Collective behaviour and driven-dissipative systems

Jonathan Keeling

SUPA

University of St Andrews
FOUNDED 1413

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Acknowledgements

GROUP (&ALUMNI):



COLLABORATORS: Fazio (Pisa & CQT), Schiro (CNRS), Tureci (Princeton), Eastham (TCD), Lovett (St Andrews).

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topnes Topological Protection and Non-Equilibrium States in Strongly Correlated Electron Systems

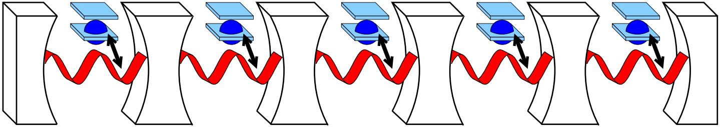
EPSRC Engineering and Physical Sciences Research Council

The Leverhulme Trust

1. Effects of dissipation on collective behaviour
   - Coherently driven JCHM (Mean-Field)
   - Parametrically driven BHM (MF and MPO)
   - Parametrically driven RHM (MF and MPO)

2. Effects of collective behaviour on dissipation
   - Coupled qubit-cavity systems
   - Collective coupling to baths

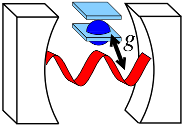
Coherently pumped JCHM



$$H = -\frac{J}{z}\sum_{ij}\psi_i^\dagger\psi_j + \sum_i \frac{\Delta}{2}\sigma_i^z + g(\psi_i^\dagger\sigma_i^- + \text{H.c.}) + f(\psi_i e^{i\omega_L t} + \text{H.c.})$$

$$\partial_t\rho = -i[H,\rho] - \frac{\kappa}{2}L_\psi[\rho] - \frac{\gamma}{2}L_{\sigma^-}[\rho]$$

Coherently pumped single cavity [Bishop *et al.* Nat. Phys '09]



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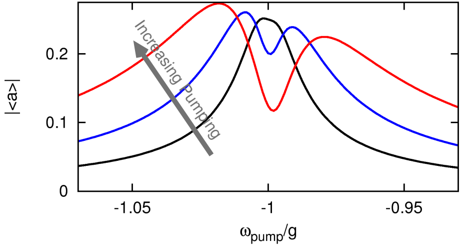
- Effective 2LS: $|\text{Empty}\rangle, |1 \text{ polariton}\rangle$



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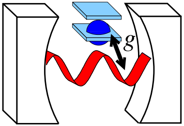

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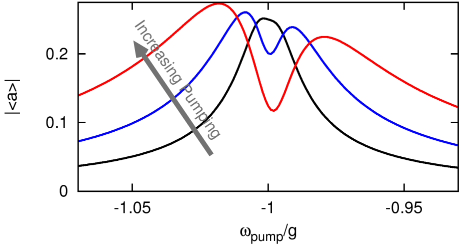
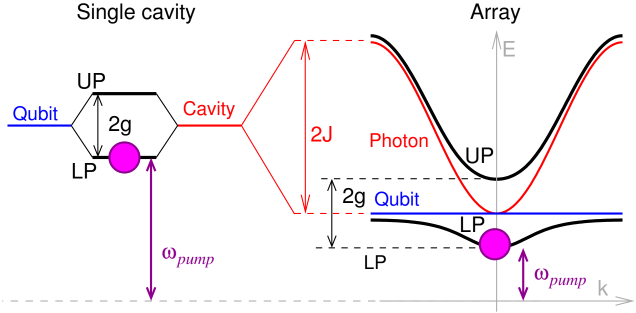


Coherently pumped dimer & array

Chose detuning *a la* Dicke model



- Bistability at intermediate J
  - More/less localised states
  - Connects to Dicke limit

[Nissen *et al.* PRL '12]

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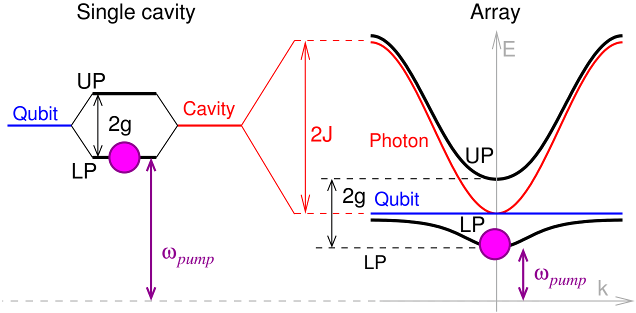

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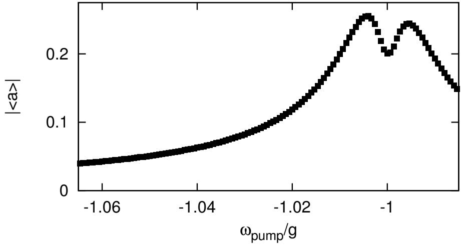

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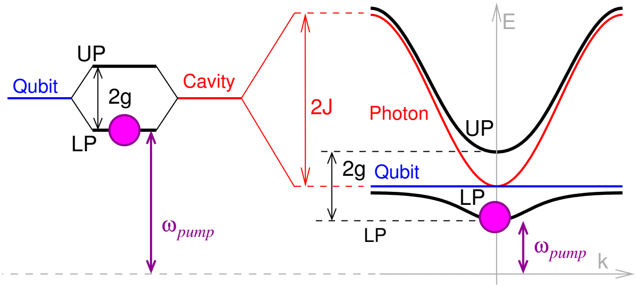

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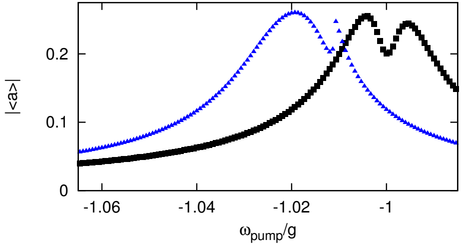

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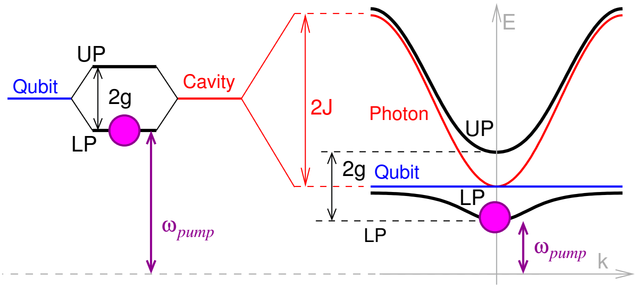

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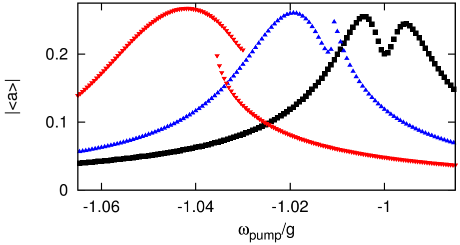

