

Consequence, Signification and Insolubles in Fourteenth-Century Logic

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Abstract. Forty years ago, Niels Green-Pedersen listed five different accounts of valid consequence, variously promoted by logicians in the early fourteenth century and discussed by Niels Drukken of Denmark in his commentary on Aristotle's *Prior Analytics*, written in Paris in the late 1330s. Two of these arguably fail to give defining conditions: truth preservation was shown by Buridan and others to be neither necessary nor sufficient; incompatibility of the opposite of the conclusion with the premises is merely circular if incompatibility is analysed in terms of consequence. Buridan proposed to define consequence in terms of preservation of signifying as things are. John Mair pinpointed a sophism which threatens to undermine this proposal. Speaking anachronistically, Bradwardine turned it around: he suggested that a necessary condition on consequence was that the premises signify everything the conclusion signifies. Dumbleton gave counterexamples to Bradwardine's postulates in which the conclusion arguably signifies more than, or even completely differently from the premises. Yet a long-standing tradition held that some species of validity depend on the conclusion being in some way contained in the premises. We explore the connection between signification and consequence and its role in solving the insolubles.

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1. Definitions of Valid Consequence

Niels Green-Pedersen (1981, pp. 53–58) finds the following varied accounts of valid inference, or consequence, in Niels Drukken of Denmark's *Questions on*

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Aristotle's 'Prior Analytics' (edited by Green-Pedersen in (Nicolaus Drukken 1997)), written in Paris in the late 1330s: a consecution¹ is valid iff

1. the premises cannot be true without the conclusion being true, or
2. it is impossible for things to be as the premises signify without their being as the conclusion signifies, or
3. the contradictory opposite of the conclusion is incompatible with the premises;

and it is formally valid iff

4. anything which is signified by the conclusion is signified by the premises (that is, the conclusion is contained or understood in the premises), or
5. it holds for any uniform substitution of terms.

All five were espoused by various contemporaries of Drukken's, and each was rejected by others.

1.1. Truth-Preservation

Does validity consist in preservation of truth from premises to conclusion, that is, that the premises cannot be true unless the conclusion is also true? John Buridan, also writing in Paris around the same time as Drukken, pointed out that on this account, any consecution would be invalid simply by not formulating the conclusion. For sentences, thought of by the medievals as concrete particulars, can only be true or false if they exist and are actually formulated. So we need at least to add 'when they are formed together' to the definition. But, Buridan argued:

... this definition is even now not good, because:

No sentence is negative, so no ass is running,
is not a sound² consecution ... because the opposite of the premise does not follow from the opposite of the conclusion, that is, this does not follow:

Some ass is running, so some sentence is negative.

But according to the definition as amended one must concede that it is sound, since it is impossible for the premise to be true, so it is impossible for it to be true and the conclusion not.³

¹Ciola (2018, p. 274) notes the ambiguity in many medieval treatises between *consequen-tia* understood, on the one hand, as the linguistic item (*oratio*) consisting of premises (*antecedens*) and conclusion (*consequens*) and, on the other, understood as the relation between these two items. I shall use 'consequence' or 'valid inference' to translate *con-sequentia* when it is the defining property or relation of validity that is in question, and 'consecution' when a particular sequence of premises and conclusion, valid or not, is being discussed.

²I shall use 'sound' to translate *bona* as applied to consecutions and 'is valid' to translate *valet*, though the medievals, while using both words, appear to have treated them as equivalent.

³John Buridan (1976, pp. 21–22). This is a somewhat free translation of the Latin, re-ordering the argument for the sake of clarity (cf. John Buridan, 2014, I 3, p. 67). As Ciola (2018, pp. 277–8) notes, Albert of Saxony and Marsilius of Inghen present similar counterexamples.

Drukken has his own counterexample:⁴

Supposing that there are many sentences and each of them is affirmative (which is possible), then one argues like this: ‘Each sentence is affirmative, therefore none is negative.’ The consecution is formally valid, and the premise is true by the scenario proposed, and the conclusion is false. Therefore a falsehood follows from a true sentence in a formally valid consecution. The premise is true by the scenario, and the conclusion is false, which is proved because this sentence, ‘No sentence is negative’, is simply false because it itself is negative, therefore, some sentence is negative. Therefore ‘No sentence is negative’ is false, and this was the conclusion. Therefore the conclusion was false.⁵

Thus truth-preservation is not sufficient for validity and the account of validity must be revised.

Roger Swyneshed, writing in Oxford a few years earlier, gave a different kind of counterexample, since he incorporated the exclusion of self-falsifying sentences into his account of truth in order to solve the insolubles (that is, the logical paradoxes):⁶

A true sentence is a sentence not falsifying itself signifying principally as things are either naturally or by imposition whereby it was last imposed to signify . . . A false sentence is an utterance falsifying itself or an utterance not falsifying itself signifying principally other than things are either naturally or by imposition whereby it was last imposed to signify. (Spade, 1979, §§18–19, pp. 185–6)

He then argues for three iconoclastic theses:

1. There is a false sentence signifying principally as things are
2. There is a formal and valid consecution in which the false follows from the true
3. Two contradictories mutually contradicting one another are both false.

The examples he gives illustrating and demonstrating all three theses are based on the insoluble ‘This sentence is false.’ In particular, the second thesis is proved by considering the consecution:

The conclusion of this consecution is false,
so the conclusion of this consecution is false.

The conclusion is false because it falsifies itself. So the premise is true since it truly says of the conclusion that it is false—and the premise does not refer

⁴The example is also found in Pseudo-Scotus (2001, p. 228) and repeated some dozen or so years later by Buridan in his *Sophismata*: John Buridan (2004, cap. 8: Primum Sophisma, p. 140), Eng.tr. in John Buridan (2001, p. 952).

⁵Nicolaus Drukken 1997, pp. 33–34, ll. 50–59. This is the fifth argument in response to Question 7 (*ibid.*, p. 32 ll. 1–2): ‘whether in any possible scenario a false conclusion follows by a formal consequence from true premises’.

