Non-equilibrium phases of coupled matter-light systems

Jonathan Keeling

University of St Andrews 600 YEARS

SUPA

Southhampton, May 2013

Coupling many atoms to light

Old question: *What happens to radiation when many atoms interact "collectively" with light.*

Superradiance — dynamical and steady state.

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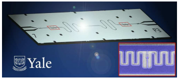
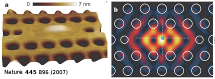
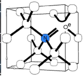
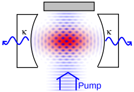
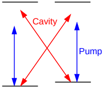
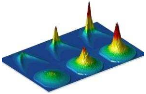
New relevance

- Superconducting qubits 

- Quantum dots & NV centres 


- Ultra-cold atoms 



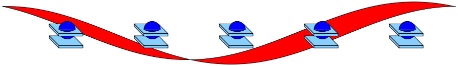
- Rydberg atoms/polaritons
- Microcavity Polaritons 

Dicke effect: Enhanced emission

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

R. H. DICKE
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey



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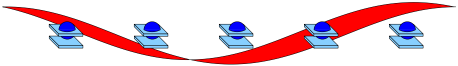

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Collective decay:

$$\frac{d\rho}{dt} = -\frac{\Gamma}{2}\left[S^+S^-\rho - S^-\rho S^+ + \rho S^+ S^- \right]$$

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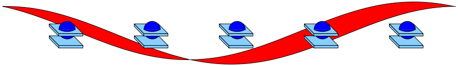

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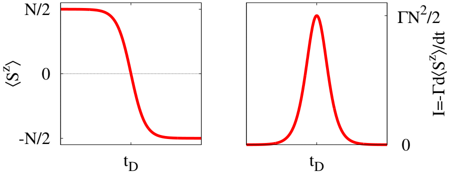
$$I \propto -\Gamma \frac{d\langle S^z \rangle}{dt} = \frac{\Gamma N^2}{4} \text{sech}^2\left[\frac{\Gamma N}{2} t\right]$$



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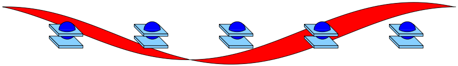

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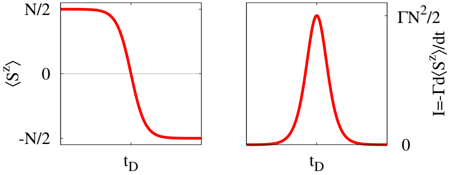
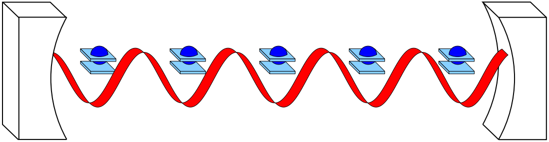


Problem: dipole interactions dephase. [Friedberg et al, Phys. Lett. 1972]

Collective radiation **with a cavity**: Dynamics



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Single cavity mode: oscillations

[Bonifacio and Preparata PRA '70]

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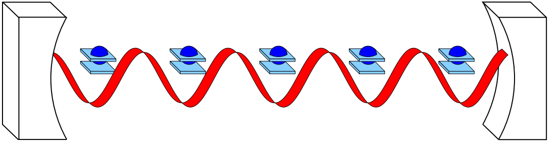

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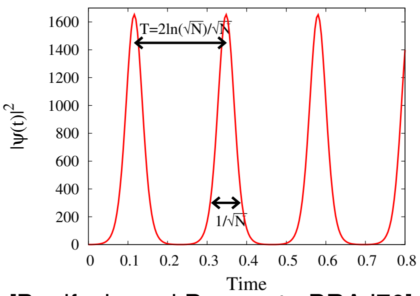

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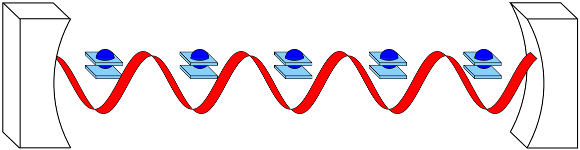
Dicke model: Equilibrium superradiance transition



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- Coherent state: $|\Psi\rangle \to e^{\lambda\psi^\dagger + \eta S^+}|\Omega\rangle$
- Small g, min at $\lambda, \eta = 0$

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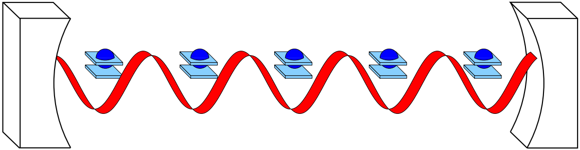

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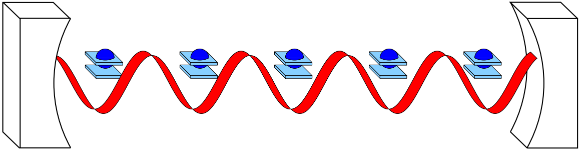

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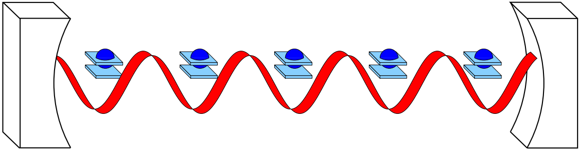

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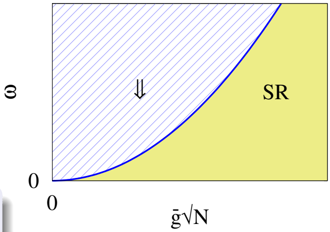

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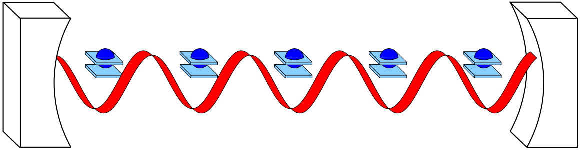
No go theorem and transition



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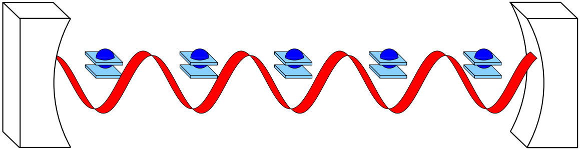

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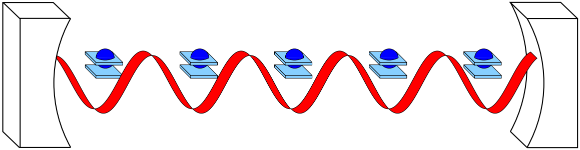

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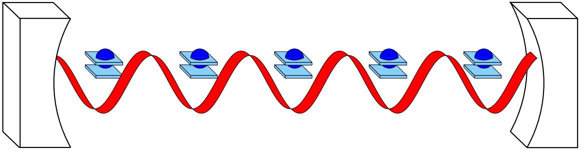

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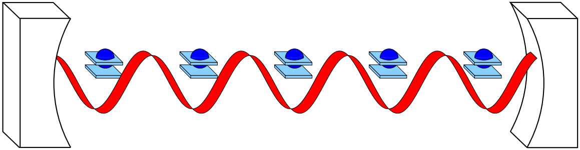

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But Thomas-Reiche-Kuhn sum rule states: $g^2/\omega_0 < 2\zeta$. **No transition**

[Rzazewski *et al* PRL '75]

Dicke phase transition: ways out

Problem: $g^2/\omega_0 < 2\zeta$ for intrinsic parameters. **Solutions**:

- **Interpretation**
  Ferroelectric transition in $\mathbf{D}\cdot\mathbf{r}$ gauge.
  [JK JPCM '07, Vukics & Domokos PRA 2012]
- Circuit QED [Nataf and Ciuti, Nat. Comm. '10; Viehmann *et al.* PRL '11]
- Grand canonical ensemble:
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- Dissociate g, ω_0.
  e.g. Raman scheme: $\omega_0 \ll \omega$.
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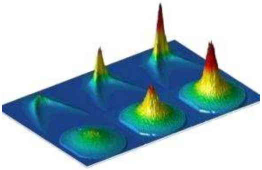

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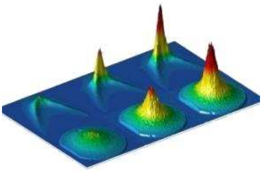

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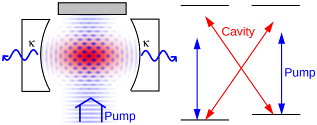

Outline

Acknowledgements

GROUP:





COLLABORATORS: Simons, Bhaseen, Schmidt, Blatter, Türeci, Krüger
EXPERIMENT: Houck, Wallraff, Fink, Mylnek

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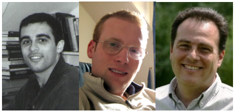
EPSRC
Engineering and Physical Sciences
Research Council

Nuffield
Foundation

Dynamics of generalized Dicke model

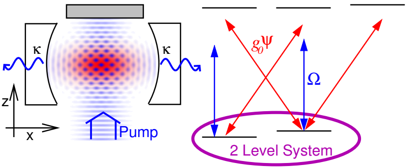


1 Introduction: Dicke model and superradiance

2 Dynamics of generalized Dicke model
- Summary of experiment and classical dynamcs
- Fixed points and dynamical phases
- Timescales and consequences for experiment
- Persistent oscillating phases

3 Jaynes Cummings Hubbard model
- JCHM vv Dicke
- Coherently driven array
- Disorder

Reminder of cold-atom extended Dicke model



2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$:

$\Downarrow$: $\Psi(x,z) = 1$

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[Baumann *et al* Nature '10]

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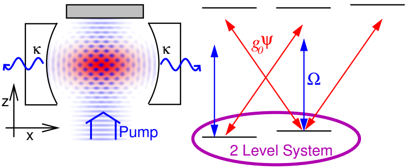