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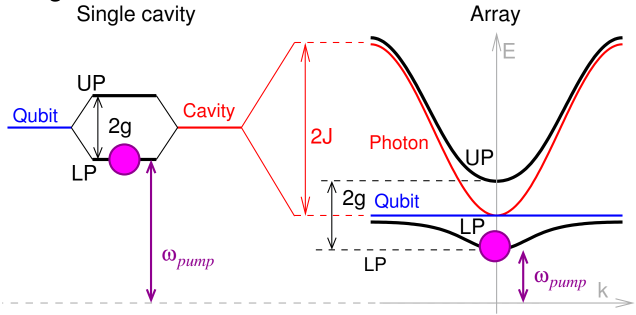

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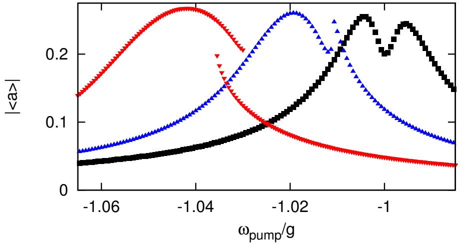

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  - Distribution of ψ – Washes out bistable jump
- Bistability near resonance — phase of ψ depends on Δ_i
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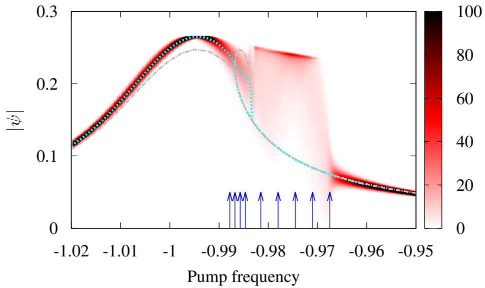

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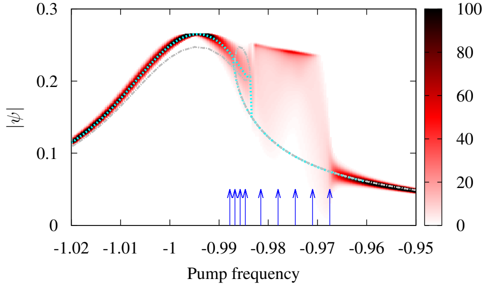

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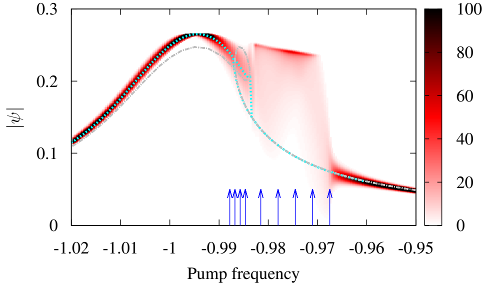

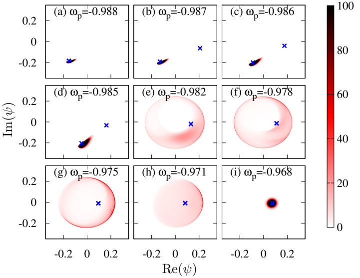

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Coherent pumping beyond Mean Field

- Crucial question: what can we expect from true ρ ?
  - No bistability (replaced by bimodality)

$$\rho_{SS} = \sum_i W_i \rho_{MF_i}$$

  - Slow approach to steady state.

[Lugiato, Prog. Opt. 1984; Mendoza-Arenas ... Jaksch `1510.06651`]

- But...
  - Density matrix is *ensemble average* of experiments
  - cf Interference fringes of BEC. [Leggett, RMP '01]
    $|\psi_1(r) + \psi_2(r)|^2 = \ldots + \sqrt{I_1 I_2} \cos(kr + \Delta\phi)$
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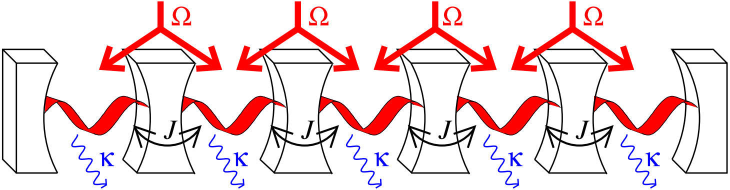
Parametrically pumped BHM



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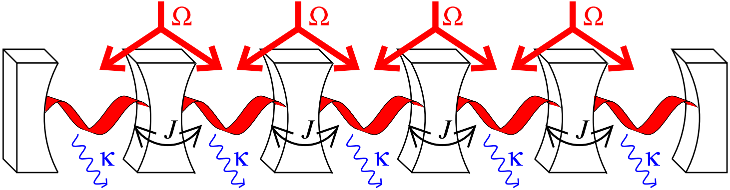

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Rotating frame, blockade approximation, rescale:

$$H = -J \sum \left[\tau_i^+ \tau_{i+1}^- + \tau_{i+1}^+ \tau_i^- + g\tau_i^z + \Delta\left(\tau_i^+ \tau_{i+1}^+ + \tau_{i+1}^- \tau_i^-\right)\right]$$

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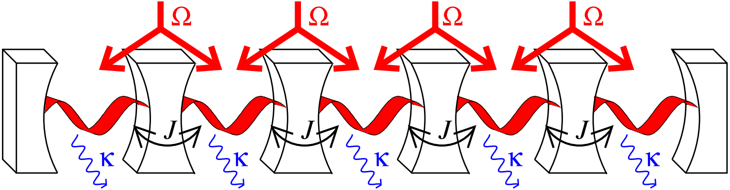

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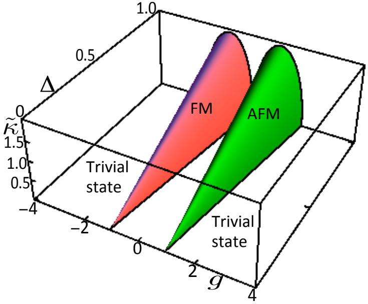
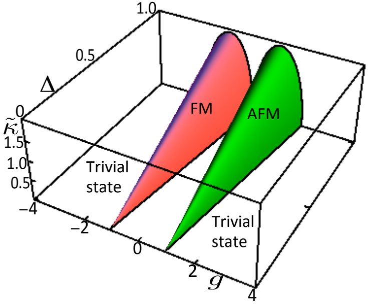


Why AFM/FM attractors



- Linear stability, fluctuation $\sim \exp(-i\nu_k t + ikr_i)$ Linear stability

$$\nu_k = -i\kappa \pm 2J\sqrt{g^2 + 2g\cos k + (1-\Delta^2)\cos^2 k}$$

- $g \ll -1$, Dissipation matches ground state
  - Most unstable mode, $k = 0$
- $g \gg +1$, Dissipation matches max energy
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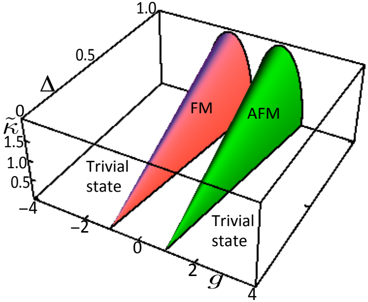

