Superfluidity and coherence in non-equilibrium condensates

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University of St Andrews 600 YEARS

SUPA

Universal themes of BEC, Leiden 2013

Acknowledgements

People:

 

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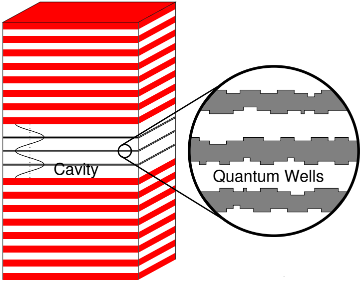
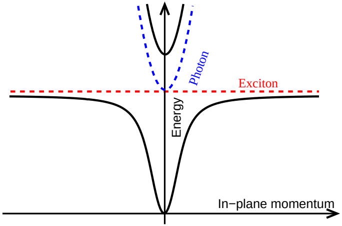
EPSRC
Engineering and Physical Sciences
Research Council

Microcavity polaritons



Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$



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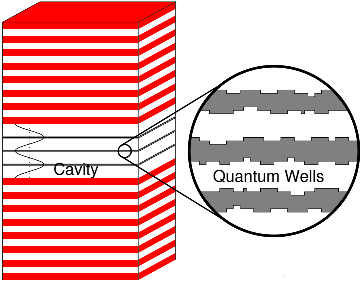

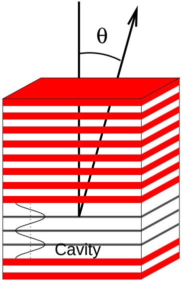

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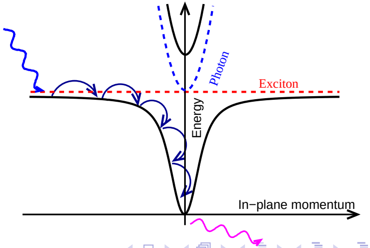
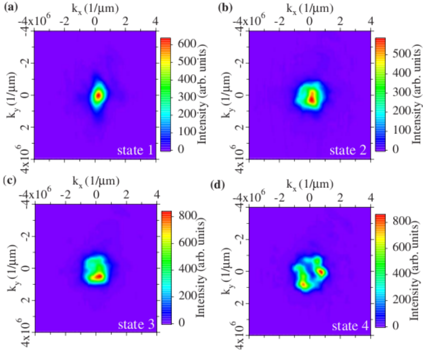
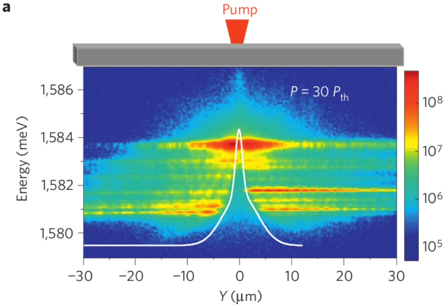


Non-equilibrium features in experiment





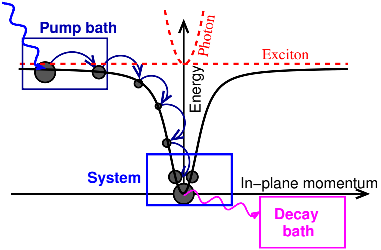
$|\psi(\mathbf{k})|^2 \neq |\psi(-\mathbf{k})|^2$:
Broken time-reversal symmetry.
[Krizhanovskii *et al.* PRB (2009)]

Flow from pumping spot
[Wertz *et al.* Nat. Phys. (2010)]

Non-equilibrium approach: Steady state, and fluctuations

$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}},$$

$$H_{\text{sys}} = \sum_k \omega_k \psi_k \psi_k^\dagger + \sum_\alpha g_\alpha (\phi_\alpha^\dagger \psi_k + \text{H.c.}) + H_{\text{ex}}[\phi_\alpha, \phi_\alpha^\dagger]$$



