From weak to ultra-strong matter-light coupling with organic materials

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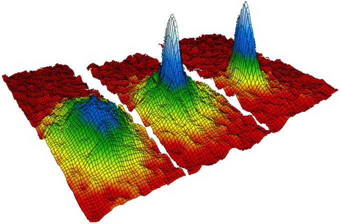
Birmingham, March 2016

Coherent states of matter and light

Atomic BEC $T \sim 10^{-7}$K



[Anderson *et al.* Science ’95]

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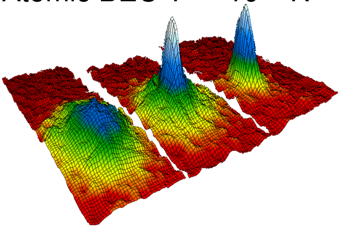
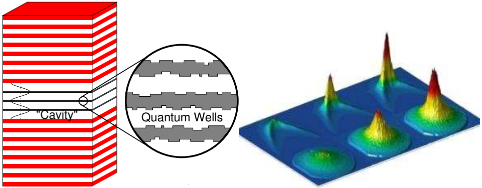


[Anderson *et al.* Science '95]

Polariton Condensate $T \sim 20$K



[Kasprzak *et al.* Nature, '06]

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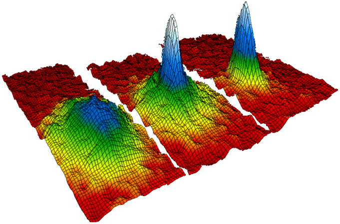

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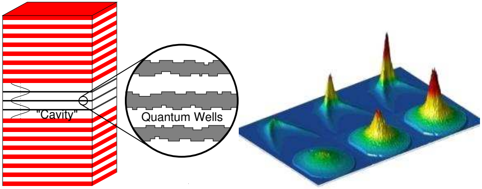

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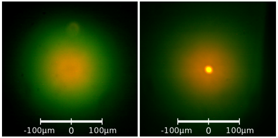
Photon Condensate
$T \sim 300$K



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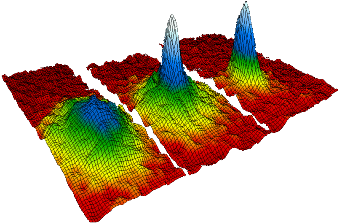

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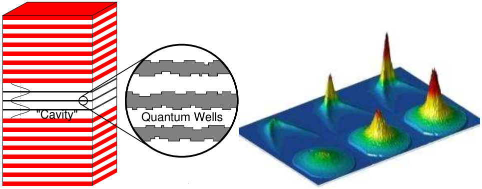

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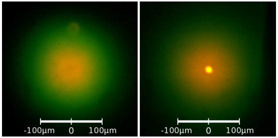

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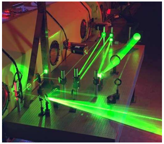
Laser
$T \sim ?, < 0, \infty$



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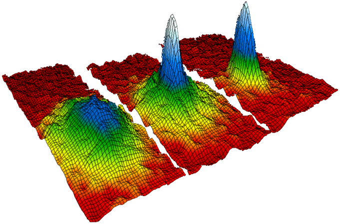

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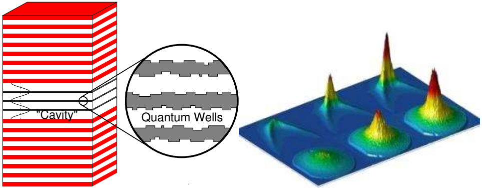

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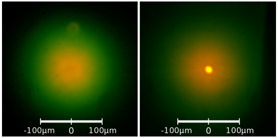

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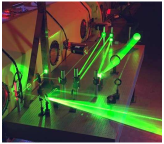
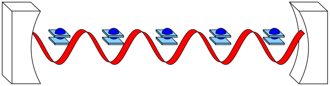


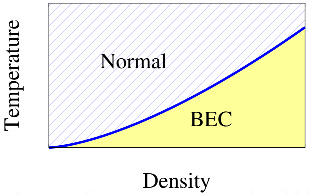
Superradiance transition
$T \sim 0$

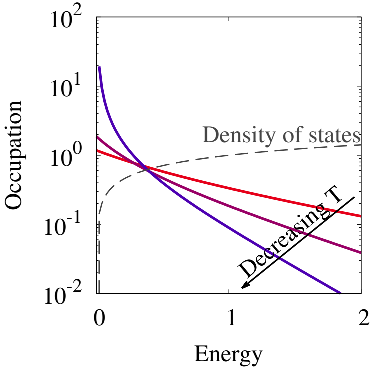


[Hepp & Lieb, Ann. Phys. '73]

“Textbook” BEC

- **Non-interacting** viewpoint
  - BE distribution: $\mu < \omega_0$
  - $T_c = \frac{2\pi\hbar^2}{m}\left(\frac{n}{\xi_d}\right)^{2/d}$





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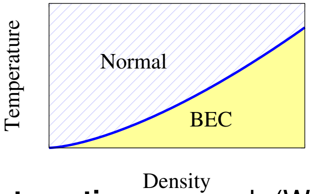

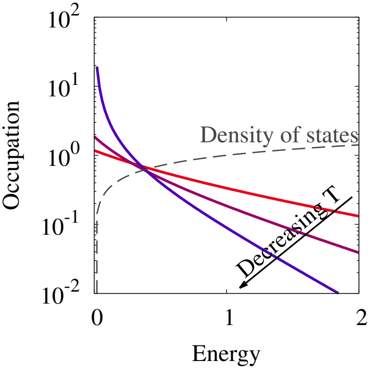

- **Interacting** approach (WIDBG)

$$H = \sum_k \omega_k \psi_k^\dagger \psi_k + \frac{g}{2V} \sum_{k,k',q} \psi_{k+q}^\dagger \psi_{k'-q}^\dagger \psi_{k+q} \psi_k$$

  - Mean field: $|\psi|^2 = (\mu - \omega_0)/g$
  - Fluctuations deplete condensate, vanishes at $T > T_c$

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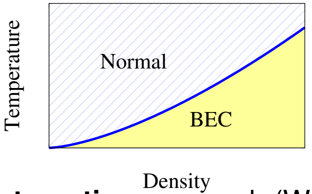

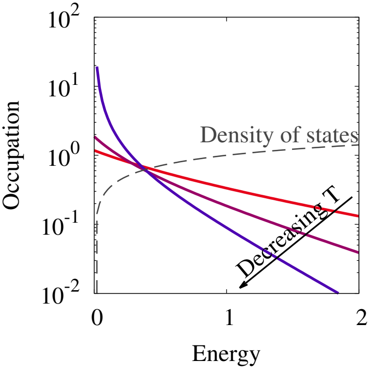

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"Textbook" Laser: Maxwell Bloch equations

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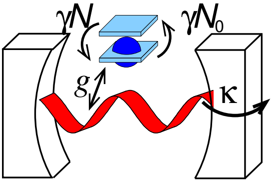

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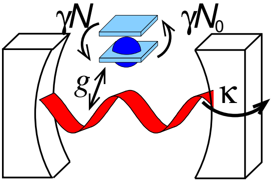

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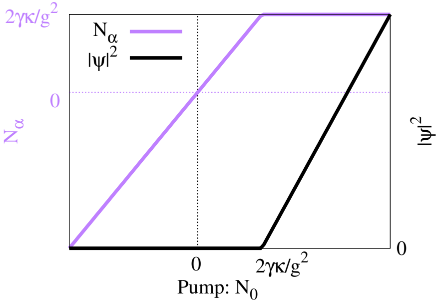
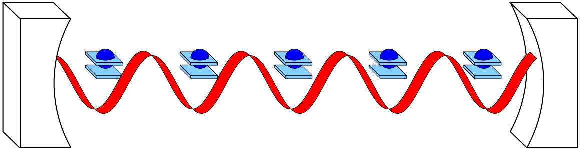


$|\psi|^2 > 0$ if $N_0 g^2 > 2\gamma\kappa$

“Textbook” Dicke-Hepp-Lieb superradiance



$$H = \omega \psi^\dagger \psi + \sum_\alpha \epsilon \sigma^z_\alpha + g \left(\psi^\dagger \sigma^-_\alpha + \psi \sigma^+_\alpha \right)$$

- Coherent state: $|\Psi\rangle \to e^{\lambda \psi^\dagger + \eta \sigma^+} |\Omega\rangle$
- Small g, min at $\lambda, \eta = 0$

[Hepp, Lieb, Ann. Phys. ’73]

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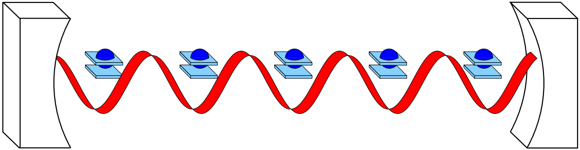

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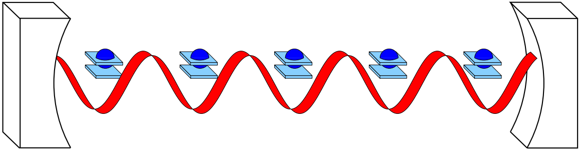

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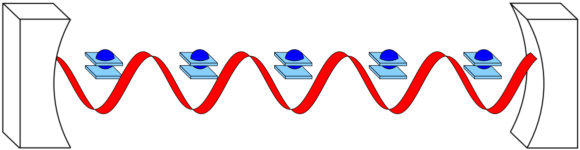

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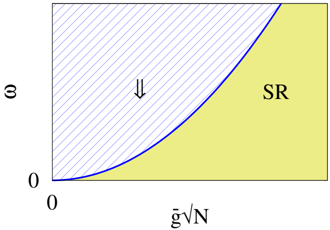

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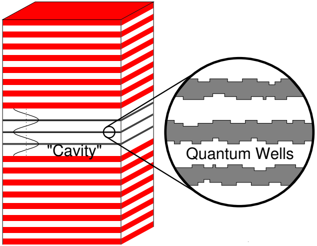
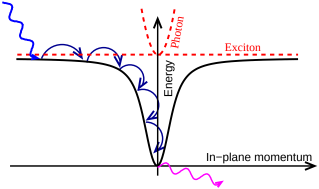
Coupled matter-light system: polaritons