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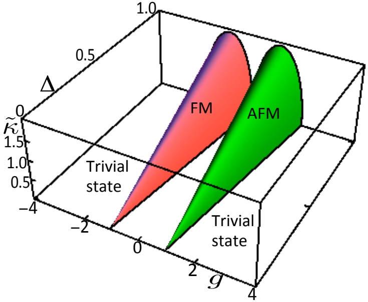

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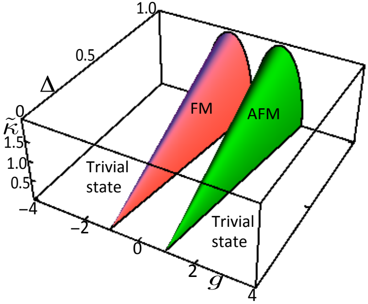

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- MPO for density matrices.
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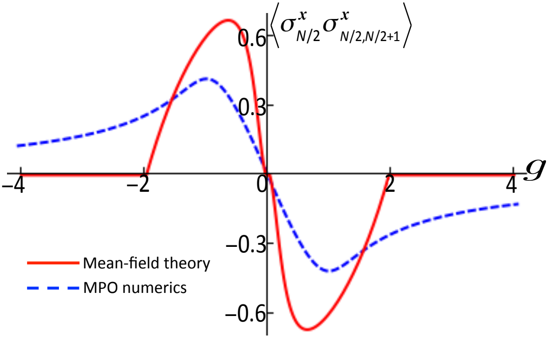

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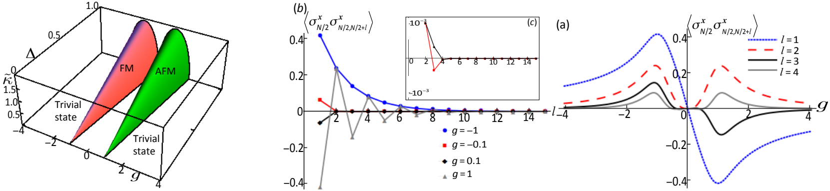
- AFM vs FM from sign of g ($\Delta = 1$)



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- $\Delta \to 0$. Analytic spin-wave, $\left|\langle \sigma_i^- \sigma_{i+l}^+ \rangle\right| \propto \exp(-\xi_c l)$

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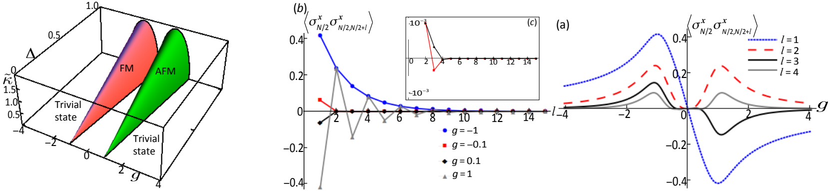

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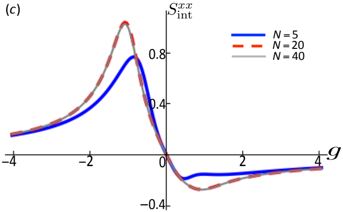

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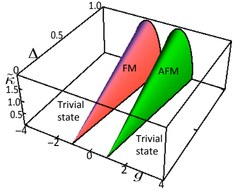

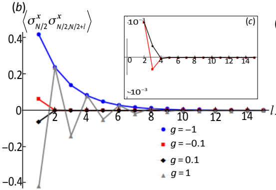

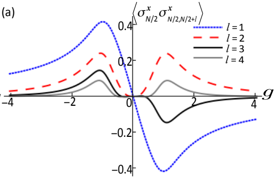

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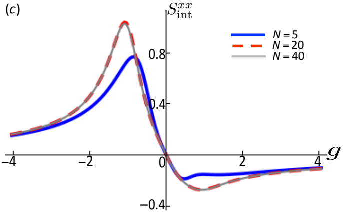

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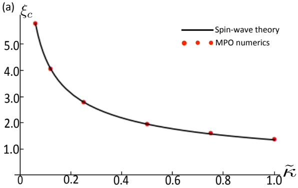

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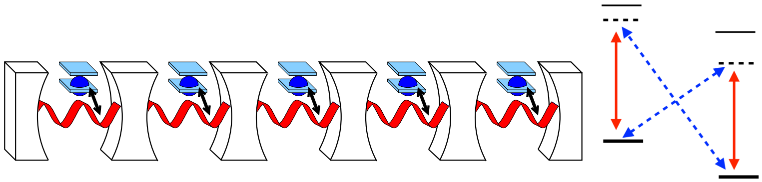
   - Coupled qubit-cavity systems
   - Collective coupling to baths

Rabi Hubbard model



$$H = -J \sum_{\langle ij \rangle} \psi_i^\dagger \psi_j + \sum_i h_i^{\text{Rabi}}$$

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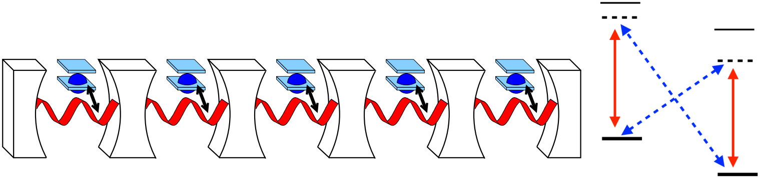

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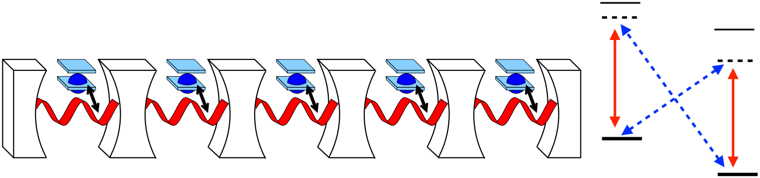

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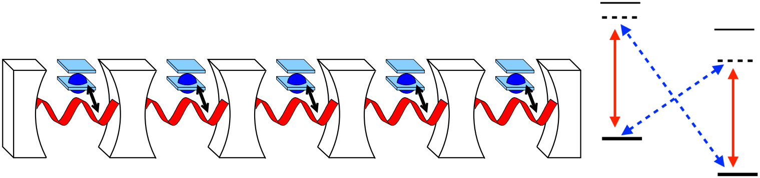

