Non-equilibrium phases of coupled matter-light systems

Jonathan Keeling

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600 YEARS

SUPA

Los Alamos, June 2013

Coupling many atoms to light

Old question: *What happens to radiation when many atoms interact "collectively" with light.*

Superradiance — dynamical and steady state.

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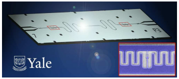
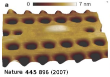
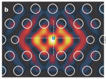
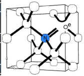
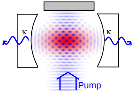
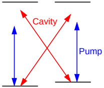
New relevance

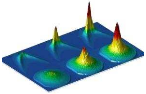
- Superconducting qubits 

- Quantum dots & NV centres 




- Ultra-cold atoms 



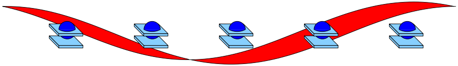
- Rydberg atoms/polaritons
- Microcavity Polaritons 

Dicke effect: Enhanced emission

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

R. H. DICKE
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey



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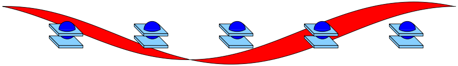

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Collective decay:

$$\frac{d\rho}{dt} = -\frac{\Gamma}{2}\left[S^+S^-\rho - S^-\rho S^+ + \rho S^+S^- \right]$$

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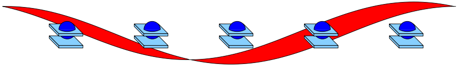

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If $S^z = |S| = N/2$ initially:

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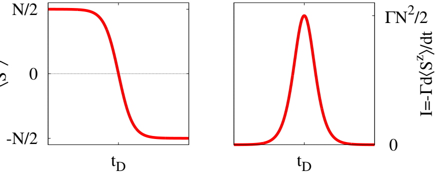

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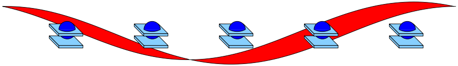

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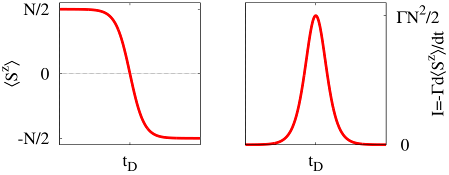
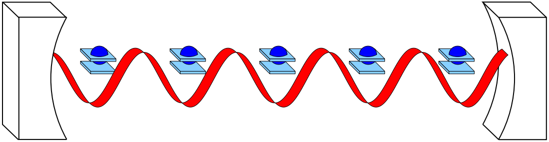


Problem: dipole interactions dephase. [Friedberg et al, Phys. Lett. 1972]

Collective radiation **with a cavity**: Dynamics



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Single cavity mode: oscillations

[Bonifacio and Preparata PRA '70]

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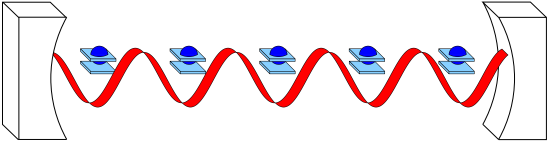

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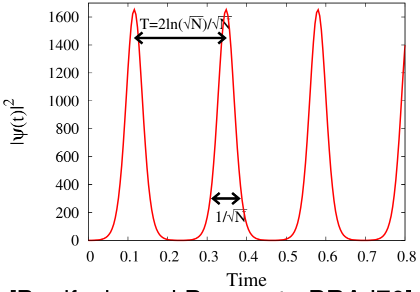

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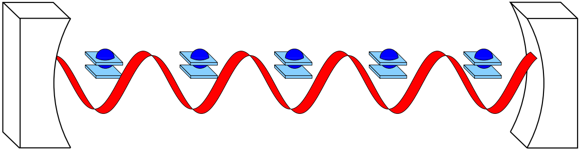
Dicke model: Equilibrium superradiance transition



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- Coherent state: $|\Psi\rangle \to e^{\lambda\psi^\dagger + \eta S^+}|\Omega\rangle$
- Small g, min at $\lambda, \eta = 0$

[Hepp, Lieb, Ann. Phys. '73]

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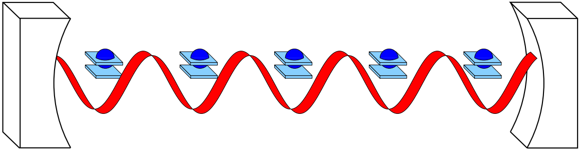

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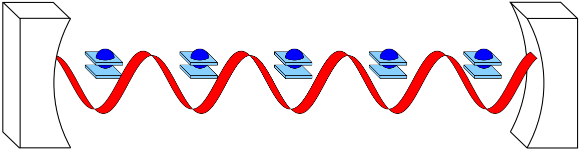

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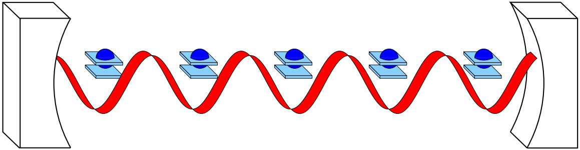

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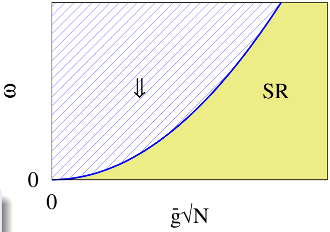

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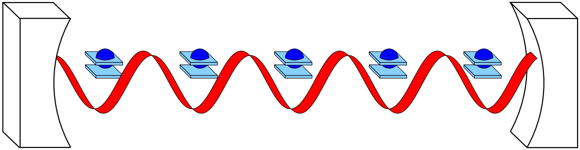
No go theorem and transition



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[Rzazewski *et al* PRL '75]

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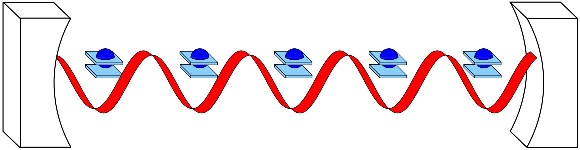

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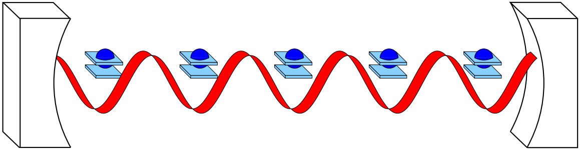

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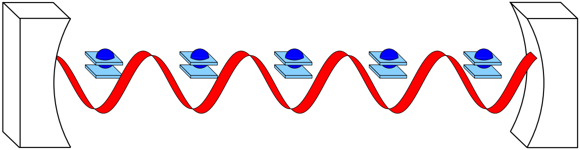

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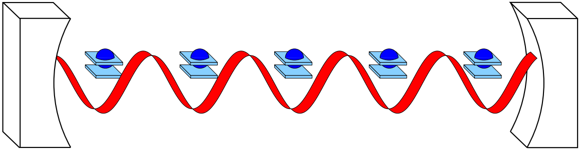

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But Thomas-Reiche-Kuhn sum rule states: $g^2/\omega_0 < 2\zeta$. **No transition**
[Rzazewski *et al* PRL '75]

Dicke phase transition: ways out

Problem: $g^2/\omega_0 < 2\zeta$ for intrinsic parameters. **Solutions**:

- **Interpretation**
  Ferroelectric transition in $\mathbf{D}\cdot\mathbf{r}$ gauge.
  [JK JPCM '07, Vukics & Domokos PRA 2012]
- Circuit QED [Nataf and Ciuti, Nat. Comm. '10; Viehmann *et al.* PRL '11]
- Grand canonical ensemble:
  - If $H \to H - \mu(S^z + \psi^\dagger\psi)$, need only: $g^2 N > (\omega - \mu)(\omega_0 - \mu)$
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  e.g. Raman scheme: $\omega_0 \ll \omega$.
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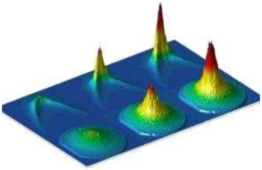

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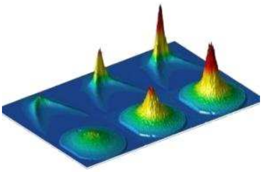

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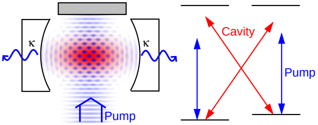

Outline

Acknowledgements

GROUP:

COLLABORATORS: Simons, Bhaseen, Schmidt, Blatter, Türeci, Krüger
EXPERIMENT: Houck, Wallraff, Fink, Mylnek

FUNDING:

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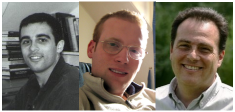
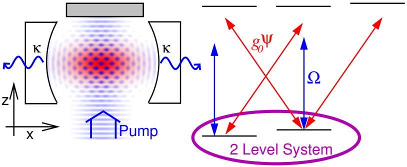
Nuffield Foundation

Dynamics of generalized Dicke model



Cold-atom extended Dicke model



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[Baumann *et al* Nature '10]

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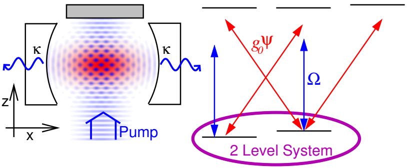

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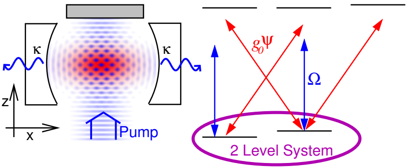

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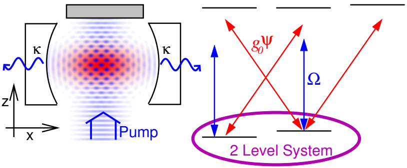

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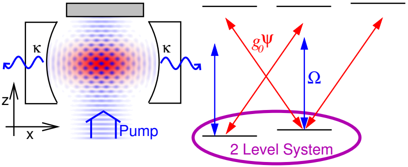
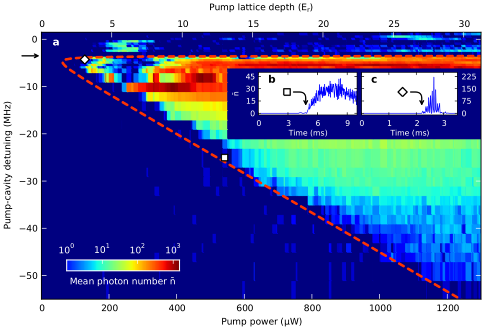
$\omega_0 \sim$ kHz $\ll \omega, \kappa, g\sqrt{N} \sim$ MHz.

