Non-equilibrium phases of matter-light systems

Jonathan Keeling

SUPA®

University of St Andrews 1413-2013

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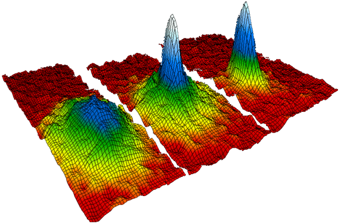
Coherent states of matter and light

Atomic BEC $T \sim 10^{-7}$K



[Anderson *et al.* Science '95]

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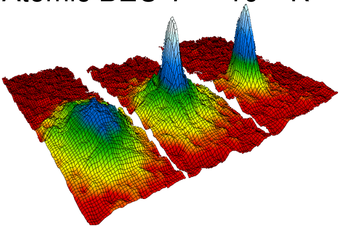
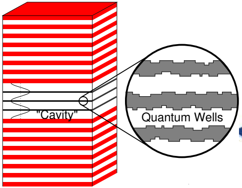
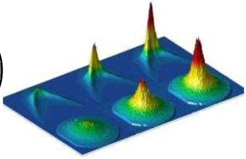


[Anderson *et al.* Science '95]

Polariton Condensate $T \sim 20$K





[Kasprzak *et al.* Nature, '06]

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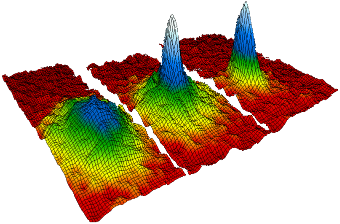

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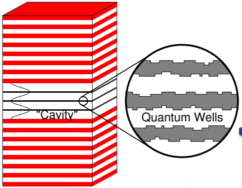

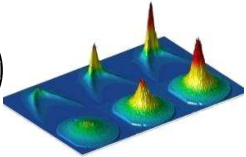

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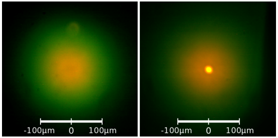
Photon Condensate
$T \sim 300$K



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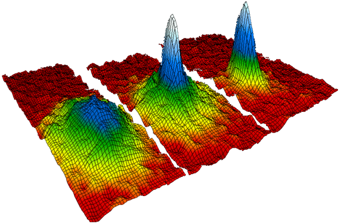

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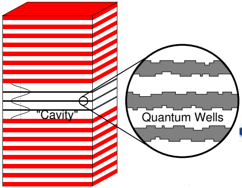

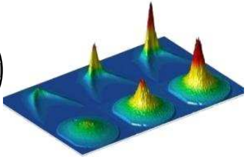

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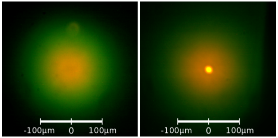
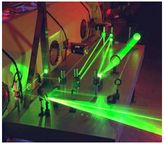


[Klaers *et al.* Nature, '10]

Laser
$T \sim ?, < 0, \infty$



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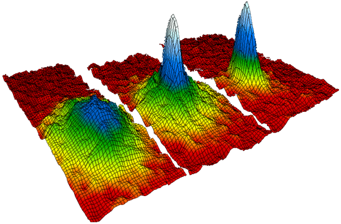

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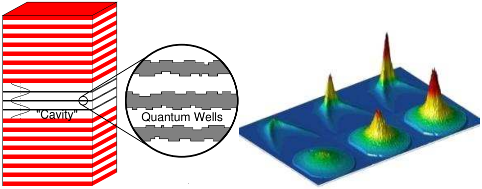

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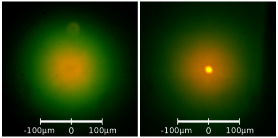

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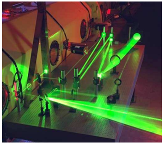
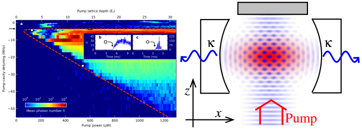


Superradiance transition
$T \sim 0$



[Baumann *et al.* Nature, '10]

Driven systems

Open quantum system

$$\partial_t \rho = -i[\hat{H}, \rho] + \sum_i \kappa_i \mathcal{L}[\hat{X}_i], \qquad \mathcal{L}[\hat{X}_i] = 2\hat{X}_i \rho \hat{X}_i^\dagger - \hat{X}_i^\dagger \hat{X}_i \rho - \rho \hat{X}_i^\dagger \hat{X}_i$$

Need drive to balance loss

1. External **coherent** drive:

$$\hat{H} \to \hat{H} + \hat{V}\cos(\Omega t)$$

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- $\tilde{\hat{H}} = e^{-i\Omega \hat{N} t} \hat{H} e^{i\Omega \hat{N} t} - \Omega \hat{N}$
- Neglect fast $e^{2i\Omega t}$ terms — fast
- Rotating frame — breaks detailed balance with bath.