⁶On the medieval theories of insolubles, see, e.g., Spade and Read (2021); and see further §2.1 below. A sentence falsifies itself if it entails its own falsehood, such as ‘This sentence is false’.

to, or falsify, itself. Thus truth-preservation is not necessary for validity—at least, not for Swyneshed.⁷ So what is the correct account of validity?

1.2. Preservation of Signifying-as-things-are

Paul Spade (1983, p. 113 n. 32) conjectured that for Swyneshed, validity was preservation of signifying as things are. This account would certainly validate the consecution above.⁸ Not only are things as the premise signifies, they are also as the conclusion signifies, since it is false, which is what it signifies—for it is false because it falsifies itself.

So does validity consist in preservation of signifying as things are, from premises to conclusion? That is, does validity mean that the premises cannot signify as things are unless the conclusion does so too?⁹ John Mair, writing in Paris in the early sixteenth century, presented a counterexample to this proposal:

The conclusion of this consecution signifies other than things are
So the conclusion of this consecution signifies other than things
are.

The consecution would seem to be valid, since premise and conclusion say the same thing. But the consecution does not preserve signifying as things are, as Mair argues:

This consecution is sound, ‘This conclusion signifies other than things are, therefore, this conclusion signifies other than things are’, referring to the conclusion each time. For it proceeds from one synonym to another, but the conclusion signifies other than things are and the premise as things are. Proof: because either the conclusion signifies as things are or other than things are. If the second, we are done; if the first, and it signifies that the conclusion signifies other than things are, and so it does (you agree), therefore the conclusion signifies other than things are. From this it is clear that the premise signifies as things are, so [the definition is bad].¹⁰

⁷Sandgren notes that if Swyneshed really thinks this inference is valid (which he claims in the proof of his second thesis), then the premise falsifies itself, since it entails the conclusion, which he claims is false. Hence his second thesis is mistaken.

⁸Read (2020, §4) argued that Swyneshed’s account of validity could not be preservation of signifying-as-things-are, although it was left as an open question, pending further research, what account he would, or did give.

⁹Preservation of signifying as things are is the account offered by John Buridan (2014, I 3) and Albert von Sachsen (2010, IV 1).

¹⁰Johannis Mair (1527, fol. 142rb): *Secundo argumentor, hec consequentia est bona, hoc consequens significat aliter esse quam est, ergo hoc consequens significat aliter esse quam est, demonstrando consequens utrobique. Proceditur enim a synonymo ad synonymum, et tamen consequens significat aliter esse quam est, et antecedens sicut est. Probatio quia vel consequens significat taliter sicut est vel aliter esse quam est. Si secundum intentum habetur, si prius et ipsum significat consequens significare aliter esse quam est, et ita est per te, ergo consequens significat aliter esse quam est. Et ex illo patet quod antecedens significat taliter esse qualiter est in re, igitur.*

The conclusion signifies other than things are because, if it signified as things are, it would signify other than things are, since that is what it signifies. Thus we have a valid consecution whose premise signifies as things are and whose conclusion does not. Hence, the correct account of validity cannot be preservation of signifying as things are.

1.3. Incompatibility

Paul of Venice, among others,¹¹ had a different account of validity. Paul spent the years 1390–93 in Oxford, and wrote in his *Logica Parva* when he returned to Italy:

A sound consecution is one in which the opposite of the conclusion is incompatible with the premises.¹²

Of course, whether this account deals with the counterexamples above depends crucially on the account of incompatibility. Paul writes in his *Logica Magna*:

Two sentences are mutually incompatible when, signifying as they do, they cannot be nor can have been nor could be true together, or at least their significates cannot be nor can have been nor could be true together. (Paulus Venetus, 1990, p. 21)

In the same vein, Gerard Odo, writing in Paris in the 1320s, noted:

In any consecution the opposite of the conclusion cannot stand with the premises. (Giraldus Odonis, 1997, p. 337)

That is, they can't both be true or at least they can't both signify as things are, and they cannot stand together—presumably, on pain of contradiction.¹³

First, take Buridan's counterexample to truth-preservation: no sentence being negative is compatible with an ass running, so the consecution is correctly deemed to be invalid. More surprisingly, Swyneshed's counterexample

¹¹See, e.g., John of Wesel (1996, §56): 'That this consecution is sound is proved: the opposite of the conclusion is contradictorily inconsistent with the premise'; and Peter of Mantua, *Logica*, cited in Pozzi (1978, 17, 281): 'A consecution indicated by "if" or "therefore" is a necessary relation of two sentences where the contradictory of the second cannot stand with the first without new imposition [of meaning] or can be convertible with one such without new imposition'.

¹²Paulus Venetus (2002, ch.III §1, p. 52). He repeats this in the *Logica Magna*: both in the 'Consequences' (see Paulus Venetus, 1990, p. 80), where Hughes clearly takes this to be a definition: 'A valid inference which signifies in accordance with the composition of its elements may be defined as one in which the contradictory of its conclusion would be incompatible with the premiss of that inference, given that these signify as they do; and by "as they do" I refer to what they customarily signify'; and in the 'Insolubles' (see Paul of Venice, 2022, §2.2.5): 'For, in agreement with everybody, I mean that if there is [a consecution] that signifies by the composition of its parts and the opposite of the conclusion is formally incompatible with the premise, that [consecution] is formally sound'.

¹³Interestingly, Ralph Strode (Seaton, 1973, §1.2.63) distinguishes being incompatible (*repugnans*) from the impossibility of standing together: 'But although the contradictory of the conclusion of a sound consecution never stands with its premises, it is not, however, necessary that the opposite of the conclusion is always incompatible with the premises'. The example he gives of sentences which cannot stand together but are not incompatible are 'You are not white' and 'Nothing exists'.

satisfies the ‘incompatibility’-criterion: for ‘The conclusion of this consecution is not false’ clearly cannot stand with ‘The conclusion of this consecution is false’ without contradiction. Finally, Mair’s consecution also satisfies the ‘incompatibility’-criterion of validity: that is, ‘The conclusion of this consecution does not signify other than things are’ evidently cannot stand with ‘The conclusion of this consecution signifies other than things are’ without contradiction.

