Superradiance and self organisation with fermionic atoms

Jonathan Keeling

University of St Andrews 600 YEARS

SUPA

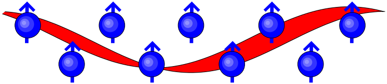
QSOE, August 2013

Dicke effect: Superradiance

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

R. H. DICKE
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey



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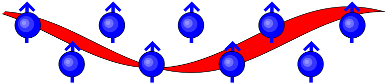

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$$\frac{d\rho}{dt} = -\frac{\Gamma}{2}\left[S^+S^-\rho - S^-\rho S^+ + \rho S^+ S^-\right]$$

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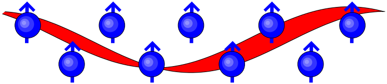

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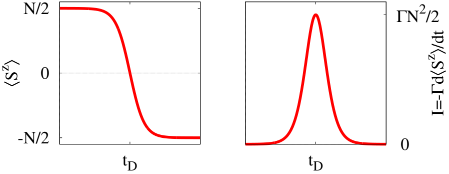
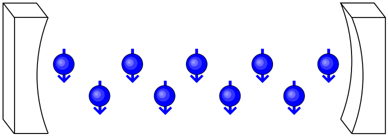
If $S^z = |S| = N/2$ initially:

$$I \propto -\Gamma \frac{d\langle S^z \rangle}{dt} = \frac{\Gamma N^2}{4}\text{sech}^2\left[\frac{\Gamma N}{2} t\right]$$



Dicke model and Dicke-Hepp-Lieb transition



$$H = \omega \psi^\dagger \psi + \sum_i \omega_0 S_i^z + g(\psi + \psi^\dagger)(S_i^+ + S_i^-)$$

Rotating wave approx → Tavis–Cummings model

- Coherent state: $|\Psi\rangle \to e^{\lambda\psi^\dagger + \eta S^+}|\Omega\rangle$
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[Hepp, Lieb, Ann. Phys. '73]

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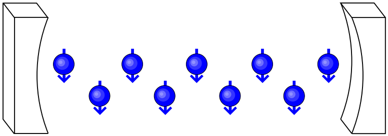

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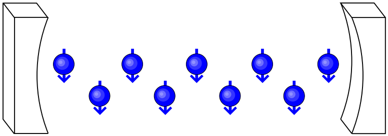

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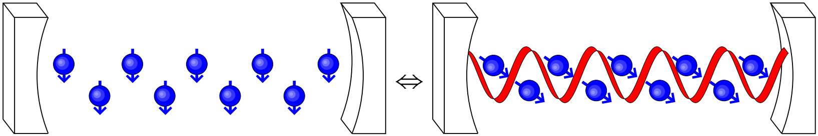

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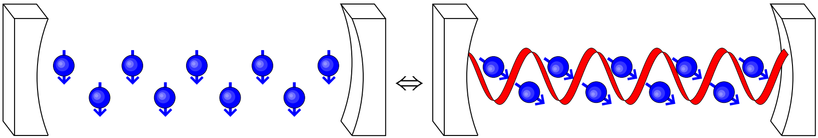

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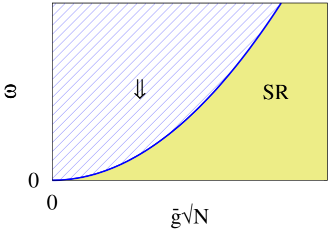

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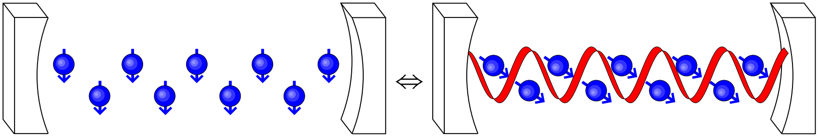
No go theorem and transition



Spontaneous polarisation if: $4Ng^2 > \omega\omega_0$

[Rzazewski *et al* PRL '75]

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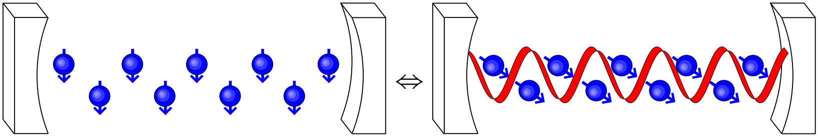

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No go theorem:. Minimal coupling $(p - eA)^2/2m$

$$-\sum_i \frac{e}{m} A \cdot p_i \Leftrightarrow g(\psi^\dagger + \psi)(S^- + S^+), \qquad \sum_i \frac{A^2}{2m} \Leftrightarrow N\zeta(\psi + \psi^\dagger)^2$$

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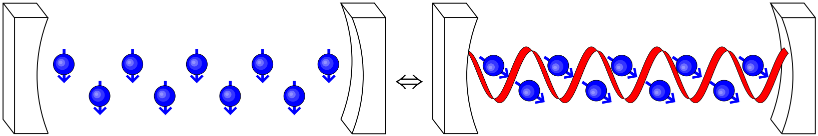

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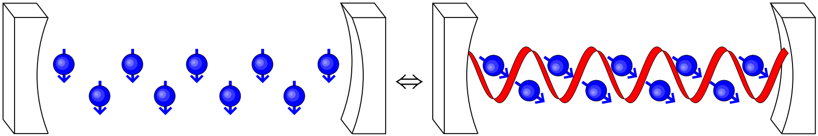

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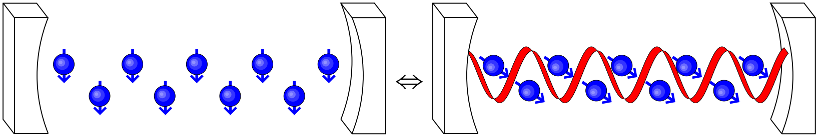

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But f-sum rule states: $g^2/\omega_0 < \zeta$. **No transition**

[Rzazewski *et al* PRL '75]

Dicke phase transition: ways out

Problem: $g^2/\omega_0 < \zeta$ for intrinsic parameters. **Solutions**:

1. Gauge/interpretation of "photon"
   Ferroelectric transition in $\mathbf{D} \cdot \mathbf{r}$ gauge.
   [JK JPCM '07, Vukics & Domokos PRA 2012]
   - Circuit QED [Nataf and Ciuti, Nat. Comm. '10; Viehmann *et al.* PRL '11]
2. Grand canonical ensemble:
   - If $H \to H - \mu(S^z + \psi^\dagger\psi)$, need only:
     $g^2 N > (\omega - \mu)(\omega_0 - \mu)$
   - Incoherent pumping — polariton condensation.
3. Dissociate g, ω_0,
   e.g. Raman scheme: $\omega_0 \ll \omega$,
   [Dimer *et al.* PRA '07; Baumann *et al.* Nature '10. Also, Black *et al.* PRL '03]

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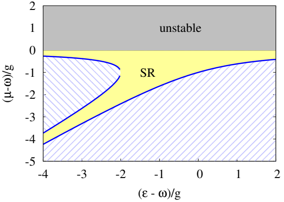

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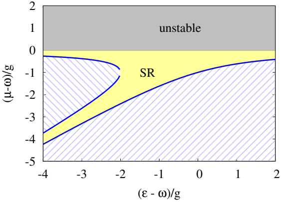

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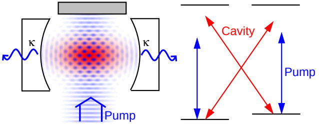

Outline

1. Dicke model, superradiance and no–go theorem
2. Superradiance and self-organisation
   - Raman scheme
   - Rayleigh scheme and hierarchies of H_{eff}
   - Generalized Dicke equilibrium theory
3. Fermionic self organisation
   - Equilibrium phase diagrams
   - Landau theory and microscopics
   - Evolution with filling
4. Open system dynamics
   - Linear stability with losses
   - Attractors of the Dicke model phases

Acknowledgements

PEOPLE:

FUNDING:

EPSRC
Engineering and Physical Sciences
Research Council

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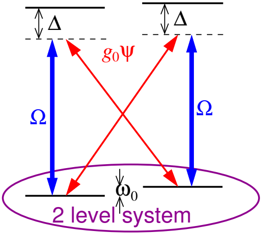

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[Dimer *et al.* PRA '07]

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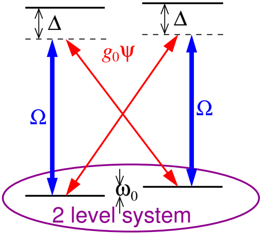

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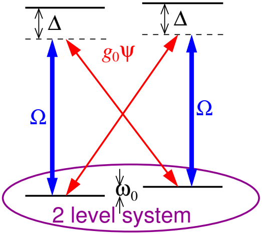

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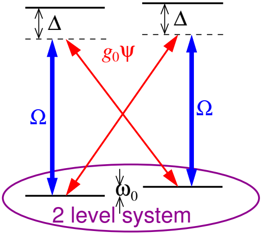

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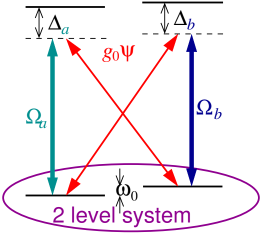

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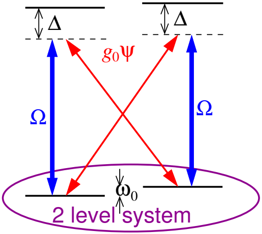

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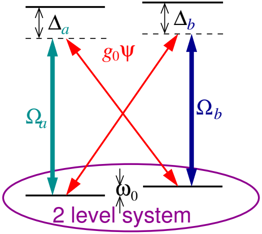

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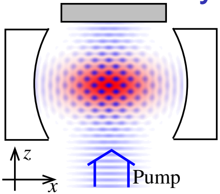
Transversely pumped cavity



Internal state → momentum states

1. Full description

$$H_0 = \omega_{\text{cavity}} \psi^\dagger \psi + \int d^2r \Big[\sum_{\alpha=e,g} c^\dagger_\alpha \left(\frac{-\nabla^2}{2m} \right) c_\alpha + \omega_{\text{atom}} c^\dagger_e c_e + E(x,z,t)(c^\dagger_e c_g + c^\dagger_g c_e) \Big]$$

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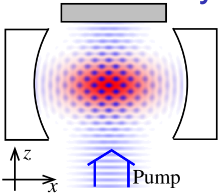

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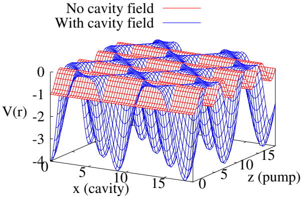
2. Eliminate e state
   - Rotating frame $\omega = \omega_{\text{cavity}} - \omega_{\text{pump}} - N\delta$



$$H = \omega \psi^\dagger \psi + \int d^2 \mathbf{r} c^\dagger(\mathbf{r}) \left(-\frac{\nabla^2}{2m} - V(\mathbf{r}) \right) c(\mathbf{r})$$

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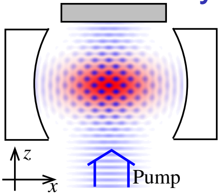
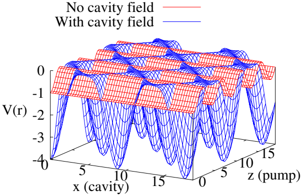
Transversely pumped cavity



Internal state → momentum states

1. Full description

$$H_0 = \omega_{\text{cavity}}\psi^\dagger\psi + \int d^2r\Big[\sum_{\alpha=e,g} c^\dagger_\alpha \left(\frac{-\nabla^2}{2m}\right) c_\alpha + \omega_{\text{atom}} c^\dagger_e c_e + E(x,z,t)(c^\dagger_e c_g + c^\dagger_g c_e)\Big]$$



