# Pairing Phases of Polaritons

Jonathan Keeling

University of St Andrews 600 YEARS

SUPA

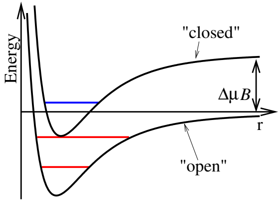
St Petersburg, March 2014

# Outline

1. Introduction
   - Pairing phases of atoms
   - Feshbach for polaritons
2. Pairing phases for polaritons
   - Phase diagram: Critical detunings
   - Signatures
   - Phase diagram: Critical temperatures

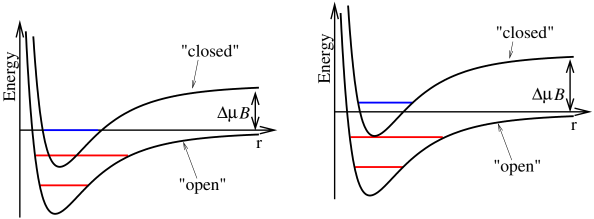
# Tuning pair interactions

- Different $F_z$ states ($F$=$J + I$), tune by $\Delta E(r \to \infty) = \text{const} + (\mu_a - \mu_b)B_z$



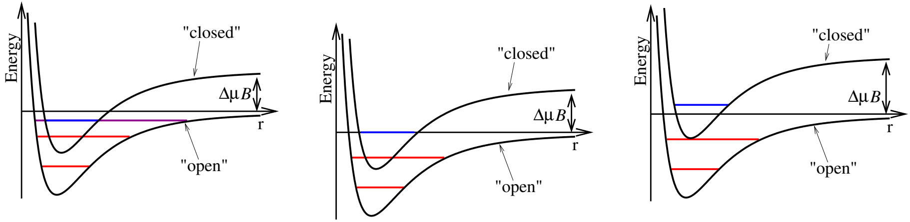
# Tuning pair interactions

- Different $F_z$ states ($F$=$J + I$), tune by $\Delta E(r \to \infty) = \text{const} + (\mu_a - \mu_b)B_z$



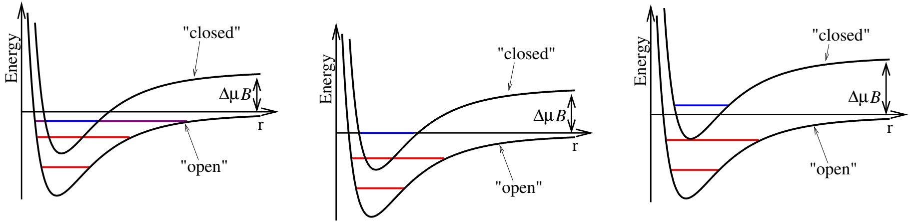
# Tuning pair interactions

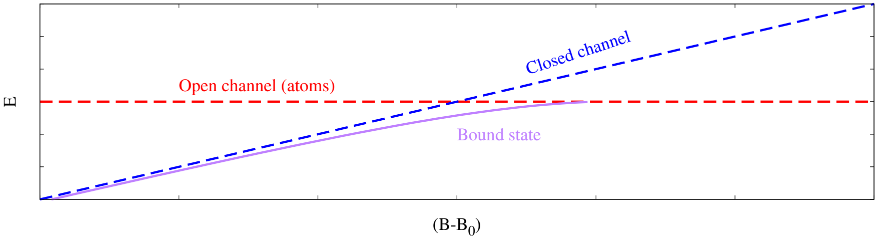
- Different $F_z$ states ($F$=$J + I$), tune by $\Delta E(r \rightarrow \infty) = \text{const} + (\mu_a - \mu_b) B_z$



# Tuning pair interactions

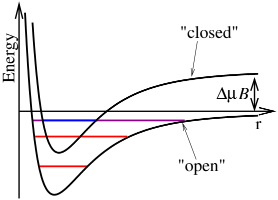
- Different $F_z$ states ($F$=$J+I$), tune by $\Delta E(r \to \infty) = \text{const} + (\mu_a - \mu_b)B_z$



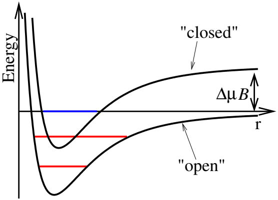



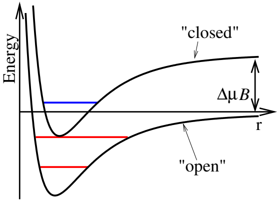
# Tuning pair interactions

- Different $F_z$ states ($F$=$J+I$), tune by $\Delta E(r \to \infty) = \text{const} + (\mu_a - \mu_b)B_z$

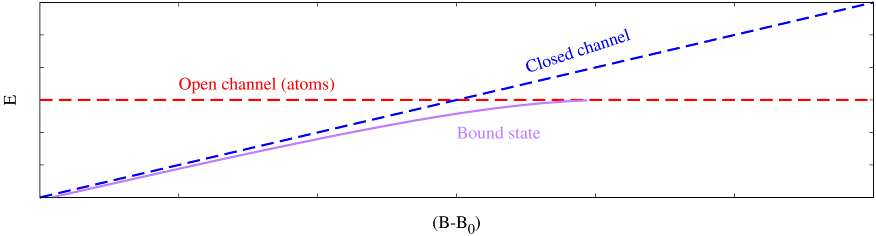



$E = 0$ Repulsive



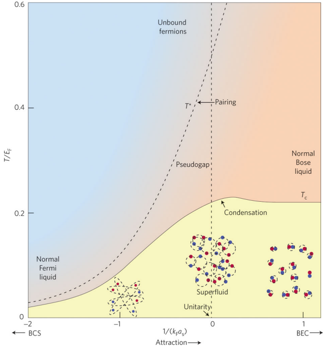




$E = 0$ Attractive



# Pairing phases of atoms

## Fermions




From Randeria, Nat. Phys. ’10

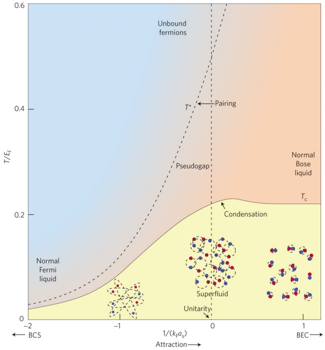
- No phase transition, BEC-BCS crossover

[Eagles, Leggett, Keldysh, Nozières, Randeria, . . . ]

- BEC of atoms or pairs
- $\hat{H} = \ldots + \hat{\psi}^\dagger_m \hat{\psi}_{a_1} \hat{\psi}_{a_2} + \text{h.c.}$
  - If $\langle \hat{\psi}_m \rangle \neq 0$, MSF
  - If $\langle \hat{\psi}_{a_1} \rangle \neq 0$, $\langle \hat{\psi}_{a_2} \rangle \neq 0$, AMSF
- High density → metastability.

# Pairing phases of atoms

## Fermions

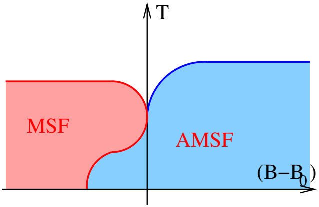



From Randeria, Nat. Phys. ’10

- No phase transition, BEC-BCS crossover

[Eagles, Leggett, Keldysh, Nozières, Randeria, . . . ]

## Bosons

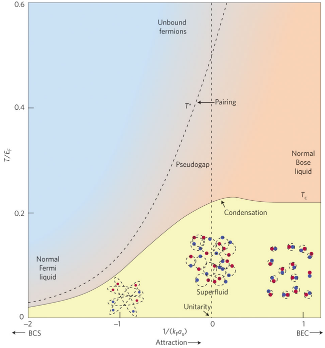



- BEC of atoms *or* pairs
- $\hat{H} = \ldots + \hat{\psi}^\dagger_m \hat{\psi}_{a_1} \hat{\psi}_{a_2} + \text{h.c.}$
  - If $\langle \hat{\psi}_m \rangle \neq 0$, MSF
  - If $\langle \hat{\psi}_{a_1} \rangle \neq 0, \langle \hat{\psi}_{a_2} \rangle \neq 0$. AMSF
- High density → metastability.

[Nozières, St James, Timmermanns, Mueller, Thouless, Radzihovsky, Stoof, Sachdev . . . ]

# Pairing phases of atoms

## Fermions

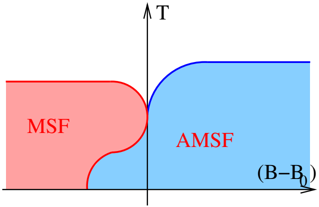



From Randeria, Nat. Phys. '10

- No phase transition, BEC-BCS crossover

[Eagles, Leggett, Keldysh, Nozières, Randeria, . . . ]

## Bosons




- BEC of atoms *or* pairs
- $\hat{H} = \ldots + \hat{\psi}^\dagger_m \hat{\psi}_{a_1} \hat{\psi}_{a_2} + \text{h.c.}$
  - If $\langle \hat{\psi}_m \rangle \neq 0$, MSF
  - If $\langle \hat{\psi}_{a_1} \rangle \neq 0, \langle \hat{\psi}_{a_2} \rangle \neq 0$. AMSF
- High density $\rightarrow$ metastability.

