Non-equilibrium states of coupled cavity arrays.

Jonathan Keeling

SUPA

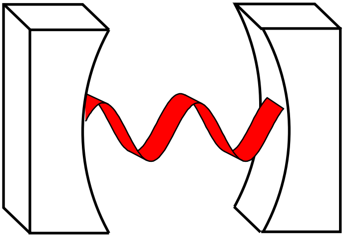
University of St Andrews
600 YEARS

ICTP, May 2014

Quantum Optics and cavity QED

- Quantum optics
- Cavity QED

$$H = \omega\psi^\dagger\psi$$



- Open system:

$$\partial_t\rho = -i[H,\rho] + \kappa\mathcal{L}[\psi] + \gamma\mathcal{L}[\sigma^-]$$
$$\mathcal{L}[X] = 2X\rho X^\dagger - X^\dagger X\rho - \rho X^\dagger X$$

- Rabi oscillations, collapse revival
- Fluorescence, Mollow triplet, power broadening, Purcell effect

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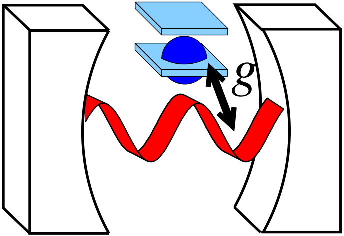

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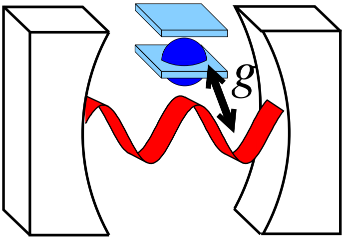

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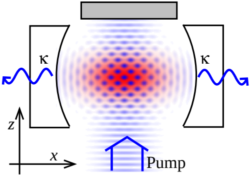
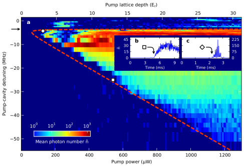
Cold atoms in cavities





[Baumann *et al.* Nature '10]

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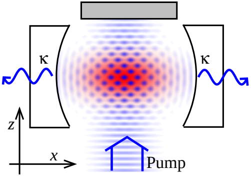

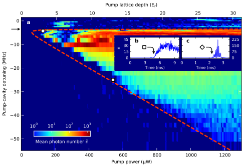

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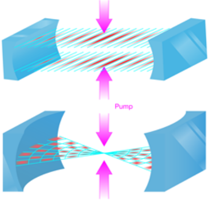

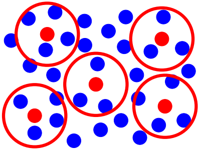

Multi-mode

[Gopalakrishnan *et al.* Nat. Phys. '09]

Rydberg states

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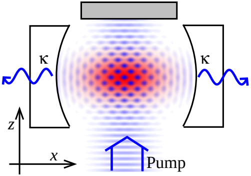

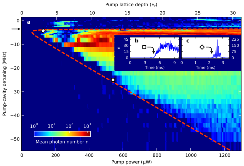

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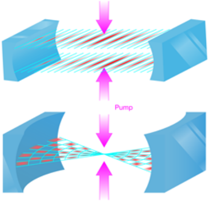

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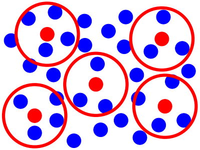
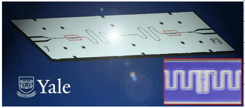
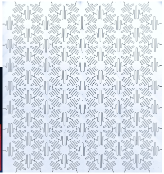


Rydberg states

Superconducting qubits





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[Review: Houck *et al.* Nat. Phys. '12]

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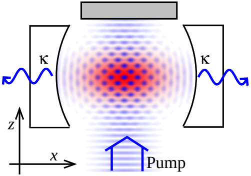

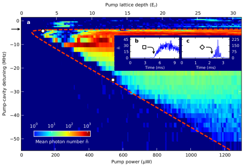

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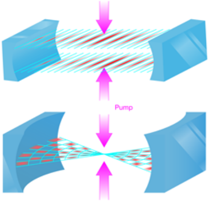

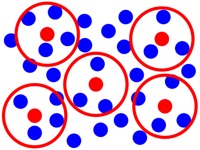

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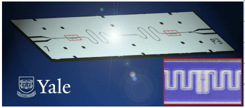

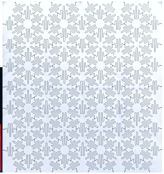

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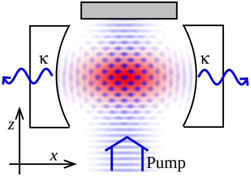

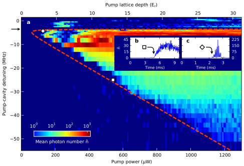

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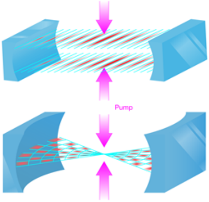

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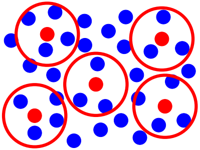

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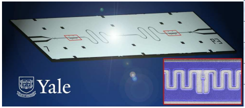

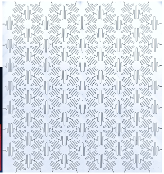

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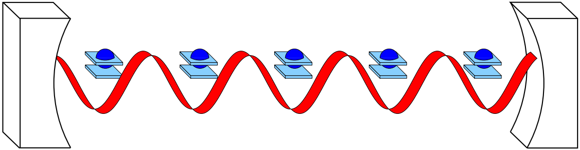
Many body quantum optics: Superradiance



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[Hepp, Lieb, Ann. Phys. '73]

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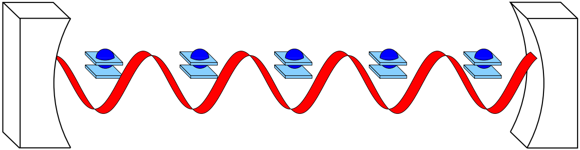

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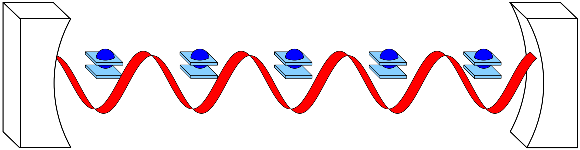

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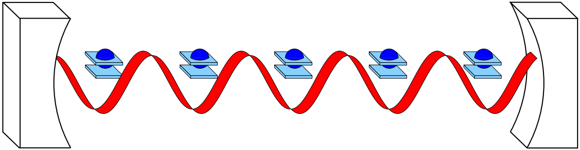

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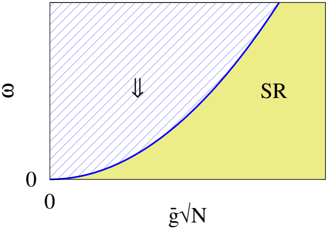

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[Eastham and Littlewood, PRB '01]

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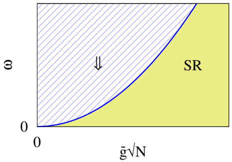

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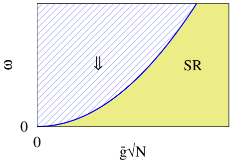

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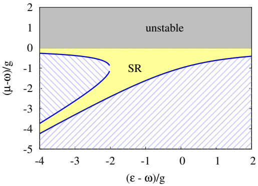

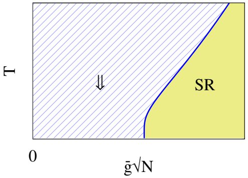

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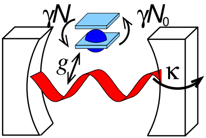

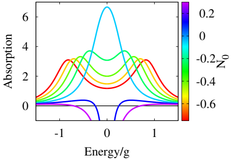

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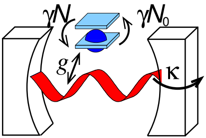