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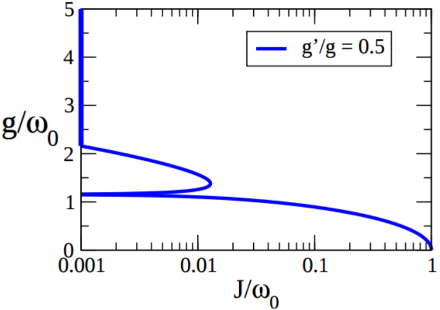
Rabi Hubbard model – equilibrium



- Discrete $\mathbb{Z}_2$ symmetry
  - Parity Mott lobes

- $g = g'$, never degenerate — never superfluid

[Schiró *et al.* PRL '12]

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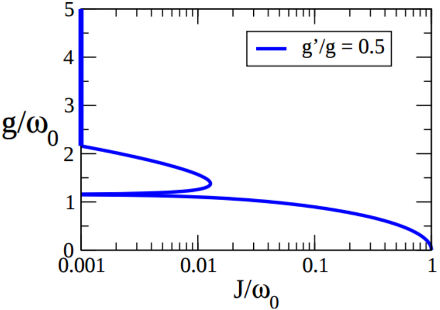

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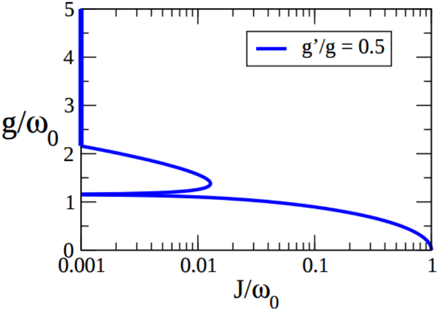

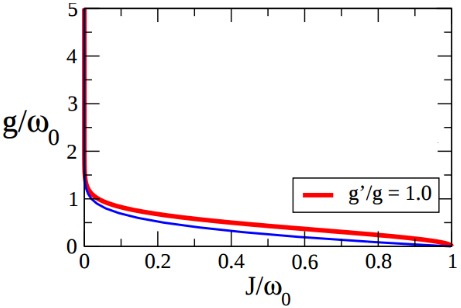

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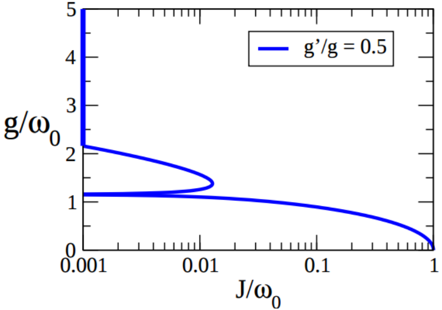

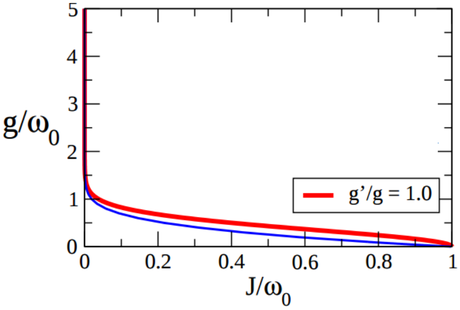

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Driven-dissipative system — linear stability

Mean field theory — still large Hilbert space.

- Normal state + fluctuations: $\rho = \otimes_n(\rho_{ss} + \sum_{\mathbf{k}} \delta\rho_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{n} - i\nu_{\mathbf{k}} t} + \text{H.c.})$
- $\nu_{\mathbf{k}}$ Eigenvalues of $M = M_0 - t_{\mathbf{k}} M_1$, $t_{\mathbf{k}} = -2J\cos(k)$
- Unstable if $\Im[\nu_{\mathbf{k}}] > 0$

- Given J, $|t_{\mathbf{k}}| < 2J$
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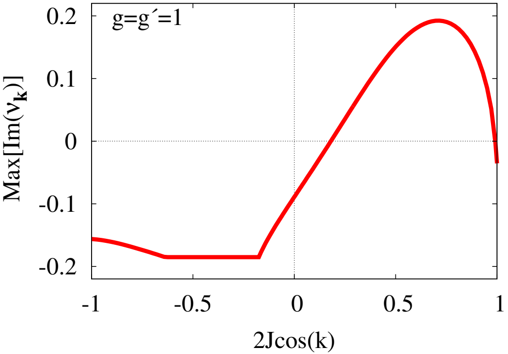

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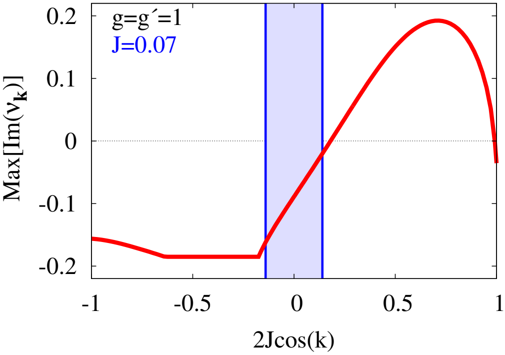

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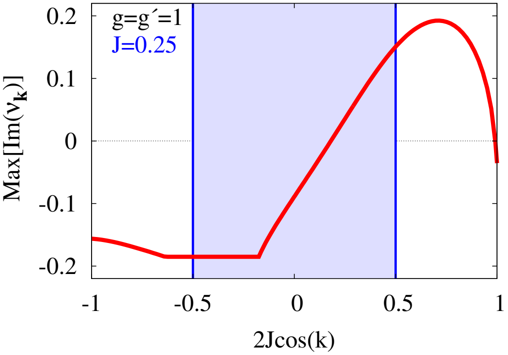

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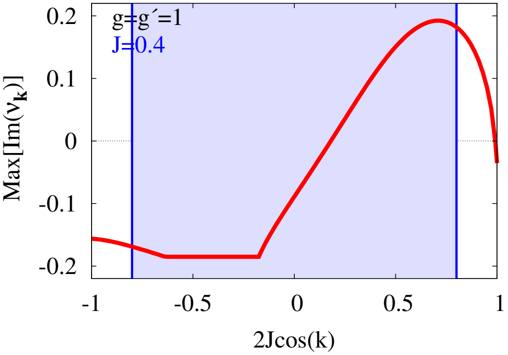

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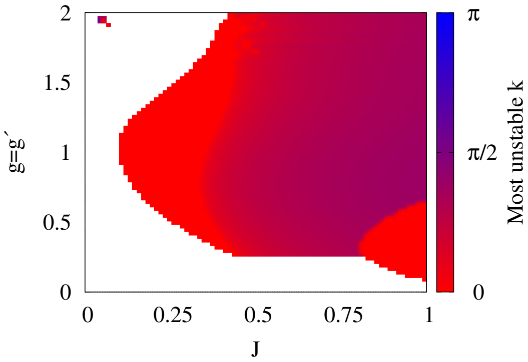
Rabi-Hubbard model — linear stability

