# Tunable ultra-strong coupling in multimode cavity QED systems

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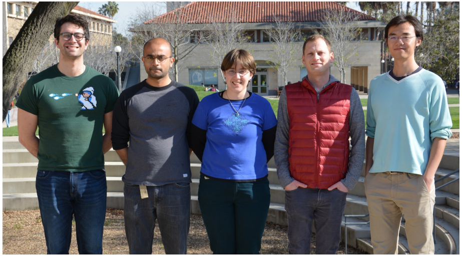
SUPA

Chichley Hall, March 2016

# Acknowledgments

Experiment (Stanford):
Benjamin Lev



Theory:

Ben Simons (Cambridge), Joe Bhaseen (KCL), James Mayoh (Southampton)

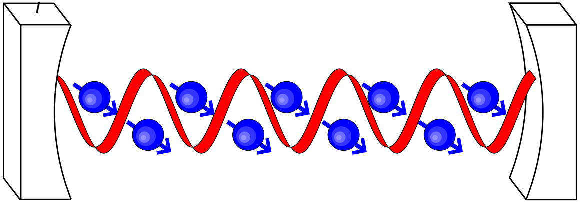
Sarang Gopalakrishnan (Caltech)
Surya Ganguli, Jordan Cotler (Stanford)
Laura Staffini, Kyle Ballantine (St Andrews)

1. Tunable Cavity QED with many atoms
2. Tunable multimode Cavity QED
   - With momentum states
   - With spin states
   - Other multimode setups
3. Tunable Cavity QED: Experimental progress

# (Multimode) cavity QED

$$H = \sum_k \omega_k a_k^\dagger a_k + \sum_n \omega_0 \sigma_n^+ \sigma_n^- + \sum_{n,k} g_{k,n}(a_k^\dagger + a_{-k})(\sigma_n^+ + \sigma_n^-)$$

$$\dot{\rho} = -i[H, \rho] + \kappa \sum_k \mathcal{L}[a_k, \rho] + \gamma \sum_i \mathcal{L}[\sigma_n^-, \rho]$$



- Compare $g$ (or $g\sqrt{N}$) vs:
  - $\kappa, \gamma$
  - bandwidth
  - $\omega_k, \omega_0$
- Physics:
  - Purcell effect, superfluorescence
  - Rabi oscillations, Polaritons,
  - Phase transitions (superradiance, lasing)
  - ...
- Problems:
  - Oscillator-strength sum rules
  - Fabrication constraints
  - Tuning parameters?
    - Chemical potential (pumping)
    - Cavity size/concentration
    - Stark/Zeeman/strain shifts

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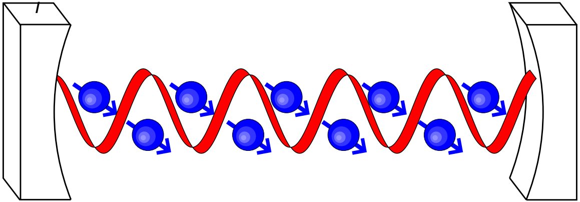

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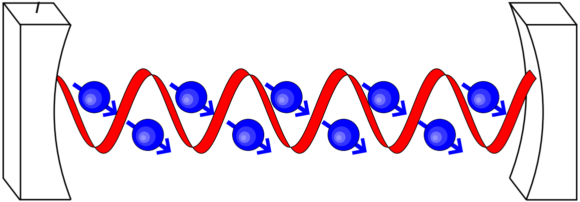

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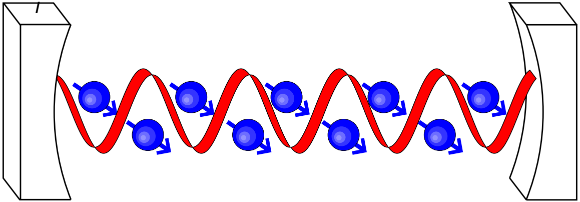

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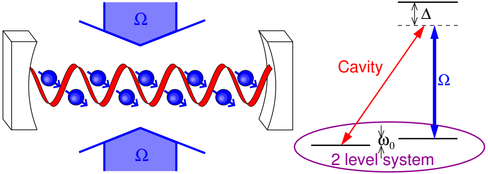
# Synthetic cavity QED: Raman driving

- Tunable coupling via Raman




$$H_{\text{eff}} = \ldots \frac{\Omega g}{\Delta} (\sigma_n^+ a + \text{H.c.})$$

- Real systems: loss $\partial_t \rho = -i[H, \rho] + \kappa \mathcal{L}[a, \rho] + \ldots$
- To balance loss, counter-rotating:

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[Dimer *et al.* PRA '07]

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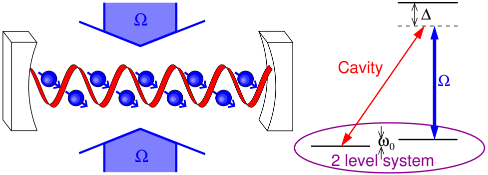


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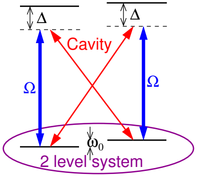


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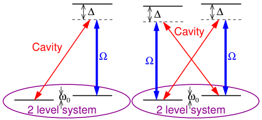
# Possibilities

- XY *vs* Ising

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- Single mode *vs* multimode
- Thermal gas *vs* BEC *vs* disorder localised

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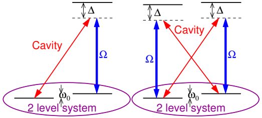


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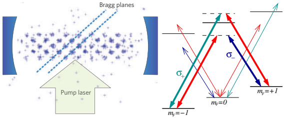


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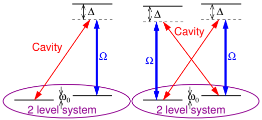


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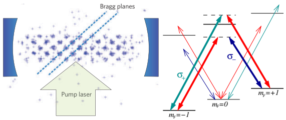