[Baumann *et al* Nature '10]

Cold-atom extended Dicke model





2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$:

$\Downarrow$: $\Psi(x,z) = 1$

$\Uparrow$: $\Psi(x,z) = \sum_{\sigma,\sigma'=\pm} e^{ik(\sigma x + \sigma' z)}$

Feedback: $U \propto \frac{g_0^2}{\omega_c - \omega_a}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+) + U S_z \psi^\dagger\psi.$$

$$\partial_t \rho = -i[H,\rho] - \kappa(\psi^\dagger\psi\rho - 2\psi\rho\psi^\dagger + \rho\psi^\dagger\psi)$$

$\omega_0 \sim \text{kHz} \ll \omega, \kappa, g\sqrt{N} \sim \text{MHz}.$

[Baumann *et al* Nature '10]

Classical dynamics of the extended Dicke model

Open dynamical system:

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- Neglects quantum fluctuations — restore via Wigner distributed initial conditions.
- Linearisation about fixed point:
  - Recover Retarded Green's function (spectrum)
  - Cannot recover occupations

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Classical EOM
($|\mathbf{S}| = N/2 \gg 1$)

$$\dot{S}^- = -i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
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- $\psi = 0, \mathbf{S} = (0, 0, \pm N/2)$ always a solution.
- If $g > g_c$, $\psi \neq 0$ too
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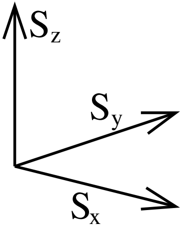

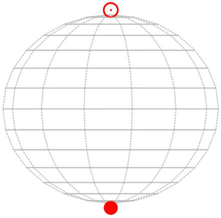

Small g: $\Uparrow, \Downarrow$ only.
($\omega = 30$MHz, $UN = -40$MHz)

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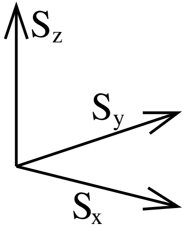

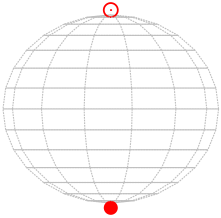

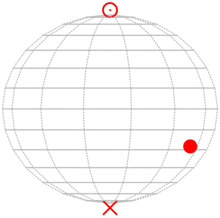

Small g: $\Uparrow, \Downarrow$ only. Larger g: SR too.

$(\omega = 30\text{MHz}, UN = -40\text{MHz})$

Steady state phase diagram

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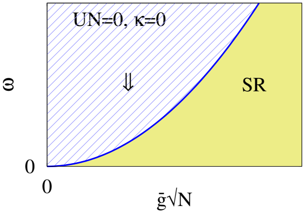

See also Domokos and Ritsch PRL '02, Domokos *et al.* PRL '10

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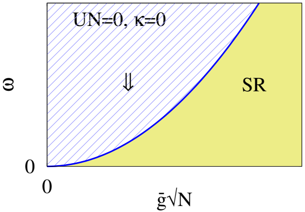

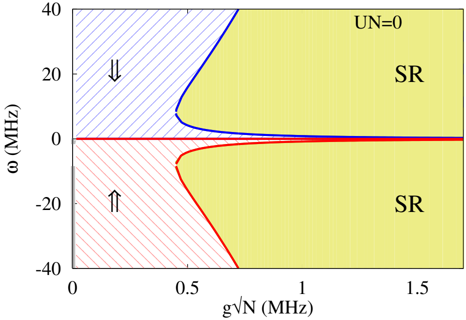

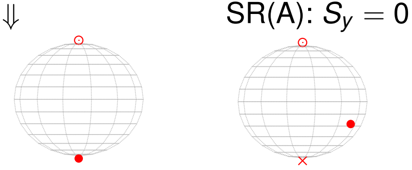

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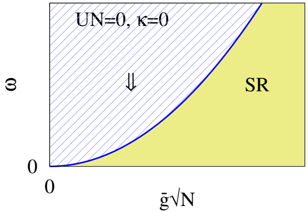

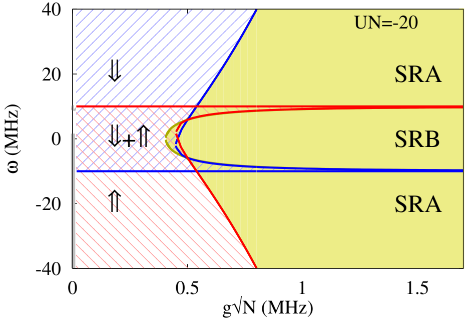

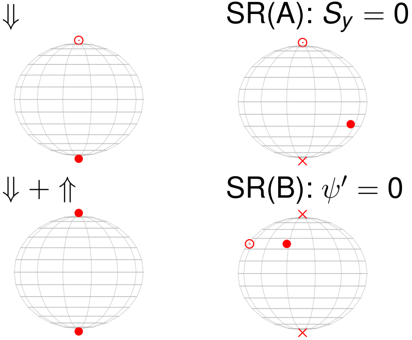

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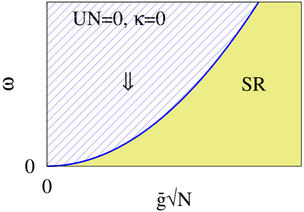

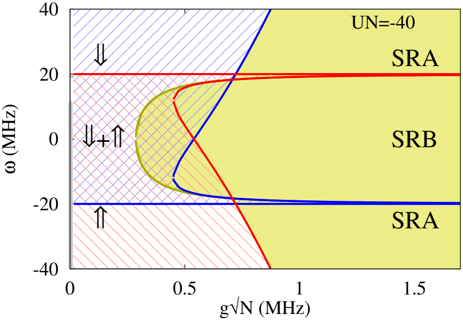

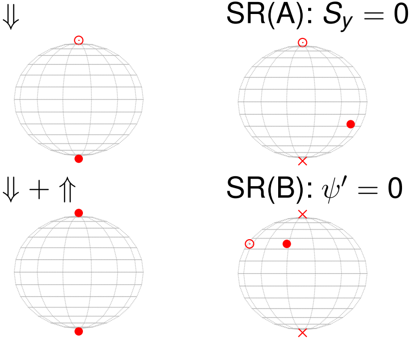

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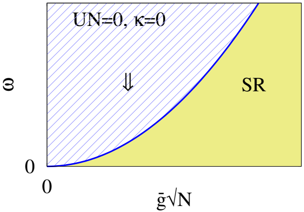

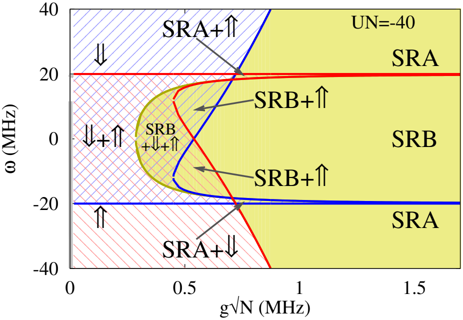

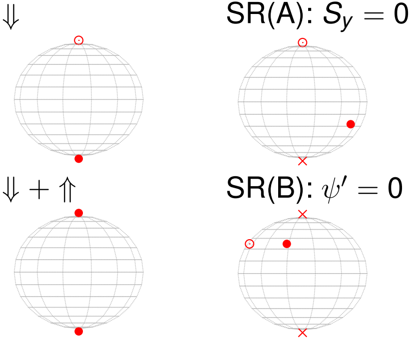

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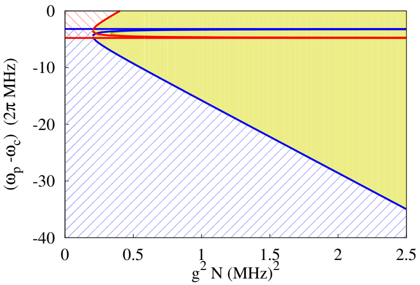
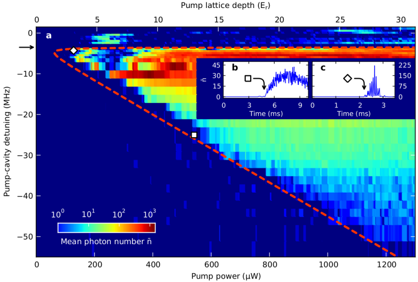
Comparison to experiment





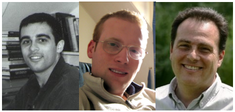
$$UN = -10\text{MHz}$$

Adapted from: [Bhaseen *et al.* PRA '12]

[Baumann *et al* Nature '10]