However, one might worry that there is a circularity in the definition. For whether things can stand together without contradiction would seem to mean ‘their standing together (or their both signifying as things are) does not entail a contradiction.’ But ‘entail’ is just another word for (the consecution being) ‘valid’. Indeed, for Burley, for example, the incompatibility is not the defining property, but a useful mark:

The fourth main rule: Whatever is compatible with the premise is compatible with the conclusion ... The third rule that follows is that in every sound inference the opposition of the conclusion is incompatible with the premises. (Walter Burley, 1955, 63)(2000, 149 §275)

Indeed, in the same vein, many authors say that a consecution is valid just if the opposite of the premises follows from the opposite of the conclusion. But clearly, this cannot be a definition, otherwise we would have a regress.¹⁴

Drukken’s fifth definition similarly presupposes a prior definition of validity:

Some say that a formal consecution is one which holds thanks to the form of the combination of the terms, so that wherever there is a similar combination of terms, however those terms signify, there is always a sound consecution in the same way. (Nicolaus Drukken 1997, Q. 14, p. 81 ll. 110–13)

Tarski used a similar account to give a reductive definition of consequence without recourse to modal notions (like necessity or incompatibility). However, the medievals, notably Parisians like Buridan, used the term-invariance test to distinguish formal validity from validity more generally, and from material validity in particular.¹⁵ Hence Drukken’s fifth definition (that is, his definition of formal validity) presupposes a prior definition of validity itself.

¹⁴Richard Ferrybridge presents a counterexample to the incompatibility criterion in his *Consequentiae* (Pozzi, 1978, 262–71): consider the consecution ‘*A* will signify precisely that everything true will be false, therefore *A* will be false’. Ferrybridge argues that the opposite of the conclusion is incompatible with the premise, but the consecution is invalid, for *A* could continue to signify that everything true will be false and still be true since everything true will be false. The example was later used by Paul of Venice in his *Quadratura* to motivate his solution to the insolubles: see Read (2022).

¹⁵See, e.g., John Buridan (2014, ‘Introduction’ §3.1). However, we will see in §2.3 that Drukken rejects the notion of material validity, so that his fifth criterion follows from his fourth.

1.4. Containment

Drukken's fourth criterion similarly sets out to define 'formal validity', but without the dependence on a prior definition of validity:

In every formal consecution the conclusion should be included in the premises so understood that the whole of what is signified by the conclusion should be signified by the premises ... [that is,] in every sound consecution what is signified by the premises and the conclusion is the same, so that whatever is signified by the conclusion should be signified by the premises. (Nicolaus Drukken 1997, Q. 4, p. 21 ll. 82–4; Q. 9, p. 50 ll. 171–3)

This account of validity is particularly associated with the English school of logic in the fourteenth century.¹⁶ Niels Drukken seems to endorse this account himself. Unsurprisingly, he was a member of the so-called 'English Nation' at the University of Paris—though he himself was from Denmark (Dacia). A consecution is not formally valid, he says, if the conclusion signifies more than the premises. Whatever the conclusion signifies must already have been signified in the premises.¹⁷

As Green-Pedersen (1981, pp. 55–6) notes, Drukken considers it impossible to prove the rule that in a formally valid consecution the premise can't be true and the conclusion at the same time false. Nonetheless, he proceeds to argue for it (Nicolaus Drukken 1997, p. 35 ll. 103–5). His argument seems to turn on the claim that there must be some connection between what the premises signify and what the conclusion signifies. For an understanding of what the premises signify naturally leads to inferring the conclusion.¹⁸ Indeed, if there were some part of the signification of the conclusion which exceeded that of the premises, inferring that part from the premises would be just as unsound. So the whole of what the conclusion signifies must be contained in the premises:

Every sound consecution is sound on the basis of what is signified by the premises and what is signified by the conclusion. Then either what is signified by the conclusion is the same as what is signified by the premises or they are disparate in some respect. If they are disparate in some respect, then the intellect would never infer the conclusion from the premises any more than it would infer, 'A man is running, therefore a stick is standing in the corner.'

¹⁶See, e.g., Dutill Novaes (2020, §3.1) and Ashworth and Spade (1992, p. 39). It was dubbed the 'Containment' condition in Martin (1987, pp. 392–3).

¹⁷Nicolaus Drukken 1997, Q. 7, p. 36 ll. 125–6. Recall that Question 7 asks 'whether in any possible scenario a false conclusion follows in a formal consecution from true premises', to which Drukken answers (p. 35 ll. 99–102): '... never in any scenario outside obligations and insolubles can a false conclusion follow from true premises such that the consecution was formal and the premises were true and the conclusion at the same time false'. We'll discuss Drukken's account of the exception in the case of insolubles in §2.1 below.

¹⁸See Read (1993) for discussion of this idea in connection with the paradoxes of implication, *ex impossibile sequitur quodlibet* and *necessarium sequitur ad quodlibet*.