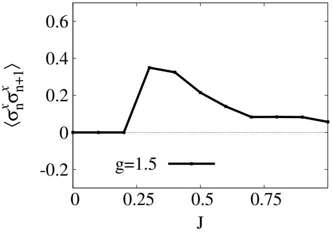
Stability phase diagram:



[Schiró *et al.* arXiv:1503.04456]

Rabi-Hubbard model — linear stability

Steady state correlations:



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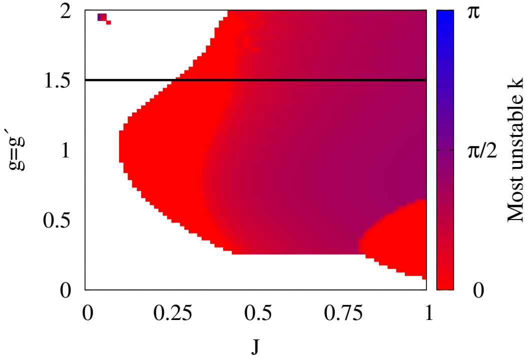

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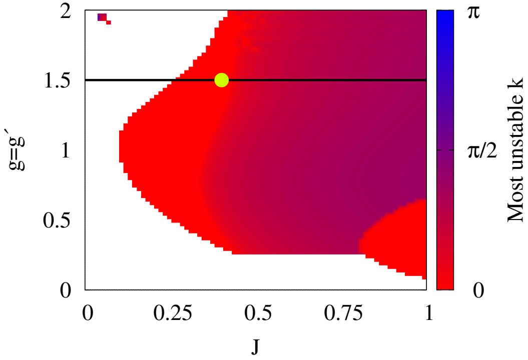

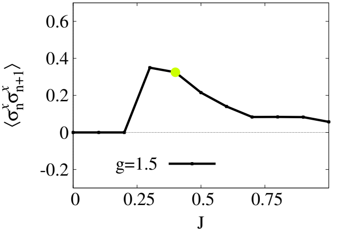

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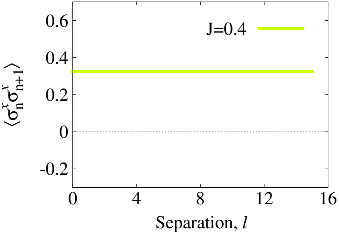

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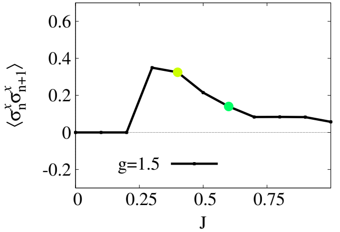

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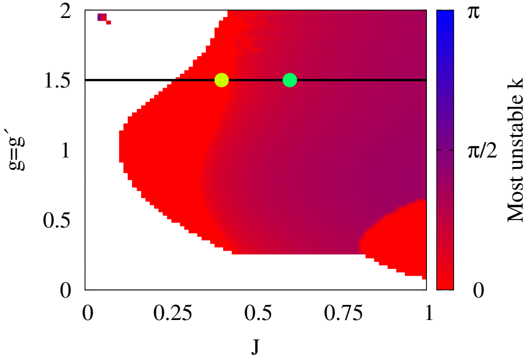

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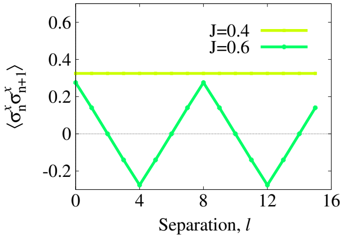

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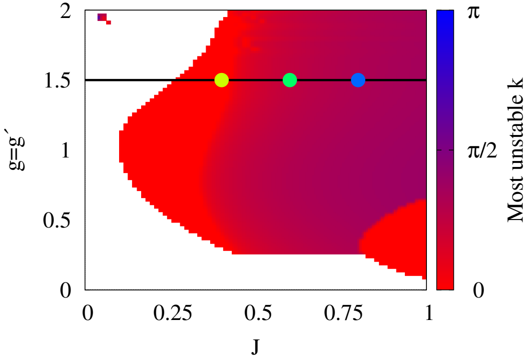

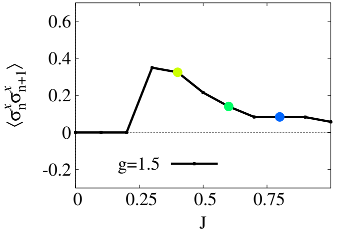

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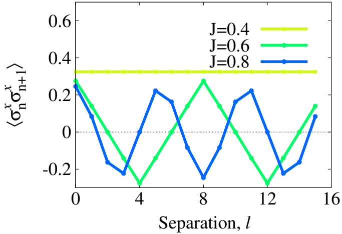

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Linear stability – limit cycles

- If $\nu_k = \pm\nu'_k + i\nu''_k$ at instability $\rightarrow$ Limit Cycle
  [Lee *et al.* PRA '11, Jin *et al.* PRL '13, Ludwig & Marquard PRL '13, Chan *et al.* arXiV:1501.00979]

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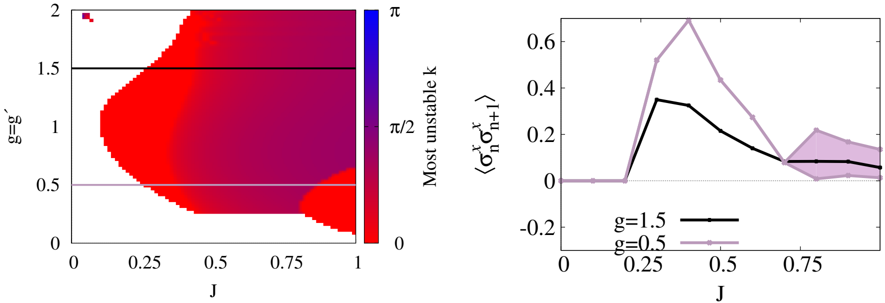

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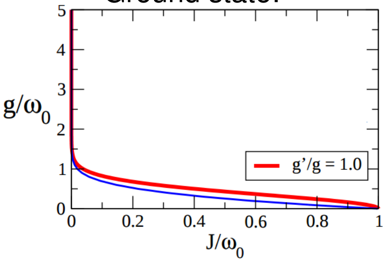
Phase-boundary Effective model

- Compare phase boundaries

Ground state:



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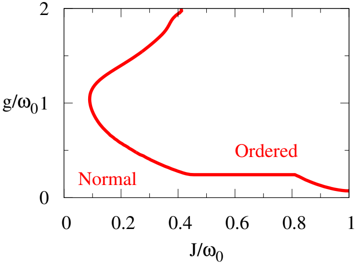