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Non-equilibrium steady state

2. External **incoherent** drive:

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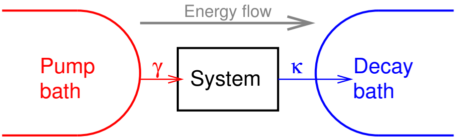

- Energy flow through system

- Not thermodynamics — attractors of dynamics
  - Stationary points — extrema of energy?
  - Nontrivial attractors

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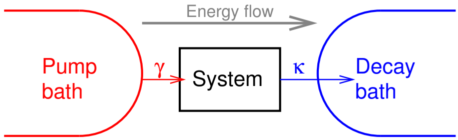

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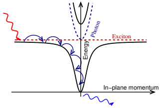

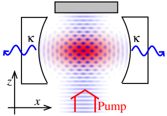

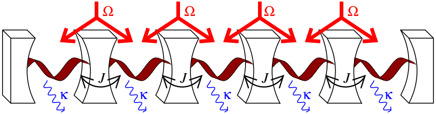

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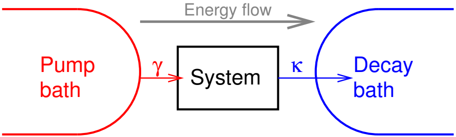

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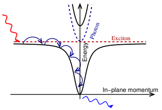

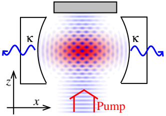

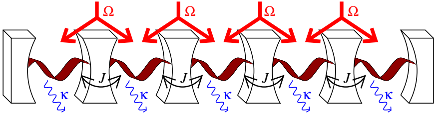

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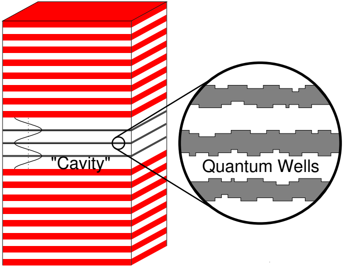

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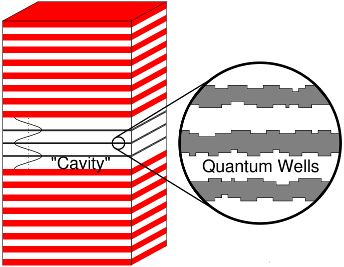

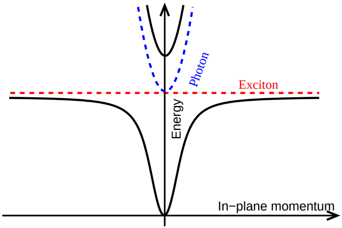

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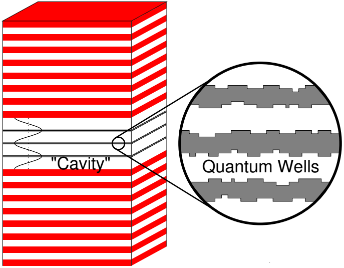

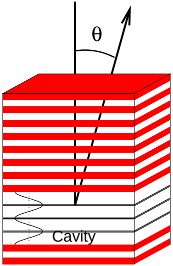

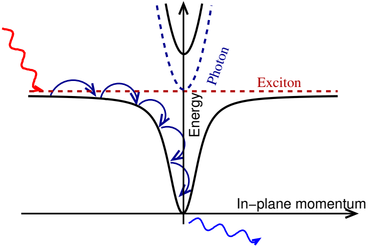

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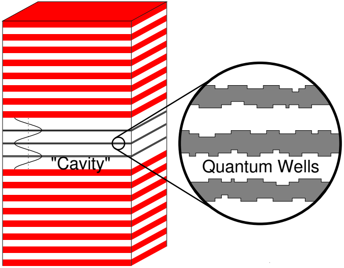

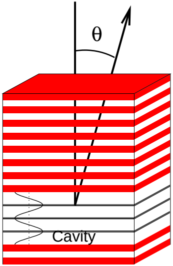

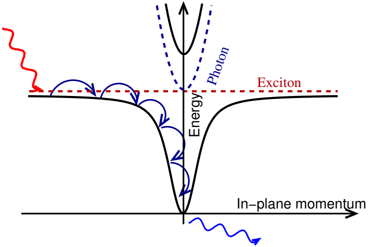

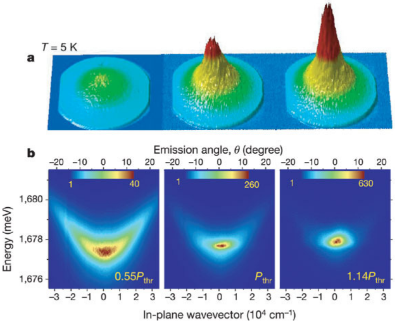

[Kasprzak, *et al.* Nature, '06]

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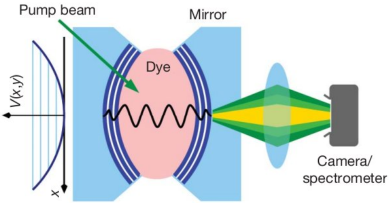

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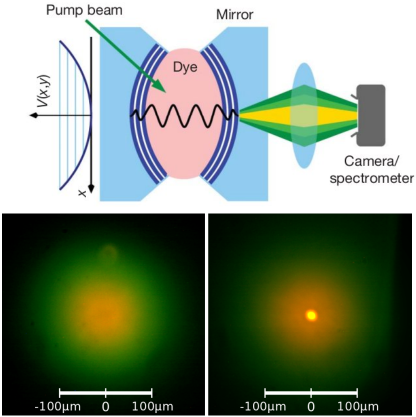

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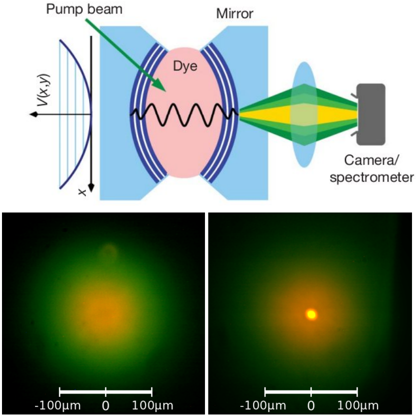