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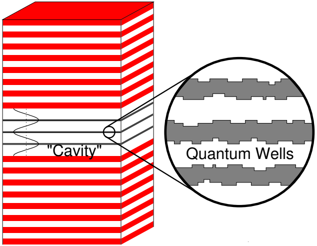
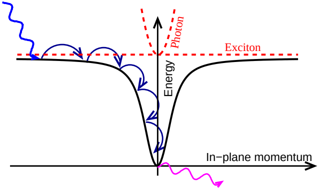
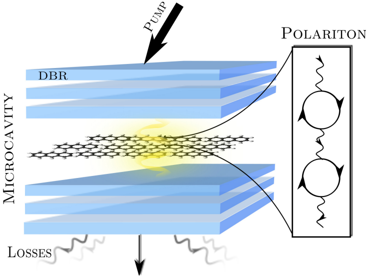
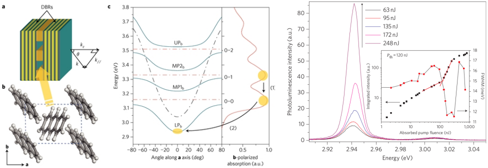






Motivation: polariton condensates

- Anthracene Polariton Lasing
  $T \sim 300$K



[Kena Cohen and Forrest, Nat. Photon '10]

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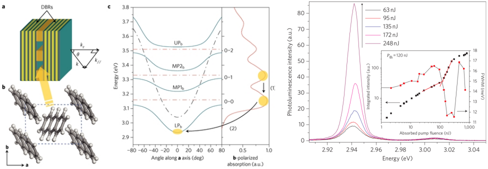

Q1. Vibrational replicas?

Q2. Relevance of disorder?

Q3. Lasing vs condensation?

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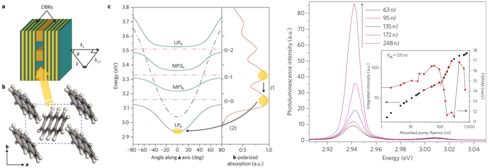

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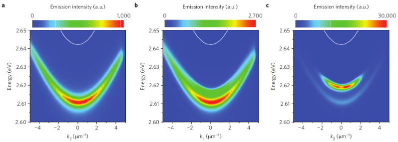

[Plumhoff *et al.* Nat. Materials '14, Daskalakis *et al.* ibid '14]

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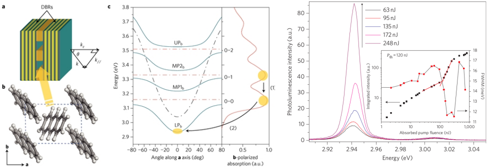

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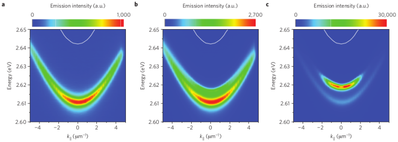


[Plumhoff *et al.* Nat. Materials '14, Daskalakis *et al.* ibid '14]

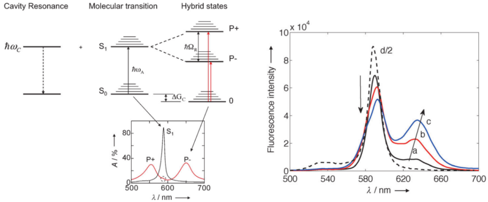
Q1. Frenkel to Wannier crossover?

Q2. Optimal vibrational properties?

Q3. Nonlinearities?

Motivation: vacuum-state strong coupling

- Linear response (no pump, no condensate): effects of matter-light coupling alone.



[Canaguier-Durand *et al.* Angew. Chem. ’13; Baumberg group]

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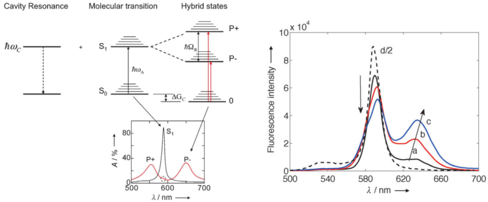


[Canaguier-Durand *et al.* Angew. Chem. '13; Baumberg group]

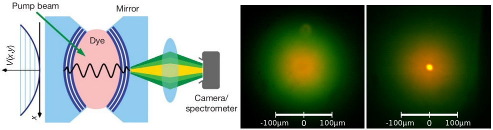
Q1. Can **ultra-strong** coupling to light change:

- charge distribution?
- vibrational configuration?
- molecular orientation?
- crystal structure?

Q2. Are changes collective ($\sqrt{N}$ factor) or not?

Motivation: photon condensates

- Photon Condensate $T \sim 300\text{K}$



[Klaers *et al.* Nature, '10, Marelic *et al.* '15]

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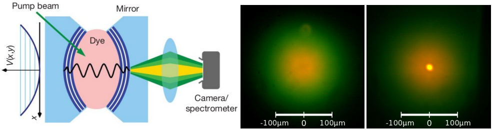


[Klaers *et al.* Nature, '10, Marelic *et al.* '15]

Q1. Relation to dye laser?

Q2. Relation to polaritons?

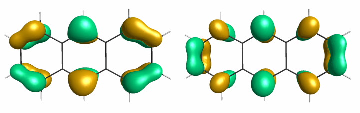
Q3. Thermalisation breakdown?

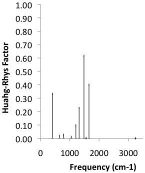
Overview

1. Introduction
   - Condensation, lasing and superradiance
   - Modelling photon BEC & organic polaritons
2. Weak coupling: Photon BEC
   - Homogeneous model & threshold
   - Spatial profile and dynamics
3. Strong coupling
   - Superradiance transition
   - Vibrational dressing in normal-state spectrum
4. Ultrastrong coupling: vibrational reconfiguration

Toy models

1. Full molecular spectra electronic structure & Raman spectrum





2. Simplified archetypal model: Dicke-Holstein

- *Each* molecule: two DoF
  - Electronic state: 2LS
  - Vibrational state: harmonic oscillator

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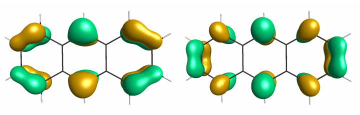

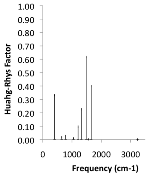

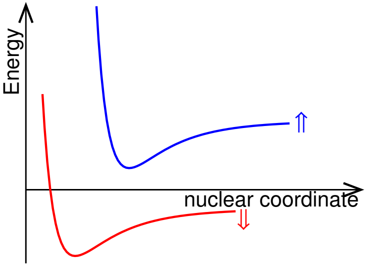

2. Focus on low-energy effective theory

- Two-level system, HOMO/LUMO
- Single DoF PES

See also [Galego, Garcia-Vidal, Feist. PRX '15]

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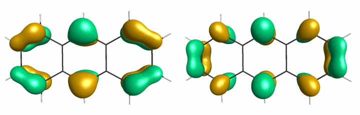
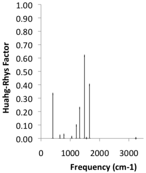
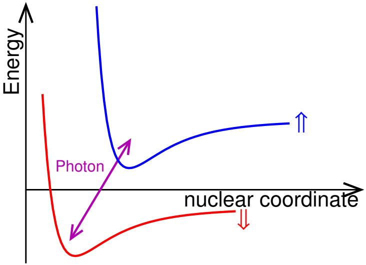




2. Focus on low-energy effective theory

- Two-level system, HOMO/LUMO
- Single DoF PES



See also [Galego, Garcia-Vidal, Feist. PRX '15]

3. Simplified archetypal model: Dicke-Holstein

- *Each* molecule: two DoF
  - Electronic state: 2LS
  - Vibrational state: harmonic oscillator

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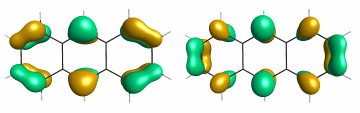

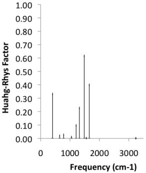

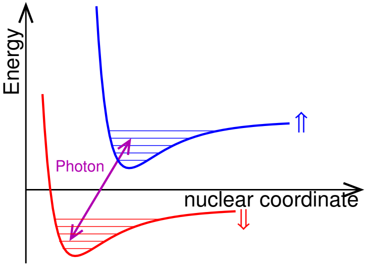

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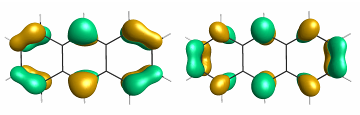

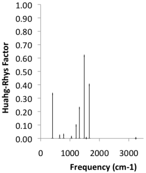

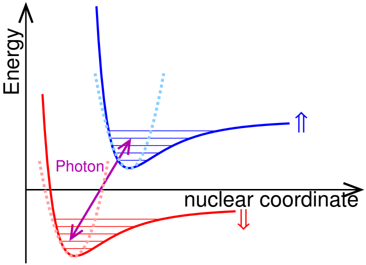

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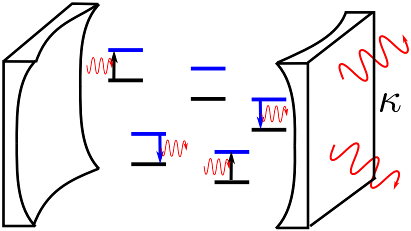
Dicke Holstein Model

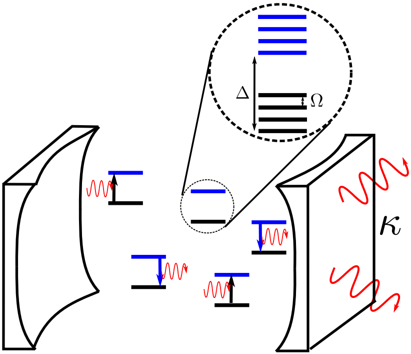
- Dicke model: 2LS $\leftrightarrow$ photons



$$H_{\text{sys}} = \omega\psi^\dagger\psi + \sum_\alpha \left[\frac{\epsilon}{2}\sigma^z_\alpha + g\left(\psi + \psi^\dagger\right)\left(\sigma^+_\alpha + \sigma^-_\alpha\right)\right]$$