2. Eliminate e state
   - Rotating frame $\omega = \omega_{\text{cavity}} - \omega_{\text{pump}} - N\delta$

$$H = \omega\psi^\dagger\psi + \int d^2\mathbf{r} c^\dagger(\mathbf{r}) \left(-\frac{\nabla^2}{2m} - V(\mathbf{r})\right) c(\mathbf{r})$$

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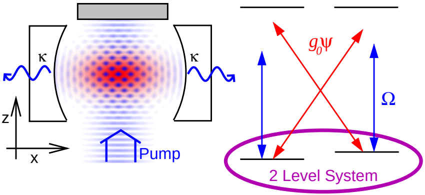
3. Dicke: project to atomic states $\phi(x,z) \propto \begin{cases} 1 \\ \cos(qz)\cos(qz) \end{cases}$

Extended Dicke model



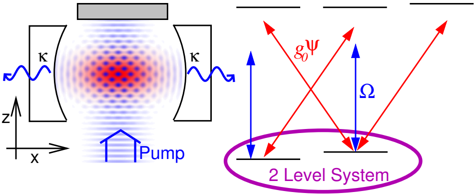
Reduced basis:

$$\phi(x,z) \propto \begin{cases} 1 & \Downarrow \\ \cos(qz)\cos(qz) & \Uparrow \end{cases}$$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+)$$

[Baumann *et al* Nature '10]

Extended Dicke model



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"Feedback" due to extra states $U = -\frac{g_0^2}{4\Delta}$

[Baumann *et al* Nature '10]

Extended Dicke phase diagram

Ground state energy, $\lambda = \langle \psi \rangle / \sqrt{N}$:

$$\frac{E}{N} = \omega \lambda^2 - \frac{N}{2} \sqrt{(\omega_0 + UN\lambda^2)^2 + 16 g^2 N \lambda^2}$$

- Superradiant transition:

$$4g^2 N > \left(\omega - \frac{UN}{2} \right) \omega_0$$

- Stability, $\lambda \to \infty$

$$E \sim \left(\omega - \frac{UN}{2} \right) \lambda^2 + \ldots$$

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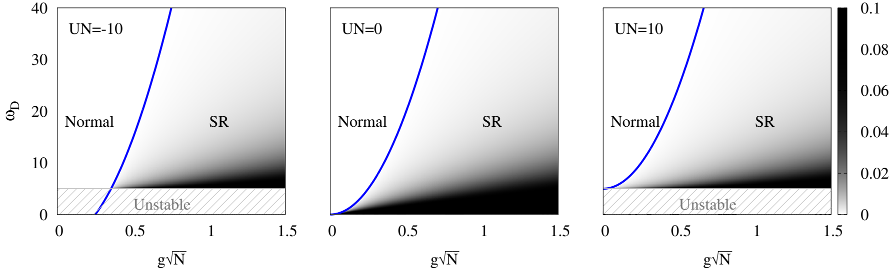
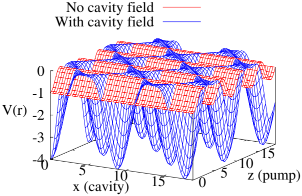


Outline

Fermions in optical cavities



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[Domokos & Ritsch, PRL '03; Black *et al.* PRL '03; Piazza, Strack, Zweger `arXiv:1305.2928`]

- Pauli blocking
- Commensurability effects
- Rescale with $\sqrt{2}q$, $\omega_r = \hbar^2 q^2/2m$
  - $n_F = N/N_L$, $\omega \to \tilde{\omega}$
  - $\Omega \to P$, $\langle\psi\rangle \to \lambda$

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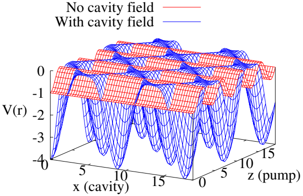

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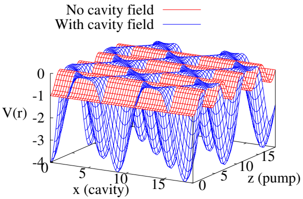

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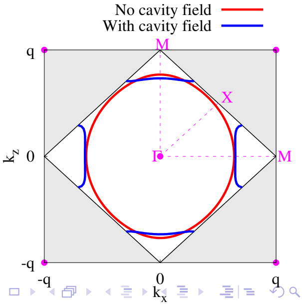

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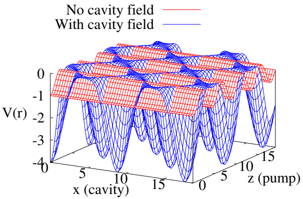

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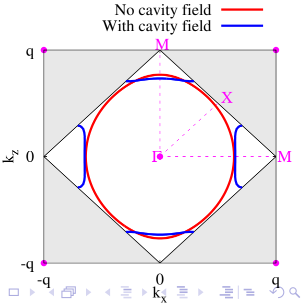

Free energy to phase diagram

- Free energy $f = F/N_L\omega_r$

$$f(\tilde{\omega}, P, n_F \to \mu; \lambda) = \tilde{\omega}\lambda^2 + \mu n_F - \frac{1}{\beta}\int_{BZ} d^2k \sum_n \ln\left[1 + e^{-\beta(\epsilon_{\mathbf{k},n}-\mu)}\right]$$

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- $n_F \to 0$, statistics irrelevant, expect SR.

