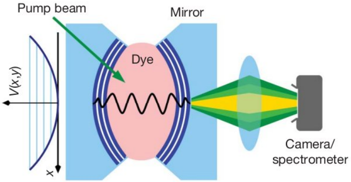
Spatial dynamics, thermalization and breakdown of thermalization in photon condenstaes

Jonathan Keeling

University of St Andrews
1413-2013

Windsor, August 2015

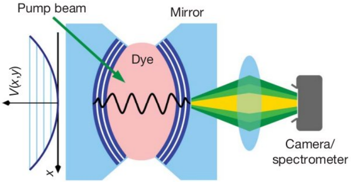
Photon BEC experiments



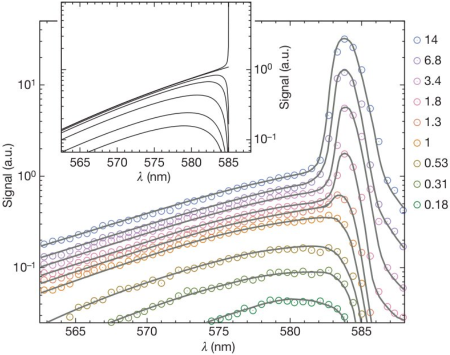
- (Curved) microcavity
- R6G dye (in solvent)
- Thermalisation of light
- Condensation at $P > P_{th}$

[Klaers et al, Nature, 2010]

Photon BEC experiments

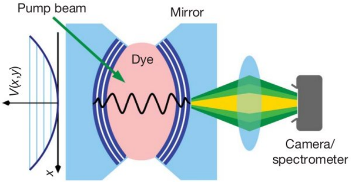


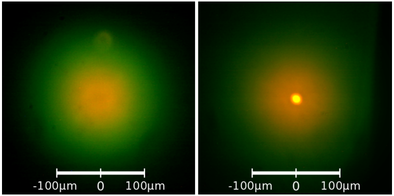
- (Curved) microcavity
- R6G dye (in solvent)
- Thermalisation of light
- Condensation at $P > P_{th}$

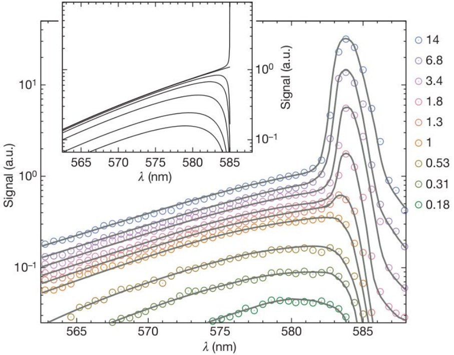


[Klaers et al, Nature, 2010]

Photon BEC experiments





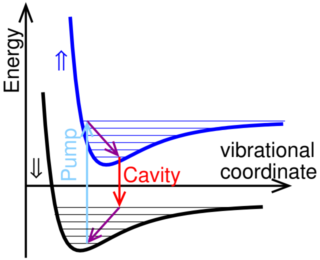


- (Curved) microcavity
- R6G dye (in solvent)
- Thermalisation of light
- Condensation at $P > P_{\text{th}}$

[Klaers et al, Nature, 2010]

Relation to dye laser

4 Level Dye Laser

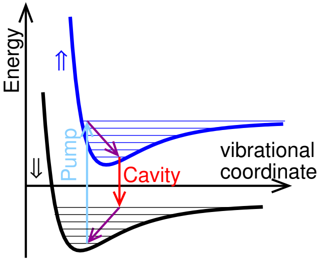


- No electronic inversion
- No strong coupling

- No single cavity mode
  - Condensate mode is not maximum gain
  - Gain/Absorption in balance
- Thermalised many-mode system

Relation to dye laser

4 Level Dye Laser

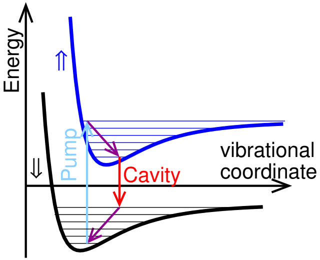


- No electronic inversion
- No strong coupling

- No single cavity mode
  - Condensate mode is not maximum gain
  - Gain/Absorption in balance
- Thermalised many-mode system

Relation to dye laser

4 Level Dye Laser



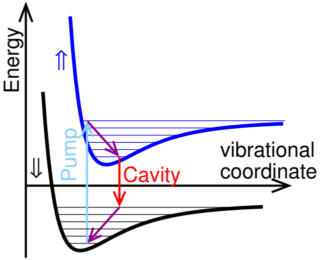
- No electronic inversion
- No strong coupling

But:

- No single cavity mode
  - Condensate mode is not maximum gain
  - Gain/Absorption in balance
- Thermalised many-mode system

Relation to dye laser

4 Level Dye Laser



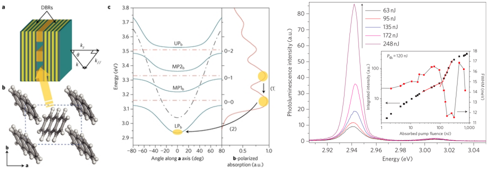
- No electronic inversion
- No strong coupling

But:

- No single cavity mode
  - Condensate mode is not maximum gain
  - Gain/Absorption in balance
- Thermalised many-mode system

Motivation: organic polariton condensates

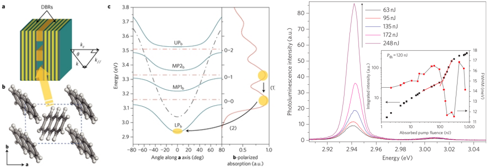
- Anthracene Polariton Lasing
  $T \sim 300$K



[Kena Cohen and Forrest, Nat. Photon '10]

Motivation: organic polariton condensates

- Anthracene Polariton Lasing
  $T \sim 300$K



Q1. Vibrational replicas?

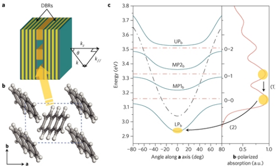
Q2. Relevance of disorder?

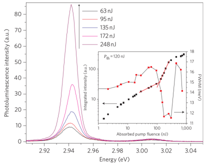
Q3. Lasing vs condensation?

[Kena Cohen and Forrest, Nat. Photon '10]

Motivation: organic polariton condensates

- Anthracene Polariton Lasing
  $T \sim 300$K





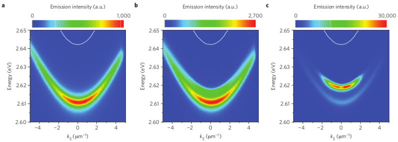
Q1. Vibrational replicas?

Q2. Relevance of disorder?

Q3. Lasing vs condensation?

[Kena Cohen and Forrest, Nat. Photon '10]

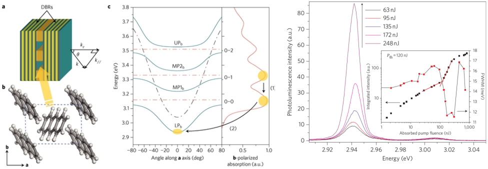
- Polariton condensates, other materials, e.g. polymers:



[Plumhoff *et al.* Nat. Materials '14, Daskalakis *et al.* ibid '14]

Motivation: organic polariton condensates

- Anthracene Polariton Lasing
  $T \sim 300$K



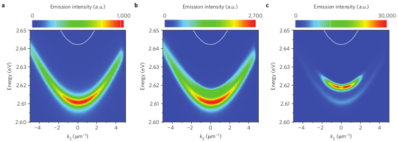
Q1. Vibrational replicas?

Q2. Relevance of disorder?

Q3. Lasing vs condensation?

[Kena Cohen and Forrest, Nat. Photon '10]

- Polariton condensates, other materials, e.g. polymers:



[Plumhoff *et al.* Nat. Materials '14, Daskalakis *et al.* ibid '14]

Q1. Frenkel to Wannier crossover?

Q2. Optimal vibrational properties?

Q3. Nonlinearities?

1. Introduction to Photon BEC

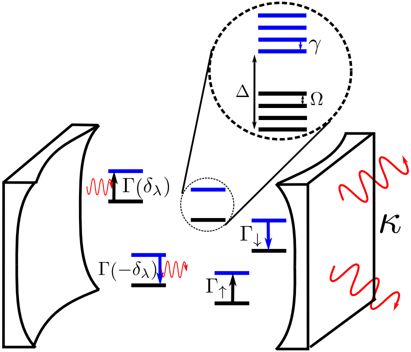
2. Modelling photon BEC
   - Steady state
   - Approach to steady state

3. Spatial profile
   - Steady state
   - Spatial oscillations

Modelling

$$H_{\text{sys}} = \sum_m \omega_m \psi_m^\dagger \psi_m + \sum_\alpha \left[\frac{\epsilon}{2} \sigma_\alpha^z + g \left(\psi_m \sigma_\alpha^+ + \text{H.c.} \right) \right]$$
$$+ \sum_\alpha \Omega \left\{ b_\alpha^\dagger b_\alpha + \sqrt{S} \sigma_\alpha^z \left(b_\alpha^\dagger + b_\alpha \right) \right\}$$

- **2D** harmonic oscillator
  $\omega_m = \omega_{\text{cutoff}} + m\omega_{H.O.}$
- Incoherent processes: excitation, decay, loss, vibrational thermalisation.
- Weak coupling, perturbative in g



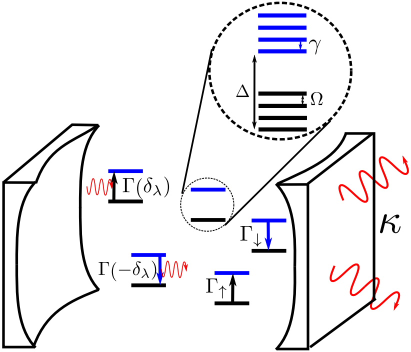
Modelling

$$H_{\text{sys}} = \sum_m \omega_m \psi_m^\dagger \psi_m + \sum_\alpha \left[\frac{\epsilon}{2} \sigma_\alpha^z + g \left(\psi_m \sigma_\alpha^+ + \text{H.c.} \right) \right]$$

$$+ \sum_\alpha \Omega \left\{ b_\alpha^\dagger b_\alpha + \sqrt{S} \sigma_\alpha^z \left(b_\alpha^\dagger + b_\alpha \right) \right\}$$

- **2D** harmonic oscillator

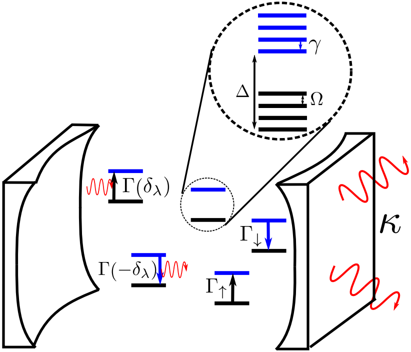
  $\omega_m = \omega_{\text{cutoff}} + m\omega_{H.O.}$
- Incoherent processes: excitation, decay, loss, vibrational thermalisation.
- Weak coupling, perturbative in g



Modelling

$$H_{\text{sys}} = \sum_m \omega_m \psi_m^\dagger \psi_m + \sum_\alpha \left[\frac{\epsilon}{2}\sigma_\alpha^z + g\left(\psi_m \sigma_\alpha^+ + \text{H.c.}\right)\right]$$
$$+ \sum_\alpha \Omega \left\{ b_\alpha^\dagger b_\alpha + \sqrt{S}\sigma_\alpha^z \left(b_\alpha^\dagger + b_\alpha\right)\right\}$$

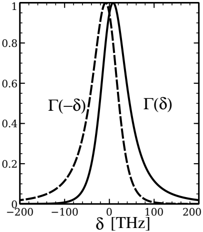
- **2D** harmonic oscillator
  $\omega_m = \omega_{\text{cutoff}} + m\omega_{H.O.}$
- Incoherent processes: excitation, decay, loss, vibrational thermalisation.
- Weak coupling, perturbative in g



Modelling

Master equation

$$\dot{\rho} = -i[H_0, \rho] + \sum_m \frac{\kappa}{2}\mathcal{L}[\psi_m] + \sum_\alpha \left[\frac{\Gamma_\uparrow}{2}\mathcal{L}[\sigma^+_\alpha] + \frac{\Gamma_\downarrow}{2}\mathcal{L}[\sigma^-_\alpha]\right]$$
$$+ \sum_{m,\alpha}\left[\frac{\Gamma(\delta_m = \omega_m - \epsilon)}{2}\mathcal{L}[\sigma^+_\alpha \psi_m] + \frac{\Gamma(-\delta_m = \epsilon - \omega_m)}{2}\mathcal{L}[\sigma^-_\alpha \psi^\dagger_m]\right]$$