[Nozières, St James, Timmermanns, Mueller, Thouless, Radzihovsky, Stoof, Sachdev . . . ]

# Polariton Feshbach

- Hybridisation of bound states:
  - Biexciton: opposite spins (two-species): $2\omega_0^X - E_b$
  - Hybridisation with photons: $2\left[\frac{1}{2}(\omega_0^C + \omega_0^X) - \frac{1}{2}\sqrt{\Omega_R^2 + \delta^2}\right]$

- Control $\delta$ change $\nu$, $m$, Interaction …

[Ivanov, Haug, Keldysh '98], [Wouters '07], [Caursotto *et al.* '10], [Deveaud-Pledran *et al.* '13]

# Polariton Feshbach

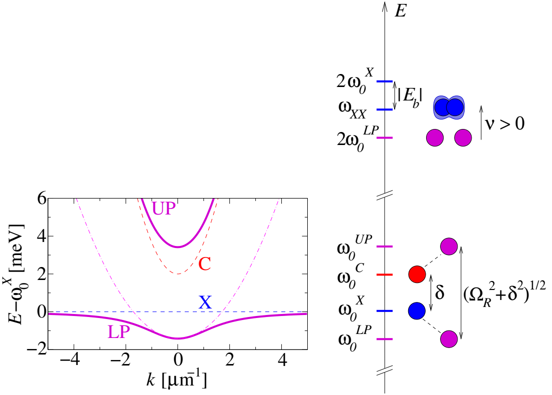
- Hybridisation of bound states:
  - Biexciton: opposite spins (two-species): $2\omega_0^X - E_b$
  - Hybridisation with photons: $2\left[\frac{1}{2}(\omega_0^C + \omega_0^X) - \frac{1}{2}\sqrt{\Omega_r^2 + \delta^2}\right]$

- Control $\delta$ change $\nu$, $m$, Interaction ...

[Ivanov, Haug, Keldysh '98], [Wouters '07], [Caursotto *et al.* '10], [Deveaud-Pledran *et al.* '13]

# Polariton Feshbach

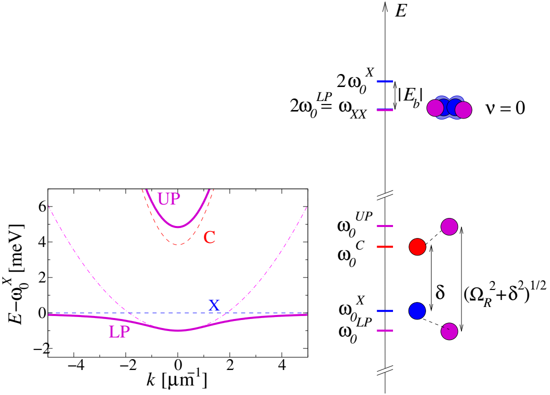
- Hybridisation of bound states:
  - Biexciton: opposite spins (two-species): $2\omega_0^X - E_b$
  - Hybridisation with photons: $2\left[\frac{1}{2}(\omega_0^C + \omega_0^X) - \frac{1}{2}\sqrt{\Omega_r^2 + \delta^2}\right]$




[Ivanov, Haug, Keldysh ’98], [Wouters ’07], [Caursotto *et al.* ’10], [Deveaud-Pledran *et al.* ’13]

# Polariton Feshbach

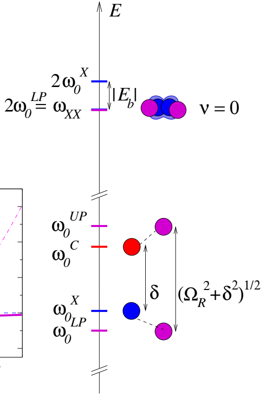
- Hybridisation of bound states:
  - Biexciton: opposite spins (two-species): $2\omega_0^X - E_b$
  - Hybridisation with photons: $2\left[\frac{1}{2}(\omega_0^C + \omega_0^X) - \frac{1}{2}\sqrt{\Omega_r^2 + \delta^2}\right]$

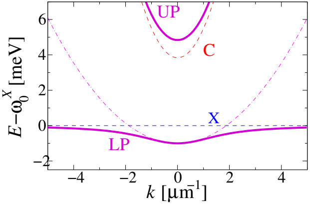



[Ivanov, Haug, Keldysh '98], [Wouters '07], [Caursotto *et al.* '10], [Deveaud-Pledran *et al.* '13]

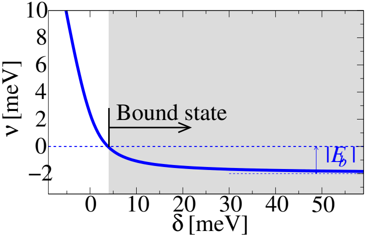
# Polariton Feshbach

- Hybridisation of bound states:
  - Biexciton: opposite spins (two-species): $2\omega_0^X - E_b$
  - Hybridisation with photons: $2\left[\frac{1}{2}(\omega_0^C + \omega_0^X) - \frac{1}{2}\sqrt{\Omega_r^2 + \delta^2}\right]$



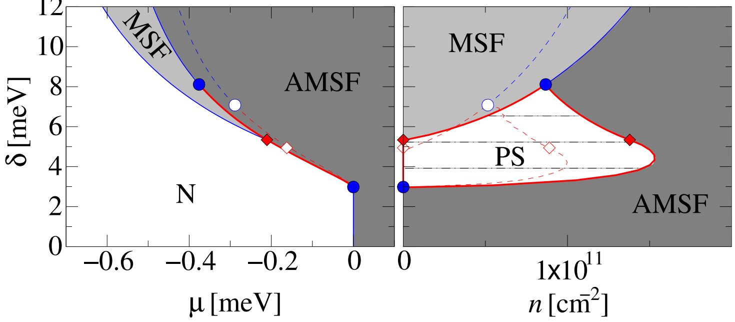




- Control $\delta$ change $\nu$, $m$, Interaction ...



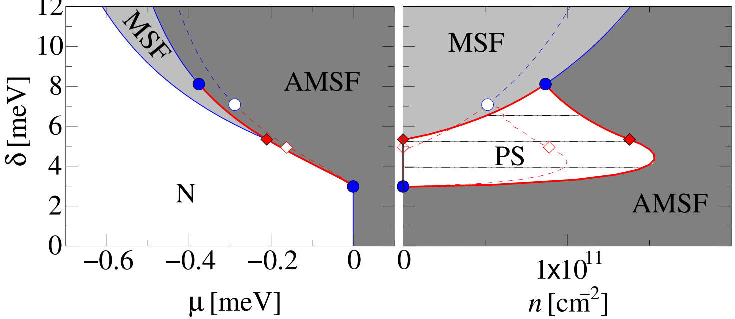

[Ivanov, Haug, Keldysh '98], [Wouters '07], [Caursotto *et al.* '10], [Deveaud-Pledran *et al.* '13]

# Phase diagram (ground state, $T = 0$)



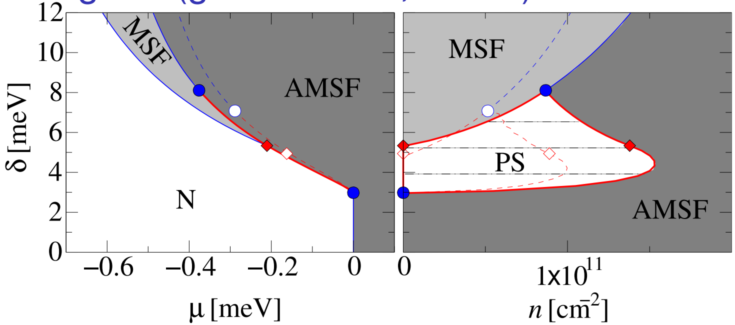

- $\delta < 0$, no biexciton physics — "standard" BEC.
- Small $|\delta|$: Fluctuations drive N–AMSF 1st order
  - Larkin-Pikin mechanism, coupling $g\psi_0^2\psi_m$
- Naive resonance $\delta_r = 3.84$,replaced by critical end-point at $\delta > \delta_r$
- Large $\delta$: MSF-AMSF transition always occurs as $U_{mm}$ renormalises energy.

# Phase diagram (ground state, $T = 0$)



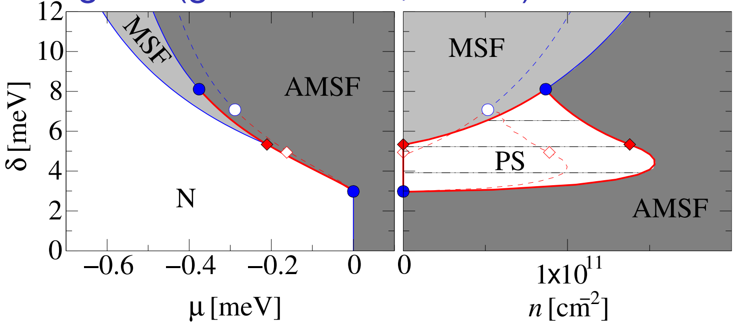

- $\delta < 0$, no biexciton physics — "standard" BEC.
- Small $|\delta|$: Fluctuations drive N–AMSF 1st order
  - Larkin-Pikin mechanism, coupling $g\psi_0^2\psi_m$
  - Large phase-separation region
- Naive resonance $\delta_r = 3.84$,replaced by critical end-point at $\delta > \delta_r$
- Large $\delta$: MSF-AMSF transition always occurs as $U_{mm}$ renormalises energy.

# Phase diagram (ground state, $T = 0$)



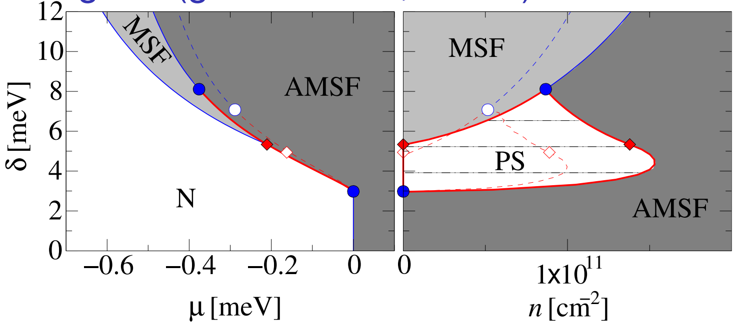

- $\delta < 0$, no biexciton physics — “standard” BEC.
- Small $|\delta|$: Fluctuations drive N–AMSF 1st order
  - Larkin-Pikin mechanism, coupling $g\psi_0^2\psi_m$
  - Large phase-separation region
- Naive resonance $\delta_r = 3.84$,replaced by critical end-point at $\delta > \delta_r$
- Large $\delta$: MSF-AMSF transition always occurs as $U_{mm}$ renormalises energy.