For it's called a consequence [i.e., valid consecution] when the understanding from its natural judgment infers the conclusion from the premises and from the understanding of the premises it understands what the conclusion signifies. If it is said that what the conclusion signifies is the same as what the premises signify, then either it is wholly the same or in some way it is and in some way not. If in some way it is and in some way not, imagine dividing it into what exceeds [the premises] and what doesn't. Then because of what exceeds the premises they do not imply the conclusion, because what exceeds is wholly disparate from what is signified by the premises. So it is necessary that the whole of what is signified by the conclusion is signified by the premises. Therefore, if the premises are true, the conclusion is true, because what they signify is entirely the same; for that reason if one sentence is true, so is the other. This is what earlier writers said, that in every sound consecution the conclusion is contained in the premises, that is, if it is formally sound, the whole of what is signified by the conclusion is signified by the premises, but not [necessarily] vice versa. (*Loc.cit.* ll. 105–24)

This account of formally valid inference goes back at least to Abelard.¹⁹ A classic example of a formally valid consecution in this sense is 'Socrates is human, so Socrates is an animal', where the whole of what the conclusion signifies (Socrates and animality) is contained in what the premise signifies (Socrates and humanity).²⁰

2. Insolubles

Bradwardine's solution to the insolubles relies on demonstrating that they mean more than at first appears. This comes out in his Second Thesis, that any sentence which signifies that it itself is false (or not true), also signifies that it is true. Consequently, any such sentence is false, since things cannot be wholly as it signifies (Thomas Bradwardine, 2010, §6.4, p. 96). His demonstration of this Thesis turns crucially on his second Postulate:

Every sentence signifies or means given how things are now or unconditionally everything which follows from it given how things are now or unconditionally. (Thomas Bradwardine, 2010, §6.3, p. 96)

Bradwardine's solution to the insolubles marks a significant shift in the generally accepted response to the insolubles. Before Bradwardine presented

¹⁹See, e.g., Martin (1987). It is the fourth account of the truth of conditionals in Sextus Empiricus (1994, II 110–12), but although there are three surviving copies of a thirteenth-century Latin translation, Sextus's work only became widely known in the Latin West in the Renaissance: see, e.g., Wittwer (2016).

²⁰See, e.g., Dutilh Novaes (2020, §2.2) and Bosman (2018).

his solution, the standard response to the insolubles was restrictionism (*restrictio*), that self-reference was impossible, at least in the context of privative terms like ‘false’ or ‘not true’. In both restrictionism and cassationism (*cassatio*), which seems to have been more often criticized than adopted, it was taken for granted that insolubles signify at most what they appear to signify, and nothing more. For example, ‘Socrates says a falsehood’ signifies at most that Socrates says a falsehood and nothing more. To avoid contradiction, restrictionism insists that in such cases, the part (in this case, ‘falsehood’) cannot supposit for the whole of which it is part, so must either supposit for some other utterance of Socrates’s, or fail to supposit. In the latter case, which is the one most often entailed by the scenario (in which this is Socrates’s only utterance), his utterance has an empty term and so is false. Cassationism similarly rejects the possibility that the part supposits for the whole, but concludes that the utterance fails to signify at all, or at least, that no significant truth-apt sentence has been uttered.

Bradwardine was not alone in claiming that insolubles signify more than at first appears, indeed, that they also signify their own truth. Burley, for example, claimed that every sentence signifies its own truth. He wrote:

Everyone saying anything asserts that what he is saying is true.
(Walter Burley, 1955, §3.02)

Walter Segrave, defending restrictionism against Bradwardine’s attacks on it, traces the idea that every sentence signifies that things are as it signifies and so (implicitly) signifies its own truth to the function of the copula, as noted by Aristotle himself:

... every sentence means things to be in reality as it signifies. This is self-evident and is clear from the Philosopher and the Commentator in comment 14 on the fifth book of the *Metaphysics* and throughout the text of that comment: for the copula in the sentence signifies being true, as is elucidated there.²¹

Other subsequent writers on insolubles resisted Bradwardine’s proposal that insolubles, perhaps all sentences, have some further hidden meaning that only the shrewd logician can discern. Among them are John Dumbleton, defending a radical solution apparently combining aspects of both restrictionism and cassationism, and Roger Swyneshed, proposing that truth require that a sentence not falsify itself, which (as we noted in §1.1) he claimed insolubles do by entailing their own falsehood. The mainstream, however, led it seems by William Heytesbury, followed Bradwardine in accepting that insolubles have an additional signification besides what they appear on their face to signify, but for the most part rejecting Bradwardine’s second Postulate and the clever proof using it to prove his Second Thesis. Heytesbury famously

²¹See Read (2023, p. 50). The reference is to Aristotle’s *Metaphysics* Δ 7, 1017a31: “to be’ and ‘is’ signify that a thing is true, and ‘not to be’ that it is not true but a falsehood, equally in the case of affirmation and of denial; as for instance that Socrates *is* artistic, that this is true, or that Socrates *is* not-pale, that it is true; and ‘a diagonal *is not* commensurable’ that it is a falsehood.” (Aristotle, 1971, p. 40)

claimed that although, on pain of contradiction, insolubles must clearly have a hidden signification that explained their being false, there was no obligation (on him, or anyone) to specify what that additional signification was:

But if someone asks what the proposition uttered by Socrates signifies in this scenario other than that Socrates says a falsehood, I say that the Respondent will not have to respond to that question, because from the scenario it follows that the proposition will signify other than that Socrates says a falsehood, but the scenario does not certify (*non certificat*) what that is and so the Respondent does not have to respond any further to what was asked.”²²

2.1. Drukken on Insolubles

From the late 1330s onwards, following publication of Heytesbury’s *Rules for Solving Sophisms* in 1335, the standard solution popular in Oxford and the English Nation at Paris is found in such authors as Ralph Strode, John of Holland and John Hunter, and in many of the collective and often anonymous treatises in the *Logica Oxoniensis*. According to this solution, the additional signification that Heytesbury claimed insolubles must have is indeed that they themselves are true. For example, in an anonymous treatise ascribed by its editor to ‘pseudo-Heytesbury’ on account of its similarity to Heytesbury’s genuine work, we read (on the assumption that ‘A falsehood exists’ is the only sentence):

It must be said that . . . ‘A falsehood exists’ signifies conjunctively, namely, that a falsehood exists and that that very same sentence is true. (Pironet, 2008, 292)

This appears to be the solution favoured by Drukken. In a further argument in Question 7, he takes Socrates’s utterance of ‘Socrates says a falsehood’ and nothing else to construct a valid consecution with true premise and false conclusion:

Sixthly, supposing that Socrates says only this true sentence: ‘Socrates says a falsehood’, then one argues like this: ‘It is the case that Socrates says a falsehood, therefore Socrates says a falsehood.’ The consecution is sound, because given the contradictory of the conclusion: ‘Socrates does not say a falsehood’, therefore it is not the case that Socrates says a falsehood, which is the opposite of the premise. Therefore the consecution was sound. Also, the premise is true in the scenario proposed, because in the scenario proposed it is the case that Socrates says a falsehood. But then I prove that the conclusion is false, namely, ‘Socrates says a falsehood’, because everything said by Socrates is false. But ‘Socrates says a falsehood’ was said by Socrates. Therefore, ‘Socrates says a falsehood’ is false. But this is the conclusion. Therefore the conclusion is false. (Nicolaus Drukken 1997, Q. 7, p. 34 ll. 60–70)

²² William Heytesbury (1987, 240). See also Pironet (2008, 286), and for an alternative translation, William Heytesbury 1979, 49–50.