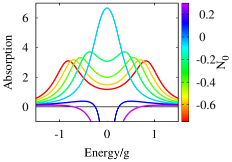

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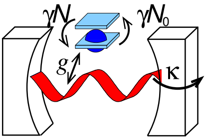

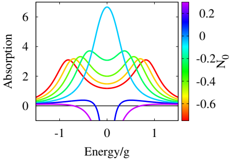

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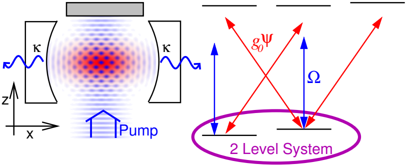
Self organisation and Dicke model



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$$\phi(x,z) \propto \begin{cases} 1 & \Downarrow \\ \cos(qz)\cos(qz) & \Uparrow \end{cases}$$

$$S = \sum_\alpha \sigma_\alpha^z/2$$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z$$

[Dimer *et al.* PRA '07][Baumann *et al* Nature '10]

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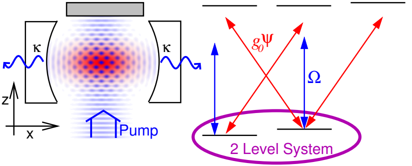

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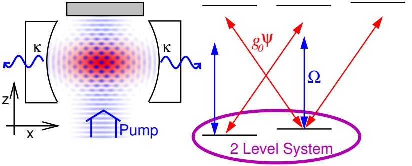

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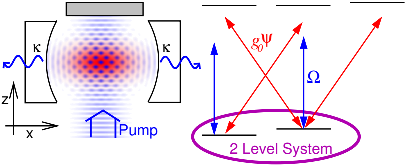

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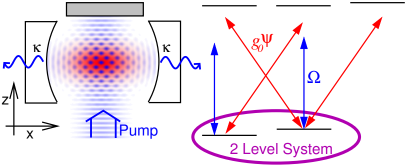
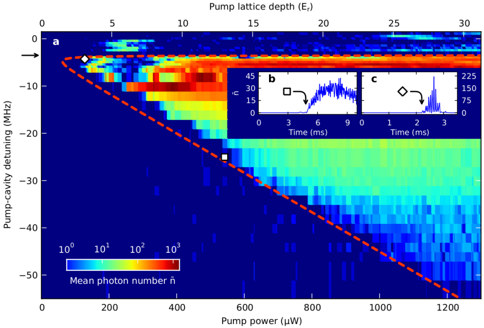
Self organisation and Dicke model





2 Level system,

$$\phi(x,z) \propto \begin{cases} 1 & \Downarrow \\ \cos(qz)\cos(qz) & \Uparrow \end{cases}$$

$$S = \sum_\alpha \sigma^z_\alpha / 2$$

Feedback: $U \propto \dfrac{g_0^2}{\omega_c - \omega_a}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+) + U S_z \psi^\dagger \psi.$$

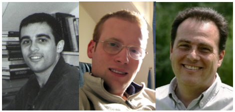
$$\partial_t \rho = -i[H,\rho] + \kappa\mathcal{L}[\psi]$$

[Dimer *et al.* PRA '07][Baumann *et al* Nature '10]

Outline

1. Many body cavity QED
   - Raman pumped Dicke model
   - From Dicke model to cavity Arrays
2. Cavity arrays: coherent pump
   - Fluoresence
   - Disorder
3. Cavity arrays: parametric pump
4. Future directions?
   - Collective dephasing

Dynamics of generalized Dicke model



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Classical dynamics of the extended Dicke model

Open dynamical system:

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Classical dynamics of the extended Dicke model

Open dynamical system:

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$$\partial_t\rho = -i[H, \rho] + \kappa\mathcal{L}[\psi]$$

Classical EOM
($|\mathbf{S}| = N/2 \gg 1$)

$$\dot{S}^- = -i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
$$\dot{S}^z = ig(\psi + \psi^*)(S^- - S^+)$$
$$\dot{\psi} = -[\kappa + i(\omega + US^z)]\psi - ig(S^- + S^+)$$

Equivalent to Maxwell-Bloch, $S^- \leftrightarrow P, S^z \leftrightarrow N$

Steady state phase diagram

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
$$0 = ig(\psi + \psi^*)(S^- - S^+)$$
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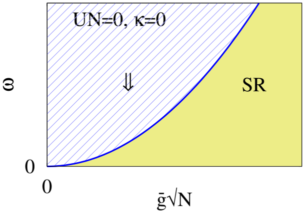

See also Domokos and Ritsch PRL '02, Domokos *et al.* PRL '10

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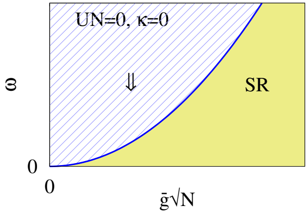
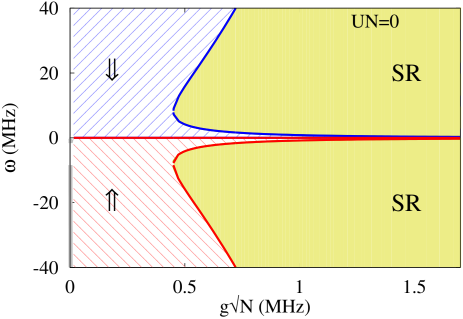
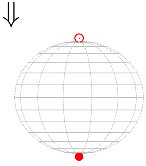
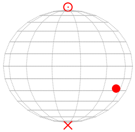






SR(A): $S_y = 0$



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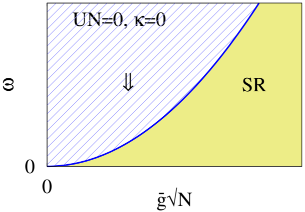

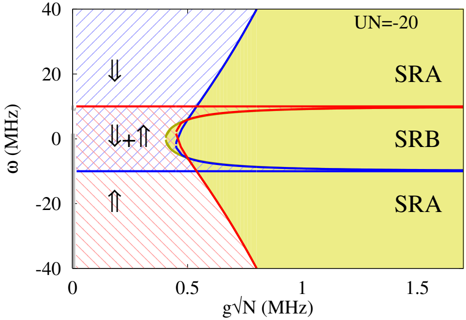

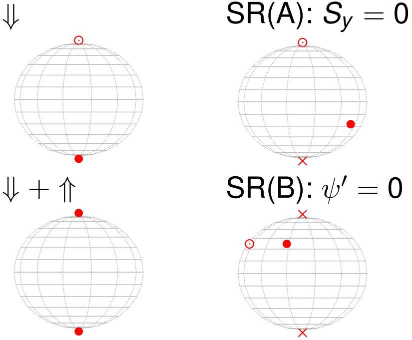

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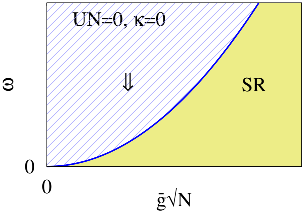

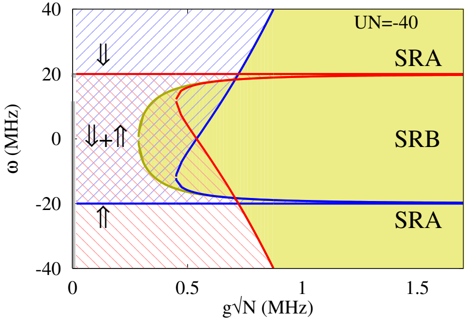

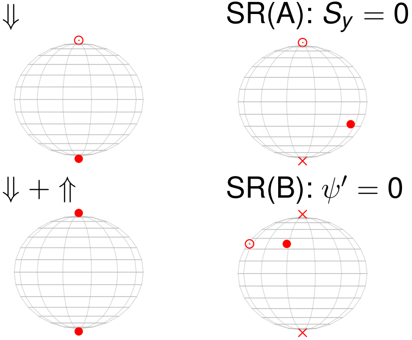

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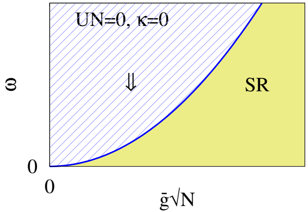
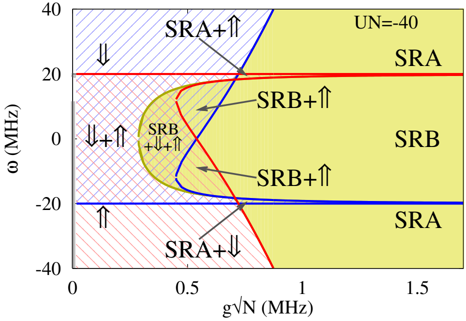
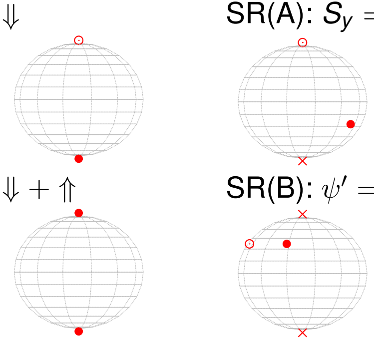