# Phase diagram (ground state, $T = 0$)




- $\delta < 0$, no biexciton physics — “standard” BEC.
- Small $|\delta|$: Fluctuations drive N–AMSF 1st order
  - Larkin-Pikin mechanism, coupling $g\psi_0^2\psi_m$
  - Large phase-separation region
- Naive resonance $\delta_r = 3.84$,replaced by critical end-point at $\delta > \delta_r$
- Large $\delta$: MSF-AMSF transition always occurs as $U_{mm}$ renormalises energy.

# Phase diagram (ground state, $T = 0$)




- $\delta < 0$, no biexciton physics — "standard" BEC.
- Small $|\delta|$: Fluctuations drive N–AMSF 1st order
  - Larkin-Pikin mechanism, coupling $g\psi_0^2\psi_m$
  - Large phase-separation region
- Naive resonance $\delta_r = 3.84$,replaced by critical end-point at $\delta > \delta_r$
- Large $\delta$: MSF-AMSF transition always occurs as $U_{mm}$ renormalises energy.

# Consequences and Signatures

- 1st-order near resonance — phase separation
- Direct access to polariton phase coherence

- Vortex structure — novel half vortices, $\psi_\uparrow = e^{im_\uparrow\theta}, \psi_\downarrow = e^{im_\downarrow\theta}$,

  MSF has $(m_\uparrow, m_\downarrow) = (1/2, 1/2)$

  Previous half-vortex $(m_\uparrow, m_\downarrow) = (1, 0)$ [Lagoudakis *et al.* Science '09]

# Consequences and Signatures

- 1st-order near resonance — phase separation
- Direct access to polariton phase coherence
  - AMSF: standard $g_\sigma^{(1)} = \langle \psi_\sigma^\dagger(r,t)\psi_\sigma(0,0)\rangle$. (2D, QLRO)
- Vortex structure — novel half vortices, $\psi_\uparrow = e^{im_\uparrow\theta}, \psi_\downarrow = e^{im_\downarrow\theta}$, MSF has $(m_\uparrow, m_\downarrow) = (1/2, 1/2)$ .

  Previous half-vortex $(m_\uparrow, m_\downarrow) = (1, 0)$ [Lagoudakis *et al.* Science '09]

# Consequences and Signatures

- 1st-order near resonance — phase separation
- Direct access to polariton phase coherence
  - AMSF: standard $g^{(1)}_\sigma = \langle \psi^\dagger_\sigma(r,t)\psi_\sigma(0,0)\rangle$. (2D, QLRO)
- Vortex structure — novel half vortices, $\psi_\uparrow = e^{im_\uparrow\theta}, \psi_\downarrow = e^{im_\downarrow\theta}$, MSF has $(m_\uparrow, m_\downarrow) = (1/2, 1/2)$

  Previous half-vortex $(m_\uparrow, m_\downarrow) = (1,0)$ [Lagoudakis *et al.* Science '09]

# Consequences and Signatures

- 1st-order near resonance — phase separation
- Direct access to polariton phase coherence
  - AMSF: standard $g_\sigma^{(1)} = \langle \psi_\sigma^\dagger(r,t)\psi_\sigma(0,0)\rangle$. (2D, QLRO)

- Vortex structure — novel half vortices, $\psi_\uparrow = e^{im_\uparrow\theta}, \psi_\downarrow = e^{im_\downarrow\theta}$,

  MSF has $(m_\uparrow, m_\downarrow) = (1/2, 1/2)$

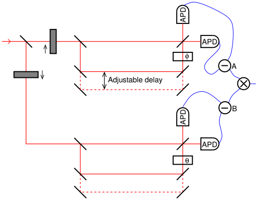
  Previous half-vortex $(m_\uparrow, m_\downarrow) = (1, 0)$ [Lagoudakis *et al.* Science '09]

# Consequences and Signatures

- 1st-order near resonance — phase separation
- Direct access to polariton phase coherence
  - AMSF: standard $g_\sigma^{(1)} = \langle \psi_\sigma^\dagger(r,t)\psi_\sigma(0,0)\rangle$. (2D, QLRO)
  - MSF: no "atomic" coherence, but:

    $$g_m^{(1)} = \langle \psi_\uparrow^\dagger(\mathbf{r},t)\psi_\downarrow^\dagger(\mathbf{r},t)\psi_\downarrow(0,0)\psi_\uparrow(0,0)\rangle$$

    **not** $g_2$ – see time/space labels




- Vortex structure — novel half vortices, $\psi_\uparrow = e^{im_\uparrow\theta}, \psi_\downarrow = e^{im_\downarrow\theta}$,

  MSF has $(m_\uparrow, m_\downarrow) = (1/2, 1/2)$

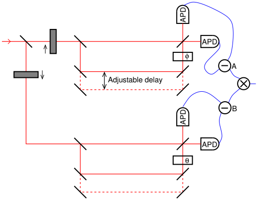
  Previous half-vortex $(m_\uparrow, m_\downarrow) = (1, 0)$ [Lagoudakis *et al.* Science '09]

# Consequences and Signatures

- 1st-order near resonance — phase separation
- Direct access to polariton phase coherence
  - AMSF: standard $g^{(1)}_\sigma = \langle \psi^\dagger_\sigma(r,t)\psi_\sigma(0,0)\rangle$. (2D, QLRO)
  - MSF: no "atomic" coherence, but:

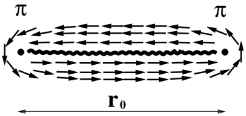
    $$g^{(1)}_m = \langle \psi^\dagger_\uparrow(\mathbf{r},t)\psi^\dagger_\downarrow(\mathbf{r},t)\psi_\downarrow(0,0)\psi_\uparrow(0,0)\rangle$$

    **not** $g_2$ – see time/space labels




- Vortex structure — novel half vortices, $\psi_\uparrow = e^{im_\uparrow\theta}, \psi_\downarrow = e^{im_\downarrow\theta}$,

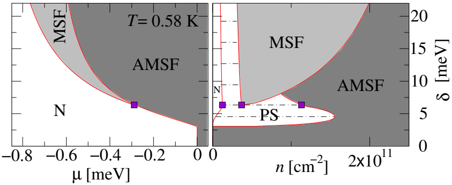
MSF has $(m_\uparrow, m_\downarrow) = (1/2, 1/2)$



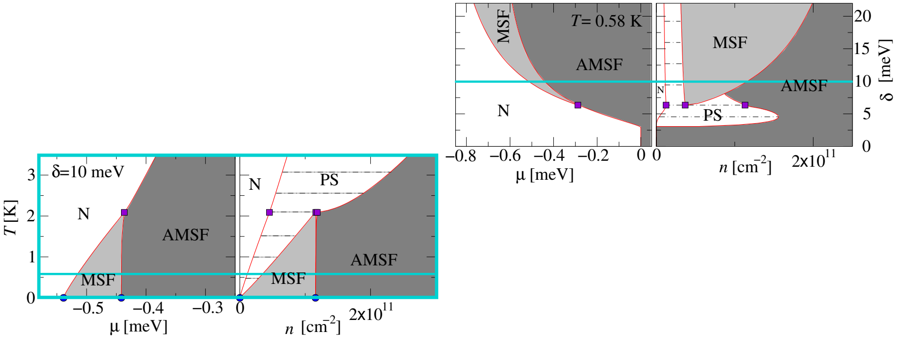

.

Previous half-vortex $(m_\uparrow, m_\downarrow) = (1, 0)$ [Lagoudakis *et al.* Science '09]