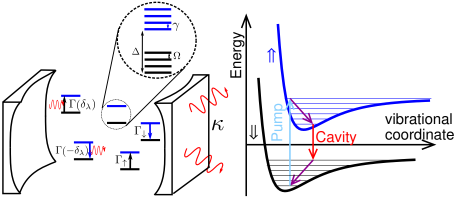

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Atoms and light

- **Cold atoms, optical lattice**: static light, dynamic matter
- **Quantum optics**: (Static) atoms, quantum dynamics of light.
- Coupling atomic motion to optical cavity

[Experiments: MIT, ETH, CQT Singapore, Stanford, Hamburg]

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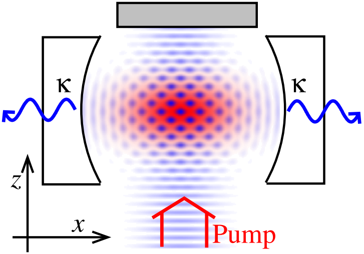

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Overview

1. Non-equilibrium phases of matter-light systems
   - Physical systems
2. Cold atoms in optical cavities – Dicke model
   - Superradiance and the Dicke model
   - Cold atoms & open Dicke model
   - Dynamics and attractors
   - Beyond Dicke: Chaotic dynamics
3. Beyond mean-field theory
   - Coupled cavity arrays
   - Multimode cavities

Cold atoms in optical cavities – Dicke model

Coupling many atoms to light

Old question: *What happens to radiation when many atoms interact "collectively" with light.*

Superradiance — dynamical and steady state.

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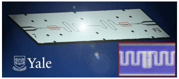
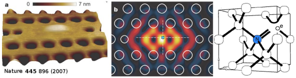
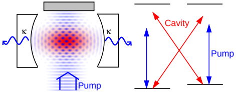
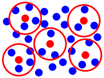
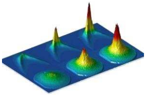
New relevance

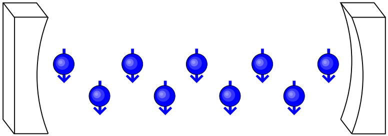
- Superconducting qubits 

- Quantum dots & NV centres 

- Ultra-cold atoms 

- Rydberg atoms/polaritons 
- Microcavity Polaritons 

Dicke model and Dicke-Hepp-Lieb transition



$$H = \omega \hat{a}^\dagger \hat{a} + \sum_i \omega_0 S_i^z + g(\hat{a} + \hat{a}^\dagger)(S_i^+ + S_i^-)$$

- Coherent state: $|\hat{a}\rangle \to e^{\alpha \hat{a}^\dagger + \eta S^+}|\Omega\rangle$
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[Hepp, Lieb, Ann. Phys. '73]

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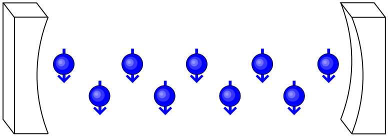

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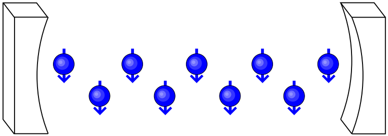

$$H = \omega \hat{a}^\dagger \hat{a} + \sum_i \omega_0 S_i^z + g(\hat{a} + \hat{a}^\dagger)(S_i^+ + S_i^-)$$
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- Coherent state: $|\hat{a}\rangle \to e^{\alpha \hat{a}^\dagger + \eta S^+}|\Omega\rangle$
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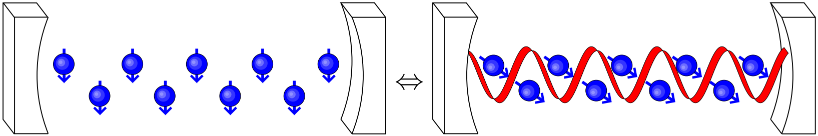

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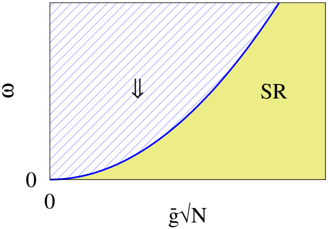
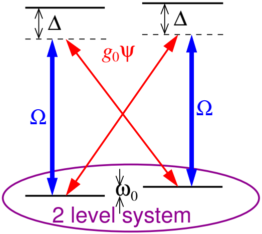
Non-zero cavity field if: $4Ng^2 > \omega\omega_0$



[Hepp, Lieb, Ann. Phys. '73]

Raman scheme, decoupling g, ω_0



$$H = \omega_0 S^z + g(\hat{a} + \hat{a}^\dagger)(S^- + S^+) + \omega \hat{a}^\dagger \hat{a}$$

- 2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$
- Coupling $g = \dfrac{g_0 \Omega}{2\Delta}$
- Rotating frame of pump, $\omega = \omega_{\text{cavity}} - \omega_{\text{pump}}$

[Dimer *et al.* PRA '07]

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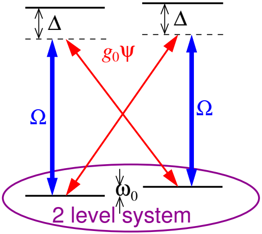

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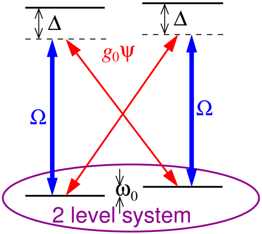