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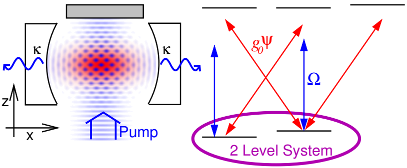

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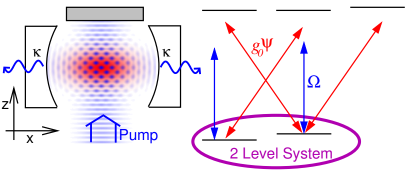

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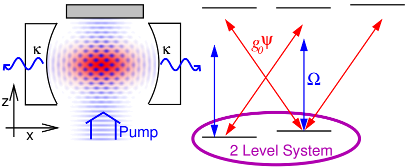
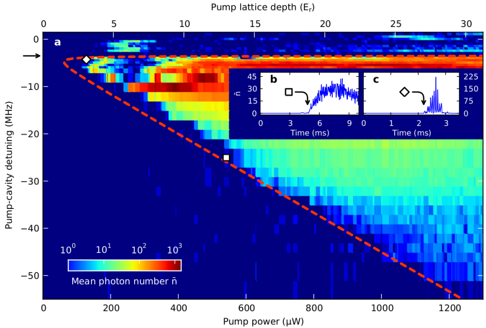
[Baumann *et al* Nature '10]

Reminder of cold-atom extended Dicke model





2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$:

$\Downarrow$: $\Psi(x,z) = 1$

$\Uparrow$: $\Psi(x,z) = \sum_{\sigma,\sigma'=\pm} e^{ik(\sigma x + \sigma' z)}$

Feedback: $U \propto \frac{g_0^2}{\omega_c - \omega_a}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+) + U S_z \psi^\dagger\psi.$$

$$\partial_t \rho = -i[H,\rho] - \kappa(\psi^\dagger\psi\rho - 2\psi\rho\psi^\dagger + \rho\psi^\dagger\psi)$$

$\omega_0 \sim \text{kHz} \ll \omega, \kappa, g\sqrt{N} \sim \text{MHz}.$

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Classical dynamics of the extended Dicke model

Open dynamical system:

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- Linearisation about fixed point:
  - Recover Retarded Green's function (spectrum)
  - Cannot recover occupations

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Classical EOM
($|\mathbf{S}| = N/2 \gg 1$)

$$\dot{S}^- = -i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$

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- $\psi = 0, \mathbf{S} = (0, 0, \pm N/2)$ always a solution.
- If $g > g_c$, $\psi \neq 0$ too
  - A $S^y = -\Im[S^-] = 0$
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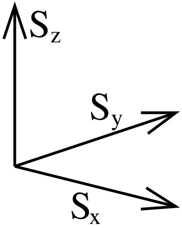

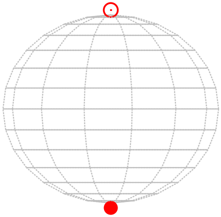

Small g: $\Uparrow, \Downarrow$ only.
($\omega = 30$MHz, $UN = -40$MHz)

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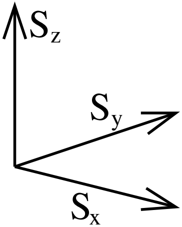

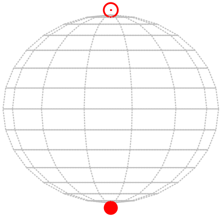

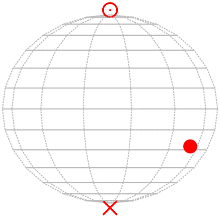

Small g: $\Uparrow, \Downarrow$ only. Larger g: SR too.
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Steady state phase diagram

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
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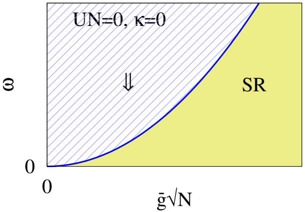

See also Domokos and Ritsch PRL '02, Domokos *et al.* PRL '10

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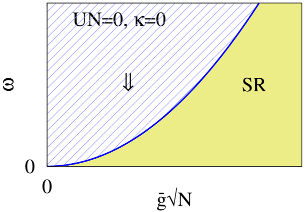

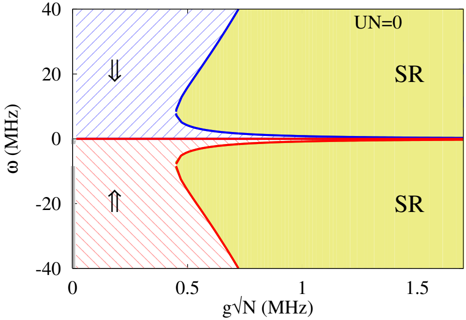

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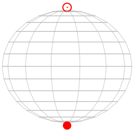
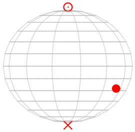


SR(A): $S_y = 0$



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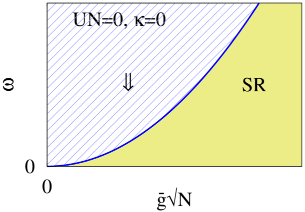

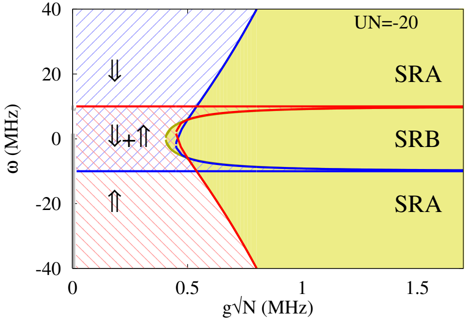

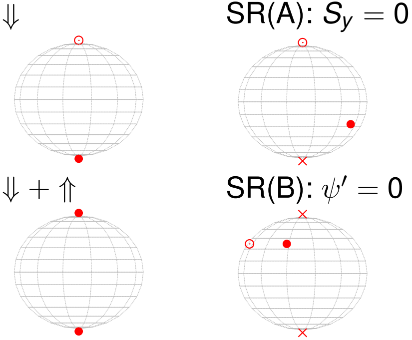

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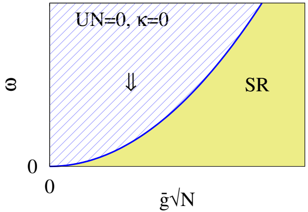

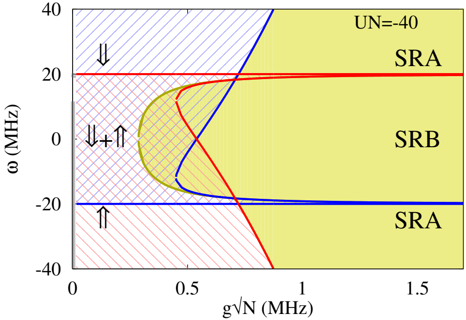

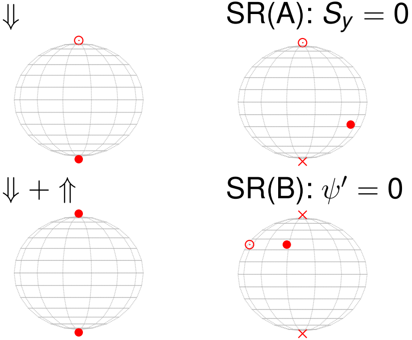

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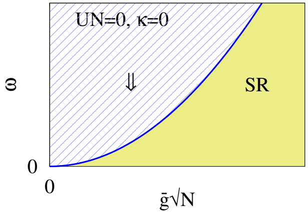

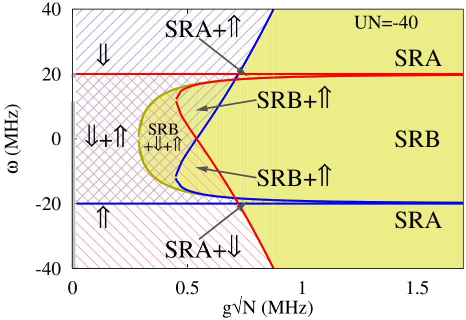

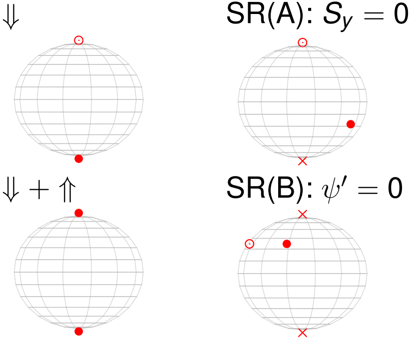

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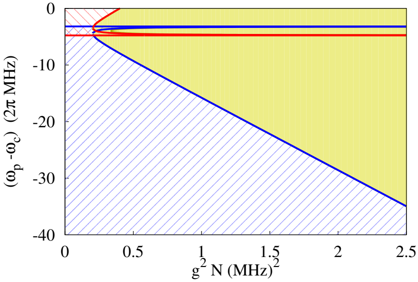
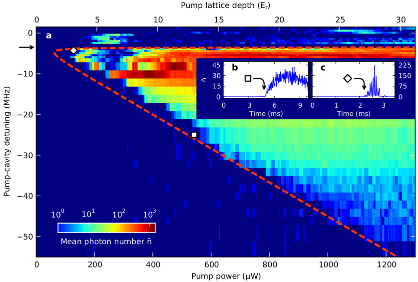
Comparison to experiment





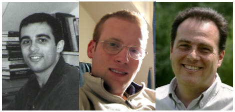
$$UN = -10\text{MHz}$$

Adapted from: [Bhaseen *et al.* PRA '12]

