Collective behaviour and driven-dissipative systems

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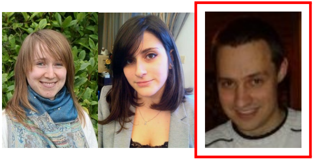
Acknowledgements

GROUP (&ALUMNI):



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Collective behaviour and driven-dissipative systems

1. Nonequilibrium quantum matter
2. Collective behaviour in driven–dissipative systems
   - Transverse field Ising
   - Rabi-Hubbard model
3. Collective dissipation
   - Coupled qubit-cavity systems
   - Bath induced coherence

Nonequilibrium quantum matter

Driven systems

Open quantum system

$$\partial_t \rho = -i[\hat{H}, \rho] + \sum_i \kappa_i \mathcal{L}[X_i], \qquad \mathcal{L}[X_i] = 2X_i \rho X_i^\dagger - X_i^\dagger X_i \rho - \rho X_i^\dagger X_i$$

Need drive to balance loss

1. External **coherent** drive:

$$\hat{H} \to \hat{H} + \hat{V}\cos(\Omega t)$$

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- $\tilde{\hat{H}} = e^{-i\Omega \hat{N} t} \hat{H} e^{i\Omega \hat{N} t} - \Omega \hat{N}$
- Neglect fast $e^{2i\Omega t}$ terms — fast
- Rotating frame — breaks detailed balance with bath.

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Non-equilibrium steady state

2. External **incoherent** drive:
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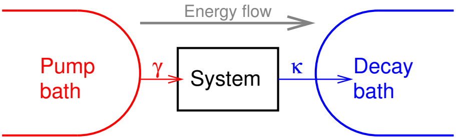

- Energy flow through system

- Not thermodynamics — attractors of dynamics
  - Stationary points — extrema of energy?
  - Nontrivial attractors

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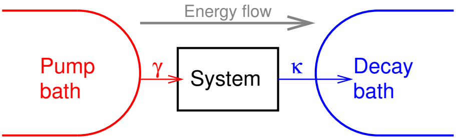

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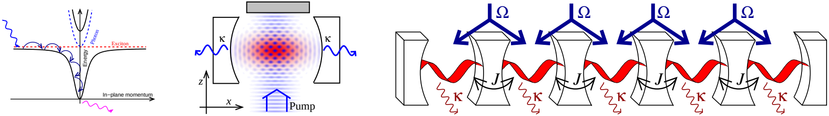

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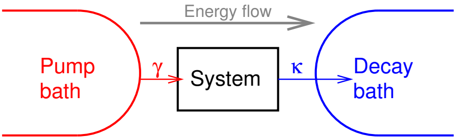

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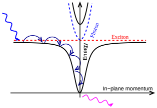

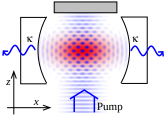

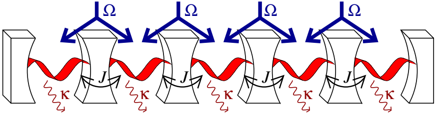

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Coupled cavity arrays

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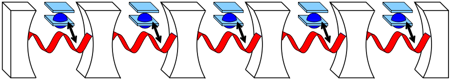

[Hartmann *et al.* Nat. Phys. '06; Greentree *et al.* *ibid* 06; Angelakis *et al.* PRA '07]

- X-Hubbard Model, $H = \sum_i H_{X,site} - J \sum_{\langle ij \rangle} \psi_i^\dagger \psi_j$

  [X=Bose, Jaynes-Cummings, Rabi, ...]

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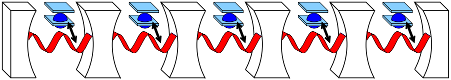

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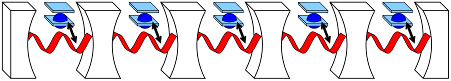

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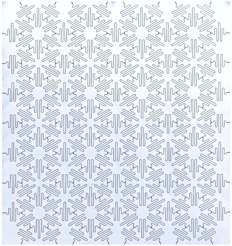
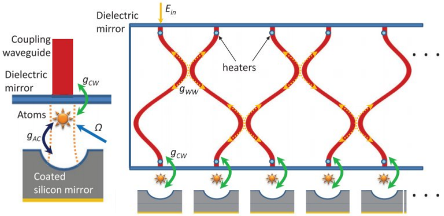


[Underwood *et al.* PRA '12; Nat. Phys '12]



[Lepert *et al.* NJP '11; APL '13]

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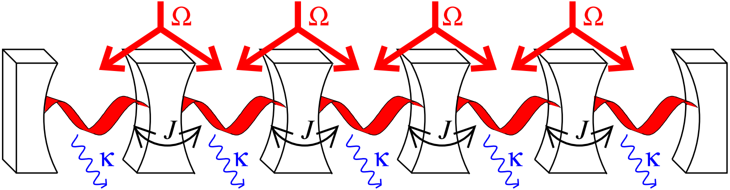
Parametrically pumped BHM



$$H = -\frac{J}{z}\sum_{<ij>}\psi_i^\dagger\psi_j + \sum_i\left[\omega_c\psi_i^\dagger\psi_i + U\psi_i^\dagger\psi_i^\dagger\psi_i\psi_i - \Omega\left(\psi_i^\dagger\psi_{i+1}^\dagger e^{-2i\omega_p t} + \text{H.c.}\right)\right]$$