SR(A): $S_y = 0$

SR(B): $\psi' = 0$



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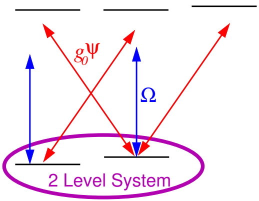
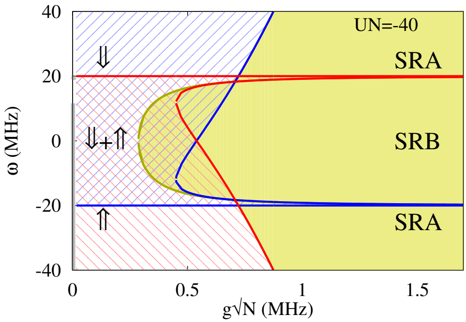
Regions without fixed points

Changing U:



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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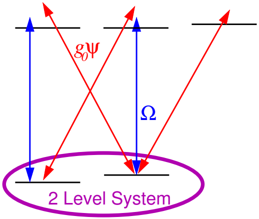

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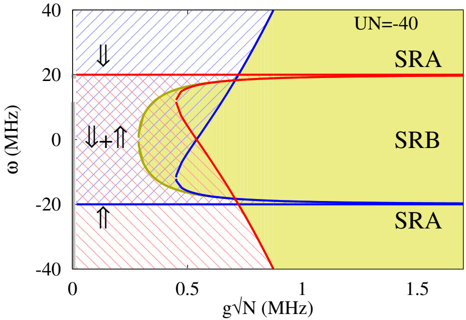
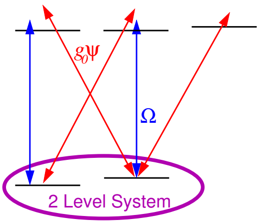


Regions without fixed points

Changing *U*:



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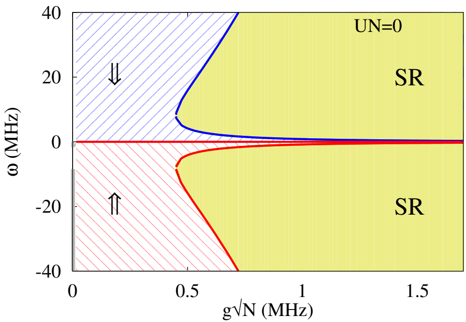

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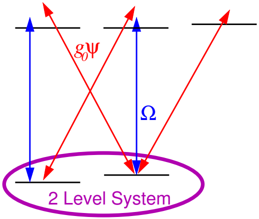

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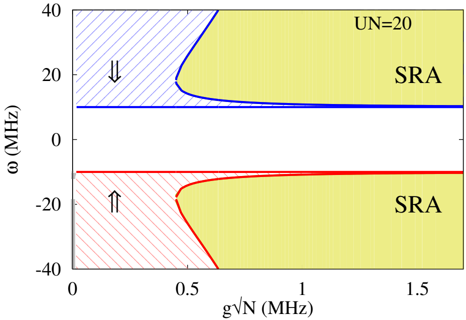

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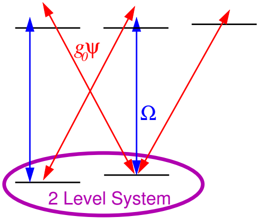

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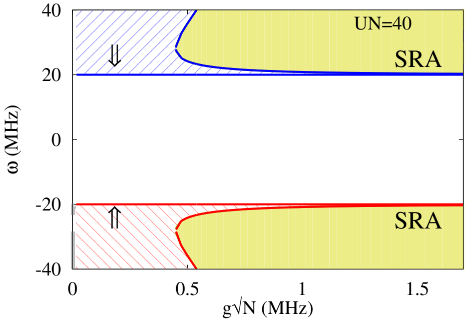

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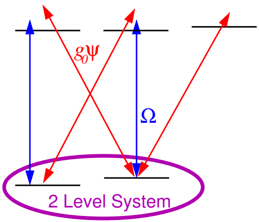

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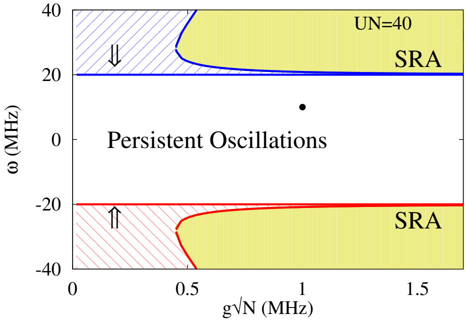
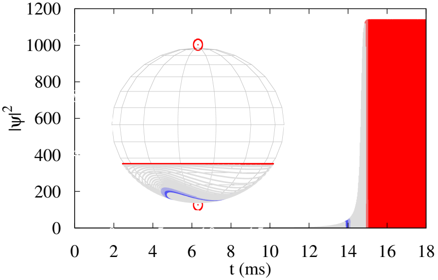
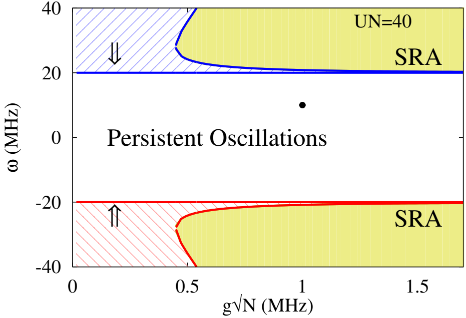
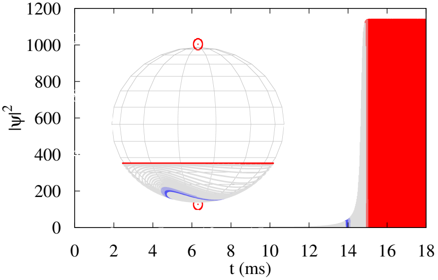
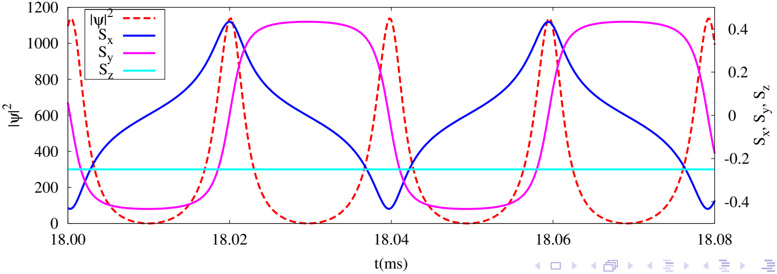




Persistent (optomechanical) oscillations







From Dicke model to Cavity Arrays

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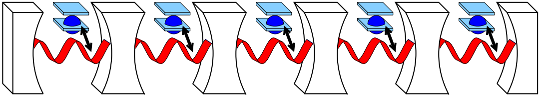
Coupled cavity arrays

- Control photon dispersion — lattice



[Hartmann *et al.* Nat. Phys. '06; Greentree *et al.* Nat. Phys. 06; Angelakis *et al.* PRA '07]

- X-Hubbard Model [X=Bose, Jaynes-Cummings, Rabi, . . .]

