Quantum Many-Body Physics with Multimode Cavity QED

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University of St Andrews
FOUNDED 1413

SUPA

CONDMAT17, Copenhagen, May 2017

What can quantum systems do?

Condensed matter physics: two types of question

What physics is needed to explain the material properties we do see

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How? Doping, strain, heterostructure growth, nanofabrication ...

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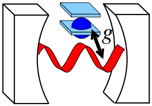
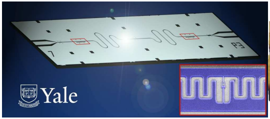
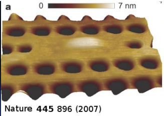
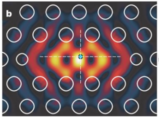
How? Doping, strain, heterostructure growth, nanofabrication . . .
Floquet driving, strong matter-light coupling . . .

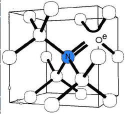
From cavity QED . . .











- Precision tests of quantum optics
  - Purcell effect, strong coupling
  - Rabi oscillations, collapse & revival
  - Resonant fluoresecence, EIT
- Many body cavity QED?
  - Phase transitions: Lasing, superfluoresence, superradiance

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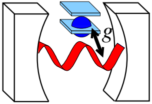

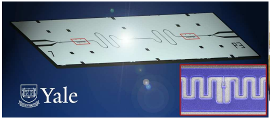

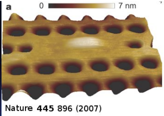

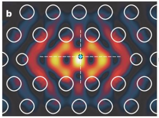

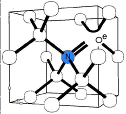

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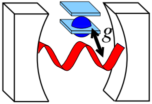
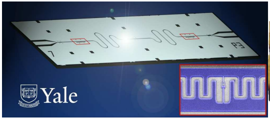

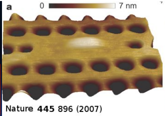

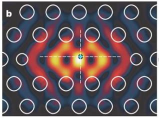

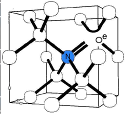

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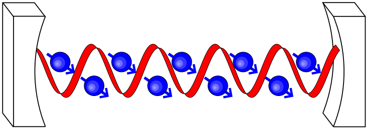
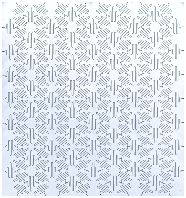
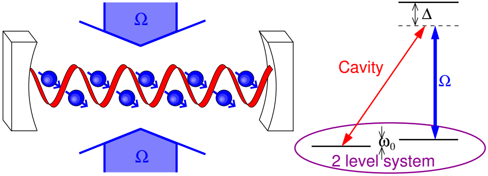



[Circuit QED array, Houck group]

  - Phase transitions: Lasing, superfluoresence, superradiance

…synthetic cavity QED: Raman driving

- Tunable coupling via Raman



$$H_{\text{eff}} = \ldots \frac{\Omega g}{\Delta}\left(\sigma_n^{+} a + \text{H.c.}\right)$$

- Real systems: loss $\partial_t \rho = -i[H, \rho] + \kappa \mathcal{L}[a, \rho] + \ldots$
- To balance loss, counter-rotating:

$$H_{\text{eff}} = \ldots \frac{\Omega g}{\Delta} \sigma_n^{x}(a + a^\dagger)$$

[Dimer *et al.* PRA ’07]

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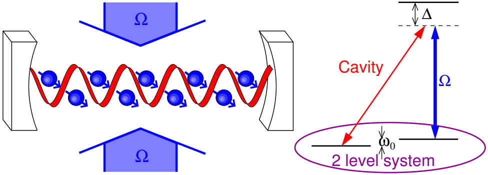

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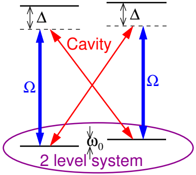

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(Multimode) cavity QED

$$H = \sum_k \omega_k a_k^\dagger a_k + \sum_n \omega_0 \sigma_n^+ \sigma_n^- + \sum_{n,k} g_{k,n}(a_k^\dagger + a_{-k})(\sigma_n^+ + \sigma_n^-)$$

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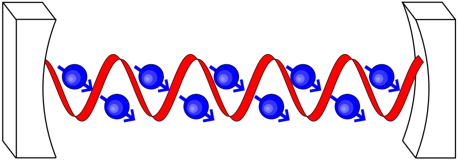

- Compare g (or $g\sqrt{N}$) vs:
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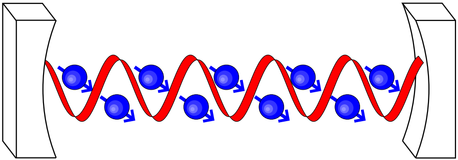

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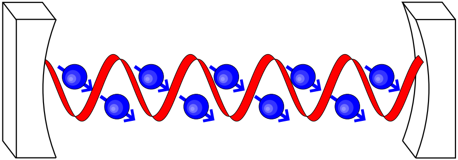
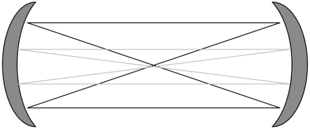
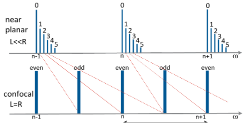


Multimode cavities

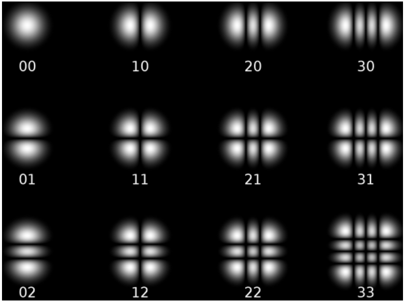
Confocal cavity $L = R$:



- Modes
  $\Xi_{l,m}(\mathbf{r}) = H_l(x)H_m(y)$,
  $l + m$ fixed parity



- Tune between degenerate *vs* non-degenerate



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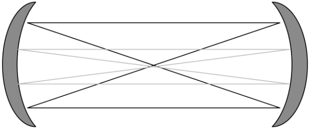