The reasoning looks rather confused. But his solution seems to turn on accepting that Socrates's utterance signifies not only that it is false, but also that it is true. For Drukken then writes:

This conclusion can also be derived through the popular solution to insoluble sentences: and it is also clear by the rule which everyone grants on the subject of insolubles, which is that every sentence signifying itself to be true and itself to be false is simply false. But this sentence, 'Socrates says a falsehood', is of this sort, because in the scenario proposed it signifies that what is said by Socrates is a falsehood, and nothing else was said by Socrates except this sentence. Therefore it signifies that it itself is false. It also signifies that things are as is signified by it, and thence it signifies itself to be true. But because it is impossible that it is at the same time true and false, therefore it is impossible that things are wholly as is signified by it. And thence if it is not wholly the case as is signified by it, it is simply false. Therefore, Aristotle's rule is true, that every sentence signifying itself to be true and false is simply false. Therefore, this sentence, 'Socrates says a falsehood', is false. And this is the original conclusion. Therefore, the conclusion was false and the premise true, as was to be proved. Therefore etc.²³

2.2. Bradwardine's Second Postulate

Ralph Strode was keen to point out that Bradwardine's second postulate should not be taken literally:

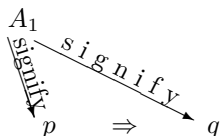
All [Bradwardine's] assumptions are true except the second, namely, 'Every sentence signifies or means given how things are now or unconditionally everything which follows from it given how things are now or unconditionally', which is false taken literally, because taken that way it follows that the sentence 'Socrates is running' would signify 'A substance exists', which is clearly false. Nevertheless, if it is understood that every sentence given how things are now or unconditionally signifies or means given how things are now or unconditionally everything signified by a sentence which follows from it given how things are now or unconditionally, then it is tolerably acceptable: e.g., this sentence 'A man is running' signifies that an animal is running and that an animal exists and that a substance exists, and so on, but it does not signify the sentences 'An animal is running', 'An animal exists', 'A substance is running', 'A substance exists', and so on. And the author understood it like that.²⁴

²³ *Loc. cit.*, pp. 34–35 ll. 71–86. Drukken repeats several times that the false only follows from the true in insolubles and obligations. See, e.g., Q. 8, p. 41 ll. 8–9. See also Q. 7, p. 35 ll. 95–102, Q. 8, p. 43 ll. 79–80 and Q. 9, p. 51 ll. 202–3 ms P.

²⁴ Ralph Strode (ms, fol. 9rb): *Omnes suppositiones sunt vere preter secundam, que falsa est de virtute sermonis, ista scilicet 'quelibet propositio significat vel denotat ut nunc vel simpliciter omne quod sequitur ad ipsam ut nunc vel simpliciter', quia sic sequitur*

Bradwardine's second postulate can be interpreted as a closure principle, that signification is closed under (simple or as-of-now) consequence, so that a sentence signifies anything that follows from anything it signifies. But as noted above, Strode and Paul of Venice give a different interpretation, one which connects interestingly with Drukken's fourth definition of validity.

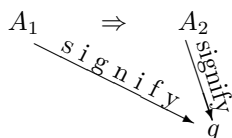
Interpreted as a closure postulate, Bradwardine's postulate can be pictured like this:²⁵



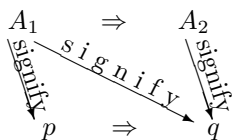
that is, if A_1 signifies that p , and p entails (\Rightarrow) q , then A_1 signifies that q . But Paul of Venice, echoing the passage from Strode cited above, writes:

I say that any sentence signifies the significate of any sentence following from it formally . . . This is how the common saying, 'Any sentence signifies whatever follows from it', should be understood. (Paulus Venetus, 1978, p. 74)

That suggests a different diagram:²⁶



i.e., if A_2 signifies that q , and A_1 entails A_2 , then A_1 signifies that q . Putting them together, we have two 'routes' from A_1 to q , a 'northern route' (with Strode and Paul of Venice) and a 'southern route' (the closure condition):



Mathematicians (in particular, category theorists) would say that the diagram commutes, that is, that the different routes from A_1 to q are essentially

quod ista propositio 'Sortes currit' significaret istam 'substantia est', quod de se patet esse falsum. Verumtamen si intelligatur quod omnis propositio ut nunc vel simpliciter significat vel denotat ut nunc vel simpliciter omne significatum propositionis ad ipsam sequens ut nunc vel simpliciter, tunc potest tolerabiliter sustineri: sicut ista propositio 'homo currit' significat quod animal currit et quod animal est et quod substantia currit et quod substantia est, et sic ulterius, non tamen significat istas propositiones 'animal currit', 'animal est', 'substantia currit', 'substantia est', et sic ultra. Et sic forte intellexit auctor.

²⁵See Thomas Bradwardine (2010, 'Introduction' §5) and Read (2015, p. 405).

²⁶Note that there is an abuse of notation here. In $p \Rightarrow q$, ' \Rightarrow ' is an operator or connective, connecting what is signified by sentences (propositions), whereas in $A_1 \Rightarrow A_2$, ' \Rightarrow ' is a relation between those sentences. On the harmless need to move back and forth between these two grammatical forms, see Anderson and Belnap (1975, §A5, pp. 80-2).

the same, depending on whether you think of consequence as relating sentences or their significates.