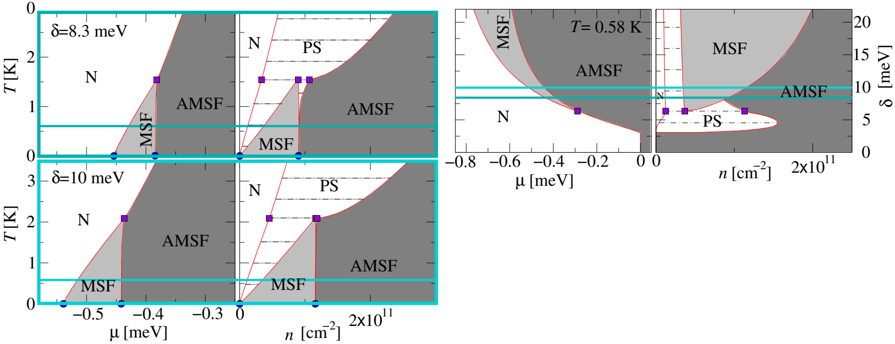
# Phase diagram, $T > 0$



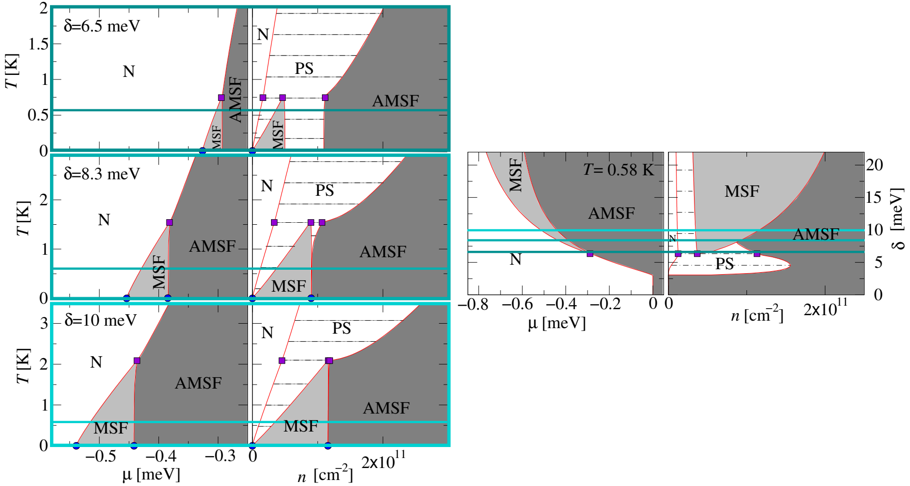
# Phase diagram, $T > 0$



# Phase diagram, $T > 0$



# Phase diagram, $T > 0$



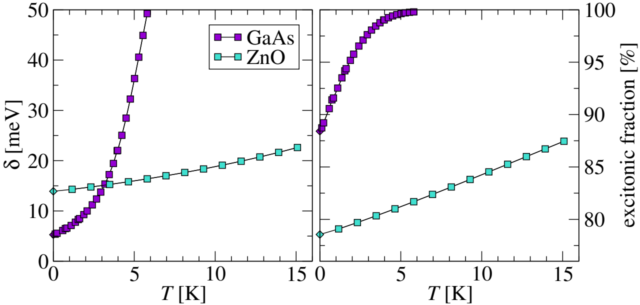
# Evolution of triple point

- Required $T, \delta$ for MSF: Triple point
- Excitonic fraction $c_0^2 = \frac{1}{2}\left[1 + \frac{\delta}{\sqrt{\delta^2 + \Omega^2}}\right]$. $\delta \gg \Omega$: Pure exciton

- GaAs, need low T/high exciton fraction
- ZnO, easy to attain MSF

# Evolution of triple point

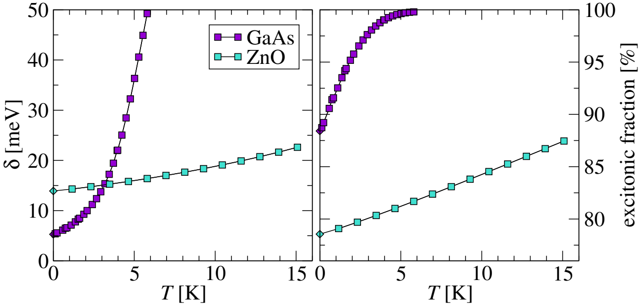
- Required $T, \delta$ for MSF: Triple point
- Excitonic fraction $c_0^2 = \frac{1}{2}\left[1 + \frac{\delta}{\sqrt{\delta^2 + \Omega^2}}\right]$. $\delta \gg \Omega$: Pure exciton




- GaAs, need low T/high exciton fraction
- ZnO, easy to attain MSF

# Evolution of triple point

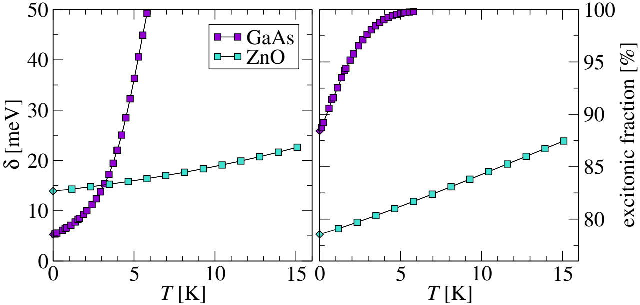
- Required $T, \delta$ for MSF: Triple point
- Excitonic fraction $c_0^2 = \frac{1}{2}\left[1 + \frac{\delta}{\sqrt{\delta^2 + \Omega^2}}\right]$. $\delta \gg \Omega$: Pure exciton




- GaAs, need low T/high exciton fraction
- ZnO, easy to attain MSF

# Evolution of triple point

- Required $T, \delta$ for MSF: Triple point
- Excitonic fraction $c_0^2 = \frac{1}{2}\left[1 + \frac{\delta}{\sqrt{\delta^2 + \Omega^2}}\right]$. $\delta \gg \Omega$: Pure exciton




- GaAs, need low T/high exciton fraction
- ZnO, easy to attain MSF

# Acknowledgements


GROUP:

COLLABORATORS:

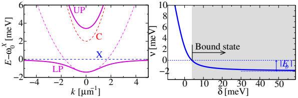
Francesca Marchetti, UAM

FUNDING: topnes Topological Protection and Non-Equilibrium States in Strongly Correlated Electron Systems

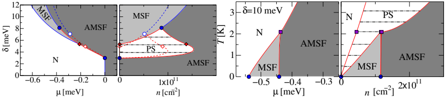
EPSRC Engineering and Physical Sciences Research Council

# Summary

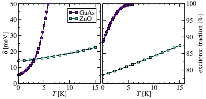
- Polariton — biexciton detuning → Feshbach resonance



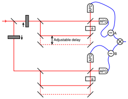

- Polaritons can show pairing phase




- MSF easily attainable for ZnO; for GaAs




- Possible signatures in coherence and vortices




[Marchetti and Keeling, `arXiv:1308.1032`]

# Exciton-photon model

- Microscopic model — coupled exciton-photon system

$$H = \sum_k \left[ \sum_{\sigma=\pm 2,\pm 1} \left( \frac{k^2}{2m_X} - \mu \right) \hat{X}^\dagger_{k\sigma} \hat{X}_{k\sigma} + \sum_{\sigma=\pm 1} \left( \delta + \frac{k^2}{2m_C} - \mu \right) \hat{C}^\dagger_{k\sigma} \hat{C}_{k\sigma} \right.$$
$$\left. + \sum_{\sigma=\pm 1} \Omega_R \left( \hat{C}^\dagger_{k\sigma} \hat{X}_{k\sigma} + \hat{X}^\dagger_{k\sigma} \hat{C}_{k\sigma} \right) \right] + \iint d^2r d^2R \sum_{\sigma,\sigma',\tau,\tau'=\pm 2,\pm 1} U^{XX}_{\sigma'\tau'\tau\sigma}(\mathbf{r})$$
$$\times \hat{X}^\dagger_{\sigma'} \left( \mathbf{R} + \frac{\mathbf{r}}{2} \right) \hat{X}^\dagger_{\tau'} \left( \mathbf{R} - \frac{\mathbf{r}}{2} \right) \hat{X}_{\tau} \left( \mathbf{R} - \frac{\mathbf{r}}{2} \right) \hat{X}_{\sigma} \left( \mathbf{R} + \frac{\mathbf{r}}{2} \right)$$

- Interaction $U^{XX}$ has exchange structure
- For large $\Omega_R$, neglect $\sigma = \pm 2$
- Interaction supports bound states in $U^{XX}_{+1,-1,-1,+1}$ channel — *bipolariton*
- NB, bipolariton, bound polaritons, *but* larger exciton fraction.

# Exciton-photon model

- Microscopic model — coupled exciton-photon system

$$H = \sum_k \left[ \sum_{\sigma=\pm2,\pm1} \left( \frac{k^2}{2m_X} - \mu \right) \hat{X}^\dagger_{k\sigma} \hat{X}_{k\sigma} + \sum_{\sigma=\pm1} \left( \delta + \frac{k^2}{2m_C} - \mu \right) \hat{C}^\dagger_{k\sigma} \hat{C}_{k\sigma} + \sum_{\sigma=\pm1} \Omega_R \left( \hat{C}^\dagger_{k\sigma} \hat{X}_{k\sigma} + \hat{X}^\dagger_{k\sigma} \hat{C}_{k\sigma} \right) \right] + \iint d^2r d^2R \sum_{\sigma,\sigma',\tau,\tau'=\pm2,\pm1} U^{XX}_{\sigma'\tau'\tau\sigma}(\mathbf{r}) \times \hat{X}^\dagger_{\sigma'}\left(\mathbf{R}+\frac{\mathbf{r}}{2}\right) \hat{X}^\dagger_{\tau'}\left(\mathbf{R}-\frac{\mathbf{r}}{2}\right) \hat{X}_{\tau}\left(\mathbf{R}-\frac{\mathbf{r}}{2}\right) \hat{X}_{\sigma}\left(\mathbf{R}+\frac{\mathbf{r}}{2}\right)$$

- Interaction $U^{XX}$ has exchange structure
- For large $\Omega_R$, neglect $\sigma = \pm 2$
- Interaction supports bound states in $U^{XX}_{+1,-1,-1,+1}$ channel — *bipolariton*
- NB, bipolariton, bound polaritons, *but* larger exciton fraction.

# Exciton-photon model

- Microscopic model — coupled exciton-photon system

$$
\begin{aligned}
H = \sum_k \Bigg[ \sum_{\sigma=\pm 2,\pm 1} \left( \frac{k^2}{2m_X} - \mu \right) \hat{X}^\dagger_{k\sigma} \hat{X}_{k\sigma} + \sum_{\sigma=\pm 1} \left( \delta + \frac{k^2}{2m_C} - \mu \right) \hat{C}^\dagger_{k\sigma} \hat{C}_{k\sigma} \\
+ \sum_{\sigma=\pm 1} \Omega_R \left( \hat{C}^\dagger_{k\sigma} \hat{X}_{k\sigma} + \hat{X}^\dagger_{k\sigma} \hat{C}_{k\sigma} \right) \Bigg] + \iint d^2r d^2R \sum_{\sigma,\sigma',\tau,\tau'=\pm 2,\pm 1} U^{XX}_{\sigma'\tau'\tau\sigma}(\mathbf{r}) \\
\times \hat{X}^\dagger_{\sigma'}\left(\mathbf{R} + \frac{\mathbf{r}}{2}\right) \hat{X}^\dagger_{\tau'}\left(\mathbf{R} - \frac{\mathbf{r}}{2}\right) \hat{X}_{\tau}\left(\mathbf{R} - \frac{\mathbf{r}}{2}\right) \hat{X}_{\sigma}\left(\mathbf{R} + \frac{\mathbf{r}}{2}\right)
\end{aligned}
$$

- Interaction $U^{XX}$ has exchange structure
- For large $\Omega_R$, neglect $\sigma = \pm 2$
- Interaction supports bound states in $U^{XX}_{+1,-1,-1,+1}$ channel — *bipolariton*
- NB, bipolariton, bound polaritons, *but* larger exciton fraction.