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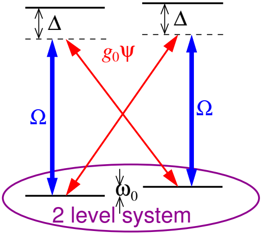

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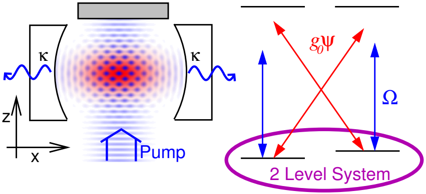
Open system Dicke transition

- $4Ng^2 > \dfrac{(\omega^2 + \kappa^2)}{\omega}\omega_0$
- Need $H = \ldots + g(S^+ a^\dagger + \text{H.c.}) + \ldots$

[Dimer *et al.* PRA '07]

Mapping transverse pumping to Dicke model



Reduced basis: $\Psi(x,z) \propto \begin{cases} 1 & \Downarrow \\ \cos(qz)\cos(qz) & \Uparrow \end{cases}$

$$H = \omega \hat{a}^\dagger \hat{a} + \omega_0 S^z + g(\hat{a} + \hat{a}^\dagger)(S^- + S^+)$$

[Baumann *et al* Nature '10]

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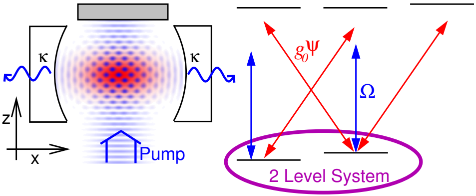

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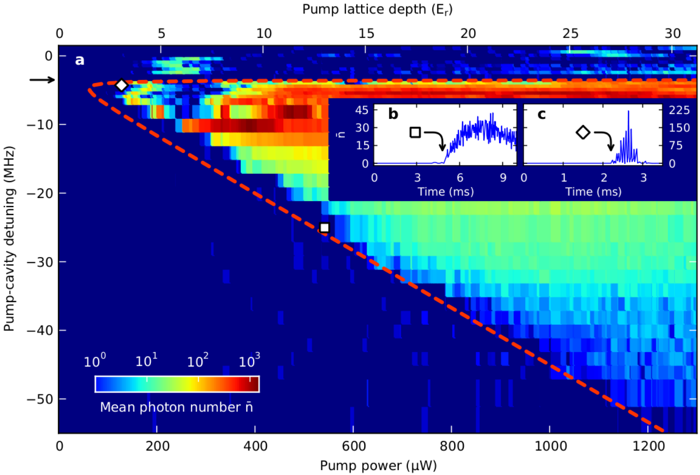
"Feedback" due to extra states $U = -\frac{g_0^2}{4\Delta}$

[Baumann *et al* Nature '10]

Experimental phase diagram



- Pump power $g \propto \sqrt{\text{Power}}$
- NB $\omega_{\text{Pump}} - \omega_{\text{cavity}} = -\omega$

[Baumann *et al* Nature '10]

Cold atoms in optical cavities – Dicke model

Classical dynamics of the extended Dicke model

Open dynamical system:

$$H = \omega \hat{a}^\dagger \hat{a} + \omega_0 S^z + g(\hat{a} + \hat{a}^\dagger)(S^- + S^+) + U S_z \hat{a}^\dagger \hat{a}.$$

$$\partial_t \rho = -i[H, \rho] + \kappa \mathcal{L}[\hat{a}]$$

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$$\partial_t \rho = -i[H, \rho] + \kappa \mathcal{L}[\hat{a}]$$

Classical EOM
($|\mathbf{S}| = N/2 \gg 1$)

$$\begin{aligned}
\dot{S}^- &= -i(\omega_0 + U|\alpha|^2)S^- + 2ig(\alpha + \alpha^*)S^z \\
\dot{S}^z &= ig(\alpha + \alpha^*)(S^- - S^+) \\
\dot{\alpha} &= -[\kappa + i(\omega + US^z)]\,\alpha - ig(S^- + S^+)
\end{aligned}$$

Steady state phase diagram

$$0 = i(\omega_0 + U|\alpha|^2)S^- + 2ig(\alpha + \alpha^*)S^z$$
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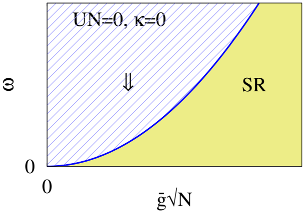

See also Domokos and Ritsch PRL '02, Domokos *et al.* PRL '10

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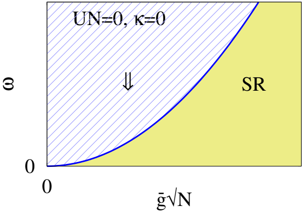
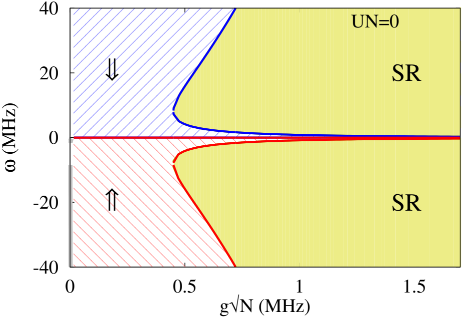
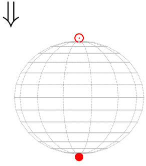
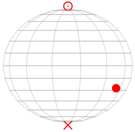






SR(A): $S_y = 0$



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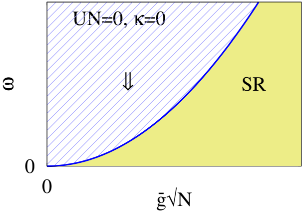