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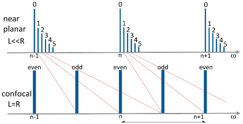

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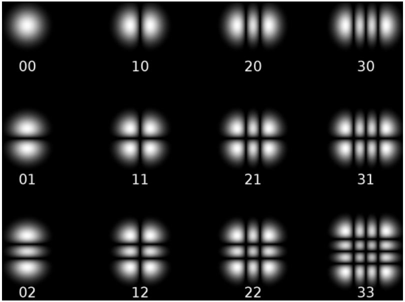
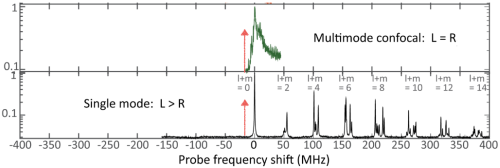
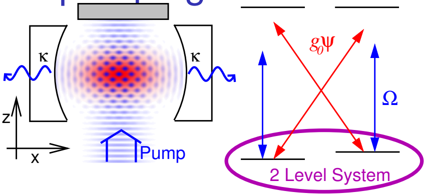




1. Introduction: Tunable multimode Cavity QED
2. Density wave polaritons
   - Superradiance transition
   - Supermode density wave polariton condensation
3. Spin wave polaritons
   - Effects of loss
   - Spin glass
4. Meissner-like effect

Mapping transverse pumping to Dicke model



- Atomic states: $\psi(\mathbf{r}) = \psi_{\Downarrow} + \psi_{\Uparrow}\cos(qx)\cos(qz)$

$$H_{\text{eff}} = \underbrace{(\omega_c - \omega_P)}_{-\Delta_c} a^\dagger a + \sum_n \frac{\omega_0}{2}\sigma_n^z + \underbrace{\frac{\Omega g_0}{\Delta}}_{g_{\text{eff}}} \sigma_n^x (a + a^\dagger)$$

- Extra "feedback" term U, cavity loss κ
- $\mathbb{Z}_2$ symmetry: phase of light

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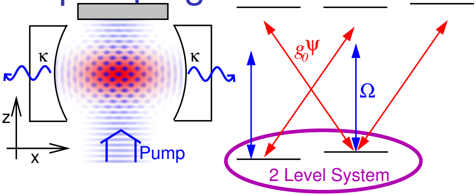

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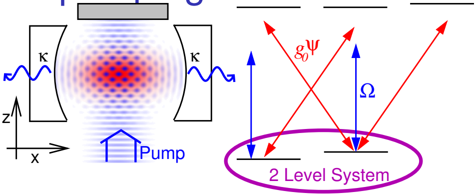

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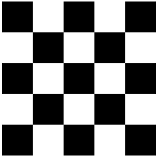
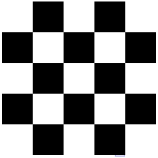
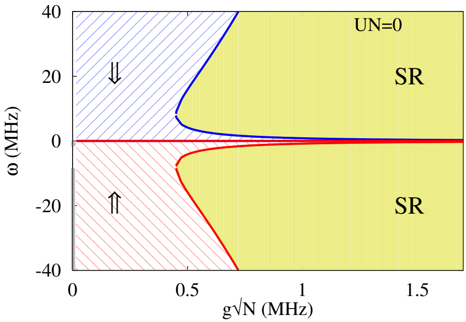
Atoms:  vs 

Classical dynamics

Changing U:

$U = 0$



[JK *et al.* PRL '10, Bhaseen *et al.* PRA '12]

Classical dynamics

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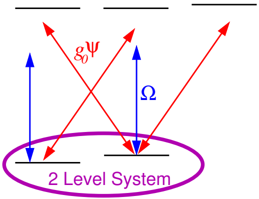

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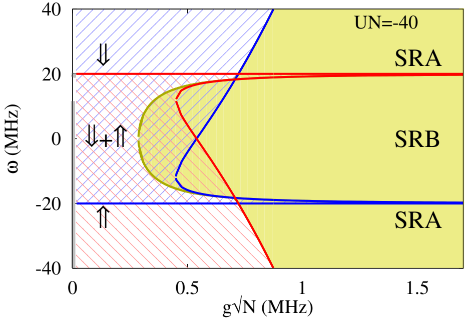

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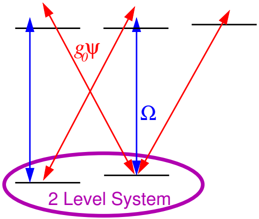

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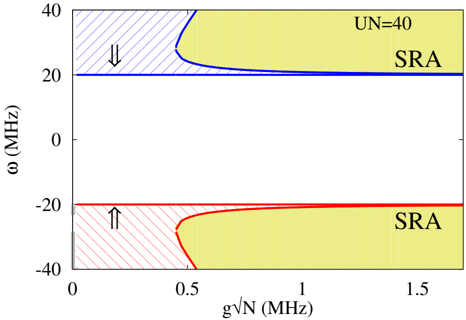

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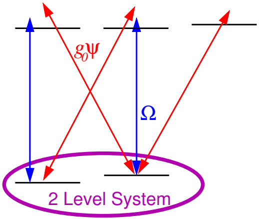

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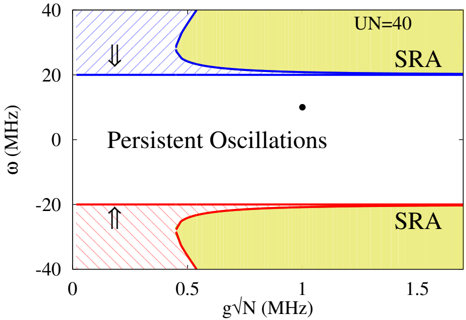
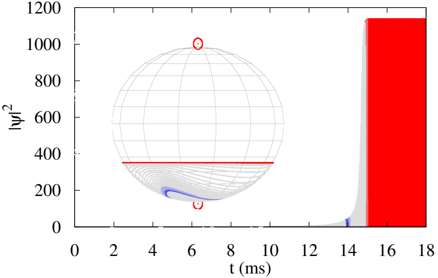
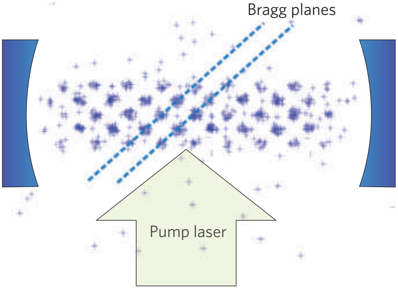