# Exciton-photon model

- Microscopic model — coupled exciton-photon system

$$H = \sum_k \left[ \sum_{\sigma=\pm 2,\pm 1} \left( \frac{k^2}{2m_X} - \mu \right) \hat{X}^\dagger_{k\sigma} \hat{X}_{k\sigma} + \sum_{\sigma=\pm 1} \left( \delta + \frac{k^2}{2m_C} - \mu \right) \hat{C}^\dagger_{k\sigma} \hat{C}_{k\sigma} + \sum_{\sigma=\pm 1} \Omega_R \left( \hat{C}^\dagger_{k\sigma} \hat{X}_{k\sigma} + \hat{X}^\dagger_{k\sigma} \hat{C}_{k\sigma} \right) \right] + \iint d^2 r d^2 R \sum_{\sigma,\sigma',\tau,\tau'=\pm 2,\pm 1} U^{XX}_{\sigma'\tau'\tau\sigma}(\mathbf{r}) \times \hat{X}^\dagger_{\sigma'} \left( \mathbf{R} + \frac{\mathbf{r}}{2} \right) \hat{X}^\dagger_{\tau'} \left( \mathbf{R} - \frac{\mathbf{r}}{2} \right) \hat{X}_{\tau} \left( \mathbf{R} - \frac{\mathbf{r}}{2} \right) \hat{X}_{\sigma} \left( \mathbf{R} + \frac{\mathbf{r}}{2} \right)$$

- Interaction $U^{XX}$ has exchange structure
- For large $\Omega_R$, neglect $\sigma = \pm 2$
- Interaction supports bound states in $U^{XX}_{+1,-1,-1,+1}$ channel — *bipolariton*
- NB, bipolariton, bound polaritons, *but* larger exciton fraction.

# Exciton-photon model

- Microscopic model — coupled exciton-photon system

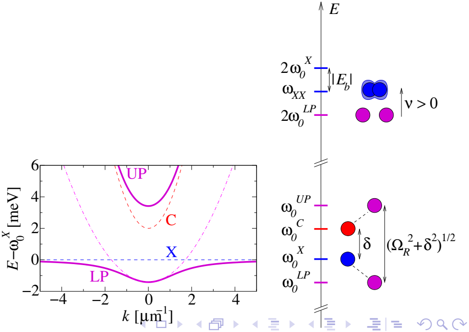
$$H = \sum_k \left[ \sum_{\sigma=\pm2,\pm1} \left( \frac{k^2}{2m_X} - \mu \right) \hat{X}^\dagger_{k\sigma} \hat{X}_{k\sigma} + \sum_{\sigma=\pm1} \left( \delta + \frac{k^2}{2m_C} - \mu \right) \hat{C}^\dagger_{k\sigma} \hat{C}_{k\sigma} \right.$$
$$\left. + \sum_{\sigma=\pm1} \Omega_R \left( \hat{C}^\dagger_{k\sigma} \hat{X}_{k\sigma} + \hat{X}^\dagger_{k\sigma} \hat{C}_{k\sigma} \right) \right] + \iint d^2r d^2R \sum_{\sigma,\sigma',\tau,\tau'=\pm2,\pm1} U^{XX}_{\sigma'\tau'\tau\sigma}(\mathbf{r})$$
$$\times \hat{X}^\dagger_{\sigma'} \left( \mathbf{R} + \frac{\mathbf{r}}{2} \right) \hat{X}^\dagger_{\tau'} \left( \mathbf{R} - \frac{\mathbf{r}}{2} \right) \hat{X}_{\tau} \left( \mathbf{R} - \frac{\mathbf{r}}{2} \right) \hat{X}_{\sigma} \left( \mathbf{R} + \frac{\mathbf{r}}{2} \right)$$

- Interaction $U^{XX}$ has exchange structure
- For large $\Omega_R$, neglect $\sigma = \pm 2$
- Interaction supports bound states in $U^{XX}_{+1,-1,-1,+1}$ channel — *bipolariton*
- NB, bipolariton, bound polaritons, *but* larger exciton fraction.

# Polariton model

$$H = \sum_k \left[ \sum_{\sigma=\uparrow,\downarrow} \left( \frac{k^2}{2m} - \mu \right) \hat{\psi}^\dagger_{\sigma k} \hat{\psi}_{\sigma k} + \left( \frac{k^2}{2m_m} + \nu - 2\mu \right) \hat{\psi}^\dagger_{mk} \hat{\psi}_{mk} \right]$$
$$+ \int d^2R \left[ \sum_{\sigma=\uparrow,\downarrow,m} \frac{U_{\sigma\sigma}}{2} \hat{\psi}^\dagger_\sigma \hat{\psi}^\dagger_\sigma \hat{\psi}_\sigma \hat{\psi}_\sigma + U_{\uparrow\downarrow} \hat{\psi}^\dagger_\downarrow \hat{\psi}^\dagger_\uparrow \hat{\psi}_\uparrow \hat{\psi}_\downarrow + \frac{g}{2} \left( \hat{\psi}^\dagger_\uparrow \hat{\psi}^\dagger_\downarrow \hat{\psi}_m + \text{h.c.} \right) \right]$$

- Polariton dispersion $m$, detuning $\nu$, interactions depend on $\delta$
- Resonance width, dispersion derived from dressed exciton $T$ matrix.

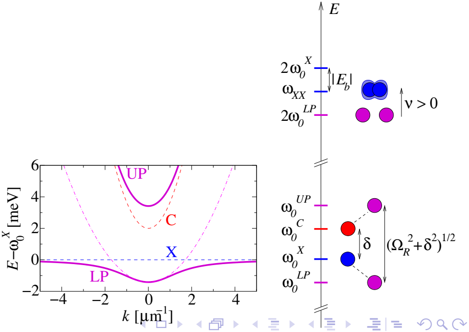


# Polariton model

$$H = \sum_k \left[ \sum_{\sigma=\uparrow,\downarrow} \left( \frac{k^2}{2m} - \mu \right) \hat{\psi}^\dagger_{\sigma k} \hat{\psi}_{\sigma k} + \left( \frac{k^2}{2m_m} + \nu - 2\mu \right) \hat{\psi}^\dagger_{mk} \hat{\psi}_{mk} \right]$$

$$+ \int d^2R \left[ \sum_{\sigma=\uparrow,\downarrow,m} \frac{U_{\sigma\sigma}}{2} \hat{\psi}^\dagger_\sigma \hat{\psi}^\dagger_\sigma \hat{\psi}_\sigma \hat{\psi}_\sigma + U_{\uparrow\downarrow} \hat{\psi}^\dagger_\downarrow \hat{\psi}^\dagger_\uparrow \hat{\psi}_\uparrow \hat{\psi}_\downarrow + \frac{g}{2} \left( \hat{\psi}^\dagger_\uparrow \hat{\psi}^\dagger_\downarrow \hat{\psi}_m + \text{h.c.} \right) \right]$$

- Polariton dispersion $m$, detuning $\nu$, interactions depend on $\delta$
- Resonance width, dispersion derived from dressed exciton $T$ matrix.

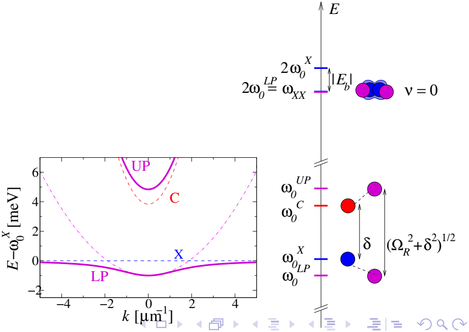


# Polariton model

$$H = \sum_k \left[ \sum_{\sigma=\uparrow,\downarrow} \left( \frac{k^2}{2m} - \mu \right) \hat{\psi}^\dagger_{\sigma k} \hat{\psi}_{\sigma k} + \left( \frac{k^2}{2m_m} + \nu - 2\mu \right) \hat{\psi}^\dagger_{mk} \hat{\psi}_{mk} \right]$$

$$+ \int d^2R \left[ \sum_{\sigma=\uparrow,\downarrow,m} \frac{U_{\sigma\sigma}}{2} \hat{\psi}^\dagger_\sigma \hat{\psi}^\dagger_\sigma \hat{\psi}_\sigma \hat{\psi}_\sigma + U_{\uparrow\downarrow} \hat{\psi}^\dagger_\downarrow \hat{\psi}^\dagger_\uparrow \hat{\psi}_\uparrow \hat{\psi}_\downarrow + \frac{g}{2} \left( \hat{\psi}^\dagger_\uparrow \hat{\psi}^\dagger_\downarrow \hat{\psi}_m + \text{h.c.} \right) \right]$$

- Polariton dispersion $m$, detuning $\nu$, interactions depend on $\delta$
- Resonance width, dispersion derived from dressed exciton $T$ matrix.