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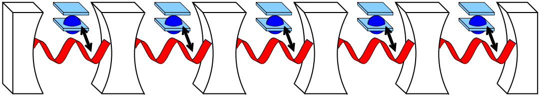

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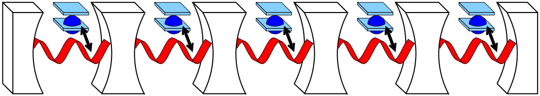

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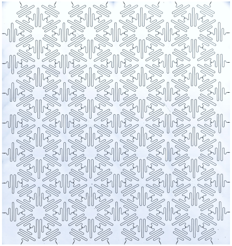
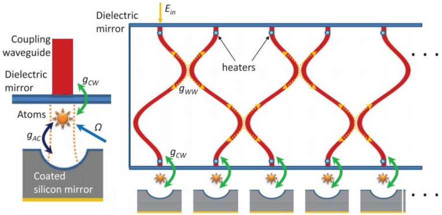


Houck

[Underwood *et al.* PRA '12; Nat. Phys '12]

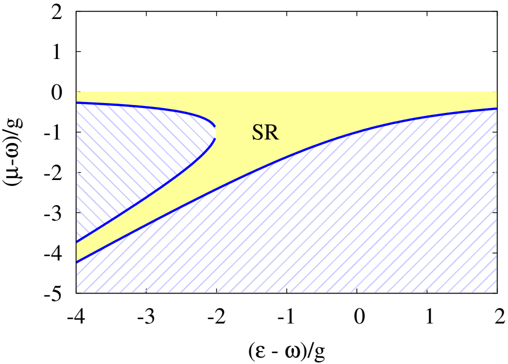


Hinds, Plenio

[Lepert *et al.* NJP '11; APL '13]

Equilibrium: Dicke model with chemical potential

$$H - \mu N = (\omega - \mu)\psi^\dagger\psi + (\omega_0 - \mu)S^z + g\left(\psi^\dagger S^- + \psi S^+\right)$$



[Eastham and Littlewood, PRB '01]

- Transition at: $g^2 N > (\omega - \mu)|\omega_0 - \mu|$
- Reduce critical g
- Unstable if $\mu > \omega$
- Inverted if $\mu > \omega_0$

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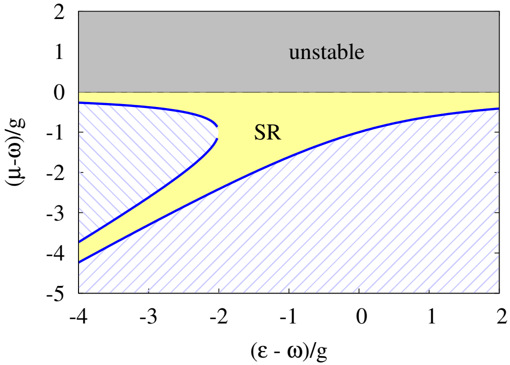

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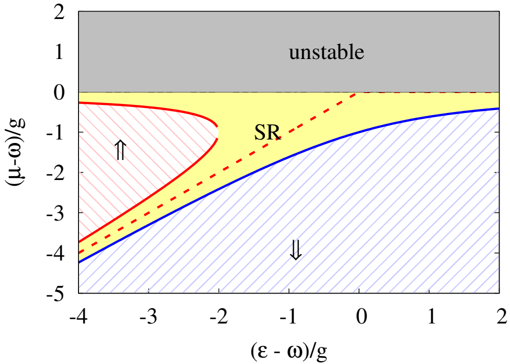

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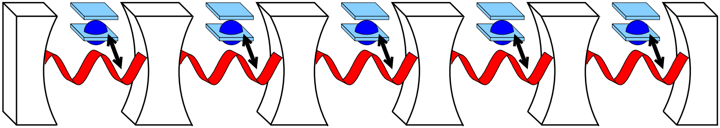
Jaynes-Cummings Hubbard model



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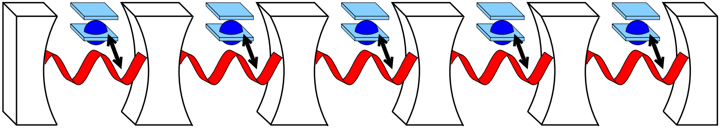

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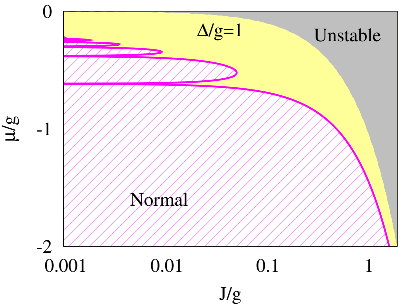
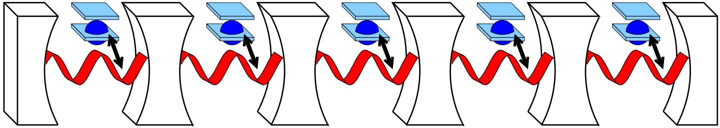


Jaynes-Cummings Hubbard model



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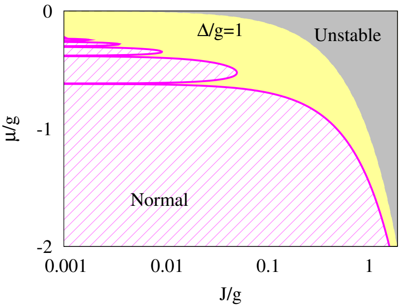
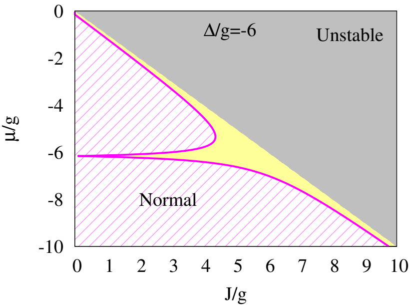
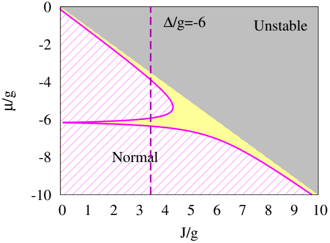


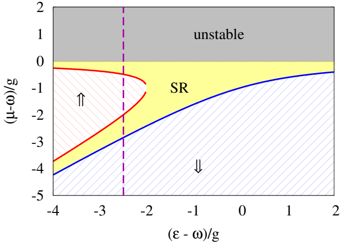


Dicke vs JCHM

JCHM



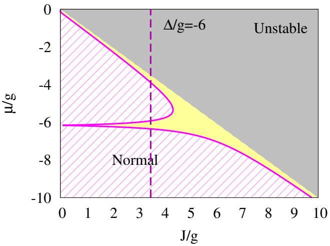
Dicke



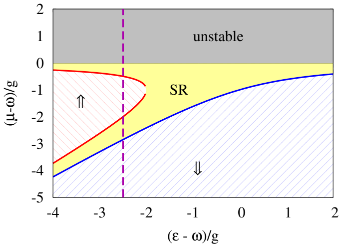
- $k = 0$ mode of JCHM ↔ Dicke photon mode
- ⇑ ↔ $n = 1$ Mott lobe

Dicke vs JCHM

JCHM

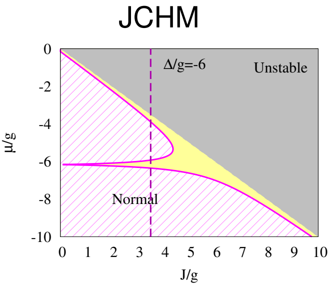


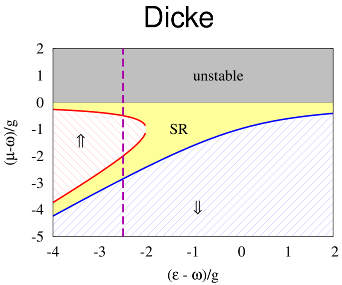
Dicke



- $k = 0$ mode of JCHM $\leftrightarrow$ Dicke photon mode
- $\Uparrow$ $\leftrightarrow$ $n = 1$ Mott lobe