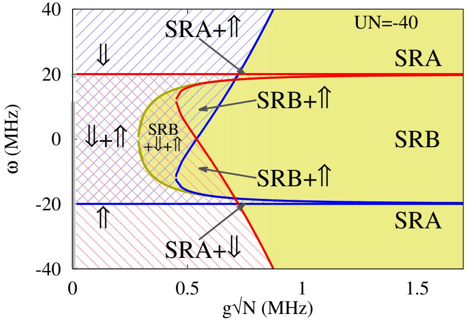

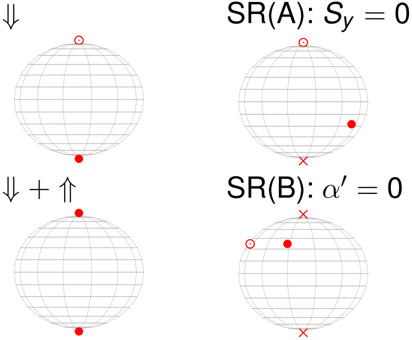

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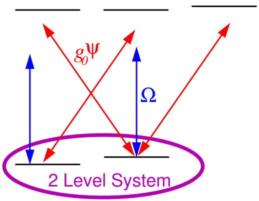
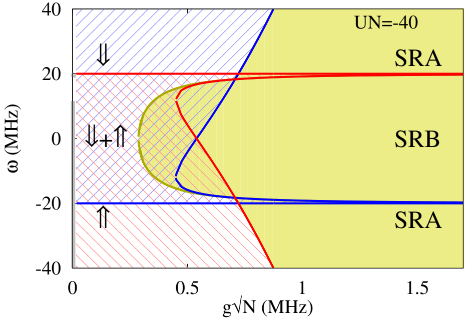
Regions without fixed points

Changing U:



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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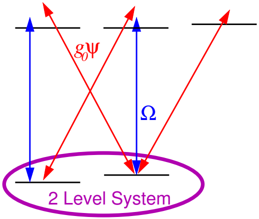

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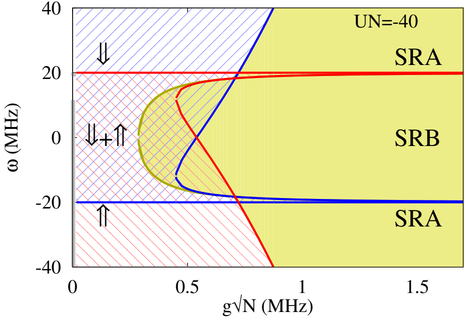
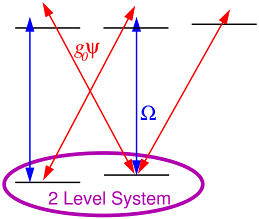


Regions without fixed points

Changing *U*:



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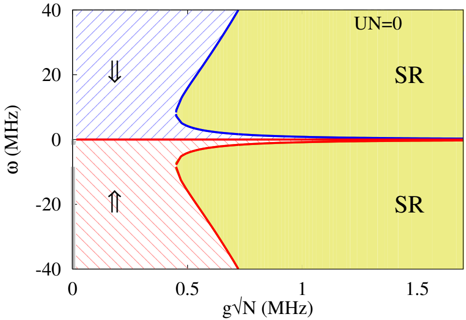

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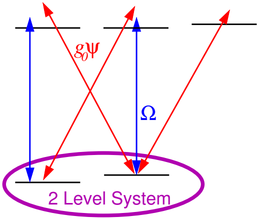

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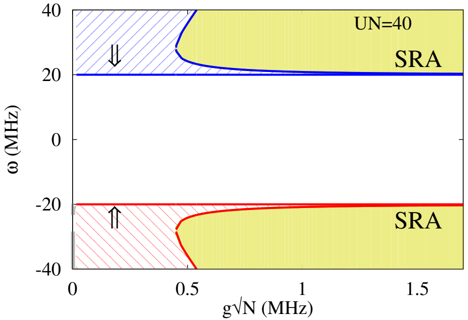

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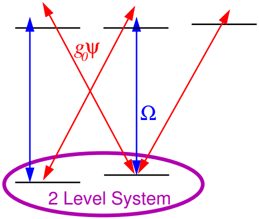

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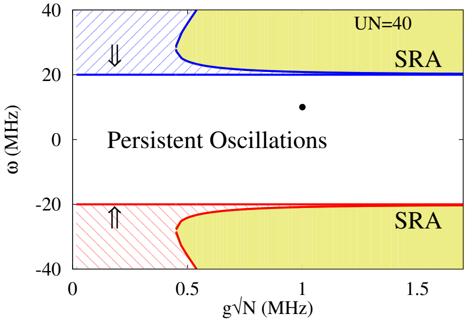
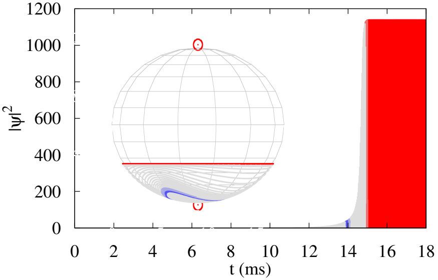
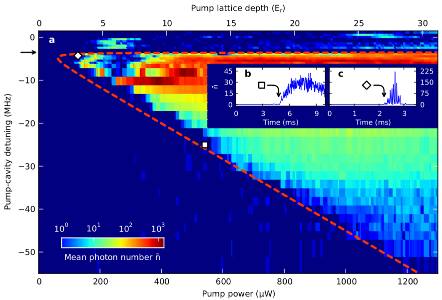




Bosons beyond Dicke — single mode

So far $\Psi(\mathbf{r}) = \chi_0 + \chi_1 2\cos(qx)\cos(qz) \longrightarrow \mathbf{S} = \chi^\dagger\vec{\sigma}\chi$.
Generally $\Psi(\mathbf{r}) = \sum_{\mathbf{n}} \chi_{\mathbf{n}} e^{iq\mathbf{n}\cdot r}$.



