Pairing Phases of Polaritons

Jonathan Keeling

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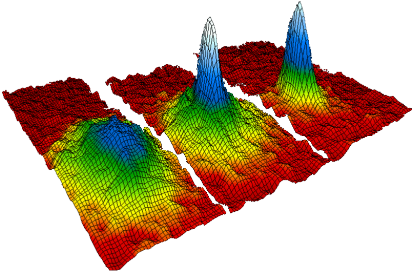
Pisa, January 2014

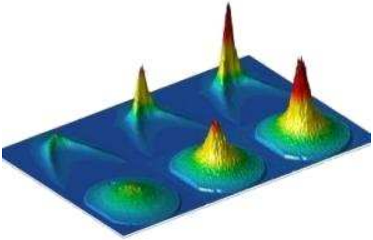
Bose-Einstein condensation: macroscopic occupation

Atoms. $\sim 10^{-7}$K



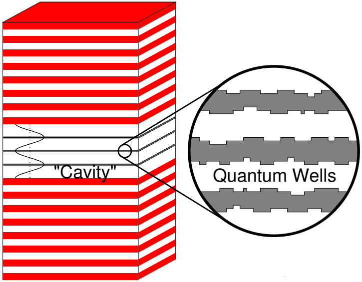
[Anderson *et al.* Science '95]

Polaritons. $\sim$ 20K



[Kasprzak *et al.* Nature, '06]

Microcavity polaritons



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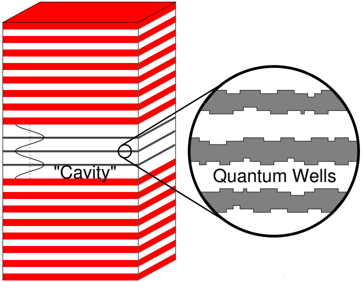
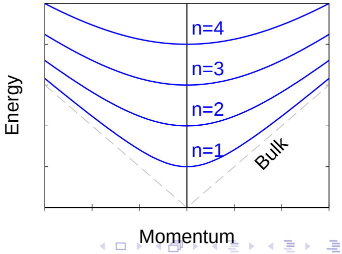


Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$



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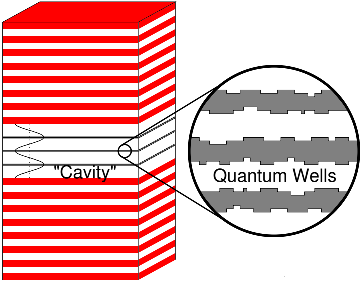

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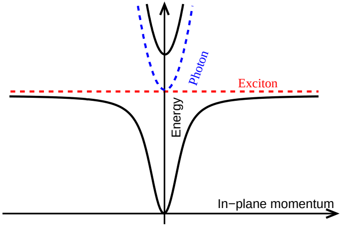

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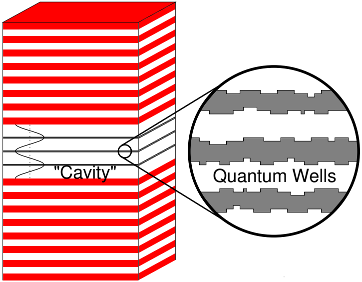

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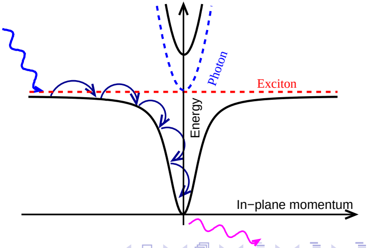

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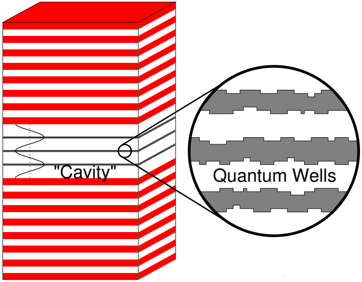

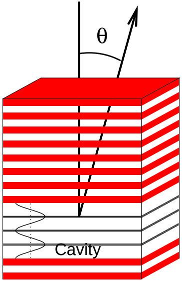

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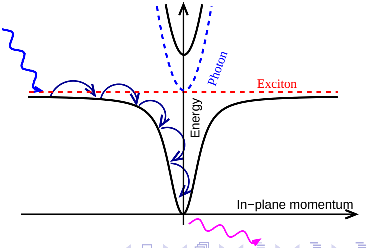
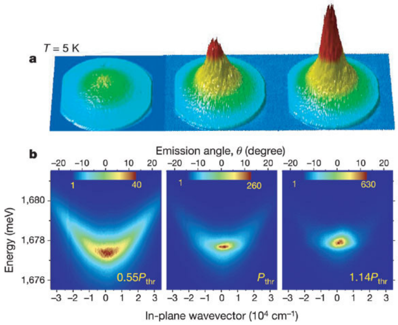
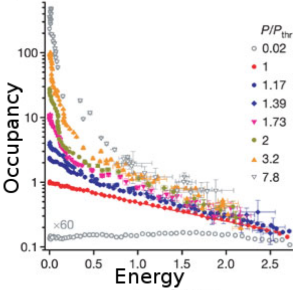


Polariton experiments: occupation and coherence





[Kasprzak, *et al.* Nature, '06]

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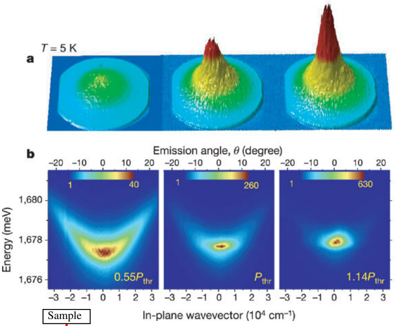
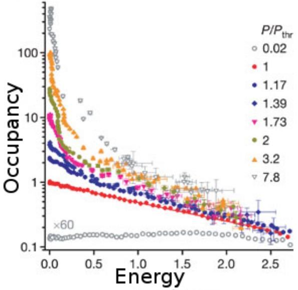
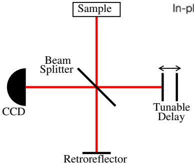
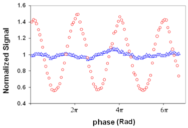
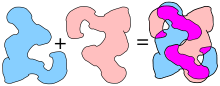