If we interpret Bradwardine's Second Postulate as Paul does, then it is entailed by Containment. For we can formalize Containment as follows:

$$A_1 \Rightarrow A_2 \text{ iff } \forall p(\text{if } \mathbf{Sig}(A_2, p) \text{ then } \mathbf{Sig}(A_1, p))$$

which from left to right, by permuting the antecedents, clearly entails:

$$\forall p(\text{if } \mathbf{Sig}(A_2, p) \text{ and } A_1 \Rightarrow A_2 \text{ then } \mathbf{Sig}(A_1, p))$$

which is a formalization of Paul's interpretation of the Second Postulate.

2.3. Dumbleton's Counterexamples

John Dumbleton claimed that Bradwardine's second postulate

should not be maintained wholly and universally without qualification, since there are some necessary consecutions whose conclusions do not signify as their premises do. And there are other formal consecutions whose conclusions do not mean the same as their premises.²⁷

Dumbleton gave two counterexamples to Bradwardine's second postulate, claiming first that:

Some man is an ass, therefore some man is a goat

is valid, since it is impossible that any man is an ass—nonetheless, there is no necessary relation between the premise and the conclusion. Secondly, claiming more generally that from every affirmative sentence it follows that God exists, presumably because the conclusion is necessary—but again, not every affirmative sentence signifies that God exists', contrary to Bradwardine's postulate.²⁸

²⁷These counterexamples to Bradwardine's postulate are found in the third of three chapters labelled '*De sophismatibus que non re sed nomine insolubilia existunt*' ('On sophisms that are insolubles in name only and not in reality') in a treatise inserted in five of the 19 manuscripts of Dumbleton's 'Insolubilia', itself contained in Part I (*Summa Logice*) of his *Summa Logice et Philosophie Naturalis*. The author writes (John Dumbleton, ms, fol. 11vb): *... est notandum quod cum in quibusdam tractatibus dicitur omnem propositionem significare quicquid sequitur ad eam ut nunc vel simpliciter, hoc non in toto et universaliter simpliciter est sustinendum cum quedam sunt consequentie necessarie quarum consequentia non significant ut antecedentia. Et alie sunt formales per quarum consequentia non denotatur idem quod per antecedentia.*

²⁸John Dumbleton (ms, fols.11vb–12ra): 'For the first case, take this consecution: 'Some man is an ass, therefore some man is a goat', which is said to be necessary since it cannot be that some man is an ass unless he is a goat, so the consecution is necessary, although there is no necessary relation between the premise and the conclusion . . . Moreover, from every affirmative sentence it follows that God exists and not every affirmative sentence signifies that God exists, as is clear from this principle: every affirmative sentence signifies (in some way) solely for that thing which is signified by its subject' (*Pro primo: capiatur talis consequentia: 'Homo est asinus, ergo homo est capra,' que necessaria dicitur cum non potest esse quod homo sit asinus nisi sit capra, quare necessaria consequentia est, licet non sit necessaria habitudo inter antecedens et consequens . . . Item ex omni propositione affirmativa sequitur deum esse et non omnis propositio affirmativa significat deum esse*

The advocate of Containment as a criterion of validity would obviously deny that these consecutions are valid, or at least, formally valid. Many authors include them as so-called ‘materially valid’ consecutions.²⁹ Others, including Drukken, dismiss them as invalid, precisely because they contravene the containment condition. Drukken writes:

But I consider that none of them is a sound consecution, because in every sound consecution what the premises and the conclusion signify is the same, so that whatever is signified by the conclusion should be signified by the premises, and then the understanding by a natural judgment infers the conclusion from the premise. (Nicolaus Drukken 1997, Q. 9, p. 50 ll. 170–75)

Consequently, he believes that every sound consecution is formal.³⁰ One might object, however, that Drukken has misidentified the necessity which the premises invoke. They don’t necessitate that one infer, assert or even consider, the conclusion. What they necessitate is that one not assert the opposite of the conclusion (as noted in the third definition, the incompatibility criterion). Given the premises, the understanding recognises that they contain the conclusion, and as such permit its assertion, but more importantly, they exclude the assertion of its contradictory.

2.4. Bradwardine’s Second Thesis

We then need to take Bradwardine’s argument for his Second Thesis (that any sentence that signifies itself to be false also signifies itself to be true) and check that it is compatible with Paul’s interpretation.

The proof has three parts: first, suppose that some sentence, A , signifies only that it itself is not true and nothing else, and suppose A is not true. Then things are not as it signifies, so it’s true, that is, $\neg TA \Rightarrow TA$. But ‘ $\neg TA$ ’ is A , so $A \Rightarrow TA$. Since TA signifies that A is true, A must also signify that it is true (by Paul’s version of Bradwardine’s postulate). So A does not signify only that it is not true, but also that it is true.³¹

That takes us to the second part of Bradwardine’s proof, which is the nub of his argument: So A signifies more than that A is not true, but also, say, that b is c . Again, suppose A is not true. Then things are not as A signifies,

ut patet per hoc principium: omnis propositio affirmativa pro re tali solum significat que significatur per eius subiectum).

²⁹Just as according to the English account, formal validity is broader than on the Parisian account, so material validity is narrower. Drukken writes (Nicolaus Drukken 1997, Q. 9, p. 49 ll. 164–7): ‘But some propose this distinction, that some simple consequence is material, some formal. And material [consequence] is what holds solely in virtue of the terms, and holds by these two rules: From the impossible anything follows; and: The necessary follows from anything’. See e.g., de Ockham (1974, III 1, p. 589), corrected against Schupp (1993): ‘Material consequence is when it holds precisely in virtue of the terms and in virtue of some extrinsic middle precisely respecting the general conditions of propositions. Of this sort are ‘If a man runs, God exists’, ‘A man is a donkey, therefore God does not exist’, and so on’.