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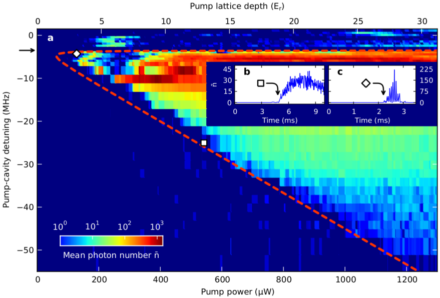

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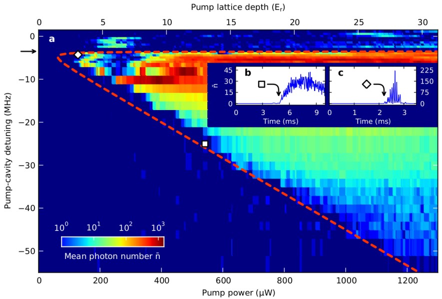

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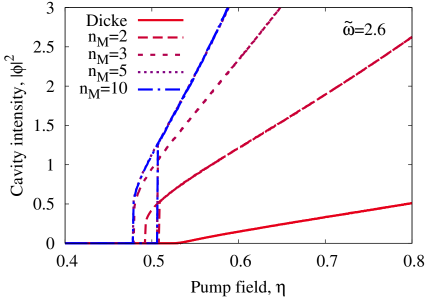
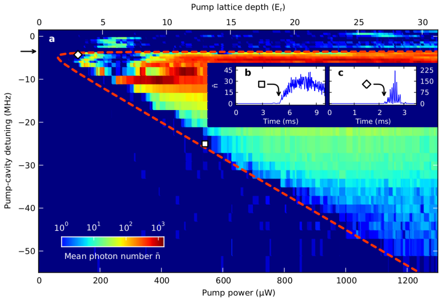
Truncate $|\mathbf{n}| < n_M$ — Hysteresis at intermediate ω





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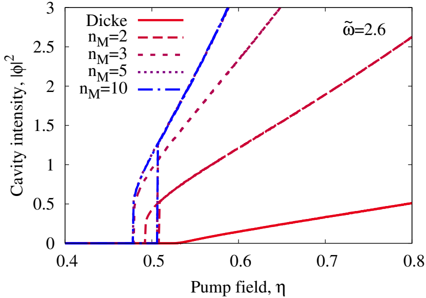
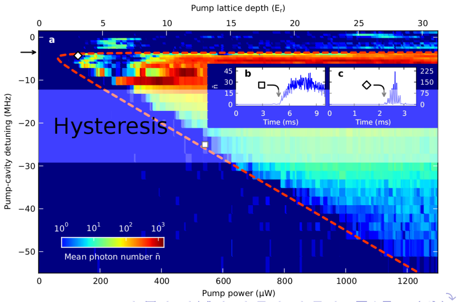
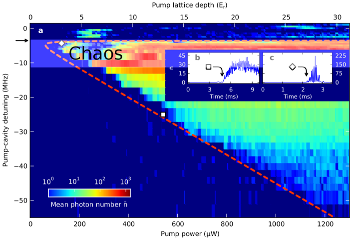
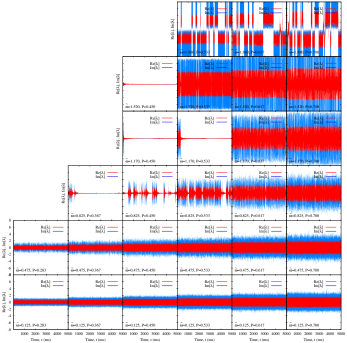




Bosons beyond Dicke — chaos

Near resonance: irregular dynamics
(NB $\omega_{\text{Pump}} - \omega_{\text{cavity}} = -\omega$)





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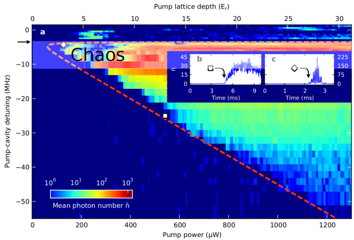

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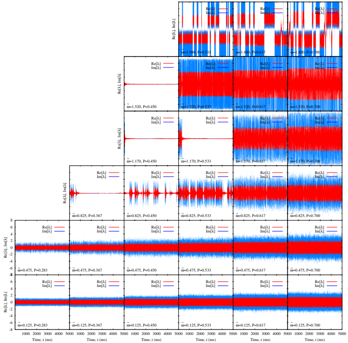
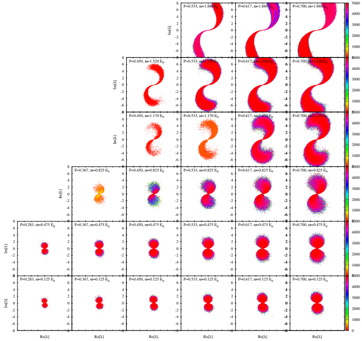
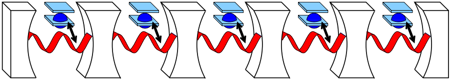




Beyond mean-field theory

Coupled cavity arrays

- Control photon dispersion — lattice



[Hartmann *et al.* Nat. Phys. '06; Greentree *et al.* *ibid* 06; Angelakis *et al.* PRA '07]

- X-Hubbard Model, $\hat{H} = \sum_i \hat{H}_{X,site} - J \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j$

  [X=Bose, Jaynes-Cummings, Rabi, ...]