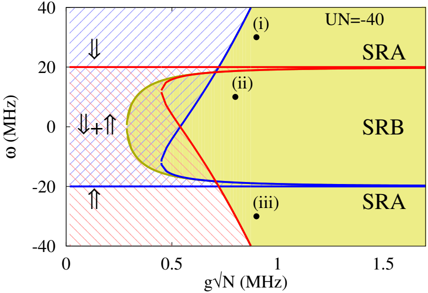
$$\omega = \omega_c - \omega_p + \frac{5}{2} UN, \qquad UN = -\frac{g_0^2}{4(\omega_a - \omega_c)}$$

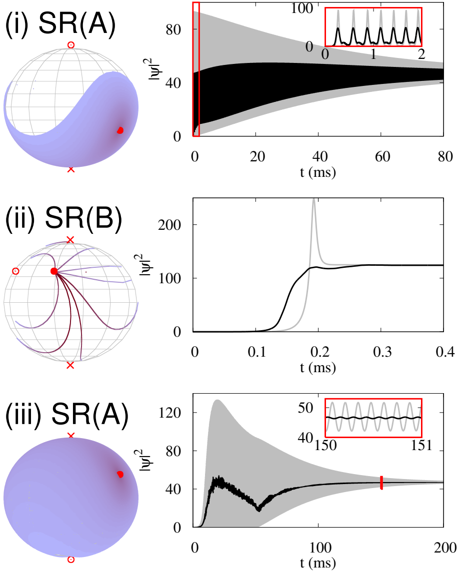
Dynamics of generalized Dicke model



Dynamics: Evolution from normal state

Gray: $\mathbf{S} = (\sqrt{N}, \sqrt{N}, -N/2)$
Black: Wigner distribution of $\mathbf{S}$, ψ



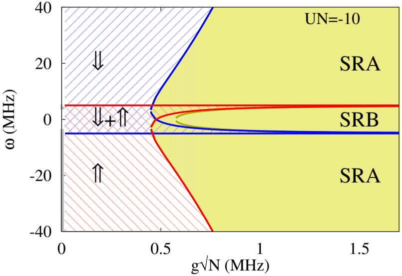


Oscillations: $\sim$ 0.1ms
Decay: 20ms, 0.1ms, 20ms

Asymptotic state: Evolution from normal state

(Near to experimental $UN = -13$MHz).

All stable attractors:



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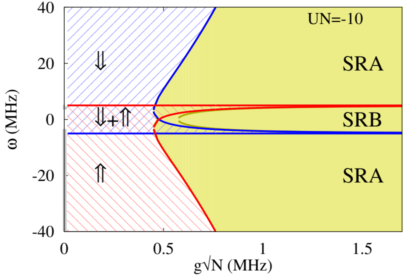
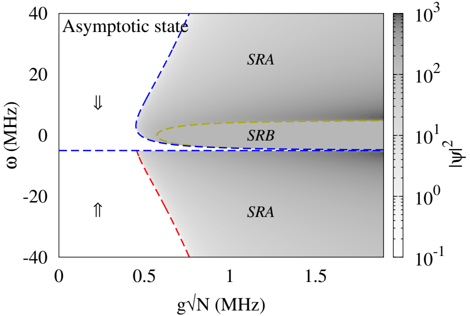
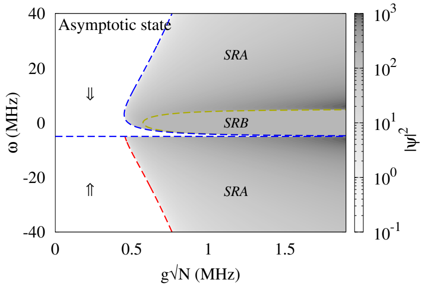


Starting from ⇓



Timescales for dynamics: Consequences for experiment



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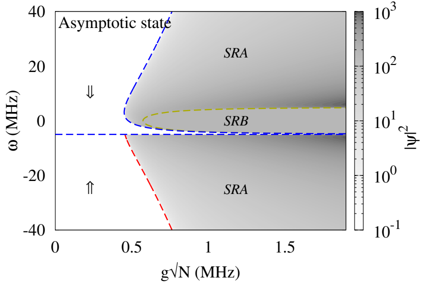

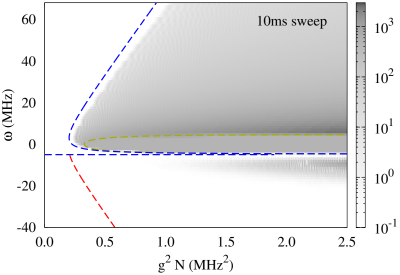

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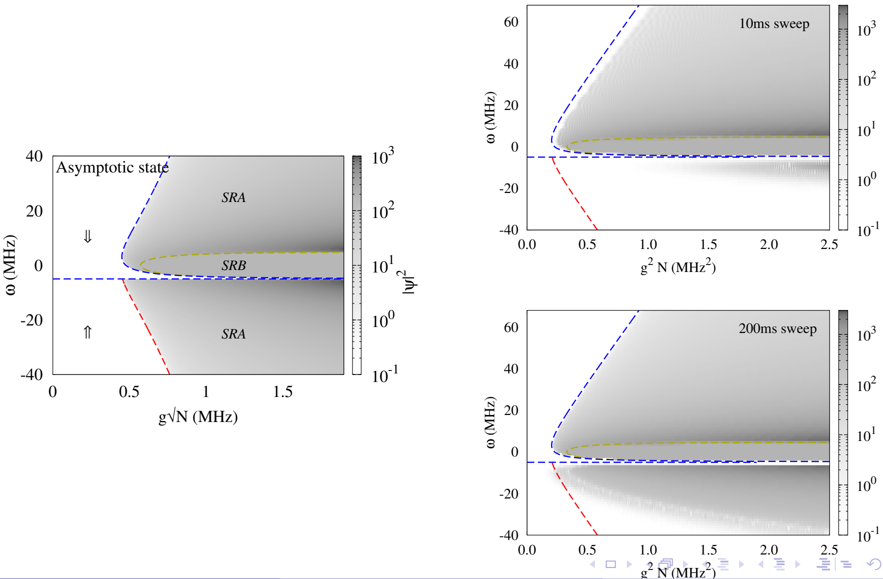
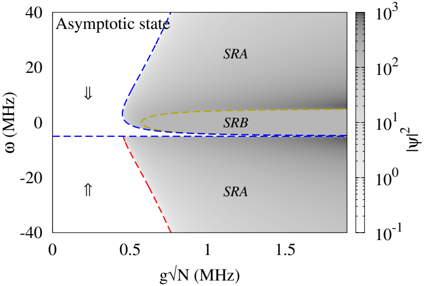
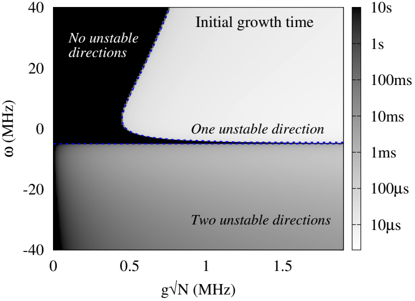


Timescales for dynamics: What are they?





Growth Most unstable eigenvalues near $\mathbf{S} = (0, 0, -N/2)$

Decay Slowest stable eigenvalues near final state

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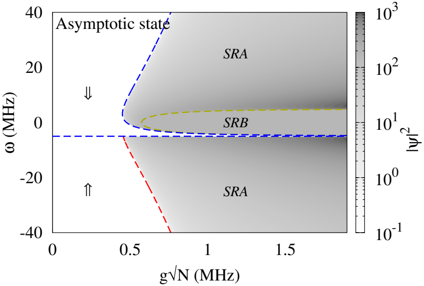

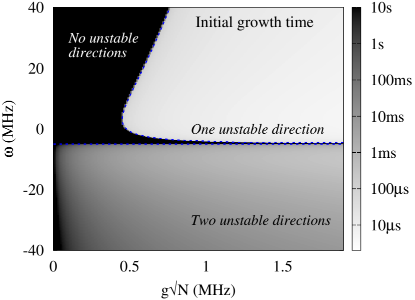

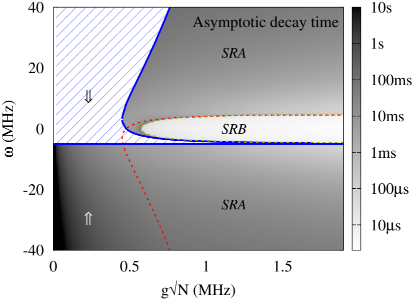

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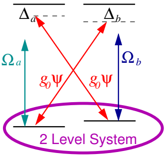
Timescales for dynamics: Why so slow and varied?