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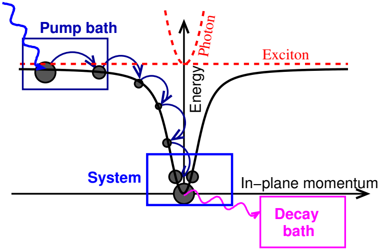

Steady state $\psi(\mathbf{r}, t) = \sqrt{\rho} e^{-i\mu t}$

Fluctuations: Green's functions:

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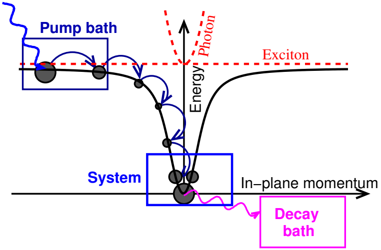

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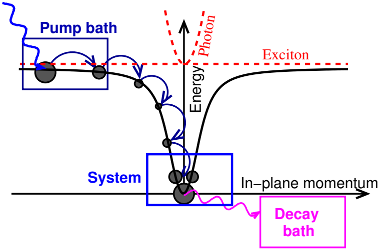

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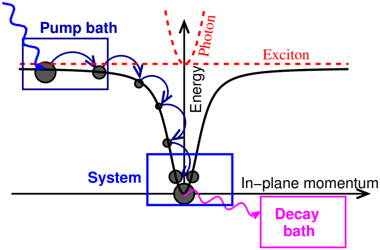

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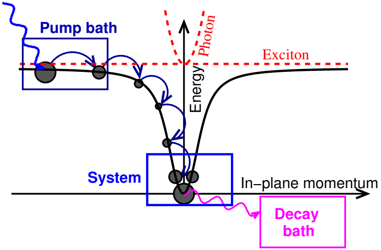

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Outline

1. Superfluidity
   - Why is there a question: spectrum
   - Experimental evidence?
2. Superfluid density
   - Response functions
   - Measuring polariton superfluid density
3. Coherence and power law decay
   - Modification of power laws?
   - Finite size and Schawlow-Townes
   - Experimental results

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Complex Gross-Pitaevskii equation

- Gross-Pitaevskii equation: energetics

$$i\partial_t\psi = \left(-\frac{\nabla^2}{2m} + V(r) + U|\psi|^2\right)\psi$$

- Introduce gain/loss from condensate mode $\gamma_{net} = \gamma - \kappa$
- Nonlinearity required for stability

- Relaxation — energy dependent gain.

- Fluctuation spectrum — Bogoliubov – de Gennes equations:

$$\psi = e^{-i\mu t}\left(\psi_0 + ue^{-i\nu t} + v^* e^{i\nu^* t}\right)$$

See [Wouters and Carusotto, PRL '07, JK and Berloff PRL '08]

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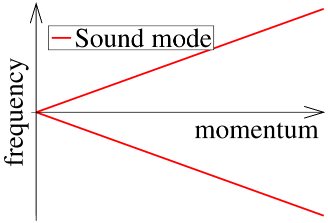
Fluctuations above transition

When condensed

$$\text{Det}\left[D^R(\omega, k)\right]^{-1} = \omega^2 - \xi_k^2$$

With $\xi_k \simeq ck$
Poles:

$$\omega^* = \xi_k$$



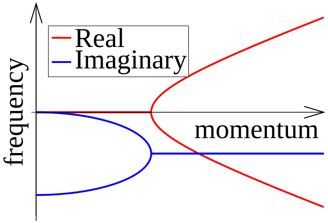
Fluctuations above transition

When condensed

$$\text{Det}\left[D^R(\omega,k)\right]^{-1} = (\omega + i\gamma_{\text{net}})^2 + \gamma_{\text{net}}^2 - \xi_k^2$$

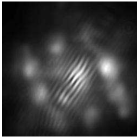
With $\xi_k \simeq ck$
Poles:

$$\omega^* = -i\gamma_{\text{net}} \pm \sqrt{\xi_k^2 - \gamma_{\text{net}}^2}$$

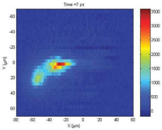


Experimental aspects of superfluidity

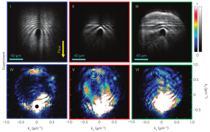
- Quantised vortices in disorder potential
  [Lagoudakis *et al.* Nature Phys. '08]