Dicke Holstein Model

- Dicke model: 2LS $\leftrightarrow$ photons
- Molecular vibrational mode
  - Phonon frequency Ω
  - Huang-Rhys parameter S — coupling strength



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Weak coupling: Photon BEC

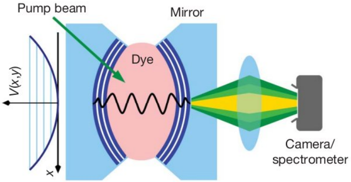
1 Introduction
- Condensation, lasing and superradiance
- Modelling photon BEC & organic polaritons

2 Weak coupling: Photon BEC
- Homogeneous model & threshold
- Spatial profile and dynamics

3 Strong coupling
- Superradiance transition
- Vibrational dressing in normal-state spectrum

4 Ultrastrong coupling: vibrational reconfiguration

Photon BEC experiments



- (Curved) microcavity
- R6G dye (in solvent)
- Thermalisation of light
- Condensation at $P > P_{th}$

[Klaers et al, Nature, 2010]

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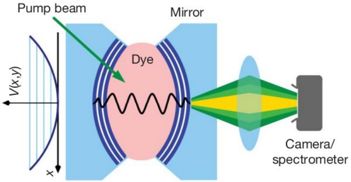

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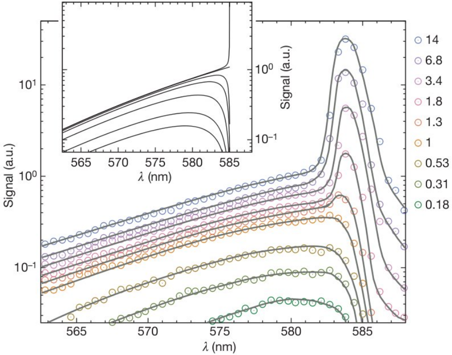

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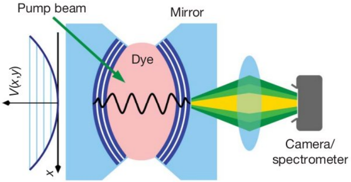

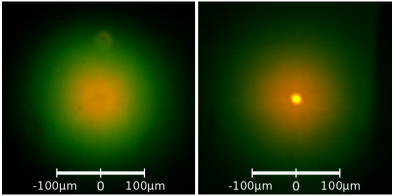

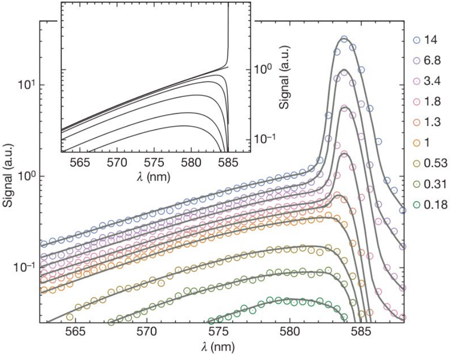

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$$H_{\text{sys}} = \sum_m \omega_m \psi_m^\dagger \psi_m + \sum_\alpha \left[\frac{\epsilon}{2} \sigma_\alpha^z + g \left(\psi_m \sigma_\alpha^+ + \text{H.c.} \right) \right]$$
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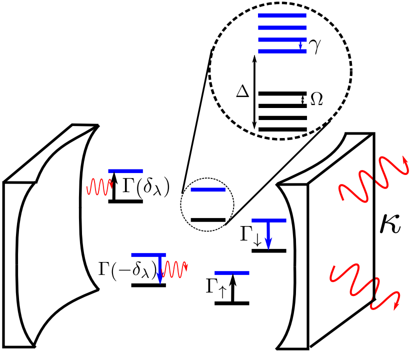
- **2D** harmonic oscillator
  $\omega_m = \omega_{\text{cutoff}} + m\omega_{H.O.}$
- Incoherent processes: excitation, decay, loss, vibrational thermalisation.
- Weak coupling, perturbative in g



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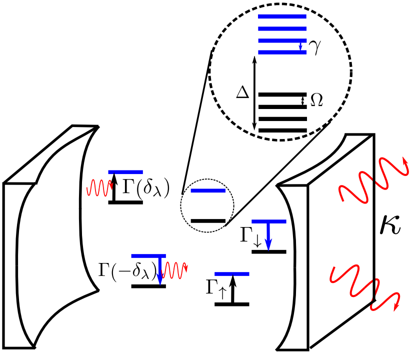

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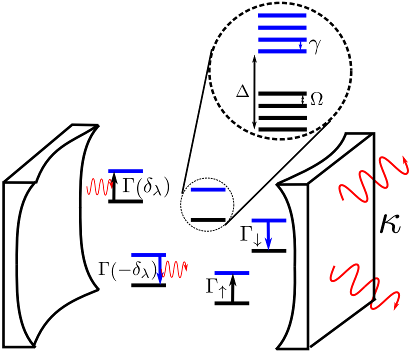

Microscopic model – all orders in S

- Polaron transform (exact), $H = \sum_m \omega_m \psi_m^\dagger \psi_m + \sum_\alpha h_\alpha$,

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- Master equation

$$\dot{\rho} = -i[H_0, \rho] + \sum_m \frac{\kappa}{2}\mathcal{L}[\psi_m] + \sum_\alpha \left[\frac{\Gamma_\uparrow}{2}\mathcal{L}[\sigma_\alpha^+] + \frac{\Gamma_\downarrow}{2}\mathcal{L}[\sigma_\alpha^-]\right]$$
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$$\Gamma(\delta) = 2g^2 \Re\left[\int dt e^{-i\delta t - (\Gamma_\uparrow + \Gamma_\downarrow)t/2}\langle D_\alpha^\dagger(t) D_\alpha(0)\rangle\right]$$

[Marthaler et al PRL '11, Kirton & JK PRL '13]

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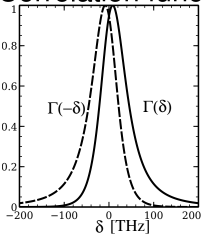

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Steady state populations and equilibrium

- Rate equation:

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[Kirton & JK PRL '13]

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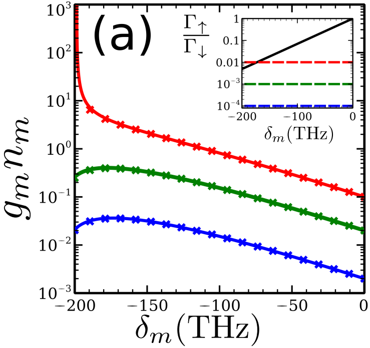

Low loss: Thermal

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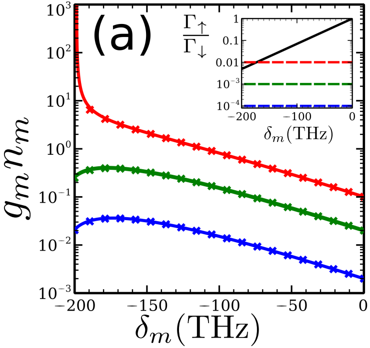

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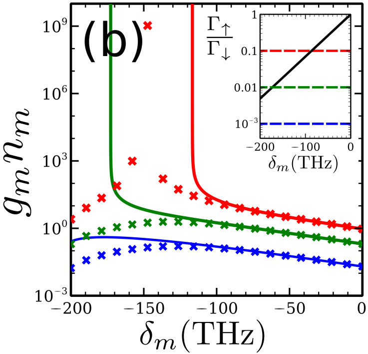
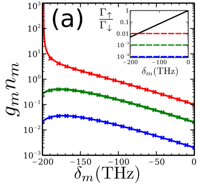


High loss → Laser

[Kirton & JK PRL '13]

Chemical potential?

- Steady state distribution:



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$$\mu = k_B T \ln[\Gamma_\uparrow/\Gamma_\downarrow]$$

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[Kirton & JK, PRA '15]

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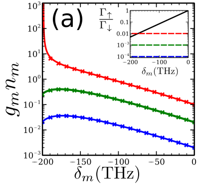

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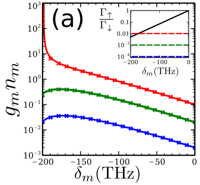

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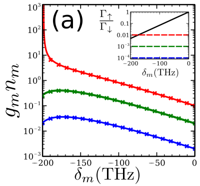


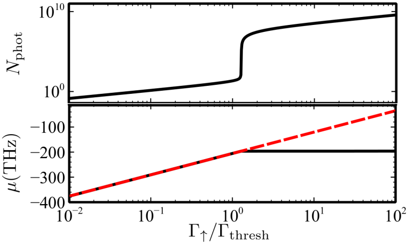
- $\kappa \ll N\Gamma(\delta)$, Kennard-Stepanov

$$\frac{n_m}{n_m+1} = e^{-\beta\delta_m+\beta\mu}, \qquad e^{\beta\mu} \equiv \frac{N_\uparrow}{N_\downarrow} = \frac{\Gamma_\uparrow + \sum_m \Gamma(\delta_m)n_m}{\Gamma_\downarrow + \sum_m \Gamma(-\delta_m)(n_m+1)}$$

- Below threshold,

$$\mu = k_B T \ln[\Gamma_\uparrow/\Gamma_\downarrow]$$

- At/above threshold, $\mu \to \delta_0$



[Kirton & JK, PRA '15]

Weak coupling: Photon BEC

1. Introduction
   - Condensation, lasing and superradiance
   - Modelling photon BEC & organic polaritons
2. Weak coupling: Photon BEC
   - Homogeneous model & threshold
   - Spatial profile and dynamics
3. Strong coupling
   - Superradiance transition
   - Vibrational dressing in normal-state spectrum
4. Ultrastrong coupling: vibrational reconfiguration

Spatially varying pump intensity

- Consider effects of pump profile, $\Gamma_{\uparrow}(\mathbf{r}) = \frac{\Gamma_{\uparrow}\exp(-r^2/2\sigma_p^2)}{(2\pi\sigma_p^2)^{d/2}}$
- Experiments: [Marelic & Nyman, PRA '15]

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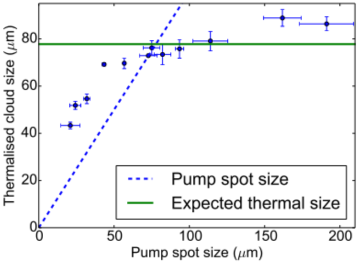

Modelling spatial profile.