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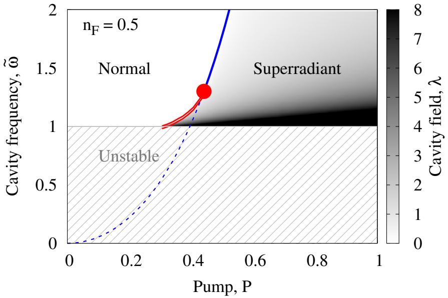

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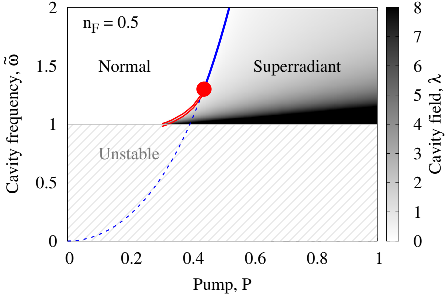

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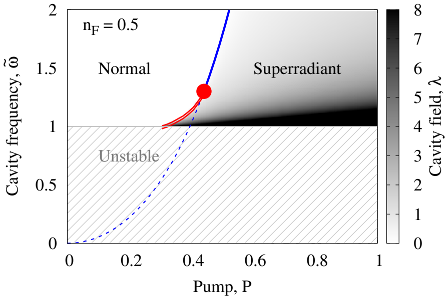

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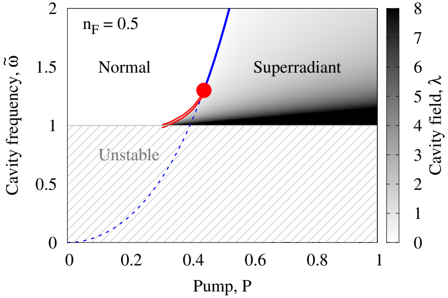
$$f = a\lambda^2 + b\lambda^4 + c\lambda^6$$

$b < 0$ at small P.

Origin of first order transition



- $\epsilon_{\mathbf{k},n}$ from $\hat{h} = k^2 \delta_{\mathbf{k},\mathbf{k}'} - V_{\mathbf{k},\mathbf{k}'}$

$$\begin{aligned} V_{\mathbf{k},\mathbf{k}'} &= \lambda^2 \sum_{s} \delta_{\mathbf{k},\mathbf{k}'+s\sqrt{2}\hat{\mathbf{x}}} \\ &+ P\lambda \sum_{s,s'} \delta_{\mathbf{k},\mathbf{k}'+\frac{s}{\sqrt{2}}\hat{\mathbf{x}}+\frac{s'}{\sqrt{2}}\hat{\mathbf{z}}} \\ &+ P^2 \sum_{s} \delta_{\mathbf{k},\mathbf{k}'+s\sqrt{2}\hat{\mathbf{z}}} \end{aligned}$$

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- Second order perturbation theory, $-\lambda^4 |m_{\mathbf{k},\mathbf{k}'}|^2/(E_{\mathbf{k}'} - E_{\mathbf{k}})$
- Jahn-Teller like distortion
- Survives to low n_F: Bosons!
  - But needs state $\phi(x, z) = \cos(\sqrt{2}x)$
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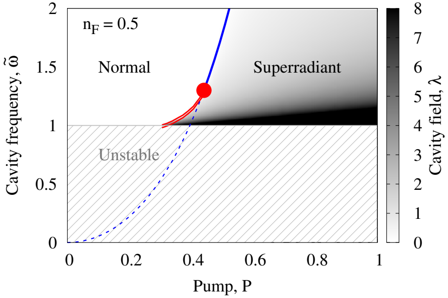

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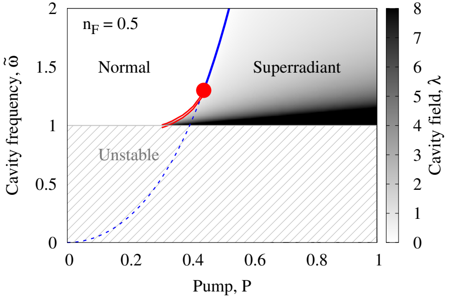

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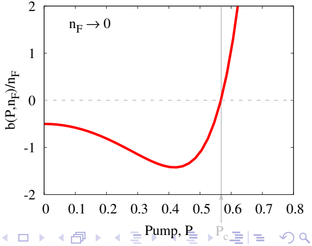

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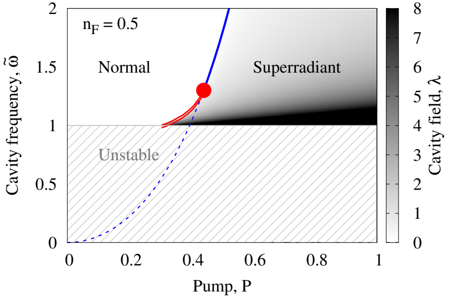

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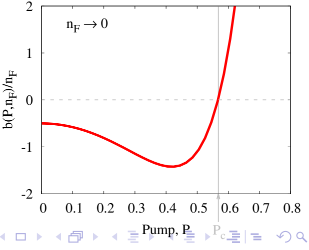
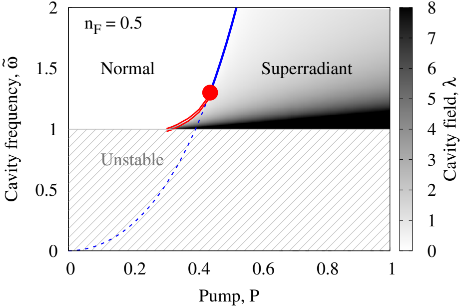


Higher fillings

$$f = a\lambda^2 + b\lambda^4 + c\lambda^6$$

- Phase diagram unchanged for $n_F < 1$
- 2nd order line $a = 0$
- Tricritical ● at $a = b = 0$



- 2nd band, new structure.
  - Critical end-point
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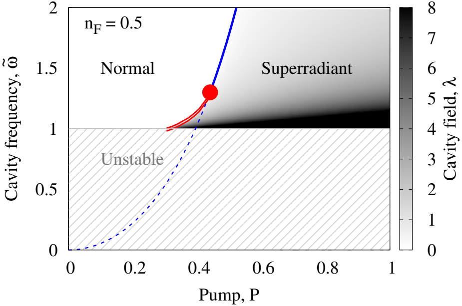