- Wavepacket propagation
  [Amo *et al.* Nature '09]



- Driven superfluidity
  [Amo *et al.* Nature Phys. ('09)



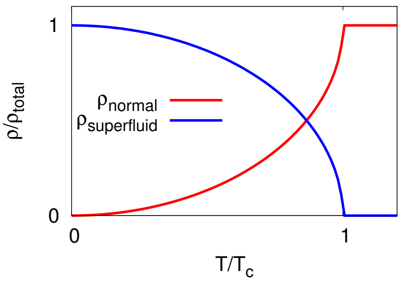
Aspects of superfluidity

	Quantised vortices	Landau critical velocity	Metastable persistent flow	Two-fluid hydrodynamics	Local thermal equilibrium	Solitary waves
Superfluid ^{4}He/cold atom Bose-Einstein condensate	✔	✔	✔	✔	✔	✔
Non-interacting Bose-Einstein condensate	✔	✗	✗	✔	✔	✗
Classical irrotational fluid	✗	✔	✗	✔	✔	✔
Parametrically pumped polariton condensates	✔	✔	✔	?	✗	✔
Incoherently pumped polariton condensates	✔	✗	?	?	✗	?

Adapted from [JK & Berloff, Nature News and Views, 2009]

Superfluid density

- Two-fluid hydrodynamics



- ρ_s, ρ_n distinguished by slow rotation

- Experimentally, rotation:

- To calculate, transverse/longitudinal:

Superfluid density

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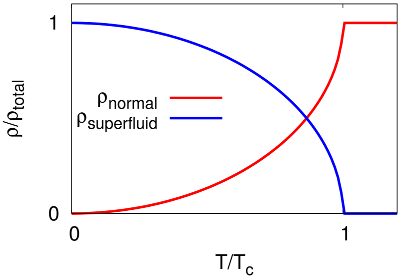

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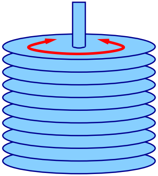

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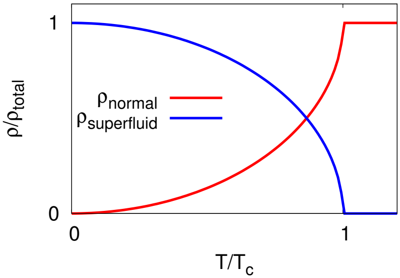

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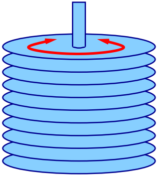

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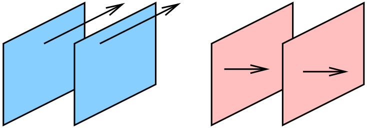

Superfluid density

- Currrent:

$$\mathbf{J}(r) = \psi^\dagger i \nabla \psi$$

$$J_i(\mathbf{q}) = \psi^\dagger_{\mathbf{k}+\mathbf{q}} \frac{2k_i + q_i}{2m} \psi_{\mathbf{k}}$$

- Response function:

$$H \to H - \sum_q \mathbf{f}(\mathbf{q}) \cdot \mathbf{J}_i(\mathbf{q}) \qquad J_i(\mathbf{q}) = \chi_{ij}(\mathbf{q}) f_j(\mathbf{q})$$

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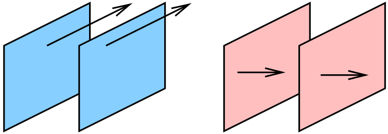

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$$\chi_{ij}(\omega = 0, \mathbf{q} \to 0) = \langle [J_i(\mathbf{q}), J_j(-\mathbf{q})] \rangle = \frac{\rho_S}{m} \frac{q_i q_j}{q^2} + \frac{\rho_N}{m} \delta_{ij}$$