- Varying excited density – differential coupling to modes

$$\partial_t n_m = -\kappa n_m + \Gamma(-\delta_m) O_m (n_m + 1) - \Gamma(\delta_m)(\rho_m - O_m) n_m$$

$$O_m = \int d\mathbf{r} \rho_\uparrow(\mathbf{r}) |\psi_m(\mathbf{r})|^2, \qquad \rho_\uparrow + \rho_\downarrow = \rho_m$$

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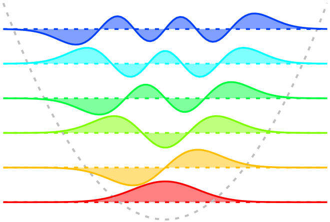

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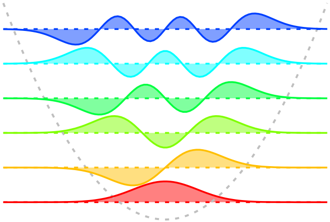

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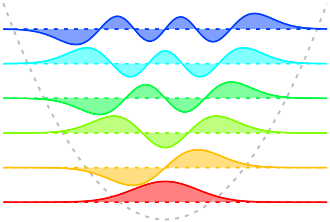
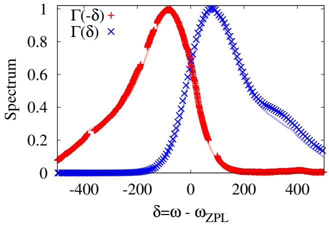


- Use exact R6G spectrum



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Spatially varying pump: below threshold

- Far below threshold:
  - If $\kappa \ll \rho_m \Gamma(\delta_m)$, $$\frac{n_m}{n_m+1} \simeq e^{-\beta\delta_m} \times \int d\mathbf{r} \rho_{\uparrow}(\mathbf{r}) |\psi_m(\mathbf{r})|^2$$
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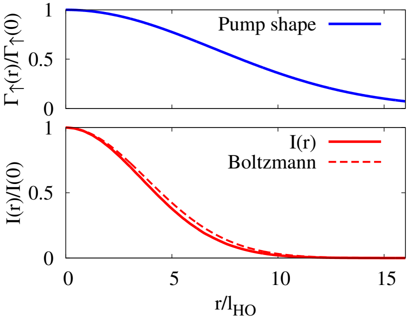

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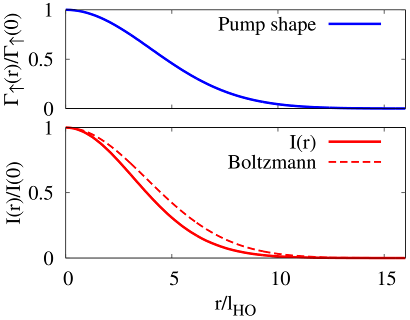

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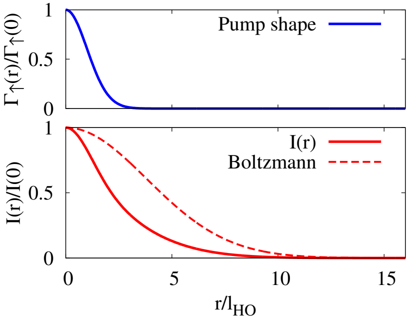

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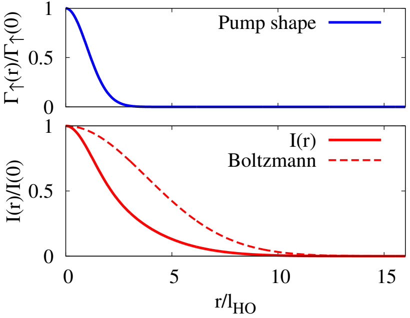
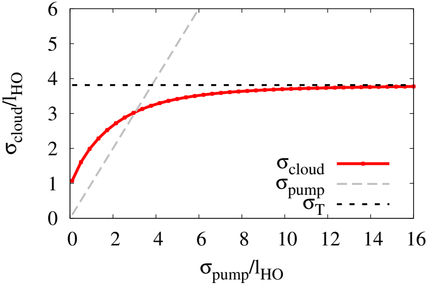
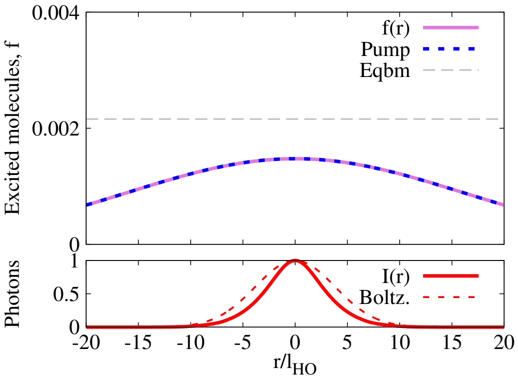




Near threshold behaviour



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- "Gain saturation" at centre
- Saturation of $f(r) = 1/(1 + e^{-\beta\mu})$ — spatial equilibriation

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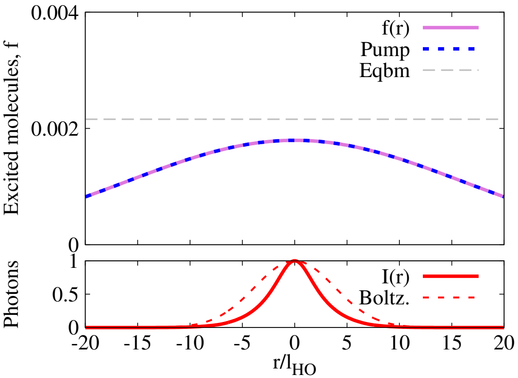

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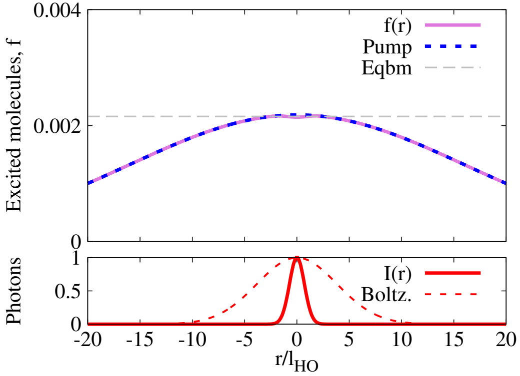

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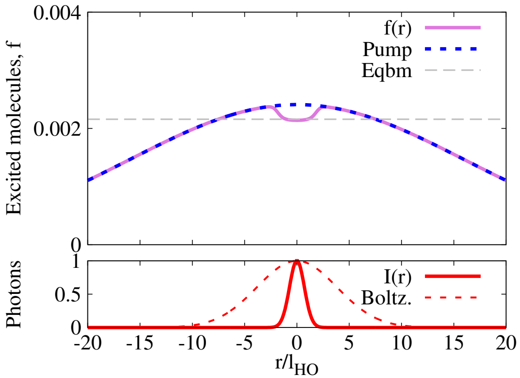

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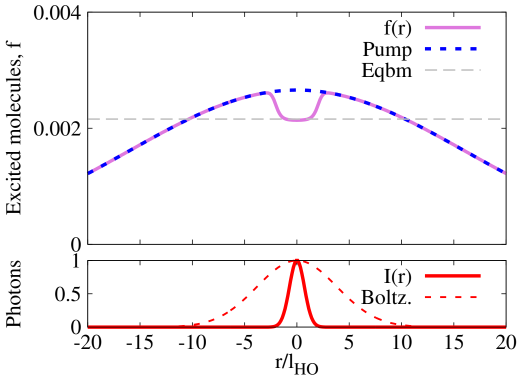

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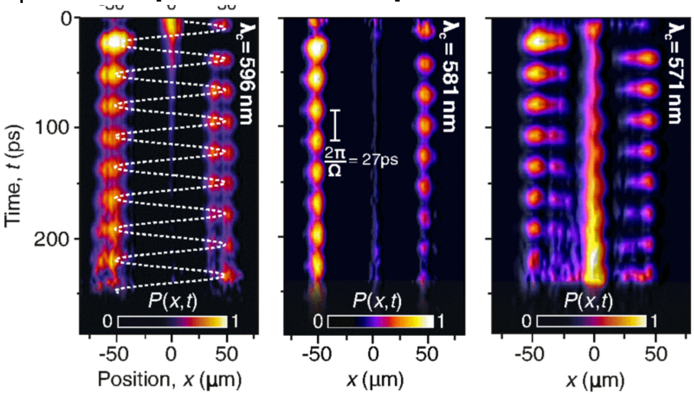
Off centre pumping; oscillations

- Experiments [Schmitt *et al.* PRA '15]



- Oscillations in space – beating of normal modes
- Thermalisation depends on cutoff

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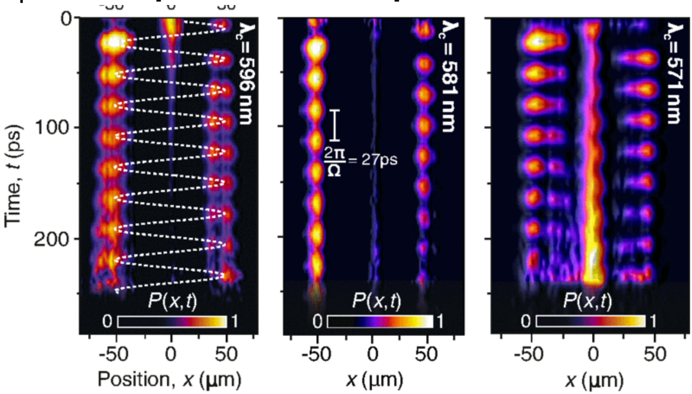