[Baumann *et al* Nature '10]

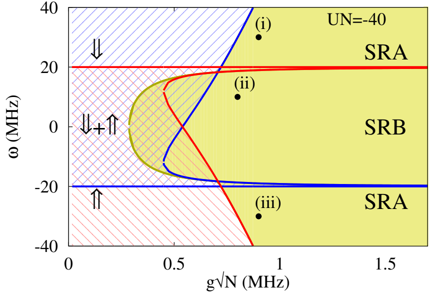
$$\omega = \omega_c - \omega_p + \frac{5}{2} UN, \qquad UN = -\frac{g_0^2}{4(\omega_a - \omega_c)}$$

Dynamics of generalized Dicke model

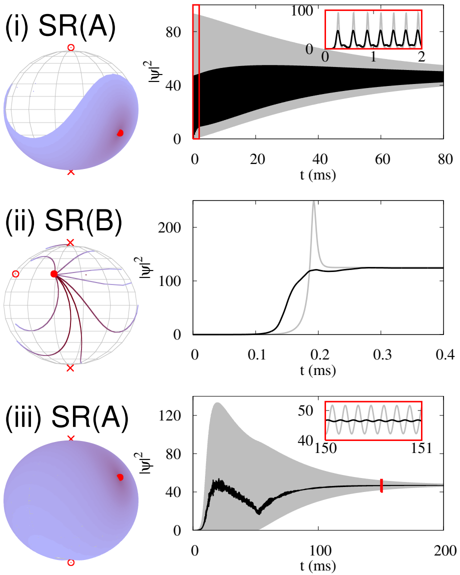


Dynamics: Evolution from normal state

Gray: $\mathbf{S} = (\sqrt{N}, \sqrt{N}, -N/2)$
Black: Wigner distribution of $\mathbf{S}$, ψ



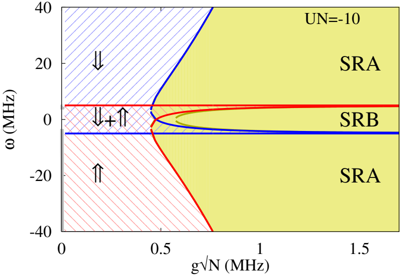
Oscillations: ~ 0.1ms
Decay: 20ms, 0.1ms, 20ms



Asymptotic state: Evolution from normal state

(Near to experimental $UN = -13$MHz).

All stable attractors:



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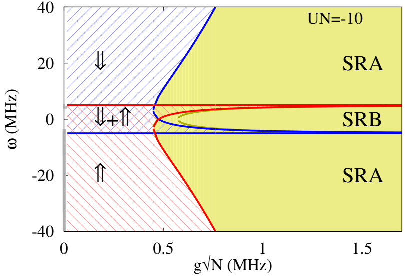
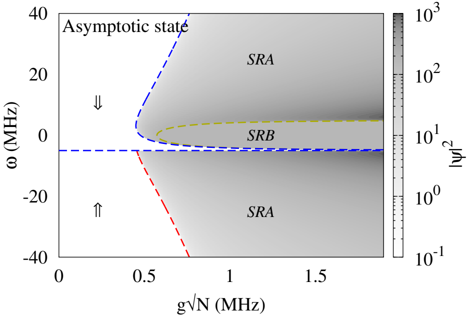
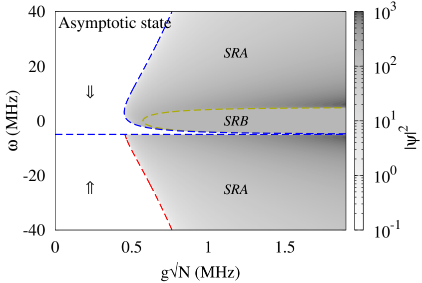


Starting from ⇓



Timescales for dynamics: Consequences for experiment



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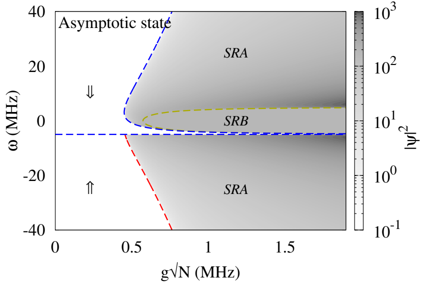

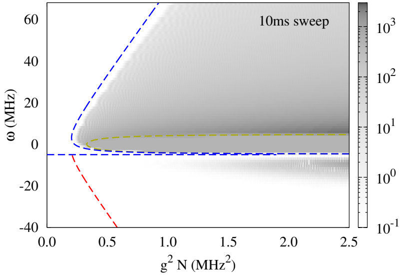

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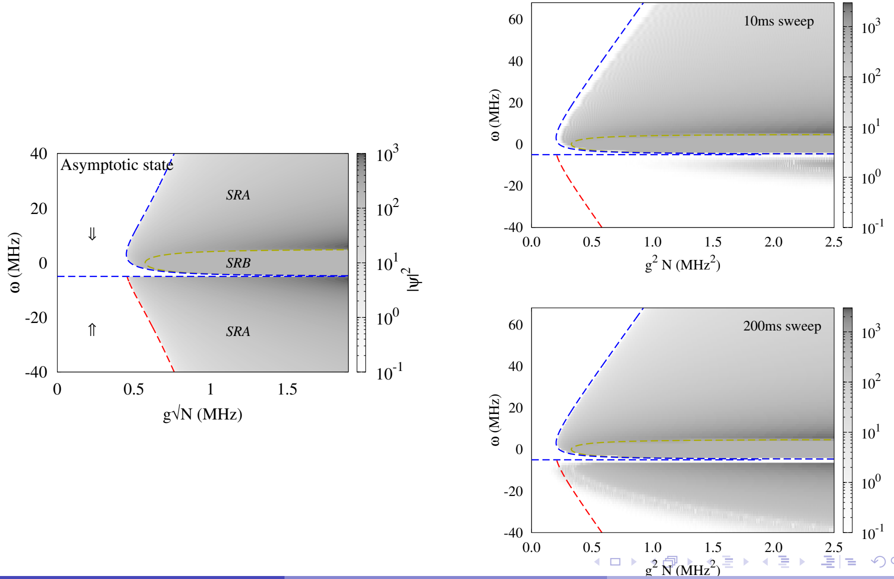
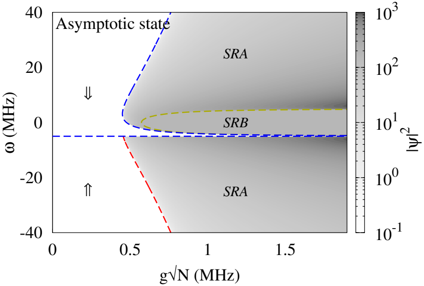
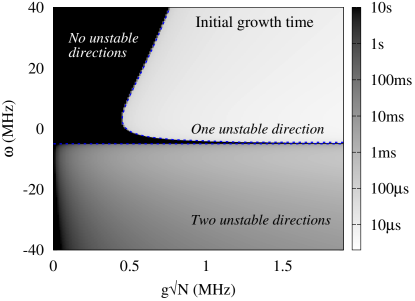


Timescales for dynamics: What are they?





Growth Most unstable eigenvalues near $\mathbf{S} = (0, 0, -N/2)$

Decay Slowest stable eigenvalues near final state

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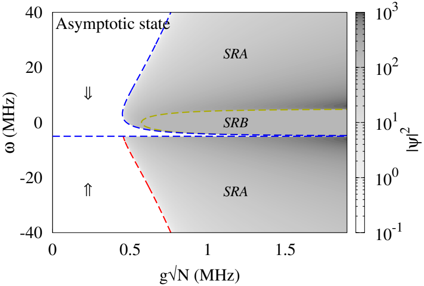

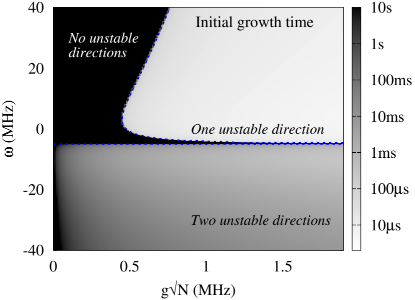

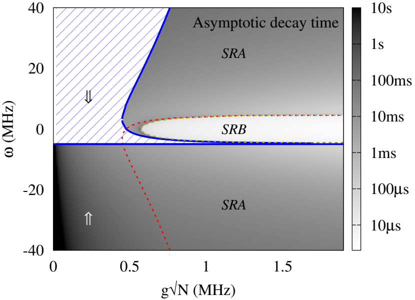

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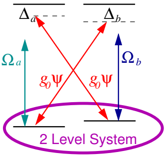
Timescales for dynamics: Why so slow and varied?