Calculating superfluid response function

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- Require vertex corrections
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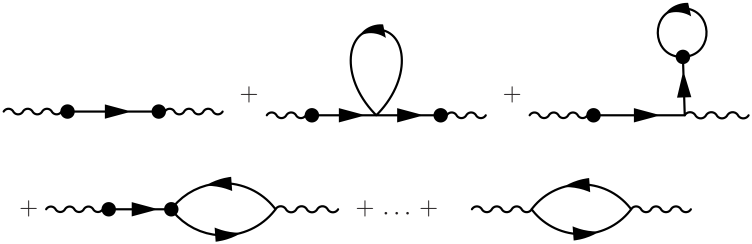

Calculating superfluid response function

- Use WIDBG model
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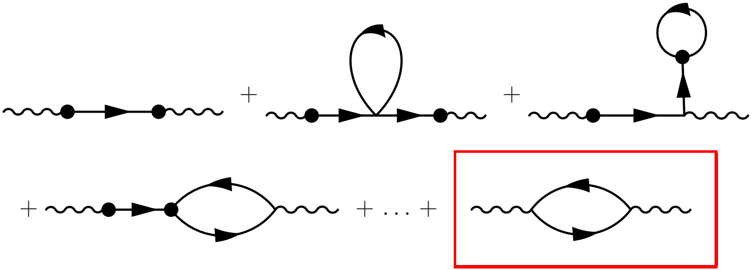
- Saddle point + fluctuations: Only one diagram for χ_N



Non-equilibrium superfluid response

- Superfluid response exists because:

$$= \frac{i\psi_0 q_i}{2m} (1, -1)\, D^R(q, \omega = 0) \begin{pmatrix} 1 \\ -1 \end{pmatrix} \frac{i\psi_0 q_j}{2m}$$

- $D^R(\omega = 0) \propto 1/\epsilon_q$ despite pumping/decay — superfluid response exists.
- Normal density:

$$\rho_N = \int d^d k\, \epsilon_k \int \frac{d\omega}{2\pi} \mathrm{Tr}\left[\sigma_z D^K \sigma_z (D^R + D^A)\right]$$

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  Does not vanish at $T \to 0$.

[JK PRL '11]

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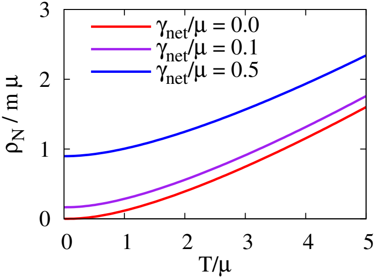


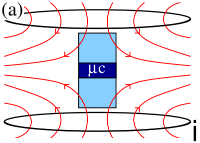
[JK PRL '11]

Measuring superfluid density

1. Effect rotating frame

Polariton polarization: $(\psi_{\circlearrowleft}, \psi_{\circlearrowright})$

$$H = \lambda \begin{pmatrix} \ell^2 & r^2 e^{2i\phi} \\ r^2 e^{-2i\phi} & -\ell^2 \end{pmatrix}$$



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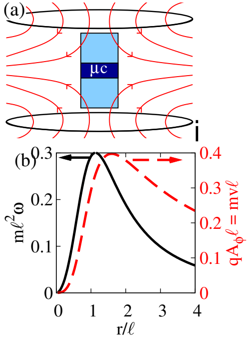
$$q\mathbf{A}_{\text{eff}} = m\omega \times \mathbf{r} = \frac{\hat{\phi}}{r}\left[1 - \frac{\ell^2}{\sqrt{r^4 + \ell^4}}\right]$$



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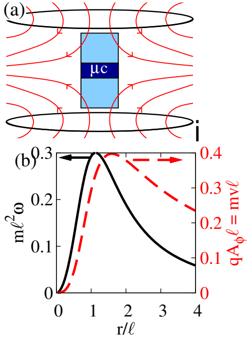
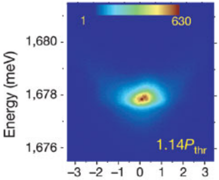