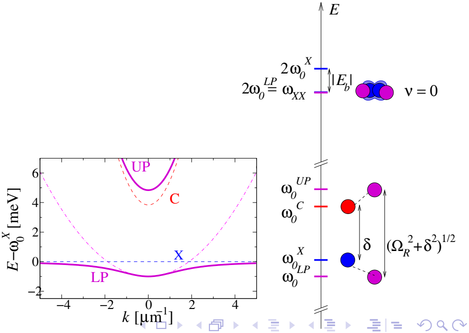


# Polariton model

$$H = \sum_k \left[ \sum_{\sigma=\uparrow,\downarrow} \left( \frac{k^2}{2m} - \mu \right) \hat{\psi}^\dagger_{\sigma k} \hat{\psi}_{\sigma k} + \left( \frac{k^2}{2m_m} + \nu - 2\mu \right) \hat{\psi}^\dagger_{mk} \hat{\psi}_{mk} \right]$$

$$+ \int d^2R \left[ \sum_{\sigma=\uparrow,\downarrow,m} \frac{U_{\sigma\sigma}}{2} \hat{\psi}^\dagger_\sigma \hat{\psi}^\dagger_\sigma \hat{\psi}_\sigma \hat{\psi}_\sigma + U_{\uparrow\downarrow} \hat{\psi}^\dagger_\downarrow \hat{\psi}^\dagger_\uparrow \hat{\psi}_\uparrow \hat{\psi}_\downarrow + \frac{g}{2} \left( \hat{\psi}^\dagger_\uparrow \hat{\psi}^\dagger_\downarrow \hat{\psi}_m + \text{h.c.} \right) \right]$$

- Polariton dispersion $m$, detuning $\nu$, interactions depend on $\delta$
- Resonance width, dispersion derived from dressed exciton $T$ matrix.

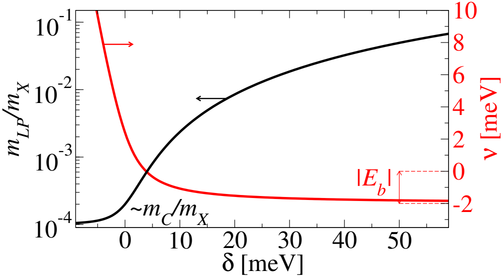


# Polariton model

$$H = \sum_k \left[ \sum_{\sigma=\uparrow,\downarrow} \left( \frac{k^2}{2m} - \mu \right) \hat{\psi}^\dagger_{\sigma k} \hat{\psi}_{\sigma k} + \left( \frac{k^2}{2m_m} + \nu - 2\mu \right) \hat{\psi}^\dagger_{mk} \hat{\psi}_{mk} \right]$$

$$+ \int d^2R \left[ \sum_{\sigma=\uparrow,\downarrow,m} \frac{U_{\sigma\sigma}}{2} \hat{\psi}^\dagger_\sigma \hat{\psi}^\dagger_\sigma \hat{\psi}_\sigma \hat{\psi}_\sigma + U_{\uparrow\downarrow} \hat{\psi}^\dagger_\downarrow \hat{\psi}^\dagger_\uparrow \hat{\psi}_\uparrow \hat{\psi}_\downarrow + \frac{g}{2} \left( \hat{\psi}^\dagger_\uparrow \hat{\psi}^\dagger_\downarrow \hat{\psi}_m + \text{h.c.} \right) \right]$$

$\nu > 0$
Biexciton in continuum



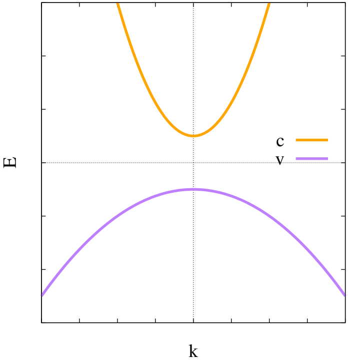

$\nu < 0$
Bound biexciton. (Excitonic limit)

# Exciton and polariton spin degrees of freedom

- Photon: two circular polarisation modes
- Exciton: bound state of electron & hole
- Exciton spin states $J_z = +2, +1, -1, -2$
- Optically active states $J_z = \pm 1$

# Exciton and polariton spin degrees of freedom

- Photon: two circular polarisation modes
- Exciton: bound state of electron & hole

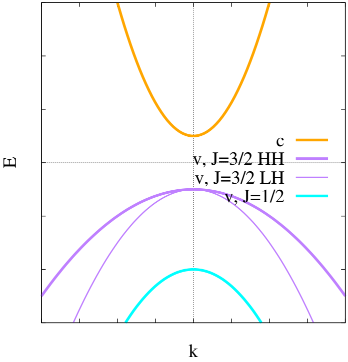



- $J = 1 \pm 1/2$ hole ($p$-orbital), $J = 1/2$ electron
- Spin orbit splits hole bands, $4 \times 2$ states.
- Quantum well fixes $k_z$ of hole $2 \times 2$ states.

- Exciton spin states $J_z = +2, +1, -1, -2$
- Optically active states $J_z = \pm 1$

# Exciton and polariton spin degrees of freedom

- Photon: two circular polarisation modes
- Exciton: bound state of electron & hole

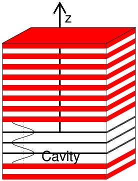


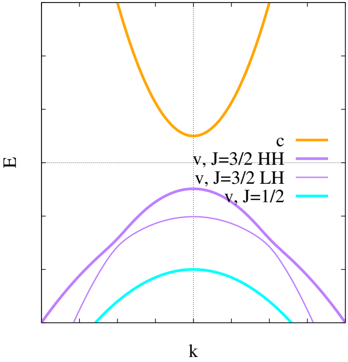

- $J = 1 \pm 1/2$ hole ($p$-orbital), $J = 1/2$ electron
- Spin orbit splits hole bands, $4 \times 2$ states.
- Quantum well fixes $k_z$ of hole $2 \times 2$ states.

- Exciton spin states $J_z = +2, +1, -1, -2$
- Optically active states $J_z = \pm 1$

# Exciton and polariton spin degrees of freedom

- Photon: two circular polarisation modes
- Exciton: bound state of electron & hole



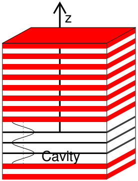



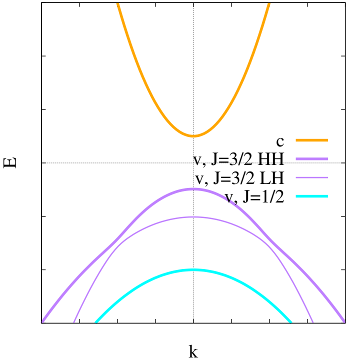

- $J = 1 \pm 1/2$ hole ($p$-orbital), $J = 1/2$ electron
- Spin orbit splits hole bands, $4 \times 2$ states.
- Quantum well fixes $k_z$ of hole $2 \times 2$ states.

- Exciton spin states $J_z = +2, +1, -1, -2$
- Optically active states $J_z = \pm 1$

# Exciton and polariton spin degrees of freedom

- Photon: two circular polarisation modes
- Exciton: bound state of electron & hole



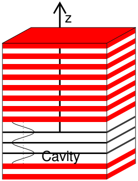



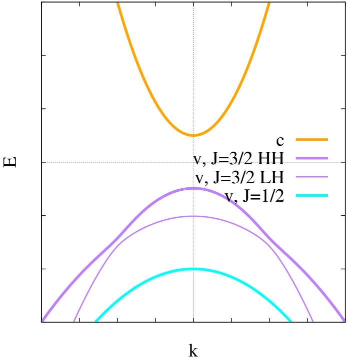

  - $J = 1 \pm 1/2$ hole ($p$-orbital), $J = 1/2$ electron
  - Spin orbit splits hole bands, $4 \times 2$ states.
  - Quantum well fixes $k_z$ of hole $2 \times 2$ states.

- Exciton spin states $J_z = +2, +1, -1, -2$
- Optically active states $J_z = \pm 1$

# Exciton and polariton spin degrees of freedom

- Photon: two circular polarisation modes
- Exciton: bound state of electron & hole




- $J = 1 \pm 1/2$ hole ($p$-orbital), $J = 1/2$ electron
- Spin orbit splits hole bands, $4 \times 2$ states.
- Quantum well fixes $k_z$ of hole $2 \times 2$ states.

- Exciton spin states $J_z = +2, +1, -1, -2$
- Optically active states $J_z = \pm 1$

# Beyond mean-field

- Fluctuation effects?
  - Polariton fluctuations irrelevant: $mU \sim 10^{-4}$.
  - Exciton fluctuations important: $m_m U \sim 1$.
- Next order theory: [Nozières & St James, J. Phys '82]

$$|\Psi\rangle \propto \exp\left( - \sum_{\sigma=\uparrow,\downarrow,m} \psi_\sigma \hat{\psi}_{k=0,\sigma} + \sum_{k,\gamma=a,b,m} \tanh(\theta_{k\gamma}) \hat{b}^\dagger_{k\gamma} \hat{b}^\dagger_{-k\gamma} \right).$$

where $\hat{b}^\dagger_{km} = \hat{\psi}^\dagger_{km}$ and $\begin{pmatrix} \hat{\psi}^\dagger_{\mathbf{k}\uparrow} \\ \hat{\psi}^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \hat{b}^\dagger_{\mathbf{k}a} \\ \hat{b}^\dagger_{-\mathbf{k}b} \end{pmatrix}$,

- Variational functional $E[\psi_0, \psi_m, \theta_{k\gamma}]$
- Can show minimum $\theta_{k\gamma}$ has form $\tanh(2\theta_{k\gamma}) = \frac{\alpha_\gamma}{\beta_\gamma + k^2/2m_\gamma}$
  - Finite only if $|\alpha_\gamma| < \beta_\gamma$

# Beyond mean-field

- Fluctuation effects?
  - Polariton fluctuations irrelevant: $mU \sim 10^{-4}$.
  - Exciton fluctuations important: $m_m U \sim 1$.
- Next order theory: [Nozières & St James, J. Phys '82]

$$|\Psi\rangle \propto \exp\left( - \sum_{\sigma=\uparrow,\downarrow,m} \psi_\sigma \hat{\psi}_{k=0,\sigma} + \sum_{k,\gamma=a,b,m} \tanh(\theta_{k\gamma}) \hat{b}^\dagger_{k\gamma} \hat{b}^\dagger_{-k\gamma} \right).$$

where $\hat{b}^\dagger_{km} = \hat{\psi}^\dagger_{km}$ and $\begin{pmatrix} \hat{\psi}^\dagger_{\mathbf{k}\uparrow} \\ \hat{\psi}^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \hat{b}^\dagger_{\mathbf{k}a} \\ \hat{b}^\dagger_{-\mathbf{k}b} \end{pmatrix}$,