Dicke vs JCHM





- $k = 0$ mode of JCHM $\leftrightarrow$ Dicke photon mode
- $\Uparrow \leftrightarrow$ $n = 1$ Mott lobe

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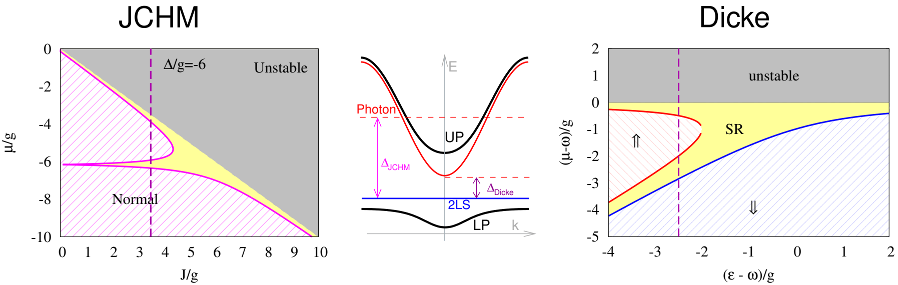

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Cavity arrays: Coherent pump

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   - Raman pumped Dicke model
   - From Dicke model to cavity Arrays
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   - Fluoresence
   - Disorder
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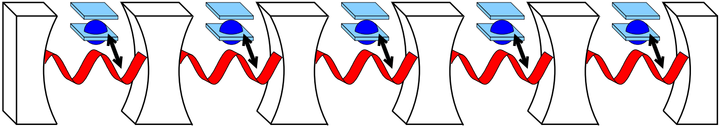
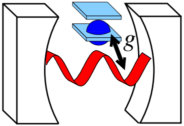
Coherently pumped JCHM



$$H = -\frac{J}{z}\sum_{ij}\psi_i^\dagger\psi_j + \sum_i \frac{\Delta}{2}\sigma_i^z + g(\psi_i^\dagger\sigma_i^- + \text{H.c.}) + f(\psi_i e^{i\omega_L t} + \text{H.c.})$$

$$\partial_t\rho = -i[H,\rho] + \frac{\kappa}{2}\mathcal{L}[\psi] + \frac{\gamma}{2}\mathcal{L}[\sigma^-]$$

Coherently pumped single cavity [Bishop *et al.* Nat. Phys '09]



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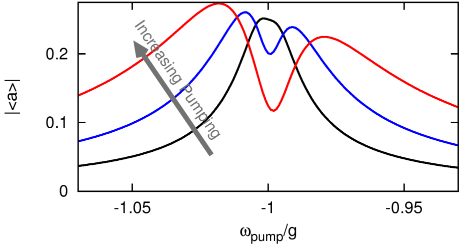
- Anti-resonance in $|\langle\psi\rangle|$.
- Effective 2LS: $|\text{Empty}\rangle, |\text{1 polariton}\rangle$

- Mollow triplet fluorescence



[Lang *et al.* PRL '11]

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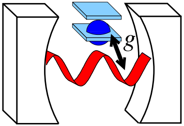

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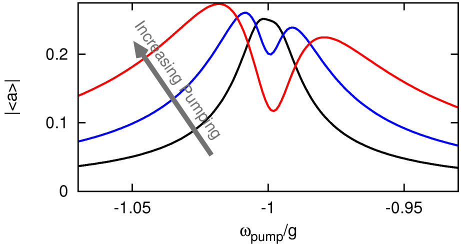

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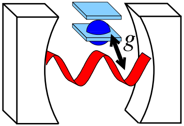

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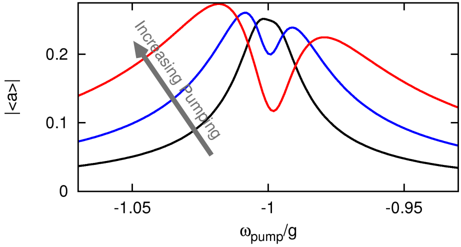

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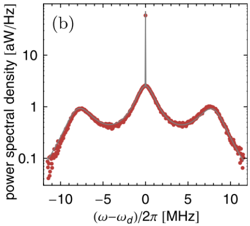

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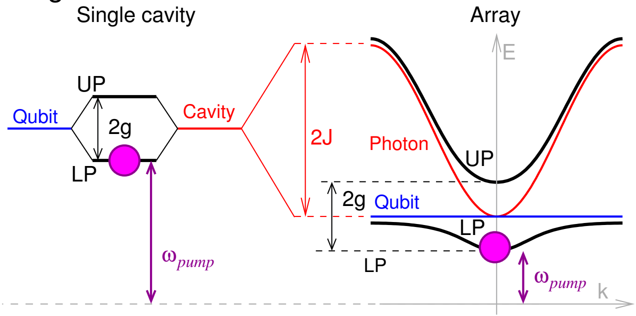
Coherently pumped dimer & array

Chose detuning *a la* Dicke model



- Bistability at intermediate J
  - More/less localised states
  - Connects to Dicke limit

[Nissen *et al.* PRL '12]

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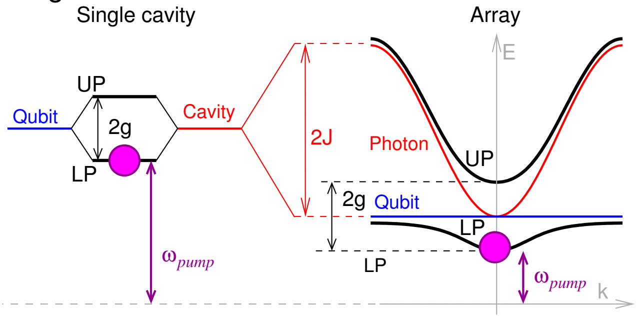

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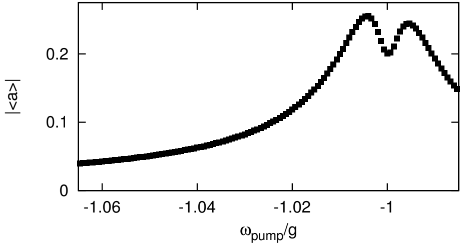

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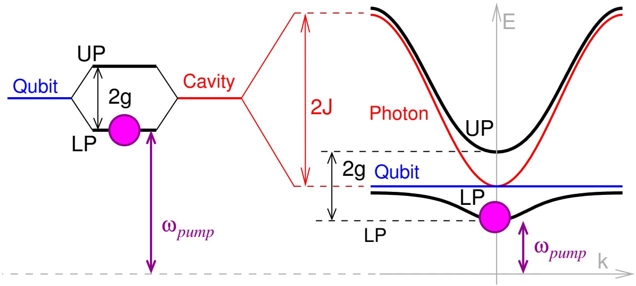

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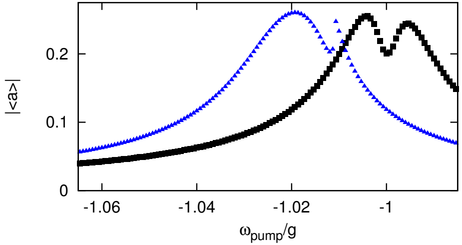

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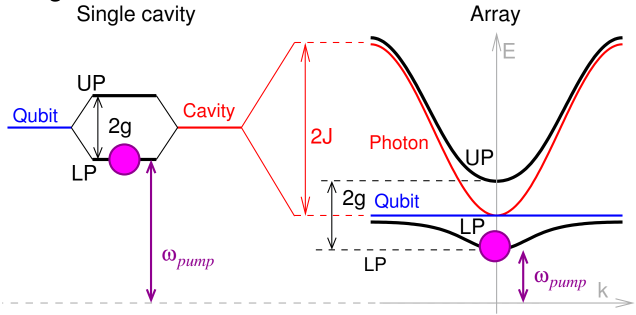