[Bardyn & Immamoglu, PRL '12]

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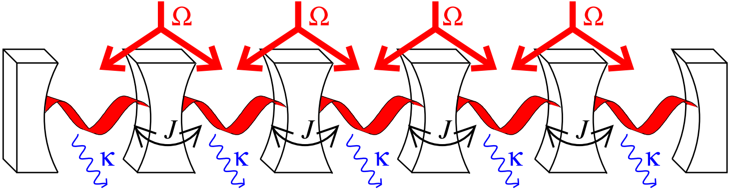

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Rotating frame, blockade approximation, rescale:

$$H = -J\sum \left[\tau_i^+ \tau_{i+1}^- + \tau_{i+1}^+ \tau_i^- + g\tau_i^z + \Delta\left(\tau_i^+ \tau_{i+1}^+ + \tau_{i+1}^- \tau_i^-\right)\right]$$

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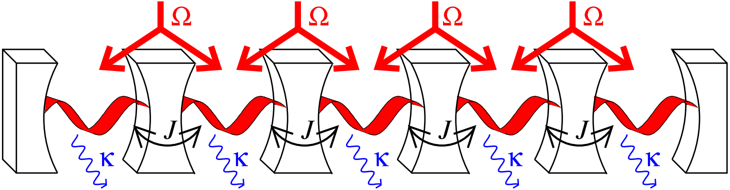

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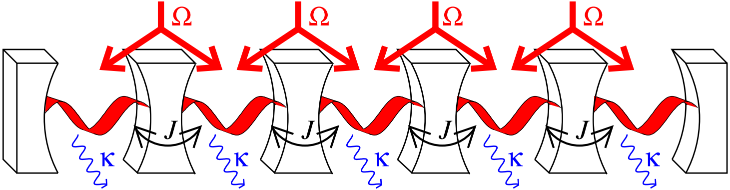
Parametric pumping – equilibrium



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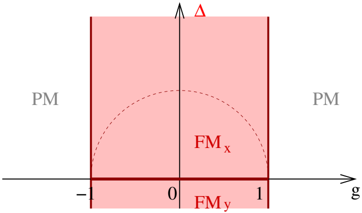
- Equilibrium – transverse field Ising model
  - g – transverse field, $g_{\text{crit}} = 1$.
  - Δ – anisotropy.
    $\Delta = 0$: XY, $|\Delta| > 0$: Ising (X,Y).



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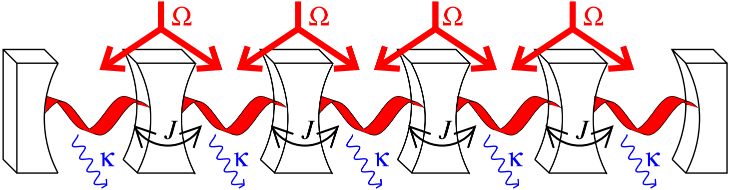

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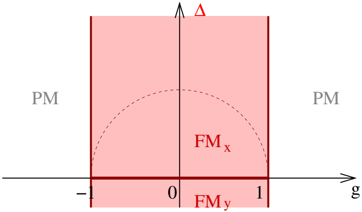

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Parametric pumping – open system

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$$\partial_t \rho = -i[H, \rho] + \sum_i \kappa \mathcal{L}[\tau_i^-]$$

- Mean-field EOM: $\partial_t \langle \tau_i^\alpha \rangle = F_\alpha(\langle \tau_{i-1}^\beta \rangle, \langle \tau_i^\beta \rangle, \langle \tau_{i+1}^\beta \rangle)$
- Dynamical attractors, linear stability:

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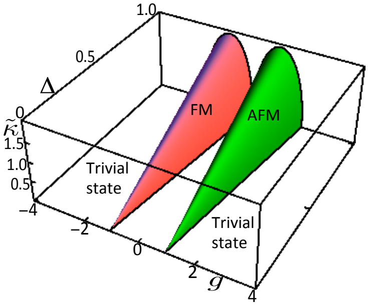
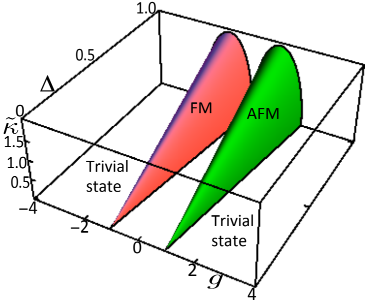


Why AFM/FM attractors



- Linear stability, fluctuation $\sim \exp(-i\nu_k t + ikr_i)$ Linear stability

$$\nu_k = -i\kappa \pm 2J\sqrt{g^2 + 2g\cos k + (1-\Delta^2)\cos^2 k}$$

- $g \ll -1$, Dissipation matches ground state
  - Most unstable mode, $k = 0$
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  - Most unstable mode, $k = \pi$

[Joshi, Nissen, Keeling, PRA '13]

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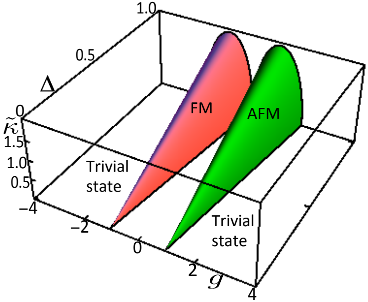