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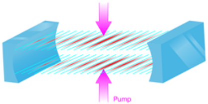


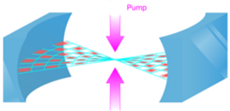


Credit: Alan Stonebreaker, Physics 3, 88 (2010)

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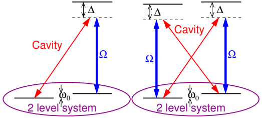


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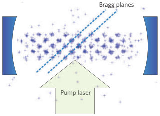


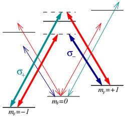


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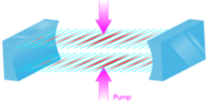


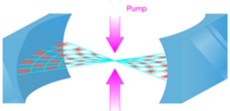


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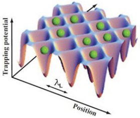


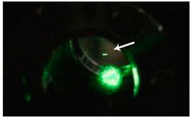

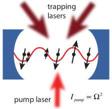

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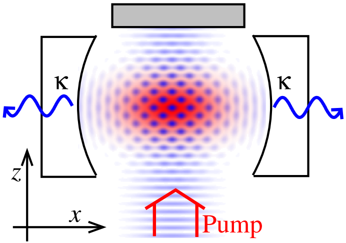


- Momentum degrees of freedom:
  $\psi(\mathbf{r}) = \psi_{\Downarrow}(\mathbf{r}) + \psi_{\Uparrow}(\mathbf{r}) \cos(kx) \cos(kz)$
- Effective 2LS $(\psi_{\Downarrow}, \psi_{\Uparrow})$

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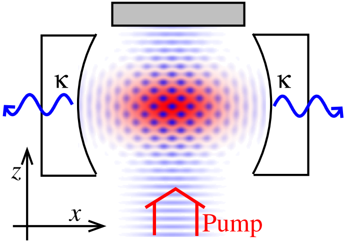


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- Extra "feedback" term
- Single mode – mean-field EOM, $\alpha = \langle\psi\rangle$, $S^i = \sum_n \sigma_n^i/2$.

$$\dot{S}^- = -i(\omega_0 + U|\alpha|^2)S^- + 2ig_{\text{eff}}(\alpha + \alpha^*)S^z$$

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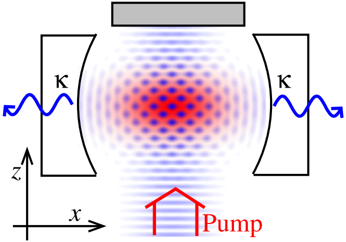


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# Classical dynamics

Changing $U$:

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$U > 0$

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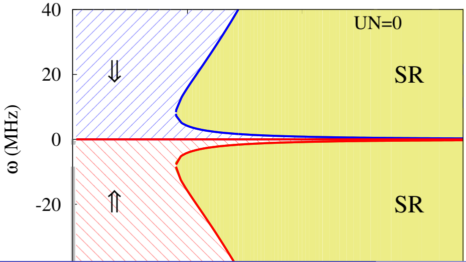

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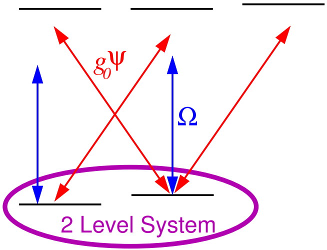
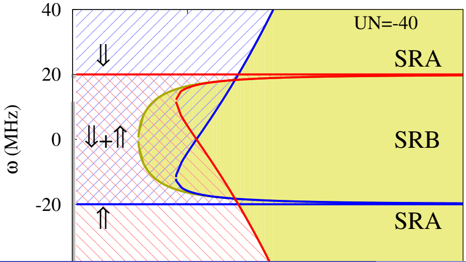



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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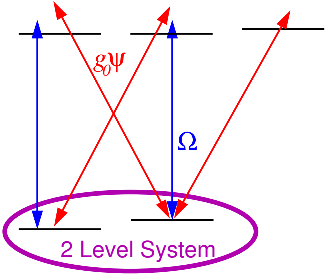


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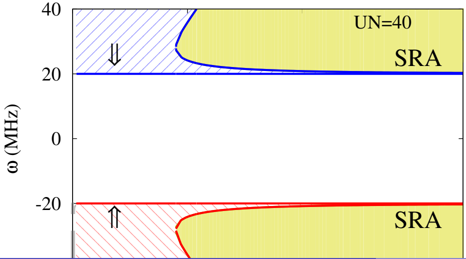

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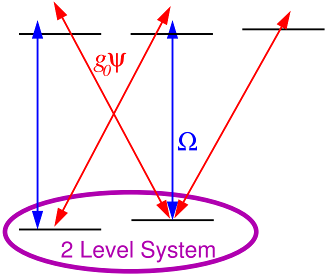


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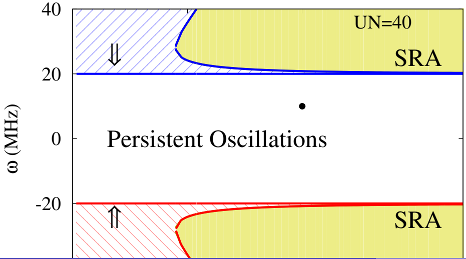
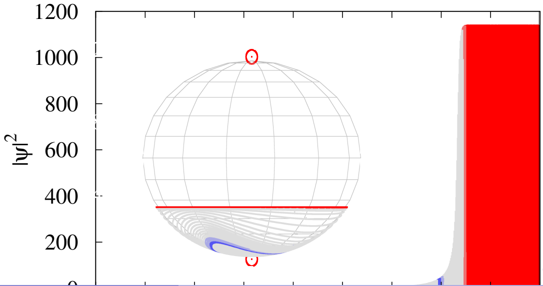
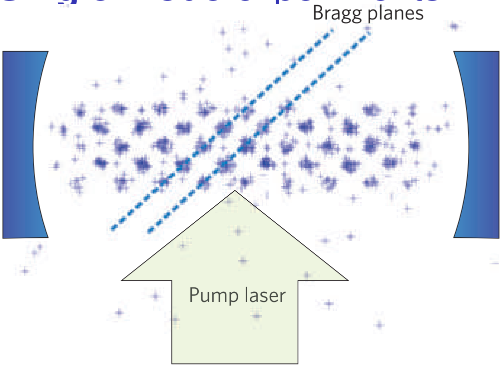