[JK *et al.* PRL '10, Bhaseen *et al.* PRA '12]

Single mode experiments



Ritsch *et al.* PRL '02

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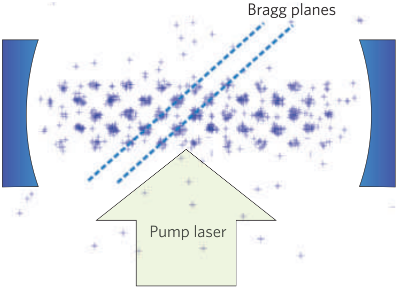
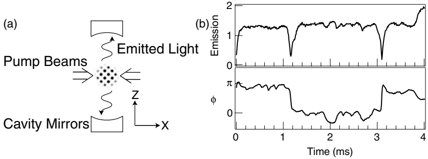


Ritsch *et al.* PRL '02

Thermal atoms, momentum state



Vuletic *et al.* PRL '03 (MIT)

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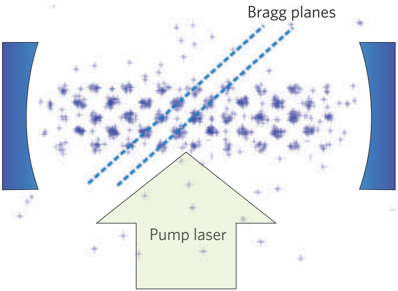

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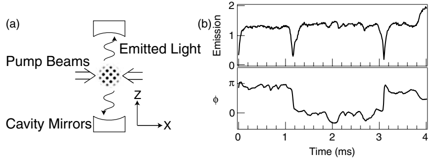

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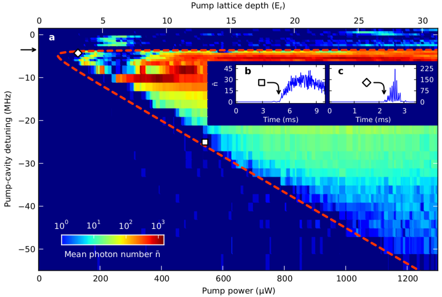
BEC, momentum state



Baumann *et al.* Nature '10 (ETH)

Kinder *et al.* PRL '15 (Hamburg)

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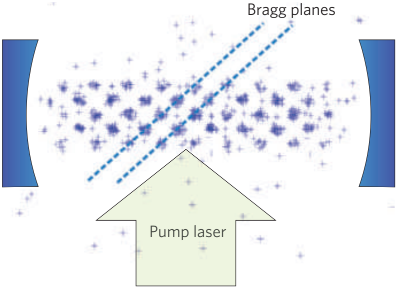

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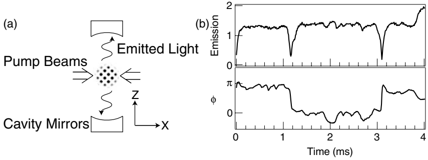

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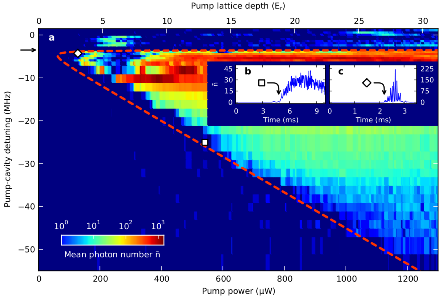


Baumann *et al.* Nature '10 (ETH)

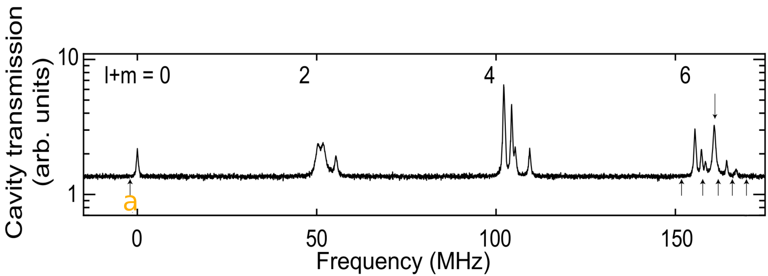
Kinder *et al.* PRL '15 (Hamburg)

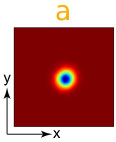
BEC, hyperfine states

Baden *et al.* PRL '14 (Singapore)

Density wave polaritons

Superradiance in multimode cavity: Even family





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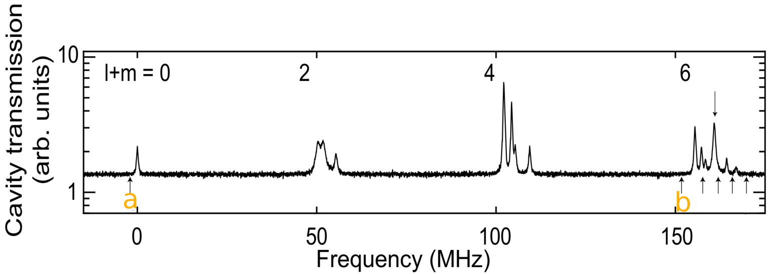