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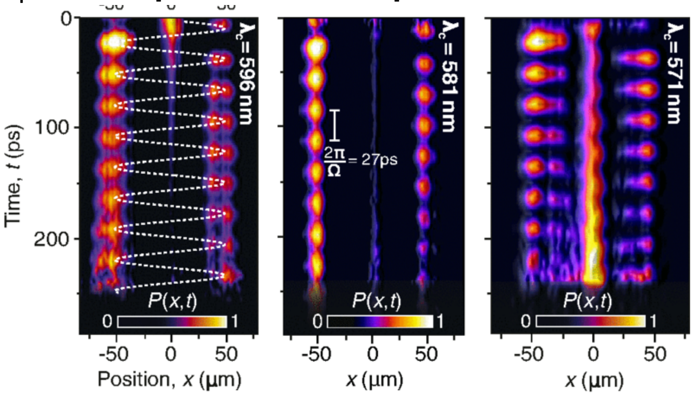

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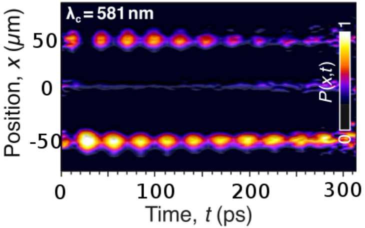
Limit of rate equations



$$\partial_t n_m = -\kappa n_m + \Gamma(-\delta_m)(n_m + 1)N_\uparrow - \Gamma(\delta_m) n_m N_\downarrow$$

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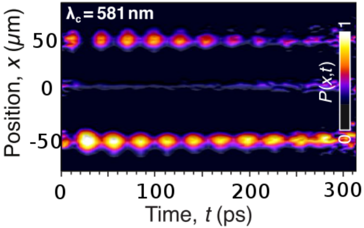

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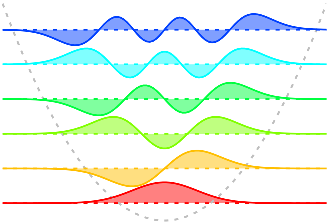
Emission into Gauss-Hermite mode m:

$$I(x) = \sum_m n_m |\psi_m(x)|^2$$



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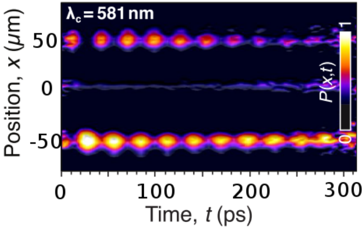

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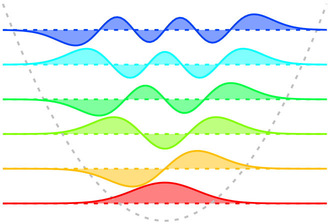

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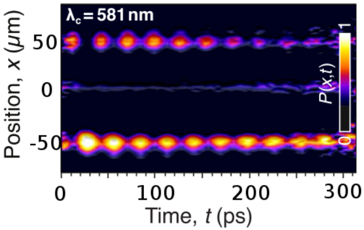

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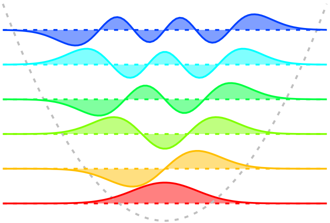

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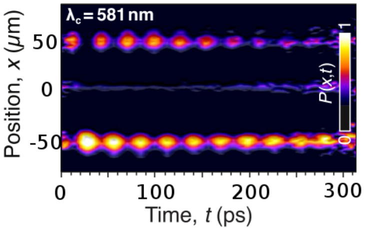

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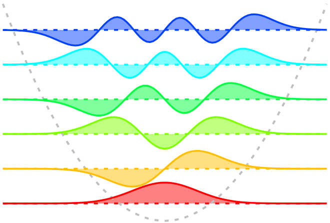

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Modelling

- Following toy model, use Redfield theory:

$$\partial_t \rho = -i \left[\sum_m \omega_m a_m^\dagger a_m, \rho \right] + \sum_{m,m',i} \psi_m^*(r_i)\psi_{m'}(r_i) \Big(K(\delta_{m'})[\hat{a}_{m'}\hat{\sigma}_i^+ \hat{\rho}, \hat{a}_m^\dagger \hat{\sigma}_i^-]$$
$$+ K(-\delta_m)[\hat{a}_m^\dagger \hat{\sigma}_i^- \hat{\rho}, \hat{a}_{m'}\hat{\sigma}_i^+] \Big) + \text{H.c.} + (\textit{pumping}, \textit{decay} \ldots),$$

- $K(\delta)$ analytic continuation of $\Gamma(\delta)$.
- Not secular approximation
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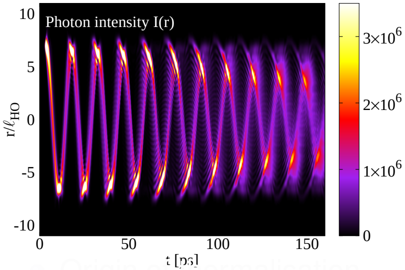
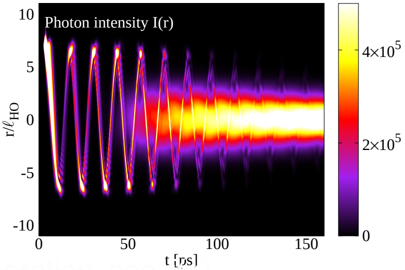
Dynamics from model

Longer cavity



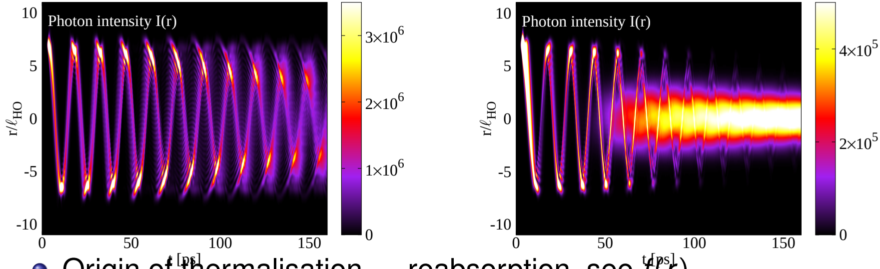
Shorter cavity



- Origin of thermalisation — reabsorption, see I(r)

Dynamics from model

Longer cavity | Shorter cavity



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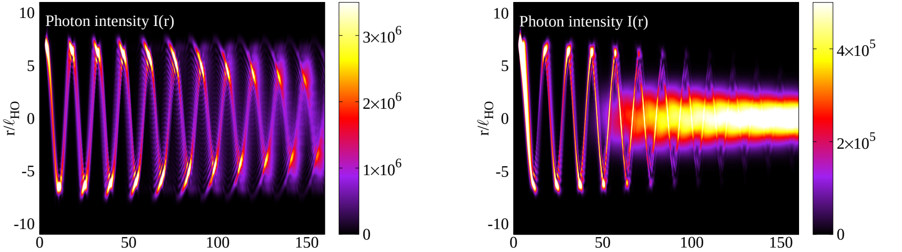

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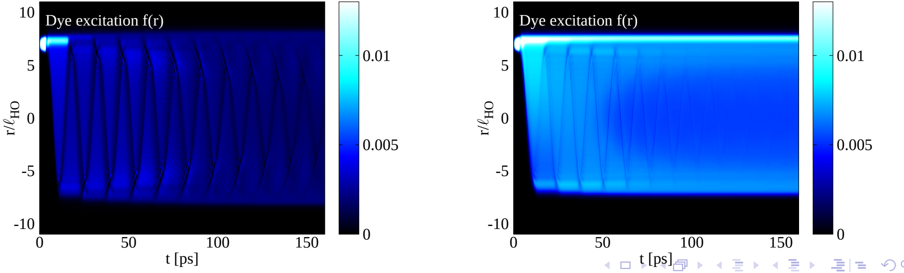
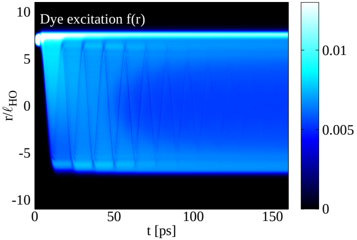


Thermalisation at late times

- Reabsorption “fills-in” excited molecules
- Reach thermal equilibrium, $f = [e^{-\beta\delta_0} + 1]^{-1}$



- Photon occupation thermalises later

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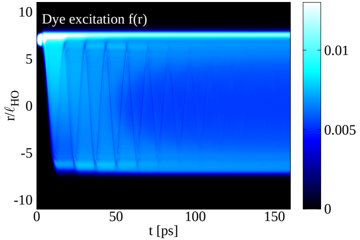

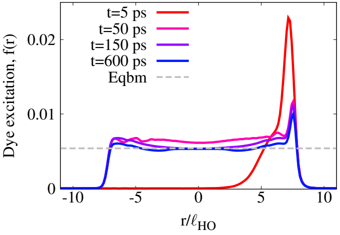

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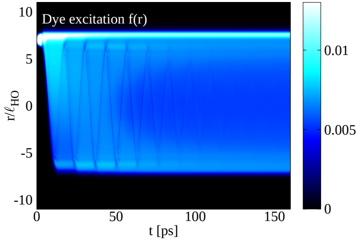

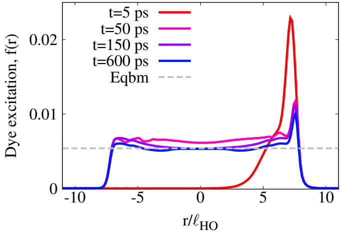

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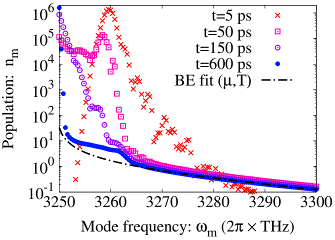

Strong coupling

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- Mean field — single photon mode

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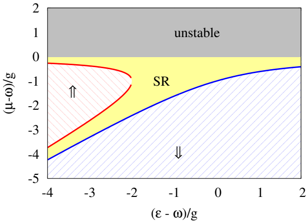
- Dicke phase diagram vs μ



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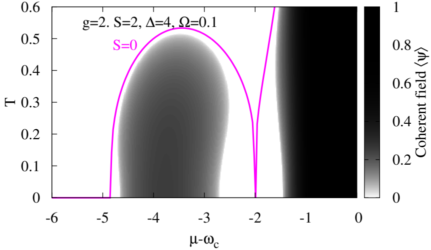

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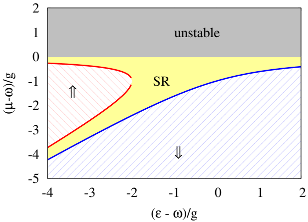

