

Supermode-density-wave-polariton condensation, and Meissner-like effect with multimode cavity-QED

Jonathan Keeling



Trento, January 2017

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What can quantum systems do?

Condensed matter physics: two types of question

What physics is needed to explain the
material properties we do see

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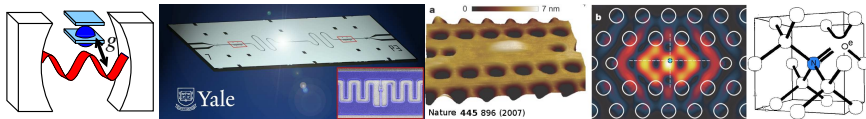
Condensed matter physics: two types of question

What physics is needed to explain the
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to

**What material properties can be possible
from quantum physics?**

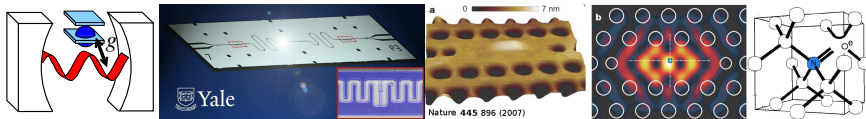
Once upon a time there was cavity QED ...



- Precision tests of quantum optics
 - Purcell effect, strong coupling
 - Rabi oscillations, collapse & revival
 - Resonant fluorescence, EIT
- Many atom physics

• Phase transitions: Lasing, superfluorescence, superradiance

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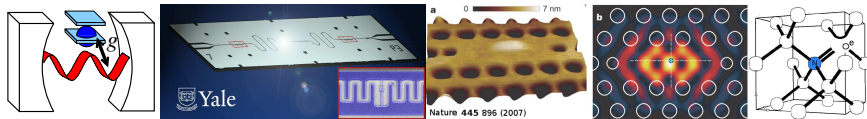
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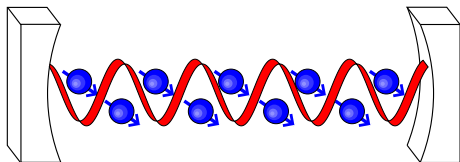
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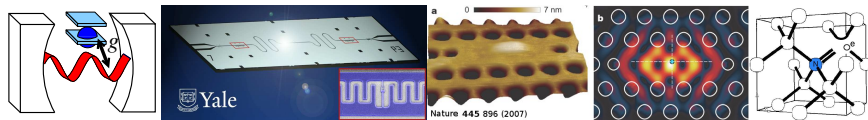


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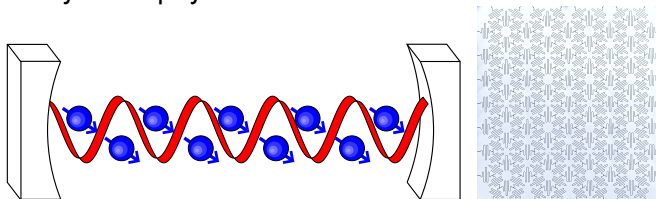


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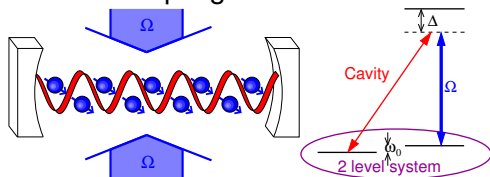
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Synthetic cavity QED: Raman driving

- Tunable coupling via Raman



$$H_{\text{eff}} = \dots \frac{\Omega g}{\Delta} (\sigma_n^+ a + \text{H.c.})$$

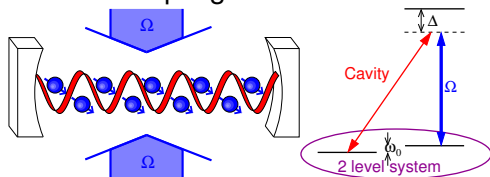
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[Dimer *et al.* PRA '07]

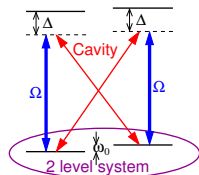
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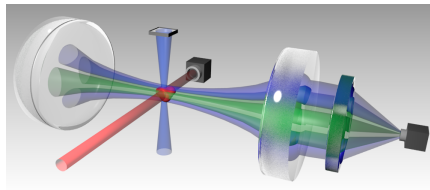
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[Dimer *et al.* PRA '07]

Multimode cavity QED



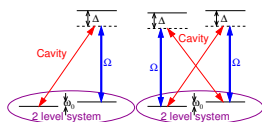
- Full model:

$$H_{\text{eff}} = \sum_{\mu} \underbrace{(\omega_{\mu} - \omega_P)}_{-\Delta_{\mu}} a_{\mu}^{\dagger} a_{\mu} + \sum_N \frac{\omega_0}{2} \sigma_n^z + \underbrace{\frac{\Omega g_0}{\Delta}}_{g_{\text{eff}}} \sum_{\mu} \Xi_{\mu}(\mathbf{r}_n) \sigma_n^x (a + a^{\dagger})$$

[Gopalakrishnan, Lev, Goldbart. Nat. Phys '09, PRA '10]

Possibilities

- XY vs Ising



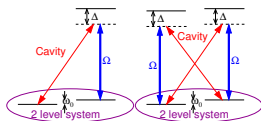
- Momentum state vs hyperfine state

- Single mode vs multimode

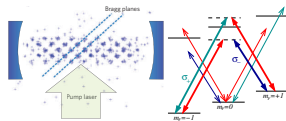
- Thermal gas vs BEC vs disorder localised

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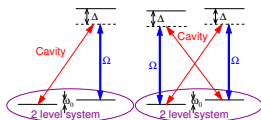


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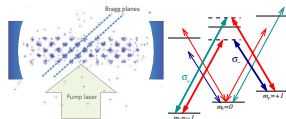
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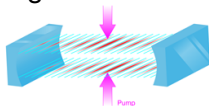
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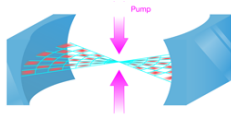
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- Single mode vs **multimode**



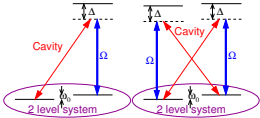
Credit: Alan Stonebreaker, Physics 3, 88 (2010).



