Non-equilibrium coherence in light-matter systems

Condensation, lasing, superradiance and more

Jonathan Keeling

University of St Andrews 600 YEARS

SUPA

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Acknowledgements

GROUP:

COLLABORATORS: Szymanska, Littlewood, Simons, Bhaseen, Schmidt, Blatter, Türeci, Krüger

EXPERIMENT: Houck, Wallraff, Fink, Mylnek

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EPSRC Engineering and Physical Sciences Research Council

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Coupling many atoms to light

Old question: *What happens to radiation when many atoms interact "collectively" with light.*

Superradiance — dynamical and steady state.

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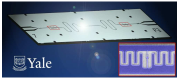
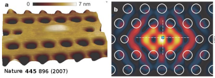
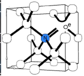
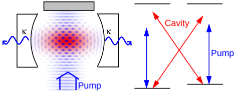
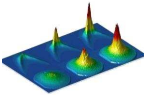
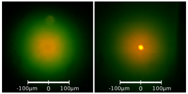
New relevance

- Superconducting qubits 

- Quantum dots & NV centres 


- Ultra-cold atoms 

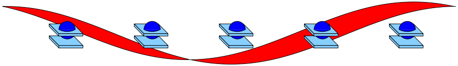
- Rydberg atoms/polaritons
- Microcavity Polaritons 
- Photon condensation 

Dicke effect: Enhanced emission

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

R. H. Dicke
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey



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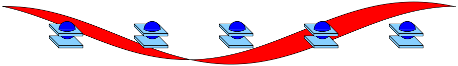

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Collective decay:

$$\frac{d\rho}{dt} = -\frac{\Gamma}{2} \left[S^+ S^- \rho - S^- \rho S^+ + \rho S^+ S^- \right]$$

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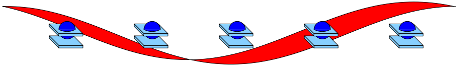

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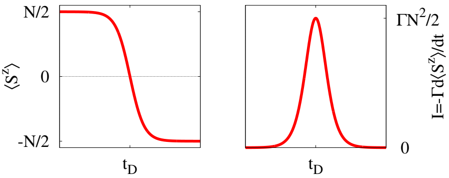

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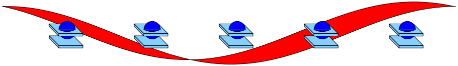

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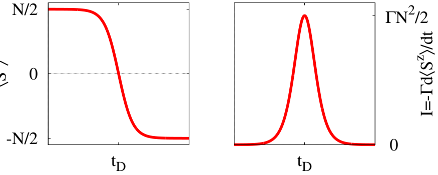
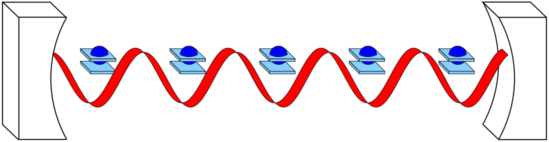


Problem: dipole interactions dephase. [Friedberg et al, Phys. Lett. 1972]

Collective radiation **with a cavity**: Dynamics



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Single cavity mode: oscillations

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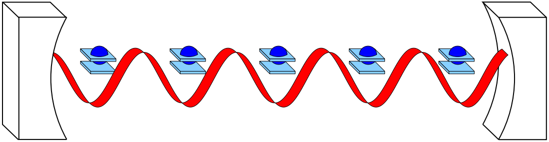

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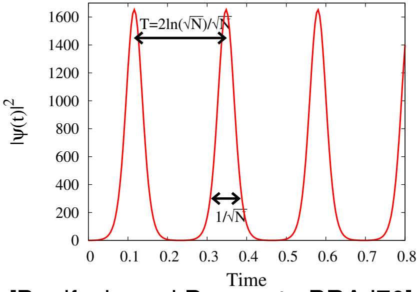

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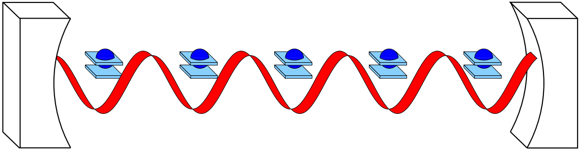
Dicke model: Equilibrium superradiance transition



$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g\left(\psi^\dagger S^- + \psi S^+\right).$$

- Coherent state: $|\Psi\rangle \to e^{\lambda\psi^\dagger + \eta S^+}|\Omega\rangle$
- Small g, min at $\lambda, \eta = 0$

[Hepp, Lieb, Ann. Phys. ’73]

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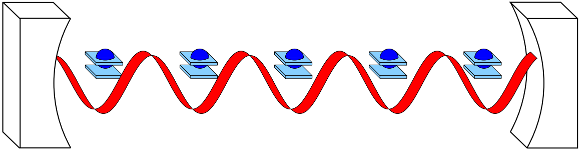