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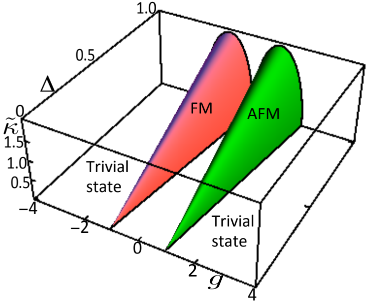

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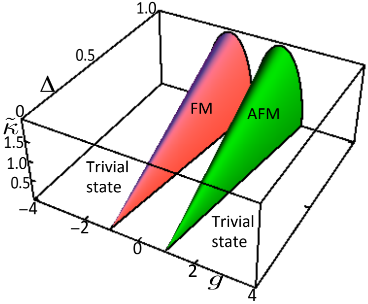

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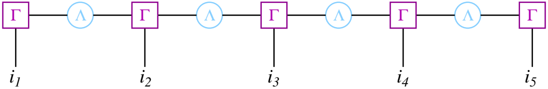
Beyond mean-field

- Matrix-product-operator representation of

$$\rho = \sum_{\{i_1, i_2, \ldots, i_N\}} \left(\sum_{\{\alpha_j\}} \Gamma^{[1]i_1}_{1,\alpha_1} \Lambda^{[1]}_{\alpha_1} \Gamma^{[2]i_2}_{\alpha_1,\alpha_2} \ldots \Gamma^{[N-1]i_{N-1}}_{\alpha_{N-2},\alpha_{N-1}} \Lambda^{[N-1]}_{\alpha_{N-1}} \Gamma^{[N]i_N}_{\alpha_{N-1},1} \right) \bigotimes_{j=1}^{N} \tau_j^{i_j}$$



Vidal, White, Schollwöck, *et al.* Density matrices: [Zwolak & Vidal, PRL ’04]

- No broken symmetry — correlators:

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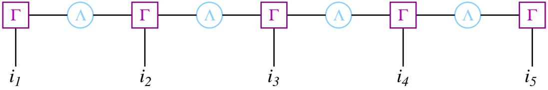

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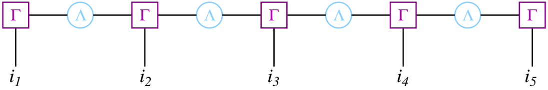

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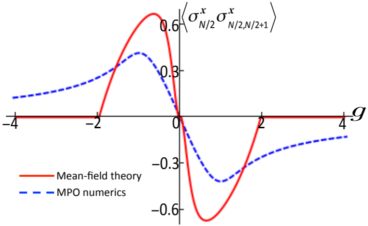
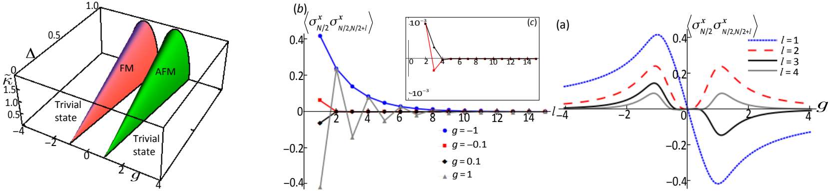


Correlations

- AFM vs FM from sign of g ($\Delta = 1$)



- $\Delta \to 0$, Analytic spin-wave, $|\langle \tau_i^- \tau_{i+l}^+ \rangle| \propto \exp(-\xi_c l)$

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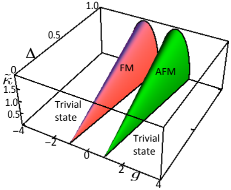

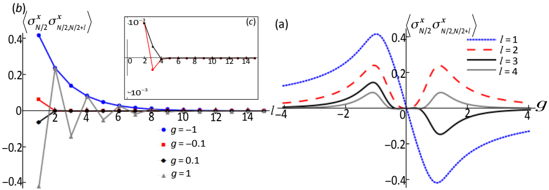

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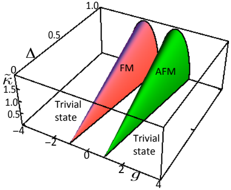

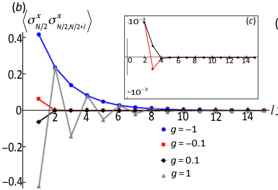

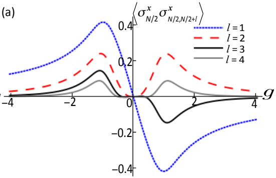

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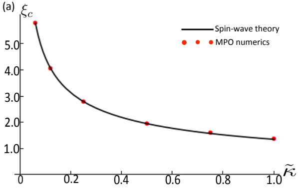
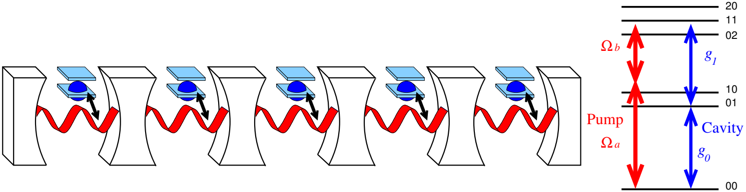