# Single mode experiments




Ritsch *et al.* PRL '02

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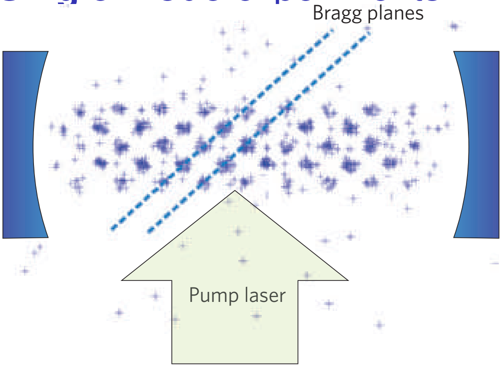


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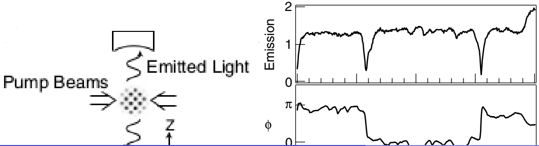
**Thermal atoms, momentum state**



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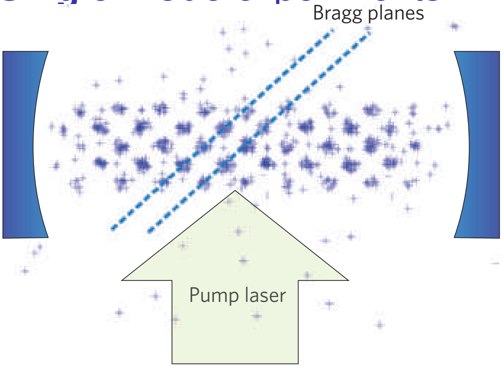


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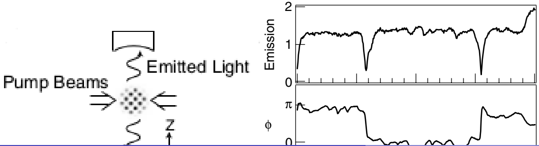


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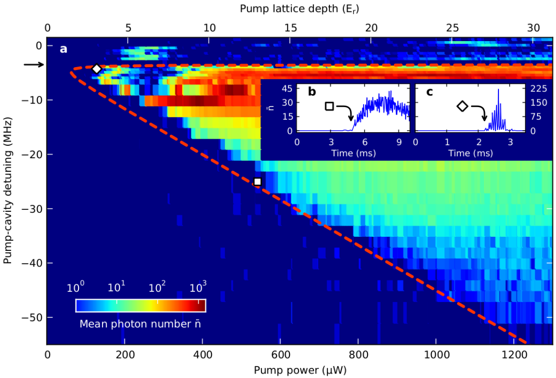


Baumann *et al.* Nature '10 (ETH)

Kinder *et al.* PRL '15 (Hamburg)

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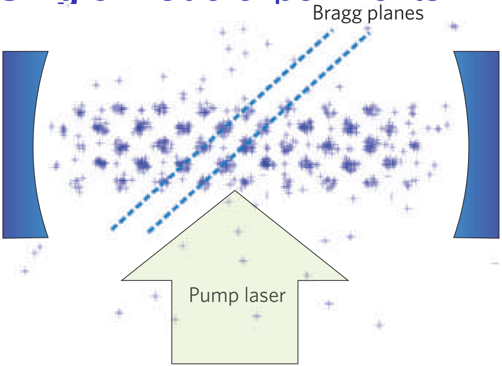


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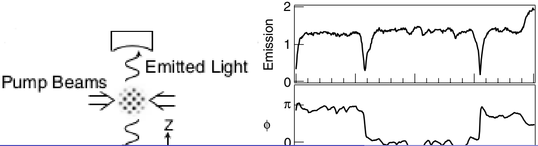


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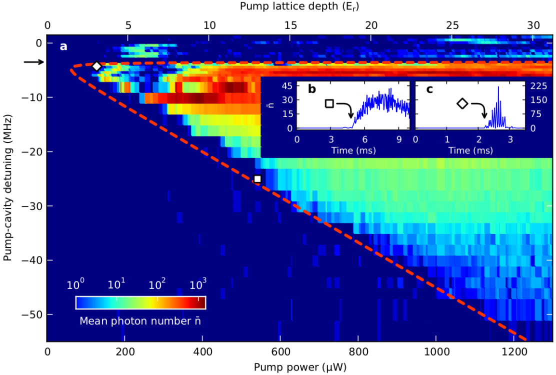


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**BEC, hyperfine states**

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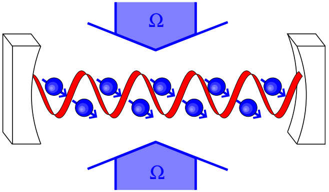
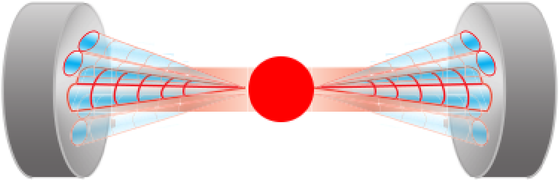
# Multimode cavity QED



Hyperfine states:

- Full model:

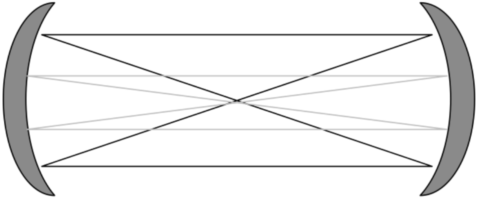
$$H_{\text{eff}} = \sum_{\mu} \underbrace{(\omega_\mu - \omega_P)}_{-\Delta_\mu} a^\dagger_\mu a_\mu + \sum_N \frac{\omega_0}{2}\sigma^z_n + \underbrace{\frac{\Omega g_0}{\Delta}}_{g_{\text{eff}}} \sum_\mu \Xi_\mu(\mathbf{r}_n)\sigma^x_n(a + a^\dagger)$$

[Gopalakrishnan, Lev, Goldbart. Nat. Phys '09, PRA '10]

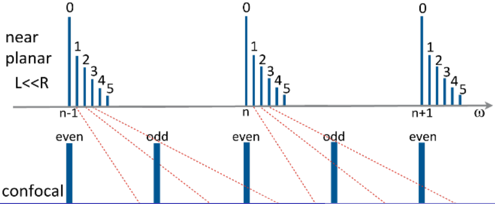
- Can reach $|\Delta_0| \ll \delta\Delta_\mu < g_{\text{eff}}$

# Multimode cavities

Confocal cavity:

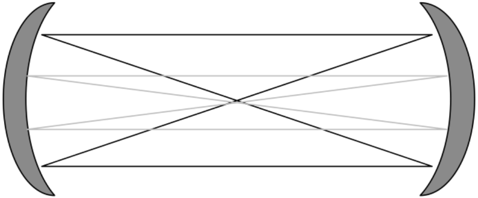


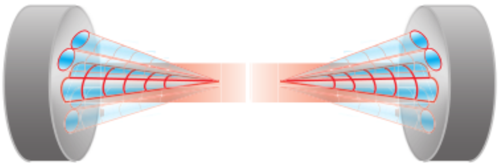
- Modes $\Xi_\mu(\mathbf{r}) = H_{\mu_x}(x)H_{\mu_y}(y)$, $\mu_x + \mu_y$ fixed parity

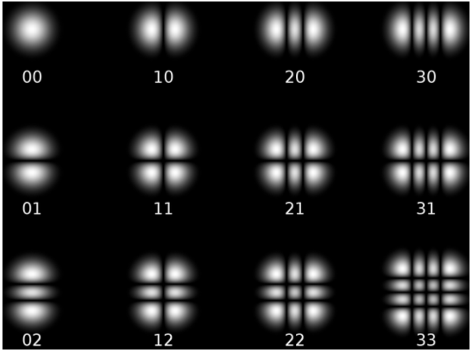


# Multimode cavities

Confocal cavity:





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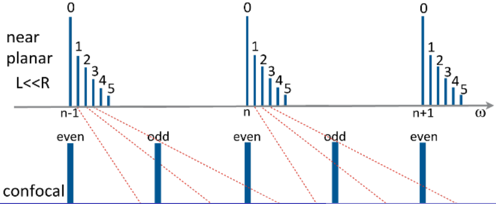

# Degenerate: Short range interactions

- Eliminate photons

$$H_{\text{eff}} = \sum_{n,m} J_{n,m} \begin{cases} \sigma_n^x \sigma_m^x & \textit{Ising} \\ \sigma_n^+ \sigma_m^- & XY \end{cases}, \qquad J_{nm} = \sum_{\mu} \frac{\Omega^2 g_0{}^2 \Xi_\mu(\mathbf{r}_n)\Xi_\mu(\mathbf{r}_m)}{\Delta^2 \Delta_\mu}$$

- If degenerate,

$$J_{nm} \propto \sum_{\mu}^{M} \Xi_\mu(\mathbf{r}_n)\Xi_\mu(\mathbf{r}_m)$$

- In general, complete set of modes, $J_{nm} \to \delta(\mathbf{r}_n - \mathbf{r}_m)$
- Gauss-Hermite: Christoffel-Darboux summation formula:

$$J_{nm} \sim \mathrm{sinc}\left(\sqrt{1+2M}|x_n - x_m|\right) \mathrm{sinc}\left(\sqrt{1+2M}|y_n - y_m|\right)$$

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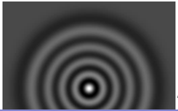
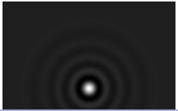
- Short range interactions

$M = 20$

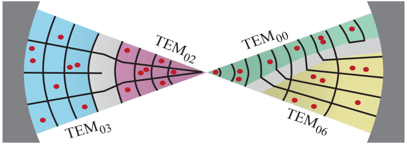


$M = 80$



# Degenerate multimode: Liquid crystal physics

- **Spatial states** of atoms $\psi(\mathbf{r}) = \psi_{\Downarrow}(\mathbf{r}) + \psi_{\Uparrow}(\mathbf{r})\cos(kx)\cos(kz)$
- Coupled dynamics of $\alpha(\mathbf{r}) = \sum_\mu \langle \hat{a}_\mu \rangle \Xi_\mu(\mathbf{r})$, and $\psi_{0,1}(\mathbf{r})$
  - Non-mean-field
  - Allow sharp structures – defects




- Degenerate limit, transverse pump:

$$i\partial_t \psi_{\mathbf{k}} = \left[\Delta + \lambda(|\mathbf{k}| - q)^2\right]\psi_{\mathbf{k}} + U_{\text{contact}} \sum_{\mathbf{k}',\mathbf{q}} \psi^*_{\mathbf{k}'+\mathbf{q}} \psi_{\mathbf{k}'} \psi_{\mathbf{k}-\mathbf{q}}$$

- Smectic Brazovskii transition

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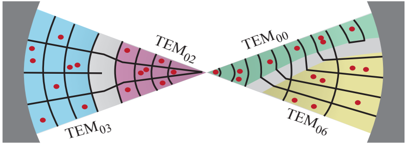


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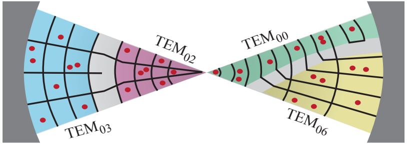