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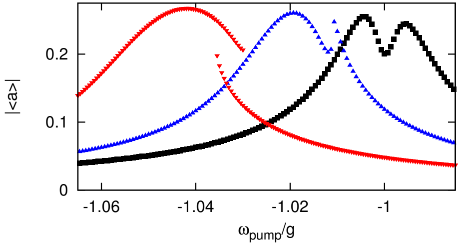

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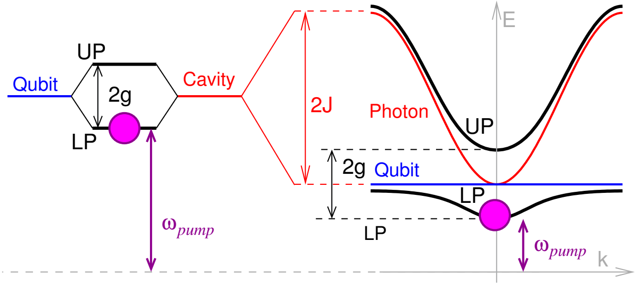

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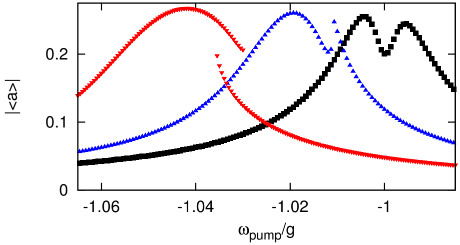

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Photon blockade picture $J \lesssim g$

- Polariton basis
- Nonlinearity $|\epsilon_2 - 2\epsilon_1| \propto g$.

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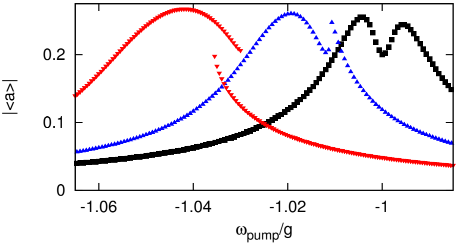

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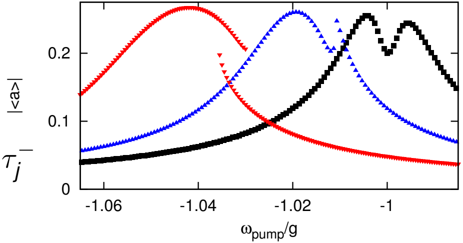

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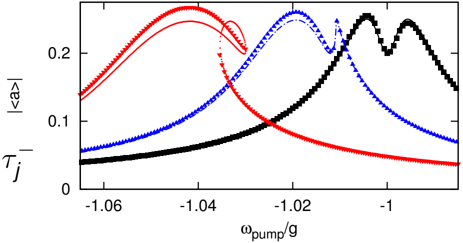

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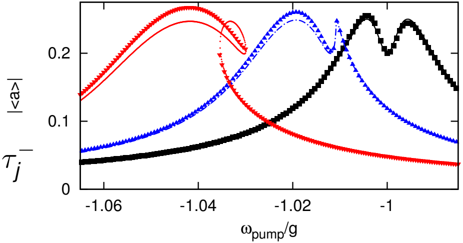

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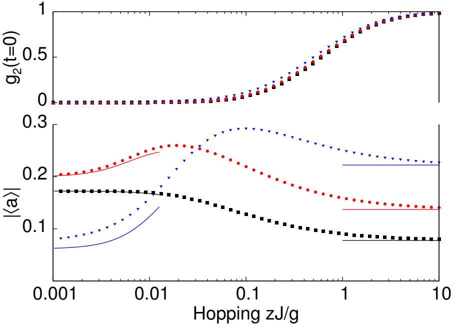
Coherently pumped array: correlations & fluorescence



Correlations

- $g_2 : 0 \rightarrow 1$ crossover.

- Small J: Mollow triplet
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  - Pump at collective resonance
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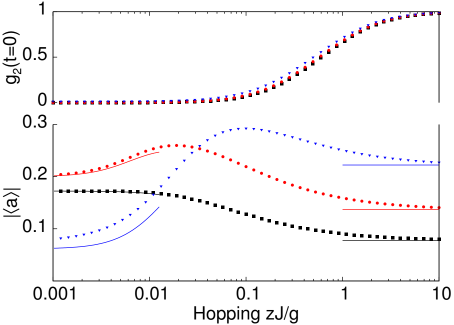

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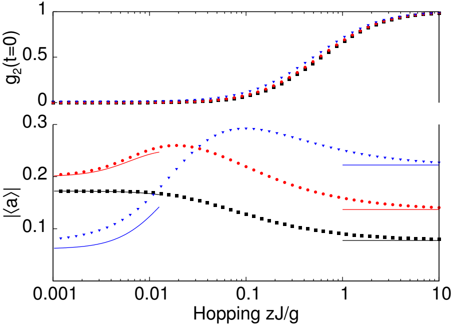

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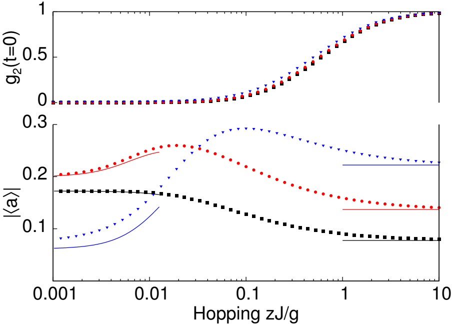

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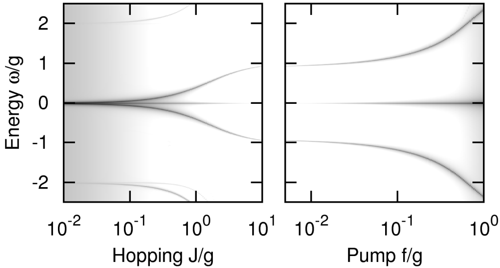

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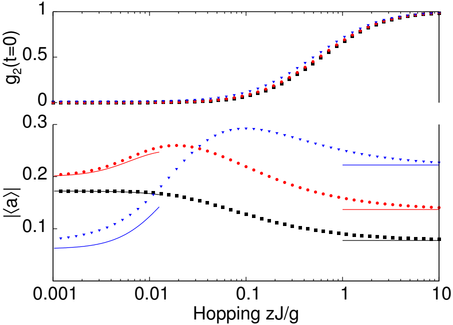

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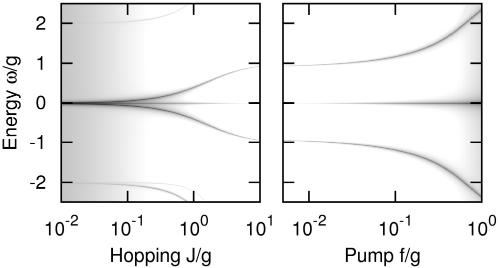

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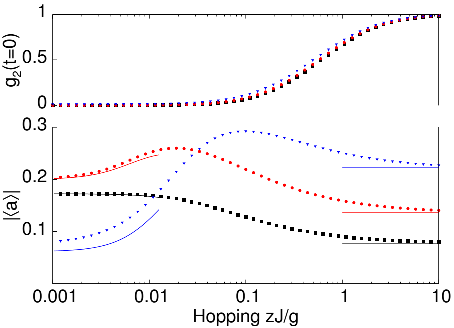

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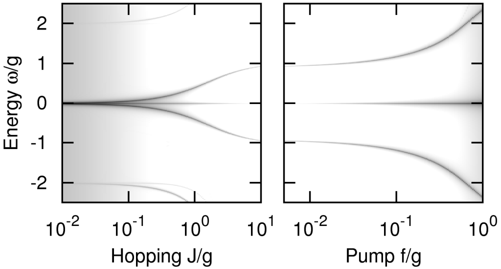