Rabi Hubbard model



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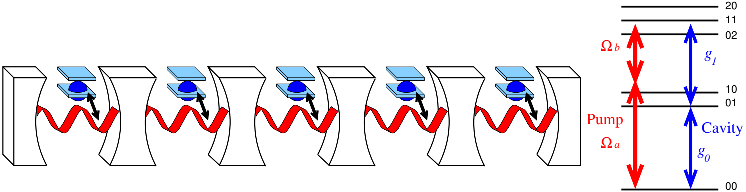

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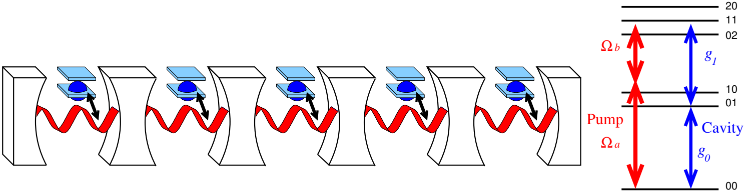

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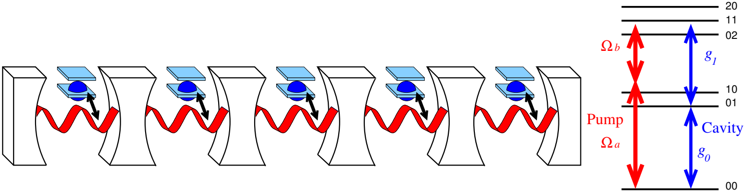

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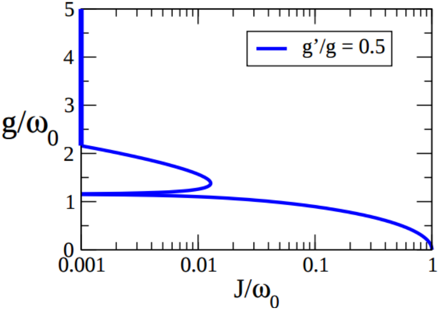
Rabi Hubbard model – equilibrium



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[Schiró *et al.* PRL '12]

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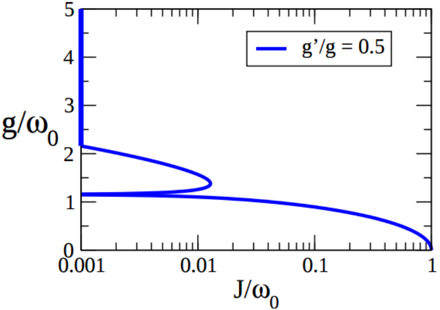

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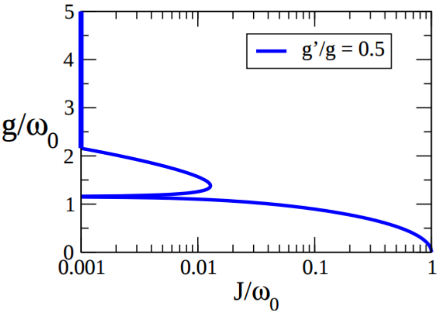

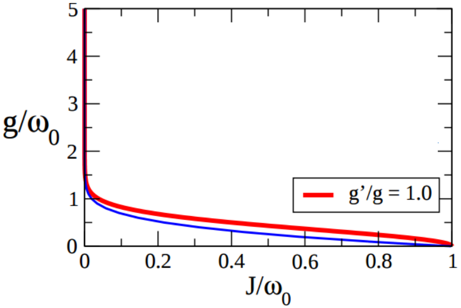

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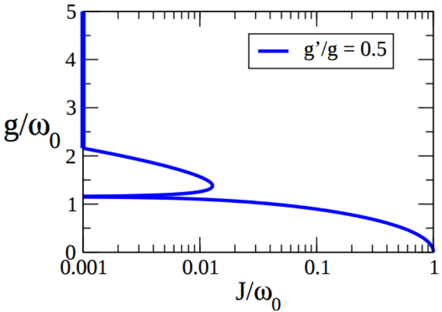

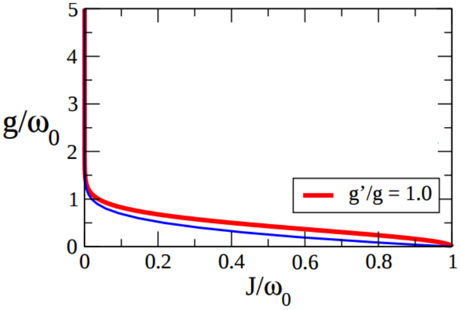

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Driven-dissipative system — linear stability

Mean field theory — still large Hilbert space.

- Normal state + fluctuations: $\rho = \otimes_n(\rho_{ss} + \sum_{\mathbf{k}} \delta\rho_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{n} - i\nu_{\mathbf{k}} t} + \text{H.c.})$
- $\nu_{\mathbf{k}}$ Eigenvalues of $M = M_0 - t_{\mathbf{k}} M_1$, $t_{\mathbf{k}} = -2J\cos(k)$
- Unstable if $\Im[\nu_{\mathbf{k}}] > 0$