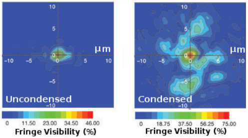






Coherence map:

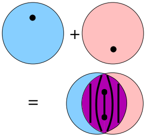


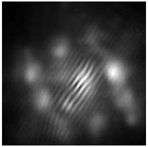


[Kasprzak, *et al.* Nature, '06]

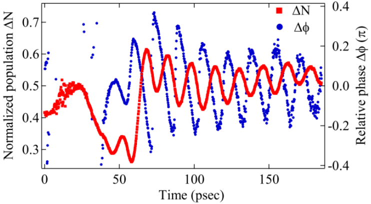
(Some) other polariton condensation experiments

- Quantised vortices
  [**Lagoudakis *et al.* Nat. Phys. ’08**. Science ’09, PRL ’10; Sanvitto *et al.* Nat. Phys. ’10; Roumpos *et al.* Nat. Phys. ’10]

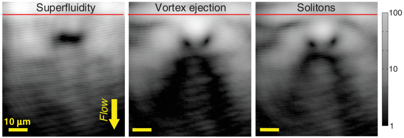




- Josephson oscillations
  [**Lagoudakis *et al.* PRL ’10**]



- Pattern formation/Hydrodynamics
  [**Amo *et al.* Science ’11**, Nature ’09; Wertz *et al.* Nat. Phys ’10]



What can you do beyond BEC (atoms)

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- Optical lattices

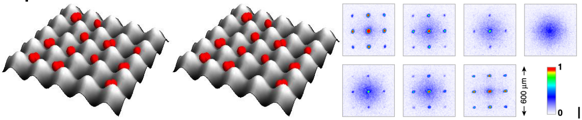
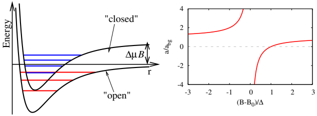
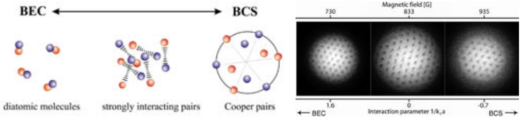


Image: Bloch group

- Feshbach resonance — strong interactions



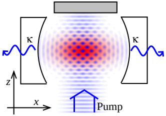
- Degenerate fermions — BEC BCS crossover

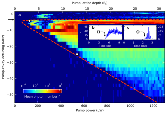


Images: Jin Group, Zweierlein group

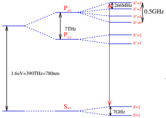
What can you do beyond BEC (atoms)

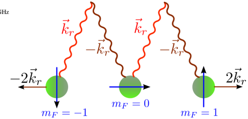
- Coupled matter-light systems





- Spinor condensates, spin-orbit, gauges





- Quenches, thermalisation, dynamics.

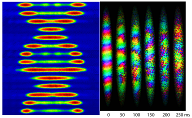
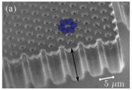


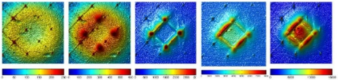
Image: Weiss group//Stamper Kurn group

What of this can polaritons do?

- Engineered potentials; lattices
  - Bloch — etched micropillars



  - Yamamoto — metal lattices



  - Snoke — stress traps
  - Krizhanovskii — Surface Acoustic Waves
- TE-TM interaction $\rightarrow$ Spin-Orbit
  - Theory: Malpuech, Rubo
- “Feshbach” resonances
  - Theory: Wouters, Carusotto
  - Deveaud, Biexciton resonance

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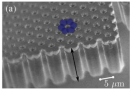

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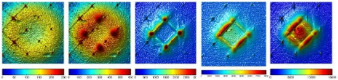

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This talk: *Spin structure, pairing phases of bosons*

Outline

Acknowledgements

GROUP:



COLLABORATORS:

Francesca Marchetti, UAM

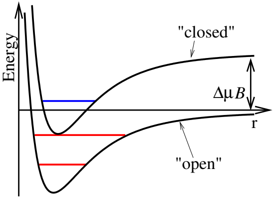
FUNDING: topnes — Topological Protection and Non-Equilibrium States in Strongly Correlated Electron Systems

EPSRC — Engineering and Physical Sciences Research Council

Outline

Tuning atomic interactions

- Open/closed channel — different F_z states (F=$J + I$)
- Magnetic field, $\Delta E(r \to \infty) = \text{const} + (\mu_a - \mu_b)B_z$
- Hybridisation by hyperfine coupling



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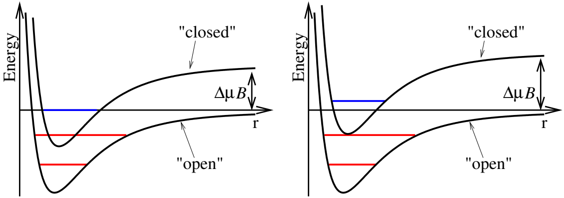

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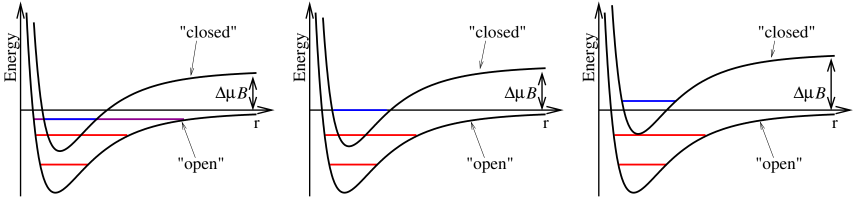