Suppose co- and counter-rotating terms differ



$$H = \ldots + g(\psi^\dagger S^- + \psi S^+) + g'(\psi^\dagger S^+ + \psi S^-) + \ldots$$

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- SR(A), SR(B) continuously connect

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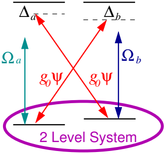

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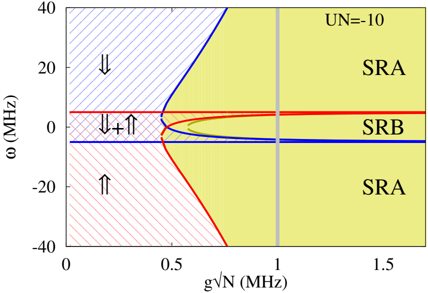

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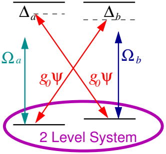

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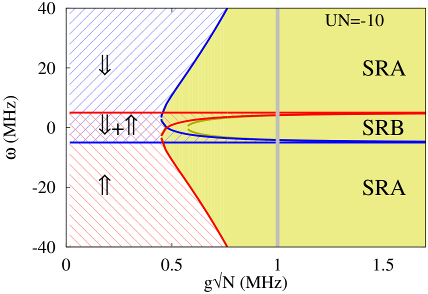

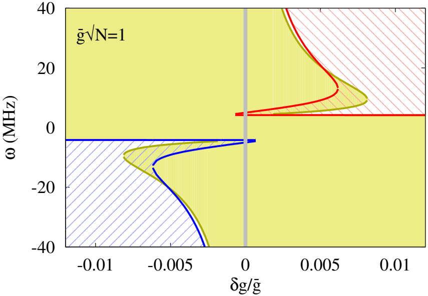

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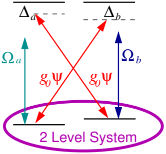

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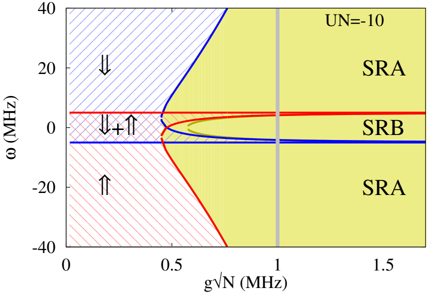

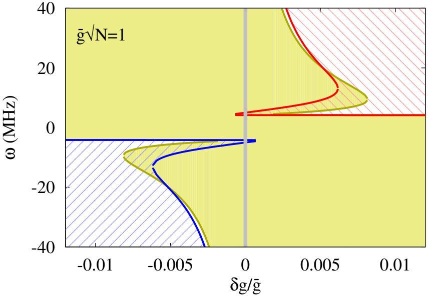

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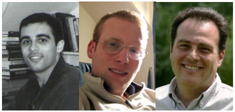
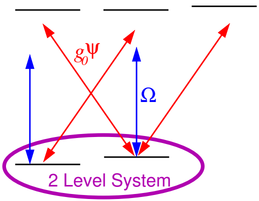
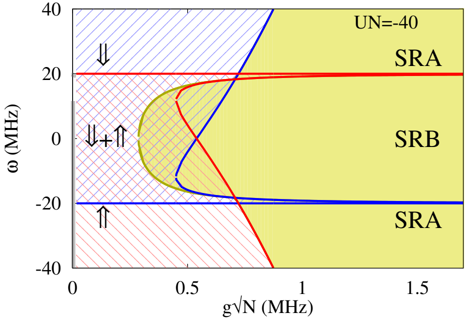


Regions without fixed points

Changing U:



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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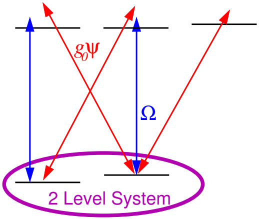

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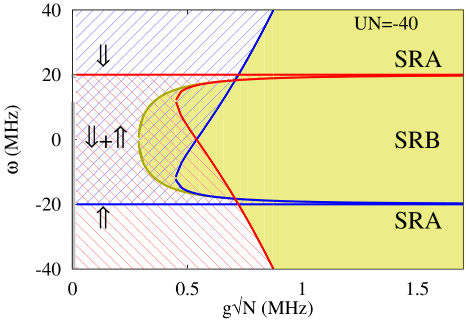

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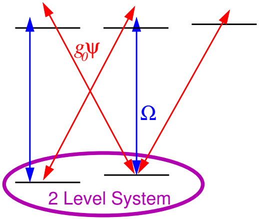

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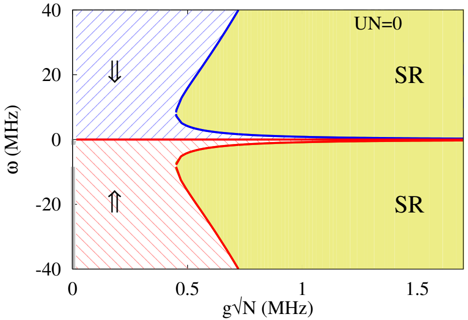

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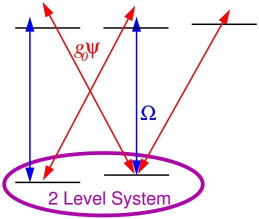

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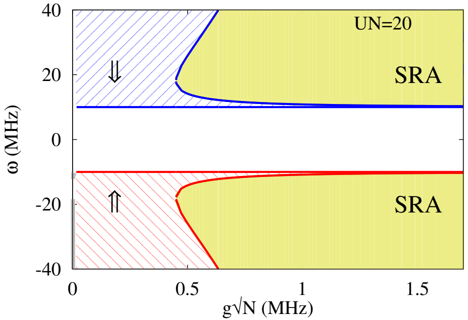
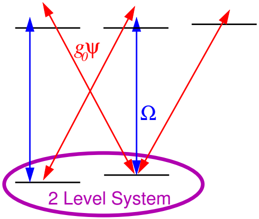


Regions without fixed points

Changing *U*:



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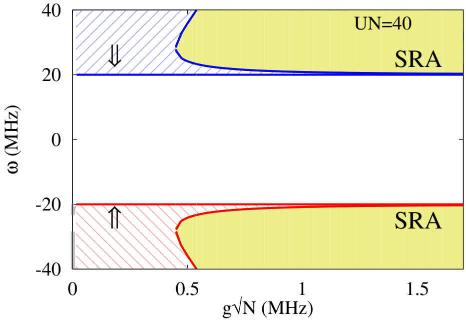

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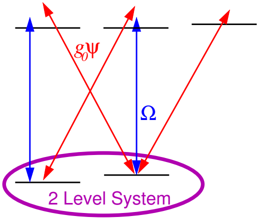

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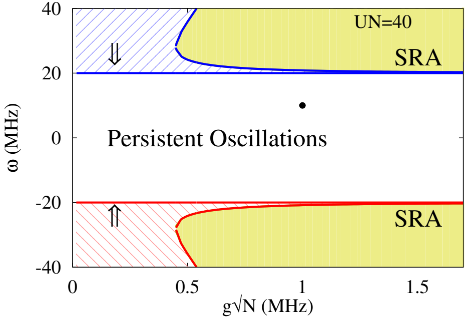
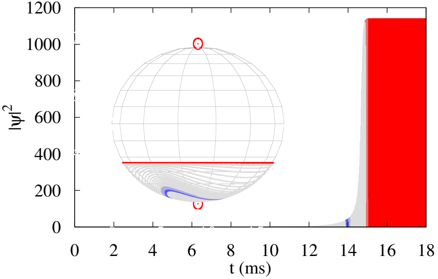
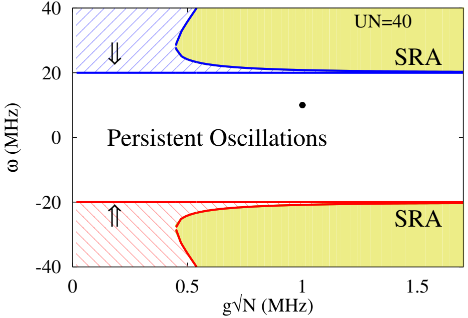
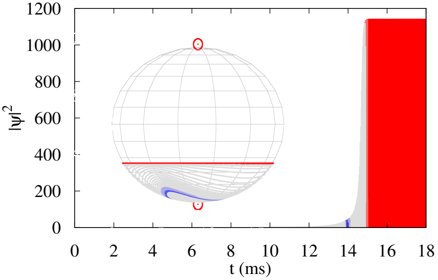
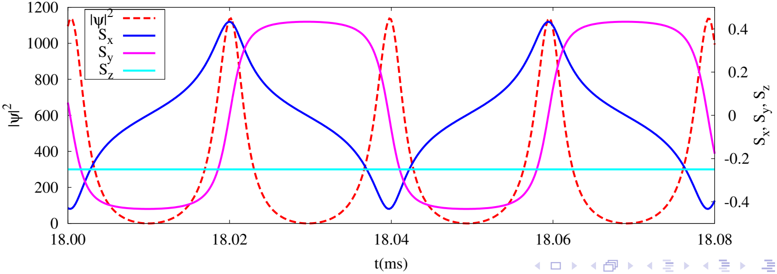




Persistent (optomechanical) oscillations





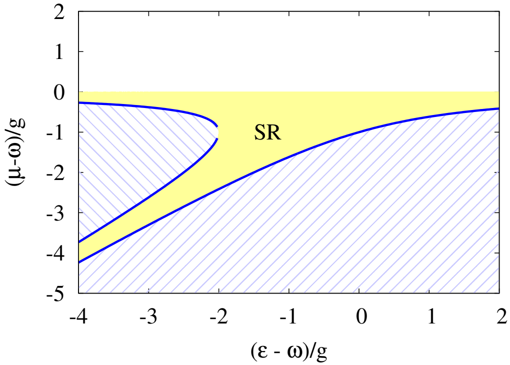


Jaynes Cummings Hubbard model




Equilibrium: Dicke model with chemical potential

$$H - \mu N = (\omega - \mu)\psi^\dagger\psi + (\omega_0 - \mu)S^z + g\left(\psi^\dagger S^- + \psi S^+\right)$$



[Eastham and Littlewood, PRB '01]

- Transition at: $g^2 N > (\omega - \mu)|\omega_0 - \mu|$
- Reduce critical g
- Unstable if $\mu > \omega$
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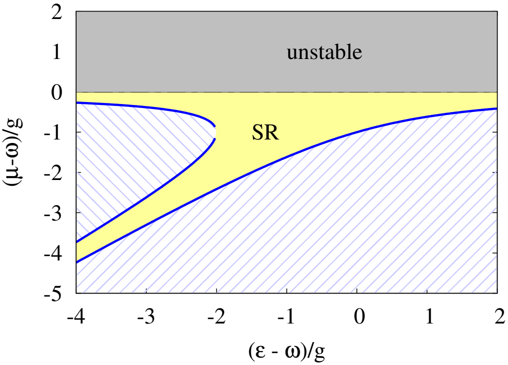