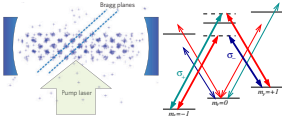
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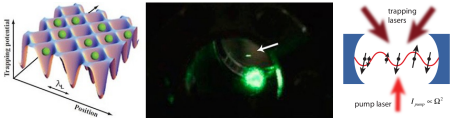


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Introduction: Tunable multimode Cavity QED

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- 2 Single mode cavity QED
 - Spin-non-conserving loss
- 3 Multimode cavity QED experiments
 - Experimental setup
 - Supermode density wave polariton condensation
- 4 Theoretical possibilities
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Single mode cavity QED

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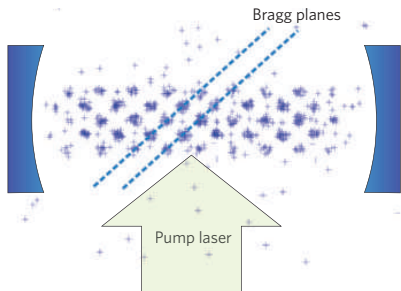
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4 Theoretical possibilities

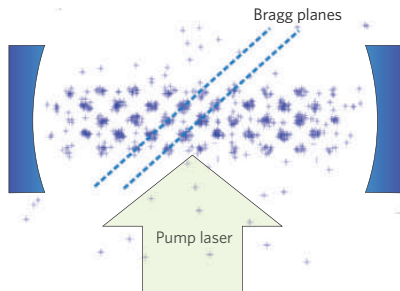
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Single mode experiments



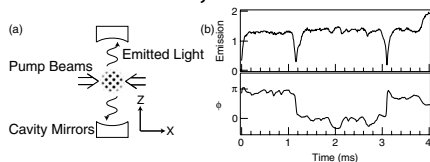
Ritsch *et al.* PRL '02

Single mode experiments



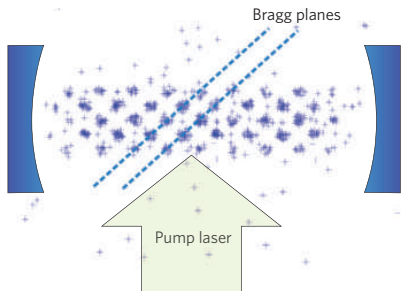
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Thermal atoms, momentum state



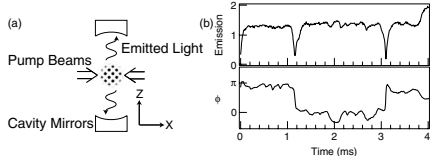
Vuletic *et al.* PRL '03 (MIT)

Single mode experiments



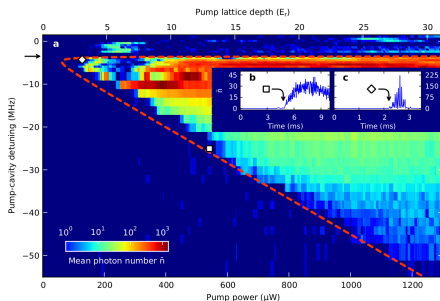
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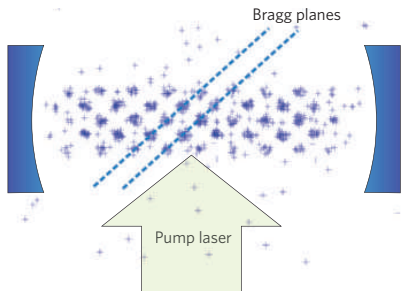
BEC, momentum state



Baumann *et al.* Nature '10 (ETH)

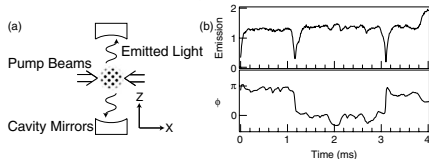
Kinder *et al.* PRL '15 (Hamburg)

Single mode experiments



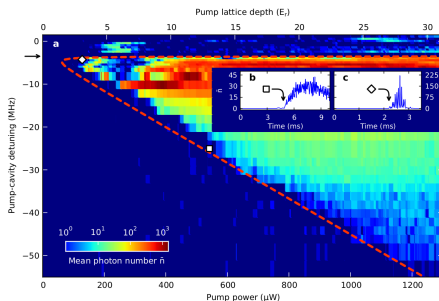
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BEC, hyperfine states

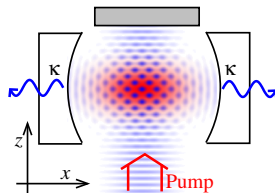
Badeen *et al.* PRL '14 (Singapore)

Single mode theory

- Momentum degrees of freedom:

$$\psi = \psi_{\downarrow} + \psi_{\uparrow} \cos(kx) \cos(kz)$$

- Effective 2LS ($\psi_{\downarrow}, \psi_{\uparrow}$)



$$H_{\text{eff}} = \underbrace{(\omega_c - \omega_p)}_{-\Delta_c} a^\dagger a + \sum_n \frac{\omega_0}{2} \sigma_n^z + \underbrace{\frac{\Omega g_0}{\Delta}}_{g_{\text{eff}}} \sigma_n^x (a + a^\dagger)$$

- Extra "feedback" term U , cavity loss κ
- Single mode – mean-field EOM, $\alpha = \langle \hat{a} \rangle$, $S^i = \sum_n \sigma_n^i / 2$.

$$\dot{S}^- = -i(\omega_0 + U|\alpha|^2) S^- + 2ig_{\text{eff}}(\alpha + \alpha^*) S^z$$

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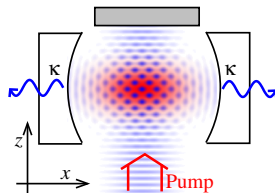
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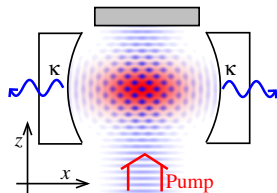
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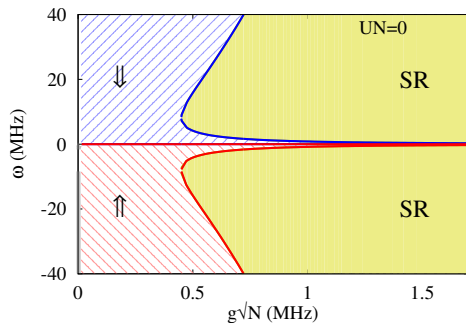
Classical dynamics

Changing U :

$$U = 0$$

$$U < 0$$

$$U > 0$$



[JK *et al.* PRL '10, Bhaseen *et al.* PRA '12]