2. Measure resulting current

Energy shift of normal state:
$\Delta E = (1/2)mv^2 = 0.08/m\ell^2 \simeq 0.1\text{meV}$

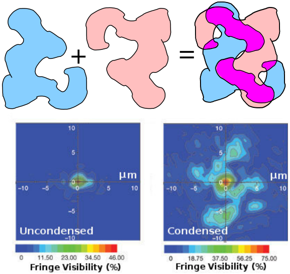


Superfluidity

Coherence in a 2D Gas

Correlations:

$$g_1(\mathbf{r},\mathbf{r}',t) = \left\langle \psi^\dagger(\mathbf{r},t)\psi(\mathbf{r}',0) \right\rangle$$



- $D^< = D^K - D^R + D^A$
- Generally, get:

$$\left\langle \psi^\dagger(\mathbf{r},t)\psi(0,0) \right\rangle \simeq |\psi_0|^2 \exp\left[-a_p \begin{cases} \ln(r/r_0) & t \simeq 0 \\ \frac{1}{2}\ln(c^2 t/\gamma_{net} r_0^2) & r \simeq 0 \end{cases} \right]$$

[Szymańska *et al.* PRL '06; PRB '07] [Wouters and Savona PRB '09]

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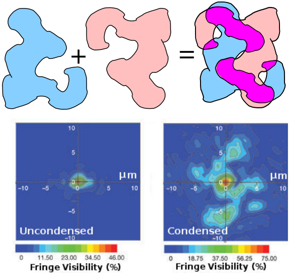

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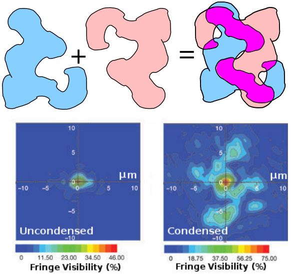

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Finite size effects: Single mode vs many mode

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$$D^<_{\phi\phi}(\mathbf{r},\mathbf{r},t) \propto \sum_n^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(\mathbf{r})|^2(1-e^{i\omega t})}{\left|(\omega+i\gamma_{\text{net}})^2+\gamma_{\text{net}}^2-\xi_n^2\right|^2}$$

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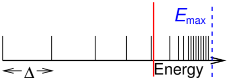

$$D^<_{\phi\phi} \sim 1 + \ln\left(E_{\rm max}\sqrt{\frac{t}{\gamma_{\rm net}}}\right)$$

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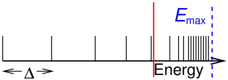

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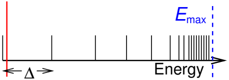
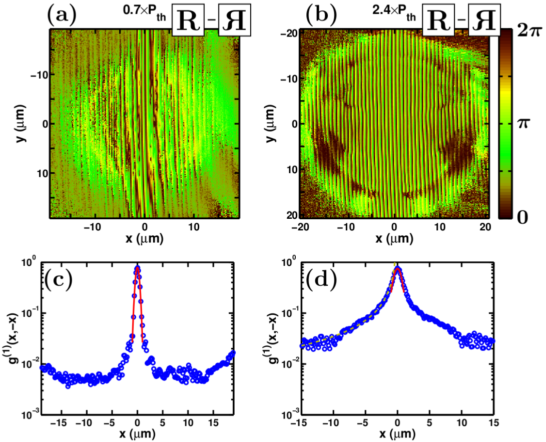


$$D^<_{\phi\phi} \sim \left(\frac{\pi C}{2\gamma_{\rm net}}\right)\left(\frac{t}{2\gamma_{\rm net}}\right)$$

(Recovers Schawlow-Townes laser linewidth)