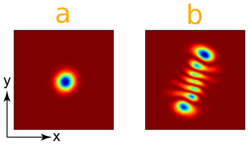

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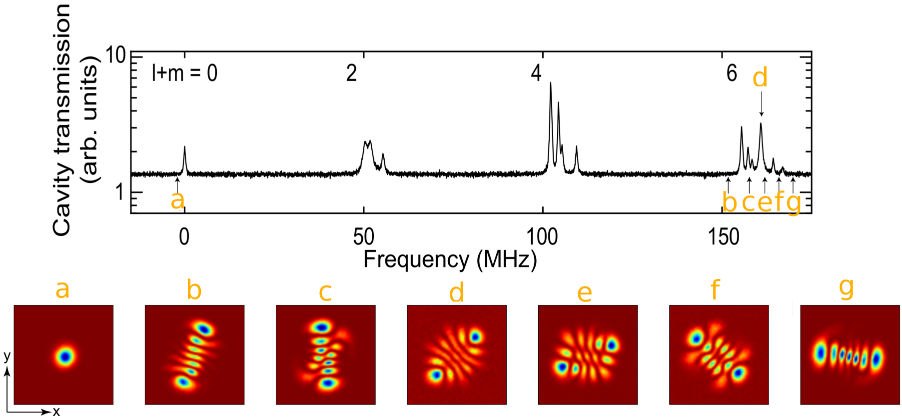
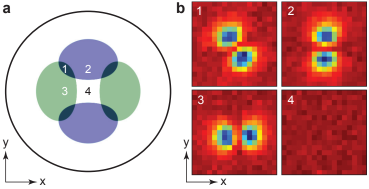


Superradiance in multimode cavity: Odd family

- Dependence on cloud position



- Atomic time-of-flight — structure factor
  $\psi(\mathbf{r}) = \psi_{\Downarrow}(\mathbf{r}) + \psi_{\Uparrow}(\mathbf{r}) \cos(qx)\cos(qz)$

- Near-degeneracy of $(1,0)$, $(0,1)$ modes broken by matter-light coupling.

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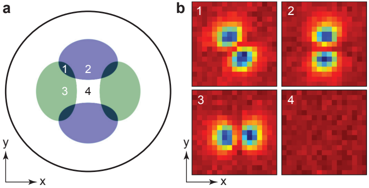

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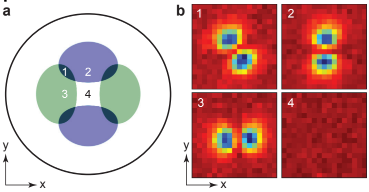

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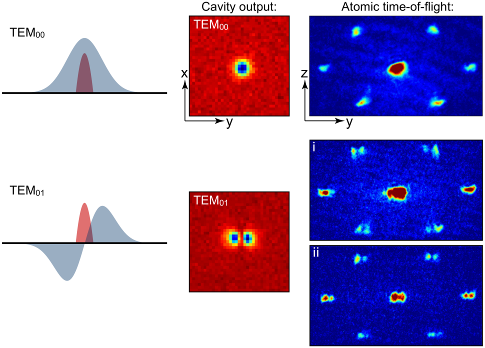

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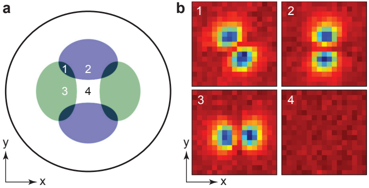

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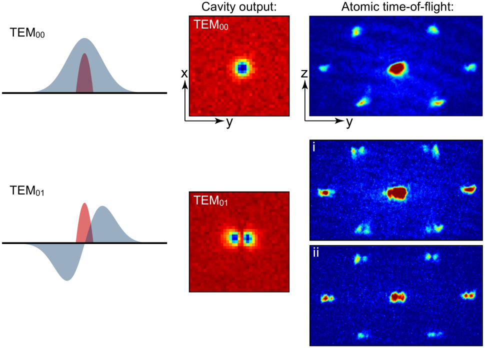
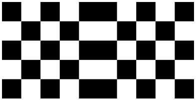
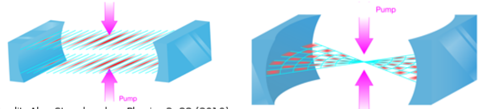




Synthetic cQED Possibilities

- Single mode *vs* **multimode**



Credit: Alan Stonebreaker, Physics 3, 88 (2010)

- Momentum state *vs* hyperfine state

- XY *vs* Ising

- Thermal gas *vs* BEC *vs* disorder localised

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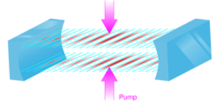

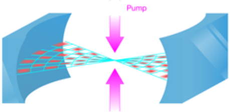

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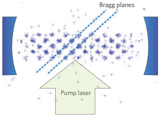

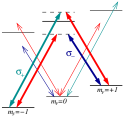

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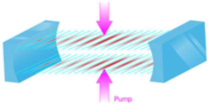

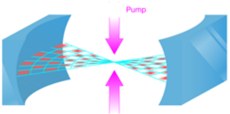

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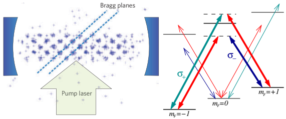

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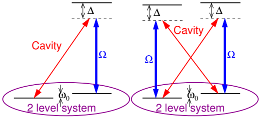

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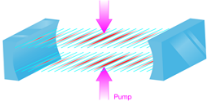

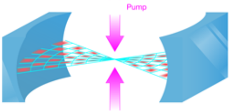

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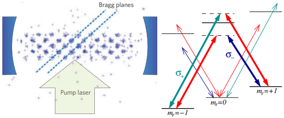

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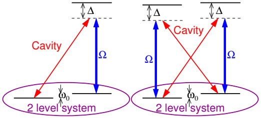

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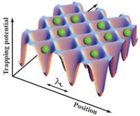

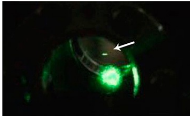
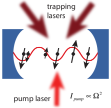

Spin wave polaritons

Internal states: Effect of particle losses

- Dicke Hamiltonian: $H = \omega a^\dagger a + \sum_i \omega_0 \sigma_i^+ \sigma_i^- + g\sigma_i^x (a^\dagger + a)$
- Adding other loss terms

$$\partial_t \rho = -i[H, \rho] + \kappa \mathcal{L}[\hat{a}] + \sum_i \Gamma_\downarrow \mathcal{L}[\sigma_i^-] + \Gamma_\phi \mathcal{L}[\sigma_i^z]$$

$$\mathcal{L}[X] = X\rho X^\dagger - (X^\dagger X \rho + \rho X^\dagger X)/2$$

- $\Gamma_\downarrow, \Gamma_\phi$ break S conservation.
- Mean field: confusing result.