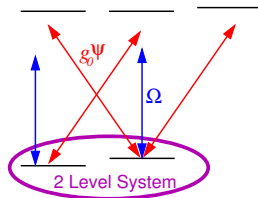
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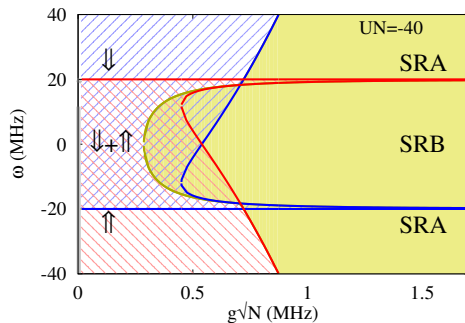
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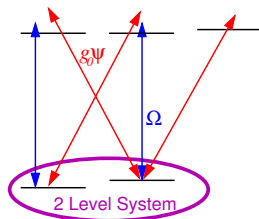
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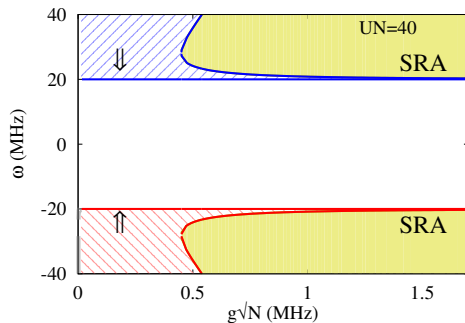
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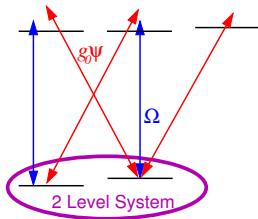
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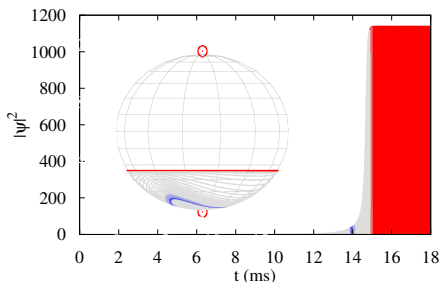
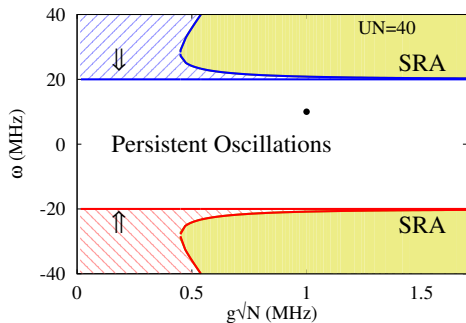
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Effect of particle losses

- Adding other loss terms

$$\partial_t \rho = -i[H, \rho] + \kappa \mathcal{L}[\hat{a}] + \sum_i \Gamma_{\downarrow} \mathcal{L}[\sigma_i^-] + \Gamma_{\phi} \mathcal{L}[\sigma_i^z]$$

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[Dalla Torre *et al.*, PRA (Rapid) 2016, Kirton & JK, arXiv:1611.03342]

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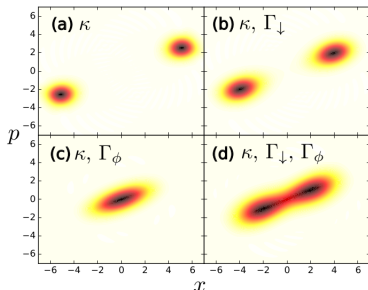
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- Finite N : no symmetry breaking

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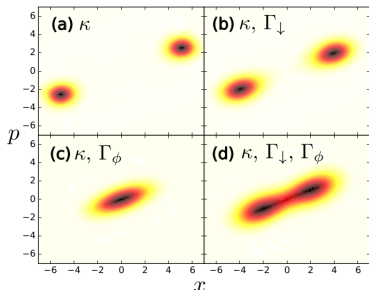
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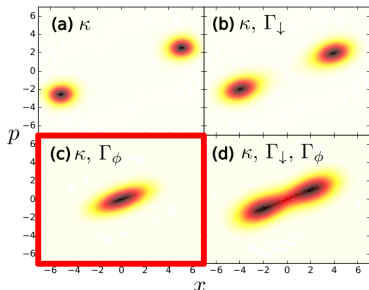


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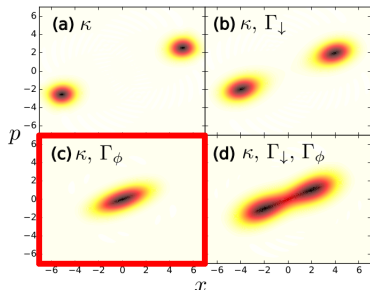
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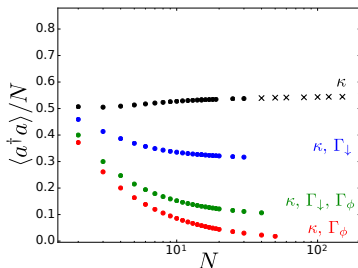
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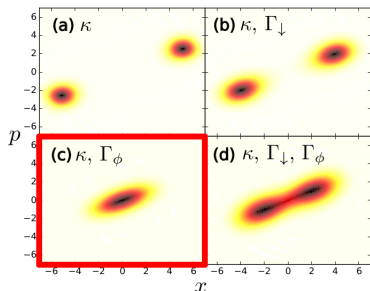
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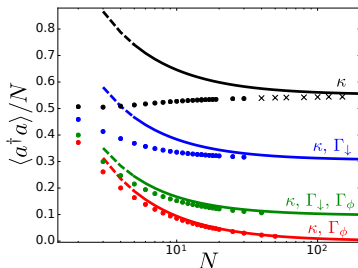
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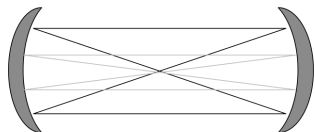


Multimode cavity QED experiments

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Multimode cavities

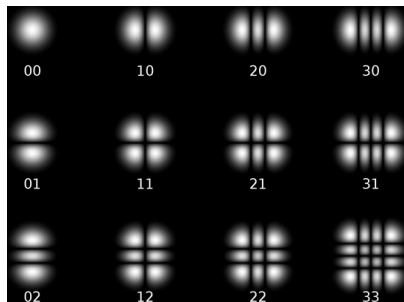
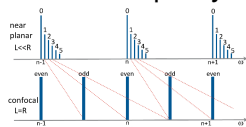
Confocal cavity $L = R$:



- Modes

$$\Xi_{l,m}(\mathbf{r}) = H_l(x)H_m(y),$$

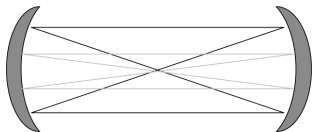
$l + m$ fixed parity



- Extra distinction: degenerate vs non-degenerate

Multimode cavities

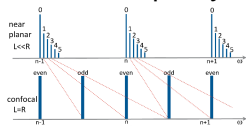
Confocal cavity $L = R$:



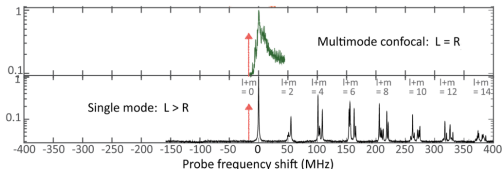
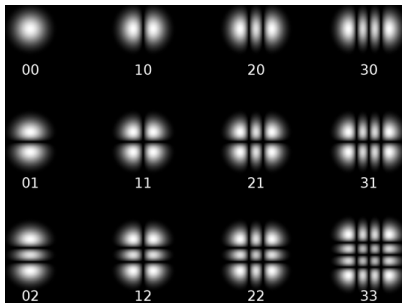
- Modes

$$\Xi_{l,m}(\mathbf{r}) = H_l(x)H_m(y),$$

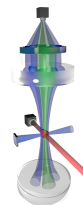
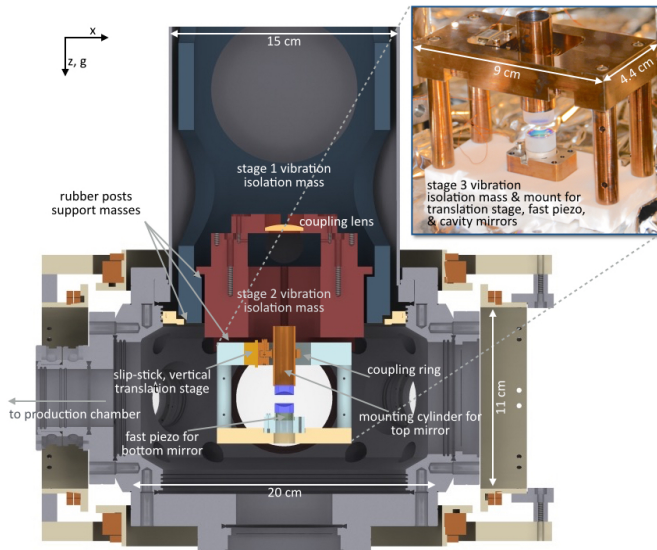
$l + m$ fixed parity



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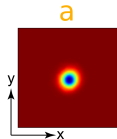
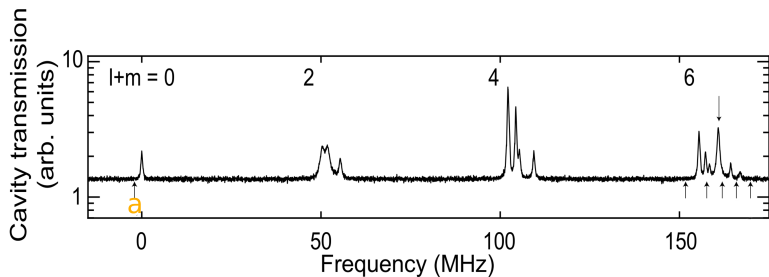


Adjustable length multimode cavity

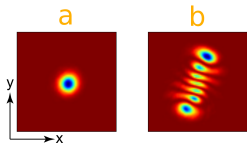
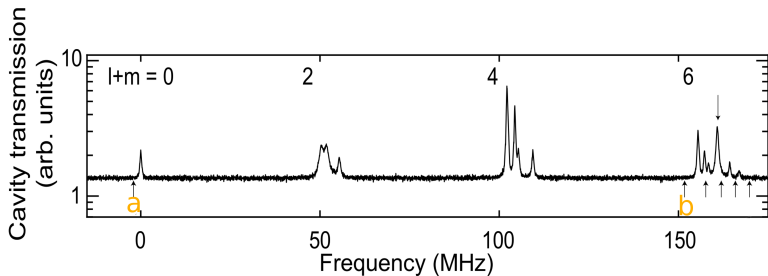


[Kollár, Papegeorge, Baumann, Armen & Lev, NJP '15]

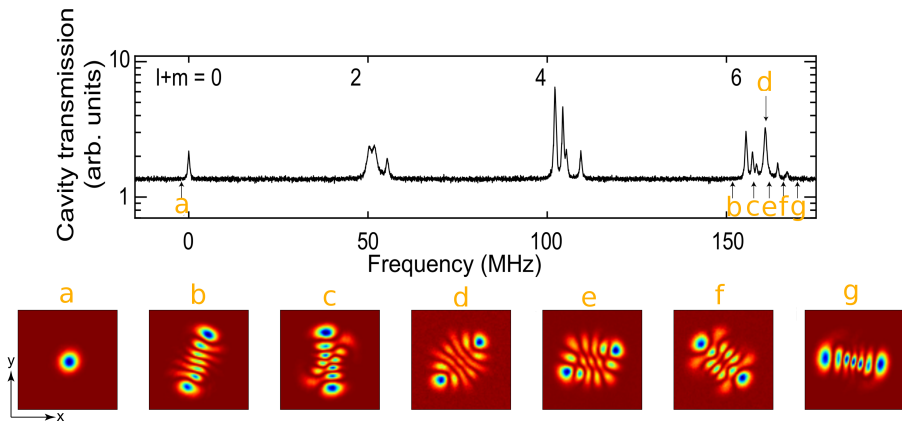
Superradiance in multimode cavity: Even family



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Supermodes vs polariton condensation

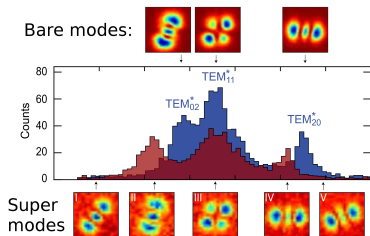
Supermode density-wave polariton:

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 - Atoms remix cavity modes \rightarrow superposition
 - Condensation of polaritons remixes again

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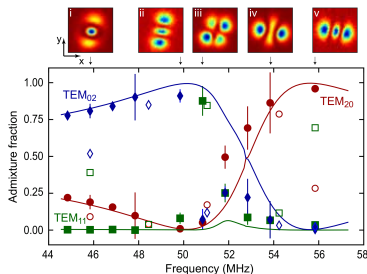
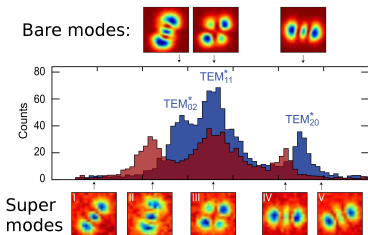
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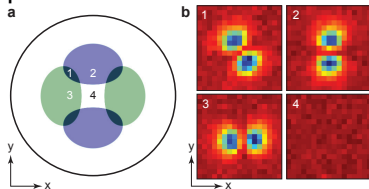
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Superradiance in multimode cavity: Odd family

- Dependence on cloud position

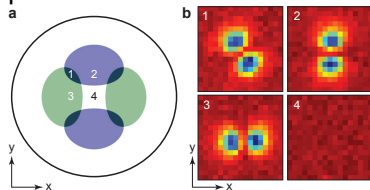


- Atomic time-of-flight — structure factor

- Near-degeneracy of $(1, 0)$, $(0, 1)$ modes broken by matter-light coupling.