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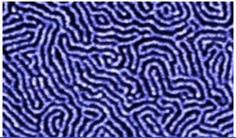

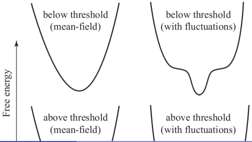


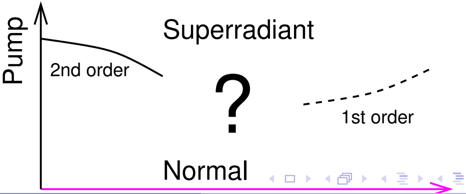

# Tunable multimode Cavity QED

1 Tunable Cavity QED with many atoms

2 Tunable multimode Cavity QED
- With momentum states
- With spin states
- Other multimode setups

3 Tunable Cavity QED: Experimental progress

# Disordered atoms

- Multimode cavity, Hyperfine states,

$$H_{\text{eff}} = -\sum_\mu \Delta_\mu a_\mu^\dagger a_\mu + \sum_n \frac{\omega_0}{2}\sigma_n^z + \frac{\Omega g_0}{\Delta}\sum_\mu \Xi_\mu(\mathbf{r}_n)\sigma_n^x(a_\mu + a_\mu^\dagger)$$

- Random atom positions – queched disorder

- Effective XY/Ising spin glass

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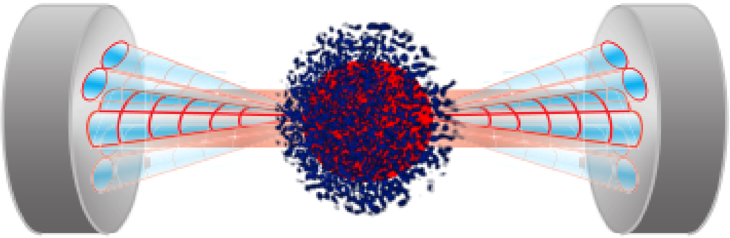

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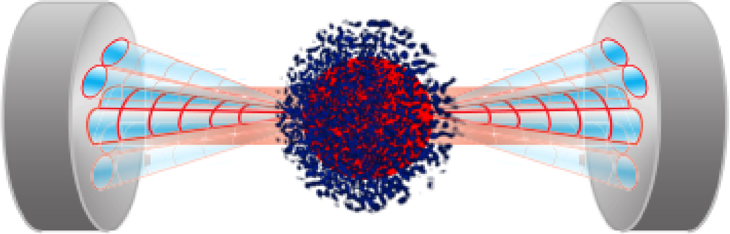

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- Tunable complexity
- Explore RSB/Droplet order
- Open system spin-glass.
  [Strack & Sachdev PRL '11]

[Gopalakrishnan, Lev and Goldbart. PRL '11, Phil. Mag. '12]

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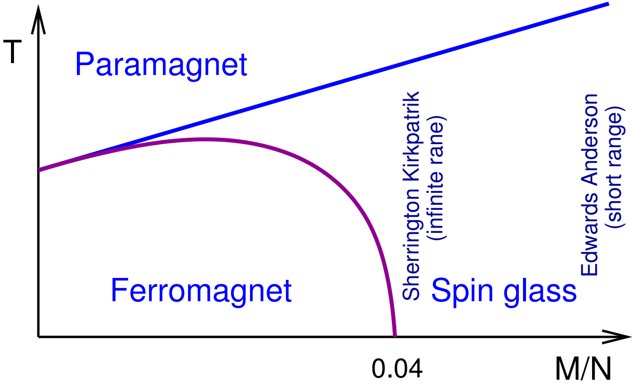


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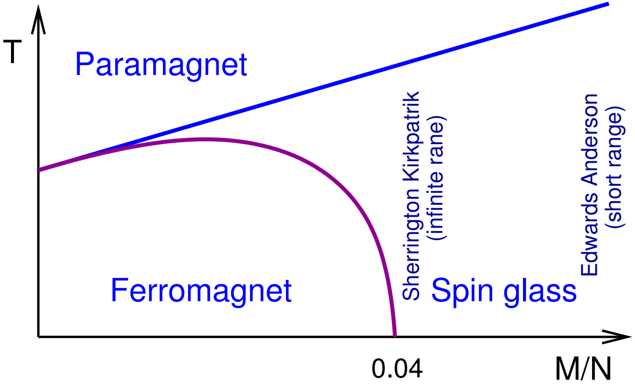


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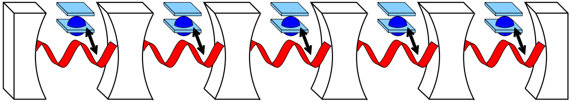
# Tunable multimode Cavity QED

# Coupled cavity arrays

- Control photon dispersion — lattice



[Hartmann *et al.* Nat. Phys. '06; Greentree *et al.* *ibid* 06; Angelakis *et al.* PRA '07]

- X-Hubbard Model, $\hat{H} = \sum_i \hat{H}_{X,site} - J \sum_{\langle ij \rangle} a_i^\dagger a_j$

  [X=Bose, Jaynes-Cummings, Rabi, ...]

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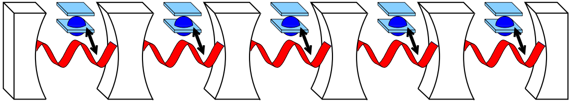

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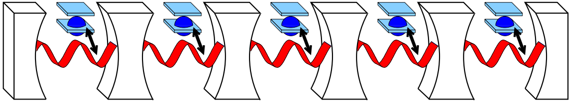

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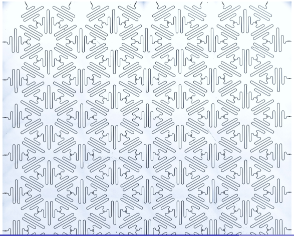
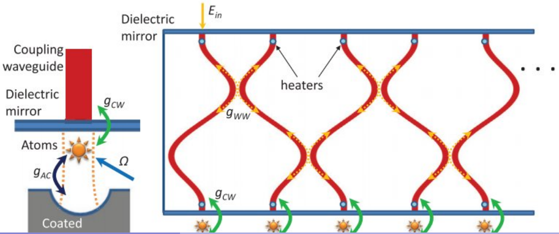
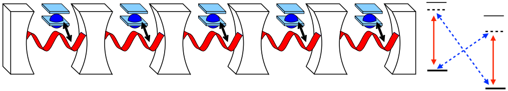