Suppose co- and counter-rotating terms differ



$$H = \ldots + g(\psi^\dagger S^- + \psi S^+) + g'(\psi^\dagger S^+ + \psi S^-) + \ldots$$

- SR(A) near phase boundary at small δg → Critical slowing down
- SR(A), SR(B) continuously connect

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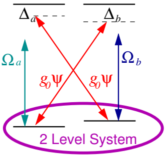

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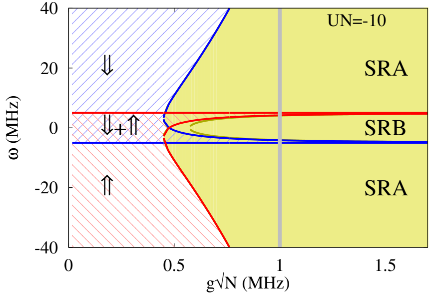

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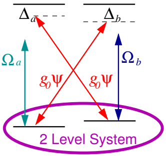

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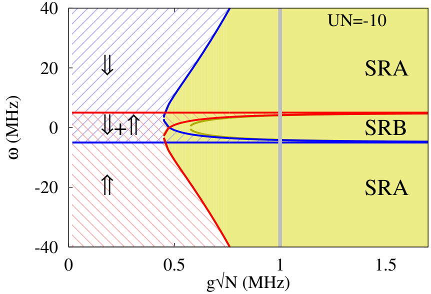

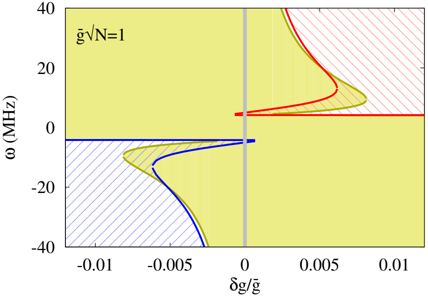

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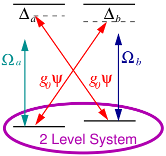

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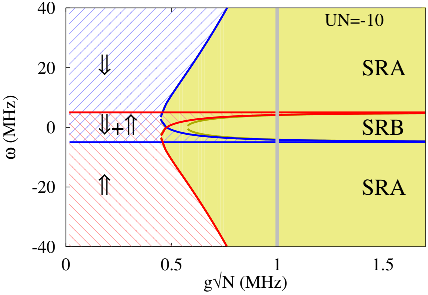

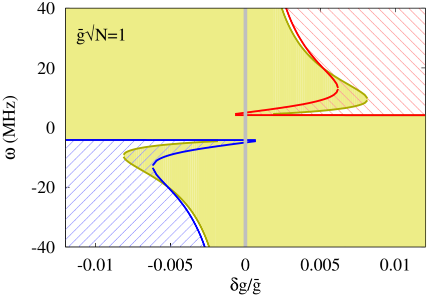

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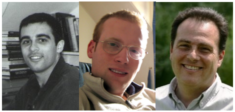
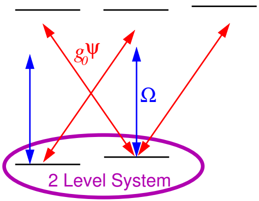
Dynamics of generalized Dicke model

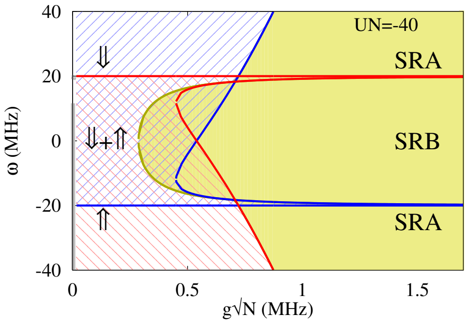


Regions without fixed points

Changing U:



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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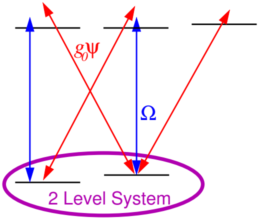

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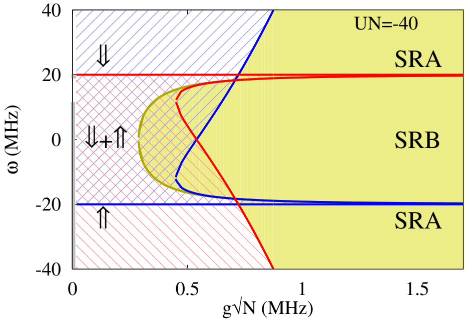

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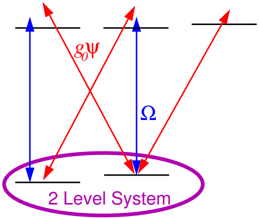

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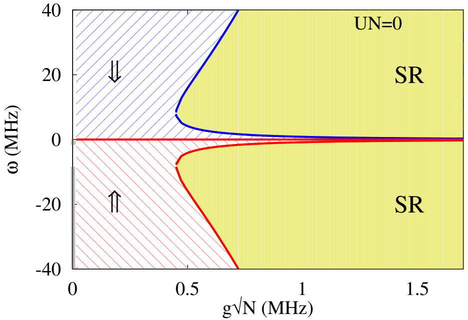

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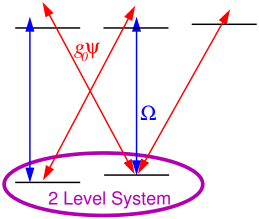

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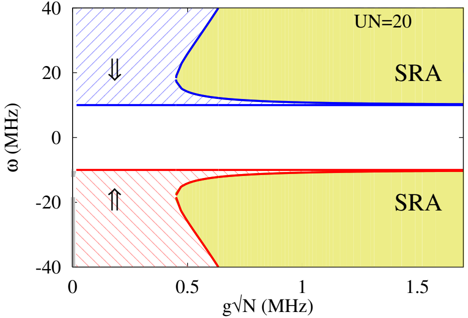

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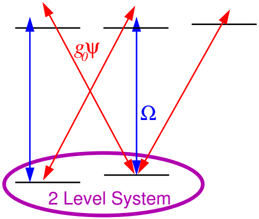

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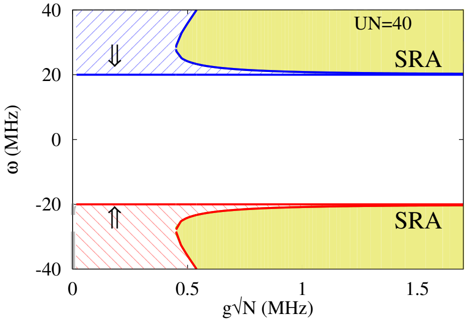

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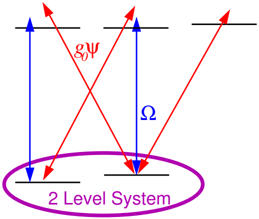

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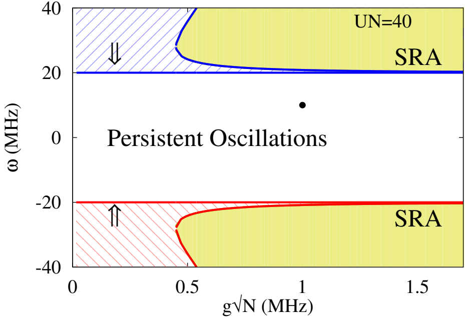
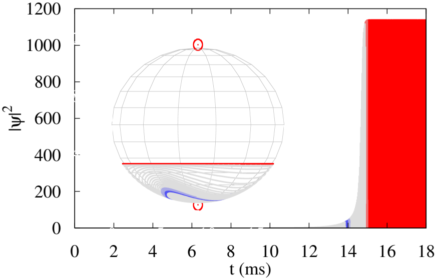
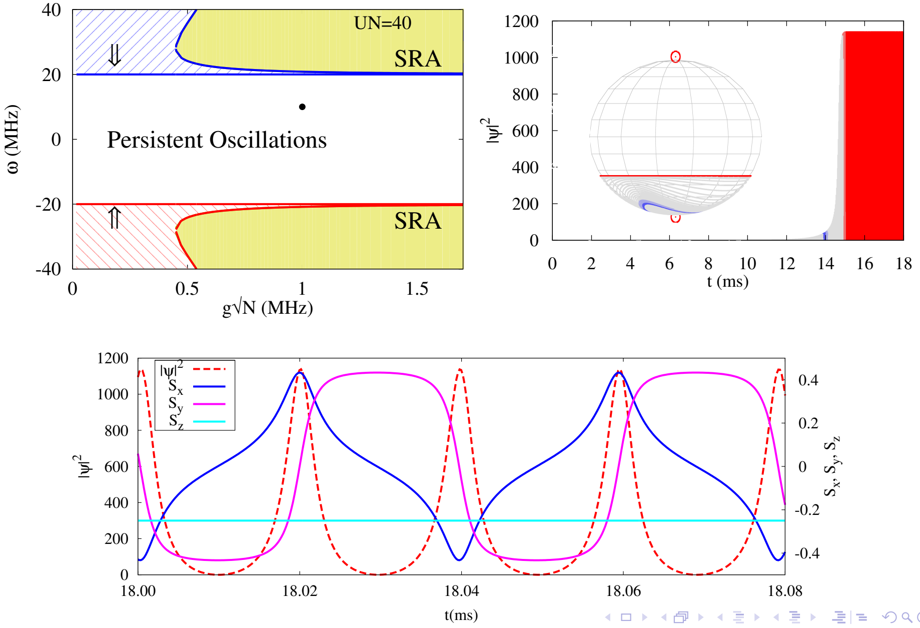




Persistent (optomechanical) oscillations



Jaynes Cummings Hubbard model

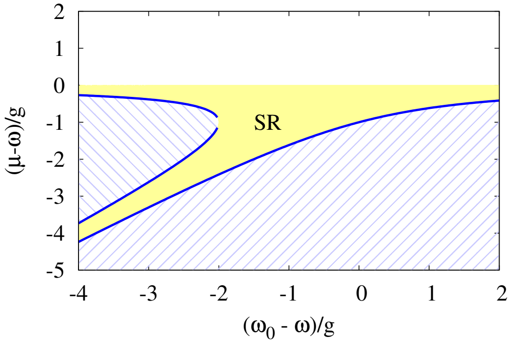
1. Introduction: Dicke model and superradiance

2. Dynamics of generalized Dicke model
   - Summary of experiment and classical dynamcs
   - Fixed points and dynamical phases
   - Timescales and consequences for experiment
   - Persistent oscillating phases