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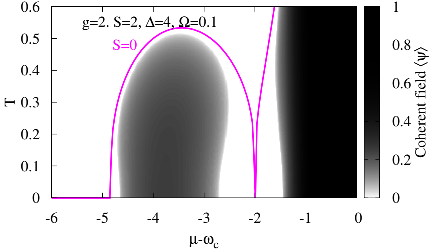

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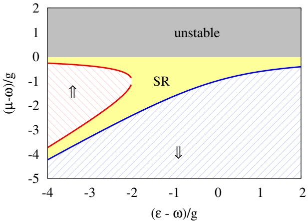

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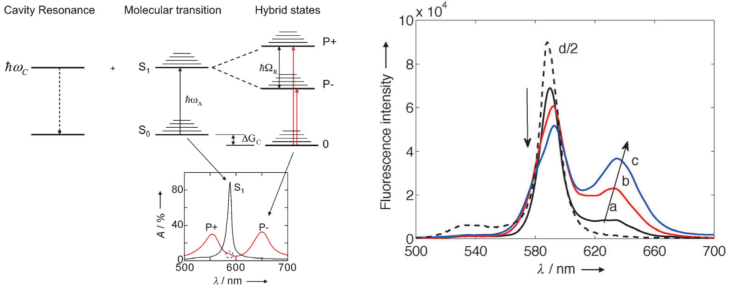

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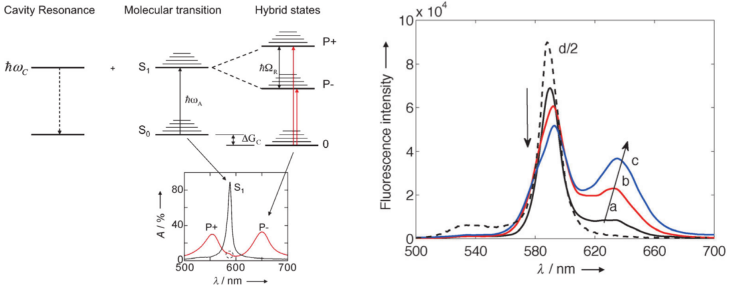

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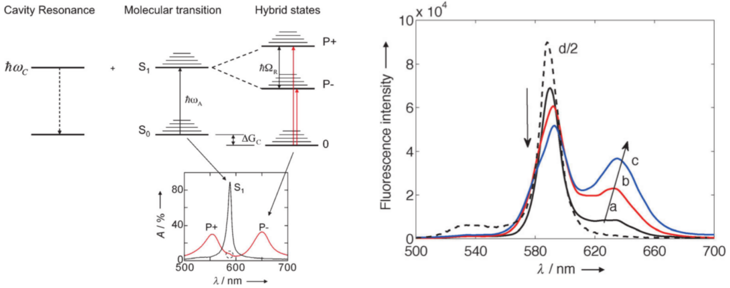

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  - Vibrationally dressed spectrum + disorder

Disordered molecules — spectrum

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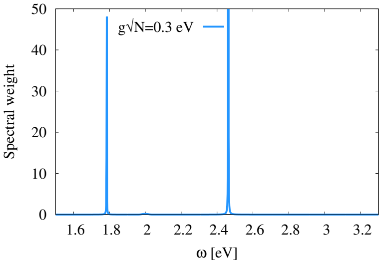

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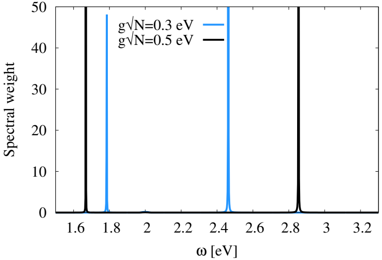

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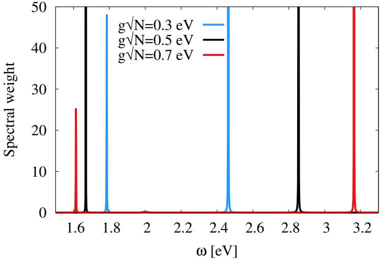

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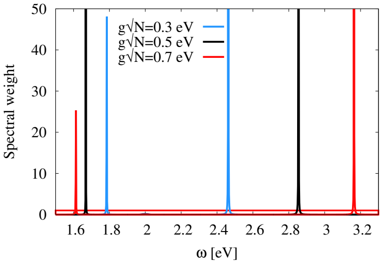

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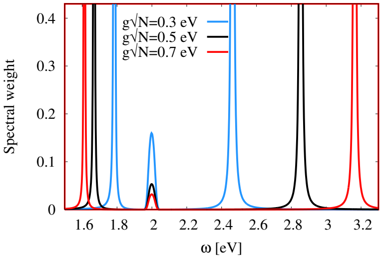

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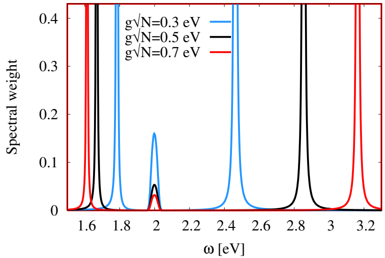

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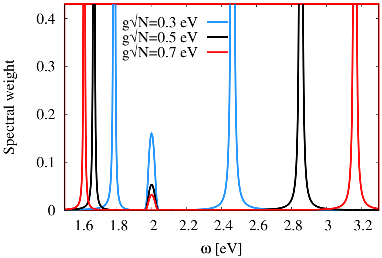

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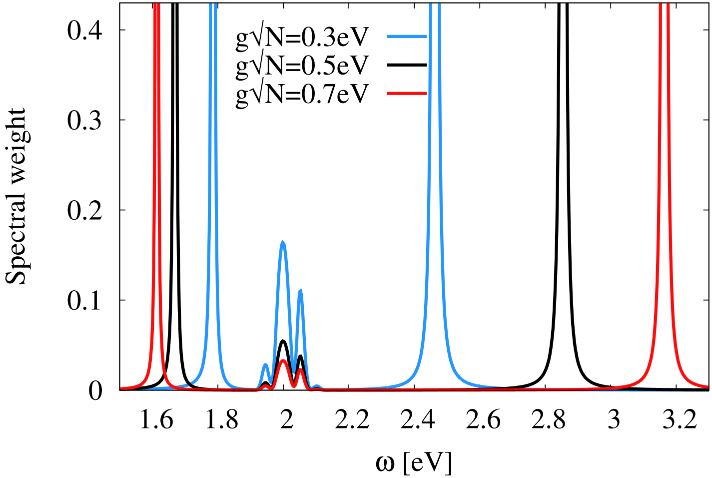

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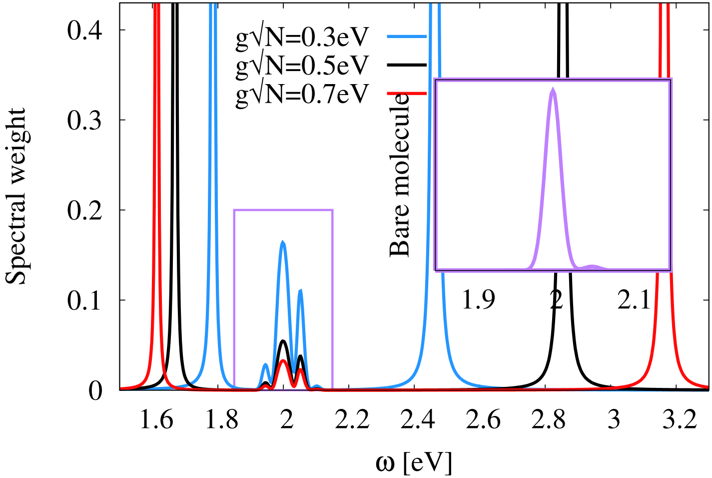
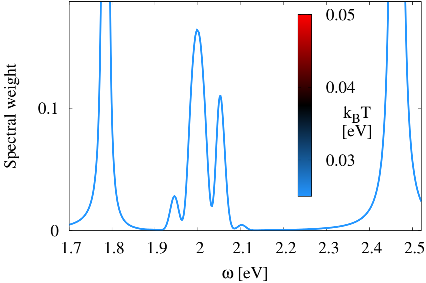


Disordered molecules + vibrations – *vs* temperature

- *vs* vs temperature
- Stronger disorder & $S = 0.5, \sigma = 0.025$eV



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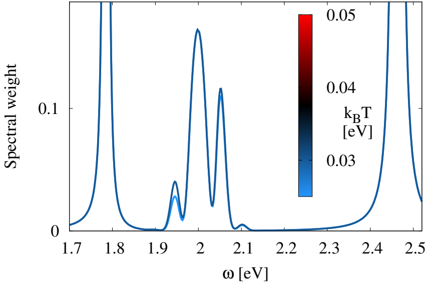

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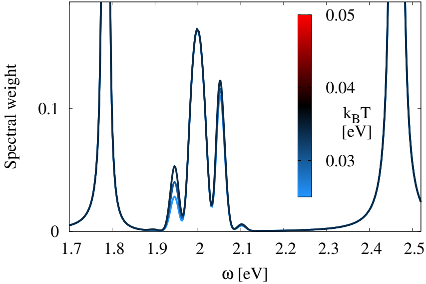

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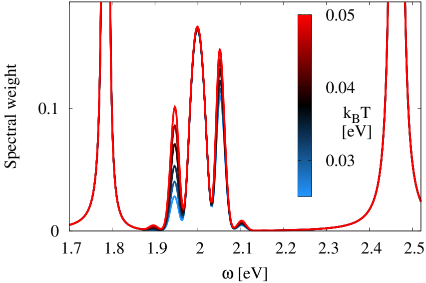

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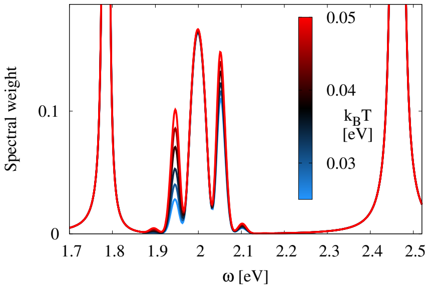