[Dalla Torre *et al.* , PRA (Rapid) 2016, Kirton & JK, PRL 2017]

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Effect of particle losses

- Wigner function $W(\hat{a} = \hat{x} + i\hat{p})$

- Γ_ϕ only: MFT $\to$ no SR
- Asymptotic scaling

- $N = 30$: no symmetry breaking

[Kirton & JK, PRL 2017]

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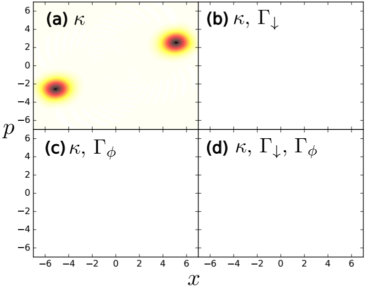

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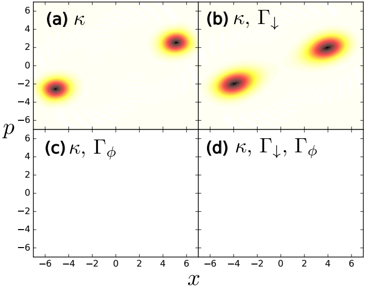

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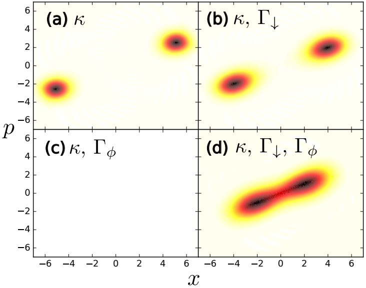

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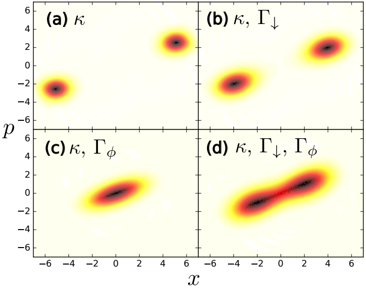

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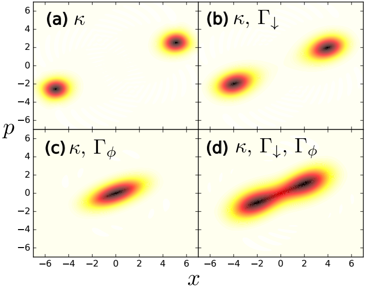

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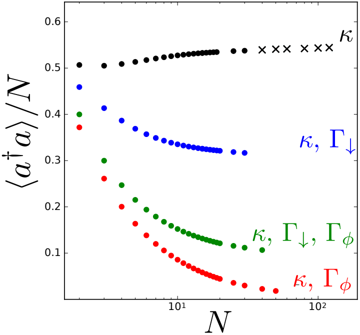

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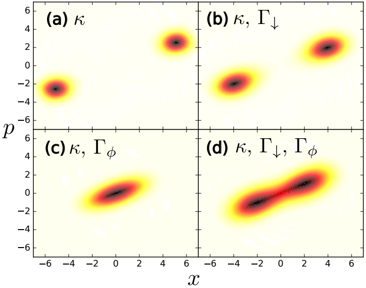

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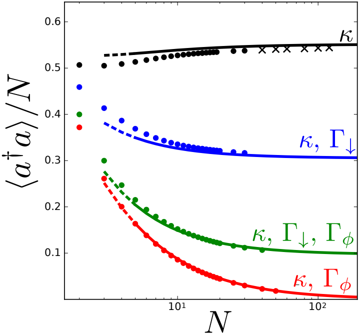

Disordered atoms

- Multimode cavity, Hyperfine states,

$$H_{\text{eff}} = -\sum_{\mu} \Delta_{\mu} a^{\dagger}_{\mu} a_{\mu} + \sum_{n} \frac{\omega_0}{2} \sigma^{z}_{n} + \frac{\Omega g_0}{\Delta} \sum_{\mu} \Xi_{\mu}(\mathbf{r}_n) \sigma^{x}_{n} (a_{\mu} + a^{\dagger}_{\mu})$$

- Random atom positions – quenched disorder

- Effective XY/Ising spin glass

$$H_{\text{eff}} = \sum_{n,m} J_{n,m} \begin{cases} \sigma^{x}_{n} \sigma^{x}_{m} & \textit{Ising} \\ \sigma^{+}_{n} \sigma^{-}_{m} & XY \end{cases}, \qquad J_{nm} = \sum_{\mu} \frac{\Omega^2 g_0{}^2 \Xi_{\mu}(\mathbf{r}_n) \Xi_{\mu}(\mathbf{r}_m)}{\Delta^2 \Delta_{\mu}}$$

[Gopalakrishnan, Lev and Goldbart. PRL ’11, Phil. Mag. ’12]

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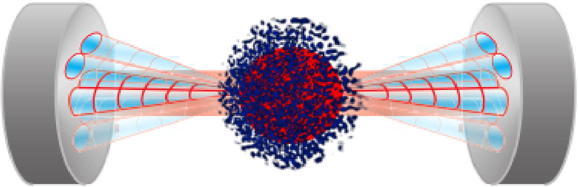

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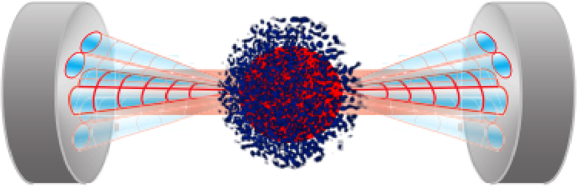