- Given J, $|t_{\mathbf{k}}| < 2J$
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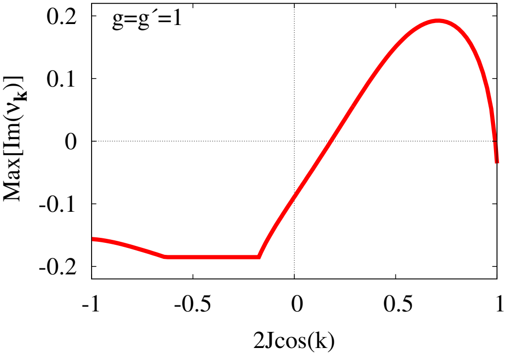

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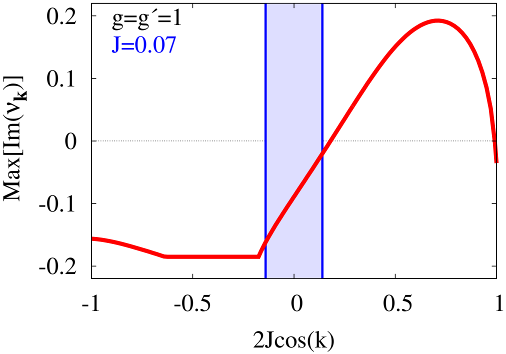

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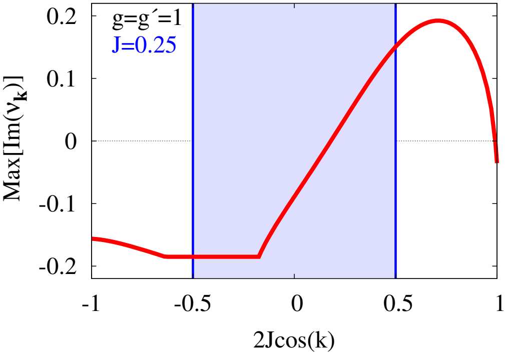

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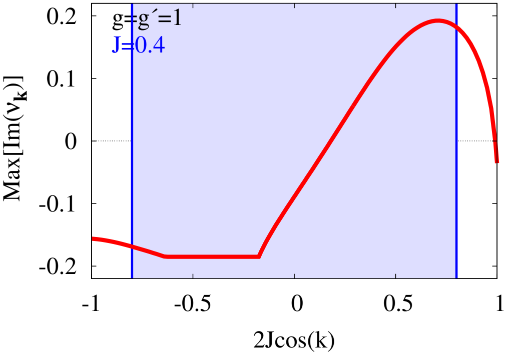

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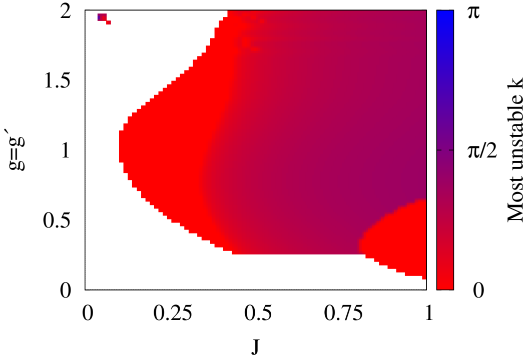
Rabi-Hubbard model — linear stability

Stability phase diagram:



[Schiró *et al.* arXiv:1503.04456]

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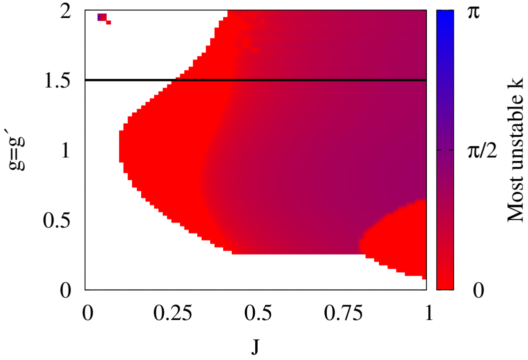
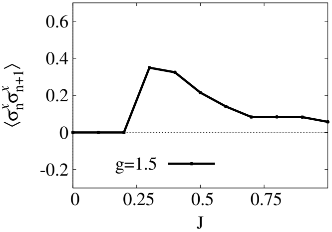


Steady state correlations:



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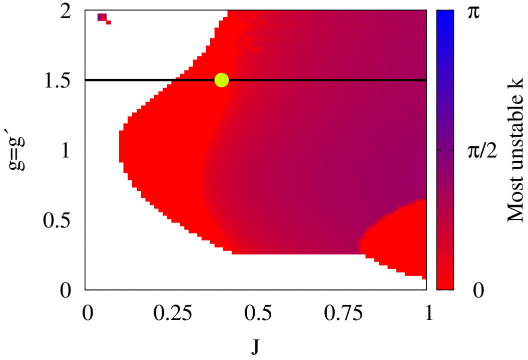

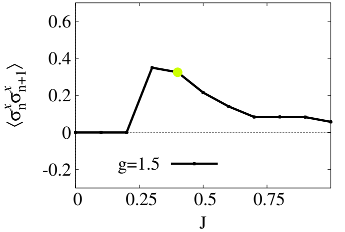

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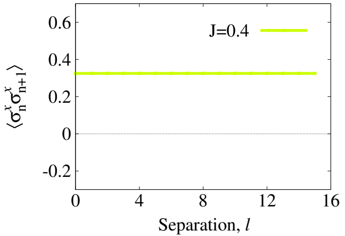