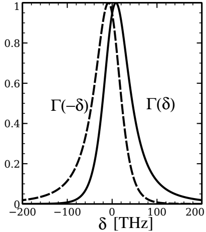
- Kennard-Stepanov $\Gamma(+\delta) \simeq \Gamma(-\delta)e^{\beta\delta}$

[Marthaler et al PRL ’11, Kirton & JK PRL ’13]

Modelling

Master equation

$$\dot{\rho} = -i[H_0, \rho] + \sum_m \frac{\kappa}{2}\mathcal{L}[\psi_m] + \sum_\alpha \left[\frac{\Gamma_\uparrow}{2}\mathcal{L}[\sigma^+_\alpha] + \frac{\Gamma_\downarrow}{2}\mathcal{L}[\sigma^-_\alpha]\right]$$
$$+ \sum_{m,\alpha}\left[\frac{\Gamma(\delta_m = \omega_m - \epsilon)}{2}\mathcal{L}[\sigma^+_\alpha \psi_m] + \frac{\Gamma(-\delta_m = \epsilon - \omega_m)}{2}\mathcal{L}[\sigma^-_\alpha \psi^\dagger_m]\right]$$



- Kennard-Stepanov $\Gamma(+\delta) \simeq \Gamma(-\delta)e^{\beta\delta}$

[Marthaler et al PRL ’11, Kirton & JK PRL ’13]

Distribution $g_m n_m$

- Master equation $\rightarrow$ Rate equation

$$\partial_t n_m = -\kappa n_m + \Gamma(-\delta_m)(n_m + 1)N_\uparrow - \Gamma(\delta_m) n_m N_\downarrow$$

- Bose-Einstein distribution without losses

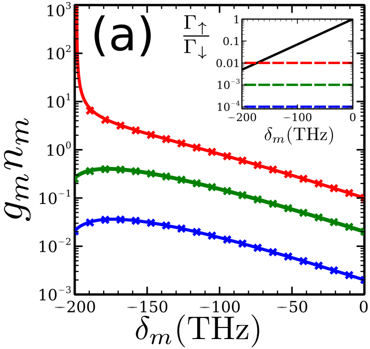
[Kirton & JK PRL '13]

Distribution $g_m n_m$

- Master equation $\rightarrow$ Rate equation

$$\partial_t n_m = -\kappa n_m + \Gamma(-\delta_m)(n_m + 1)N_\uparrow - \Gamma(\delta_m) n_m N_\downarrow$$

- Bose-Einstein distribution without losses



Low loss: Thermal

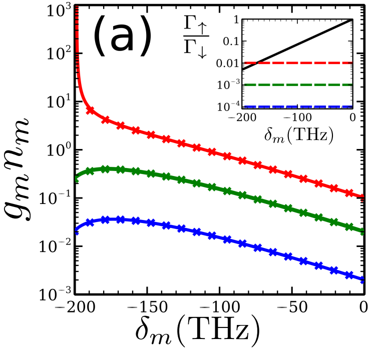
[Kirton & JK PRL '13]

Distribution $g_m n_m$

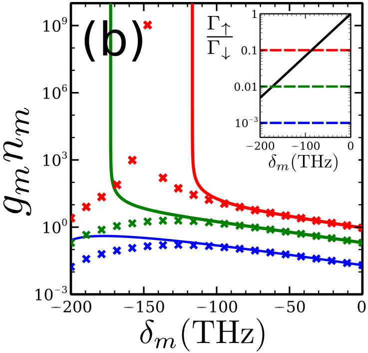
- Master equation → Rate equation

$$\partial_t n_m = -\kappa n_m + \Gamma(-\delta_m)(n_m + 1)N_\uparrow - \Gamma(\delta_m) n_m N_\downarrow$$

- Bose-Einstein distribution without losses



Low loss: Thermal



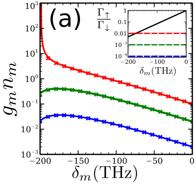
High loss → Laser

[Kirton & JK PRL '13]

Chemical potential?

- Steady state distribution:

$$\frac{n_m}{n_m+1} = \frac{\Gamma(-\delta_m)N_\uparrow}{\kappa + \Gamma(\delta_m)N_\downarrow}$$



- $\kappa \ll N\Gamma(\delta)$, Kennard-Stepanov

$$\frac{n_m}{n_m+1} = e^{-\beta\delta_m+\beta\mu}, \qquad e^{\beta\mu} = \frac{N_\uparrow}{N_\downarrow} = \frac{\Gamma_\uparrow + \sum_m \Gamma(\delta_m) n_m}{\Gamma_\downarrow + \sum_m \Gamma(-\delta_m)(n_m+1)}$$

- Below threshold,

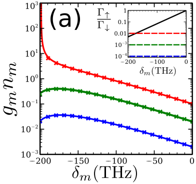
$$\mu = k_B T \ln[\Gamma_\uparrow/\Gamma_\downarrow]$$

- At/above threshold, $\mu \to \delta_0$

Chemical potential?

- Steady state distribution:

$$\frac{n_m}{n_m+1} = \frac{\Gamma(-\delta_m)N_\uparrow}{\kappa + \Gamma(\delta_m)N_\downarrow}$$



- $\kappa \ll N\Gamma(\delta)$, Kennard-Stepanov

$$\frac{n_m}{n_m+1} = e^{-\beta\delta_m+\beta\mu}, \qquad e^{\beta\mu} \equiv \frac{N_\uparrow}{N_\downarrow} = \frac{\Gamma_\uparrow + \sum_m \Gamma(\delta_m)n_m}{\Gamma_\downarrow + \sum_m \Gamma(-\delta_m)(n_m+1)}$$

- Below threshold,

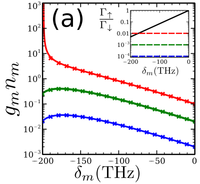
$$\mu = k_B T \ln[\Gamma_\uparrow/\Gamma_\downarrow]$$

- At/above threshold, $\mu \to \delta_0$

Chemical potential?

- Steady state distribution:

$$\frac{n_m}{n_m+1} = \frac{\Gamma(-\delta_m)N_\uparrow}{\kappa + \Gamma(\delta_m)N_\downarrow}$$



- $\kappa \ll N\Gamma(\delta)$, Kennard-Stepanov

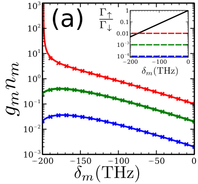
$$\frac{n_m}{n_m+1} = e^{-\beta\delta_m+\beta\mu}, \qquad e^{\beta\mu} \equiv \frac{N_\uparrow}{N_\downarrow} = \frac{\Gamma_\uparrow + \sum_m \Gamma(\delta_m)n_m}{\Gamma_\downarrow + \sum_m \Gamma(-\delta_m)(n_m+1)}$$

- Below threshold,

$$\mu = k_B T \ln[\Gamma_\uparrow/\Gamma_\downarrow]$$

- At/above threshold, $\mu \to \delta_0$

Chemical potential?



- Steady state distribution:

$$\frac{n_m}{n_m+1} = \frac{\Gamma(-\delta_m)N_\uparrow}{\kappa + \Gamma(\delta_m)N_\downarrow}$$

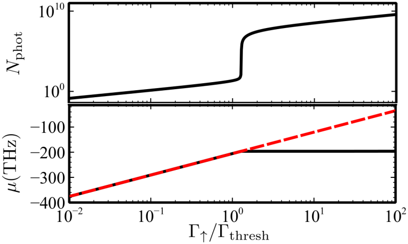
- $\kappa \ll N\Gamma(\delta)$, Kennard-Stepanov

$$\frac{n_m}{n_m+1} = e^{-\beta\delta_m+\beta\mu}, \qquad e^{\beta\mu} \equiv \frac{N_\uparrow}{N_\downarrow} = \frac{\Gamma_\uparrow + \sum_m \Gamma(\delta_m)n_m}{\Gamma_\downarrow + \sum_m \Gamma(-\delta_m)(n_m+1)}$$

- Below threshold,

$$\mu = k_B T \ln[\Gamma_\uparrow/\Gamma_\downarrow]$$

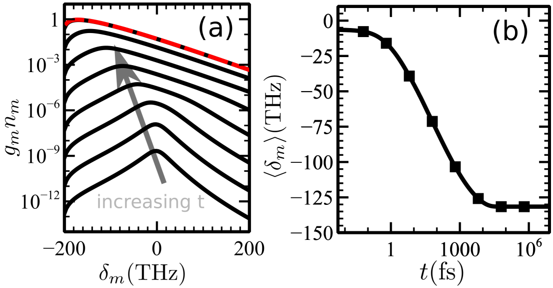
- At/above threshold, $\mu \to \delta_0$



Modelling photon BEC

Time evolution

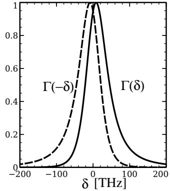
- Initial state: excited molecules
- Initial emission, follows gain peak
- Thermalisation by repeated absorption

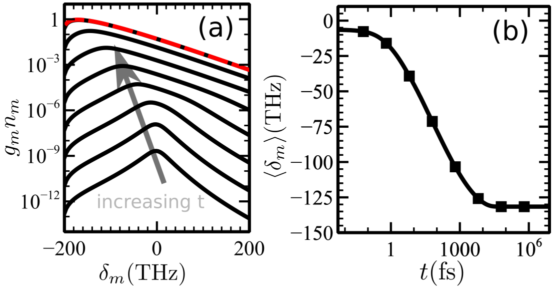


[Kirton & JK PRA '15]

Time evolution

- Initial state: excited molecules
- Initial emission, follows gain peak
- Thermalisation by repeated absorption

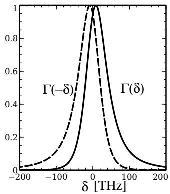


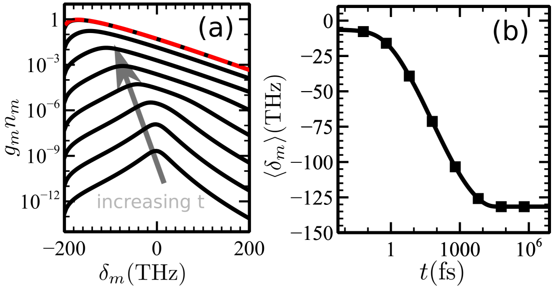


[Kirton & JK PRA ’15]

Time evolution

- Initial state: excited molecules
- Initial emission, follows gain peak
- Thermalisation by repeated absorption





[Kirton & JK PRA ’15]

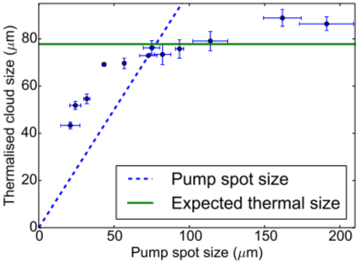
Spatial profile

Spatially varying pump intensity

- Consider effects of pump profile, $\Gamma_{\uparrow}(\mathbf{r}) = \dfrac{\Gamma_{\uparrow} \exp(-r^2/2\sigma_p^2)}{(2\pi\sigma_p^2)^{d/2}}$
- Experiments: [Marelic & Nyman, PRA '15]

Spatially varying pump intensity

- Consider effects of pump profile, $\Gamma_\uparrow(\mathbf{r}) = \frac{\Gamma_\uparrow \exp(-r^2/2\sigma_p^2)}{(2\pi\sigma_p^2)^{d/2}}$
- Experiments: [Marelic & Nyman, PRA '15]



Modelling spatial profile.

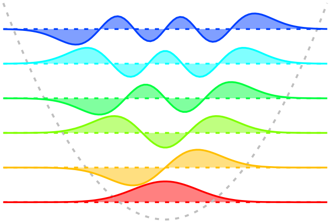
- Varying excited density – differential coupling to modes

$$\partial_t n_m = -\kappa n_m + \Gamma(-\delta_m) O_m (n_m + 1) - \Gamma(\delta_m)(\rho_m - O_m) n_m$$

$$O_m = \int d\mathbf{r} \rho_\uparrow(\mathbf{r}) |\psi_m(\mathbf{r})|^2, \qquad \rho_\uparrow + \rho_\downarrow = \rho_m$$

Modelling spatial profile.