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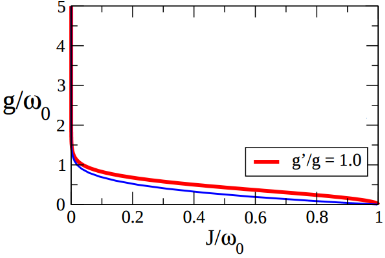

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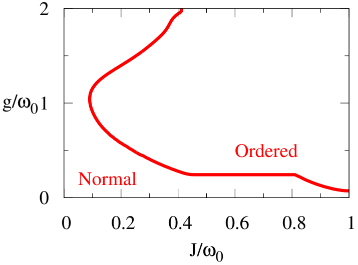

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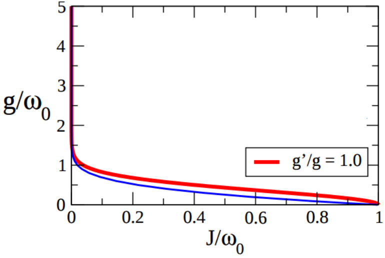

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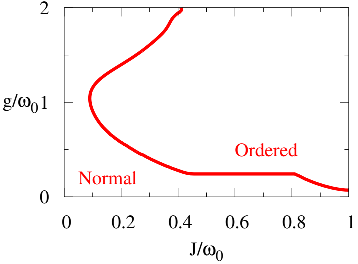

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$$H = \sum_i \frac{\Delta}{2}\tau_i^z - \sum_{\langle ij \rangle} \tilde{J}_x \tau_i^x \tau_i^x + \tilde{J}_y \tau_i^y \tau_i^y, \qquad \dot{\rho} = -i[H, \rho] + \ldots$$

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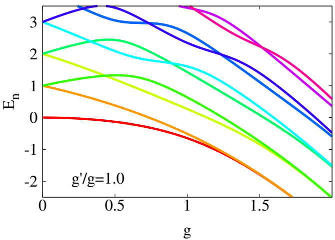

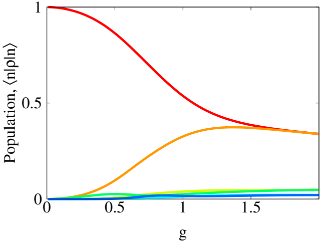

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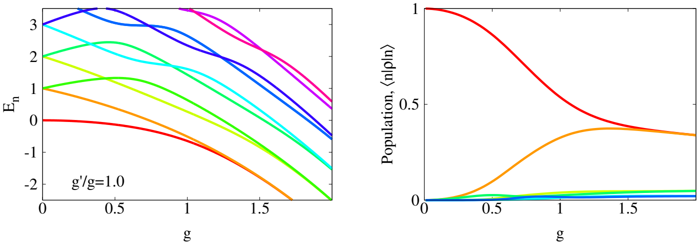

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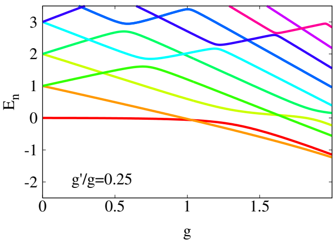
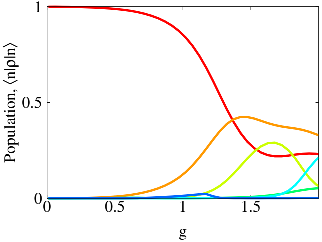
$g' \neq g$, Level crossings

- For $g' \neq g$, Δ can swap sign





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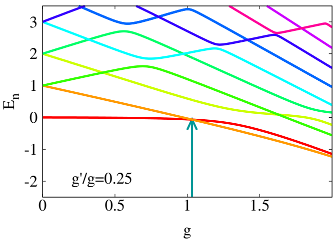

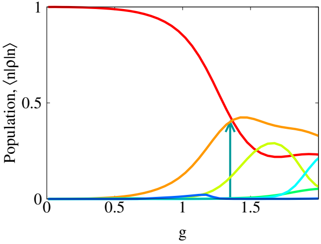

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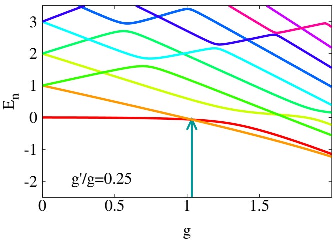

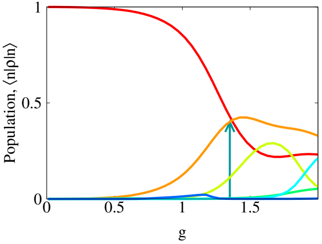

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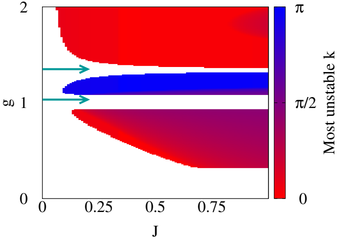
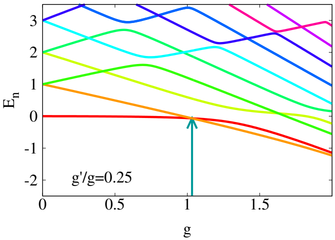
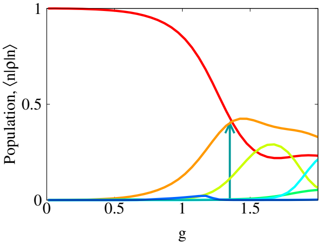


$g' \neq g$, Level crossings

- For $g' \neq g$, Δ can swap sign . . . and loss can invert populatoin





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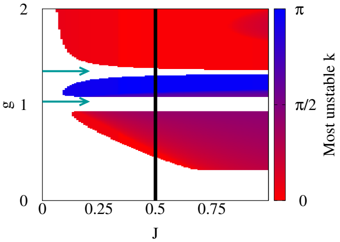

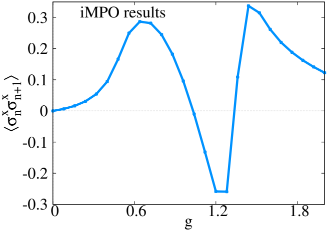

Collective dissipation

1. Effects of dissipation on collective behaviour
   - Coherently driven JCHM (Mean-Field)
   - Parametrically driven BHM (MF and MPO)
   - Parametrically driven RHM (MF and MPO)

2. Effects of collective behaviour on dissipation
   - Coupled qubit-cavity systems
   - Collective coupling to baths

Collective dephasing

- Real environment is not Markovian
  - [Carmichael & Walls JPA '73] Requirements for correct equilibrium
  - [Ciuti & Carusotto PRA '09] Dicke SR and emission
- Cannot assume fixed κ, γ
- Phase transition $\rightarrow$ soft modes

- Bath density of states $J(\nu) = \sum_q \gamma_q^2 \delta(\nu - \beta_q) \propto 1/\nu$
- Spectrum ϵ_α of H_0: Linewidth $\propto J(\epsilon_\alpha - \epsilon_\beta)$