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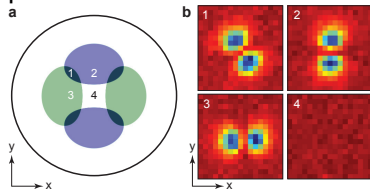


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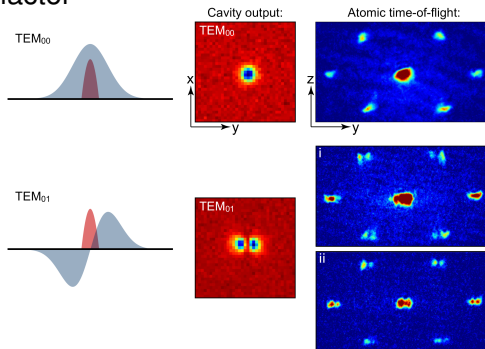
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 - Spin glass, Hopfield memory
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Disordered atoms

- Multimode cavity, Hyperfine states,

$$H_{\text{eff}} = - \sum_{\mu} \Delta_{\mu} a_{\mu}^{\dagger} a_{\mu} + \sum_n \frac{\omega_0}{2} \sigma_n^z + \frac{\Omega g_0}{\Delta} \sum_{\mu} \Xi_{\mu}(\mathbf{r}_n) \sigma_n^x (a_{\mu} + a_{\mu}^{\dagger})$$

- Random atom positions – quenched disorder

- Effective XY/Ising spin glass

$$H_{\text{eff}} = \sum_{n,m} J_{n,m} \begin{cases} \sigma_n^x \sigma_m^x & \text{Ising} \\ \sigma_n^{\pm} \sigma_m^{\mp} & \text{XY} \end{cases}, \quad J_{nm} = \sum_{\mu} \frac{\Omega^2 g_0^2 \Xi_{\mu}(\mathbf{r}_n) \Xi_{\mu}(\mathbf{r}_m)}{\Delta^2 \Delta_{\mu}}$$

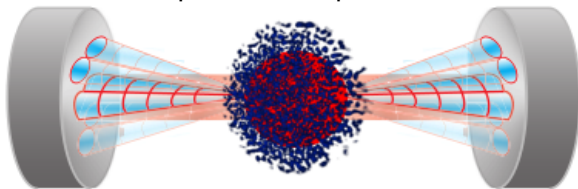
[Gopalakrishnan, Lev and Goldbart. PRL '11, Phil. Mag. '12]

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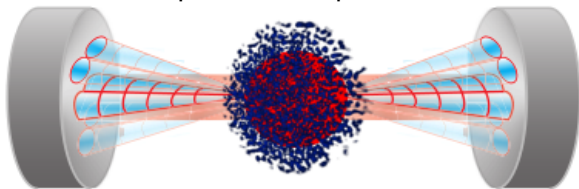
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- Tunable complexity
- Explore RSB/Droplet order
- Open system spin-glass.
[Strack & Sachdev PRL '11]

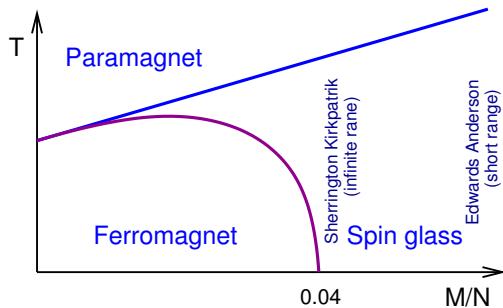
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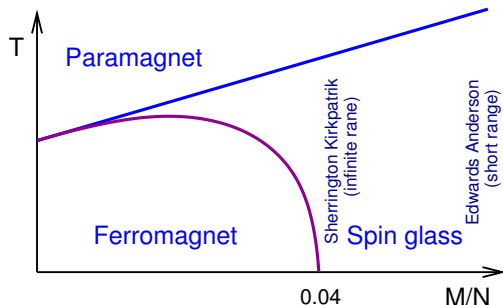


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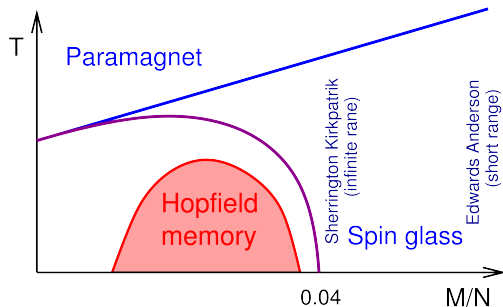


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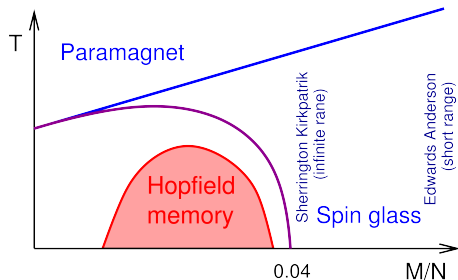


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Hopfield memory

- Between Mean-Field and Spin-Glass

- Multiple fixed points
- Recover corrupted image



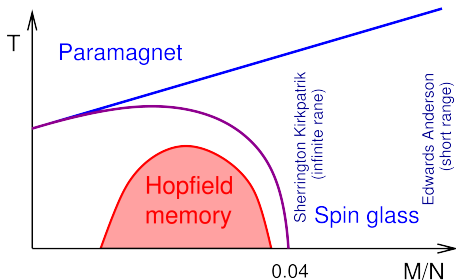
Low dimensional cartoon:

- Neurons \rightarrow Spins
- Synapses \rightarrow Modes
- Plasticity \rightarrow Atom movement
- Need $|s_i| = 1$ (hard spins)