Experimental observation of power-law decay



G. Rompos, *et al.* PNAS ’12

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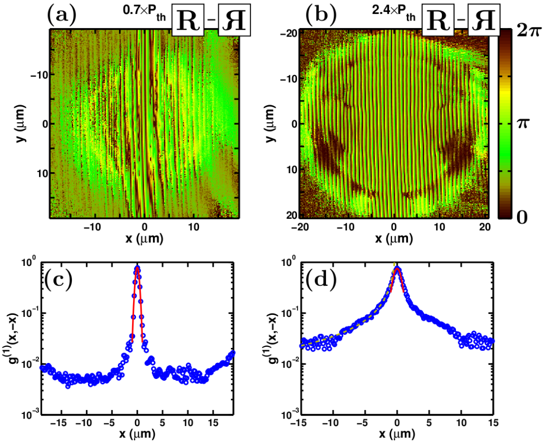
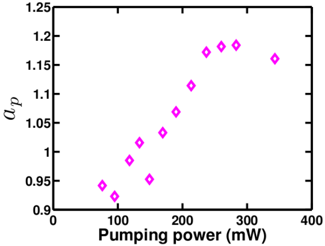


G. Rompos, *et al.* PNAS '12

$$g_1(\mathbf{r}, -\mathbf{r}) \propto \left(\frac{r}{r_0}\right)^{-a_p}$$



Exponent in a non-equilibrium 2D gas

$$\lim_{r\to\infty}\left\langle \psi^\dagger(\mathbf{r},0)\psi(-\mathbf{r},0)\right\rangle = |\psi_0|^2 \exp\left[-D^{<}_{\phi\phi}(\mathbf{r},-\mathbf{r})\right] \propto \exp\left[-a_p \ln\left(\frac{2r}{r_0}\right)\right]$$

- Experimentally, $a_P \simeq 1.2$
- In equilibrium $a_p = \dfrac{mk_BT}{2\pi\hbar^2 n_s} < \dfrac{1}{4}$ (BKT transition)
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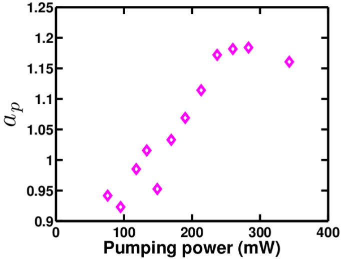

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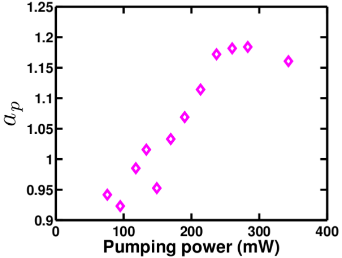

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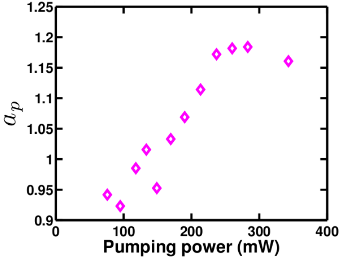

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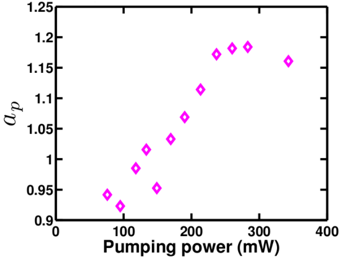

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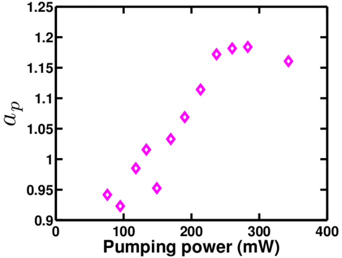
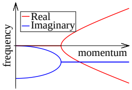
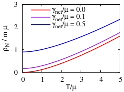


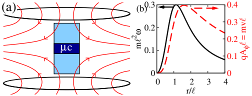
Conclusions

- Survival of superfluid response

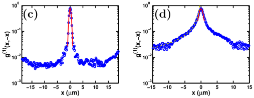


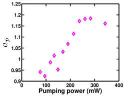


- Possibilities for measurement?