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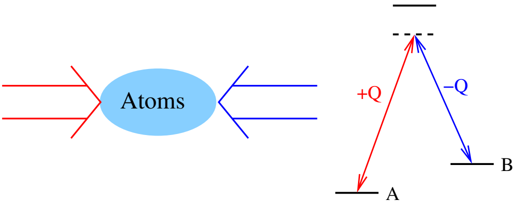
Meissner-like effect

Cavity QED and synthetic gauge fields

- [Spielman, PRA '09] scheme, hyperfine states A, B

$$H = \begin{pmatrix} \psi_A & \psi_B \end{pmatrix} \begin{pmatrix} E_A + (\nabla - Q\hat{x})^2 & \Omega/2 \\ \Omega/2 & E_B + (\nabla + Q\hat{x})^2 \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$$



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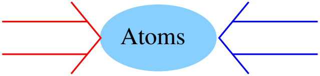

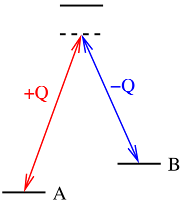

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$$E_-(\mathbf{k}) \simeq \frac{\left(\mathbf{k} - \frac{(E_B - E_A)Q\hat{\mathbf{x}}}{\Omega}\right)^2}{2m^*}$$

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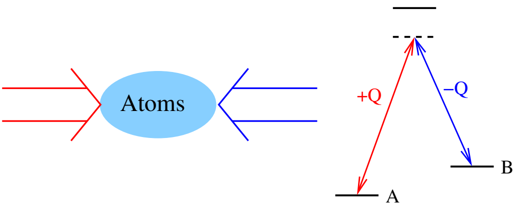

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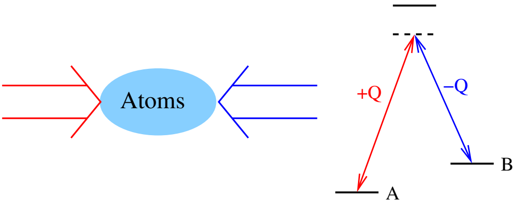
Cavity QED and synthetic gauge fields

- [Spielman, PRA '09] scheme, hyperfine states A, B

$$H = \begin{pmatrix} \psi_A & \psi_B \end{pmatrix} \begin{pmatrix} E_A + (\nabla - Q\hat{x})^2 & \Omega/2 \\ \Omega/2 & E_B + (\nabla + Q\hat{x})^2 \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$$



Ground state:

$$E_-(\mathbf{k}) \simeq \frac{(\mathbf{k} - \frac{(E_B - E_A)Q\hat{\mathbf{x}}}{\Omega})^2}{2m^*}$$

- Feedback
  - Why?
    - ★ Meissner effect, Anderson-Higgs mechanism, confinement-deconfinement transition.
  - How?
    - ★ Multimode cavity QED

Meissner-like physics: idea

- Follow Spielman scheme

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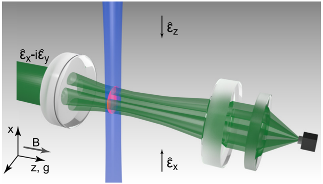

- $E_A, E_B \propto |\varphi|^2$ from cavity Stark shift
- Ground state $E_-(\mathbf{k}) \propto (\mathbf{k} - Q\hat{x}|\varphi|^2)^2$

- Multimode cQED → local matter-light coupling
- Variable profile synthetic gauge field?
- Reciprocity: matter affects field

[Ballantine *et al.* PRL 2017]

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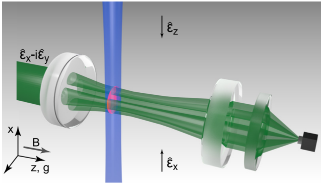

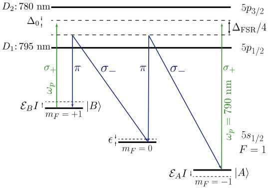

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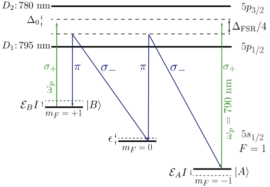
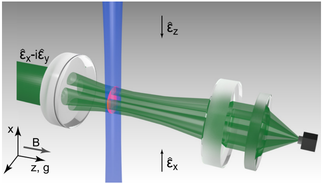


[Ballantine *et al.* PRL 2017]



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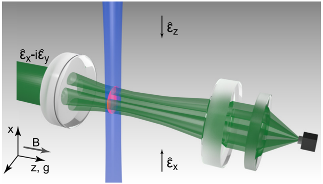

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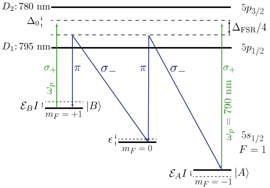

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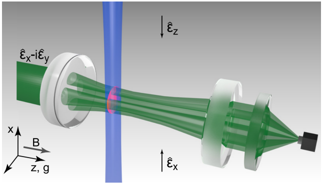

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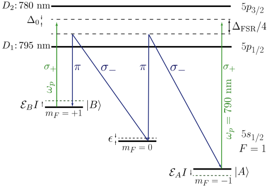

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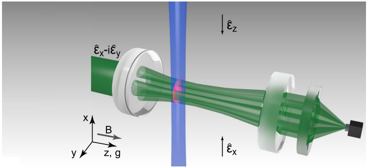

- Atoms:

$$i\partial_t \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \left[-\frac{\nabla^2}{2m} + \begin{pmatrix} -\mathcal{E}_\Delta |\varphi|^2 + i\frac{q}{m}\partial_x & \Omega/2 \\ \Omega/2 & \mathcal{E}_\Delta |\varphi|^2 - i\frac{q}{m}\partial_x \end{pmatrix} + \cdots \right] \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}.$$

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$$i\partial_t \varphi = \left[\frac{\delta}{2} \left(-l^2 \nabla^2 + \frac{r^2}{l^2} \right) - \Delta_0 - i\kappa - N\mathcal{E}_\Delta (|\psi_A|^2 - |\psi_B|^2) \right] \varphi$$