- Gauss-Hermite modes
  $I(\mathbf{r}) = \sum_m n_m |\psi_m(\mathbf{r})|^2$



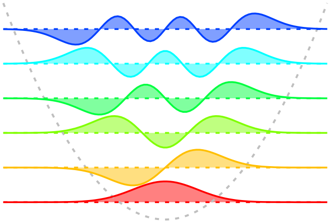
- Varying excited density – differential coupling to modes

$$\partial_t n_m = -\kappa n_m + \Gamma(-\delta_m) O_m (n_m + 1) - \Gamma(\delta_m)(\rho_m - O_m) n_m$$

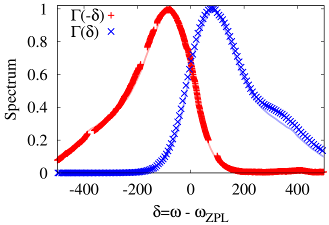
$$O_m = \int d\mathbf{r} \rho_\uparrow(\mathbf{r}) |\psi_m(\mathbf{r})|^2, \qquad \rho_\uparrow + \rho_\downarrow = \rho_m$$

Modelling spatial profile.

- Gauss-Hermite modes
  $I(\mathbf{r}) = \sum_m n_m |\psi_m(\mathbf{r})|^2$



- Use exact R6G spectrum



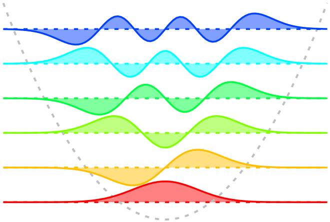
- Varying excited density – differential coupling to modes

$$\partial_t n_m = -\kappa n_m + \Gamma(-\delta_m) O_m (n_m + 1) - \Gamma(\delta_m)(\rho_m - O_m) n_m$$

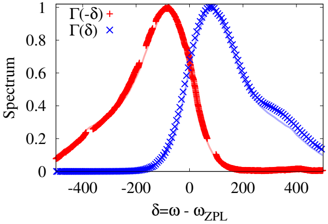
$$O_m = \int d\mathbf{r} \rho_\uparrow(\mathbf{r}) |\psi_m(\mathbf{r})|^2, \qquad \rho_\uparrow + \rho_\downarrow = \rho_m$$

Modelling spatial profile.

- Gauss-Hermite modes
  $I(\mathbf{r}) = \sum_m n_m |\psi_m(\mathbf{r})|^2$



- Use exact R6G spectrum



- Varying excited density – differential coupling to modes

$$\partial_t n_m = -\kappa n_m + \Gamma(-\delta_m) O_m (n_m + 1) - \Gamma(\delta_m)(\rho_m - O_m) n_m$$

$$O_m = \int d\mathbf{r} \rho_\uparrow(\mathbf{r}) |\psi_m(\mathbf{r})|^2, \qquad \rho_\uparrow + \rho_\downarrow = \rho_m$$

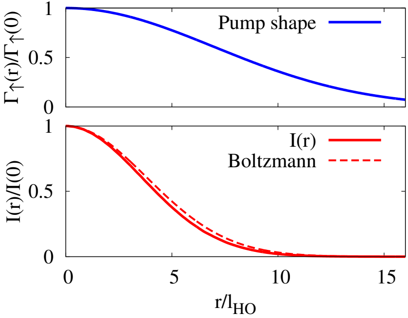
$$\partial_t \rho_\uparrow(\mathbf{r}) = -\tilde{\Gamma}_\downarrow(\mathbf{r})\rho_\uparrow(\mathbf{r}) + \tilde{\Gamma}_\uparrow(\mathbf{r})\rho_\downarrow(\mathbf{r}))$$

Spatially varying pump: below threshold

- Far below threshold:
  - Excitation: $\rho_\uparrow(\mathbf{r}) \simeq \rho_m \frac{\Gamma_\uparrow(\mathbf{r})}{\Gamma_\downarrow} \ll \rho_m$
  - If $\kappa \ll \rho_m \Gamma(\delta_m)$, $$\frac{n_m}{n_m+1} \simeq e^{-\beta\delta_m} \times \int d\mathbf{r} \frac{\Gamma_\uparrow(\mathbf{r})}{\Gamma_\downarrow} |\psi_m(\mathbf{r})|^2$$
- Resulting profile, $I(\mathbf{r}) = \sum_m n_m |\psi_m(r)|^2$

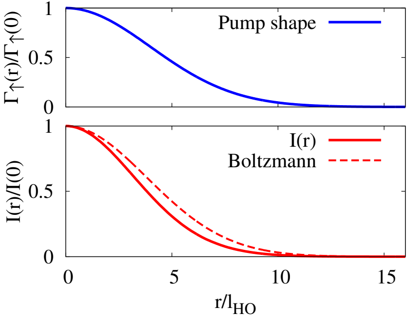
Spatially varying pump: below threshold

- Far below threshold:
  - Excitation: $\rho_\uparrow(\mathbf{r}) \simeq \rho_m \frac{\Gamma_\uparrow(\mathbf{r})}{\Gamma_\downarrow} \ll \rho_m$
  - If $\kappa \ll \rho_m \Gamma(\delta_m)$, $\quad \frac{n_m}{n_m+1} \simeq e^{-\beta\delta_m} \times \int d\mathbf{r} \frac{\Gamma_\uparrow(\mathbf{r})}{\Gamma_\downarrow} |\psi_m(\mathbf{r})|^2$
- Resulting profile, $I(\mathbf{r}) = \sum_m n_m |\psi_m(r)|^2$



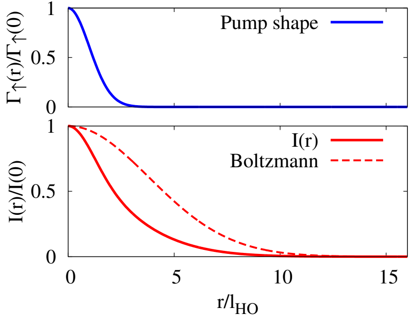
Spatially varying pump: below threshold

- Far below threshold:
  - Excitation: $\rho_\uparrow(\mathbf{r}) \simeq \rho_m \frac{\Gamma_\uparrow(\mathbf{r})}{\Gamma_\downarrow} \ll \rho_m$
  - If $\kappa \ll \rho_m \Gamma(\delta_m)$, $\quad \frac{n_m}{n_m + 1} \simeq e^{-\beta\delta_m} \times \int d\mathbf{r} \frac{\Gamma_\uparrow(\mathbf{r})}{\Gamma_\downarrow} |\psi_m(\mathbf{r})|^2$
- Resulting profile, $I(\mathbf{r}) = \sum_m n_m |\psi_m(r)|^2$



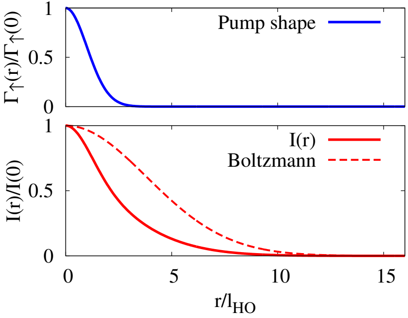
Spatially varying pump: below threshold

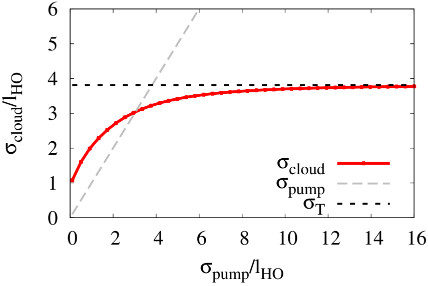
- Far below threshold:
  - Excitation: $\rho_\uparrow(\mathbf{r}) \simeq \rho_m \frac{\Gamma_\uparrow(\mathbf{r})}{\Gamma_\downarrow} \ll \rho_m$
  - If $\kappa \ll \rho_m \Gamma(\delta_m)$, $\quad \frac{n_m}{n_m + 1} \simeq e^{-\beta\delta_m} \times \int d\mathbf{r} \frac{\Gamma_\uparrow(\mathbf{r})}{\Gamma_\downarrow} |\psi_m(\mathbf{r})|^2$
- Resulting profile, $I(\mathbf{r}) = \sum_m n_m |\psi_m(r)|^2$



Spatially varying pump: below threshold

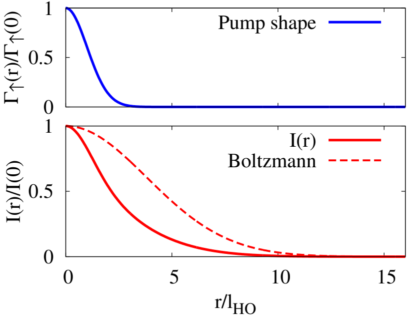
- Far below threshold:
  - Excitation: $\rho_\uparrow(\mathbf{r}) \simeq \rho_m \dfrac{\Gamma_\uparrow(\mathbf{r})}{\Gamma_\downarrow} \ll \rho_m$
  - If $\kappa \ll \rho_m \Gamma(\delta_m)$, $\quad \dfrac{n_m}{n_m + 1} \simeq e^{-\beta\delta_m} \times \int d\mathbf{r} \dfrac{\Gamma_\uparrow(\mathbf{r})}{\Gamma_\downarrow} |\psi_m(\mathbf{r})|^2$
- Resulting profile, $I(\mathbf{r}) = \sum_m n_m |\psi_m(r)|^2$

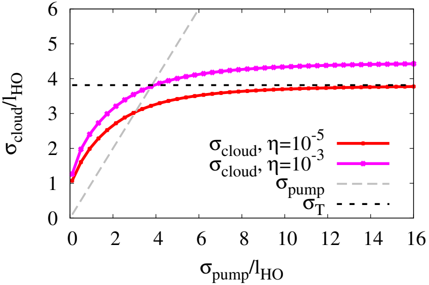




Spatially varying pump: below threshold

- Far below threshold:
  - Excitation: $\rho_\uparrow(\mathbf{r}) \simeq \rho_m \frac{\Gamma_\uparrow(\mathbf{r})}{\Gamma_\downarrow} \ll \rho_m$
  - If $\kappa \ll \rho_m \Gamma(\delta_m)$, $\qquad \frac{n_m}{n_m+1} \simeq e^{-\beta\delta_m} \times \int d\mathbf{r} \frac{\Gamma_\uparrow(\mathbf{r})}{\Gamma_\downarrow} |\psi_m(\mathbf{r})|^2$
- Resulting profile, $I(\mathbf{r}) = \sum_m n_m |\psi_m(r)|^2$

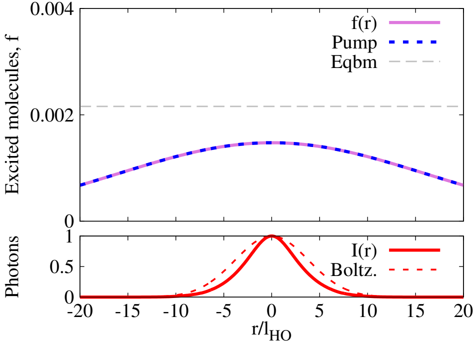




  - $\eta = \frac{\kappa}{\rho_m \max[\Gamma(\delta)]}$

Near threshold behaviour

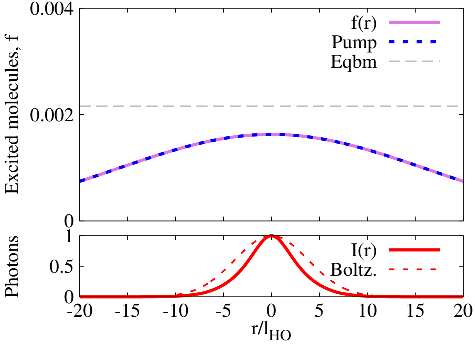
- Large spot, $\sigma_p \gg l_{HO}$
- Non Boltzmann peak — BEC
- "Gain saturation" at centre



- Saturation of $f(r) = 1/(1 + e^{-\beta\mu})$ — spatial equilibriation

Near threshold behaviour

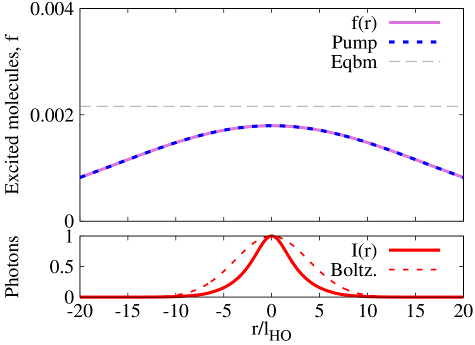
- Large spot, $\sigma_p \gg l_{HO}$
- Non Boltzmann peak — BEC
- "Gain saturation" at centre