- Power law decay of correlations and crossover

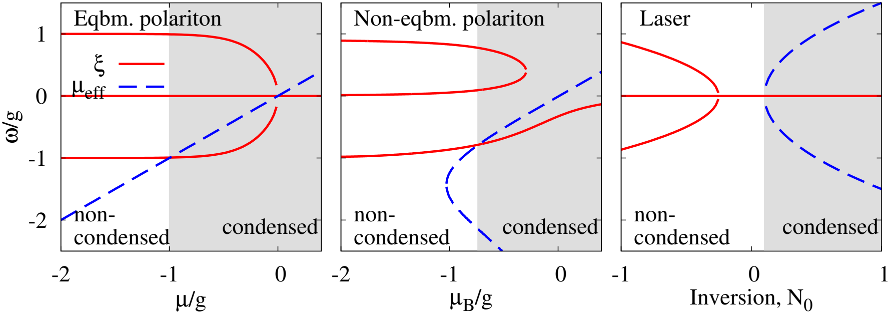




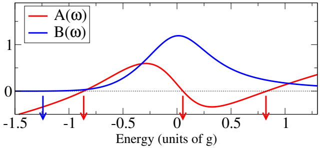
4 Relation to laser

5 Superfluid density

Strong coupling and lasing — low temperature phenomenon



- Laser: Uniformly invert TLS
- Non-equilibrium polaritons: Cold bath
- If $T_B \gg \gamma \rightarrow$ Laser limit



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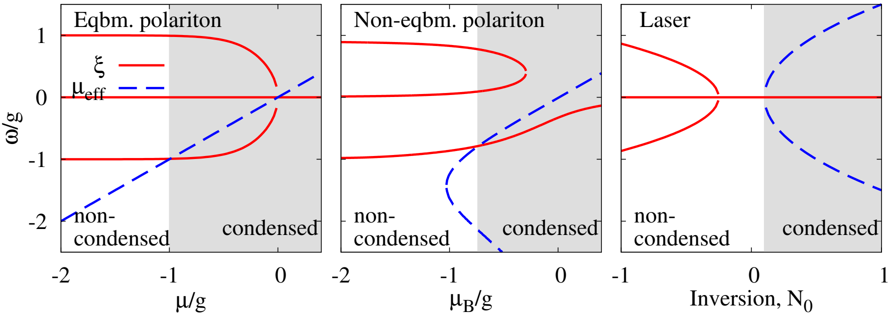

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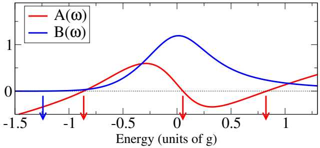

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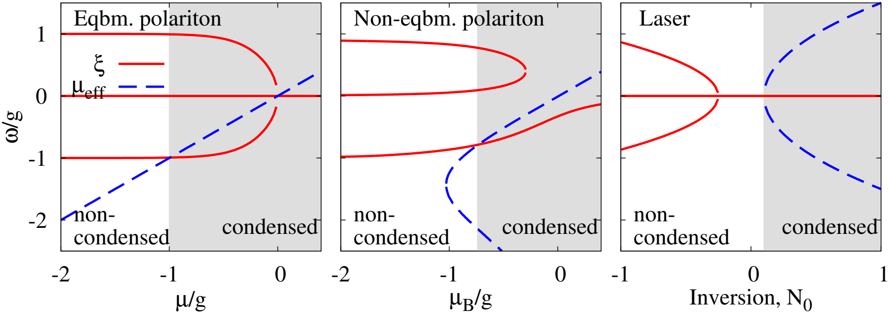

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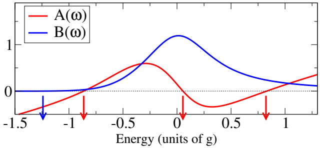
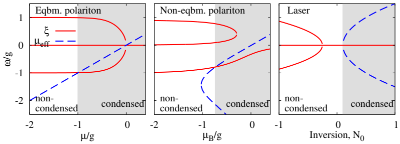


Coherence, inversion, strong-coupling

Polariton condensation:



- Inversionless
- **allows** strong coupling
- **requires** low $T \leftrightarrow$ condensation
- NB **NOT** thresholdless/single atom lasing.