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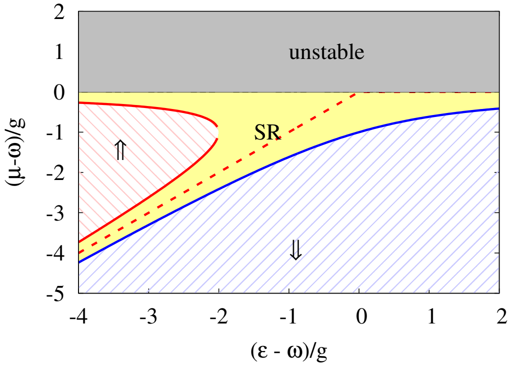

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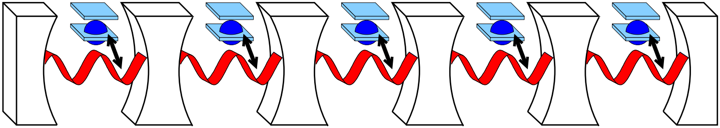
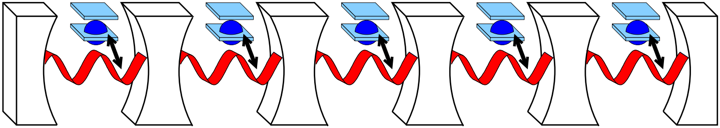
Jaynes-Cummings Hubbard model



$$H = -\frac{J}{z}\sum_{ij}\psi_i^\dagger\psi_j + \sum_i \frac{\Delta}{2}\sigma_i^z + g(\psi_i^\dagger\sigma_i^- + \text{H.c.})$$

Jaynes-Cummings Hubbard model



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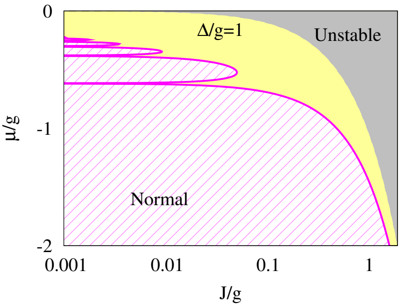

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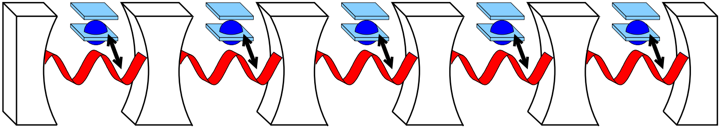

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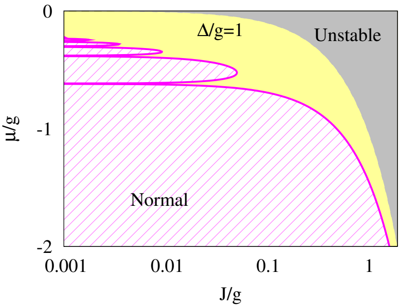
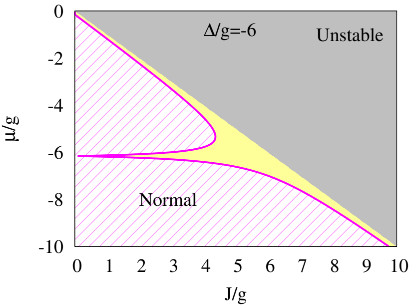
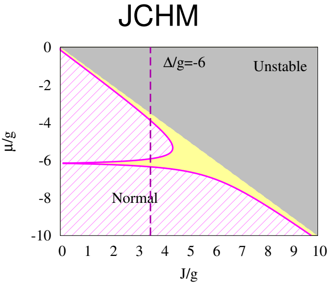
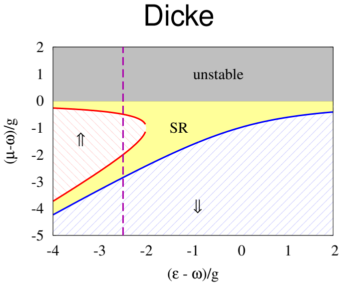




Dicke vs JCHM

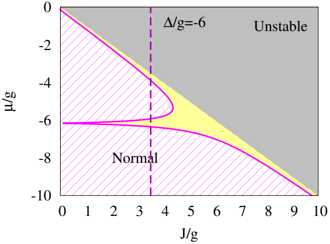




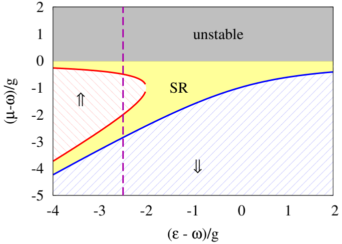
[Schmidt, Blatter, JK `arXiv:1306.????`]

Dicke vs JCHM

JCHM



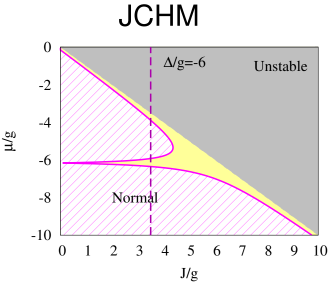
Dicke

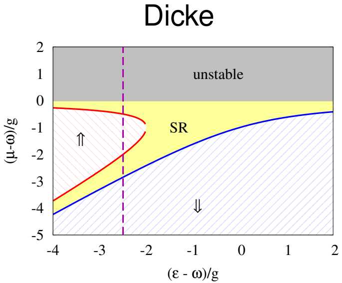


- $k = 0$ mode of JCHM $\leftrightarrow$ Dicke photon mode

[Schmidt, Blatter, JK `arXiv:1306.????`]

Dicke vs JCHM





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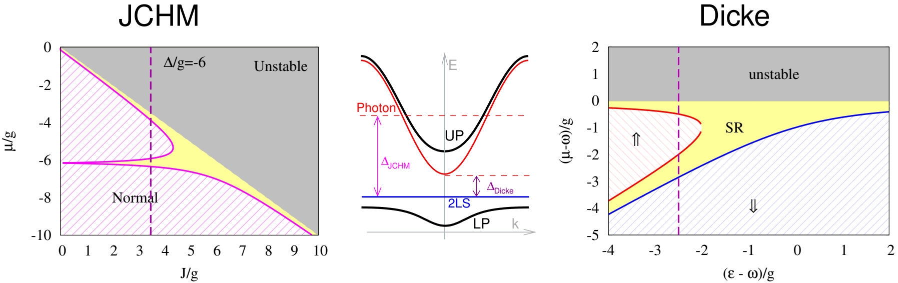

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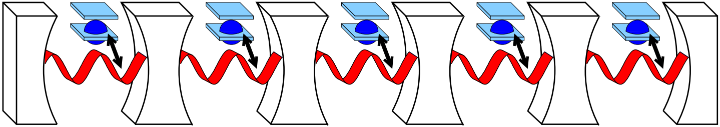
Jaynes Cummings Hubbard model

1. Introduction: Dicke model and superradiance
2. Dynamics of generalized Dicke model
   - Summary of experiment and classical dynamcs
   - Fixed points and dynamical phases
   - Timescales and consequences for experiment
   - Persistent oscillating phases
3. Jaynes Cummings Hubbard model
   - JCHM vv Dicke
   - Coherently driven array
   - Disorder

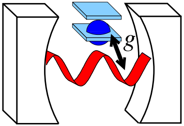
Coherently pumped JCHM



$$H = -\frac{J}{z}\sum_{ij}\psi_i^\dagger\psi_j + \sum_i \frac{\Delta}{2}\sigma_i^z + g(\psi_i^\dagger\sigma_i^- + \text{H.c.}) + f(\psi_i e^{i\omega_L t} + \text{H.c.})$$

$$\partial_t\rho = -i[H,\rho] - \frac{\kappa}{2}L_\psi[\rho] - \frac{\gamma}{2}L_{\sigma^-}[\rho]$$

Coherently pumped single cavity [Bishop *et al.* Nat. Phys '09]



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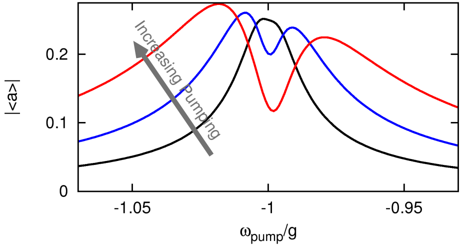
- Effective 2LS: $|\text{Empty}\rangle, |1\text{ polariton}\rangle$

- Mollow triplet fluorescence



[Lang *et al.* PRL '11]

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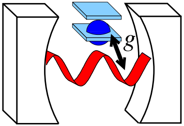

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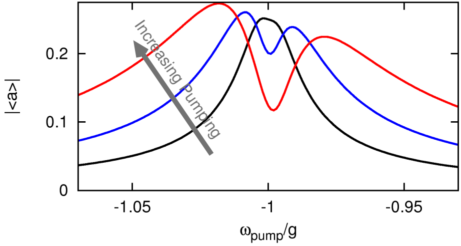

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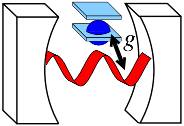

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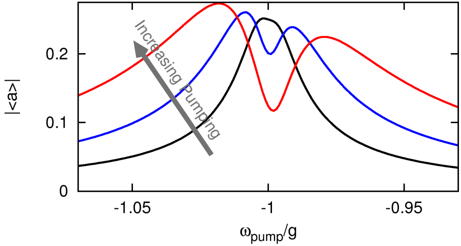

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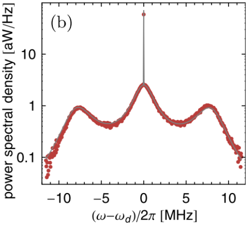