- Saturation of $f(r) = 1/(1 + e^{-\beta\mu})$ — spatial equilibriation

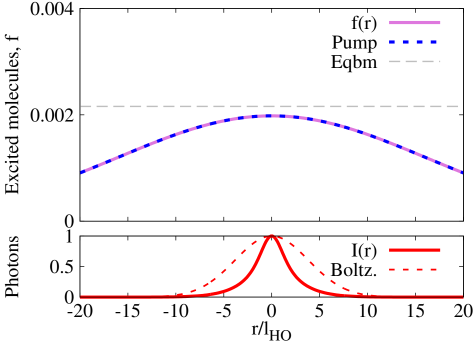
Near threshold behaviour

- Large spot, $\sigma_p \gg l_{HO}$



Near threshold behaviour

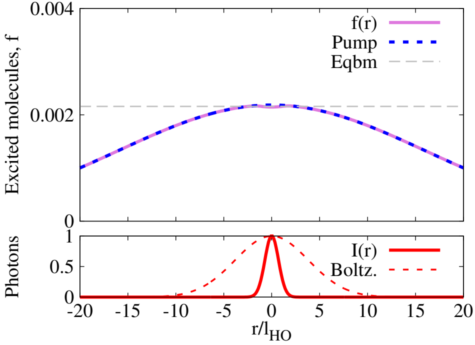
- Large spot, $\sigma_p \gg l_{HO}$
- Non Boltzmann peak — BEC
- "Gain saturation" at centre



- Saturation of $f(r) = 1/(1 + e^{-\beta\mu})$ — spatial equilibriation

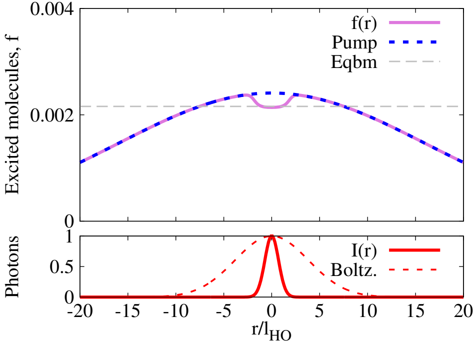
Near threshold behaviour

- Large spot, $\sigma_p \gg l_{HO}$
- Non Boltzmann peak — BEC



Near threshold behaviour

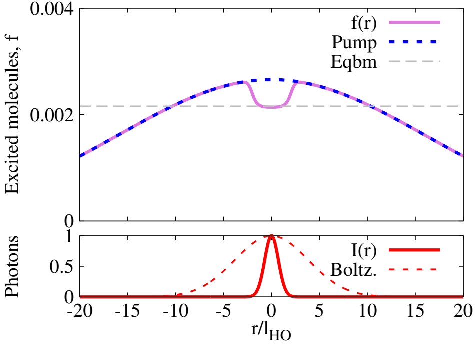
- Large spot, $\sigma_p \gg l_{HO}$
- Non Boltzmann peak — BEC
- “Gain saturation” at centre



- Saturation of $f(r) = 1/(1 + e^{-\beta\mu})$ — spatial equilibriation

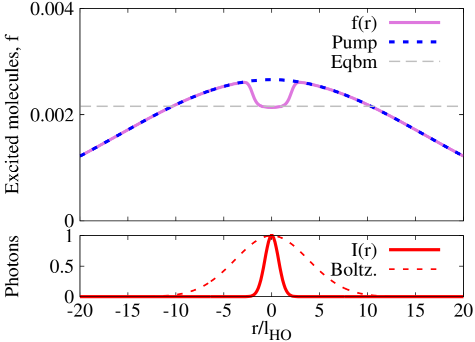
Near threshold behaviour

- Large spot, $\sigma_p \gg l_{HO}$
- Non Boltzmann peak — BEC
- “Gain saturation” at centre



Near threshold behaviour

- Large spot, $\sigma_p \gg l_{HO}$
- Non Boltzmann peak — BEC
- "Gain saturation" at centre

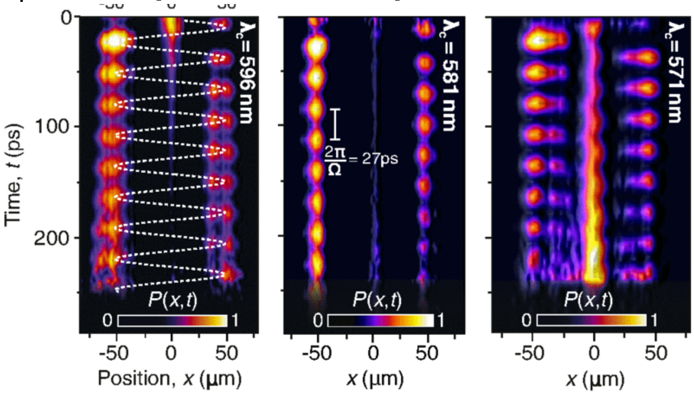


- Saturation of $f(r) = 1/(1 + e^{-\beta\mu})$ — spatial equilibriation

Spatial oscillations

Off centre pumping; oscillations

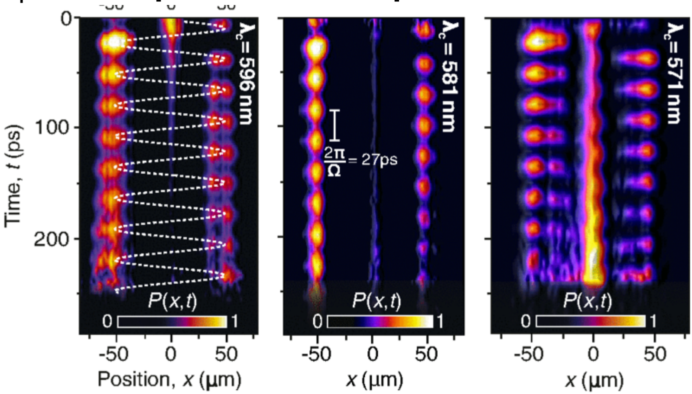
- Experiments [Schmitt *et al.* PRA '15]



- Oscillations in space – beating of normal modes
- Thermalisation depends on cutoff

Off centre pumping; oscillations

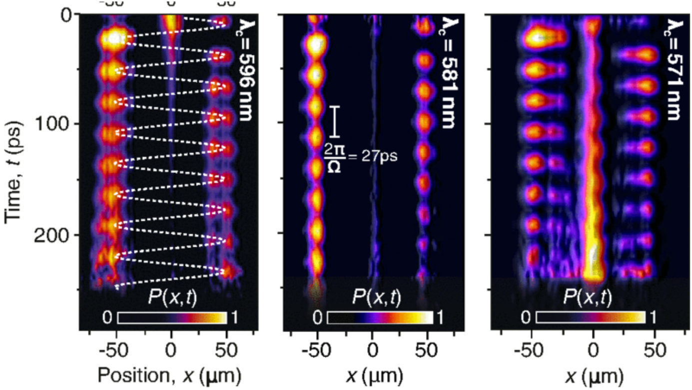
- Experiments [Schmitt *et al.* PRA '15]



- Oscillations in space – beating of normal modes
- Thermalisation depends on cutoff

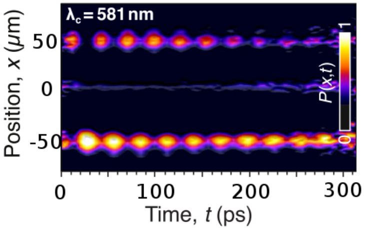
Off centre pumping; oscillations

- Experiments [Schmitt *et al.* PRA '15]



- Oscillations in space – beating of normal modes
- Thermalisation depends on cutoff

Limit of rate equations



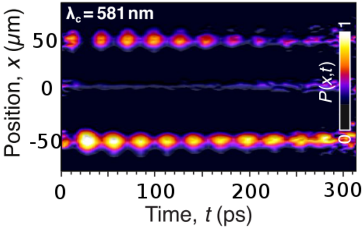
$$\partial_t n_m = -\kappa n_m + \Gamma(-\delta_m)(n_m + 1)N_\uparrow - \Gamma(\delta_m) n_m N_\downarrow$$

- Describes emission into Gauss-Hermite mode m

$$I(x) = \sum_m n_m |\psi_m(x)|^2$$

- Oscillations: beating of modes.
- Need $I(x) = \sum_{m,m'} n_{m,m'} \psi_m(x) \psi_{m'}(x)$

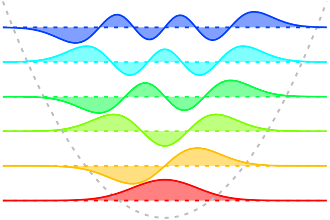
Limit of rate equations



$$\partial_t n_m = -\kappa n_m + \Gamma(-\delta_m)(n_m + 1)N_\uparrow - \Gamma(\delta_m) n_m N_\downarrow$$

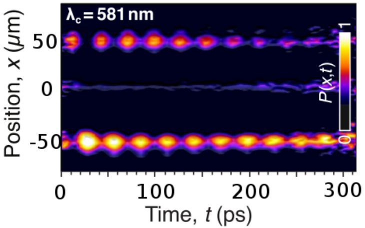
- Describes emission into Gauss-Hermite mode m

$$I(x) = \sum_m n_m |\psi_m(x)|^2$$



- Oscillations: beating of modes.
- Need $I(x) = \sum_{m,m'} n_{m,m'} \psi_m(x) \psi_{m'}(x)$

Limit of rate equations

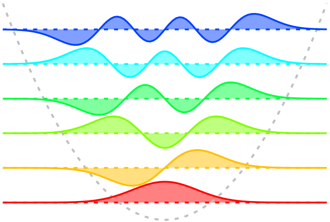


$$\partial_t n_m = -\kappa n_m + \Gamma(-\delta_m)(n_m + 1)N_\uparrow - \Gamma(\delta_m) n_m N_\downarrow$$

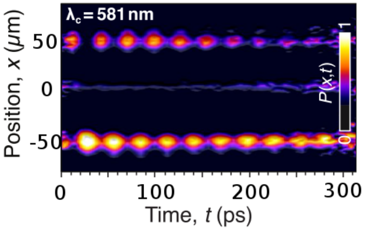
- Describes emission into Gauss-Hermite mode m

$$I(x) = \sum_m n_m |\psi_m(x)|^2$$

- Oscillations: beating of modes.
- Need $I(x) = \sum_{m,m'} n_{m,m'} \psi_m(x) \psi_{m'}(x)$



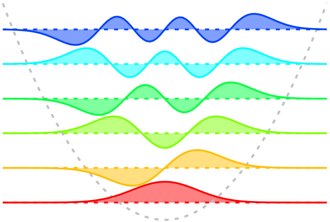
Limit of rate equations



$$\partial_t n_m = -\kappa n_m + \Gamma(-\delta_m)(n_m + 1)N_\uparrow - \Gamma(\delta_m) n_m N_\downarrow$$

- Describes emission into Gauss-Hermite mode m

$$I(x) = \sum_m n_m |\psi_m(x)|^2$$



- Oscillations: beating of modes.
- Need $I(x) = \sum_{m,m'} n_{m,m'} \psi_m(x) \psi_{m'}(x)$

Emission must create coherence between non-degenerate modes.

Modelling

- Full master equation required

$$\partial_t \rho = -i \left[\sum_m \omega_m a^\dagger_m a_m, \rho \right] + \sum_{m,m',i} \psi^*_m(r_i)\psi_{m'}(r_i) \Big(K(\delta_{m'})[\hat{a}_{m'} \hat{\sigma}^+_i \hat{\rho}, \hat{a}^\dagger_m \hat{\sigma}^-_i] + K(-\delta_m)[\hat{a}^\dagger_m \hat{\sigma}^-_i \hat{\rho}, \hat{a}_{m'} \hat{\sigma}^+_i] \Big) + \text{H.c.} + (\textit{pumping}, \textit{decay} \ldots),$$

- Not secular approximation
- Semiclassical equations for $n_{m,m'} = \langle a^\dagger_m a_{m'} \rangle$ and $f(r)$.

Modelling

- Full master equation required

$$\partial_t \rho = -i\left[\sum_m \omega_m a_m^\dagger a_m, \rho\right] + \sum_{m,m',i} \psi_m^*(r_i)\psi_{m'}(r_i)\Big(K(\delta_{m'})[\hat{a}_{m'}\hat{\sigma}_i^+\hat{\rho}, \hat{a}_m^\dagger\hat{\sigma}_i^-]$$

$$+ K(-\delta_m)[\hat{a}_m^\dagger\hat{\sigma}_i^-\hat{\rho}, \hat{a}_{m'}\hat{\sigma}_i^+]\Big) + \text{H.c.} + (\textit{pumping}, \textit{decay}\ldots),$$

- Not secular approximation
  - **Must** have emission into m, m' superposition
  - **Must** have $K = K(\delta_m)$ (Kennard-Stepanov)
- Semiclassical equations for $n_{m,m'} = \langle a_m^\dagger a_{m'}\rangle$ and $f(r)$.