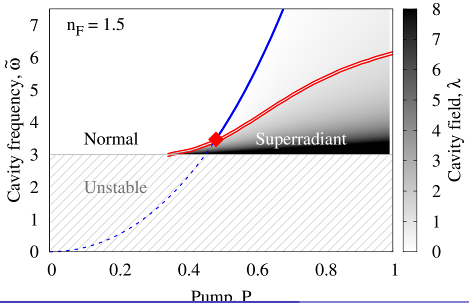

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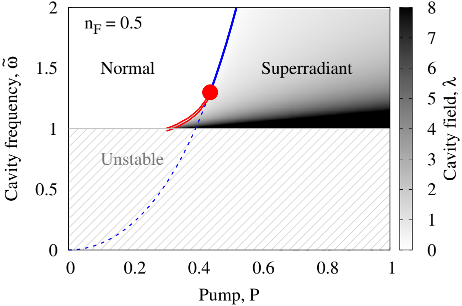

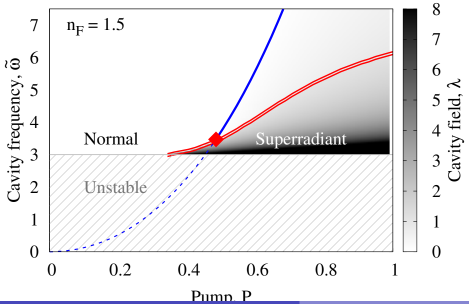

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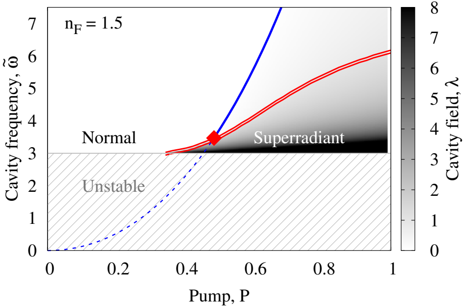
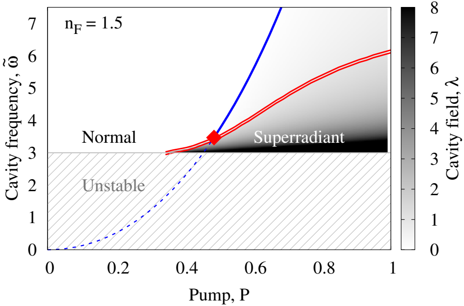


Liquid–gas transition

- $f(\lambda) \rightarrow$ multiple minima
- Higher orders in λ

- Plot bands $\inf_k[\epsilon_{k,n}]$
- Contribution of 2nd band
- Non-trivial form:
  - p_x, p_z orbitals cross at $P = \lambda$
  - $n > 1$ bands initially go up

Why liquid gas?



Liquid–gas transition

- $f(\lambda) \to$ multiple minima
- Higher orders in λ

- Plot bands $\inf_k [\epsilon_{\mathbf{k},n}]$
- Contribution of 2nd band
- Non-trivial form:
  - p_x, p_z orbitals cross at $P = \lambda$
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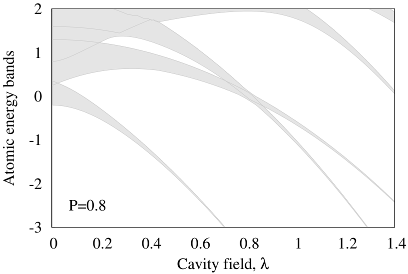

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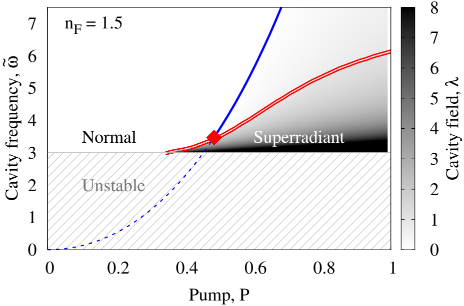

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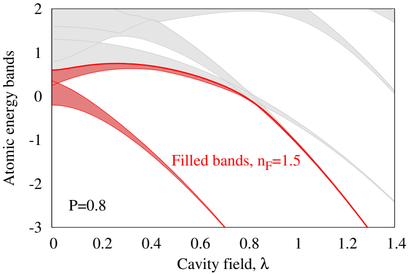

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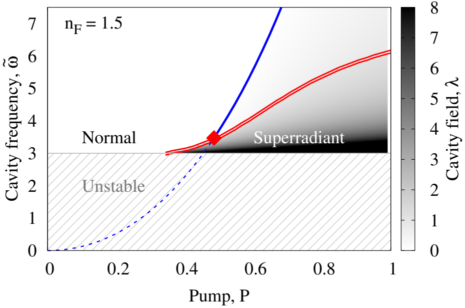

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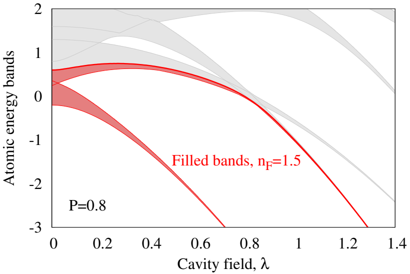
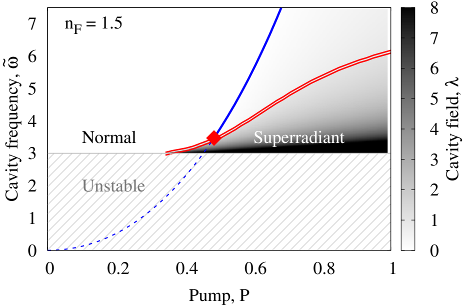
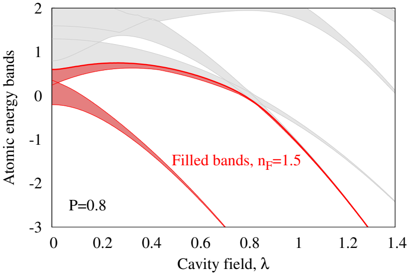
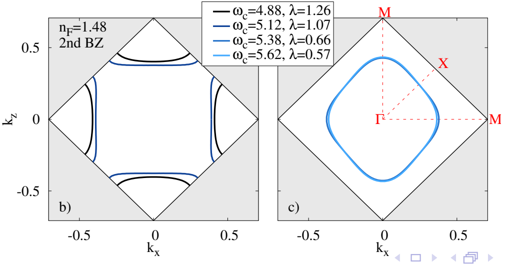