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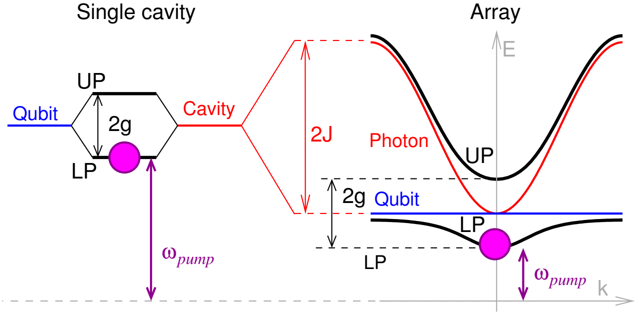
Coherently pumped dimer & array

Chose detuning *a la* Dicke model



- Bistability at intermediate J
  - More/less localised states
  - Connects to Dicke limit

[Nissen *et al.* PRL '12]

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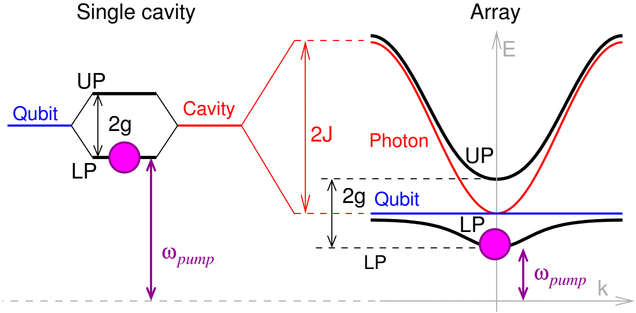

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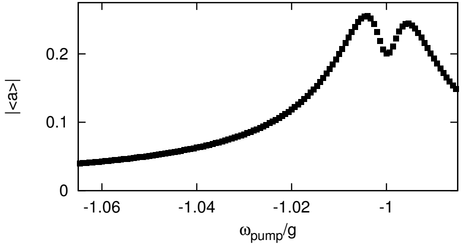

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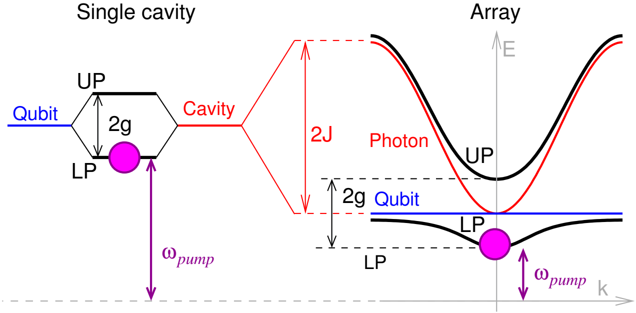

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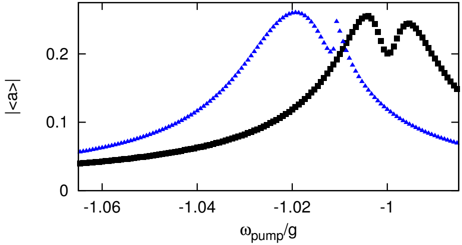

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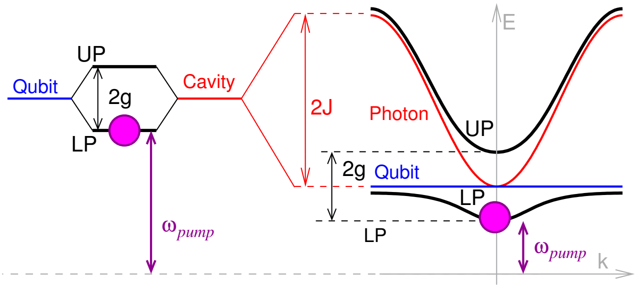

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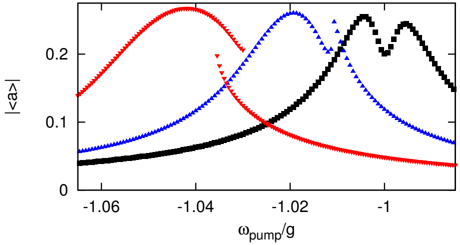

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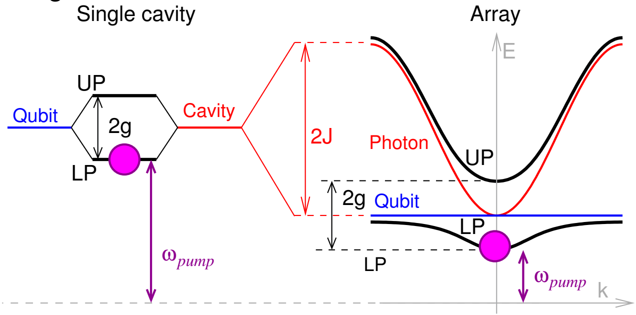

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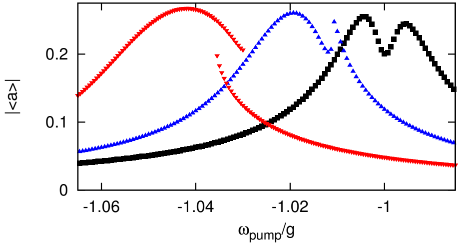

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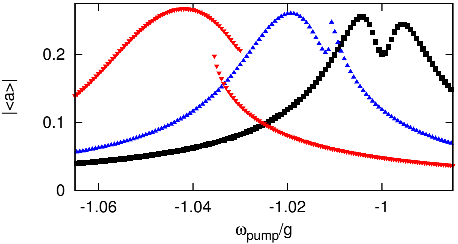
[Nissen *et al.* PRL '12]

Photon blockade picture $J \lesssim g$

- Polariton basis
- Nonlinearity $|\epsilon_2 - 2\epsilon_1| \propto g$.

$$H = \sum_i \left(\frac{\epsilon}{2} \tau_i^z + \tilde{f} \tau_i^x \right)$$



[Nissen *et al.* PRL '12]

- Decouple hopping: $\tau_i^+ \tau_j^- \to \psi \tau^+ + \psi^* \tau^-$
- Bistability for

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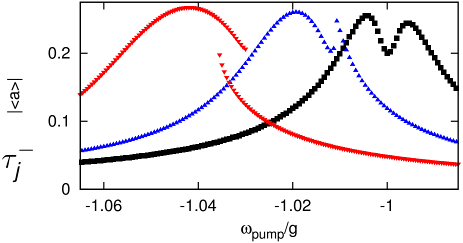

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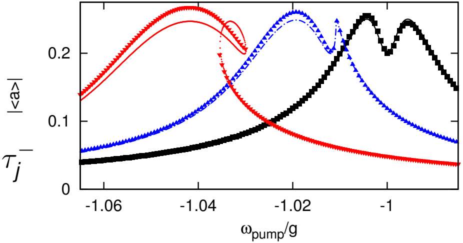


[Nissen *et al.* PRL '12]

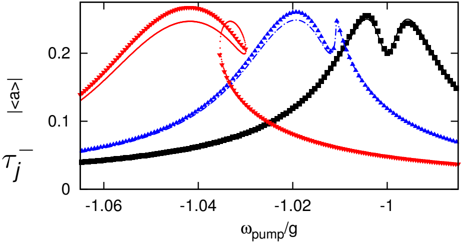
- Decouple hopping:
  $\tau_i^+ \tau_j^- \to \psi \tau^+ + \psi^* \tau^-$
- Bistability for

$$J > J_c = \frac{4}{\tilde{f}^2} \left(\frac{2\tilde{f}^2 + (\kappa/2)^2}{3} \right)^{3/2}$$

Photon blockade picture $J \lesssim g$

- Polariton basis
- Nonlinearity $|\epsilon_2 - 2\epsilon_1| \propto g$.

$$H = \sum_i \left(\frac{\epsilon}{2}\tau_i^z + \tilde{f}\tau_i^x \right) - \frac{\tilde{J}}{z} \sum_{\langle ij \rangle} \tau_i^+ \tau_j^-$$



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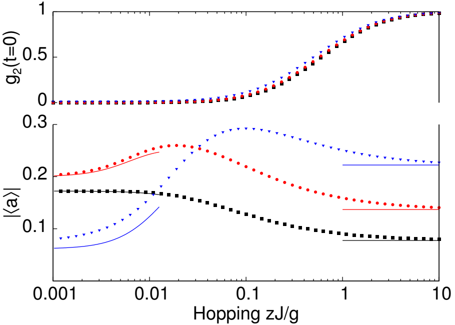
Coherently pumped array: correlations & fluorescence



Correlations

- $g_2 : 0 \rightarrow 1$ crossover.