Modelling

- Full master equation required

$$\partial_t \rho = -i\left[\sum_m \omega_m a_m^\dagger a_m, \rho\right] + \sum_{m,m',i} \psi_m^*(r_i)\psi_{m'}(r_i)\Big(K(\delta_{m'})[\hat{a}_{m'}\hat{\sigma}_i^+\hat{\rho}, \hat{a}_m^\dagger\hat{\sigma}_i^-] + K(-\delta_m)[\hat{a}_m^\dagger\hat{\sigma}_i^-\hat{\rho}, \hat{a}_{m'}\hat{\sigma}_i^+]\Big) + \text{H.c.} + (pumping, decay\ldots),$$

- Not secular approximation
  - **Must** have emission into m, m' superposition
  - **Must** have $K = K(\delta_m)$ (Kennard-Stepanov)
- Semiclassical equations for $n_{m,m'} = \langle a_m^\dagger a_{m'} \rangle$ and $f(r)$.

Modelling

- Full master equation required

$$\partial_t \rho = -i\left[\sum_m \omega_m a^\dagger_m a_m, \rho\right] + \sum_{m,m',i} \psi^*_m(r_i)\psi_{m'}(r_i)\Big(K(\delta_{m'})[\hat{a}_{m'}\hat{\sigma}^+_i \hat{\rho}, \hat{a}^\dagger_m \hat{\sigma}^-_i] + K(-\delta_m)[\hat{a}^\dagger_m \hat{\sigma}^-_i \hat{\rho}, \hat{a}_{m'}\hat{\sigma}^+_i]\Big) + \text{H.c.} + (\textit{pumping}, \textit{decay} \ldots),$$

- Not secular approximation
  - **Must** have emission into m, m' superposition
  - **Must** have $K = K(\delta_m)$ (Kennard-Stepanov)
- Semiclassical equations for $n_{m,m'} = \langle a^\dagger_m a_{m'} \rangle$ and $f(r)$.

Modelling

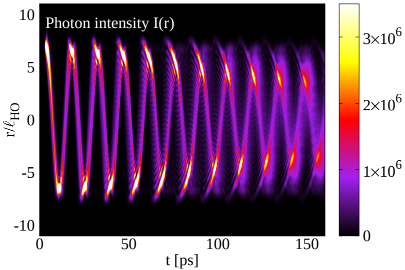
- Full master equation required

$$\partial_t \rho = -i\left[\sum_m \omega_m a^\dagger_m a_m, \rho\right] + \sum_{m,m',i} \psi^*_m(r_i)\psi_{m'}(r_i)\Big(K(\delta_{m'})[\hat{a}_{m'}\hat{\sigma}^+_i\hat{\rho}, \hat{a}^\dagger_m\hat{\sigma}^-_i]$$
$$+ K(-\delta_m)[\hat{a}^\dagger_m\hat{\sigma}^-_i\hat{\rho}, \hat{a}_{m'}\hat{\sigma}^+_i]\Big) + \text{H.c.} + (\textit{pumping}, \textit{decay}\ldots),$$

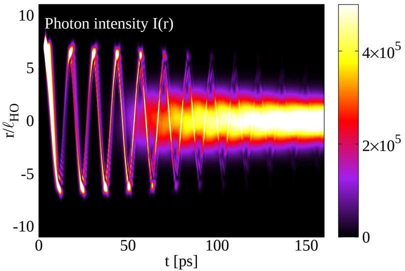
- Not secular approximation
  - **Must** have emission into m, m' superposition
  - **Must** have $K = K(\delta_m)$ (Kennard-Stepanov)
- Semiclassical equations for $n_{m,m'} = \langle a^\dagger_m a_{m'}\rangle$ and $f(r)$.

Dynamics from model

Smaller ω_c



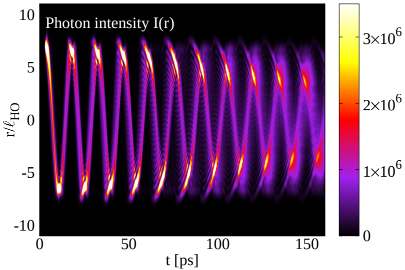
Larger ω_c



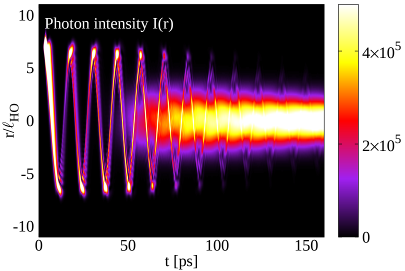
- Origin of thermalisation — reabsorption, see $f(r)$

Dynamics from model

Smaller ω_c



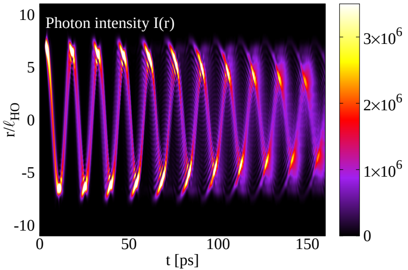
Larger ω_c



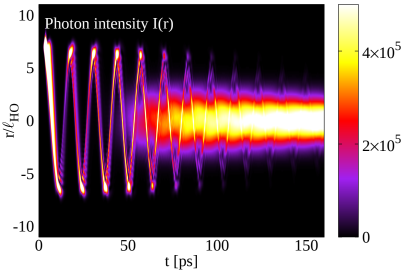
- Origin of thermalisation — reabsorption, see $f(r)$

Dynamics from model

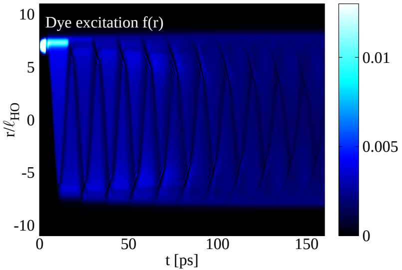
Smaller ω_c

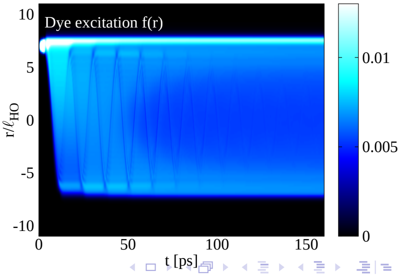


Larger ω_c



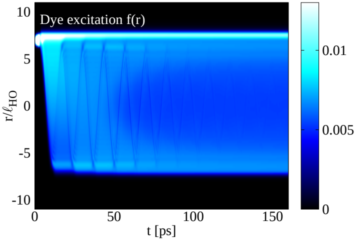
- Origin of thermalisation — reabsorption, see $f(r)$





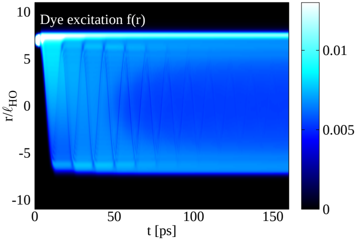
Thermalisation at late times

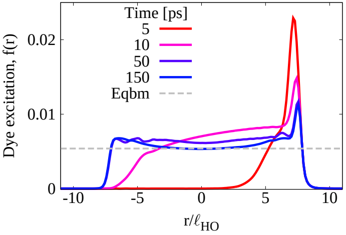
- Reabsorption “fills-in” excited molecules
- Reach thermal equilibrium, $f = [e^{-\beta\delta_0} + 1]^{-1}$



Thermalisation at late times

- Reabsorption “fills-in” excited molecules
- Reach thermal equilibrium, $f = [e^{-\beta\delta_0} + 1]^{-1}$





Classical and Quantum matrix product states:

Exploring the structure of non-equilibrium states

10th-11th September 2015, Higgs Centre for Theoretical Physics

Speakers:
M. Banuls (MPQ, Garching), X. Chen (Caltech), B. Derrida (ENS, Paris), K. Mallick (Saclay), J. Moore (UC Berkeley), T. Prosen (Ljubljana), E. Ragoucy (Annecy), G. Schutz (Julich), F. Verstraete (Vienna), S. White (UC Irvine)

The aim of this workshop is to bring together those working on quantum and classical concepts involving matrix product states, with a particular focus on non-equilibrium states.

To register, visit the Higgs centre website `https://higgs.ph.ed.ac.uk`

This meeting is made possible by funding from the EPSRC UK network on emergence and physics far from Equilibrium, the Higgs Centre for Theoretical Physics, and the EPSRC TOPNES programme grant.

EPSRC
Engineering and Physical Sciences Research Council

topnes
Topological Protection and Non-Equilibrium States in Strongly Correlated Electron Systems

Centre for Theoretical Physics Higgs

Condensates of Light

13th–15th January 2016, Chicheley Hall, Buckinghamshire, UK

The scope of the workshop covers all physical realisations of condensates of light, including:

- **Bose-Einstein condensation of light in dye-filled microcavities**
- **Condensates of semiconductor exciton-polaritons**
- **Organic polariton condensation**
- **Classical condensation phenomena in optics**
- **Superfluid light**

Keynote speakers:

- **Jason Fleischer (Princeton, USA)**
- **Elisabeth Giacobino (LKB, Paris, France)**
- **Stephane Kena-Cohen (Polytechnique, Montreal, Canada)**
- **Henk Stoof (Utrecht, Netherlands)**
- **Martin Weitz (Bonn, Germany)**

Invited speakers:

- **Alberto Amo (Marcoussis, France)**
- **Iacopo Carusotto (Trento, Italy)**
- **Natalia Berloff (Cambridge, UK/Moscow)**
- **Baruch Fischer (Technion, Haifa, Israel)**
- **Jonathan Keeling (St Andrews, UK)**
- **Pavlos Lagoudakis (Southampton, UK)**
- **Rainer Mahrt (IBM Zurich, Switzerland)**
- **Gian-Luca Oppo (Strathclyde, UK)**
- **Antonio Picozzi (Bourgogne, France)**
- **Daniele Sanvitto (Lecce, Italy)**
- **David Snoke (Pittsburgh, USA)**
- **Marzena Szymanska (UCL, UK)**
- **Jacob Taylor (Maryland, USA)**

Important Dates:

- **31st October 2015: abstract submission deadline**
- **30th November 2015: registration deadline**

Organising committee:

- **Robert Nyman (Imperial College, London)**
- **Peter Kirton (St Andrews)**
- **Jonathan Keeling (St Andrews)**
- **Martin Weitz (Bonn)**

http://condensates-of-light.org

Acknowledgements

GROUP:

FUNDING:

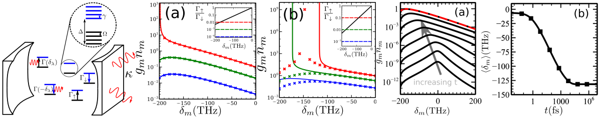
topnes
Topological Protection and Non-Equilibrium States in Strongly Correlated Electron Systems

EPSRC
Engineering and Physical Sciences Research Council

The Leverhulme Trust

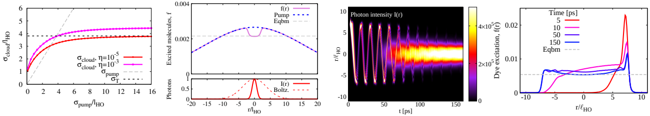
Summary

- Photon condensation and thermalisation



[Kirton & JK, PRL '13, PRA '15]

- Photon condensation, pattern formation physics



[JK & Kirton, arXiv:1506:00280]

Extra Slides

Microscopic model – calculating $\Gamma(\delta)$

How to calculate $\Gamma(\delta)$

- Polaron transform (exact)

$$h_\alpha = \frac{\epsilon}{2}\sigma^z_\alpha + g\left(\psi_m \sigma^+_\alpha D_\alpha + \text{H.c.}\right) + \Omega b^\dagger_\alpha b_\alpha,$$

$$D_\alpha = \exp\left[2\sqrt{S}(b^\dagger_\alpha - b_\alpha)\right]$$

- Correlation function:

$$\Gamma(\delta) = 2g^2 \Re \int dt \langle D^\dagger_\alpha(t) D_\alpha(0)\rangle \exp\left[-(\Gamma_\uparrow + \Gamma_\downarrow)\frac{t}{2}\right] e^{-i\delta t}$$

- Exponential of bosonic correlations $\langle D^\dagger_\alpha(t) D_\alpha(0)\rangle$