³⁰Nicolaus Drukken 1997, Q. 14, p. 81 ll. 118–9. See also Q. 4, p. 21 ll. 80–4.

³¹Cf. Thomas Bradwardine (2010, §6.6.1, pp. 102–3).

so either it's true or b is not c , that is, $\neg TA \Rightarrow (TA \vee b \text{ is not } c)$. Now A is ' $\neg TA \wedge b \text{ is } c$ ', and given that $\neg TA \Rightarrow (TA \vee b \text{ is not } c)$, $(\neg TA \wedge b \text{ is } c) \Rightarrow ((TA \vee b \text{ is not } c) \wedge b \text{ is } c)$, and so A (that is, $\neg TA \wedge b \text{ is } c$) $\Rightarrow TA$. Hence, as before, A must also signify that it is true (by Paul's version of Bradwardine's postulate). (*Loc.cit.*)

Thus any sentence that signifies that it is itself not true, also signifies that it is true, and so, being implicitly contradictory, must be false. Finally, suppose A signifies that A is false. If A is false then it's not true, so if it signifies that it is false, it must also signify that it is not true, by Bradwardine's postulate, and so by what has just been shown, it also signifies that it is true and is false.³²

Consequently, any sentence that signifies itself to be false also signifies itself to be true. Not everything such an insoluble sentence signifies can obtain, and so every insoluble is false.

2.5. Revenge

Solutions to the insolubles such as Bradwardine's sail close to the wind, however, and one might suspect that paradox has returned in that argument of Bradwardine's, when he claims that $A \Rightarrow TA$ (from which he infers **Sig**(A, TA)). For A is just FA (or $\neg TA$), and Bradwardine claims that A is false (that is, FA), and from $FA \Rightarrow TA$ and FA it follows immediately that TA , that is, that A is not only false, but also true, and paradox has returned.

This is a general problem for all those solutions, including Bradwardine's, the modified Heytesbury, Albert of Saxony's, Marsilius of Inghen's and even John Buridan's,³³ which propose that each insoluble, or perhaps every sentence, additionally signifies or, in Buridan's case, implies its own truth. For any sentence which signifies its own truth must surely entail it too. Somehow, a revenge paradox must be avoided, whereby accepting any form of what has come to be known as Capture, that is, $\alpha \Rightarrow T\alpha$ (see, e.g., Beall 2007, pp. 1–3) contradicts the verdict that every self-falsifying sentence is false.

This problem does not affect the first leg of Bradwardine's proof of his Second Thesis, despite appearances. For the premise of the application of the Second Postulate, of the form $A \Rightarrow TA$ (that is, $\neg TA \Rightarrow TA$, since A is $\neg TA$), depends on the, as it turns out, false assumption that A signifies only that it itself is not true. However, that is not so in the second leg of the argument, where it appears to be asserted categorically that $A \Rightarrow TA$. Bradwardine considers the objection and responds as follows, applied to the specific case where Socrates utters 'Socrates says a falsehood':

But if it is true that Socrates utters a falsehood, and Socrates says that, then Socrates utters a truth.

³² Thomas Bradwardine (2010, §6.6.2, pp. 102–3). The astute reader will spot that Bradwardine seems here to be taking his second postulate in the closure sense, not Paul's. We can, however, cast it in Paul's terms: $FA \Rightarrow \neg TA$ and $\neg TA$ signifies that A is not true, so FA also signifies that A is not true.

³³On these solutions, see Spade and Read (2021, §§3.1, 3.5, 3.8, 3.9, 4.1).

The reply is that the minor premise is false, because he does not utter the sentence presented by you, and conceded by me, but another (sentence) like it ... (Thomas Bradwardine, 2010, §7.1.1–ad 7.1.1)

As we noted in §2.4, under Paul's interpretation, what entails TA is ' $\neg TA \wedge b$ is c ', where clearly ' b is c ' (perhaps better written predicatively as ' B is C ', or in fact ' A is T ' or ' TA ') includes the further signification that Bradwardine's reasoning has revealed. The sentence Socrates utters is self-referential and so signifies differently from Bradwardine's own equiform utterance, which is not self-referential and refers not to itself but to Socrates's utterance. Having a different signification, their truth-conditions are unsurprisingly different, the one false, the other true.

Drukken makes the same distinction in his response to the sixth puzzle in Question 7:

Turning to the proof, you say: 'This is false: "Socrates says a falsehood'." I grant that this sentence, which Socrates says, is simply false, and with this, I grant that that sentence is not the conclusion from that initial premise. And for this reason, if then there was no one except Socrates in the scenario proposed, then no mental consecution would be sound because there would be only one sentence alone in Socrates's mind. And for this reason I readily grant that this consecution which I make, that it would be sound. But then both the premise and the conclusion supposit for the sentence which Socrates says, and each is true, although Socrates's sentence is simply false. And for this reason, when one argues: 'everything said by Socrates is false,' I grant it, and 'Socrates's sentence' etc., I grant what was concluded. Then further: 'This sentence is false, and this was the conclusion.' I deny the minor premise, because it was not the conclusion, but another sentence which is mine, and mine is similar to the one which is Socrates's. But mine is true, and Socrates's is false. And thus in no such scenario does it follow. Whence etc. (Nicolaus Drukken 1997, Q. 7 ll. 237–51, p. 40. See also Q. 9, p. 55 ll. 321–36)

So too in the case of the version of Bradwardine's proof of his Second Thesis in §2.4: Bradwardine makes the assumption that A is not true. That assumption is not A itself, but a meta-linguistic statement about A . He is not assuming that his assumption is not true, but that Socrates's utterance is not true. Bradwardine then shows that A , that is ' $\neg TA \wedge b$ is c ', entails that A is true, and so, by the Second Postulate, it follows A signifies that A is true, and so, signifying contradictory things, A must be false.

2.6. Signification Given How Things Are Now

There is one final complication, however. Recall Bradwardine's statement of his second postulate:

Every sentence signifies or means given how things are now or unconditionally everything which follows from it given how things are now or unconditionally.