- Small J: Mollow triplet
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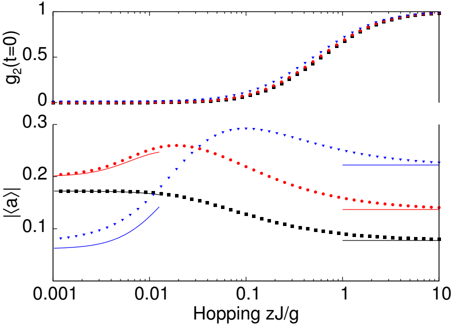

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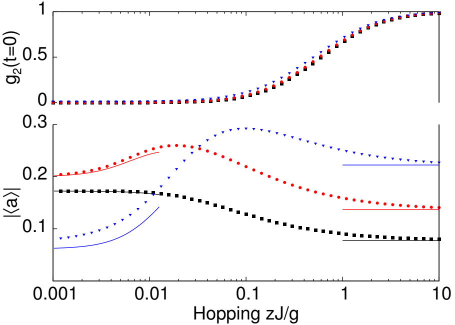

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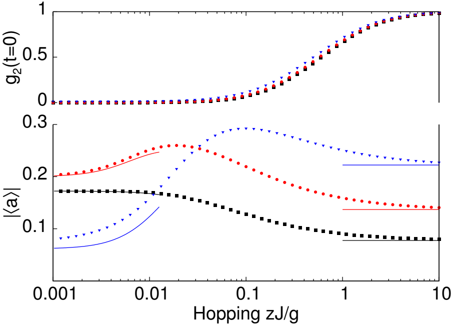

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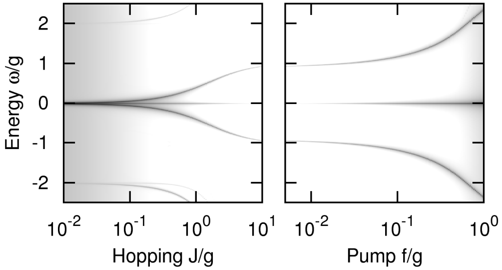

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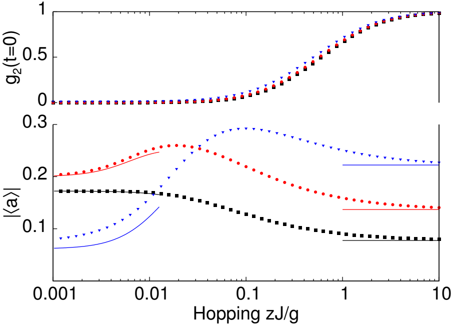

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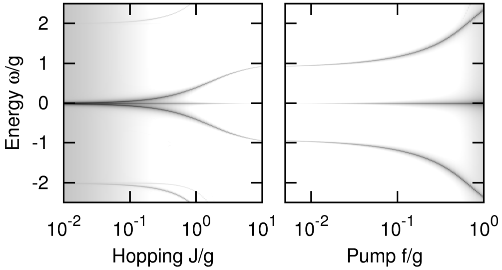

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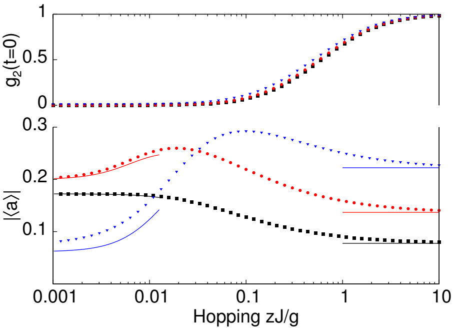

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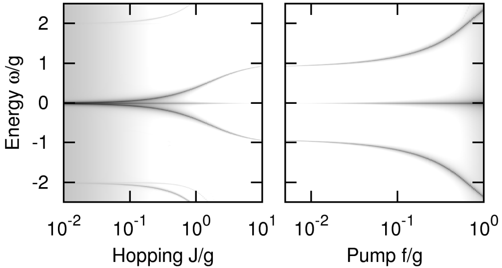

Coherent pumped array – disorder

- Effect of disorder, $\Delta \to \Delta_i$
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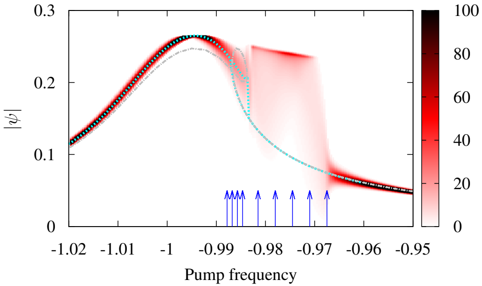

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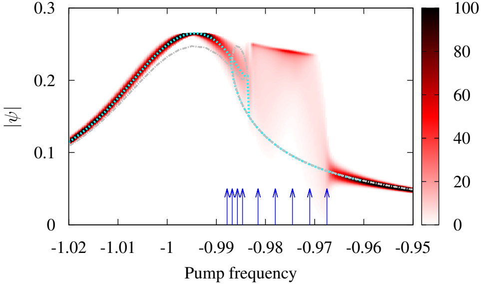

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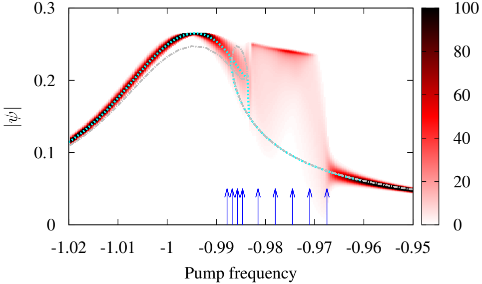

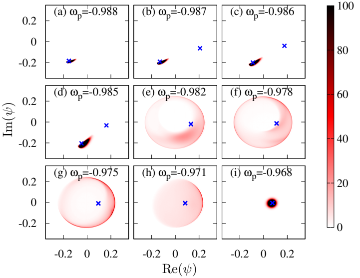

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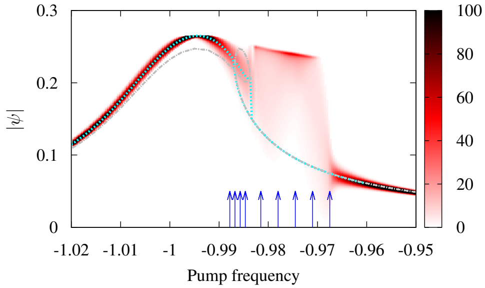
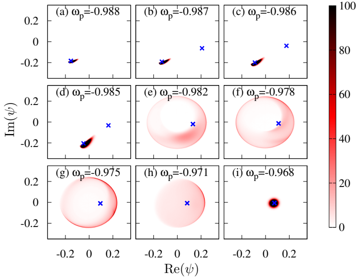
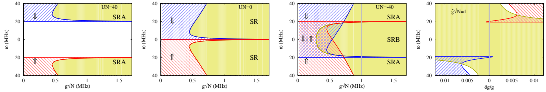




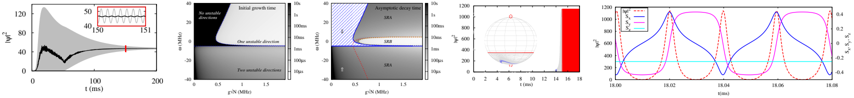
[Kulaitis *et al.* PRA, '13]

Summary

- Wide variety of dynamical phases

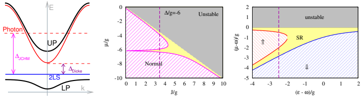


- Slow dynamics for $U < 0$ & Persistent oscillations for $U > 0$

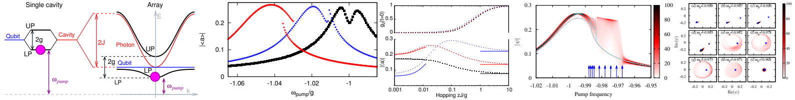


JK *et al.* PRL '10, Bhaseen *et al.* PRA '12

- Dicke model and JCHM: connection at $J \to \infty$



- Coherently pumped coupled cavity array



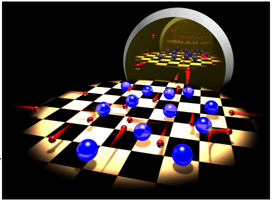
Nissen *et al.* PRL '12, Kulaitis *et al.* PRA '13

Many body quantum optics and correlated states of light

9:00 am on Monday 28 October 2013 – 5:00 pm on Tuesday 29 October 2013

at:The Royal Society at Chicheley Hall, home of the Kavli Royal Society International Centre, Buckinghamshire

Theo Murphy international scientific meeting organised by Dr Jonathan Keeling, Professor Steven Girvin, Dr Michael Hartmann and Professor Peter Littlewood FRS.



List of speakers and chairs

Professor Iacoppo Carusotto, Professor Andrew Cleland, Professor Hui Deng, Professor Tilman Esslinger, Professor Rosario Fazio, Professor Ed Hinds, Professor Andrew Houck, Professor Ataç İmamoğlu, Professor Jens Koch, Professor Misha Lukin, Professor Martin Plenio, Professor Arno Rauschenbeutel, Professor Timothy Spiller, Professor Jacob Taylor, Professor Hakan Tureci, Professor Andreas Wallraff

Attending this event

This is a residential conference which allows for increased discussion and networking. It is free to attend, however participants need to cover their accommodation and catering costs if required. Places are limited and therefore pre-registration is essential.

Ferroelectric transition

Atoms in Coulomb gauge

$$H = \sum \omega_k a_k^\dagger a_k + \sum_i [p_i - eA(r_i)]^2 + V_{\text{coul}}$$

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$$H = \omega_0 S^z + \omega \psi^\dagger \psi + g(S^+ + S^-)(\psi + \psi^\dagger) + N\zeta(\psi + \psi^\dagger)^2 - \eta(S^+ - S^-)^2$$

(nb $g^2, \zeta, \eta \propto 1/V$).