# CCA e.g. Raman pumping → Rabi-Hubbard model



$$H = \sum_i \omega \psi_i^\dagger \psi_i + \frac{\omega_0}{2}\sigma_i^z - J\psi_i^\dagger \psi_{i+1} + \left[\psi_i^\dagger (g\sigma_i^- + g'\sigma_i^+) + \text{H.c.}\right]$$

- Incommensurate ordering
- Level inversion — FM/AFM switch

[Schiró *et al.* arXiv:1503.04456]

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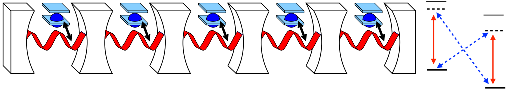

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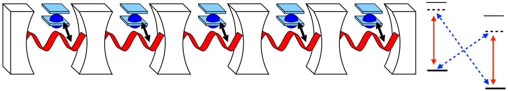

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# Tunable Cavity QED: Experimental progress

1. Tunable Cavity QED with many atoms

2. Tunable multimode Cavity QED
   - With momentum states
   - With spin states
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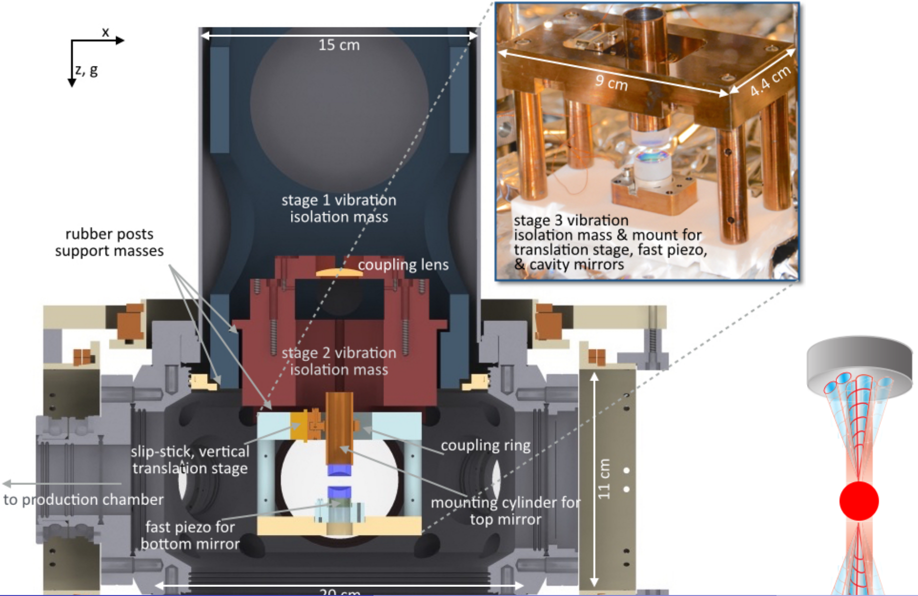
3. Tunable Cavity QED: Experimental progress

# Adjustable length multimode cavity



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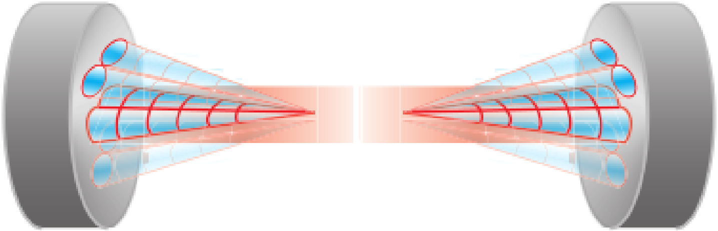

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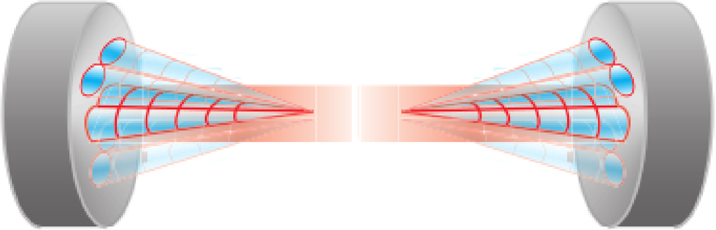