Microscopic model – calculating $\Gamma(\delta)$

How to calculate $\Gamma(\delta)$

- Polaron transform (exact)

$$h_\alpha = \frac{\epsilon}{2}\sigma^z_\alpha + g\left(\psi_m \sigma^+_\alpha D_\alpha + \text{H.c.}\right) + \Omega b^\dagger_\alpha b_\alpha,$$

$$D_\alpha = \exp\left[2\sqrt{S}(b^\dagger_\alpha - b_\alpha)\right]$$

- Correlation function:

$$\Gamma(\delta) = 2g^2 \Re \int dt \langle D^\dagger_\alpha(t) D_\alpha(0)\rangle \exp\left[-(\Gamma_\uparrow + \Gamma_\downarrow)\frac{t}{2}\right] e^{-i\delta t}$$

- Exponential of bosonic correlations $\langle D^\dagger_\alpha(t) D_\alpha(0)\rangle$

Microscopic model – calculating $\Gamma(\delta)$

How to calculate $\Gamma(\delta)$

- Polaron transform (exact)

$$h_\alpha = \frac{\epsilon}{2}\sigma^z_\alpha + g\left(\psi_m \sigma^+_\alpha D_\alpha + \text{H.c.}\right) + \Omega b^\dagger_\alpha b_\alpha,$$
$$D_\alpha = \exp\left[2\sqrt{S}(b^\dagger_\alpha - b_\alpha)\right]$$

- Correlation function:

$$\Gamma(\delta) = 2g^2\Re\int dt \langle D^\dagger_\alpha(t) D_\alpha(0)\rangle \exp\left[-(\Gamma_\uparrow + \Gamma_\downarrow)\frac{t}{2}\right] e^{-i\delta t}$$

- Exponential of bosonic correlations $\langle D^\dagger_\alpha(t) D_\alpha(0)\rangle$

Microscopic model – requirements for Kennard-Stepanov

- Correlation function

$$\Gamma(\delta) = 2g^2 \Re \int dt \langle D^\dagger_\alpha(t) D_\alpha(0)\rangle \exp\left[-(\Gamma_\uparrow + \Gamma_\downarrow)\frac{t}{2}\right] e^{-i\delta t}$$

- Kubo-Martin-Schwinger condition:

$$\langle D^\dagger_\alpha(t) D_\alpha(0)\rangle = \langle D^\dagger_\alpha(-t - i\beta) D_\alpha(0)\rangle$$

- $\Gamma(+\delta) = \Gamma(-\delta)e^{\beta\delta}$

Microscopic model – requirements for Kennard-Stepanov

- Correlation function

$$\Gamma(\delta) = 2g^2 \Re \int dt \langle D^\dagger_\alpha(t) D_\alpha(0) \rangle \exp\left[-(\Gamma_\uparrow + \Gamma_\downarrow)\frac{t}{2} \right] e^{-i\delta t}$$

- Kubo-Martin-Schwinger condition:

$$\langle D^\dagger_\alpha(t) D_\alpha(0) \rangle = \langle D^\dagger_\alpha(-t - i\beta) D_\alpha(0) \rangle$$

- $\Gamma(+\delta) = \Gamma(-\delta) e^{\beta\delta}$

Microscopic model – requirements for Kennard-Stepanov

- Correlation function

$$\Gamma(\delta) = 2g^2 \Re \int dt \langle D_\alpha^\dagger(t) D_\alpha(0) \rangle \exp\left[-(\Gamma_\uparrow + \Gamma_\downarrow)\frac{t}{2}\right] e^{-i\delta t}$$

- Kubo-Martin-Schwinger condition:

$$\langle D_\alpha^\dagger(t) D_\alpha(0) \rangle = \langle D_\alpha^\dagger(-t - i\beta) D_\alpha(0) \rangle$$

- $\Gamma(+\delta) = \Gamma(-\delta) e^{\beta\delta}$

Microscopic model – requirements for Kennard-Stepanov

- Correlation function

$$\Gamma(\delta) = 2g^2 \Re \int dt \langle D^\dagger_\alpha(t) D_\alpha(0)\rangle \exp\left[-(\Gamma_\uparrow + \Gamma_\downarrow)\frac{t}{2}\right] e^{-i\delta t}$$

- Kubo-Martin-Schwinger condition:

$$\langle D^\dagger_\alpha(t) D_\alpha(0)\rangle = \langle D^\dagger_\alpha(-t - i\beta) D_\alpha(0)\rangle$$

- $\Gamma(+\delta) = \Gamma(-\delta)e^{\beta\delta}$

Threshold condition

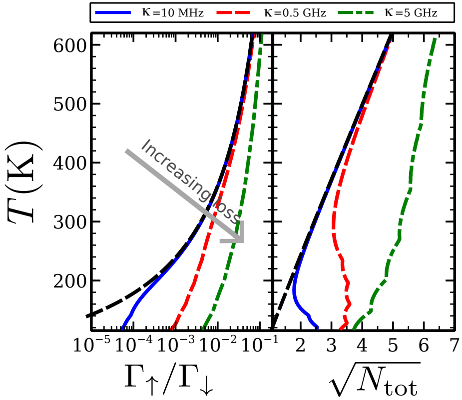
Use: $\max[n_m] = 1/(\beta\epsilon) \quad \rightarrow \quad k_B T_c = \sqrt{6/\pi^2}\epsilon\sqrt{N}$.

- Pump rate (Laser)
- Critical density (condensate)

- Thermal at low κ/high temperature
- High loss, κ competes with $\Gamma(\pm\delta_0)$
- Low temperature, $\Gamma(\pm\delta_0)$ shrinks

Threshold condition

Use: $\max[n_m] = 1/(\beta\epsilon) \quad \rightarrow \quad k_B T_c = \sqrt{6/\pi^2}\epsilon\sqrt{N}$.



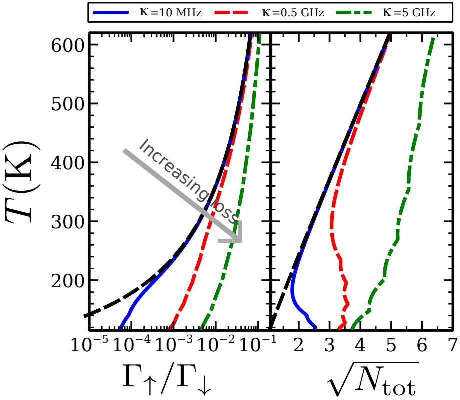
Compare threshold:

- Pump rate (Laser)
- Critical density (condensate)

- Thermal at low κ/high temperature
- High loss, κ competes with $\Gamma(\pm\delta_0)$
- Low temperature, $\Gamma(\pm\delta_0)$ shrinks

Threshold condition

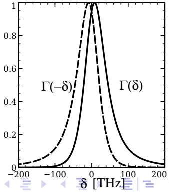
Use: $\max[n_m] = 1/(\beta\epsilon) \quad \rightarrow \quad k_B T_c = \sqrt{6/\pi^2}\epsilon\sqrt{N}$.



Compare threshold:

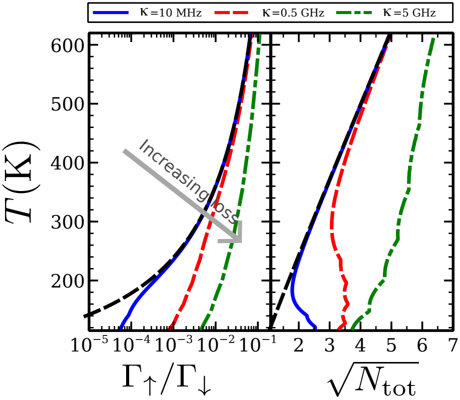
- Pump rate (Laser)
- Critical density (condensate)

- Thermal at low κ/high temperature
- High loss, κ competes with $\Gamma(\pm\delta_0)$
- Low temperature, $\Gamma(\pm\delta_0)$ shrinks



Threshold condition

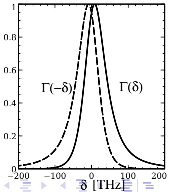
Use: $\max[n_m] = 1/(\beta\epsilon) \quad \rightarrow \quad k_B T_c = \sqrt{6/\pi^2}\epsilon\sqrt{N}$.



Compare threshold:

- Pump rate (Laser)
- Critical density (condensate)

- Thermal at low κ/high temperature
- High loss, κ competes with $\Gamma(\pm\delta_0)$
- Low temperature, $\Gamma(\pm\delta_0)$ shrinks



Threshold condition

- Lasing threshold, dependence on spot size.

  - Equilibrium: $\mu = \delta_c$
  - Gives $\Gamma_\uparrow(\mathbf{r}=0) = \Gamma_\downarrow e^{\beta\delta_c}$

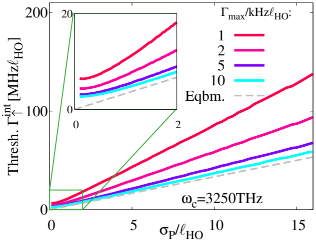
- Dependence on ω_c — experimental spectrum

Threshold condition

- Lasing threshold, dependence on spot size.
  - Equilibrium: $\mu = \delta_c$
  - Gives $\Gamma_\uparrow(\mathbf{r} = 0) = \Gamma_\downarrow e^{\beta \delta_c}$
- Dependence on ω_c — experimental spectrum

Threshold condition

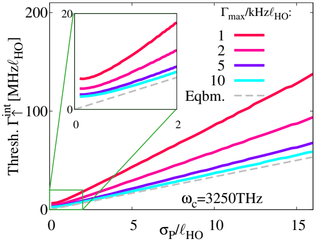
- Lasing threshold, dependence on spot size.
  - Equilibrium: $\mu = \delta_c$
  - Gives $\Gamma_{\uparrow}(\mathbf{r}=0) = \Gamma_{\downarrow} e^{\beta \delta_c}$



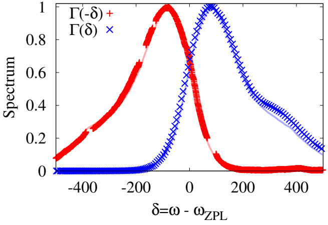
- Dependence on ω_c — experimental spectrum

Threshold condition

- Lasing threshold, dependence on spot size.
  - Equilibrium: $\mu = \delta_c$
  - Gives $\Gamma_\uparrow(\mathbf{r}=0) = \Gamma_\downarrow e^{\beta\delta_c}$

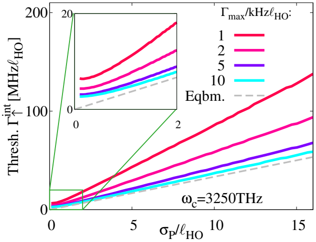


- Dependence on ω_c — experimental spectrum

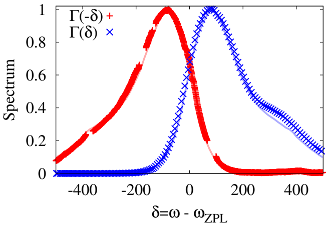


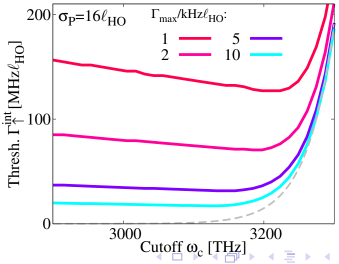
Threshold condition

- Lasing threshold, dependence on spot size.
  - Equilibrium: $\mu = \delta_c$
  - Gives $\Gamma_\uparrow(\mathbf{r}=0) = \Gamma_\downarrow e^{\beta\delta_c}$



- Dependence on ω_c — experimental spectrum





Quantum model, linewidth

Full Master equation:

$$\dot{\rho} = -i[H_0, \rho] - \frac{\kappa}{2}\mathcal{L}[\psi] - \sum_\alpha \left[\frac{\Gamma_\uparrow}{2}\mathcal{L}[\sigma^+_\alpha] + \frac{\Gamma_\downarrow}{2}\mathcal{L}[\sigma^-_\alpha]\right]$$

$$- \sum_\alpha \left[\frac{\Gamma(\delta = \omega - \epsilon)}{2}\mathcal{L}[\sigma^+_\alpha \psi] + \frac{\Gamma(-\delta = \epsilon - \omega)}{2}\mathcal{L}[\sigma^-_\alpha \psi^\dagger]\right]$$

- Factorise $\rho(t) \simeq \rho_{ph}(t) \bigotimes_i \rho_{m,i}(t)$
- Quantum regression theorm $\rightarrow$ linewidth