3. Jaynes Cummings Hubbard model
   - JCHM vv Dicke
   - Coherently driven array
   - Disorder

Equilibrium: Dicke model with chemical potential

$$H - \mu N = (\omega - \mu)\psi^\dagger\psi + (\omega_0 - \mu)S^z + g\left(\psi^\dagger S^- + \psi S^+\right)$$



- Transition at:
  $g^2 N > (\omega - \mu)|\omega_0 - \mu|$
- Reduce critical g
- Unstable if $\mu > \omega$
- Inverted if $\mu > \omega_0$

[Eastham and Littlewood, PRB '01]

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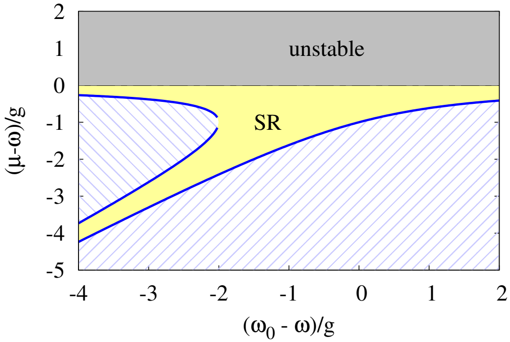

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[Eastham and Littlewood, PRB '01]

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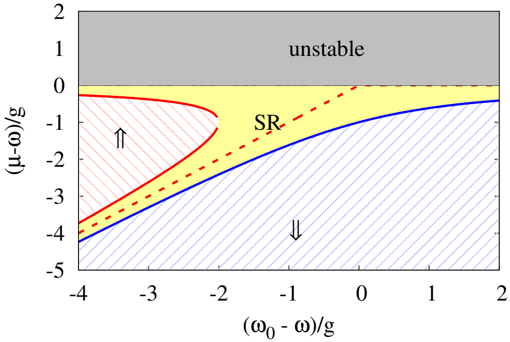

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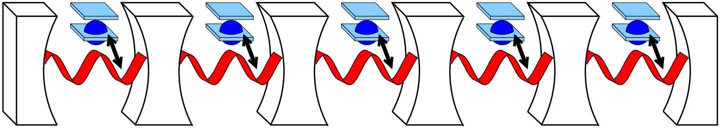
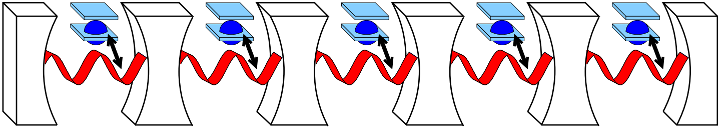
[Eastham and Littlewood, PRB '01]

Jaynes-Cummings Hubbard model



$$H = -\frac{J}{z}\sum_{ij}\psi_i^\dagger\psi_j + \sum_i \frac{\Delta}{2}\sigma_i^z + g(\psi_i^\dagger\sigma_i^- + \text{H.c.})$$

Jaynes-Cummings Hubbard model



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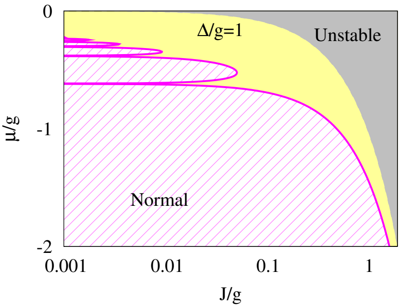
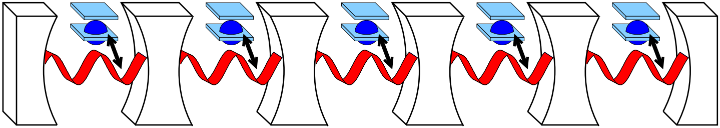


Jaynes-Cummings Hubbard model



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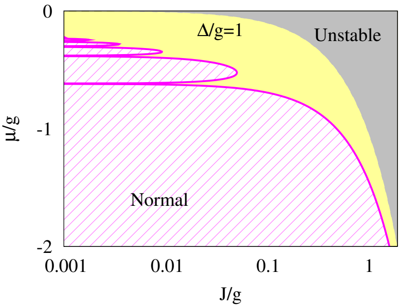
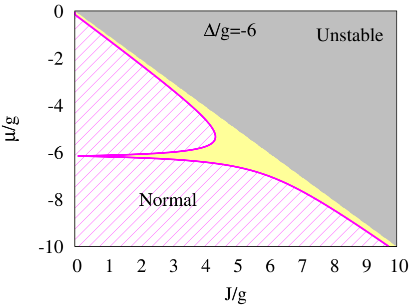
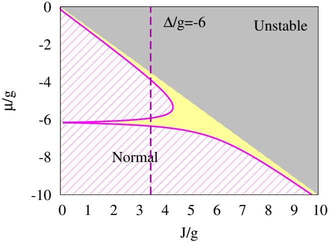


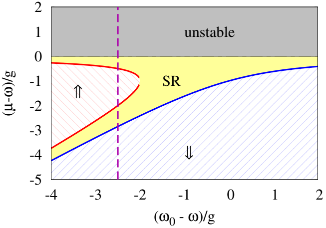


Dicke vs JCHM

JCHM



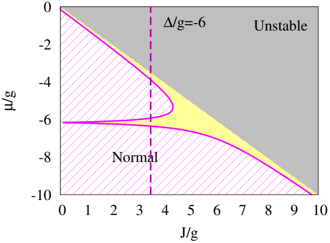
Dicke



- $k = 0$ mode of JCHM $\leftrightarrow$ Dicke photon mode
- $\Uparrow$ $\leftrightarrow$ $n = 1$ Mott lobe

Dicke vs JCHM

JCHM



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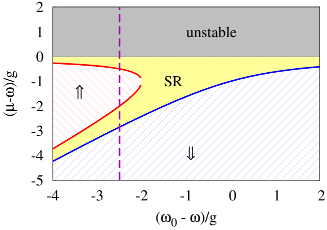

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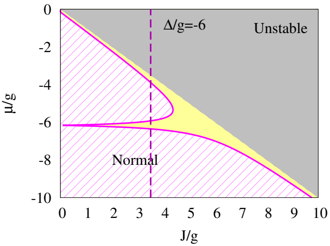
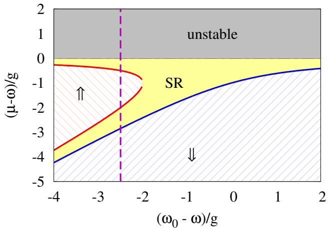


Dicke



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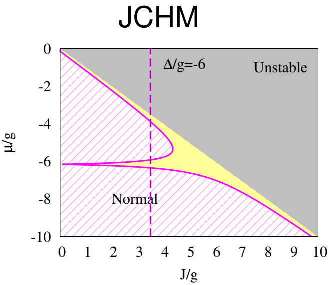

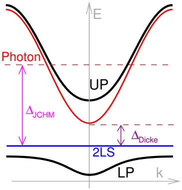

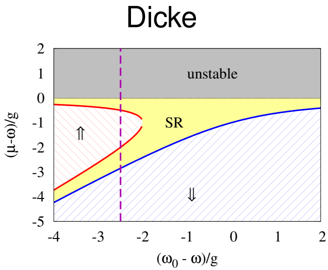

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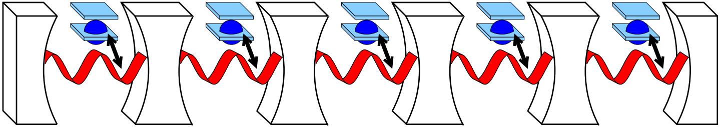
Jaynes Cummings Hubbard model

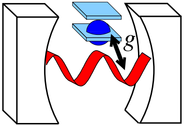
Coherently pumped JCHM



$$H = -\frac{J}{z}\sum_{ij}\psi_i^\dagger\psi_j + \sum_i \frac{\Delta}{2}\sigma_i^z + g(\psi_i^\dagger\sigma_i^- + \text{H.c.}) + f(\psi_i e^{i\omega_L t} + \text{H.c.})$$

$$\partial_t\rho = -i[H,\rho] - \frac{\kappa}{2}L_\psi[\rho] - \frac{\gamma}{2}L_{\sigma^-}[\rho]$$

Coherently pumped single cavity [Bishop *et al.* Nat. Phys '09]



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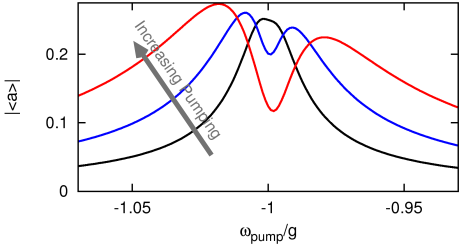

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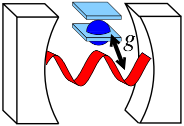

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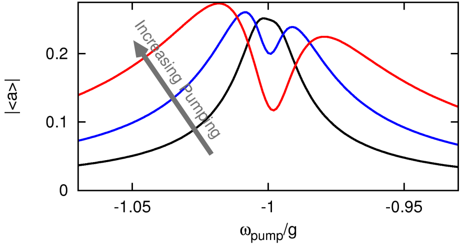
- Effective 2LS: $|\text{Empty}\rangle, |\text{1 polariton}\rangle$



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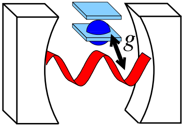