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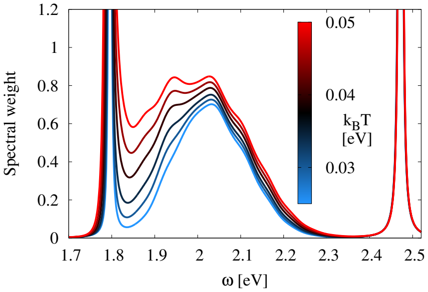

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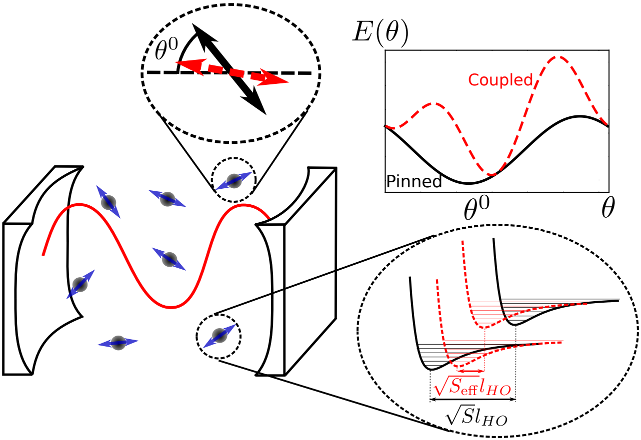
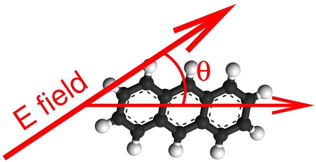


Rotational reorientation

- Rotational degrees of freedom



- Effective Hamiltonian

$$H = \ldots + \sum_{\alpha} \left[\ldots + g_{\alpha,k} \cos(\theta_\alpha)(\psi_k^\dagger + \psi_{-k})\sigma_\alpha^x + E_0(\theta_\alpha) \right]$$

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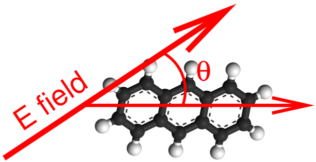

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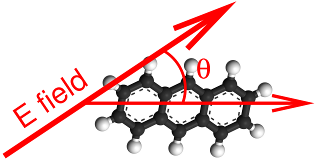

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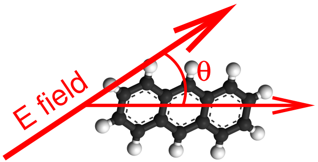

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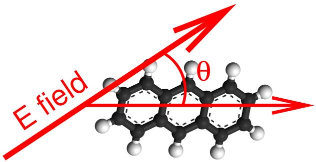

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$$H_{\text{eff}} = H_0 - \frac{g^2 N}{2(\epsilon + \omega)} \left\{ 1 - \frac{\Omega\sqrt{S}(b + b^\dagger)}{\epsilon + \omega} + \mathcal{O}\left[\left(\frac{\Omega}{\epsilon}\right)^2, \frac{g\sqrt{N}}{\epsilon} \right] \right\}$$

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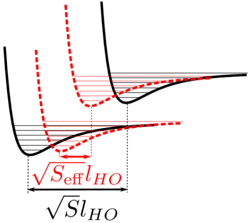

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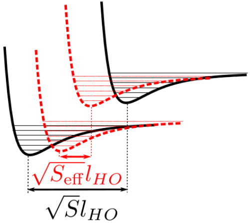

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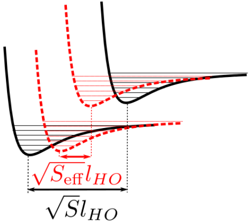

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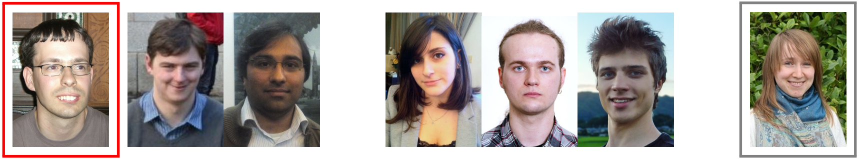


COLLABORATORS: S. De Liberato (Southhampton)

FUNDING:

EPSRC
Engineering and Physical Sciences Research Council

topnes
Topological Protection and Non-Equilibrium States in Strongly Correlated Electron Systems

Hybrid Polaritonics

The Leverhulme Trust

ICSCE8

Edinburgh, 25th–29th April, 2016.

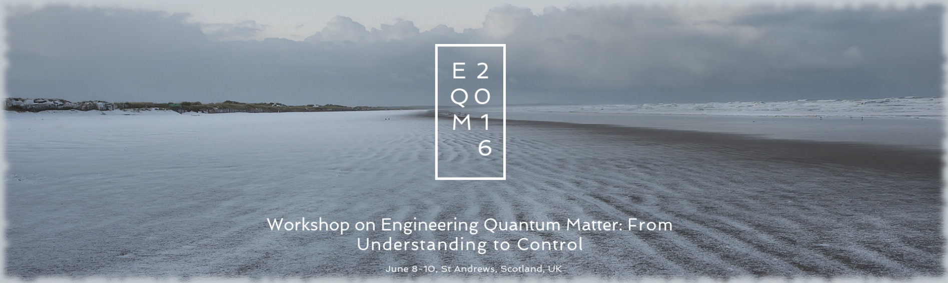
Plenary speakers: Ataç İmamoğlu, Peter Zoller.

Invited speakers: Alberto Amo, Mete Atatüre, Natasha Berloff, Charles Bardyn, Cristiano Ciuti, Thomas Ebbesen, Thiery Giamarchi, Jan Klärs, Dmitry Krizhanovskii, Xiaoqin (Elaine) Li, Peter Littlewood, Allan MacDonald, Francesca Marchetti, Keith Nelson, Pavlos Lagoudakis, Lukas Sieberer, Vivien Zapf.

Early-bird registration & abstract deadline: 31st January 2016.

Final registration deadline: March 2016.

`http://www.st-andrews.ac.uk/~icsce8`



http://eqm2016.co.uk

Summary

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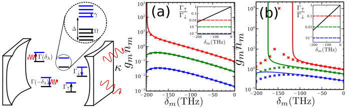
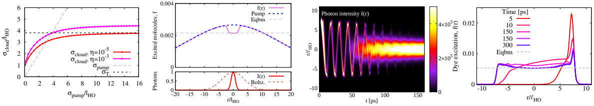


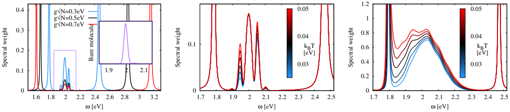
[Kirton & JK, PRL '13, PRA '15]

- Photon condensation, pattern formation physics



[JK & Kirton, PRA '16]

- Vibrational dressing, and possible reconfiguration



[Cwik *et al.* PRA '16]

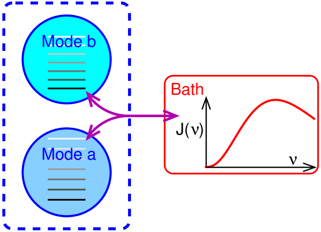
Extra Slides

5. Toy problem – two bosonic modes

6. First order transitions due to phonons

Toy problem: two bosonic modes

- Basic problem: Emission from thermal bath



$$H = \omega_a \hat{\psi}_a^\dagger \hat{\psi}_a + \omega_b \hat{\psi}_b^\dagger \hat{\psi}_b + H_{\text{Bath}}$$
$$+ (\varphi_a^* \hat{\psi}_a^\dagger + \varphi_b^* \hat{\psi}_b^\dagger) \sum_i g_i \hat{c}_i + \text{H.c.}$$

Toy problem: naïve solutions

- Two "expected" behaviours:
  - At resonance: "weak lasing" — coupling to bath dominates

$$\frac{d}{dt}\rho = \Gamma^{\downarrow}\mathcal{L}[\varphi_a\hat{\psi}_a + \varphi_b\hat{\psi}_b] + \Gamma^{\uparrow}\mathcal{L}[\varphi_a^*\hat{\psi}_a^\dagger + \varphi_b^*\hat{\psi}_b^\dagger]$$

  - Far from resonance: pointer states are eigenstates

$$\frac{d}{dt}\rho = \sum_{i=a,b} \Gamma_i^{\downarrow}\mathcal{L}[\hat{\psi}_i] + \Gamma_i^{\uparrow}\mathcal{L}[\hat{\psi}_i^\dagger]$$

- Explicit derivation → Redfield theory

$$\partial_t\rho = -i[\hat{H},\rho] + \sum_{ij} L_{ij}^{\downarrow}\left(2\hat{\psi}_j\rho\hat{\psi}_i^\dagger - [\rho,\hat{\psi}_i^\dagger\hat{\psi}_j]_+\right) + \sum_{ij} L_{ij}^{\uparrow}\left(2\hat{\psi}_j^\dagger\rho\hat{\psi}_i - [\rho,\hat{\psi}_i\hat{\psi}_j^\dagger]_+\right).$$

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Toy problem: exact solution

- Solve via Laplace transform. Find $F_{ij}(t) = \langle \hat{\psi}_i^\dagger(t) \hat{\psi}_j(t) \rangle$
- Steady state:
- Time evolution —
  $F_{ab}(t) \sim \exp(-\alpha \Delta^2 t)$
- Always some coherence
  - (individual always wrong)
- $F_{ab} \sim F_{aa}, F_{bb}$ only at $\Delta = 0$

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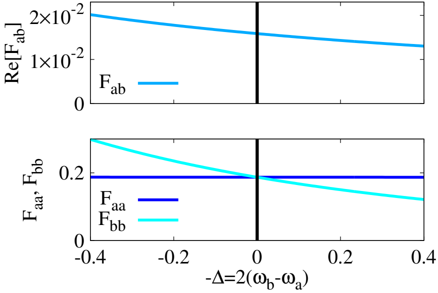

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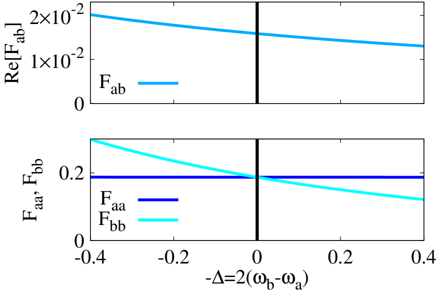