Quantum model, linewidth

Full Master equation:

$$\dot{\rho} = -i[H_0, \rho] - \frac{\kappa}{2}\mathcal{L}[\psi] - \sum_{\alpha}\left[\frac{\Gamma_{\uparrow}}{2}\mathcal{L}[\sigma^{+}_{\alpha}] + \frac{\Gamma_{\downarrow}}{2}\mathcal{L}[\sigma^{-}_{\alpha}]\right]$$
$$- \sum_{\alpha}\left[\frac{\Gamma(\delta = \omega - \epsilon)}{2}\mathcal{L}[\sigma^{+}_{\alpha}\psi] + \frac{\Gamma(-\delta = \epsilon - \omega)}{2}\mathcal{L}[\sigma^{-}_{\alpha}\psi^{\dagger}]\right]$$

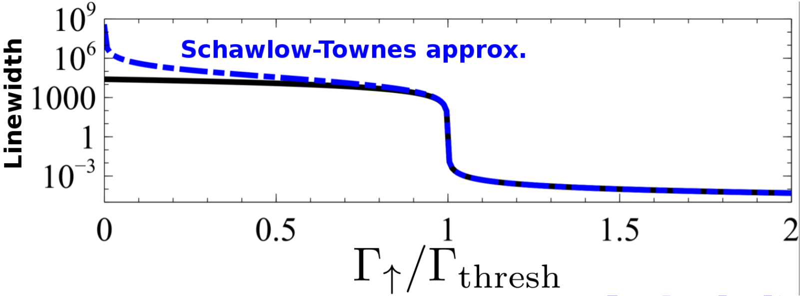
- Factorise $\rho(t) \simeq \rho_{\text{ph}}(t) \bigotimes_i \rho_{m,i}(t)$
- Quantum regression theorm $\rightarrow$ linewidth

Quantum model, linewidth

Full Master equation:

$$\dot{\rho} = -i[H_0, \rho] - \frac{\kappa}{2}\mathcal{L}[\psi] - \sum_\alpha \left[\frac{\Gamma_\uparrow}{2}\mathcal{L}[\sigma_\alpha^+] + \frac{\Gamma_\downarrow}{2}\mathcal{L}[\sigma_\alpha^-]\right]$$
$$- \sum_\alpha \left[\frac{\Gamma(\delta = \omega - \epsilon)}{2}\mathcal{L}[\sigma_\alpha^+\psi] + \frac{\Gamma(-\delta = \epsilon - \omega)}{2}\mathcal{L}[\sigma_\alpha^-\psi^\dagger]\right]$$

- Factorise $\rho(t) \simeq \rho_{\text{ph}}(t) \bigotimes_i \rho_{m,i}(t)$
- Quantum regression theorm $\rightarrow$ linewidth



Thermalisation of spectrum:

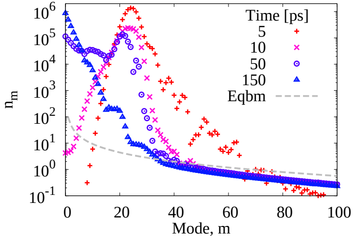
- Thermalisation of spectrum:

Thermalisation of spectrum:

- Thermalisation of spectrum:

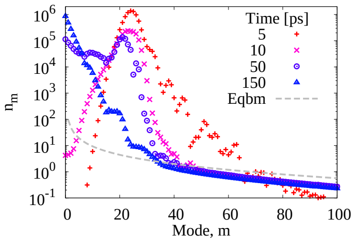
Thermalisation of spectrum:

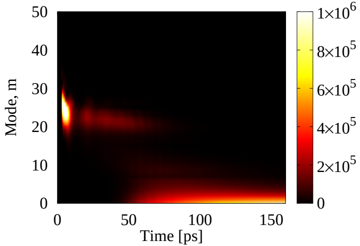
- Thermalisation of spectrum:



Thermalisation of spectrum:

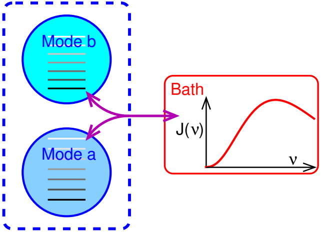
- Thermalisation of spectrum:





Toy problem: two bosonic modes

- Basic problem: Emission from thermal bath



$$H = \omega_a \hat{\psi}_a^\dagger \hat{\psi}_a + \omega_b \hat{\psi}_b^\dagger \hat{\psi}_b + H_{\text{Bath}}$$
$$+ (\varphi_a^* \hat{\psi}_a^\dagger + \varphi_b^* \hat{\psi}_b^\dagger) \sum_i g_i \hat{c}_i + \text{H.c.}$$

Toy problem: naïve solutions

Two “expected” behaviours:

- At resonance: “weak lasing” — coupling to bath dominates

$$\frac{d}{dt}\rho = \Gamma^{\downarrow}\mathcal{L}[\varphi_a\hat{\psi}_a + \varphi_b\hat{\psi}_b] + \Gamma^{\uparrow}\mathcal{L}[\varphi_a^*\hat{\psi}_a^\dagger + \varphi_b^*\hat{\psi}_b^\dagger]$$

- Far from resonance: pointer states are eigenstates

$$\frac{d}{dt}\rho = \sum_{i=a,b} \Gamma_i^{\downarrow}\mathcal{L}[\hat{\psi}_i] + \Gamma_i^{\uparrow}\mathcal{L}[\hat{\psi}_i^\dagger]$$

- Questions:
  - How does crossover work?
  - Are these actually right?

Toy problem: naïve solutions

Two "expected" behaviours:

- At resonance: "weak lasing" — coupling to bath dominates

$$\frac{d}{dt}\rho = \Gamma^{\downarrow}\mathcal{L}[\varphi_a\hat{\psi}_a + \varphi_b\hat{\psi}_b] + \Gamma^{\uparrow}\mathcal{L}[\varphi_a^*\hat{\psi}_a^\dagger + \varphi_b^*\hat{\psi}_b^\dagger]$$

- Far from resonance: pointer states are eigenstates

$$\frac{d}{dt}\rho = \sum_{i=a,b}\Gamma_i^{\downarrow}\mathcal{L}[\hat{\psi}_i] + \Gamma_i^{\uparrow}\mathcal{L}[\hat{\psi}_i^\dagger]$$

- Questions:
  - How does crossover work?
  - Are these actually right?

Toy problem: naïve solutions

Two "expected" behaviours:

- At resonance: "weak lasing" — coupling to bath dominates

$$\frac{d}{dt}\rho = \Gamma^{\downarrow}\mathcal{L}[\varphi_a\hat{\psi}_a + \varphi_b\hat{\psi}_b] + \Gamma^{\uparrow}\mathcal{L}[\varphi_a^*\hat{\psi}_a^\dagger + \varphi_b^*\hat{\psi}_b^\dagger]$$

- Far from resonance: pointer states are eigenstates

$$\frac{d}{dt}\rho = \sum_{i=a,b} \Gamma_i^{\downarrow}\mathcal{L}[\hat{\psi}_i] + \Gamma_i^{\uparrow}\mathcal{L}[\hat{\psi}_i^\dagger]$$

- Questions:
  - How does crossover work?
  - Are these actually right?

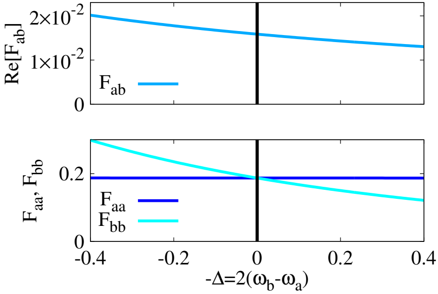
Toy problem: exact solution

- Solve via Laplace transform. Find $F_{ij}(t) = \langle \hat{\psi}_i^\dagger(t) \hat{\psi}_j(t) \rangle$
- Steady state:
- Time evolution —
  $F_{ab}(t) \sim \exp(-\alpha\Delta^2 t)$

- Always some coherence
  - (individual always wrong)
- $F_{ab} \sim F_{aa}, F_{bb}$ only at $\Delta = 0$
  - (collective almost always wrong)

Toy problem: exact solution

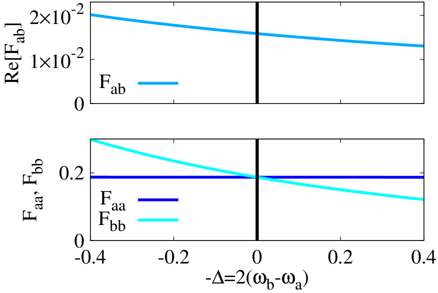
- Solve via Laplace transform. Find $F_{ij}(t) = \langle \hat{\psi}_i^\dagger(t) \hat{\psi}_j(t) \rangle$
- Steady state:
  - Singular at $\Delta = 0$
- Time evolution — $F_{ab}(t) \sim \exp(-\alpha \Delta^2 t)$



- Always some coherence
  - (individual always wrong)
- $F_{ab} \sim F_{aa}, F_{bb}$ only at $\Delta = 0$
  - (collective almost always wrong)

Toy problem: exact solution

- Solve via Laplace transform. Find $F_{ij}(t) = \langle \hat{\psi}_i^\dagger(t) \hat{\psi}_j(t) \rangle$
- Steady state:
  - Singular at $\Delta = 0$

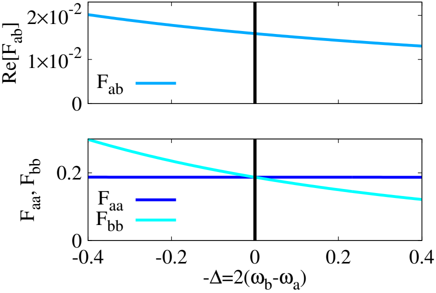


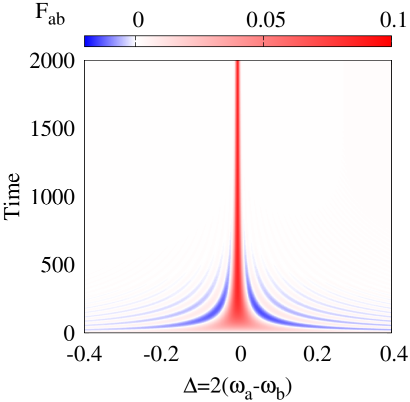
- Time evolution — $F_{ab}(t) \sim \exp(-\alpha \Delta^2 t)$

- Always some coherence
  - (individual always wrong)
- $F_{ab} \sim F_{aa}, F_{bb}$ only at $\Delta = 0$
  - (collective almost always wrong)

Toy problem: exact solution

- Solve via Laplace transform. Find $F_{ij}(t) = \langle \hat{\psi}_i^\dagger(t) \hat{\psi}_j(t) \rangle$
- Steady state:
  - Singular at $\Delta = 0$
- Time evolution — $F_{ab}(t) \sim \exp(-\alpha \Delta^2 t)$

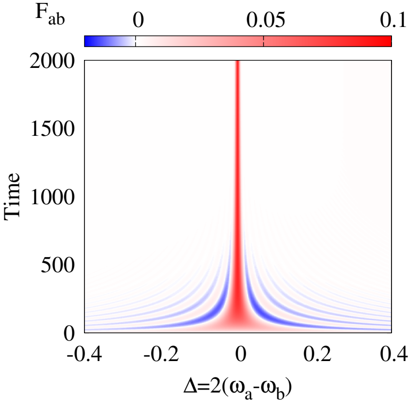


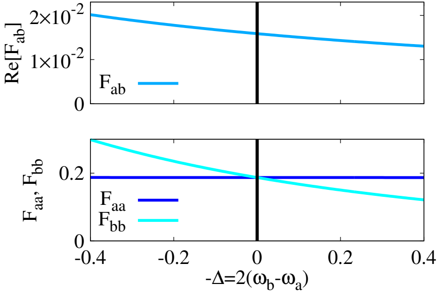


- Always some coherence
  - (individual always wrong)
- $F_{ab} \sim F_{aa}, F_{bb}$ only at $\Delta = 0$
  - (collective almost always wrong)

Toy problem: exact solution

- Solve via Laplace transform. Find $F_{ij}(t) = \langle \hat{\psi}_i^\dagger(t) \hat{\psi}_j(t) \rangle$
- Steady state:
  - Singular at $\Delta = 0$
- Time evolution — $F_{ab}(t) \sim \exp(-\alpha\Delta^2 t)$