Signatures: Changing FS topology



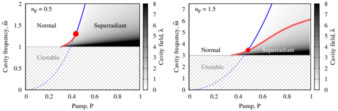


Near half-filled 2nd band — Pomeranchuk transition



Evolution with density

- Phase topology change:



- Fix P, plot vs n_F
- SR–SR after critical point
- Peak in 2nd order line $0 = a(\tilde{\omega}, n_F, P) = \tilde{\omega} + \xi(P, n_F)$
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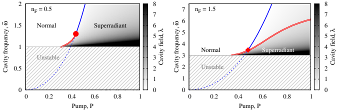

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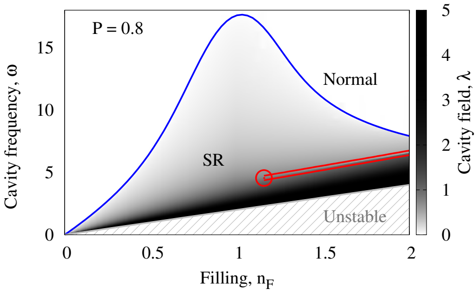

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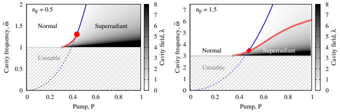

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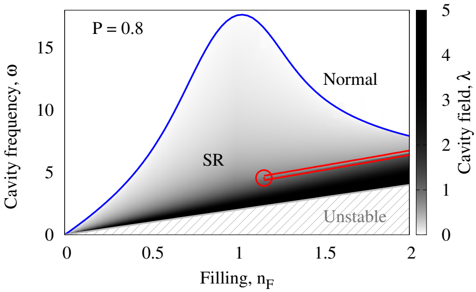

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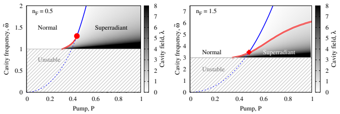

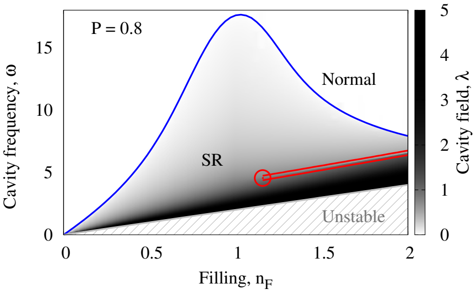

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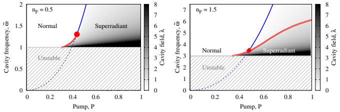

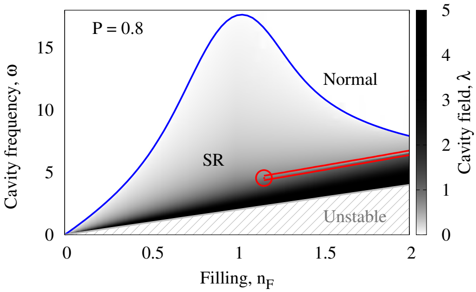

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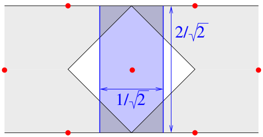
- Nesting of $\cos\left(\frac{x}{\sqrt{2}}\right) \cos\left(\frac{z}{\sqrt{2}}\right)$ at $n_F = 1$



Outline

Open system: Stable attractors

- Open system, $\dot{\rho} = -i[H, \rho] - \kappa\mathcal{L}[\psi]$
- Instead of $\min(F) \rightarrow$, stable attractors
- What survives
  - Normal–SR boundary: Consider $\psi = ue^{-i\nu t} + v^* e^{i\nu^* t}$,
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  - First order transitions/multistability?
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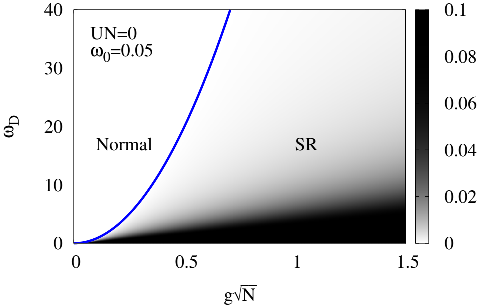
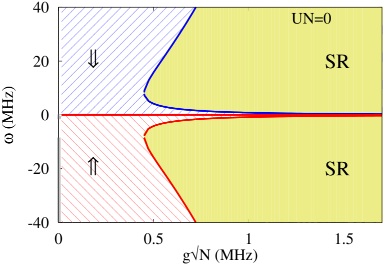
Equilibrium Dicke vs open phase diagram, $UN = 0$





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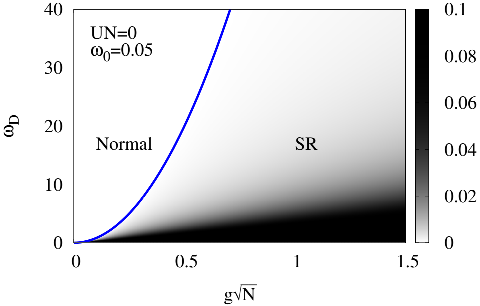

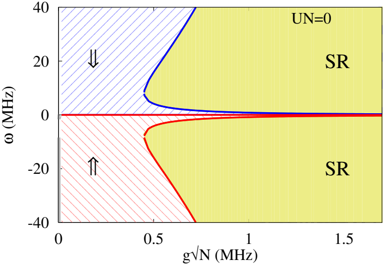

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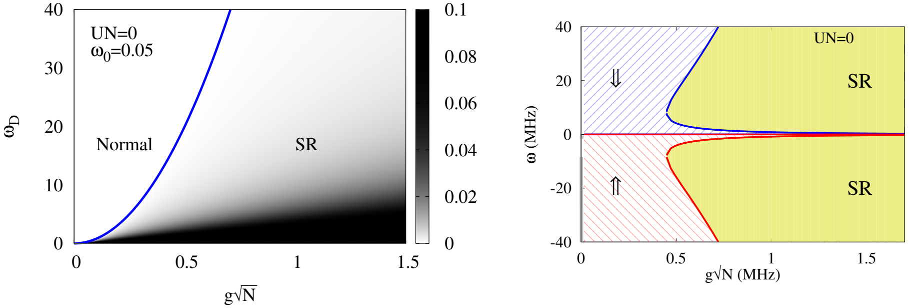