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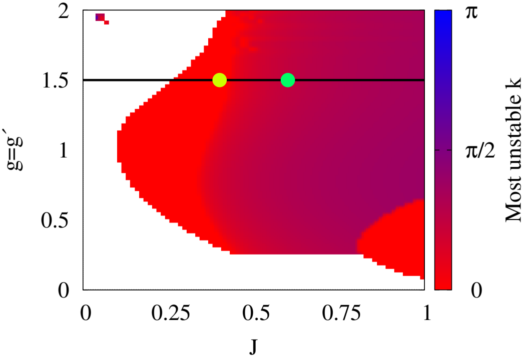

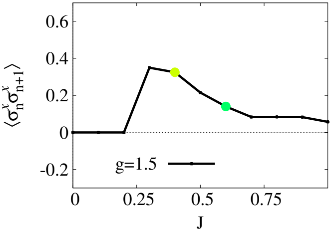

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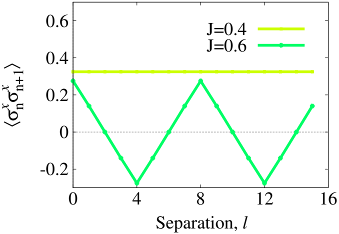

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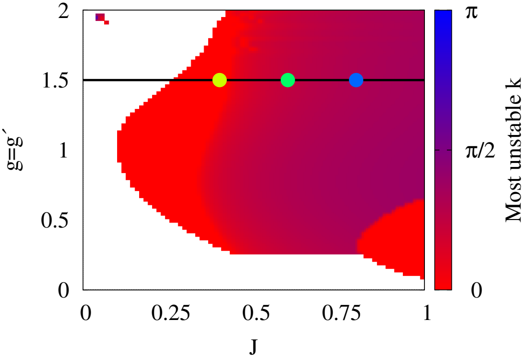

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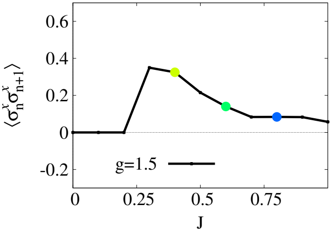

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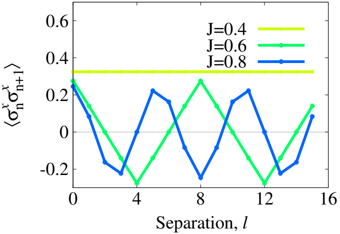

Linear stability – limit cycles

- If $\nu_k = \pm\nu'_k + i\nu''_k$ at instability $\rightarrow$ Limit Cycle
  [Lee *et al.* PRA '11, Jin *et al.* PRL '13, Ludwig & Marquard PRL '13, Chan *et al.* arXiV:1501.00979]

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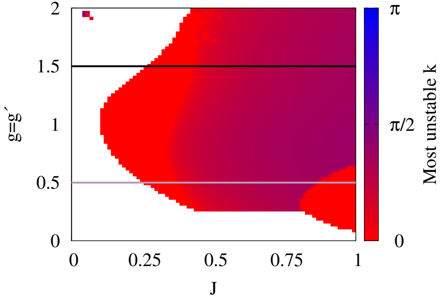

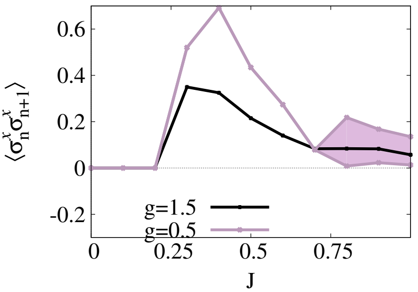

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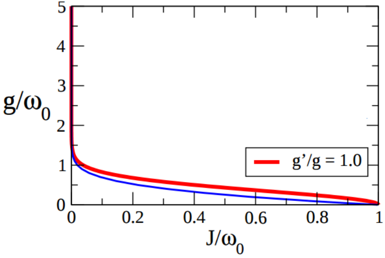
Phase-boundary Effective model

- Compare phase boundaries

Ground state:



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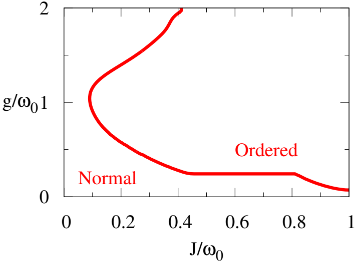

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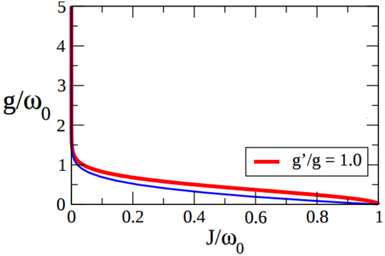

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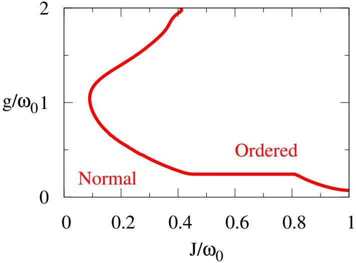

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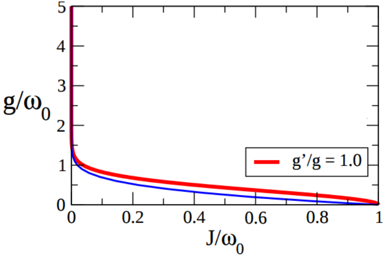