Containment, at least as stated by Drukken, applies only to unconditional consequence (*consequentia simplex* or *simpliciter*), since Drukken rejects the notion of consequence ‘given how things are now’ (*ut nunc*). The reason he rejects it is the same as his reason for rejecting material consequence, namely, that consequence *ut nunc* is enthymematic, and so our understanding of what is signified by the premises does not suffice for the natural judgment needed to infer the conclusion. Take the claim that the consecution:

Socrates is not running now. Therefore, a man is not running,
is valid given how things are now, supposing that Socrates is now a man, since ‘for some present moment, the premise cannot be true unless the conclusion is true, so the consecution is sound given how things are now.’ (Nicolaus Drukken 1997, Q. 9, p. 48 ll. 131–5) Drukken responds:

if your explanation were valid, namely, that the premises cannot be true without the conclusion, then for the same reason you would have to concede that any true sentence would imply any other truth, and so ‘You are sitting, therefore your friend is sitting’ would be a sound consecution, and so on for others. (Nicolaus Drukken 1997, Q. 9, p. 49 ll. 146–9)

Consequently, he says, ‘All consequence is unconditional,’ (*loc.cit.*) that is, every valid consecution follows unconditionally and only unconditional consecutions are valid.

But including signification given how things are now in his second postulate is crucial to Bradwardine’s solution to the insolubles, in order to deal with contingent paradox. For example, if Socrates says only ‘Socrates says a falsehood,’ it follows only given how things are now that it is itself false, in virtue of the contingent fact given how things are now that it is the only sentence he utters. Bradwardine infers from its following given how things are now that it is true that it also signifies given how things are now that it is true. Robert Eland similarly connects the two notions when he characterises convertibility given how things are now:

Sentences are convertible given how things are now when one signifies just as the other given how things are now.³⁴

This suggests a characterisation of consequence given how things are now in terms of containment of what the conclusion signifies given how things are now in what the premises signify given how things are now. Accordingly, following unconditionally and following given how things are now would, between them, support the full force of Bradwardine’s second postulate for application to every insoluble, contingent or not.

But recall that consequence given how things are now is enthymematic. For most authors that is unproblematic: consequence is an objective matter

³⁴Spade (1976, §5, p. 58). On the identity of Eland, see Read and Thakkar (2016).

for those who characterise it in terms of truth-preservation, or preservation of signifying as things are, or even incompatibility. But for someone like Drukken who defines consequence in terms of containment of the understanding of the conclusion in that of the premises, the idea that hidden and unstated additional facts might validly connect premises and conclusion is anathema. Even more so is it with signification given how things are now. He agrees with what Dummett once said about meaning and understanding, that meaning should be transparent: if two expressions which one understands mean the same, then one must know that they mean the same.³⁵ That ability extends for Drukken to the ability to articulate the soundness of a consecution in a ‘natural judgment [that] infers the conclusion from the premises.’

This is a thoroughly internalist conception of signification and consequence. It stands in contrast to the implicit semantic externalism of Bradwardine’s conception of those notions, which accepts, indeed proclaims, that what follows from our assertions, and indeed, even what they signify, may extend well beyond our understanding and awareness. Socrates innocently said that what he said was false. He thought he knew what his utterance signified: he knew its usual signification. But it took theorists such as Aristotle,³⁶ and Bradwardine and his successors to claim that, to Socrates’s, and the man on the omnibus’s astonishment, it also signified its own truth. They may or may not have been right; but a presupposition of their approach to the insolubles was that they could claim, and hopefully, back with reasons, that part of what ordinary speakers signified by their utterances was beyond their ken, at least until it was pointed out and explained to them. ‘Meanings just ain’t in the head’, as Putnam quipped.³⁷

3. Summary

Niels Drukken of Denmark discussed five different ways of defining validity, or consequence. Buridan and Swyneshed, in their different ways, cast doubt on truth-preservation as a defining feature of validity. Buridan proposed preservation of signifying-as-things-are as the basis of validity. But John Mair presented a further puzzle that seemed to show that preservation of signifying as things are would not suffice. Many authors, including Girard Odo and Paul of Venice, suggested that the incompatibility of the opposite of the conclusion with the premises was the correct mark of validity. But that seems circular,

³⁵Dummett (1978a, 131) argued that Frege distinguished sense (*Sinn*) from reference (*Be-deutung*) because the former is transparent whereas the latter is not: ‘It is an undeniable feature of the notion of meaning [i.e. sense, *Sinn*]—obscure as that notion is—that meaning is *transparent* in the sense that, if someone attaches a meaning to each of two words, he must know whether these meanings are the same.’

³⁶When surveying the various meanings of ‘is’, Aristotle (*Metaphysics* Δ 7, 1017a31–35) observes: ‘Again, ‘to be’ and ‘is’ signify that a thing is true, and ‘not to be’ that it is not true but a falsehood, equally in the case of affirmation and of denial; as for instance that Socrates *is* artistic, that this is true, or that Socrates is not-pale, that it is true; and ‘a diagonal *is not* commensurable’ that it is a falsehood.’

³⁷See Putnam (1975, 227).

since incompatibility itself is defined in terms of validity. Finally, the containment of the conclusion in the premises, that is, that the premises should signify whatever is signified by the conclusion, seems to capture validity.

Interestingly, Bradwardine's famous Second Postulate, as interpreted by Paul of Venice, follows from the characterization of validity by the Containment condition. Although John Dumbleton argued against this postulate, his objections assume the validity of the strict implication paradoxes, which many, including Drukken, reject; and Bradwardine's proof of his Second Thesis, solving the logical paradoxes (that is, the insolubles), is consistent with Paul's interpretation of the postulate. Nonetheless, such solutions as Bradwardine's must avoid a commitment to Capture, whereby any sentence entails its own truth, on pain of contradiction. Moreover, Bradwardine's inclusion of consequence given how things are now in his postulate shows how far his externalist conception of signification lies from Drukken's and perhaps others' reason for endorsing the containment criterion.

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