Coupled cavity arrays

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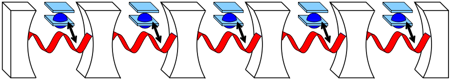


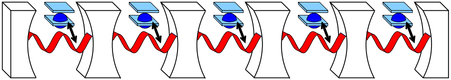
[Hartmann *et al.* Nat. Phys. '06; Greentree *et al. ibid* 06; Angelakis *et al.* PRA '07]

- X-Hubbard Model, $\hat{H} = \sum_i \hat{H}_{X,site} - J \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j$

  [X=Bose, Jaynes-Cummings, Rabi, ...]

Coupled cavity arrays

- Control photon dispersion — lattice



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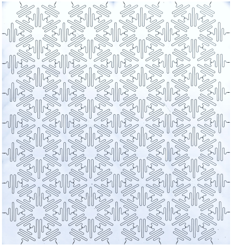
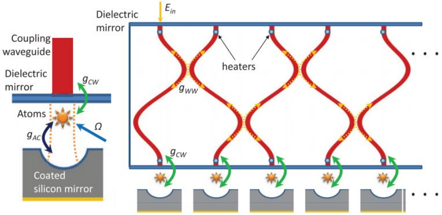
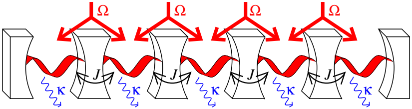


[Underwood *et al.* PRA '12; Nat. Phys '12]



[Lepert *et al.* NJP '11; APL '13]

e.g. Parametric pumping → Ising model



[Bardyn & Immamoglu, PRL '12]

$$\hat{H} = -J \sum \Big[\hat{\tau}_i^+ \hat{\tau}_{i+1}^- + \hat{\tau}_{i+1}^+ \hat{\tau}_i^- + g \hat{\tau}_i^z + \Delta \left(\hat{\tau}_i^+ \hat{\tau}_{i+1}^+ + \hat{\tau}_{i+1}^- \hat{\tau}_i^- \right) \Big]$$

[Joshi, Nissen, Keeling. PRA '13]

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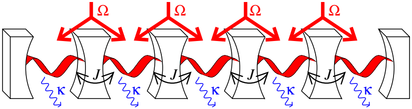

[Bardyn & Immamoglu, PRL '12]

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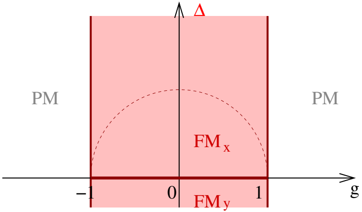
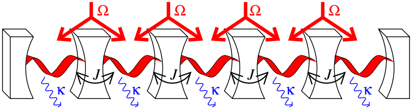
Ground state:



[Joshi, Nissen, Keeling. PRA '13]

e.g. Parametric pumping → Ising model



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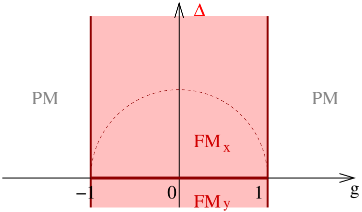
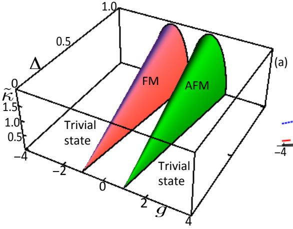
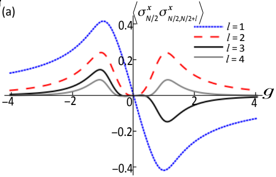


[Joshi, Nissen, Keeling. PRA '13]

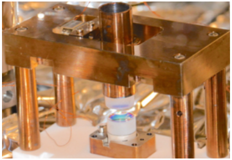
Open system:

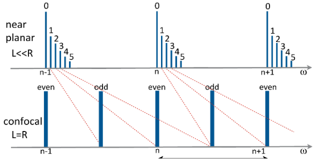
$$\partial_t\rho = -i[\hat{H},\rho] + \sum_i \kappa\mathcal{L}[\hat{\tau}_i^-]$$





Multimode cavity





[Kollár *et al.* NJP ’15]

- Degenerate limit, on-axis pump

$$i\partial_t \Psi = -\frac{\nabla^2}{2m}\Psi + \int d\mathbf{r}'\, U(\mathbf{r}-\mathbf{r}')\left[|\Psi(\mathbf{r}')|^2 + |\Psi(-\mathbf{r}')|^2\right]\Psi(\mathbf{r}),$$

with $U(\mathbf{s}) = U_0\left(\delta(\mathbf{s}) + \frac{\sin(s^2/\ell^2)}{\pi\ell^2}\right)$,

[Staffini, Lev, Keeling *et al.* in prep.]