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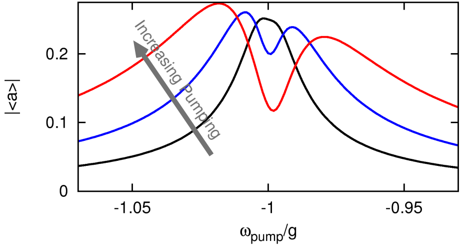
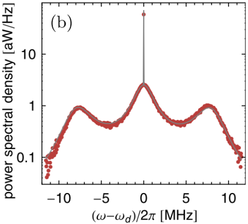


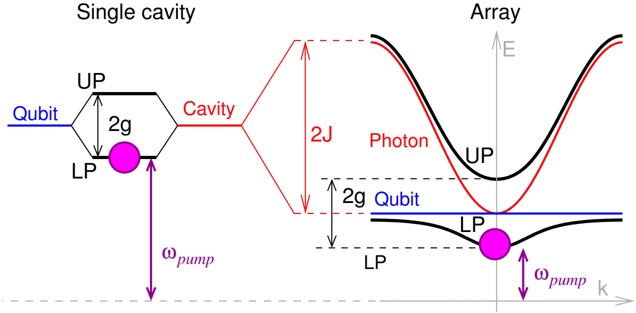
- Mollow triplet fluorescence



[Lang *et al.* PRL '11]

Coherently pumped dimer & array

Chose detuning *a la* Dicke model

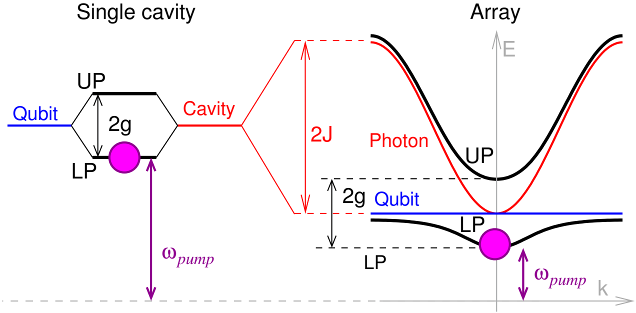


- Bistability at intermediate J
  - More/less localised states
  - Connects to Dicke limit

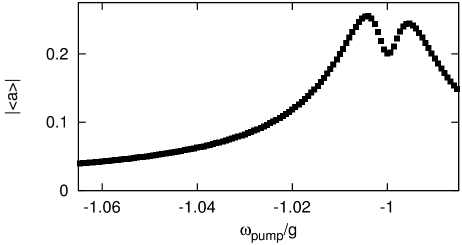
[Nissen *et al.* PRL '12]

Coherently pumped dimer & array

Chose detuning *a la* Dicke model



Evolution of anti-resonance vs *J*.



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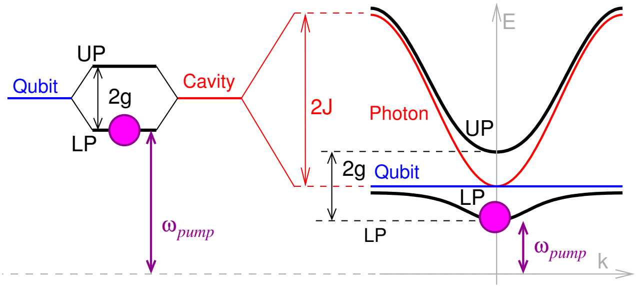

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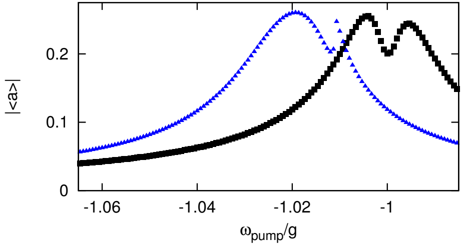

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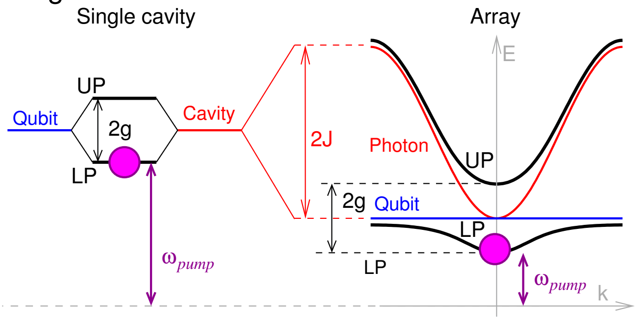

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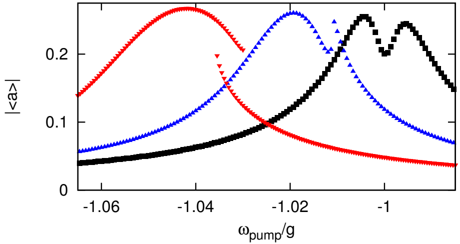

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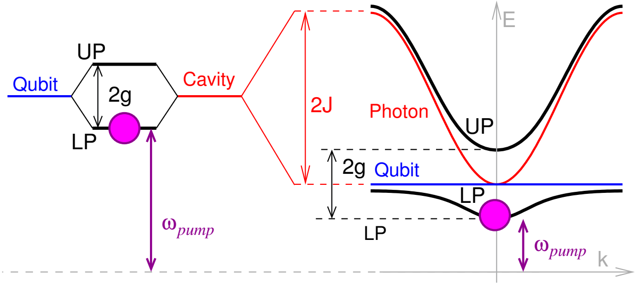

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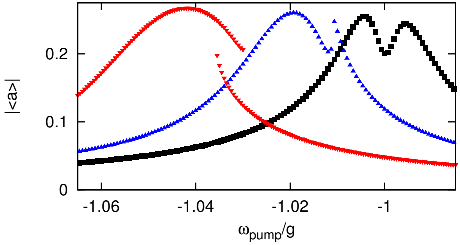

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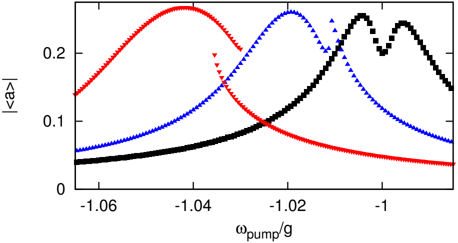
[Nissen *et al.* PRL '12]

Photon blockade picture $J \lesssim g$

- Polariton basis
- Nonlinearity $|\epsilon_2 - 2\epsilon_1| \propto g$.

$$H = \sum_i \left(\frac{\epsilon}{2}\tau_i^z + \tilde{f}\tau_i^x \right)$$



[Nissen *et al.* PRL '12]

- Decouple hopping: $\tau_i^+\tau_j^- \to \psi\tau^+ + \psi^*\tau^-$
- Bistability for

$$J > J_c = \frac{4}{\tilde{f}^2}\left(\frac{2\tilde{f}^2 + (\kappa/2)^2}{3}\right)^{3/2}$$

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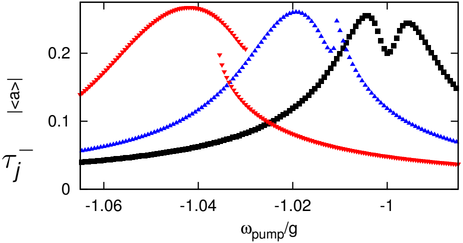

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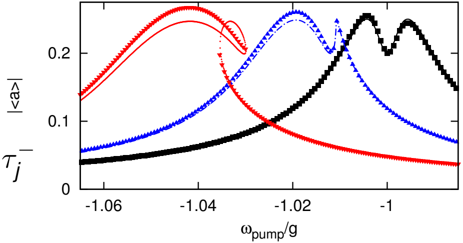

[Nissen *et al.* PRL '12]

- Decouple hopping:
$\tau_i^+ \tau_j^- \rightarrow \psi \tau^+ + \psi^* \tau^-$
- Bistability for

$$J > J_c = \frac{4}{\tilde{f}^2}\left(\frac{2\tilde{f}^2 + (\kappa/2)^2}{3}\right)^{3/2}$$

Photon blockade picture $J \lesssim g$

- Polariton basis
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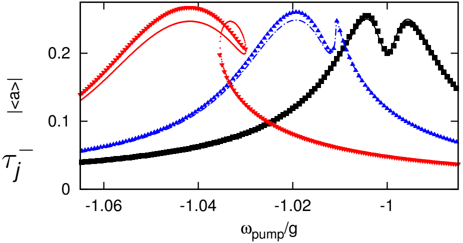

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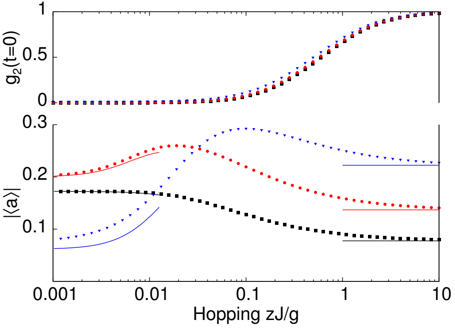
Coherently pumped array: correlations & fluorescence



Correlations

- $g_2 : 0 \rightarrow 1$ crossover.