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Gauge transform to dipole gauge $\mathbf{D} \cdot \mathbf{r}$

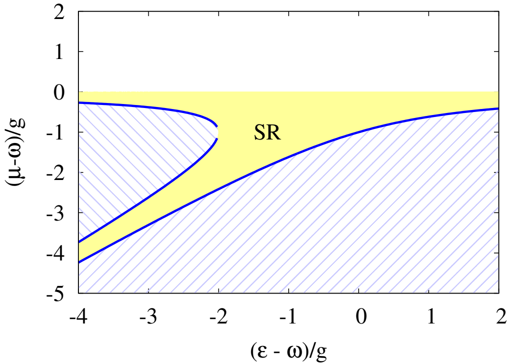
$$H = \omega_0 S^z + \omega\psi^\dagger\psi + \bar{g}(S^+ - S^-)(\psi - \psi^\dagger)$$

"Dicke" transition at $\omega_0 < N\bar{g}^2/\omega \equiv 2\eta N$

But, ψ describes **electric displacement**

Equilibrium: Dicke model with chemical potential

$$H - \mu N = (\omega - \mu)\psi^\dagger\psi + (\omega_0 - \mu)S^z + g\left(\psi^\dagger S^- + \psi S^+\right)$$



[Eastham and Littlewood, PRB '01]

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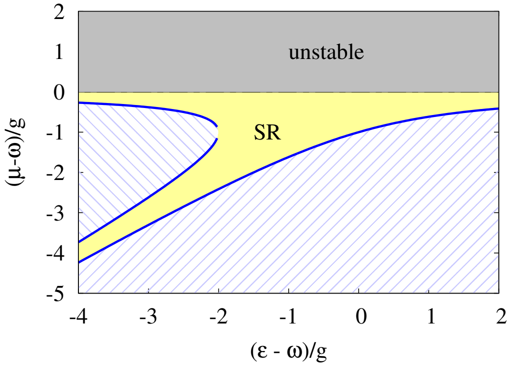

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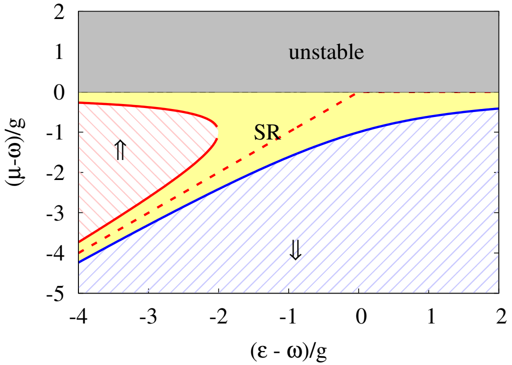

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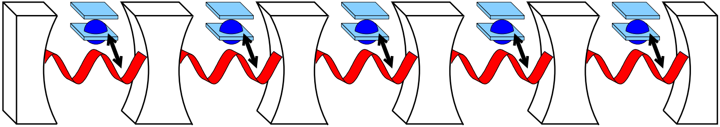

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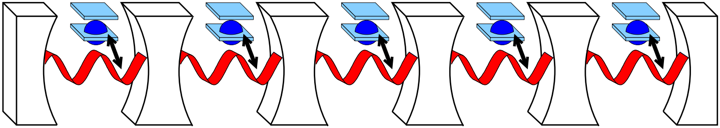

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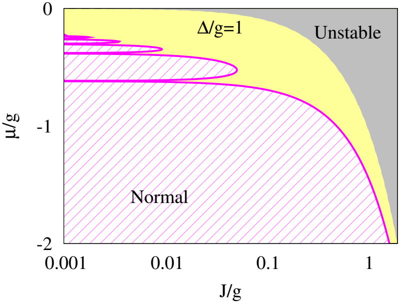

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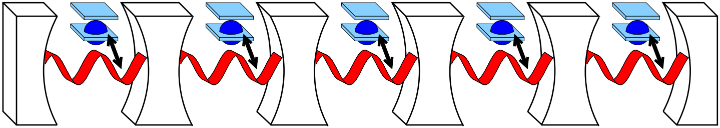

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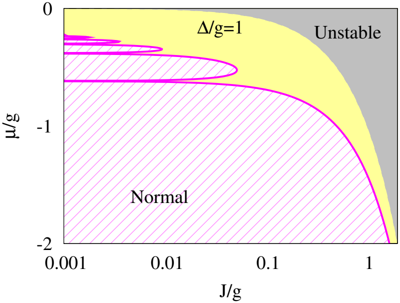
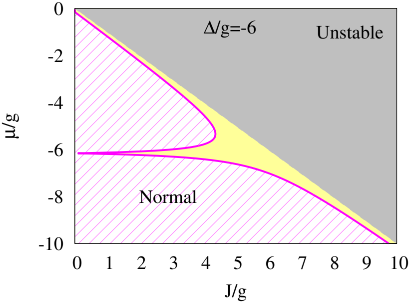
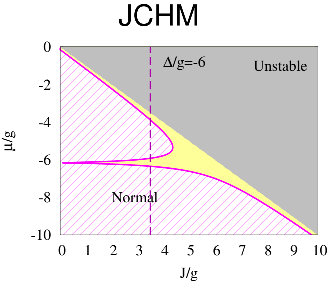
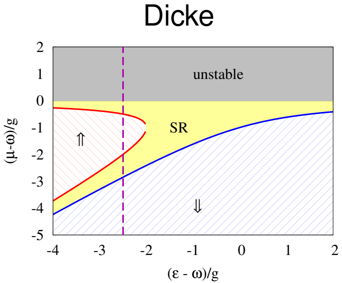




Dicke vs JCHM





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[Schmidt, Blatter, JK `arXiv:1306.????`]

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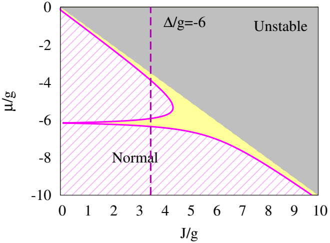

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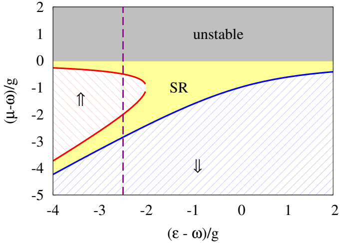

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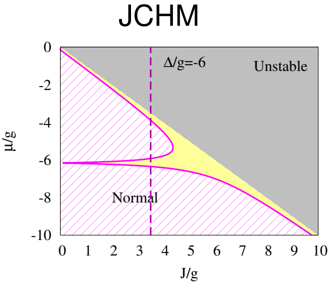

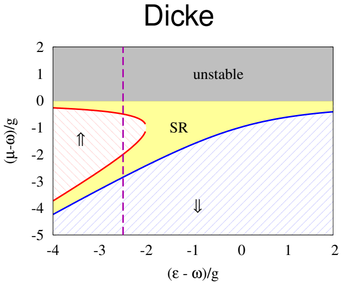

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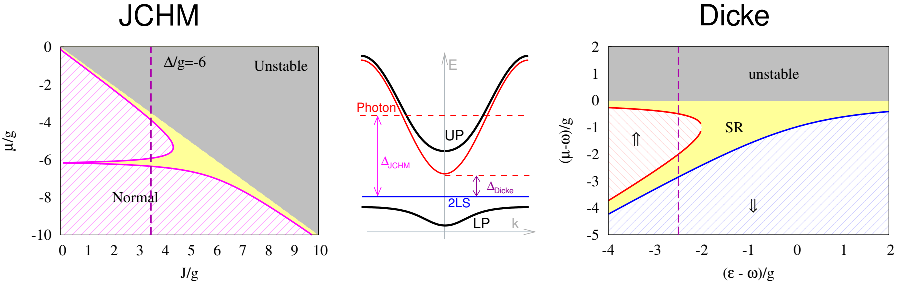

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  - Atom proposal [Dimer *et al.* PRA '07]
  - Atom experiment [Baumann *et al.* Nature '10]
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JK, Türeci, Houck in progress

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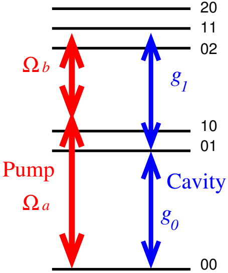

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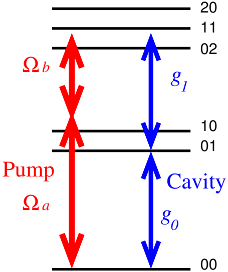

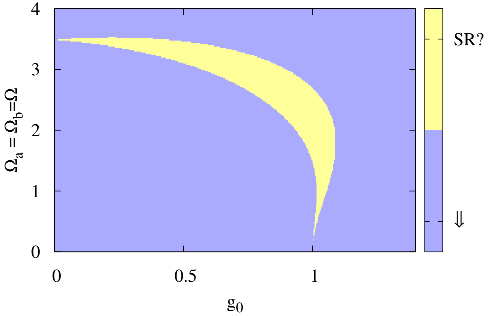

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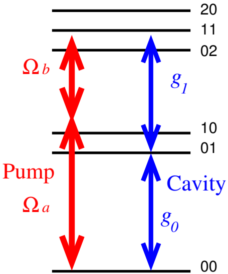

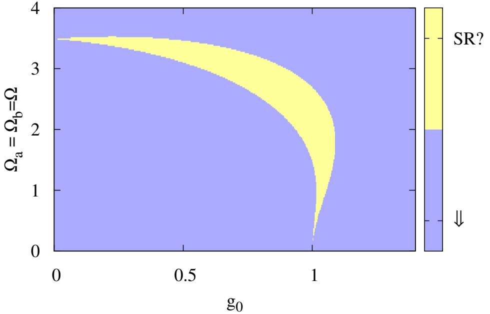

JK, Türeci, Houck in progress

Collective dephasing

- Real environment is not Markovian
  - [Carmichael & Walls JPA '73] Requirements for correct equilibrium
  - [Ciuti & Carusotto PRA '09] Dicke SR and emission
- Cannot assume fixed κ, γ
- Phase transition $\rightarrow$ soft modes
- Strong coupling $\rightarrow$ varying decay

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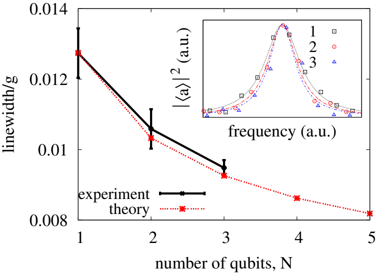

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