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$$|\psi_A|^2 - |\psi_B|^2 = \frac{Q}{im\Omega} \left(\psi_-^* \partial_x \psi_- - \psi_- \partial_x \psi_-^* \right) + 2\frac{\mathcal{E}_\Delta}{\Omega} |\psi_-|^2 |\varphi|^2$$

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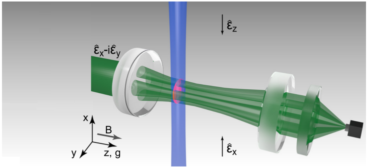

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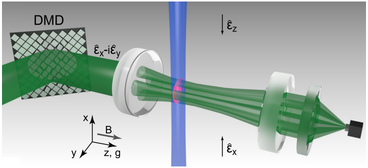

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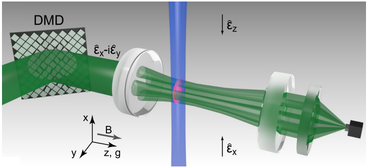

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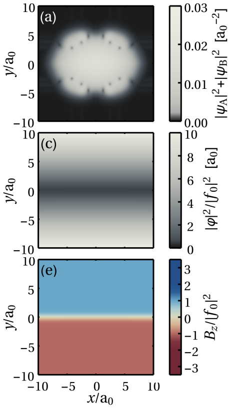
[Ballantine *et al.* PRL 2017]

Meissner-like physics: numerical simulations

Atoms

Cavity light

Synthetic field



[Ballantine *et al.* PRL 2017]

- Consider $f(\mathbf{r})$ such that $|\varphi|^2 \propto y$.
- Without feedback ($\mathcal{E}_\Delta = 0$) for field
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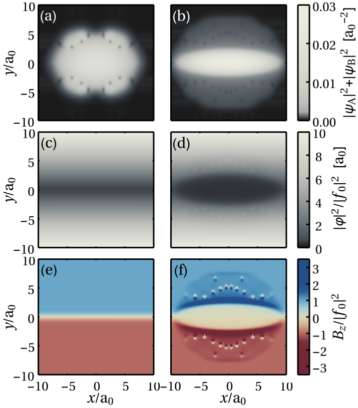

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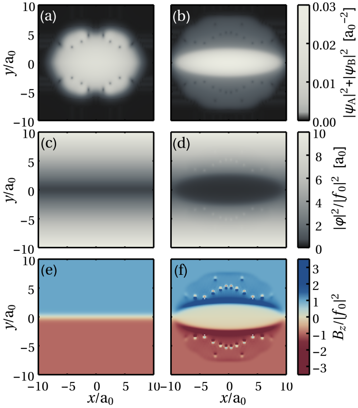

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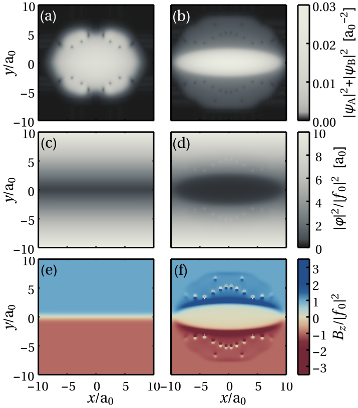

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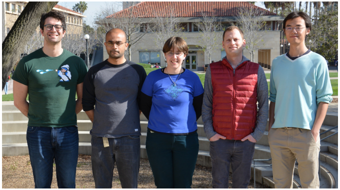
Acknowledgments

Experiment (Stanford):
Benjamin Lev



STANFORD UNIVERSITY

Theory:

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Sarang Gopalakrishnan (CUNY)
Surya Ganguli, Jordan Cotler (Stanford)
Peter Kirton, **Kyle Ballantine**, Laura Staffini (St Andrews)

The Leverhulme Trust

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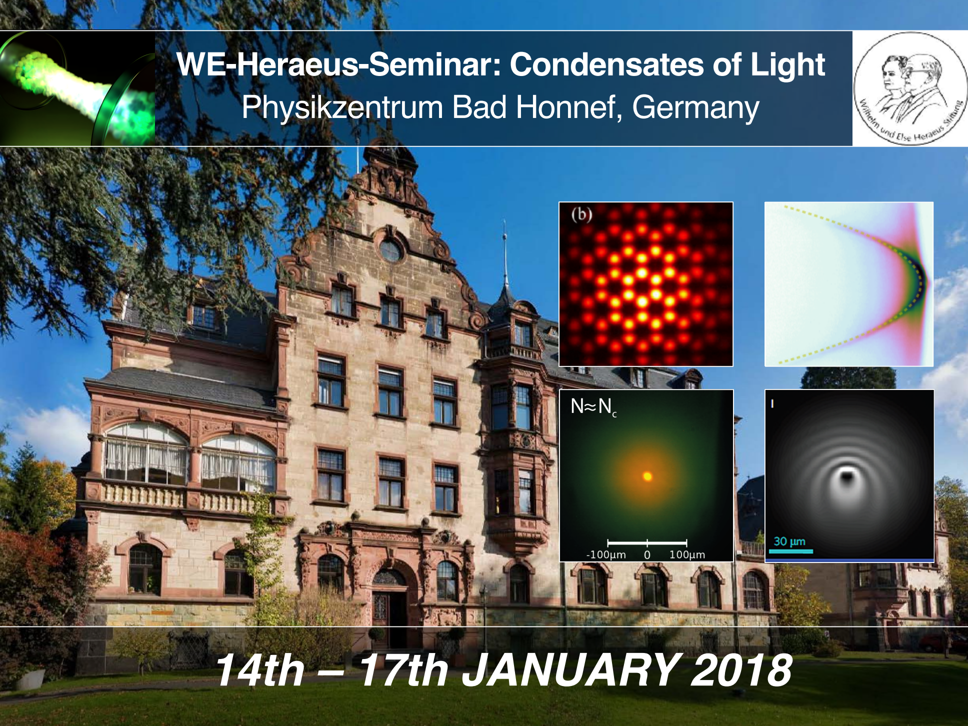
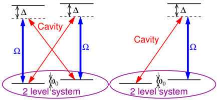
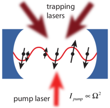
Topological Protection and Non-Equilibrium States in Strongly Correlated Electron Systems