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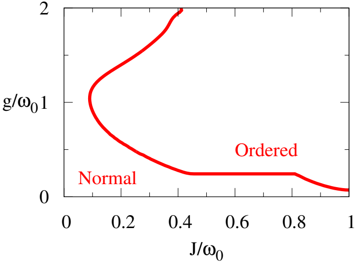

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$$H = \sum_i \frac{\Delta}{2}\tau_i^z - \sum_{\langle ij \rangle} \tilde{J}_x \tau_i^x \tau_i^x + \tilde{J}_y \tau_i^y \tau_i^y, \qquad \dot{\rho} = -i[H, \rho] + \ldots$$

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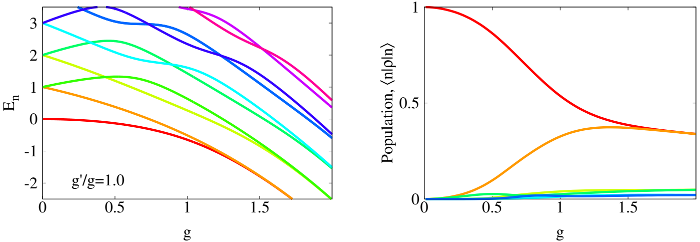

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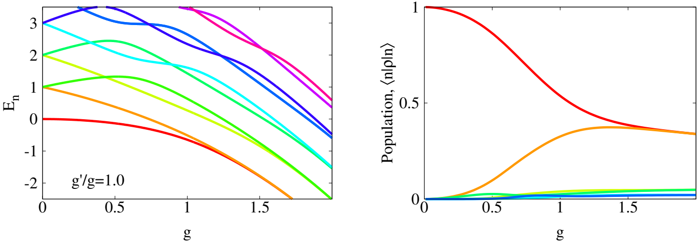

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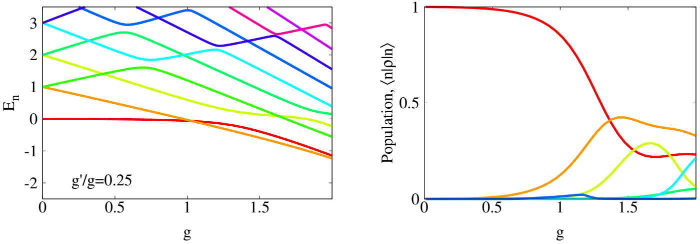
$g' \neq g$, Level crossings

- For $g' \neq g$, Δ can swap sign



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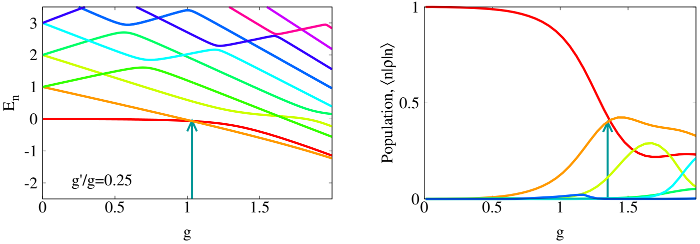

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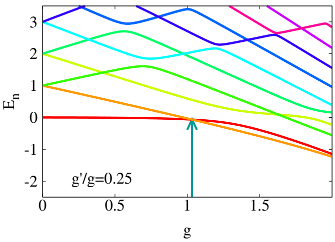

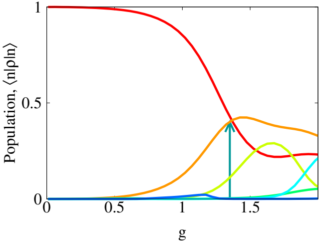

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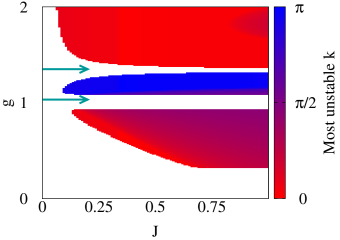

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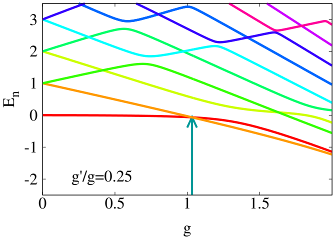

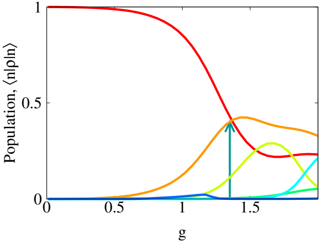

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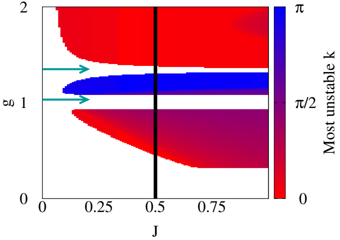

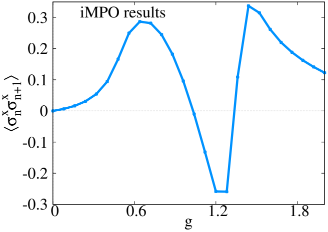

Collective dissipation

1. Nonequilibrium quantum matter

2. Collective behaviour in driven–dissipative systems
   - Transverse field Ising
   - Rabi-Hubbard model

3. Collective dissipation
   - Coupled qubit-cavity systems
   - Bath induced coherence

Collective dephasing

- Real environment is not Markovian
  - [Carmichael & Walls JPA '73] Requirements for correct equilibrium
  - [Ciuti & Carusotto PRA '09] Dicke SR and emission
- Cannot assume fixed κ, γ
- Phase transition $\rightarrow$ soft modes