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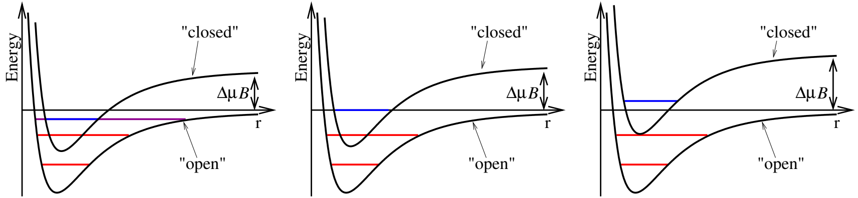
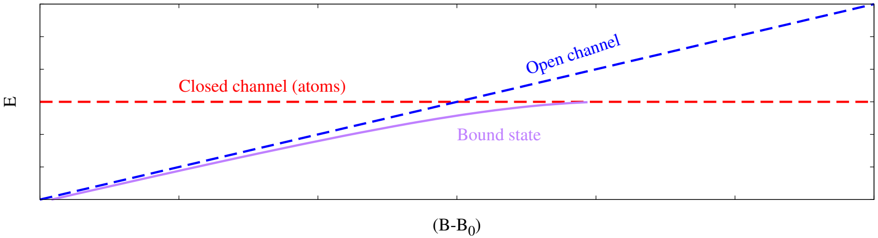
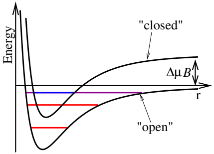




Feshbach resonance

$B < B_0$



Open channel $a > 0$ (repulsive)

True bound state exists.

Resonance — $a \to \infty$

Open channel $a < 0$ (attractive)

Hybridisation with continuum: Fano–Feshbach resonance

Feshbach resonance

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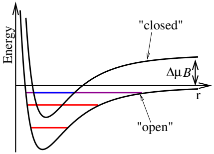

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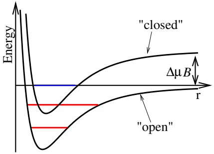

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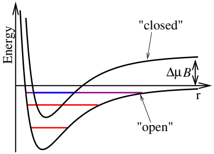

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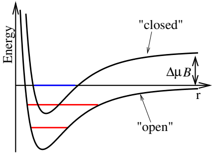

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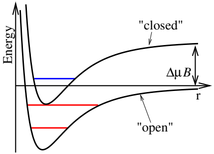

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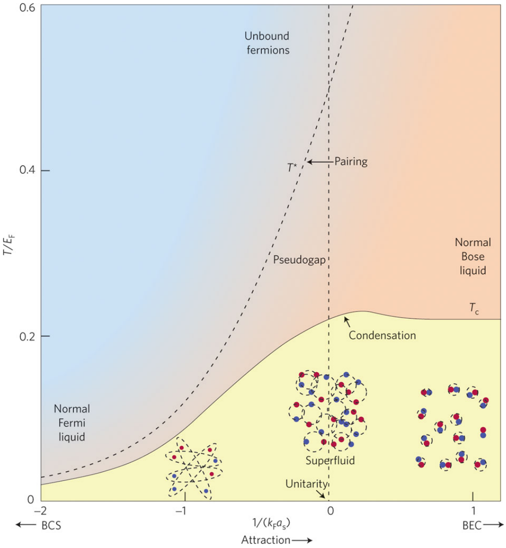
Pairing phases of Fermions



From Randeria, Nat. Phys. News & Views '10

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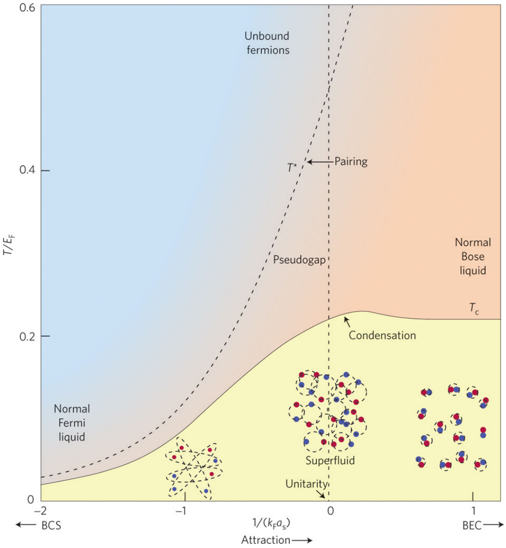

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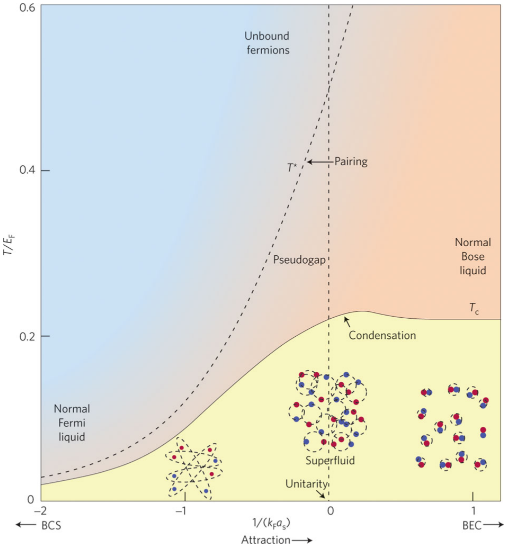

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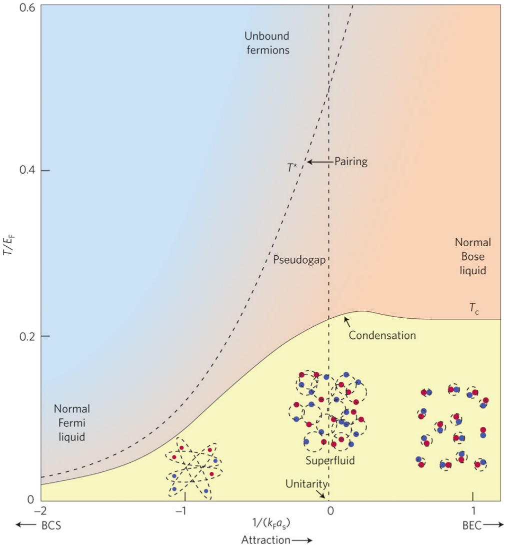

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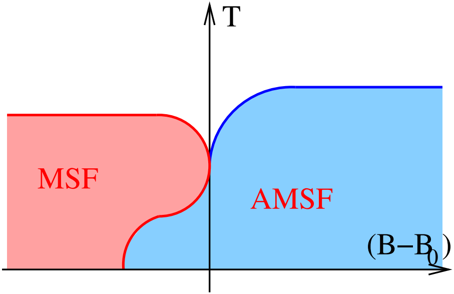

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[Radzihovsky *et al.* PRL '04, Romans *et al.* PRL '04, building on Nozières & St James, Timmermanns, Mueller, Thouless . . .]