Coherent pump with disorder

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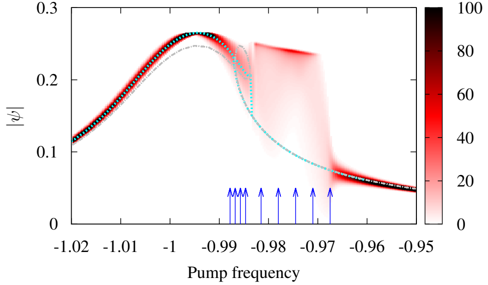
- Superfluid phases in driven system? [Janot *et al.* PRL '13]



[Kulaitis *et al.* PRA, '13]

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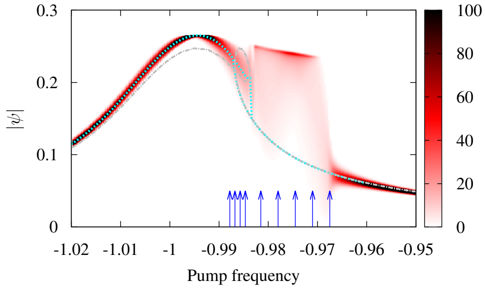

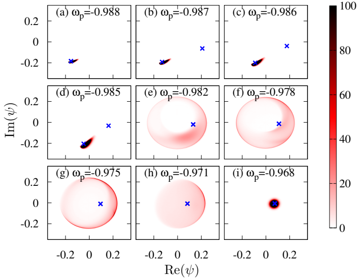

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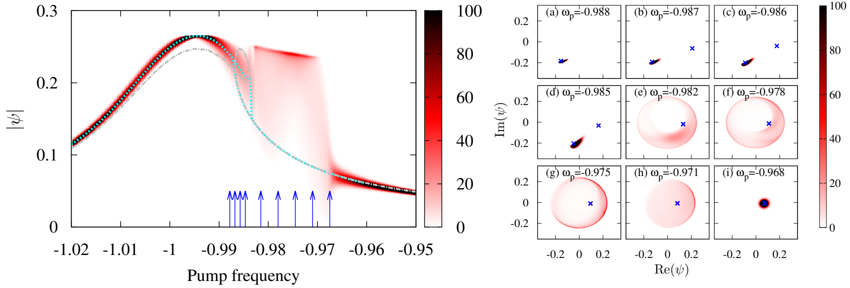
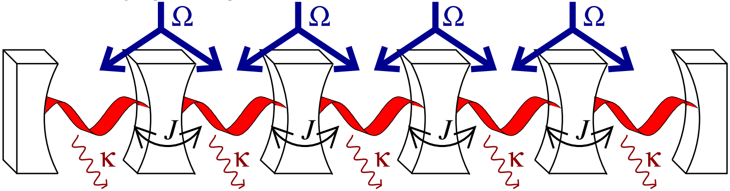


[Kulaitis *et al.* PRA, '13]

Cavity arrays: Parametric pump

Parametrically pumped JCHM



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[Bardyn & Immamoglu, PRL '12]

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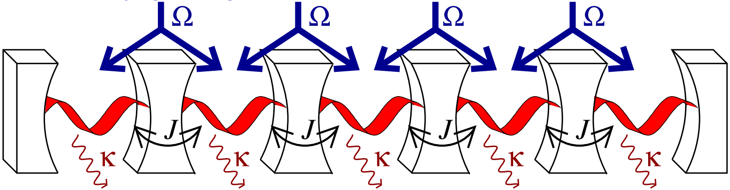

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Rotating frame, blockade approximation, rescale:

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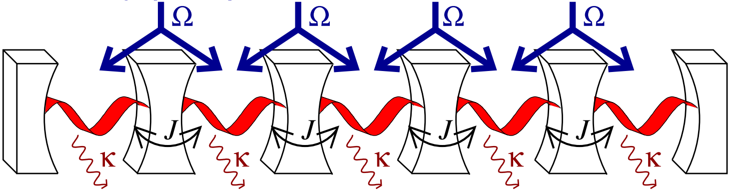

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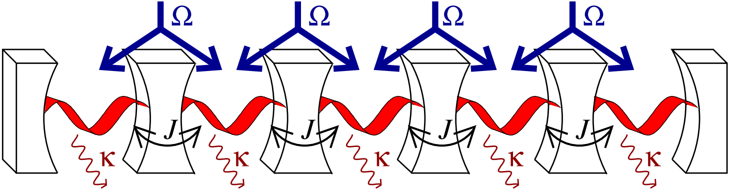
Parametric pumping – equilibrium



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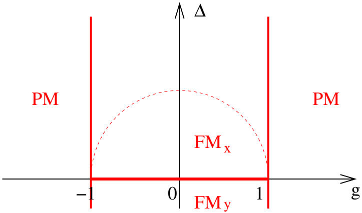
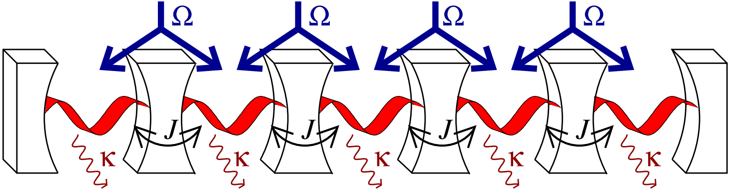


[Bardyn & Immamoglu, PRL '12]

Parametric pumping – equilibrium



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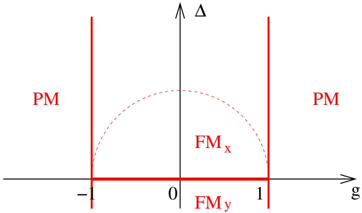

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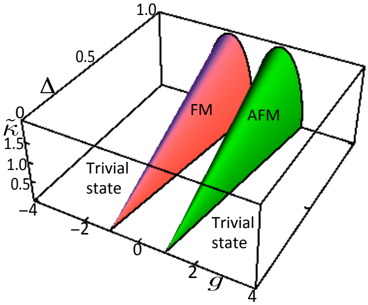
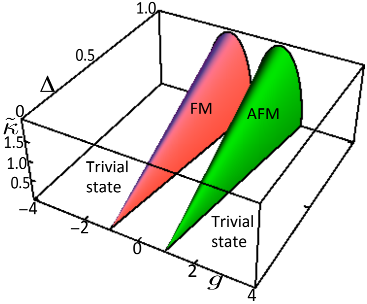


Why AFM/FM attractors



- Linear stability, fluctuation $\sim \exp(-i\nu_k t + ikr_i)$ Linear stability

$$\nu_k = -i\kappa \pm 2J\sqrt{g^2 + 2g\cos k + (1-\Delta^2)\cos^2 k}$$

- $g \ll -1$, Dissipation matches ground state
  - Most unstable mode, $k = 0$
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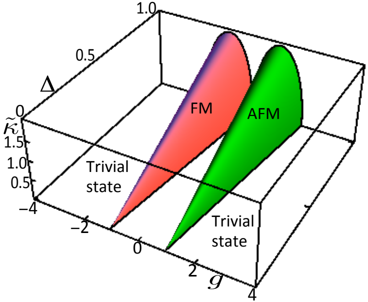

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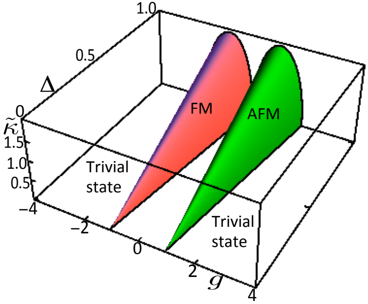

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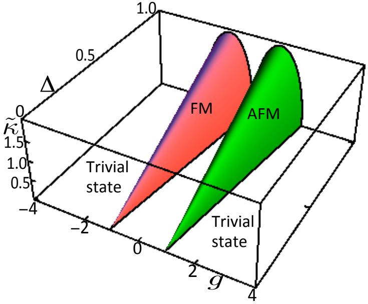

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Beyond mean-field

- Matrix-product-operator representation of

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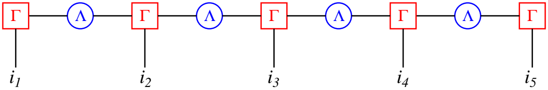