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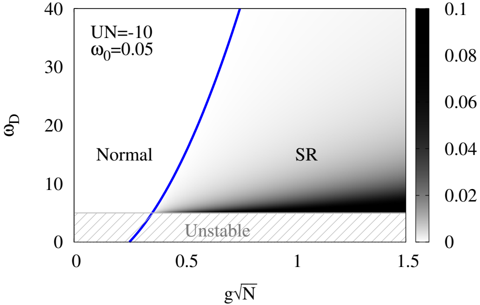
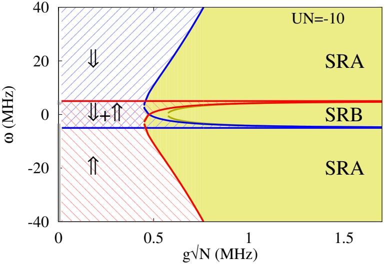
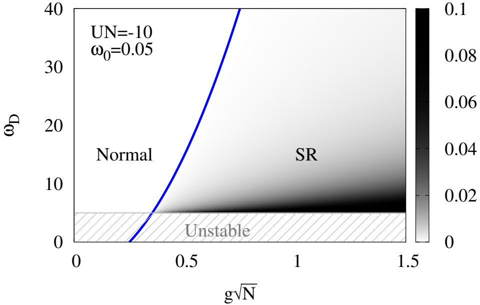
... Dicke ... $UN = -10\text{MHz}$

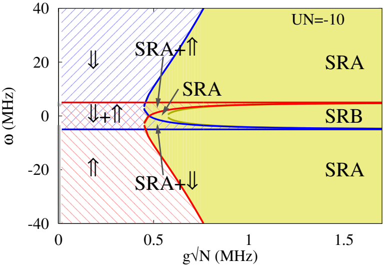




- Coexistence regions
- Unstable → SRB

. . . Dicke . . . $UN = -10\text{MHz}$





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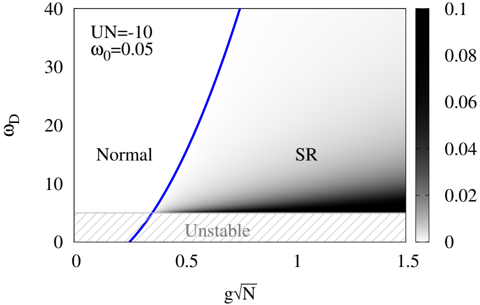

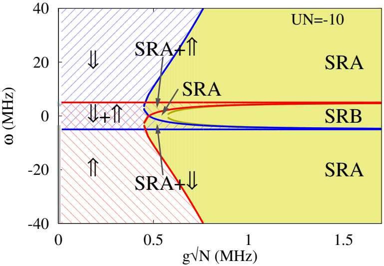

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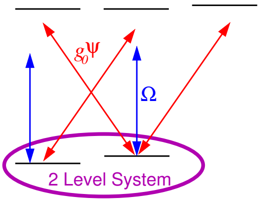
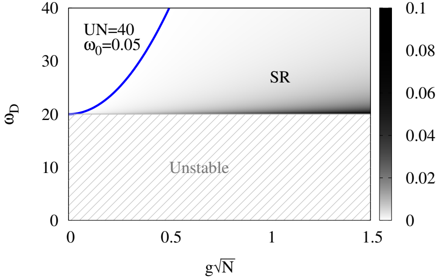
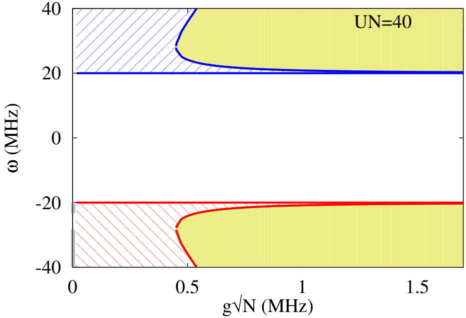
Changing *U*:







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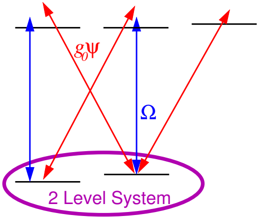