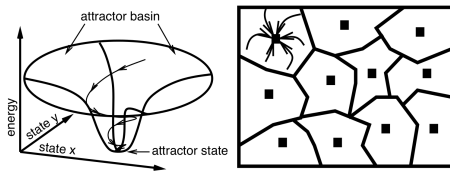
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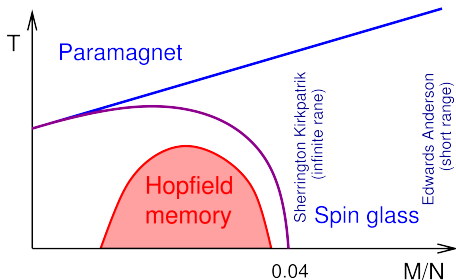
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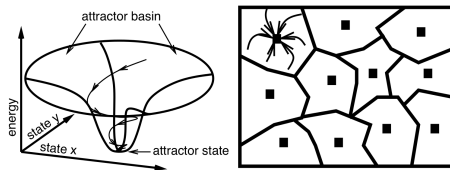
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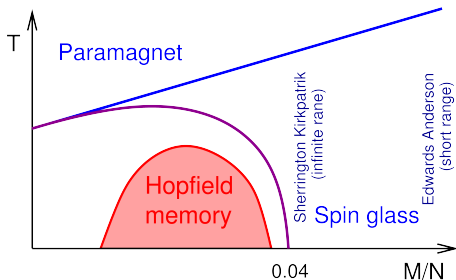
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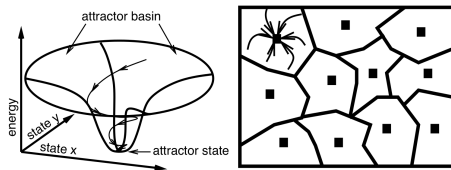
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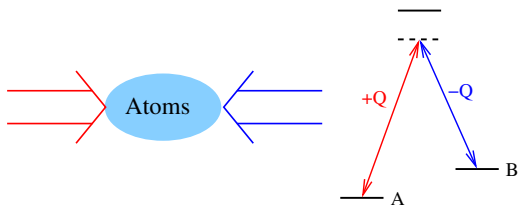
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Cavity QED and synthetic gauge fields

- [Spielman, PRA '09] scheme, hyperfine states A, B

$$H = (\psi_A \quad \psi_B) \begin{pmatrix} E_a + (\nabla - Q\hat{x})^2 & \Omega/2 \\ \Omega/2 & E_B + (\nabla + Q\hat{x})^2 \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$$



• Feedback

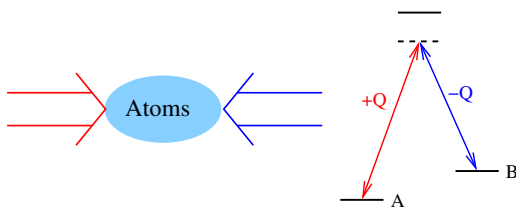
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Ground state:

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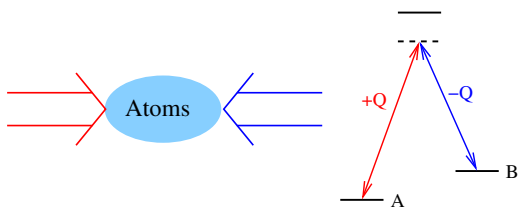
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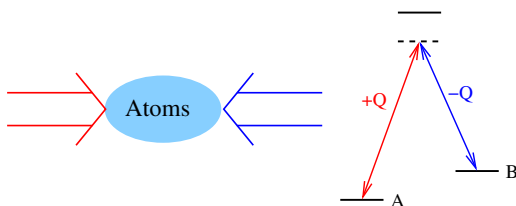
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Meissner-like physics: idea

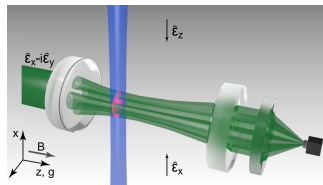
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• $E_A, E_B \propto |\psi|^2$ from cavity Stark shift

• Ground state $E_-(\mathbf{k}) \propto (\mathbf{k} - Q\hat{x}|\psi|^2)^2$

- Multimode cQED \rightarrow local matter-light coupling
- Variable profile synthetic gauge field?
- Reciprocity: matter affects field



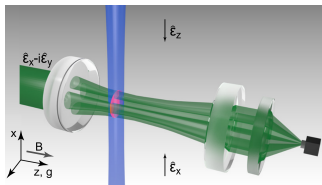
[Ballantine *et al.* arXiv:1608.07246]

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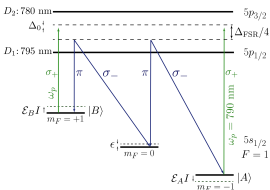
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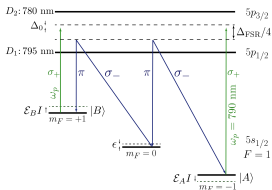
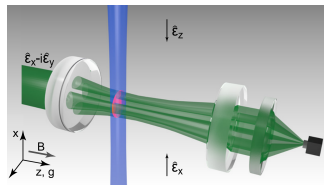
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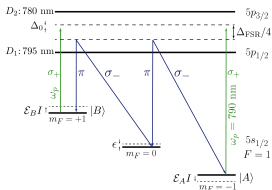
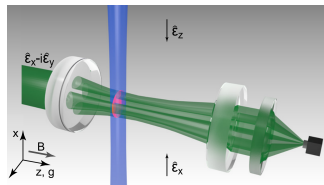
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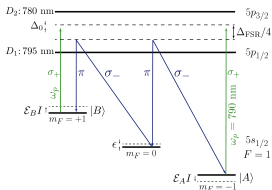
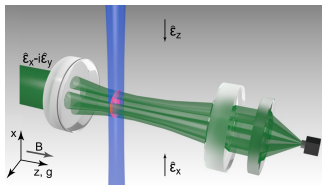
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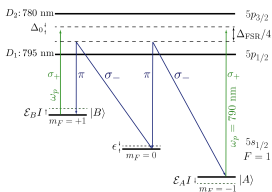
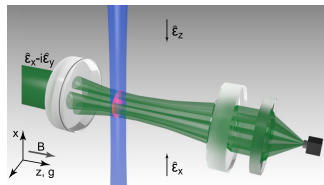
Meissner-like physics: idea

- Follow Spielman scheme

$$\begin{pmatrix} E_A + (\nabla - Q\hat{x})^2 & \Omega/2 \\ \Omega/2 & E_B + (\nabla + Q\hat{x})^2 \end{pmatrix}$$

- $E_A, E_B \propto |\varphi|^2$ from **cavity** Stark shift

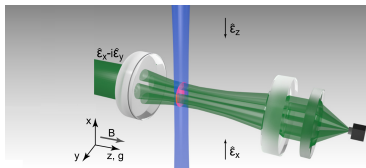
- Ground state $E_-(\mathbf{k}) \propto (\mathbf{k} - Q\hat{x}|\varphi|^2)^2$