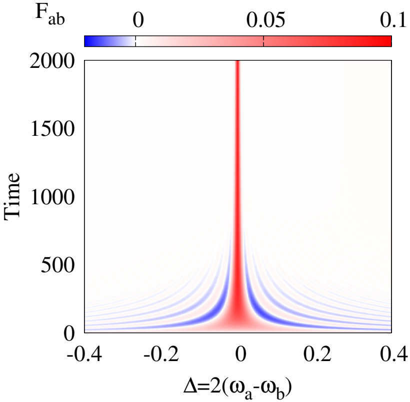


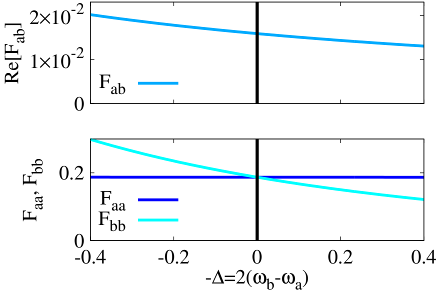


- Always some coherence
  - (individual always wrong)
- $F_{ab} \sim F_{aa}, F_{bb}$ only at $\Delta = 0$
  - (collective almost always wrong)

Toy problem: exact solution

- Solve via Laplace transform. Find $F_{ij}(t) = \langle \hat{\psi}_i^\dagger(t) \hat{\psi}_j(t) \rangle$
- Steady state:
  - Singular at $\Delta = 0$
- Time evolution —
  $F_{ab}(t) \sim \exp(-\alpha \Delta^2 t)$





- Always some coherence
  - (individual always wrong)
- $F_{ab} \sim F_{aa}, F_{bb}$ only at $\Delta = 0$
  - (collective almost always wrong)

Toy problem: Bloch-Redfield theory

Unsecularised Bloch-Redfield theory:

$$\partial_t \rho = -i[\hat{H}, \rho] + \sum_{ij} L^{\downarrow}_{ij} \varphi^*_i \varphi_j \left(2\hat{\psi}_j \rho \hat{\psi}^\dagger_i - [\rho, \hat{\psi}^\dagger_i \hat{\psi}_j]_+ \right) + \sum_{ij} L^{\uparrow}_{ij} \varphi^*_i \varphi_j \left(2\hat{\psi}^\dagger_j \rho \hat{\psi}_i - [\rho, \hat{\psi}_i \hat{\psi}^\dagger_j]_+ \right).$$

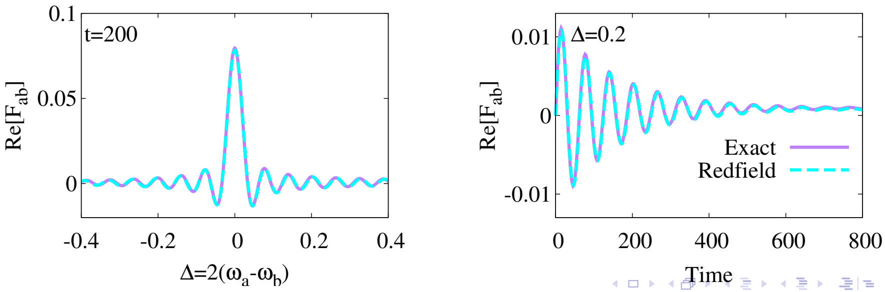
- Compare to exact solution:

Toy problem: Bloch-Redfield theory

Unsecularised Bloch-Redfield theory:

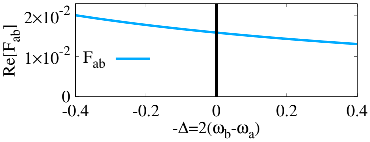
$$\partial_t \rho = -i[\hat{H}, \rho] + \sum_{ij} L^{\downarrow}_{ij} \varphi^*_i \varphi_j \left(2\hat{\psi}_j \rho \hat{\psi}^\dagger_i - [\rho, \hat{\psi}^\dagger_i \hat{\psi}_j]_+ \right) + \sum_{ij} L^{\uparrow}_{ij} \varphi^*_i \varphi_j \left(2\hat{\psi}^\dagger_j \rho \hat{\psi}_i - [\rho, \hat{\psi}_i \hat{\psi}^\dagger_j]_+ \right).$$

- Compare to exact solution:



Toy problem: Secularisation

- Secularisation (in eigenbasis of $\hat{H}$): $L_{ij}^{\uparrow,\downarrow} \to L_{ii}^{\uparrow,\downarrow}\delta_{ij}$



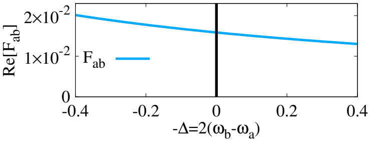
- Leads to $F_{ab}(t \to \infty) = 0$. Exact:
- Secularisation often invoked to cure negative eigenvalues of $L_{ij}^{\uparrow,\downarrow}$.
- Check stability: consider $f = (F_{aa}, F_{bb}, \Re[F_{ab}], \Im[F_{ab}])$

$$\partial_t \mathbf{f} = -\mathbf{M}\mathbf{f} + \mathbf{f}_0$$

- Eigenvalues of **M** exist in closed form:
  - Unstable (negative only if $dJ(\nu)/d\nu \gg 1$ — Markov breakdown)

Toy problem: Secularisation

- Secularisation (in eigenbasis of $\hat{H}$): $L_{ij}^{\uparrow,\downarrow} \to L_{ii}^{\uparrow,\downarrow}\delta_{ij}$
- Leads to $F_{ab}(t \to \infty) = 0$. Exact:



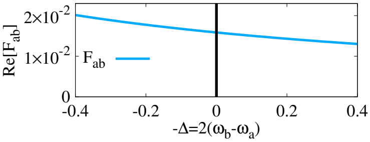
- Secularisation often invoked to cure negative eigenvalues of $L_{ij}^{\uparrow,\downarrow}$.
- Check stability: consider $f = (F_{aa}, F_{bb}, \Re[F_{ab}], \Im[F_{ab}])$

$$\partial_t \mathbf{f} = -\mathbf{M}\mathbf{f} + \mathbf{f}_0$$

- Eigenvalues of **M** exist in closed form:
  - Unstable (negative only if $dJ(\nu)/d\nu \gg 1$ — Markov breakdown)

Toy problem: Secularisation

- Secularisation (in eigenbasis of $\hat{H}$): $L_{ij}^{\uparrow,\downarrow} \to L_{ii}^{\uparrow,\downarrow}\delta_{ij}$
- Leads to $F_{ab}(t \to \infty) = 0$. Exact:



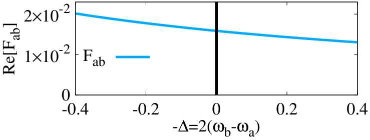
- Secularisation often invoked to cure negative eigenvalues of $L_{ij}^{\uparrow,\downarrow}$.
  - → Non-positivity of density matrix,
  - → Unstable (unbounded growth).
- Check stability: consider $f = (F_{aa}, F_{bb}, \Re[F_{ab}], \Im[F_{ab}])$

$$\partial_t \mathbf{f} = -\mathbf{M}\mathbf{f} + \mathbf{f}_0$$

- Eigenvalues of $\mathbf{M}$ exist in closed form:
  - Unstable (negative only if $dJ(\nu)/d\nu \gg 1$ — Markov breakdown)

Toy problem: Secularisation

- Secularisation (in eigenbasis of $\hat{H}$): $L^{\uparrow,\downarrow}_{ij} \rightarrow L^{\uparrow,\downarrow}_{ii}\delta_{ij}$
- Leads to $F_{ab}(t \rightarrow \infty) = 0$. Exact:



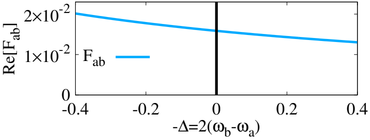
- Secularisation often invoked to cure negative eigenvalues of $L^{\uparrow,\downarrow}_{ij}$.
  - $\rightarrow$ Non-positivity of density matrix,
  - $\rightarrow$ Unstable (unbounded growth).
- Check stability: consider $\mathbf{f} = (F_{aa}, F_{bb}, \Re[F_{ab}], \Im[F_{ab}])$

$$\partial_t \mathbf{f} = -\mathbf{M}\mathbf{f} + \mathbf{f}_0$$

- Eigenvalues of $\mathbf{M}$ exist in closed form:
  - Unstable (negative only if $dJ(\nu)/d\nu \gg 1$ — Markov breakdown)

Toy problem: Secularisation

- Secularisation (in eigenbasis of $\hat{H}$): $L_{ij}^{\uparrow,\downarrow} \rightarrow L_{ii}^{\uparrow,\downarrow}\delta_{ij}$
- Leads to $F_{ab}(t \rightarrow \infty) = 0$. Exact:



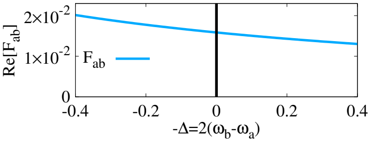
- Secularisation often invoked to cure negative eigenvalues of $L_{ij}^{\uparrow,\downarrow}$.
  - $\rightarrow$ Non-positivity of density matrix,
  - $\rightarrow$ Unstable (unbounded growth).
- Check stability: consider $f = (F_{aa}, F_{bb}, \Re[F_{ab}], \Im[F_{ab}])$

$$\partial_t \mathbf{f} = -\mathbf{M}\mathbf{f} + \mathbf{f}_0$$

- Eigenvalues of **M** exist in closed form:
  - Unstable (negative only if $dJ(\nu)/d\nu \gg 1$
    — Markov breakdown)

Toy problem: Secularisation

- Secularisation (in eigenbasis of $\hat{H}$): $L_{ij}^{\uparrow,\downarrow} \rightarrow L_{ii}^{\uparrow,\downarrow}\delta_{ij}$
- Leads to $F_{ab}(t \rightarrow \infty) = 0$. Exact:



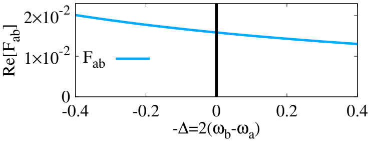
- Secularisation often invoked to cure negative eigenvalues of $L_{ij}^{\uparrow,\downarrow}$.
  - $\rightarrow$ Non-positivity of density matrix,
  - $\rightarrow$ Unstable (unbounded growth).
- Check stability: consider $f = (F_{aa}, F_{bb}, \Re[F_{ab}], \Im[F_{ab}])$

$$\partial_t \mathbf{f} = -\mathbf{M}\mathbf{f} + \mathbf{f}_0$$

- Eigenvalues of **M** exist in closed form:
  - Unstable (negative only if $dJ(\nu)/d\nu \gg 1$
  - — Markov breakdown)

Toy problem: Secularisation

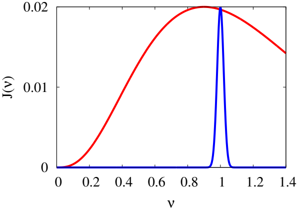
- Secularisation (in eigenbasis of $\hat{H}$): $L_{ij}^{\uparrow,\downarrow} \rightarrow L_{ii}^{\uparrow,\downarrow}\delta_{ij}$
- Leads to $F_{ab}(t \rightarrow \infty) = 0$. Exact:



- Secularisation often invoked to cure negative eigenvalues of $L_{ij}^{\uparrow,\downarrow}$.
  - $\rightarrow$ Non-positivity of density matrix,
  - $\rightarrow$ Unstable (unbounded growth).
- Check stability: consider $f = (F_{aa}, F_{bb}, \Re[F_{ab}], \Im[F_{ab}])$

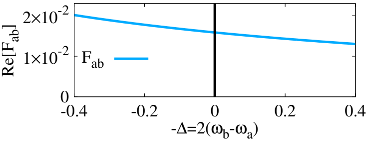
$$\partial_t \mathbf{f} = -\mathbf{M}\mathbf{f} + \mathbf{f}_0$$

- Eigenvalues of **M** exist in closed form:
  - Unstable (negative only if $dJ(\nu)/d\nu \gg 1$ — Markov breakdown)



Toy problem: Secularisation

- Secularisation (in eigenbasis of $\hat{H}$): $L_{ij}^{\uparrow,\downarrow} \to L_{ii}^{\uparrow,\downarrow}\delta_{ij}$
- Leads to $F_{ab}(t \to \infty) = 0$. Exact:



- Secularisation often invoked to cure negative eigenvalues of $L_{ij}^{\uparrow,\downarrow}$.
  - $\to$ Non-positivity of density matrix,
  - $\to$ Unstable (unbounded growth).
- Check stability: consider $f = (F_{aa}, F_{bb}, \Re[F_{ab}], \Im[F_{ab}])$

$$\partial_t \mathbf{f} = -\mathbf{M}\mathbf{f} + \mathbf{f}_0$$

- Eigenvalues of **M** exist in closed form:
  - Unstable (negative only if $dJ(\nu)/d\nu \gg 1$ — Markov breakdown)