- Variational functional $E[\psi_0, \psi_m, \theta_{k\gamma}]$
- Can show minimum $\theta_{k\gamma}$ has form $\tanh(2\theta_{k\gamma}) = \dfrac{\alpha_\gamma}{\beta_\gamma + k^2/2m_\gamma}$
  - Finite only if $|\alpha_\gamma| < \beta_\gamma$

# Beyond mean-field

- Fluctuation effects?
  - Polariton fluctuations irrelevant: $mU \sim 10^{-4}$.
  - Exciton fluctuations important: $m_m U \sim 1$.
- Next order theory: [Nozières & St James, J. Phys '82]

$$|\Psi\rangle \propto \exp\left( - \sum_{\sigma=\uparrow,\downarrow,m} \psi_\sigma \hat{\psi}_{k=0,\sigma} + \sum_{k,\gamma=a,b,m} \tanh(\theta_{k\gamma}) \hat{b}^\dagger_{k\gamma} \hat{b}^\dagger_{-k\gamma} \right).$$

where $\hat{b}^\dagger_{km} = \hat{\psi}^\dagger_{km}$ and $\begin{pmatrix} \hat{\psi}^\dagger_{\mathbf{k}\uparrow} \\ \hat{\psi}^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \hat{b}^\dagger_{\mathbf{k}a} \\ \hat{b}^\dagger_{-\mathbf{k}b} \end{pmatrix}$ ,

- Variational functional $E[\psi_0, \psi_m, \theta_{k\gamma}]$
- Can show minimum $\theta_{k\gamma}$ has form $\tanh(2\theta_{k\gamma}) = \dfrac{\alpha_\gamma}{\beta_\gamma + k^2/2m_\gamma}$
  - Finite only if $|\alpha_\gamma| < \beta_\gamma$
  - Variational function $E(\psi_0, \psi_m, \alpha_\gamma, \beta_\gamma)$

# Beyond mean-field

- Fluctuation effects?
  - Polariton fluctuations irrelevant: $mU \sim 10^{-4}$.
  - Exciton fluctuations important: $m_m U \sim 1$.
- Next order theory: [Nozières & St James, J. Phys '82]

$$|\Psi\rangle \propto \exp\left(-\sum_{\sigma=\uparrow,\downarrow,m} \psi_\sigma \hat{\psi}_{k=0,\sigma} + \sum_{k,\gamma=a,b,m} \tanh(\theta_{k\gamma}) \hat{b}^\dagger_{k\gamma} \hat{b}^\dagger_{-k\gamma}\right).$$

$$\text{where } \hat{b}^\dagger_{km} = \hat{\psi}^\dagger_{km} \text{ and } \begin{pmatrix} \hat{\psi}^\dagger_{\mathbf{k}\uparrow} \\ \hat{\psi}^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \hat{b}^\dagger_{\mathbf{k}a} \\ \hat{b}^\dagger_{-\mathbf{k}b} \end{pmatrix},$$

- Variational functional $E[\psi_0, \psi_m, \theta_{k\gamma}]$
- Can show minimum $\theta_{k\gamma}$ has form $\tanh(2\theta_{k\gamma}) = \dfrac{\alpha_\gamma}{\beta_\gamma + k^2/2m_\gamma}$
  - Finite only if $|\alpha_\gamma| < \beta_\gamma$
  - Variational function $E(\psi_0, \psi_m, \alpha_\gamma, \beta_\gamma)$

# Finite T calculation

- Finite $T$ — minimize free energy
- Use Feynman-Jensen inequality:

$$F = -k_B T \ln\left[\mathrm{Tr} e^{-\hat{H}/k_B T}\right] \le F_{\mathrm{MF}} + \langle \hat{H} - \hat{H}_{\mathrm{MF}} \rangle_{\mathrm{MF}}$$

  Where $\langle \ldots \rangle_{\mathrm{MF}}$ calculated using $\rho = e^{(F_{\mathrm{MF}} - \hat{H}_{\mathrm{MF}})/k_B T}$

- Ansatz $\hat{H}_{\mathrm{MF}} \to$ Variational $F(\psi_0, \psi_m, \alpha_\gamma, \beta_\gamma)$.

$$\hat{H}_{\mathrm{MF}} = \sum_\gamma \Bigg\{ -\sqrt{A}\psi_\gamma (\alpha_\gamma + \beta_\gamma)\left(\hat{b}^\dagger_{0\gamma} + \hat{b}_{0\gamma}\right) + \frac{1}{2}\sum_{\mathbf{k}} \begin{pmatrix} \hat{b}^\dagger_{\mathbf{k}\gamma} & \hat{b}_{-\mathbf{k}\gamma} \end{pmatrix} \begin{pmatrix} \epsilon_{\mathbf{k}\gamma} + \beta_\gamma & \alpha_\gamma \\ \alpha_\gamma & \epsilon_{\mathbf{k}\gamma} + \beta_\gamma \end{pmatrix} \begin{pmatrix} \hat{b}_{\mathbf{k}\gamma} \\ \hat{b}^\dagger_{-\mathbf{k}\gamma} \end{pmatrix} \Bigg\}.$$

# Finite T calculation

- Finite $T$ — minimize free energy
- Use Feynman-Jensen inequality:

$$F = -k_B T \ln\left[\mathrm{Tr} e^{-\hat{H}/k_B T}\right] \leq F_{\mathrm{MF}} + \langle \hat{H} - \hat{H}_{\mathrm{MF}} \rangle_{\mathrm{MF}}$$

Where $\langle \ldots \rangle_{\mathrm{MF}}$ calculated using $\rho = e^{(F_{\mathrm{MF}} - \hat{H}_{\mathrm{MF}})/k_B T}$

- Ansatz $\hat{H}_{\mathrm{MF}} \to$ Variational $F(\psi_0, \psi_m, \alpha_\gamma, \beta_\gamma)$.

$$\hat{H}_{\mathrm{MF}} = \sum_\gamma \Bigg\{ -\sqrt{A}\psi_\gamma (\alpha_\gamma + \beta_\gamma) \left(\hat{b}^\dagger_{0\gamma} + \hat{b}_{0\gamma}\right) + \frac{1}{2} \sum_{\mathbf{k}} \begin{pmatrix} \hat{b}^\dagger_{\mathbf{k}\gamma} & \hat{b}_{-\mathbf{k}\gamma} \end{pmatrix} \begin{pmatrix} \epsilon_{\mathbf{k}\gamma} + \beta_\gamma & \alpha_\gamma \\ \alpha_\gamma & \epsilon_{\mathbf{k}\gamma} + \beta_\gamma \end{pmatrix} \begin{pmatrix} \hat{b}_{\mathbf{k}\gamma} \\ \hat{b}^\dagger_{-\mathbf{k}\gamma} \end{pmatrix} \Bigg\}.$$

# Finite T calculation

- Finite $T$ — minimize free energy
- Use Feynman-Jensen inequality:

$$F = -k_B T \ln \left[ \mathrm{Tr} e^{-\hat{H}/k_B T} \right] \leq F_{\mathrm{MF}} + \langle \hat{H} - \hat{H}_{\mathrm{MF}} \rangle_{\mathrm{MF}}$$

Where $\langle \ldots \rangle_{\mathrm{MF}}$ calculated using $\rho = e^{(F_{\mathrm{MF}} - \hat{H}_{\mathrm{MF}})/k_B T}$

- Ansatz $\hat{H}_{\mathrm{MF}} \to$ Variational $F(\psi_0, \psi_m, \alpha_\gamma, \beta_\gamma)$.

$$\hat{H}_{\mathrm{MF}} = \sum_\gamma \left\{ -\sqrt{\mathcal{A}} \psi_\gamma (\alpha_\gamma + \beta_\gamma) \left( \hat{b}^\dagger_{0\gamma} + \hat{b}_{0\gamma} \right) + \frac{1}{2} \sum_{\mathbf{k}} \begin{pmatrix} \hat{b}^\dagger_{\mathbf{k}\gamma} & \hat{b}_{-\mathbf{k}\gamma} \end{pmatrix} \begin{pmatrix} \epsilon_{\mathbf{k}\gamma} + \beta_\gamma & \alpha_\gamma \\ \alpha_\gamma & \epsilon_{\mathbf{k}\gamma} + \beta_\gamma \end{pmatrix} \begin{pmatrix} \hat{b}_{\mathbf{k}\gamma} \\ \hat{b}^\dagger_{-\mathbf{k}\gamma} \end{pmatrix} \right\}.$$

# Variational MFT for WIDBG

- Test validity. WIDBG $\hat{H} = \sum_k \frac{k^2}{2m} \psi_k^\dagger \psi_k + \frac{U}{2} \int d^2 r \psi^\dagger \psi^\dagger \psi \psi$
- VMFT for WIDBG:

$$\hat{H}_{\text{MF}} = -\sqrt{\mathcal{A}}\psi(\alpha+\beta)(\hat{b}_0^\dagger + \hat{b}_0)$$
$$+\frac{1}{2}\sum_{\mathbf{k}} \begin{pmatrix} \hat{b}_{\mathbf{k}}^\dagger & \hat{b}_{-\mathbf{k}} \end{pmatrix} \begin{pmatrix} \epsilon_{\mathbf{k}} + \beta & \alpha \\ \alpha & \epsilon_{\mathbf{k}} + \beta \end{pmatrix} \begin{pmatrix} \hat{b}_{\mathbf{k}} \\ \hat{b}_{-\mathbf{k}}^\dagger \end{pmatrix}.$$