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Dicke model linewidth:

$$H = \omega\psi^\dagger\psi + \sum_{i=1}^{N} \frac{\epsilon_i}{2}\sigma_i^z + g\left(\sigma_i^+\psi + \mathrm{h.c.}\right)$$

$$+\sum_i \sigma_i^z \sum_q \gamma_q \left(b_q^\dagger + b_q\right) + \sum_q \beta_q b_{iq}^\dagger b_q.$$

[Nissen, Fink *et al.* PRL '13]

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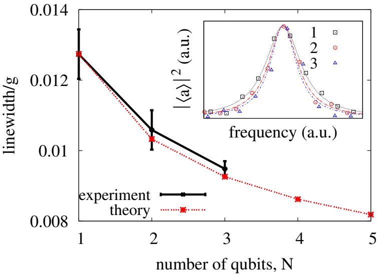

[Nissen, Fink *et al.* PRL '13]

Collective dephasing & weak lasing

- Weak lasing, [Aleiner, Altshuler, Rubo PRB '12] — dissipation selects collective modes.
- Toy problem:

$$\hat{H} = \omega_a a^\dagger a + \omega_b b^\dagger b + (a^\dagger + b^\dagger) \sum_i \xi_i c_i + \text{H.c} + H_{\text{Bath}}$$

- Standard picture:

$$\dot{\rho} = -i[H_0, \rho] + \begin{cases} \gamma_\uparrow \mathcal{L}[a^\dagger + b^\dagger] + \gamma_\downarrow \mathcal{L}[a + b] + & \text{degenerate} \\ \gamma_{\uparrow,a} \mathcal{L}[a^\dagger] + \gamma_{\uparrow,b} \mathcal{L}[b^\dagger] + \ldots & \text{secularised} \end{cases}$$

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Bath induced coherence

- Steady state:
  - If $\omega_a = \omega_b$, then $\langle a^\dagger b \rangle = \langle a^\dagger a \rangle = \langle b^\dagger b \rangle$
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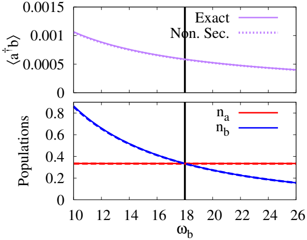

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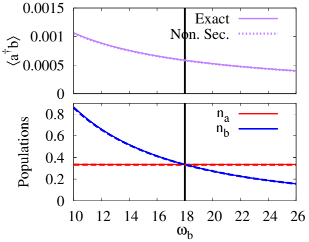

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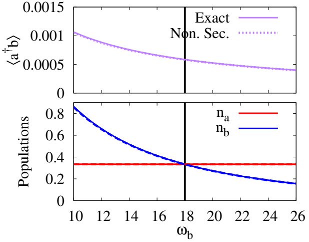

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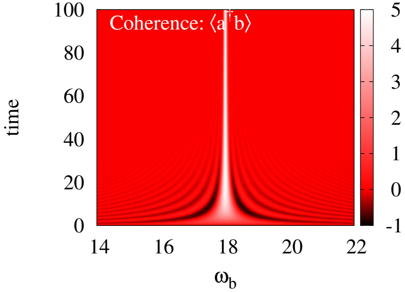

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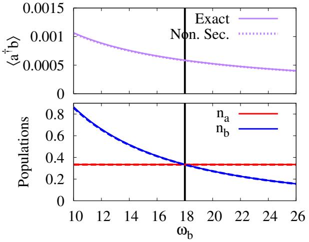

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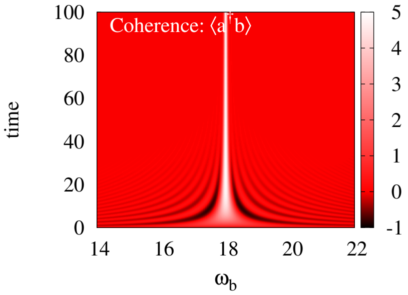
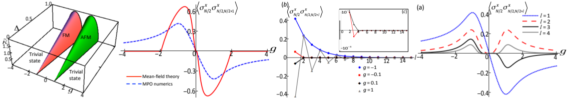


$$\langle a^\dagger b \rangle \simeq \exp\left[-C(\omega_a - \omega_b)^2 t\right] + \langle a^\dagger b \rangle_{t\to\infty}$$

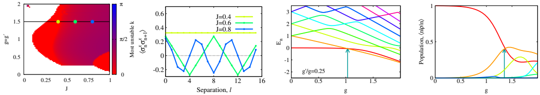
Summary

- Parametric pumping — non-equilibrium “phases” of transverse field Ising model



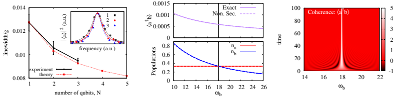
Joshi *et al.* PRA ’13

- Rabi Hubbard model — exotic attractors.



Schiró *et al.* arXiv:1503.04456

- Collective effects in dephasing



Nissen *et al.* PRL ’13