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Example: Dicke model linewidth:

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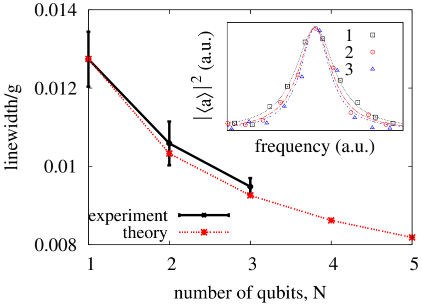
Collective dephasing of transmons



- Expt: collective bath
- Many baths — cross terms, non-monotonic

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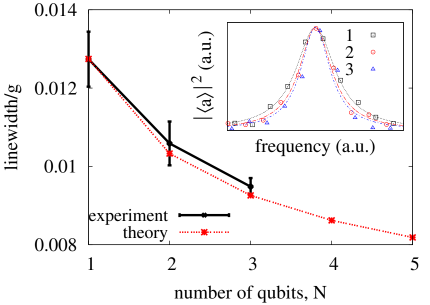
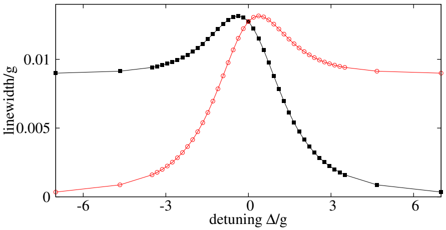


- Detuing dependence of linewidths — $\sqrt{\Delta^2 + Ng^2}$.



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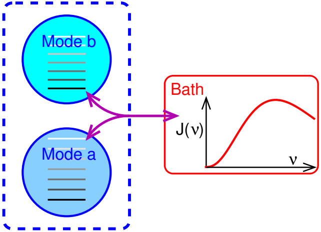
Toy problem: two bosonic modes

- Basic problem: Emission from thermal bath



$$H = \omega_a \hat{\psi}_a^\dagger \hat{\psi}_a + \omega_b \hat{\psi}_b^\dagger \hat{\psi}_b + H_{\text{Bath}} + (\varphi_a^* \hat{\psi}_a^\dagger + \varphi_b^* \hat{\psi}_b^\dagger) \sum_i g_i \hat{c}_i + \text{H.c.}$$

Toy problem: naïve solutions

- Two "expected" behaviours:
  - At resonance: "weak lasing" — coupling to bath dominates

$$\frac{d}{dt}\rho = \Gamma^{\downarrow}\mathcal{L}[\varphi_a\hat{\psi}_a + \varphi_b\hat{\psi}_b] + \Gamma^{\uparrow}\mathcal{L}[\varphi_a^*\hat{\psi}_a^\dagger + \varphi_b^*\hat{\psi}_b^\dagger]$$

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- Explicit derivation → Redfield theory

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- Solve via Laplace transform. Find $F_{ij}(t) = \langle \hat{\psi}_i^\dagger(t) \hat{\psi}_j(t) \rangle$
- Steady state:
- Time evolution — $F_{ab}(t) \sim \exp(-\alpha \Delta^2 t)$

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  - (individual always wrong)
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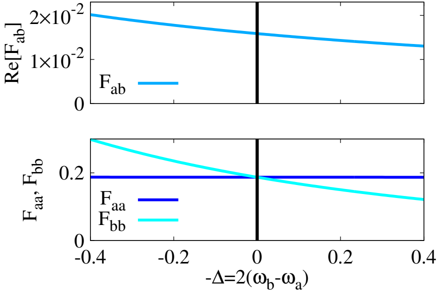

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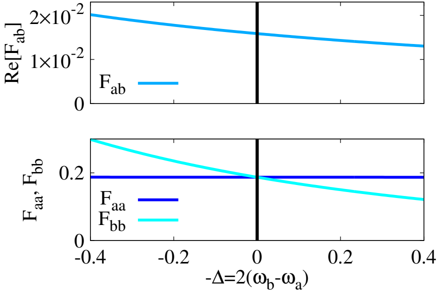

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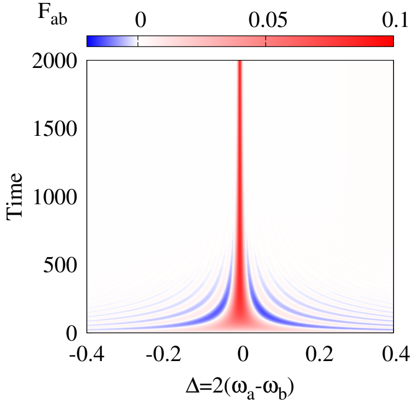

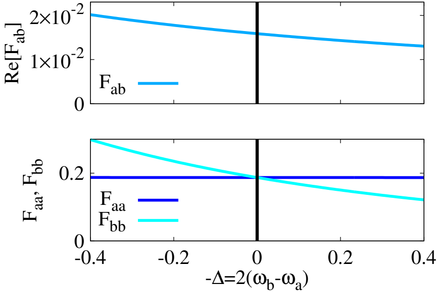

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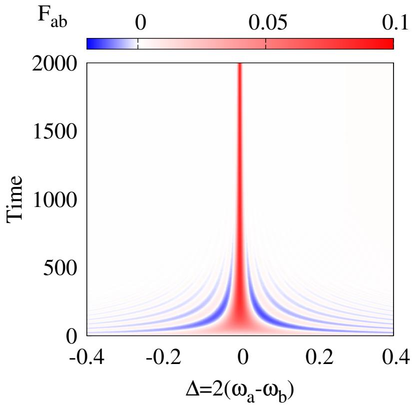

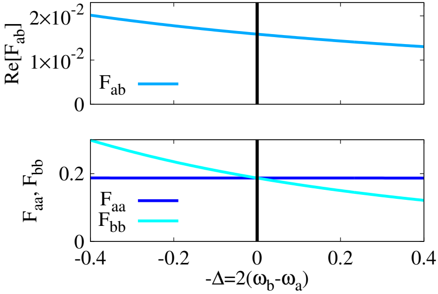

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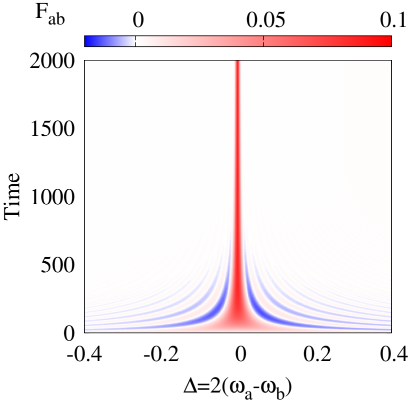

