suppose that some conjunction is true. Then things are however the conjunction signifies. Suppose its first conjunct signifies that, say, p. Then by Paul's principle, the conjunction also signifies that p, since any conjunction entails its first conjunct. But things are however the conjunction signifies. So p. That is, things are however the first conjunct signifies. So the first conjunct is true, and similarly for the second conjunct. Hence, if a conjunction is true, so are each of its conjuncts, and if either is false, and so not true, then the conjunction is not true, but false.

For the converse, we need to generalise (P2) a little further. Recall Bradwardine's proof of (T2). Assuming that the proposition signifying itself not to be true signified something else as well, call it q, Bradwardine showed that it signifies that either it is true or not q. He concluded that it signifies that it is true, since we have assumed that it signifies that q. This does not follow strictly from (P2). Rather, we need to know that if a proposition signifies that p_1 and signifies that p_2, it signifies that p_1 and p_2.[26] This may seem obvious. In the present context, we need a somewhat similar converse principle, namely, that whatever a conjunction signifies is entailed (jointly) by things signified by each conjunct.

Now suppose each conjunct of some conjunction is true. Then by our new principle, whatever the conjunction signifies is entailed by something signified by each conjunct. But since the conjuncts are true, each of those obtains, and so whatever the conjunction signifies must obtain too. So things are however the conjunction signifies, and so a conjunction is true whenever

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[25]Del Punta and McCord Adams (1978, p. 74), my translation.

[26]We can capture this in a generalisation of (P2): if s signifies that p_1 and signifies that p_2, and p_1 and p_2 (jointly) entail r, then s signifies that r.

each conjunct is true. Thus we have established the compositionality principle for conjunctions which Bradwardine states in (P6), that if a conjunction is true, each conjunct is true and conversely, and if it is false, at least one conjunct is false and conversely.

What of the distribution of truth over disjunction? Take a disjunction, and suppose one disjunct is true, that is, whatever the disjunct signifies obtains. By Paul's principle, the disjunct signifies whatever the disjunction signifies, since a disjunction is entailed by each disjunct. So whatever the disjunction signifies obtains, and so the disjunction is true.

Conversely, suppose each disjunct is false. Then something each disjunct signifies fails to obtain. It's reasonable to assume that a disjunction signifies the disjunction of anything its disjuncts severally signify. So the disjunction signifies something disjunctive neither part of which obtains, and so which does not obtain as a whole. So the disjunction is also false. Contraposing, if a disjunction is true then one or other disjunct is true. Putting it all together, we have the compositional principle for disjunction that Bradwardine states in (P6): a disjunction is true if at least one disjunct is true and conversely; and a disjunction is false if both disjuncts are false and conversely.

4 Conclusion

Implicit in their responses to the sophism 'If I say you are an ass, I say you are an animal, and if I say you are an animal I say something true, so if I say you are an ass I say something true, so you are an ass', is a shared acceptance by the Magister Abstractionum and Walter Burley that saying that, or signification, is closed under at least some form of consequence. That closure principle lies at the heart of Thomas Bradwardine's idea for solving the semantic paradoxes, together with the idea that a proposition is true only if things are wholly as it signifies, that is, only if everything it signifies obtains.

Bradwardine uses the closure principle to show that any proposition which signifies its own falsity also signifies its own truth, and so not everything it signifies can obtain, whence it must be simply false. In fact, it turns out that the closure principle implies that any proposition whatever signifies its own truth. But this does not mean that nothing is true. Establishing that something is true may be defeasible, in that one cannot check that everything it signifies obtain, but knowledge and certainty are still possible. What might at first seem more problematic is the failure of (T-IN), that if p then $\ulcorner p \urcorner$ is true. This must fail in general since the Liar, 'This proposition is false', is an immediate counter-example. As a result, the usual derivation of the compositionality principles distributing truth over negation, conjunction, disjunction and the conditional can no longer be completed. In fact, the Liar shows that truth does not distribute over negation, and Curry's paradox shows that it does not distribute over the conditional either.

The compositional principles for conjunction and disjunction, however, can be derived by invoking other persuasive principles, including an alter-

native interpretation of the closure postulate. Bradwardine's solution is thus found to preserve those truth principles which are unaffected by the paradoxes, without sacrificing any logical principles, and so constitutes an attractive and viable solution.

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