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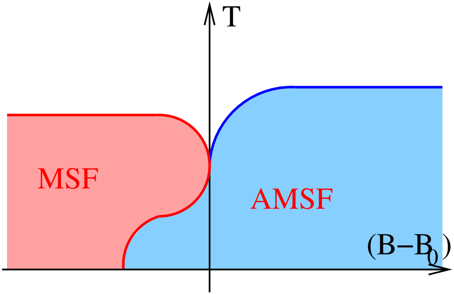

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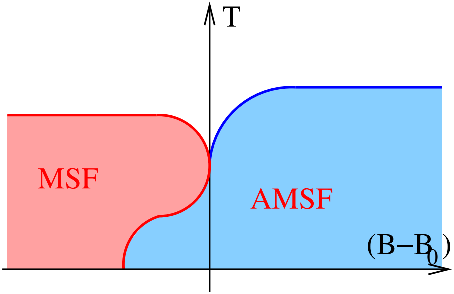

- Heteronuclear: $\hat{H} = \ldots + \hat{\psi}^\dagger_m \hat{\psi}_{a_1} \hat{\psi}_{a_2} + \text{h.c.}$
  - If $\langle \hat{\psi}_m \rangle \neq 0$, free atomic phase. MSF
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[Radzihovsky *et al.* PRL '04, Romans *et al.* PRL '04, building on Nozières & St James, Timmermanns, Mueller, Thouless . . .]

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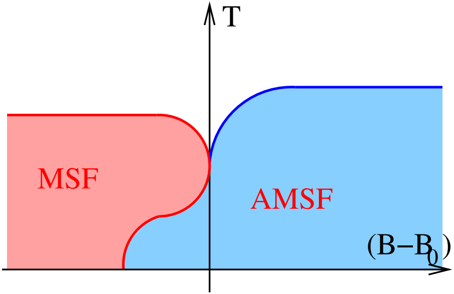

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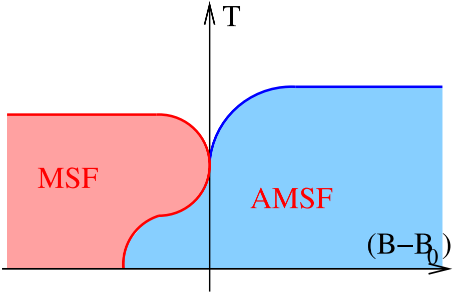
Problems with cold atoms

- Signatures?
  - Momentum distribution — indirect
  - Half vortices



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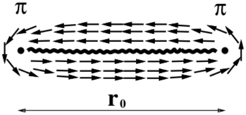

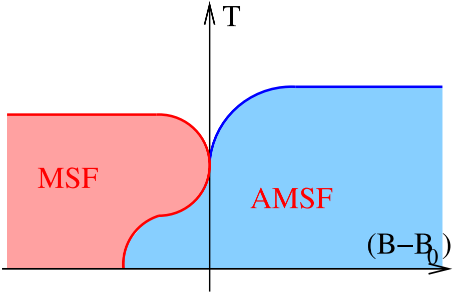

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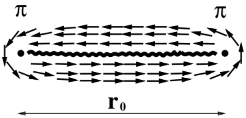

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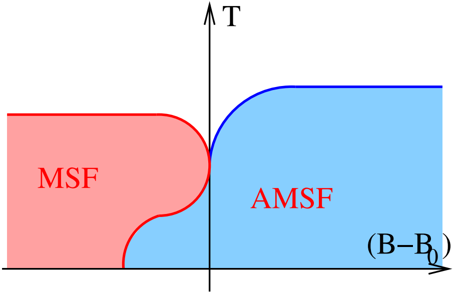

Outline

Exciton and polariton spin degrees of freedom

- Photon: two circular polarisation modes
- Exciton: bound state of electron & hole

- Exciton spin states $J_z = +2, +1, -1, -2$
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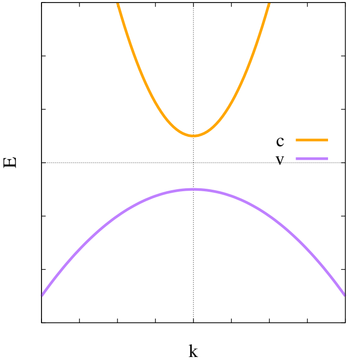

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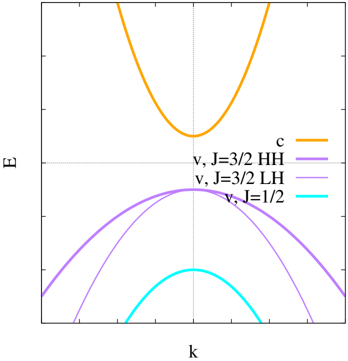

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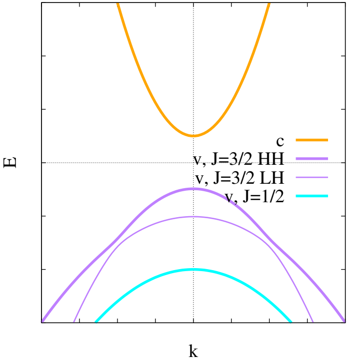

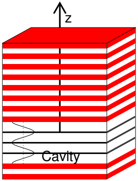

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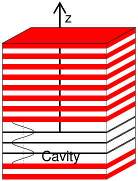

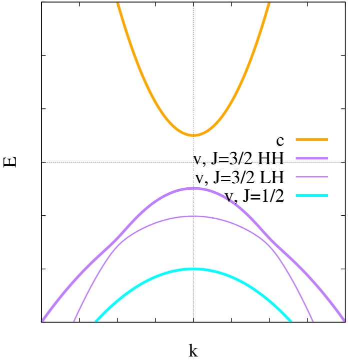

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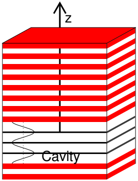