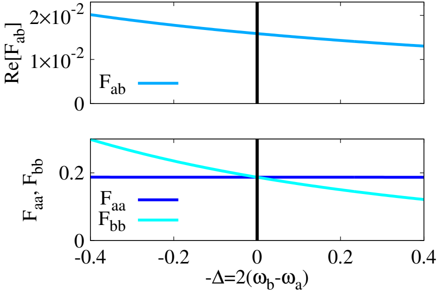

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Unsecularised Redfield theory:

$$\partial_t \rho = -i[\hat{H}, \rho] + \sum_{ij} \varphi_i^* \varphi_j \Big[K_{ij}^{\downarrow} \left(2\hat{\psi}_j \rho \hat{\psi}_i^\dagger - [\rho, \hat{\psi}_i^\dagger \hat{\psi}_j]_+ \right) + K_{ij}^{\uparrow} \left(2\hat{\psi}_j^\dagger \rho \hat{\psi}_i - [\rho, \hat{\psi}_i \hat{\psi}_j^\dagger]_+ \right) \Big].$$

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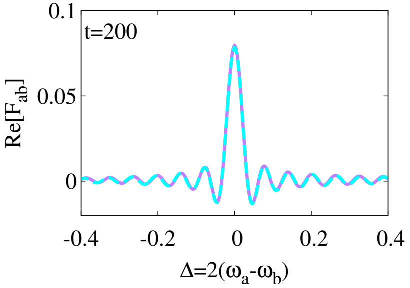

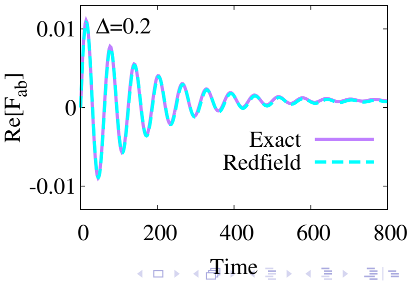

Toy problem: Secularisation and stability

- Non-Linblad form: negative eigenvalues of $L_{ij}^{\uparrow,\downarrow}$.
- Check stability: consider $f = (F_{aa}, F_{bb}, \Re[F_{ab}], \Im[F_{ab}])$

$$\partial_t \mathbf{f} = -\mathbf{M}\mathbf{f} + \mathbf{f}_0$$

- Eigenvalues of $\mathbf{M}$ exist in closed form:
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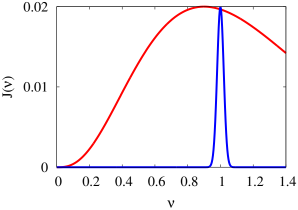

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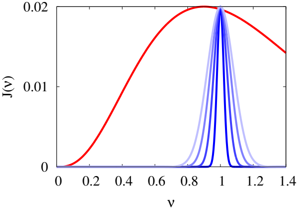

Beyond Redfield: Schrödinger picture Bloch Redfield

- Is BR the best (time-local) theory we can find?
- Hints it is not:
  - Eigenvalues of $\mathbf{M}$ vs exact sol'n near $\Delta = 0$.
  - Sum rule [Salmilehto *et al.* PRA '12; Hell *et al.* PRB '14]:

    "For $\hat{X}$ s.t. $[\hat{X}, \hat{H}_{\text{system-bath}}] = 0$, then $\partial_t \langle \hat{X} \rangle$ should match closed system."
- Alternate approach:
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  - BR assumes $\tilde{\rho}(t)$ is "slow" in interaction picture
  - Asymptotically $\rho(t)$ is steady in Schrödinger picture
  - Assume instead $\rho(t)$ is slow in Schrödinger picture
- "Schrödinger picture Bloch Redfield:"
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  - Satisfies sum rule

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    "For $\hat{X}$ s.t. $[\hat{X}, \hat{H}_{\text{system-bath}}] = 0$, then $\partial_t \langle \hat{X} \rangle$ should match closed system."

  - Here, $\langle \hat{X} \rangle = \varphi_b^2 F_{aa} + \varphi_a^2 F_{bb} - 2\varphi_a \varphi_b F'_{ab}$. **Fails**
- Alternate approach:
  - BR assumes $\tilde{\rho}(t)$ is "slow" in interaction picture
  - Asymptotically $\rho(t)$ is steady in Schrödinger picture
  - Assume instead $\rho(t)$ is slow in Schrödinger picture
- "Schrödinger picture Bloch Redfield:"
  - Correct Δ^2 expansion
  - Satisfies sum rule

Beyond Redfield: Schrödinger picture Bloch Redfield

- Is BR the best (time-local) theory we can find?
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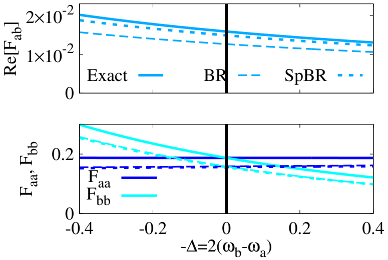

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ICSCE8

Edinburgh, 25th–29th April, 2016.

Plenary speakers: Ataç İmamoğlu, Peter Zoller.

Invited speakers: Ehud Altman, Mete Atatüre, Natasha Berloff, Charles Bardyn, Jacqueline Bloch, Iacopo Carusotto, Cristiano Ciuti, Michele Devoret†, Thomas Ebbesen, Thiery Giamarchi, Jan Klärs, Dmitry Krizhanovskii, Xiaoqin (Elaine) Li, Peter Littlewood, Allan MacDonald, Francesca Marchetti, Keith Nelson, Pavlos Lagoudakis, Vivien Zapf. († To be confirmed)

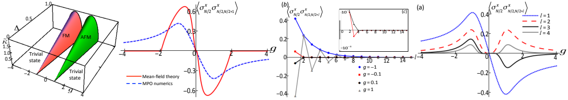
Early-bird registration & abstract deadline: 31st January 2016.

Final registration deadline: 31st March 2016.

`http://www.st-andrews.ac.uk/~icsce8`

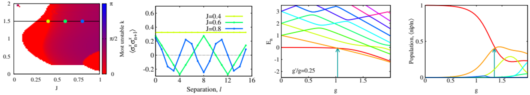
Summary

- Transverse field Ising model



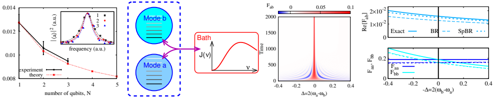
Joshi *et al.* PRA '13

- Rabi Hubbard model



Schiró *et al.* arXiv:1503.04456

- Collective effects in dephasing



Nissen *et al.* PRL '13; Eastham *et al.* arXiv:1508.04744

Questions

- Collective dynamics beyond local dissipation.
  - Many site analogues of Spin-Boson transitions?
  - Critical behaviour in open lattice models — demonstrate non Hohenberg-Halperin classes of models.
- Bistability, limit cycles, beyond mean-field
- Organizing principles of driven-dissipative system attractors