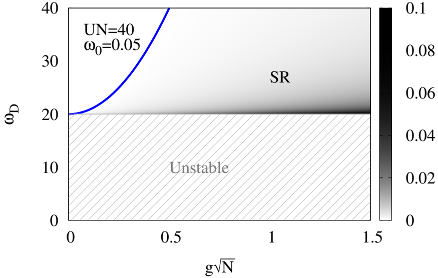

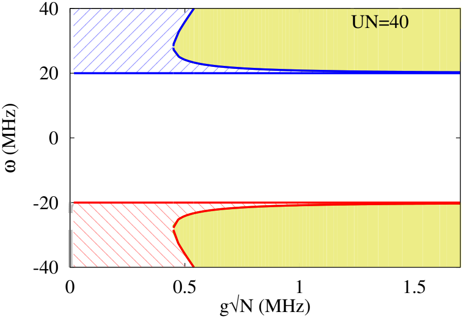

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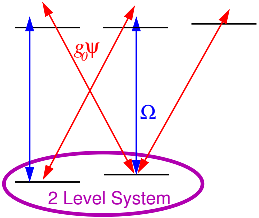
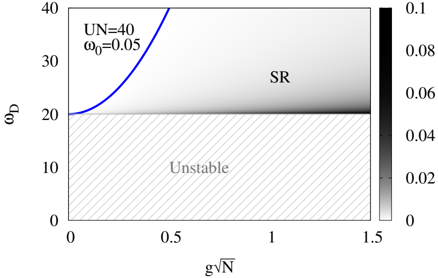
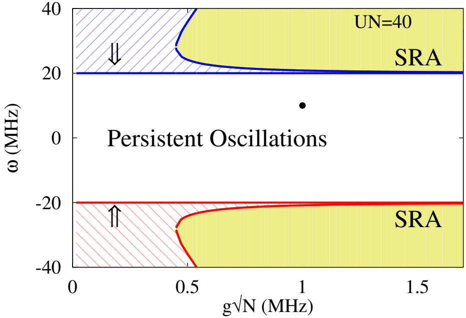
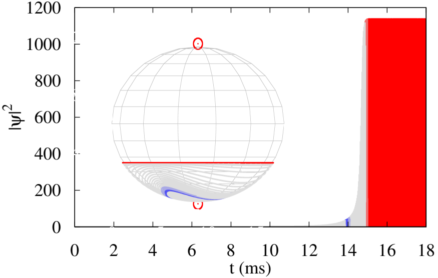


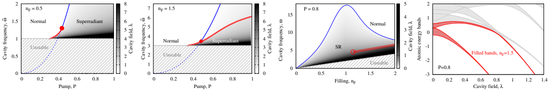




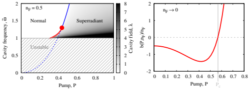


Summary

- Fermions self organisation, liquid gas, and multicritical points

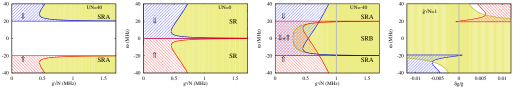


- First order transitions for bosons, outside Dicke model



JK, Bhassen, Simons *et al.* `arXiv:1308.????`

- Dicke model shows many dynamical phases



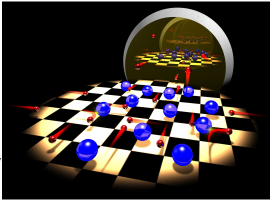
JK *et al.* PRL ’10, Bhaseen *et al.* PRA ’12

Many body quantum optics and correlated states of light

9:00 am on Monday 28 October 2013 – 5:00 pm on Tuesday 29 October 2013

at:The Royal Society at Chicheley Hall, home of the Kavli Royal Society International Centre, Buckinghamshire

Theo Murphy international scientific meeting organised by Dr Jonathan Keeling, Professor Steven Girvin, Dr Michael Hartmann and Professor Peter Littlewood FRS.



List of speakers and chairs

Professor Iacoppo Carusotto, Professor Andrew Cleland, Professor Hui Deng, Professor Tilman Esslinger, Professor Rosario Fazio, Professor Ed Hinds, Professor Andrew Houck, Professor Ataç İmamoğlu, Professor Jens Koch, Professor Misha Lukin, Professor Martin Plenio, Professor Arno Rauschenbeutel, Professor Timothy Spiller, Professor Jacob Taylor, Professor Hakan Tureci, Professor Andreas Wallraff

Attending this event

This is a residential conference which allows for increased discussion and networking. It is free to attend, however participants need to cover their accommodation and catering costs if required. Places are limited and therefore pre-registration is essential.

Classical dynamics of the extended Dicke model

Open dynamical system:

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+) + U S_z \psi^\dagger \psi.$$

$$\partial_t \rho = -i[H, \rho] - \kappa(\psi^\dagger\psi\rho - 2\psi\rho\psi^\dagger + \rho\psi^\dagger\psi)$$

- Neglects quantum fluctuations
- Linearisation about fixed point → stability, spectrum

[JK *et al.* PRL ’10, Bhaseen *et al.* PRA ’12]

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Classical EOM
($|\mathbf{S}| = N/2 \gg 1$)

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Fixed points (steady states)

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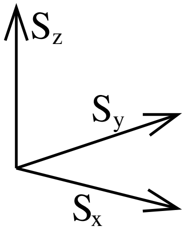

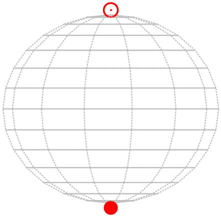

Small g: $\Uparrow, \Downarrow$ only.
($\omega = 30$MHz, $UN = -40$MHz)

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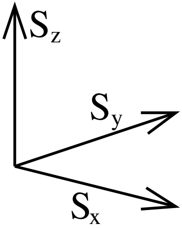

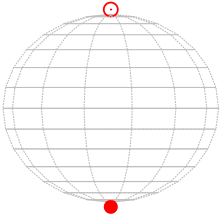

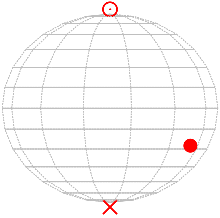

Small g: $\Uparrow, \Downarrow$ only. Larger g: SR too.
$(\omega = 30\text{MHz}, UN = -40\text{MHz})$

Ferroelectric transition

Atoms in Coulomb gauge

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Gauge transform to dipole gauge $\mathbf{D} \cdot \mathbf{r}$

$$H = \omega_0 S^z + \omega\psi^\dagger\psi + \bar{g}(S^+ - S^-)(\psi - \psi^\dagger)$$

"Dicke" transition at $\omega_0 < N\bar{g}^2/\omega \equiv 2\eta N$

But, ψ describes **electric displacement**

Grand canonical ensemble

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[Eastham and Littlewood, PRB '01]

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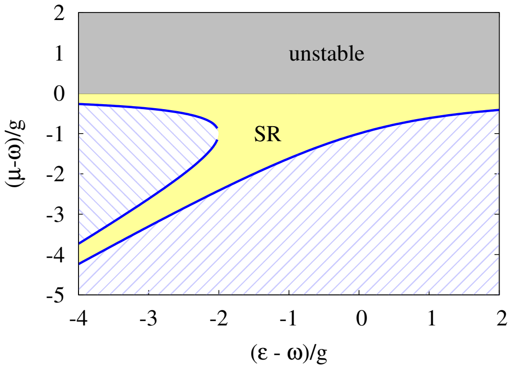

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