- Degenerate limit, transverse pump:

$$i\partial_t \psi_{\mathbf{k}} = \left[\Delta + \lambda(|\mathbf{k}| - q)^2\right]\psi_{\mathbf{k}} + U_{\text{contact}} \sum_{\mathbf{k}',\mathbf{q}} \psi^\dagger_{\mathbf{k}'+\mathbf{q}} \psi_{\mathbf{k}'} \psi_{\mathbf{k}-\mathbf{q}}$$

[Gopalakrishnan, Lev, Goldbart. N. Phys ’09]

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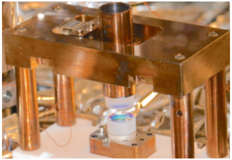
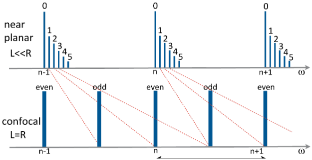
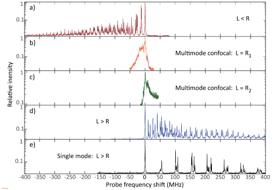


[Kollár *et al.* NJP ’15]





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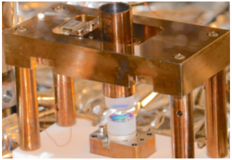

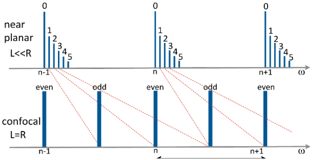

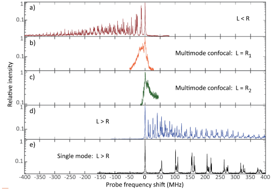

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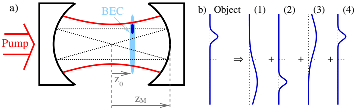

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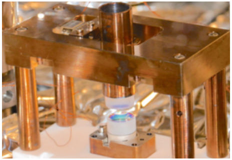

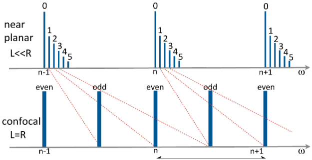

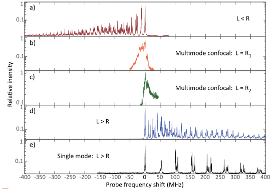

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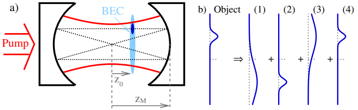

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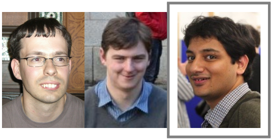
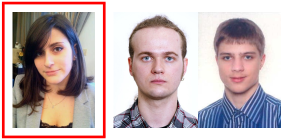
[Gopalakrishnan, Lev, Goldbart. N. Phys '09]

Acknowledgements

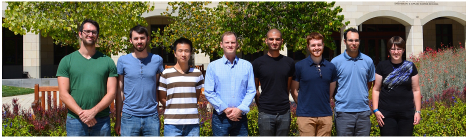
GROUP:





COLLABORATORS: Bhaseen (KCL), Simons (Cambridge),

Lev group (Stanford)



FUNDING:

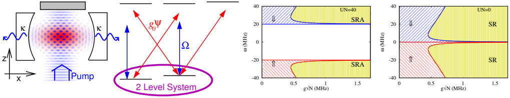
topnes Topological Protection and Non-Equilibrium States in Strongly Correlated Electron Systems

EPSRC Engineering and Physical Sciences Research Council

The Leverhulme Trust

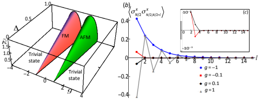
Summary

- Collective behaviour in driven-disspative systems
- Open system dynamics of Dicke model



JK *et al.* PRL '10, Bhaseen *et al.* PRA '12

- Beyond Dicke model: hysteresis, chaos
  [Staffini *et al.* in preparation]
- CCA — non-equilibrium transverse field Ising model

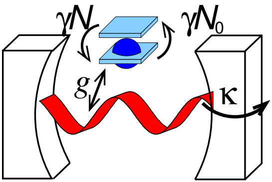


Joshi *et al.* PRA '13, Schiro *et al.* arXiv

- Multimode physics $\rightarrow$ beyond mean-field theory
  [Staffini, Lev *et al.* in preparation]

"Textbook" Laser: Maxwell Bloch equations

$$H = \omega_0 \hat{a}^\dagger \hat{a} + \sum_\alpha \epsilon_\alpha \sigma^z_\alpha + g_{\alpha,\mathbf{k}} \left(\hat{a}\sigma^+_\alpha + \hat{a}^\dagger \sigma^-_\alpha \right)$$



Maxwell-Bloch eqns:

$\alpha = \langle \hat{a} \rangle, P = -i\langle \sigma^- \rangle, N = 2\langle \sigma^z \rangle$

$$\partial_t \alpha = -i\omega_0 \alpha - \kappa\alpha + \sum_\alpha g_\alpha P_\alpha$$

$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P_\alpha + g_\alpha \alpha N_\alpha$$

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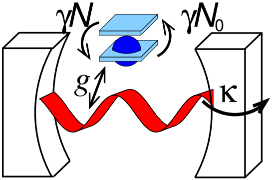

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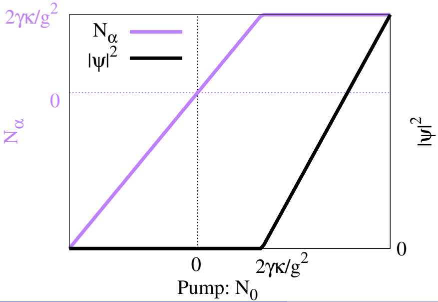

$|\alpha|^2 > 0$ if $N_0 g^2 > 2\gamma\kappa$