- Small J: Mollow triplet
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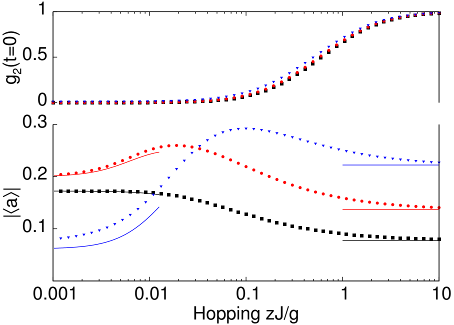

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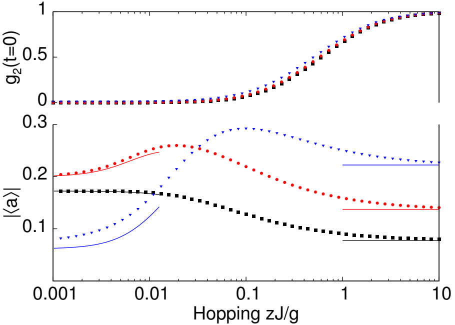

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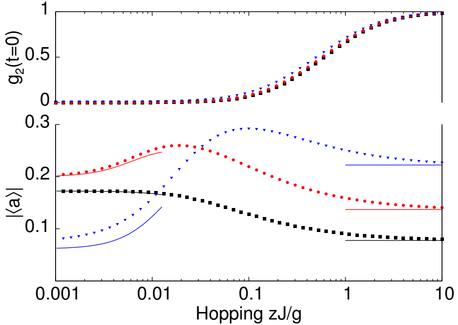

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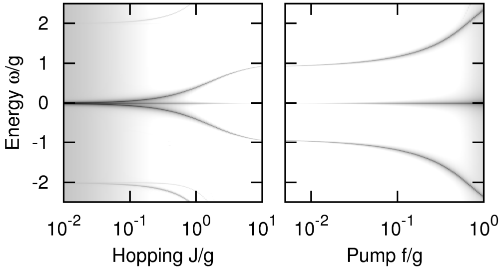

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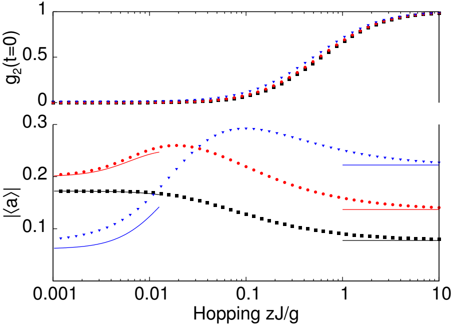

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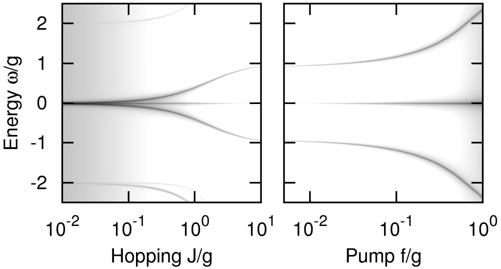

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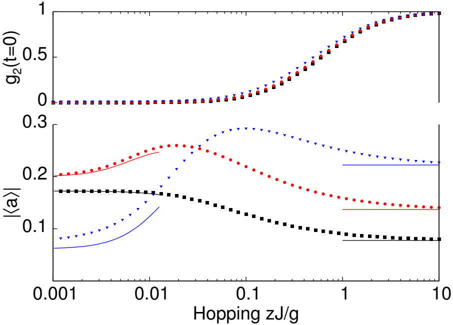

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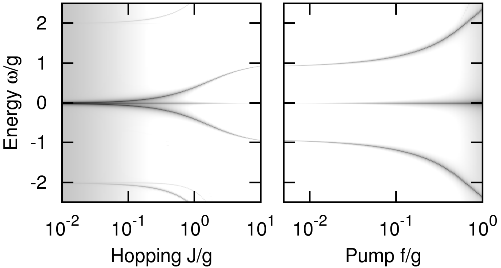

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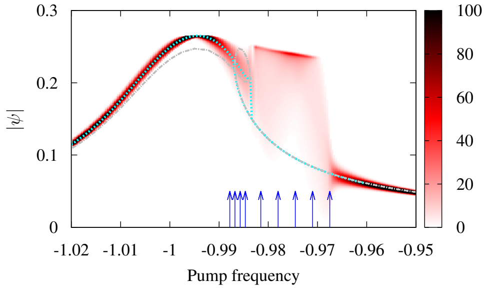

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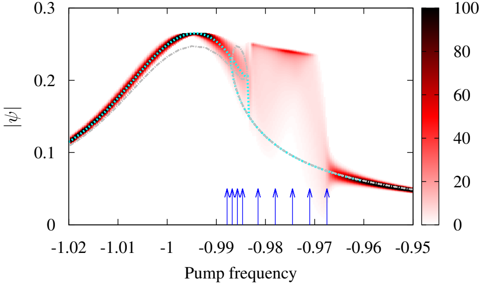

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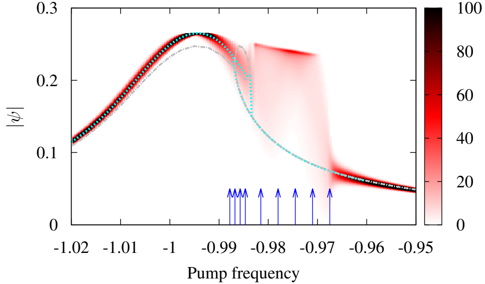

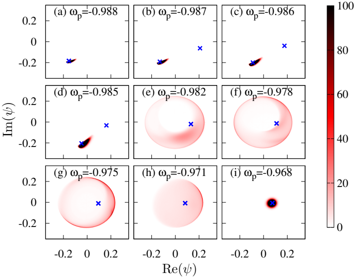

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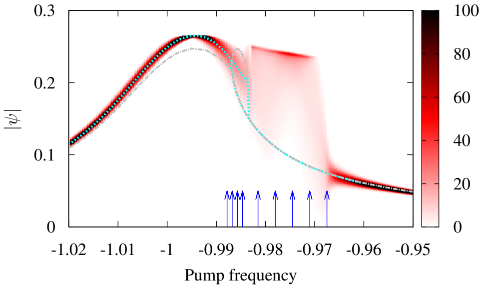
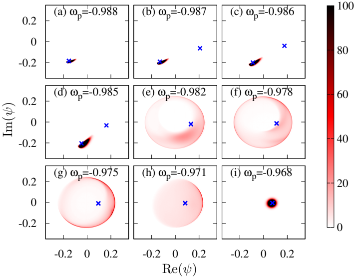
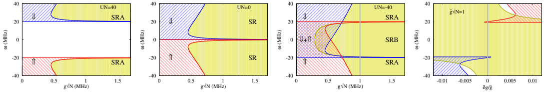




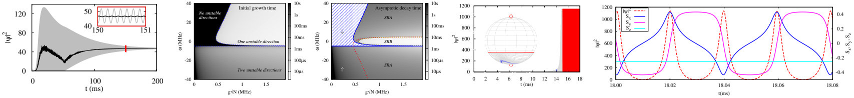
[Kulaitis *et al.* PRA, '13]

Summary

- Wide variety of dynamical phases

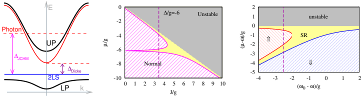


- Slow dynamics for $U < 0$ & Persistent oscillations for $U > 0$

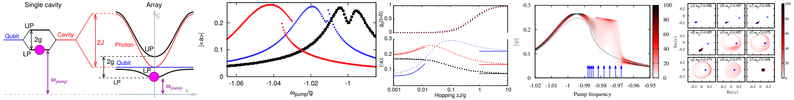


JK *et al.* PRL '10, Bhaseen *et al.* PRA '12

- Dicke model and JCHM: connection at $J \to \infty$



- Coherently pumped coupled cavity array



Nissen *et al.* PRL '12, Kulaitis *et al.* PRA '13

Ferroelectric transition

Atoms in Coulomb gauge

$$H = \sum \omega_k a_k^\dagger a_k + \sum_i [p_i - eA(r_i)]^2 + V_{\text{coul}}$$

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Two-level systems — dipole-dipole coupling

$$H = \omega_0 S^z + \omega\psi^\dagger\psi + g(S^+ + S^-)(\psi + \psi^\dagger) + N\zeta(\psi + \psi^\dagger)^2 - \eta(S^+ - S^-)^2$$

(nb $g^2, \zeta, \eta \propto 1/V$).

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Ferroelectric polarisation if $\omega_0 < 2\eta N$

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Gauge transform to dipole gauge $\mathbf{D} \cdot \mathbf{r}$

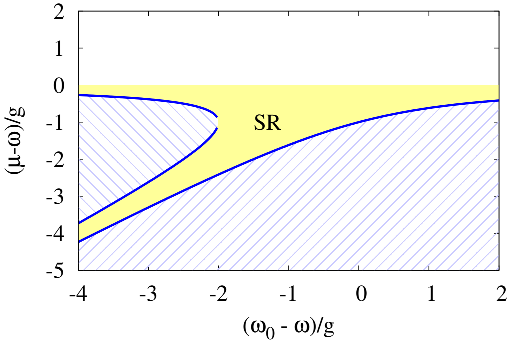
$$H = \omega_0 S^z + \omega \psi^\dagger \psi + \bar{g}(S^+ - S^-)(\psi - \psi^\dagger)$$

"Dicke" transition at $\omega_0 < N\bar{g}^2/\omega \equiv 2\eta N$

But, ψ describes **electric displacement**

Equilibrium: Dicke model with chemical potential

$$H - \mu N = (\omega - \mu)\psi^\dagger\psi + (\omega_0 - \mu)S^z + g\left(\psi^\dagger S^- + \psi S^+\right)$$



- Transition at: $g^2 N > (\omega - \mu)|\omega_0 - \mu|$
- Reduce critical g
- Unstable if $\mu > \omega$
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[Eastham and Littlewood, PRB ’01]

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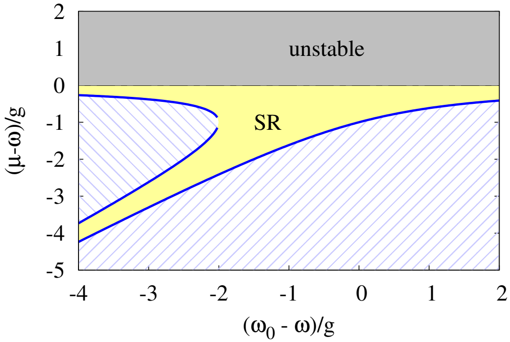