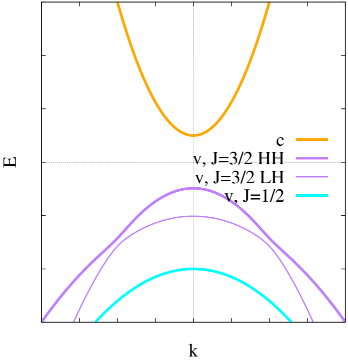

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Exciton-photon model

- Microscopic model — coupled exciton-photon system

$$H = \sum_k \left[\sum_{\sigma=\pm2,\pm1} \left(\frac{k^2}{2m_X} - \mu \right) \hat{X}^\dagger_{k\sigma} \hat{X}_{k\sigma} + \sum_{\sigma=\pm1} \left(\delta + \frac{k^2}{2m_C} - \mu \right) \hat{C}^\dagger_{k\sigma} \hat{C}_{k\sigma} + \sum_{\sigma=\pm1} \Omega_R \left(\hat{C}^\dagger_{k\sigma} \hat{X}_{k\sigma} + \hat{X}^\dagger_{k\sigma} \hat{C}_{k\sigma} \right) \right] + \iint d^2r d^2R \sum_{\sigma,\sigma',\tau,\tau'=\pm2,\pm1} U^{XX}_{\sigma'\tau'\tau\sigma}(\mathbf{r}) \times \hat{X}^\dagger_{\sigma'}\left(\mathbf{R}+\frac{\mathbf{r}}{2}\right) \hat{X}^\dagger_{\tau'}\left(\mathbf{R}-\frac{\mathbf{r}}{2}\right) \hat{X}_{\tau}\left(\mathbf{R}-\frac{\mathbf{r}}{2}\right) \hat{X}_{\sigma}\left(\mathbf{R}+\frac{\mathbf{r}}{2}\right)$$

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$$H = \sum_k \left[\sum_{\sigma=\uparrow,\downarrow} \left(\frac{k^2}{2m} - \mu \right) \hat{\psi}^\dagger_{\sigma k} \hat{\psi}_{\sigma k} + \left(\frac{k^2}{2m_m} + \nu - 2\mu \right) \hat{\psi}^\dagger_{mk} \hat{\psi}_{mk} \right]$$

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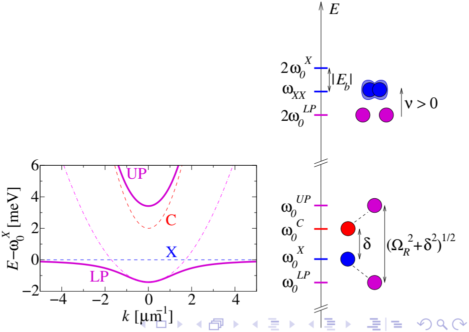
- Polariton dispersion m, detuning ν, interactions depend on δ
- Resonance width, dispersion derived from dressed exciton T matrix.



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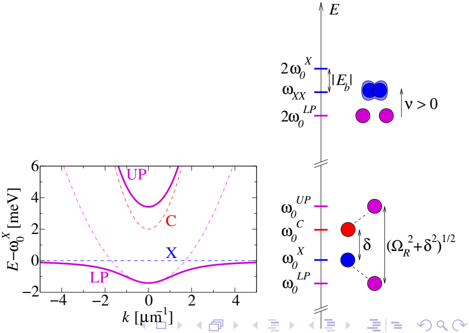

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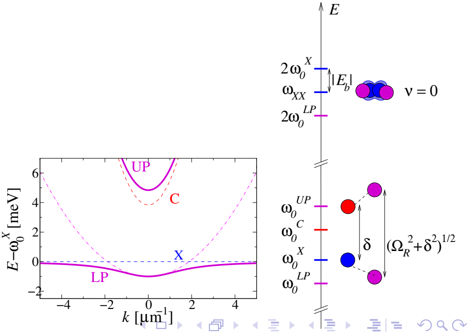

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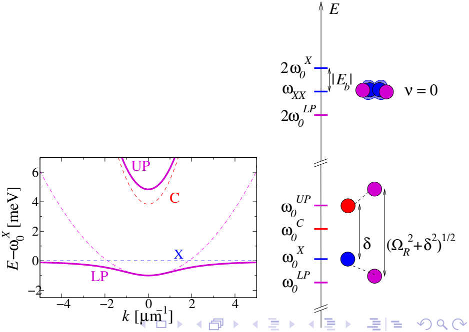

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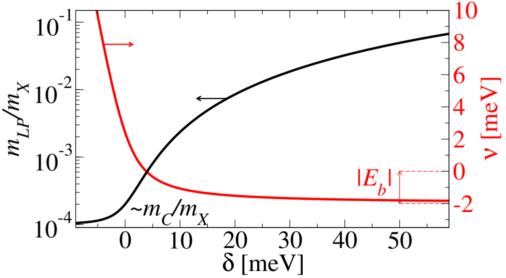
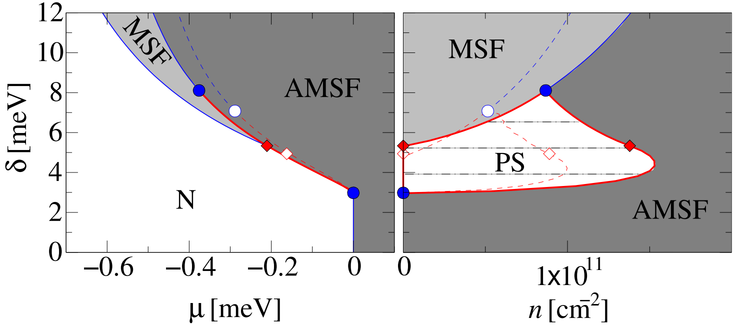
$\nu > 0$
Biexciton in continuum



$\nu < 0$
Bound biexciton.
(Excitonic limit)