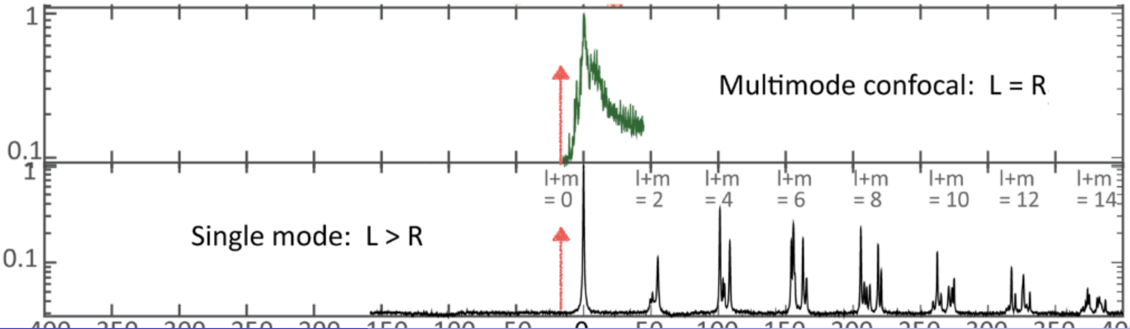

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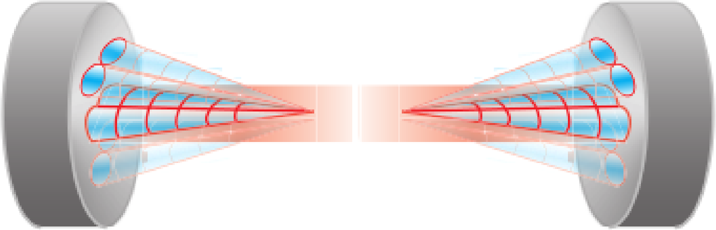
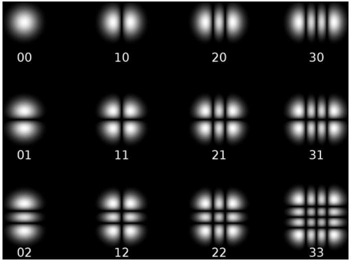
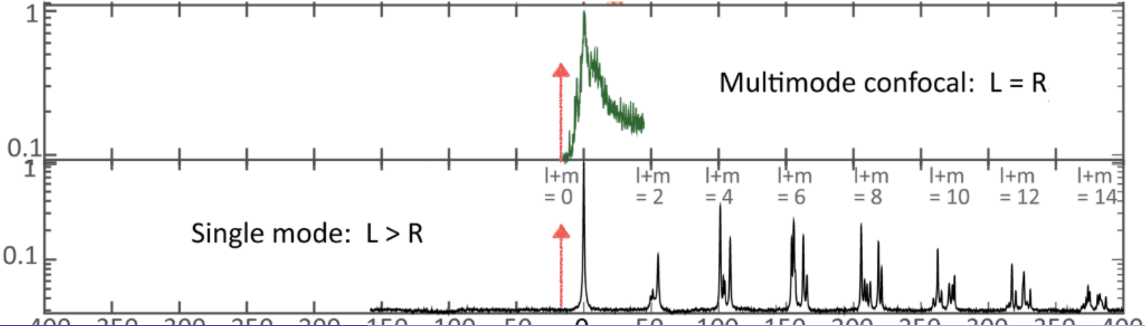
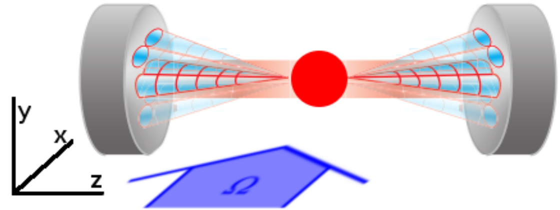


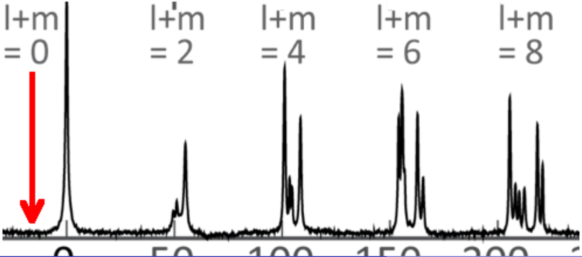


# Superradiance in multimode cavity




Pump (red of) 0,0 mode:



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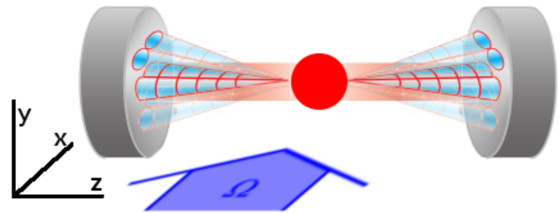


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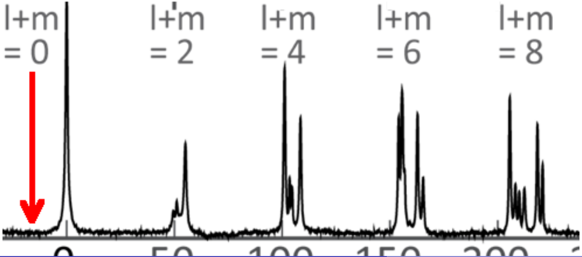
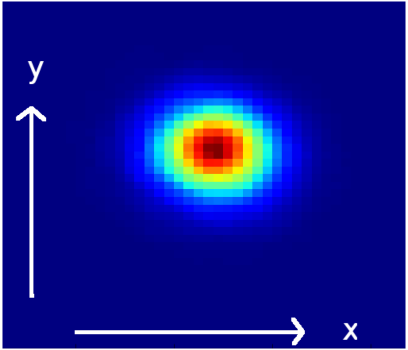



Cavity Light:



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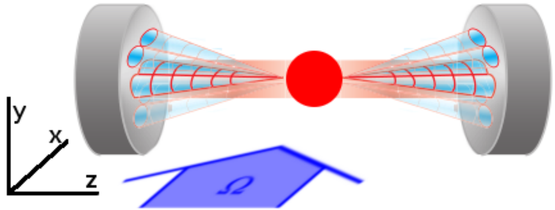


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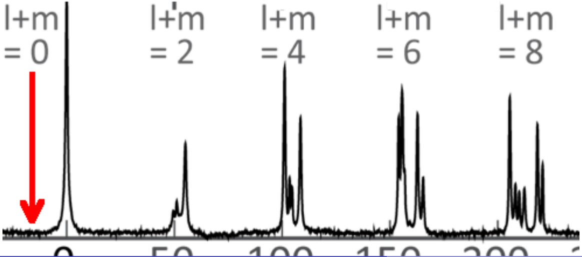
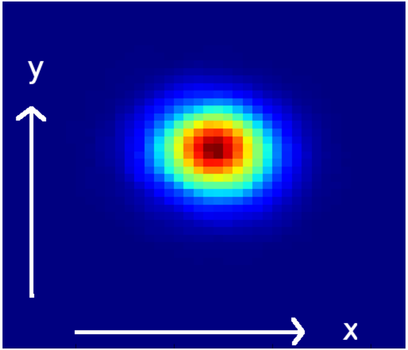
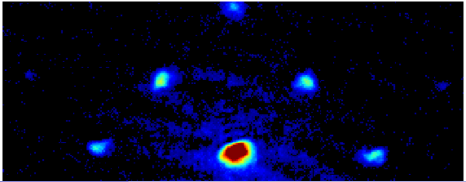