- Compare to 2D EOS, $\rho(\mu) = Tf(\mu/T)$
- CF Hartree-Fock-Popov-Bogoluibov method, include $U\rho$ in $\Sigma$

# Variational MFT for WIDBG

- Test validity. WIDBG $\hat{H} = \sum_k \frac{k^2}{2m} \psi_k^\dagger \psi_k + \frac{U}{2} \int d^2 r \psi^\dagger \psi^\dagger \psi \psi$
- VMFT for WIDBG:

$$\hat{H}_{\mathrm{MF}} = -\sqrt{\mathcal{A}}\psi(\alpha + \beta)(\hat{b}_0^\dagger + \hat{b}_0)$$
$$+\frac{1}{2}\sum_{\mathbf{k}} \begin{pmatrix} \hat{b}_{\mathbf{k}}^\dagger & \hat{b}_{-\mathbf{k}} \end{pmatrix} \begin{pmatrix} \epsilon_{\mathbf{k}} + \beta & \alpha \\ \alpha & \epsilon_{\mathbf{k}} + \beta \end{pmatrix} \begin{pmatrix} \hat{b}_{\mathbf{k}} \\ \hat{b}_{-\mathbf{k}}^\dagger \end{pmatrix}.$$

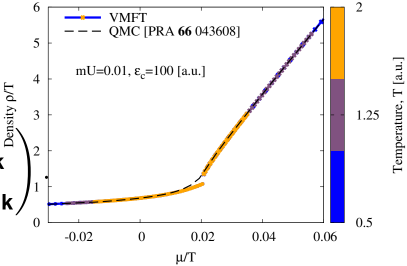
- Compare to 2D EOS, $\rho(\mu) = Tf(\mu/T)$
- CF Hartree-Fock-Popov-Bogoluibov method, include $U\rho$ in $\Sigma$

# Variational MFT for WIDBG

- Test validity. WIDBG $\hat{H} = \sum_k \frac{k^2}{2m} \psi_k^\dagger \psi_k + \frac{U}{2} \int d^2 r \psi^\dagger \psi^\dagger \psi \psi$
- VMFT for WIDBG:

$$\hat{H}_{\mathrm{MF}} = -\sqrt{\mathcal{A}}\psi(\alpha+\beta)(\hat{b}_0^\dagger + \hat{b}_0)$$

$$+\frac{1}{2}\sum_{\mathbf{k}} \begin{pmatrix} \hat{b}_{\mathbf{k}}^\dagger & \hat{b}_{-\mathbf{k}} \end{pmatrix} \begin{pmatrix} \epsilon_{\mathbf{k}} + \beta & \alpha \\ \alpha & \epsilon_{\mathbf{k}} + \beta \end{pmatrix} \begin{pmatrix} \hat{b}_{\mathbf{k}} \\ \hat{b}_{-\mathbf{k}}^\dagger \end{pmatrix}$$

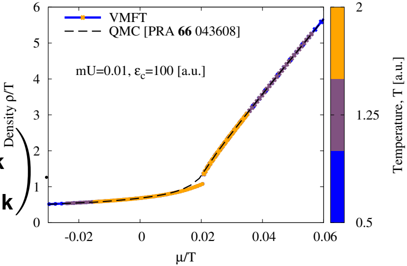



- Compare to 2D EOS, $\rho(\mu) = T f(\mu/T)$
- CF Hartree-Fock-Popov-Bogoluibov method, include $U\rho$ in $\Sigma$

# Variational MFT for WIDBG

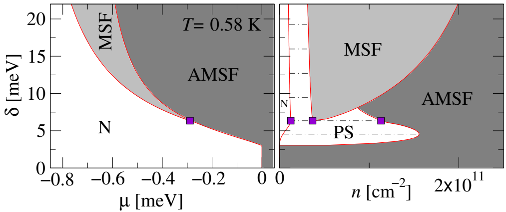
- Test validity. WIDBG $\hat{H} = \sum_k \frac{k^2}{2m} \psi_k^\dagger \psi_k + \frac{U}{2} \int d^2 r \psi^\dagger \psi^\dagger \psi \psi$
- VMFT for WIDBG:

$$\hat{H}_{\mathrm{MF}} = -\sqrt{\mathcal{A}}\psi(\alpha+\beta)(\hat{b}_0^\dagger + \hat{b}_0)$$
$$+\frac{1}{2}\sum_{\mathbf{k}} \begin{pmatrix} \hat{b}_{\mathbf{k}}^\dagger & \hat{b}_{-\mathbf{k}} \end{pmatrix} \begin{pmatrix} \epsilon_{\mathbf{k}} + \beta & \alpha \\ \alpha & \epsilon_{\mathbf{k}} + \beta \end{pmatrix} \begin{pmatrix} \hat{b}_{\mathbf{k}} \\ \hat{b}_{-\mathbf{k}}^\dagger \end{pmatrix}$$

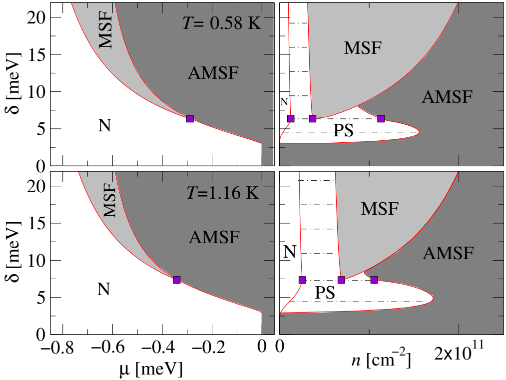



- Compare to 2D EOS, $\rho(\mu) = Tf(\mu/T)$
- CF Hartree-Fock-Popov-Bogoluibov method, include $U\rho$ in $\Sigma$

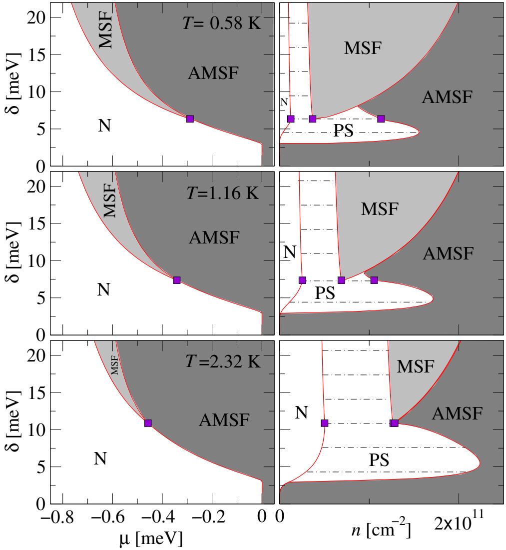
# Phase diagram, finite temperature



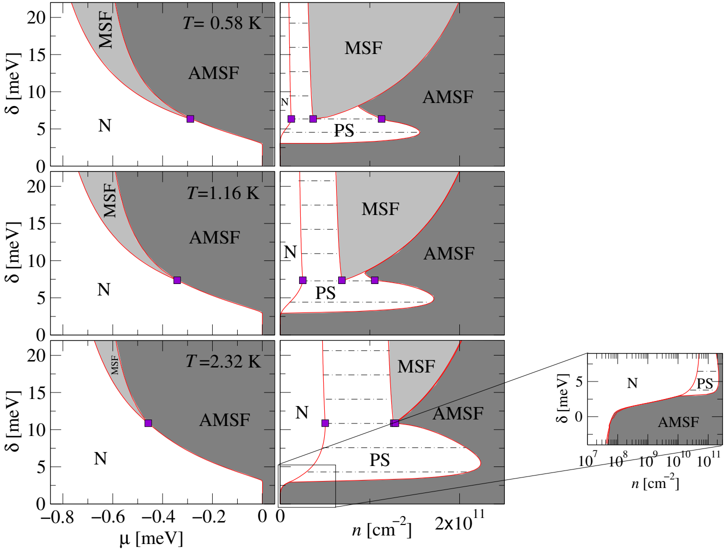
# Phase diagram, finite temperature



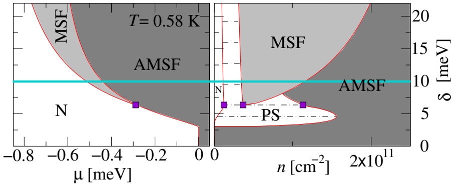
# Phase diagram, finite temperature

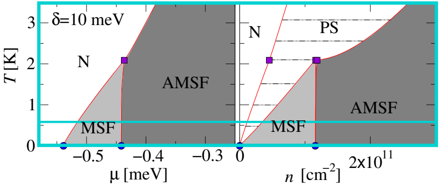


# Phase diagram, finite temperature

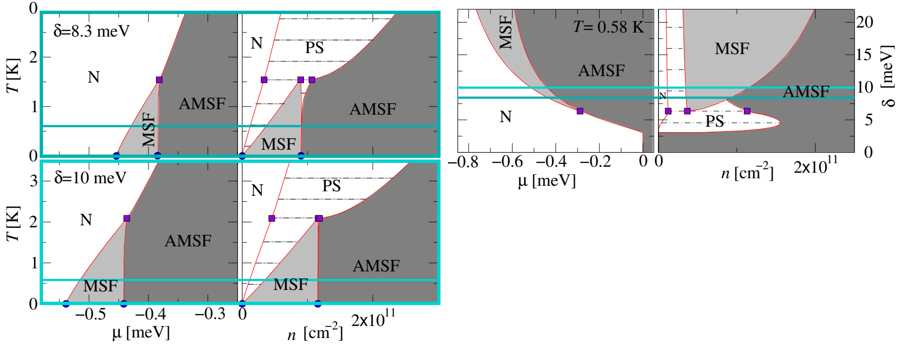


# Phase diagram, vs temperature



# Phase diagram, vs temperature



# Phase diagram, vs temperature