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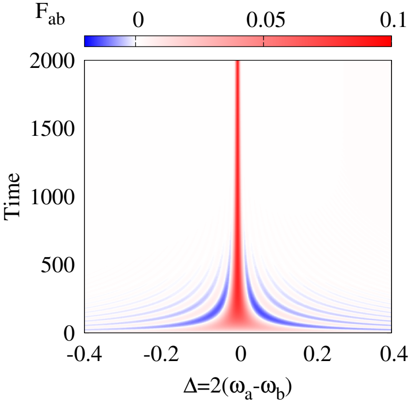

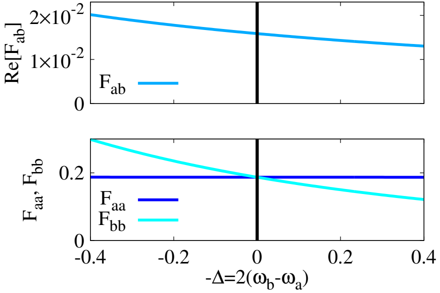

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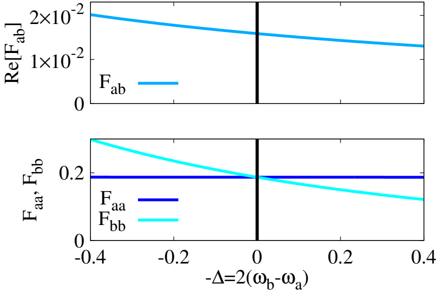

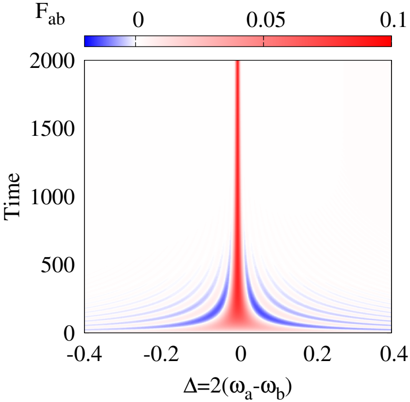

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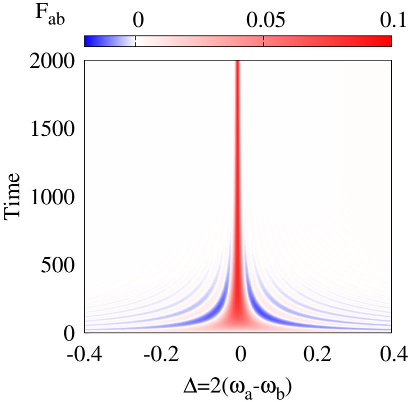

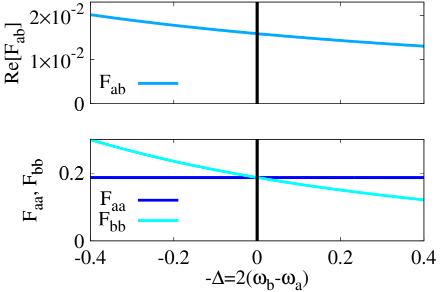

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Toy problem: Redfield theory

Unsecularised Redfield theory:

$$\partial_t \rho = -i[\hat{H}, \rho] + \sum_{ij} \varphi_i^* \varphi_j \left[K_{ij}^{\downarrow} \left(2\hat{\psi}_j \rho \hat{\psi}_i^\dagger - [\rho, \hat{\psi}_i^\dagger \hat{\psi}_j]_+ \right) \right.$$
$$\left. + K_{ij}^{\uparrow} \left(2\hat{\psi}_j^\dagger \rho \hat{\psi}_i - [\rho, \hat{\psi}_i \hat{\psi}_j^\dagger]_+ \right) \right].$$

- Compare to exact solution: $F_{ij} = \langle \hat{\psi}_i^\dagger \hat{\psi}_j \rangle$

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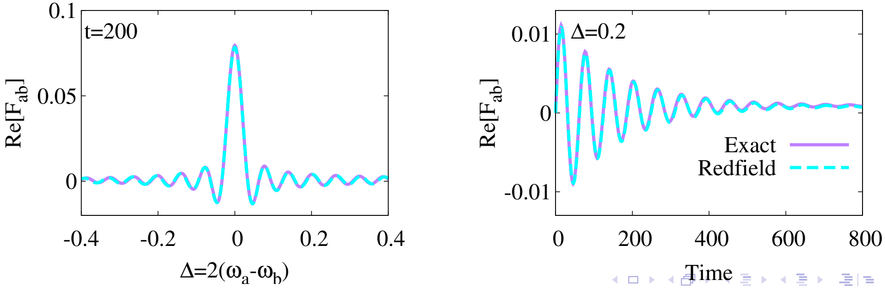

Toy problem: Secularisation

- Secularisation (in eigenbasis of $\hat{H}$): $L_{ij}^{\uparrow,\downarrow} \to L_{ii}^{\uparrow,\downarrow}\delta_{ij} \to F_{ab} = 0$
- Secularisation often invoked to cure negative eigenvalues of $L_{ij}^{\uparrow,\downarrow}$.
- Check stability: consider $f = (F_{aa}, F_{bb}, \Re[F_{ab}], \Im[F_{ab}])$

$$\partial_t \mathbf{f} = -\mathbf{M}\mathbf{f} + \mathbf{f}_0$$

- Eigenvalues of $\mathbf{M}$ exist in closed form:
  - Unstable (negative only if $dJ(\nu)/d\nu \gg 1$ — Markov breakdown)

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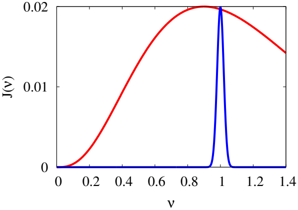

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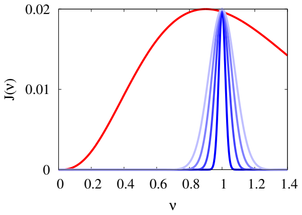
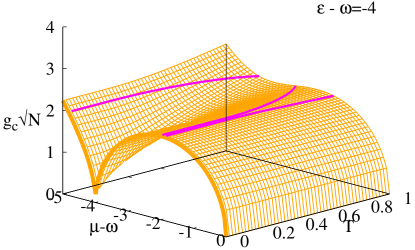


Critical coupling with increasing S

- Re-orient phase diagram
- *g* vs μ, *T*
- Colors $\rightarrow$ Jump of $\langle\psi\rangle$



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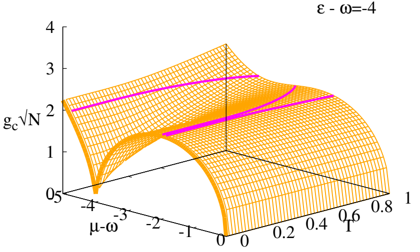

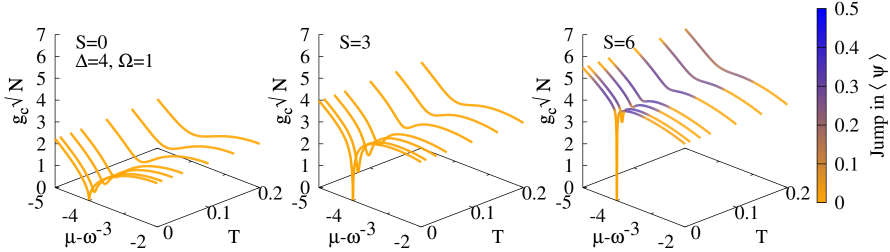

Explanation: Polaron formation

- Unitary transform

$$H_\alpha \to \tilde{H}_\alpha = e^{K_\alpha} H_\alpha e^{-K_\alpha} \qquad K = \sqrt{S} S^z_\alpha (b^\dagger_\alpha - b_\alpha)$$

- Coupling moves to $S^\pm$

$$\tilde{H}_\alpha = \text{const.} + \epsilon S^z_\alpha + \Omega b^\dagger_\alpha b_\alpha + g \left[\psi S^+_\alpha e^{\sqrt{S}(b^\dagger_\alpha - b_\alpha)} + \text{H.c.} \right]$$

- Optimal phonon displacements, $\sim \sqrt{S}$
- Reduced $g_{\text{eff}} \sim g \times \exp(-S/2)$
- For $\psi \neq 0$, competition
  Variational MFT $|\psi\rangle_\alpha \sim \exp(-\eta K_\alpha - \zeta b^\dagger_\alpha)|0, S\rangle_\alpha$

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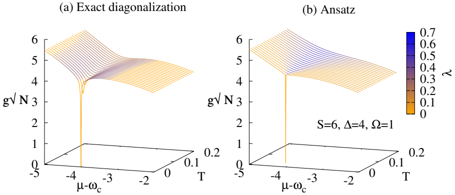
Collective polaron formation

- Compares well at $S \gg 1$
- Coherent bosonic state



- Feedback: Large/small $g_{\text{eff}} \leftrightarrow \lambda = \langle\psi\rangle$
- Variational free energy

$$F = (\omega_c - \mu)\lambda^2 + N\left\{\Omega\left[\zeta^2 - S\frac{\eta(2-\eta)}{4}\right] - T\ln\left[2\cosh\left(\frac{\xi}{T}\right)\right]\right\}$$

Effective 2LS energy in field:

$$\xi^2 = \left(\frac{\epsilon-\mu}{2} + \Omega\sqrt{S}(1-\eta)\zeta\right)^2 + g^2\lambda^2 e^{-S\eta^2}$$

[Cwik *et al.* EPL '14]

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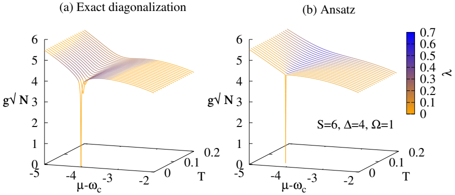

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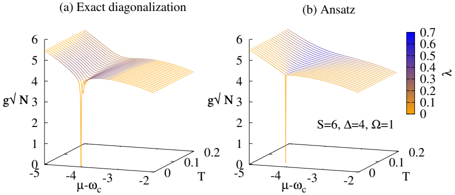

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