- Circuit QED [Marthaler *et al.* PRL '11]
  - Noise-assisted
  - Off-resonant cavity
  - Emission/absorption $\Gamma^{\pm} \sim 2n_B(\pm\delta\omega) + 1$
  - Low $T \rightarrow$ inversionless threshold
- Photon condensation [Klaers *et al.* Nature '10]
  - Vibrational modes $\rightarrow$ thermalisation
  - Inversionless weak coupling lasing

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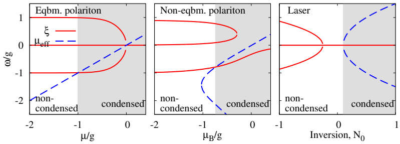

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Related *weak-coupling* **inversionless** lasing:

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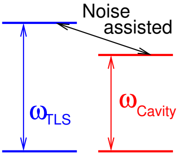

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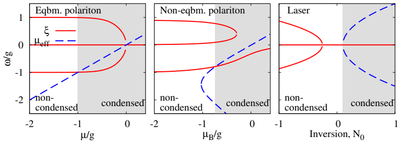

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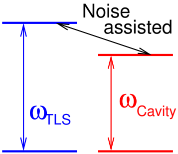

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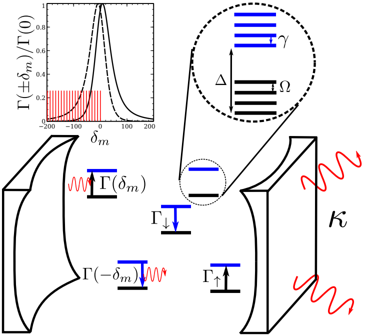
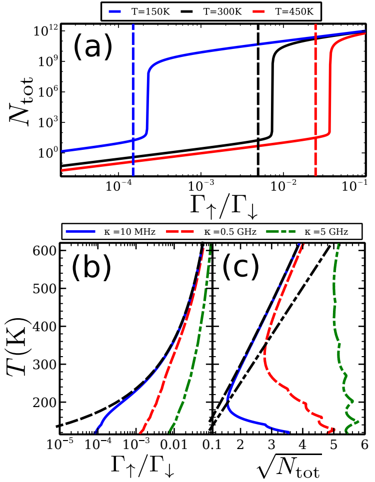
Photon condensation



$$\dot{\rho} = -i[H_0, \rho] - \sum_{i,m} \left\{ \frac{\kappa}{2}\mathcal{L}[a_m] + \frac{\Gamma_\uparrow}{2}\mathcal{L}[\sigma_i^+] + \frac{\Gamma_\downarrow}{2}\mathcal{L}[\sigma_i^-] + \frac{\Gamma(-\delta_m)}{2}\mathcal{L}[a_m^\dagger \sigma_i^-] + \frac{\Gamma(\delta_m)}{2}\mathcal{L}[a_m \sigma_i^+] \right\} \rho.$$

[Kirton & JK, on arXiv later this week]

Photon condensation



[Kirton & JK, on arXiv later this week]

Details of Superfluid density

- Generic structure of Green's function:

$$[D^R]^{-1} = \begin{pmatrix} \omega + i\gamma_{\text{net}} - \epsilon_k - \mu & i\gamma_{\text{net}} - \mu \\ -i\gamma_{\text{net}} - \mu & -\omega - i\gamma_{\text{net}} - \epsilon_k - \mu \end{pmatrix}$$

- Using Keldysh generating functional

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- f, θ couple as force/response current.

$$S[f,\theta] = S + \sum_{k,q} \begin{pmatrix} \bar\psi_{cl} & \bar\psi_{q} \end{pmatrix}_{k+q} \begin{pmatrix} \theta_i & f_i + \theta_i \\ f_i - \theta_i & -\theta_i \end{pmatrix}_q \frac{2k_i + q_i}{2m} \begin{pmatrix} \psi_{cl} \\ \psi_{q} \end{pmatrix}_k$$

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