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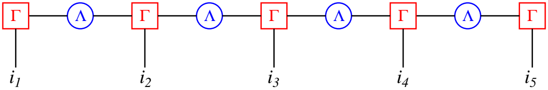

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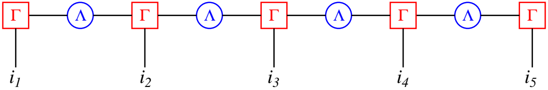

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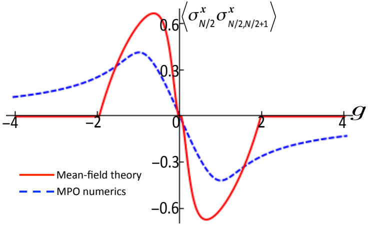

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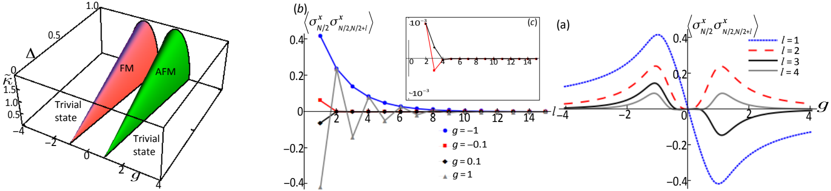

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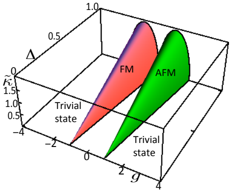

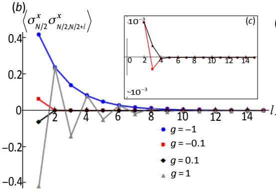

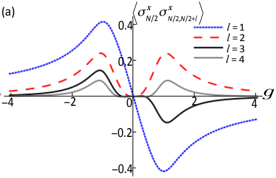

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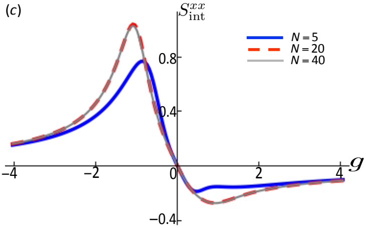

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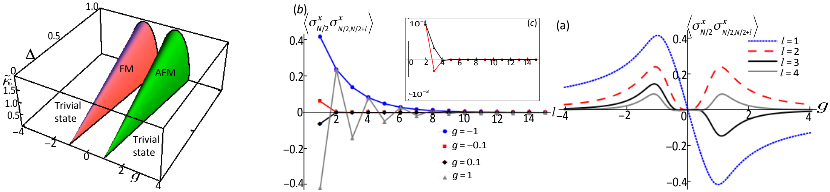

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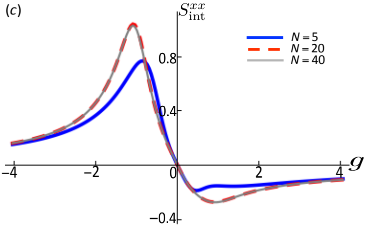

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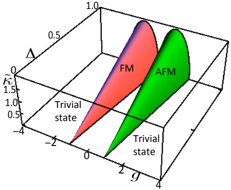

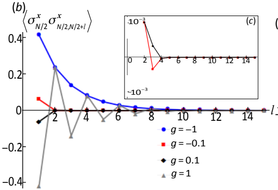

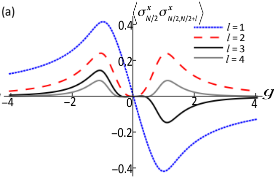

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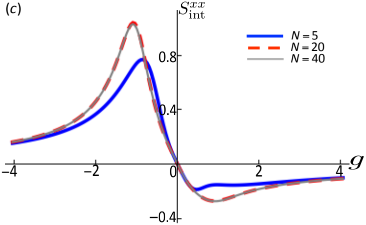

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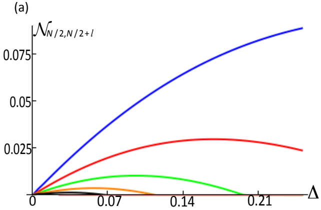

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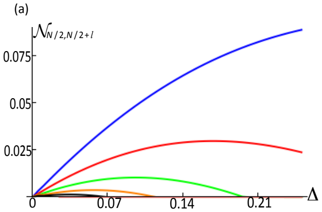

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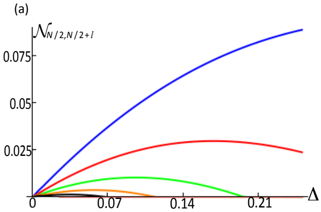

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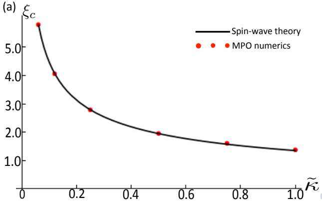

Future directions?

1. Many body cavity QED
   - Raman pumped Dicke model
   - From Dicke model to cavity Arrays
2. Cavity arrays: coherent pump
   - Fluoresence
   - Disorder
3. Cavity arrays: parametric pump
4. Future directions?
   - Collective dephasing

Collective effects and dissipation

- Real environment is not Markovian
  - [Carmichael & Walls JPA '73] Requirements for correct equilibrium
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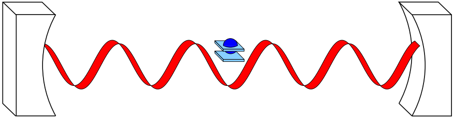

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[Nissen, Fink *et al.* PRL '13]

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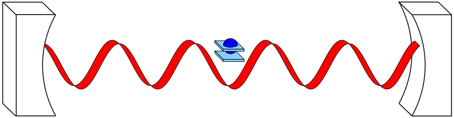

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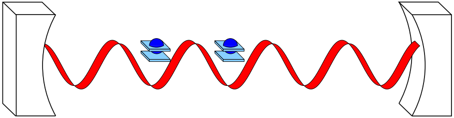

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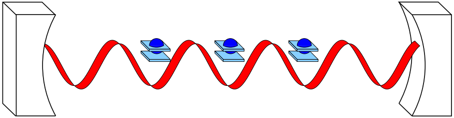

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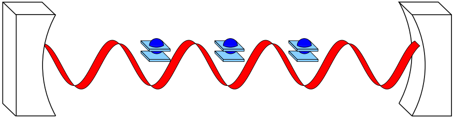

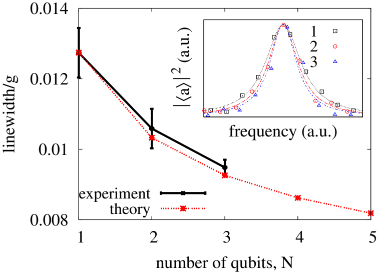

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Acknowledgements

GROUP:

COLLABORATORS:

FUNDING:

topnes
Topological Protection and Non-Equilibrium States in Strongly Correlated Electron Systems

EPSRC
Engineering and Physical Sciences Research Council

Summary

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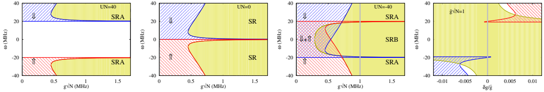
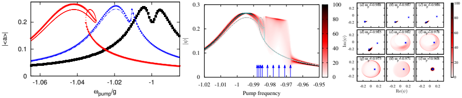


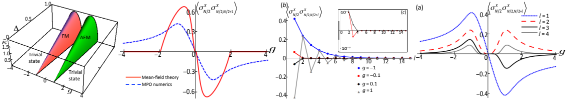
JK *et al.* PRL '10, Bhaseen *et al.* PRA '12

- Pumped coupled cavity array — bistability and disorder



Nissen *et al.* PRL '12

- Parametric pumping — non-equilibrium "phases" of transverse field Ising model



Joshi *et al.* PRA '13