Outline

Phase diagram (ground state, $T = 0$)



- Parameters for GaAs $\Omega_R = 4.4$meV, $E_b = 2$meV.
- Dashed: Mean-field. Solid: Next order fluctuations
- Two condensed phases: MSF, $\langle\hat{\psi}_\uparrow\rangle = \langle\hat{\psi}_\downarrow\rangle = 0$, $\langle\hat{\psi}_m\rangle \neq 0$. AMSF $\forall_\sigma$: $\langle\hat{\psi}_\sigma\rangle \neq 0$.
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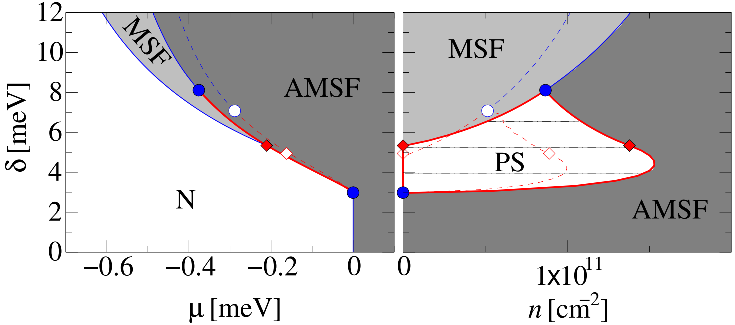

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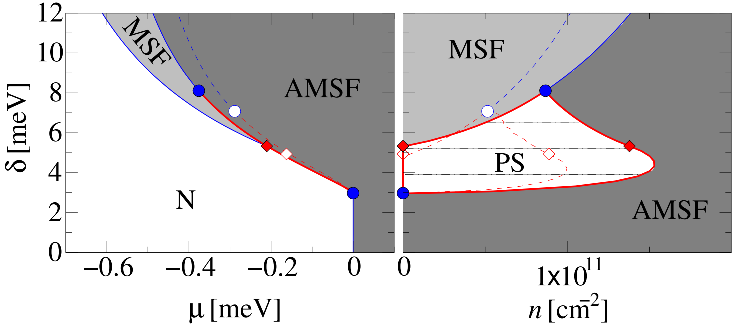

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Mean field ansatz

- Hamiltonian: Weakly Interacting Bose Gas
- First ansatz: mean-field theory [Zhou *et al.* PRA '08]

$$|\Psi\rangle \propto \exp\left(-\sum_{\sigma=\uparrow,\downarrow,m} \psi_\sigma \hat{\psi}_{k=0,\sigma}\right).$$

- No magnetic field, take $\psi_\uparrow = \psi_\downarrow = \psi_0$ (real):

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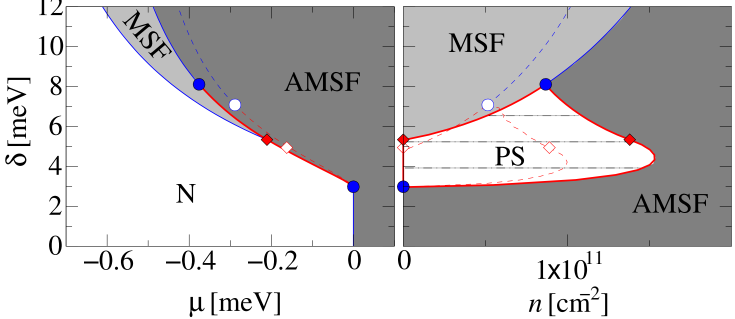

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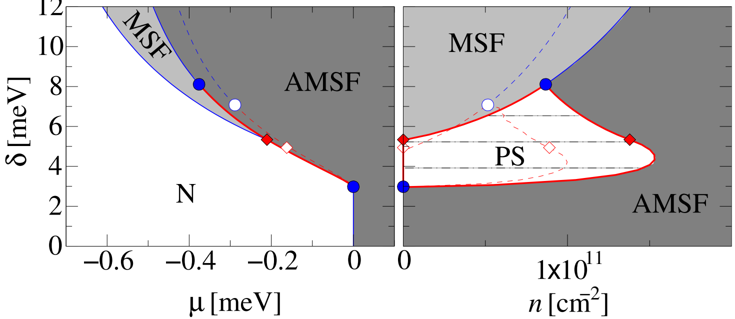

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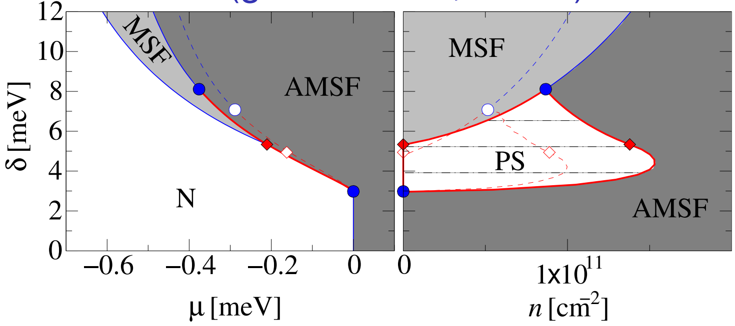

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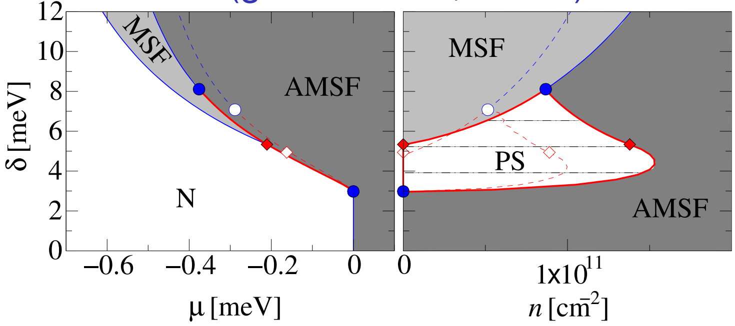

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Outline

Detecting MSF vs AMSF

- Phase separation
- Direct access to polariton phase coherence

- Vortex structure — half vortices, $\psi_\uparrow = e^{im_\uparrow\theta}, \psi_\downarrow = e^{im_\downarrow\theta}$,
  MSF has $(m_\uparrow, m_\downarrow) = (1/2, 1/2)$.