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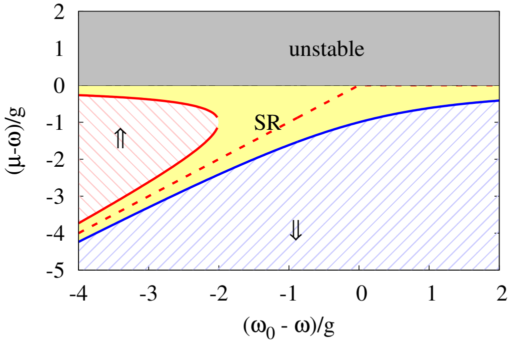

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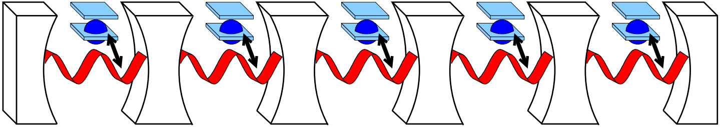
Jaynes-Cummings Hubbard model



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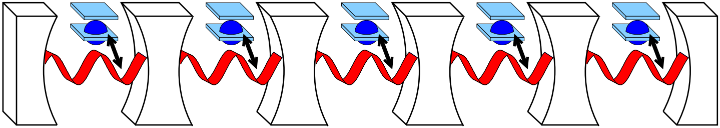

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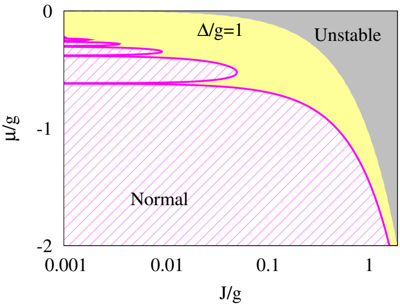

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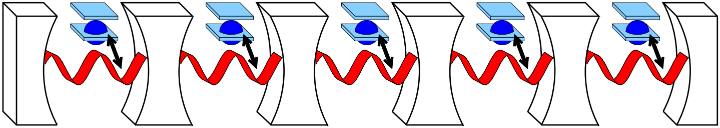

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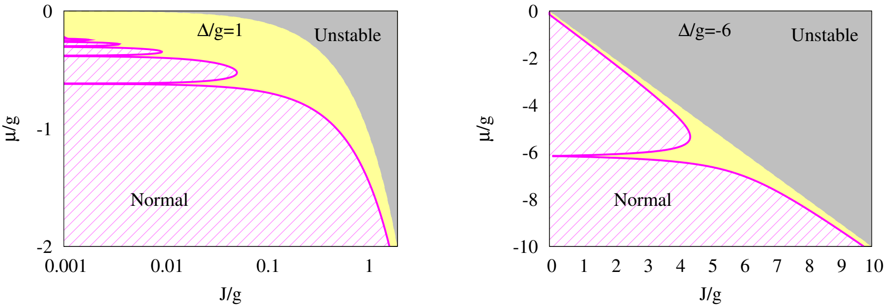
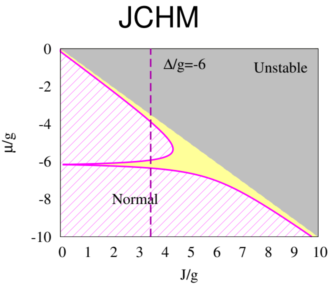
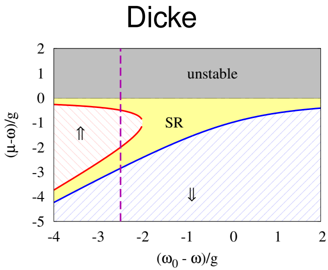


Dicke vs JCHM





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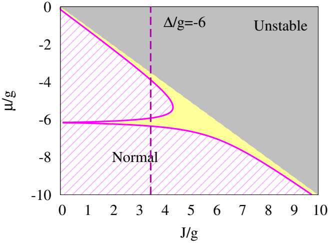

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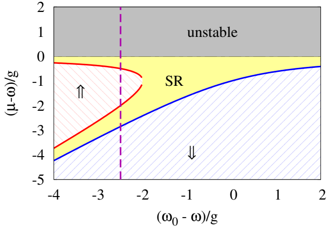

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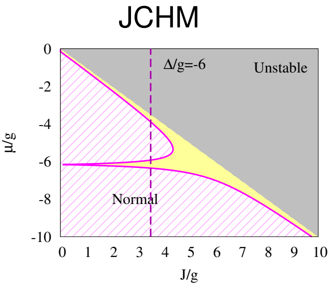

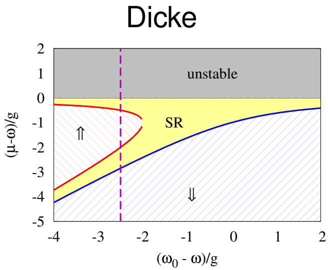

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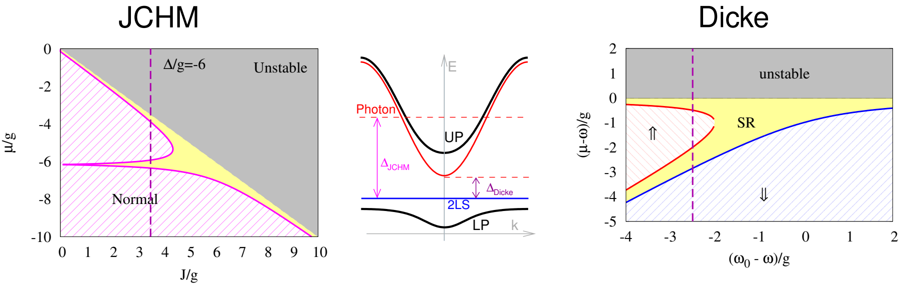

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JK, Türeci, Houck in progress

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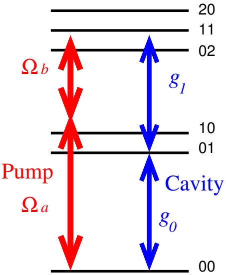

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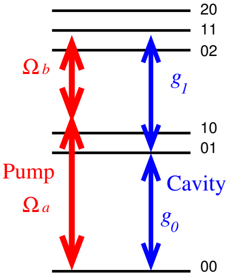

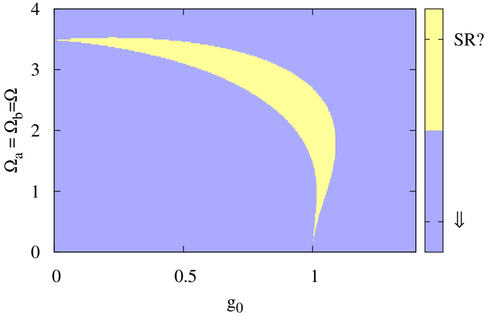

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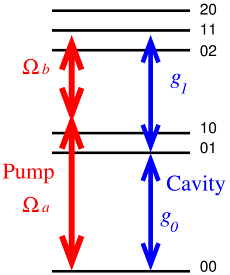

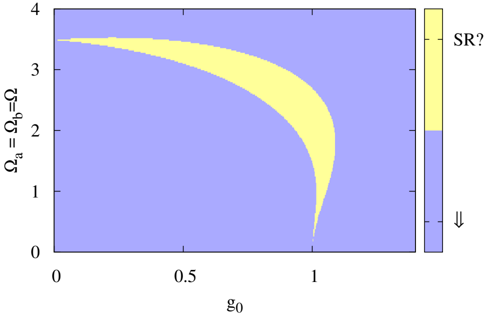

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  - [Carmichael & Walls JPA '73] Requirements for correct equilibrium
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- Phase transition $\rightarrow$ soft modes
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Dicke model linewidth:

$$H = \omega\psi^\dagger\psi + \sum_{i=1}^{N} \frac{\epsilon_i}{2}\sigma_i^z + g\left(\sigma_i^+\psi + \mathrm{h.c.}\right)$$

$$+\sum_i \sigma_i^z \sum_q \gamma_q \left(b_q^\dagger + b_q\right) + \sum_q \beta_q b_{iq}^\dagger b_q.$$

[Nissen, Fink *et al.* arXiv:1302.0665]

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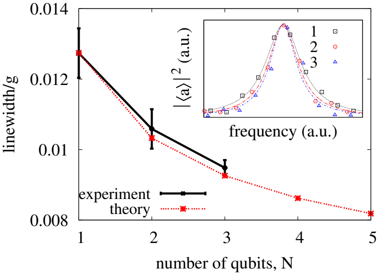
- Real environment is not Markovian
  - [Carmichael & Walls JPA '73] Requirements for correct equilibrium
  - [Ciuti & Carusotto PRA '09] Dicke SR and emission
- Cannot assume fixed κ, γ
- Phase transition $\rightarrow$ soft modes
- Strong coupling $\rightarrow$ varying decay

Dicke model linewidth:

$$H = \omega \psi^\dagger \psi + \sum_{i=1}^{N} \frac{\epsilon_i}{2} \sigma_i^z + g \left(\sigma_i^+ \psi + \text{h.c.} \right)$$

$$+ \sum_i \sigma_i^z \sum_q \gamma_q \left(b_q^\dagger + b_q \right) + \sum_q \beta_q b_{iq}^\dagger b_q.$$



[Nissen, Fink *et al.* arXiv:1302.0665]