Cavity Light:




Atom TOF



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  - Hybrid cavity photon and atomic density wave
  - Composition varies with $\Delta$ (unlike static atoms)
- Even mode (20,11,02) family
- Odd parity modes, (10,01)

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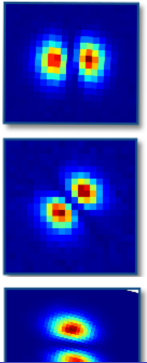
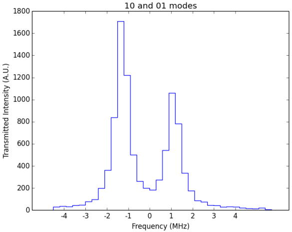
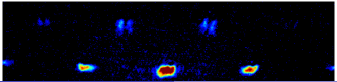





Atomic structure factor



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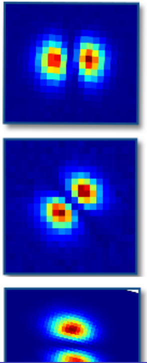

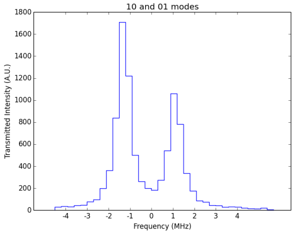


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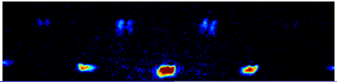

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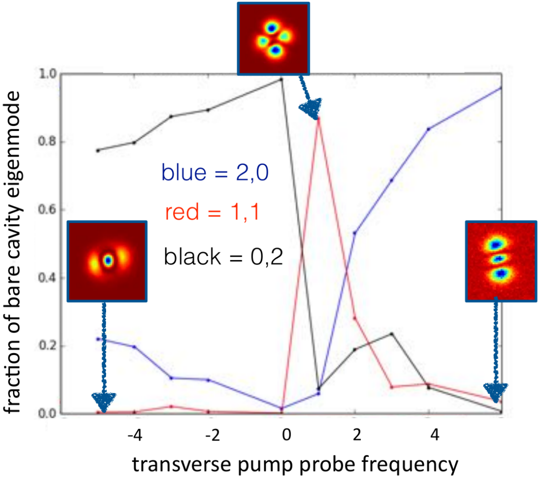

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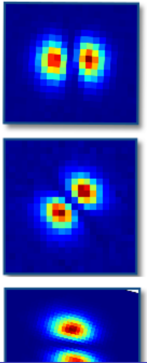

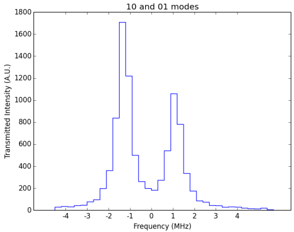


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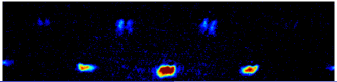

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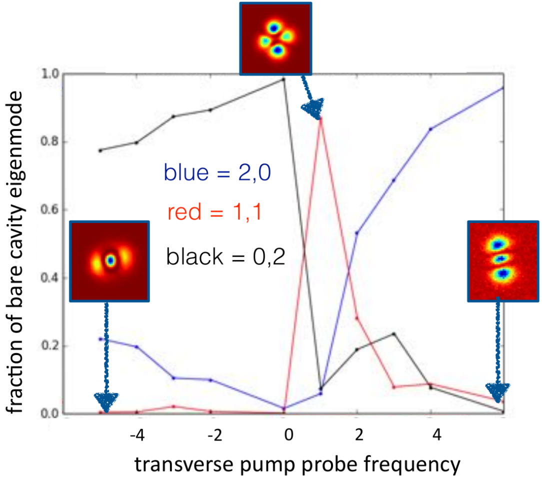

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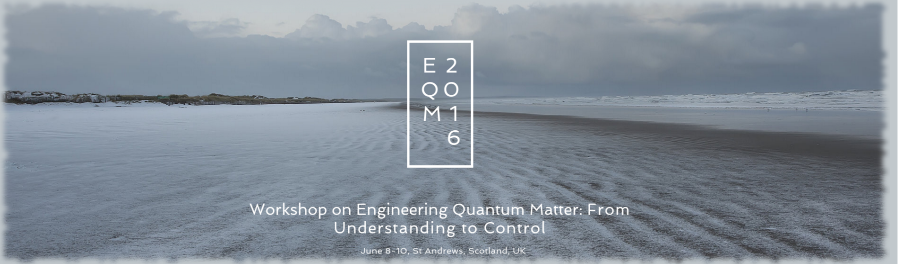
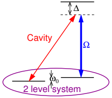
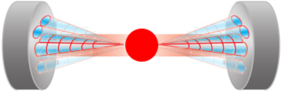
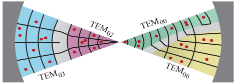


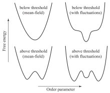

http://eqm2016.co.uk

# Summary

- Many possibilities of multimode cavity QED
- Spin glass (XY/Ising) and soft-matter physics with spatial DoF



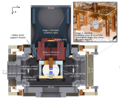
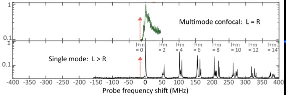


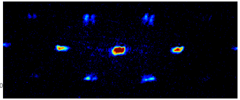

[Gopalakrishnan, Lev and Goldbart. PRL ’11, Phil. Mag. ’12,Gopalakrishnan, Lev, Goldbart. Nat. Phys ’09, PRA ’10]

- CCA — non-equilibrium lattice models

Schiro *et al.* arXiv:1503.04456

- Working multimode cavity



[Kollár, *et al.* NJP ’15; Kollár *et al.* in preparation]