- ▶ Multimode cQED \rightarrow local matter-light coupling
- ▶ Variable profile synthetic gauge field?
- ▶ Reciprocity: matter affects field

[Ballantine *et al.* arXiv:1608.07246]

Meissner-like physics: setup



- Atoms:

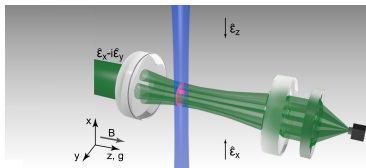
$$i\partial_t \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \left[-\frac{\nabla^2}{2m} + \begin{pmatrix} -\mathcal{E}_\Delta |\varphi|^2 + i\frac{g}{m}\partial_x & \Omega/2 \\ \Omega/2 & \mathcal{E}_\Delta |\varphi|^2 - i\frac{g}{m}\partial_x \end{pmatrix} + \dots \right] \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}.$$

- Light:

$$i\partial_t \varphi = \left[\frac{\delta}{2} \left(-P\nabla^2 + \frac{P^2}{P} \right) - \Delta_0 - i\kappa - N\mathcal{E}_\Delta (|\psi_A|^2 - |\psi_B|^2) \right] \varphi.$$

[Ballantine *et al.* arXiv:1608.07246]

Meissner-like physics: setup



- Atoms:

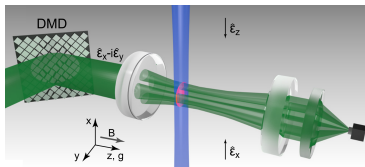
$$i\partial_t \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \left[-\frac{\nabla^2}{2m} + \begin{pmatrix} -\mathcal{E}_\Delta |\varphi|^2 + i\frac{q}{m}\partial_x & \Omega/2 \\ \Omega/2 & \mathcal{E}_\Delta |\varphi|^2 - i\frac{q}{m}\partial_x \end{pmatrix} + \dots \right] \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}.$$

- Light:

$$i\partial_t \varphi = \left[\frac{\delta}{2} \left(-\rho^2 \nabla^2 + \frac{r^2}{\rho^2} \right) - \Delta_0 - i\kappa - N\mathcal{E}_\Delta (|\psi_A|^2 - |\psi_B|^2) \right] \varphi.$$

[Ballantine *et al.* arXiv:1608.07246]

Meissner-like physics: setup



- Atoms:

$$i\partial_t \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \left[-\frac{\nabla^2}{2m} + \begin{pmatrix} -\mathcal{E}_\Delta |\varphi|^2 + i\frac{q}{m}\partial_x & \Omega/2 \\ \Omega/2 & \mathcal{E}_\Delta |\varphi|^2 - i\frac{q}{m}\partial_x \end{pmatrix} + \dots \right] \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}.$$

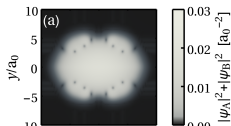
- Light:

$$i\partial_t \varphi = \left[\frac{\delta}{2} \left(-\rho^2 \nabla^2 + \frac{r^2}{\rho^2} \right) - \Delta_0 - i\kappa - N\mathcal{E}_\Delta (|\psi_A|^2 - |\psi_B|^2) \right] \varphi + f(\mathbf{r}).$$

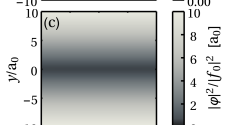
[Ballantine *et al.* arXiv:1608.07246]

Meissner-like physics: numerical simulations

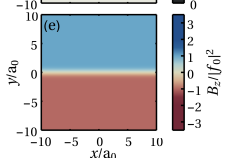
Atoms



Cavity light



Synthetic field



- Consider $f(\mathbf{r})$ such that $|\varphi|^2 \propto y$.
- Without feedback ($\mathcal{E}_\Delta = 0$) for field
- With feedback

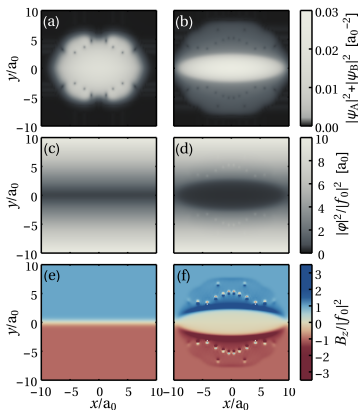
[Ballantine *et al.* arXiv:1608.07246]

Meissner-like physics: numerical simulations

Atoms

Cavity light

Synthetic field



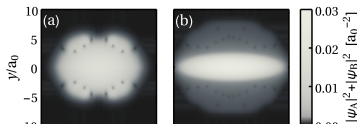
- Consider $f(\mathbf{r})$ such that $|\varphi|^2 \propto y$.
- Without feedback ($\mathcal{E}_\Delta = 0$) for field
- With feedback

Field expelled
Cloud shrinks

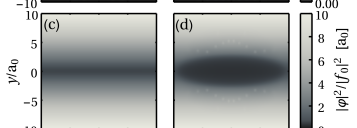
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Meissner-like physics: numerical simulations

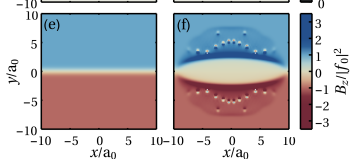
Atoms



Cavity light



Synthetic field



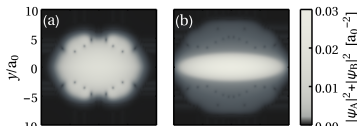
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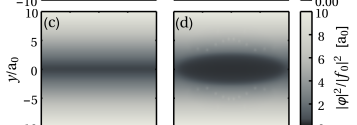
[Ballantine *et al.* arXiv:1608.07246]

Meissner-like physics: numerical simulations

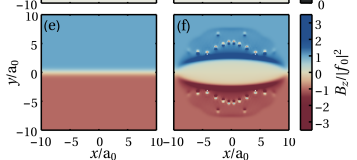
Atoms



Cavity light



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[Ballantine *et al.* arXiv:1608.07246]

Acknowledgments

Experiment (Stanford):
Benjamin Lev



Theory:



Ben Simons (Cambridge), Joe Bhaseen (KCL), James Mayoh (Southampton)



Sarang Gopalakrishnan (CUNY)
Surya Ganguli, Jordan Cotler (Stanford)
Peter Kirton, Kyle Ballantine, Laura Staffini
(St Andrews)