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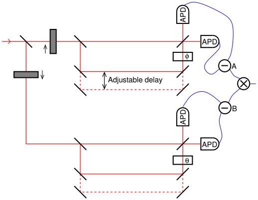

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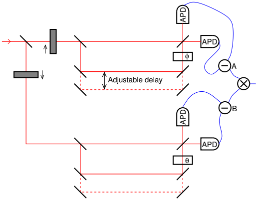

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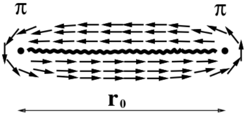
MSF has $(m_\uparrow, m_\downarrow) = (1/2, 1/2)$  .

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1. Introduction
2. Pairing phases of atoms: review
3. Modelling polariton pairing
   - Exciton spin structure
   - Microscopic and effective Hamiltonian
4. Ground state phase diagram
   - Origin of multicritical behaviour
   - Signatures of phases
5. Finite temperature phase diagram
   - Variational MFT
   - Required temperatures, detuning
   - Candidate material systems

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- Finite T — minimize free energy
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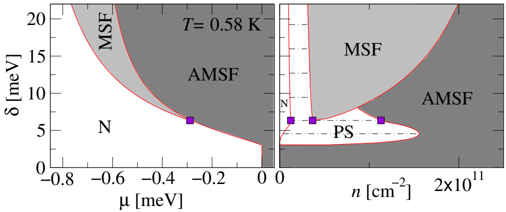

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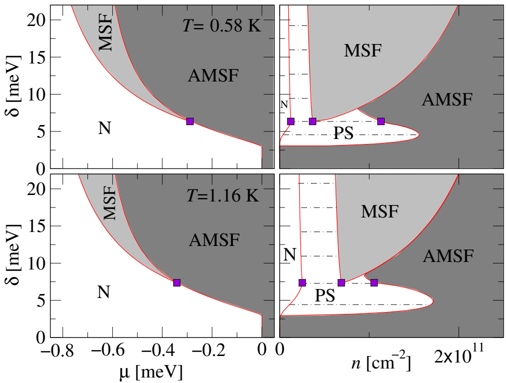

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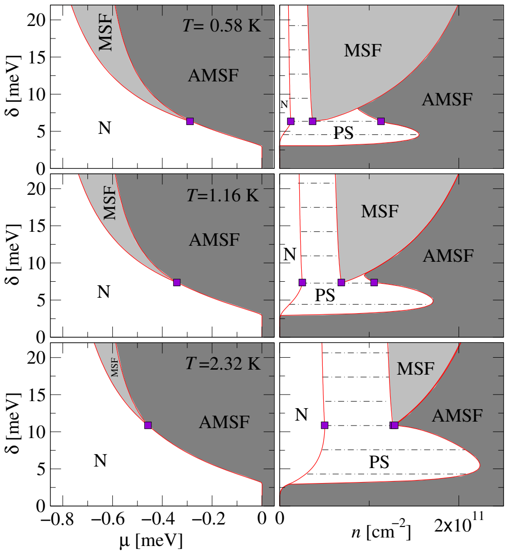

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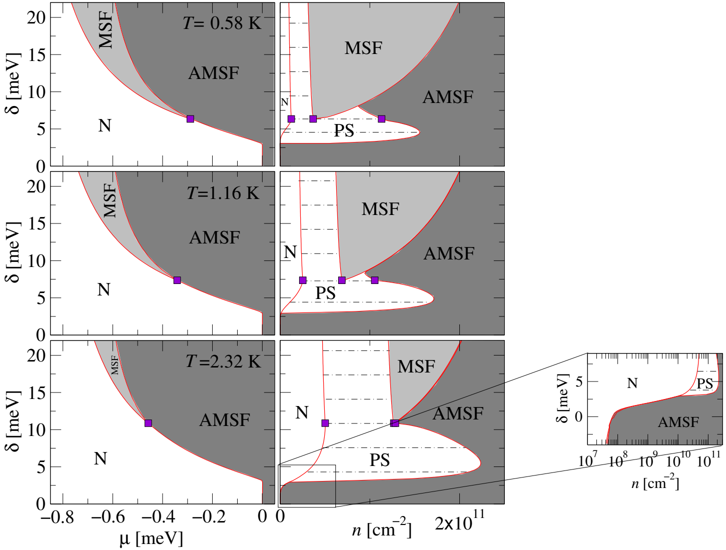

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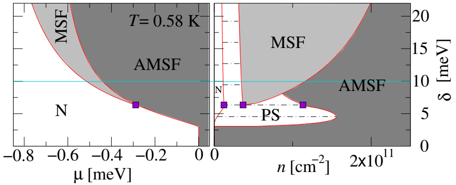

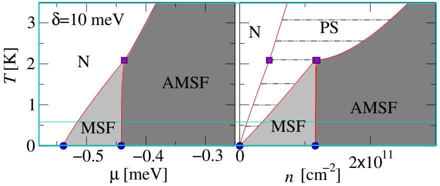

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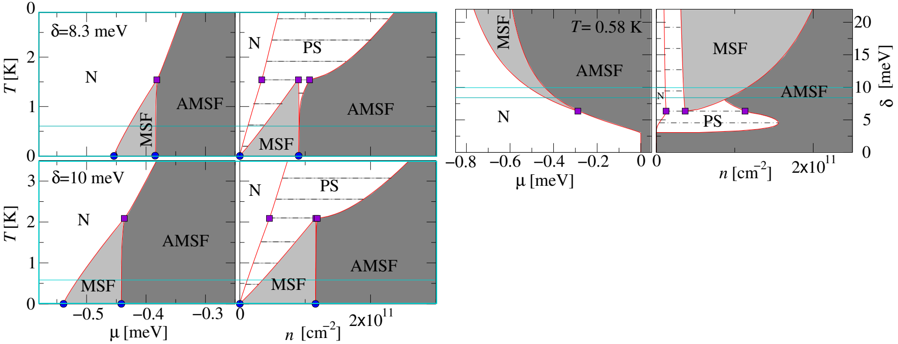