WE-Heraeus-Seminar: Condensates of Light
Physikzentrum Bad Honnef, Germany
(b)
N≈N$_c$
-100μm 0 100μm
30 μm
14th – 17th JANUARY 2018

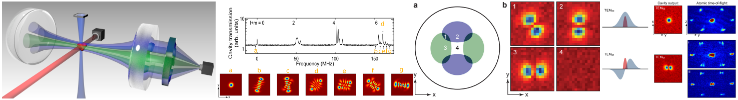
Summary

- Many possibilities of multimode cavity QED

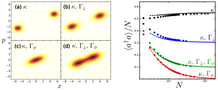




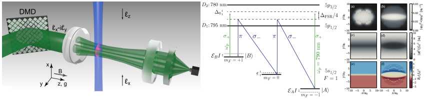
- Supermode polariton condensation [Kollár *et al.* Nat. Comms. 2017]



- Open Dicke model, $\kappa, \Gamma_\phi, \Gamma_\downarrow$ [Kirton & JK, PRL 2017]



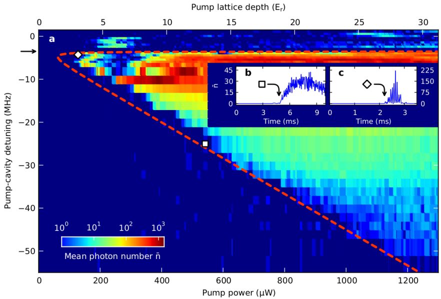
- Meissner like effect [Ballantine *et al.* PRL 2017]



- Beyond Dicke: Chaotic dynamics

Bosons beyond Dicke — single mode

So far $\Psi(\mathbf{r}) = \chi_0 + \chi_1 2\cos(qx)\cos(qz) \longrightarrow \mathbf{S} = \chi^\dagger \sigma \chi$.
Generally $\Psi(\mathbf{r}) = \sum_{\mathbf{n}} \chi_{\mathbf{n}} e^{iq\mathbf{n}\cdot r}$.



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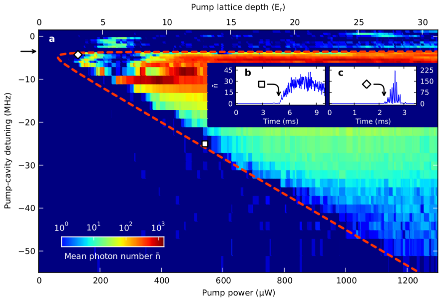

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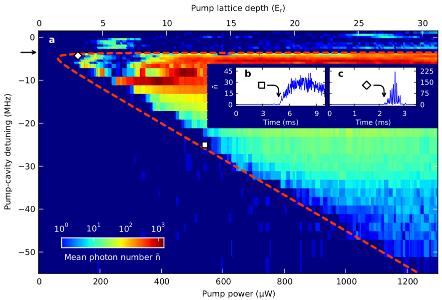

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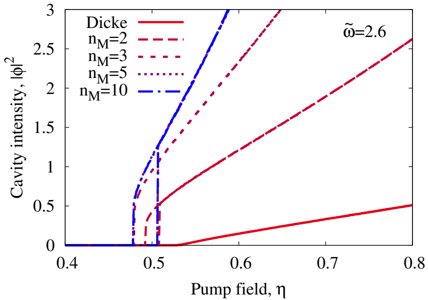
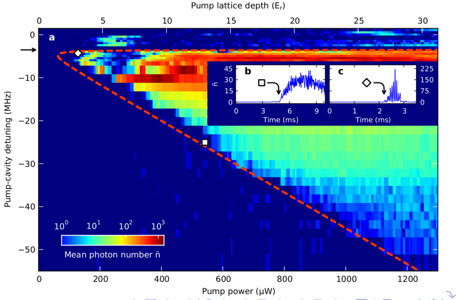
Truncate $|\mathbf{n}| < n_M$ — Hysteresis at intermediate ω





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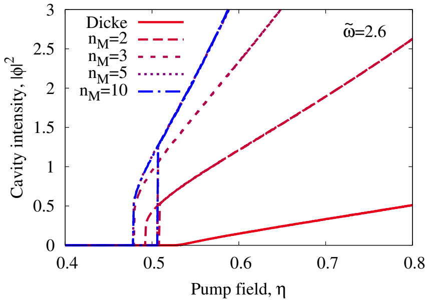
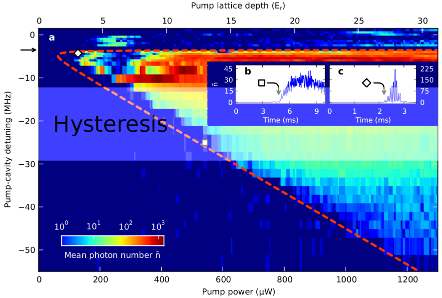
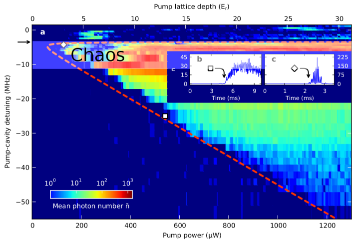


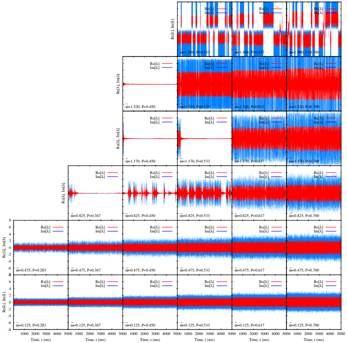


Bosons beyond Dicke — chaos



Near resonance: irregular dynamics

(NB $\omega_{\text{Pump}} - \omega_{\text{cavity}} = -\omega$)



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