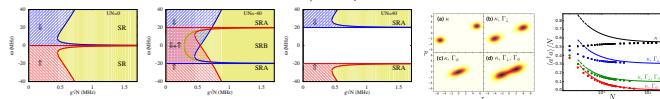
The Leverhulme Trust

EPSRC

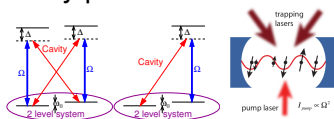
Engineering and Physical Sciences
Research Council

Summary

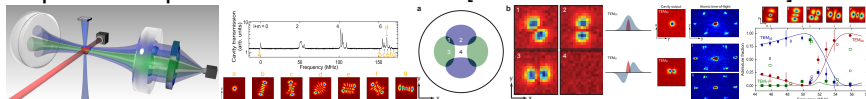
- Open Dicke model, κ , Γ_ϕ , Γ_\downarrow [Kirton & JK, arXiv:1611.03342]



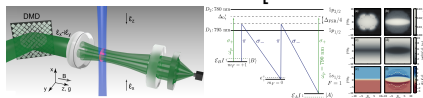
- Many possibilities of multimode cavity QED



- Supermode polariton condensation [Kollár *et al.* arXiv:1606.04127]



- Meissner like effect [Ballantine *et al.* arXiv:1608.07246]



- 1 Introduction: Tunable multimode Cavity QED
- 2 Single mode cavity QED
 - Spin-non-conserving loss
- 3 Multimode cavity QED experiments
 - Experimental setup
 - Supermode density wave polariton condensation
- 4 Theoretical possibilities
 - Spin glass, Hopfield memory
 - Meissner-like effect

5 Training Hopfield

How to train your atoms

- Input/output by cavity modes, $Q_\mu = \langle \hat{a}_\mu \rangle$

$$H_{\text{eff}} = - \sum_{\mu} \Delta_{\mu} a_{\mu}^{\dagger} a_{\mu} + \sum_n \frac{\omega_0}{2} \sigma_n^z + E_P \sum_{\mu, n} \Xi_{\mu}(\mathbf{r}_n) \sigma_n^x (a_{\mu} + a_{\mu}^{\dagger}) + \sum_{\mu} f_{\mu} a_{\mu}^{\dagger} + \text{H.c.}$$

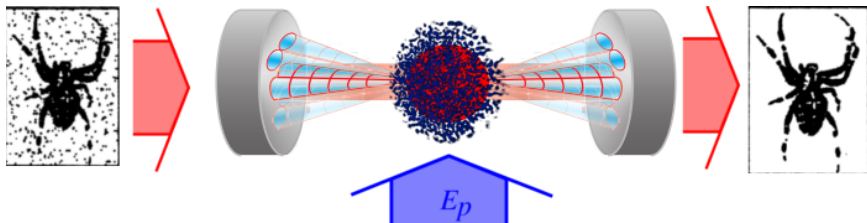
- Effective problem:

$$H_{\text{eff}} = -E_P \sum_{\mu} (f_{\mu} + Q_{\mu})^2, \quad Q_{\mu} = \sum_n \Xi_{\mu}(\mathbf{r}_n) \sigma_n^x$$

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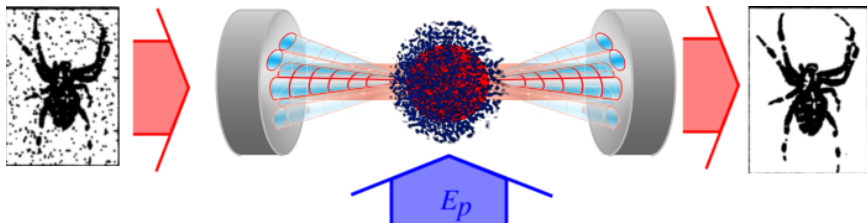
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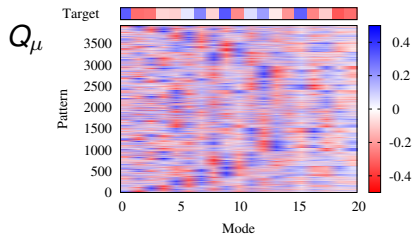
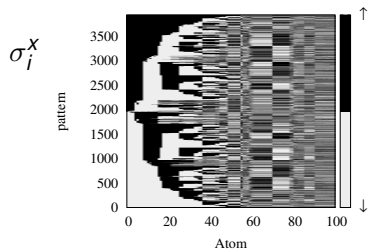


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How to train your atoms

- Before training, many fixed points

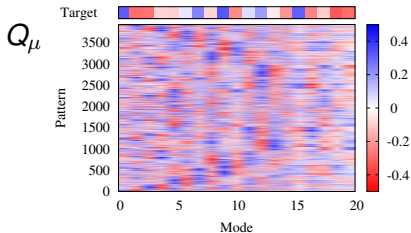
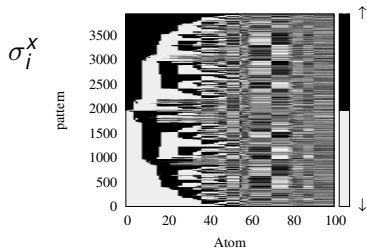


• Train by moving atoms:

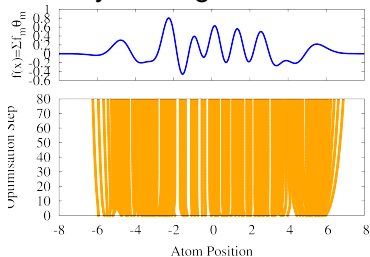
• Recover corrupted image

How to train your atoms

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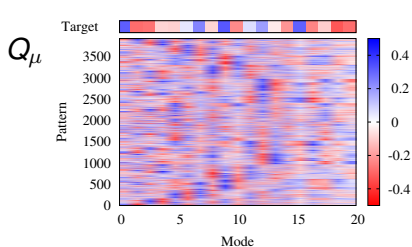
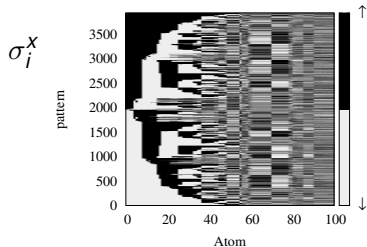
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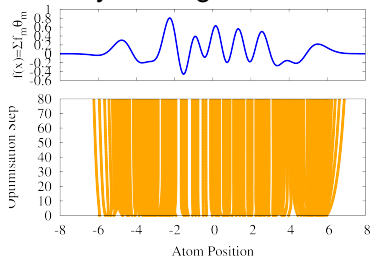
- Recover corrupted image

How to train your atoms

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- Recover corrupted image