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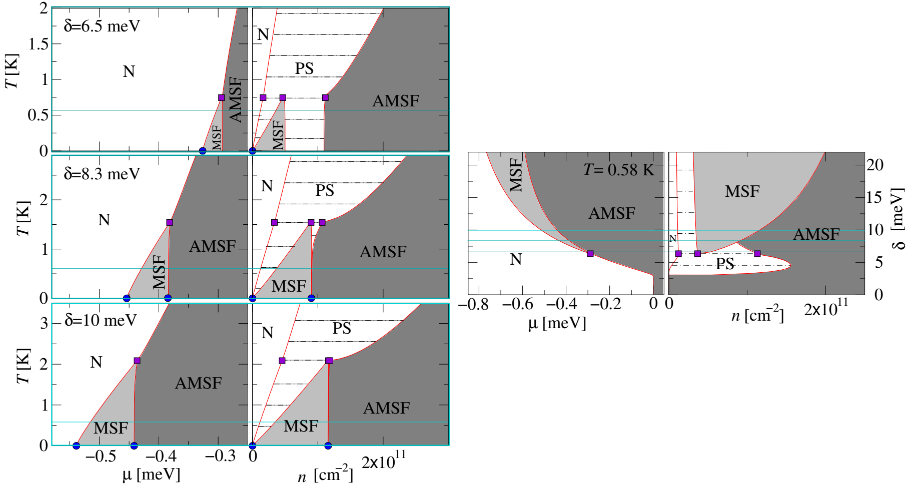

Evolution of triple point

- Required T, δ for MSF: Triple point
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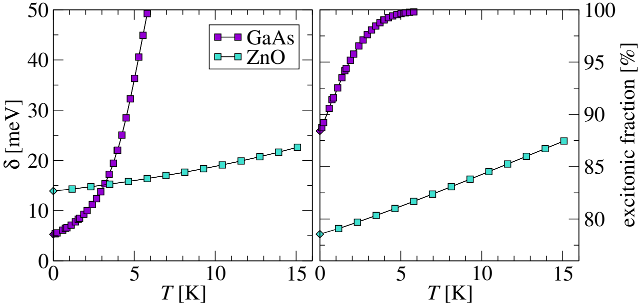

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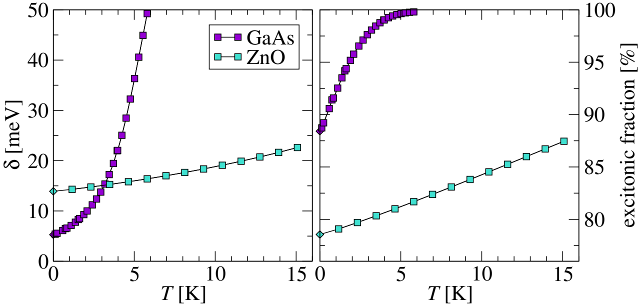

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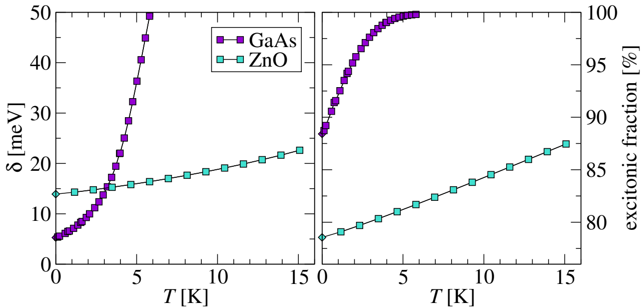

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Summary

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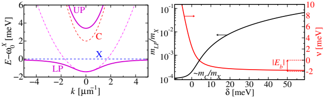
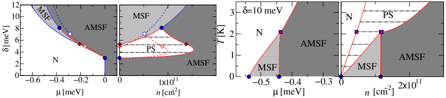
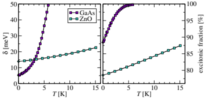


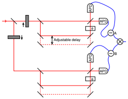
- Polaritons can show pairing phase



- MSF easily attainable for ZnO; ”possible” for GaA”

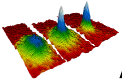
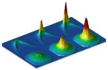


- Possible signatures in coherence and vortices



[Marchetti and Keeling, `arXiv:1308.1032`]

Differences & new opportunities

 Atoms	 Polaritons
Metastable, isolated	Finite lifetime, in semiconductor
Fixed mass $\sim 10^3 m_0$	Tunable mass $\sim 10^{-4} m_0$
Couples to light	Is part light
Can be 3D	Always 2D or less.
Spin: $S_z = -S, -S+1, \ldots, S$	Light polarisations $S_z = \pm 1$

Variational MFT for WIDBG

- Test validity. WIDBG $\hat{H} = \sum_k \frac{k^2}{2m}\psi^\dagger_k\psi_k + \frac{U}{2}\int d^2r\psi^\dagger\psi^\dagger\psi\psi$
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$$\hat{H}_{\rm MF} = -\sqrt{\mathcal{A}}\psi(\alpha+\beta)(\hat{b}^\dagger_0+\hat{b}_0)$$
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- Compare to 2D EOS, $\rho(\mu) = Tf(\mu/T)$
- CF Hartree-Fock-Popov-Bogoluibov method, include $U\rho$ in Σ

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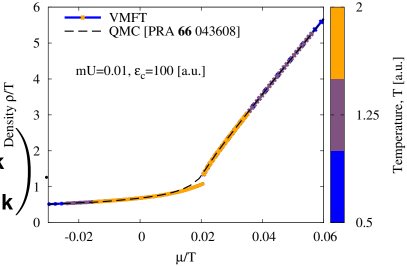

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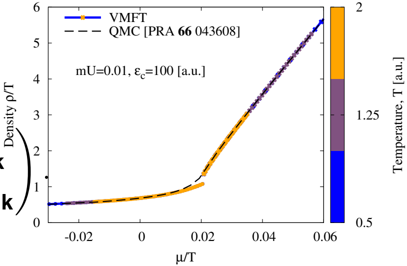

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