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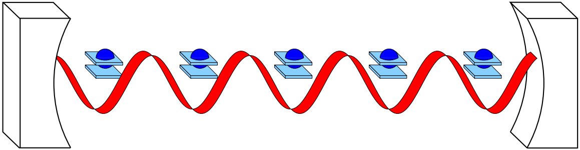

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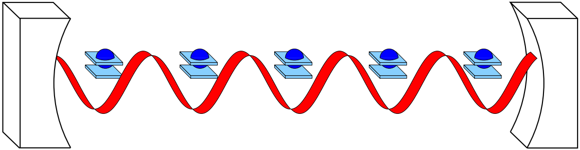

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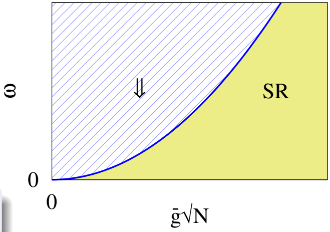

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Dicke model: Superradiance at $T \neq 0$

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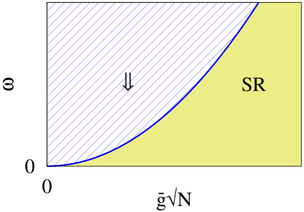

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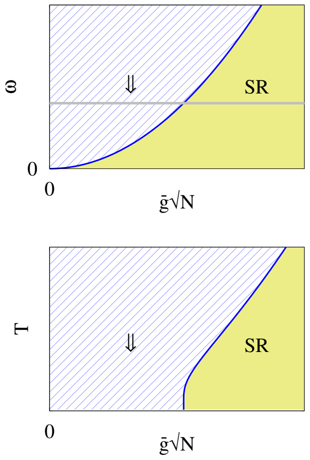

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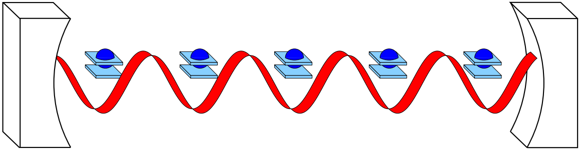
No go theorem and transition



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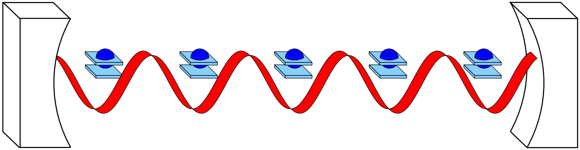

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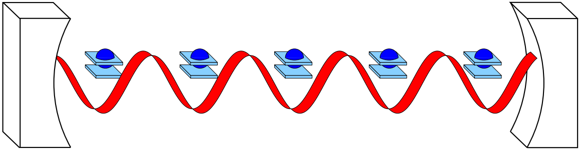

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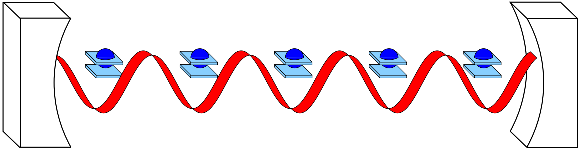

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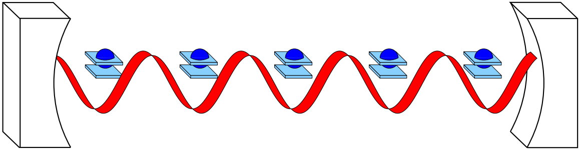

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But Thomas-Reiche-Kuhn sum rule states: $g^2/\omega_0 < 2\zeta$. **No transition**

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Dicke phase transition: ways out

Problem: $g^2/\omega_0 < 2\zeta$ for intrinsic parameters. **Solutions**:

- **Interpretation**
  Ferroelectric transition in $\mathbf{D}\cdot\mathbf{r}$ gauge.
  [JK JPCM '07, Vukics & Domokos PRA 2012]
- Circuit QED [Nataf and Ciuti, Nat. Comm. '10; Viehmann *et al.* PRL '11]
- Grand canonical ensemble:
  - If $H \to H - \mu(S^z + \psi^\dagger\psi)$, need only: $g^2 N > (\omega - \mu)(\omega_0 - \mu)$
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- Dissociate g, ω_0,
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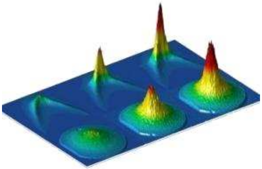

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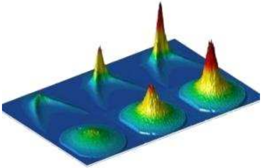

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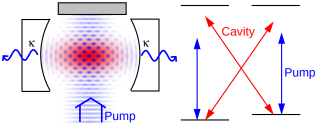

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   - Photon condensation
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   - JCHM vv Dicke
   - Coherently driven array
   - Disorder
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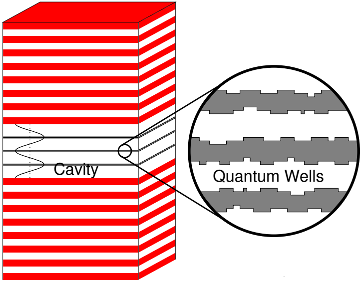

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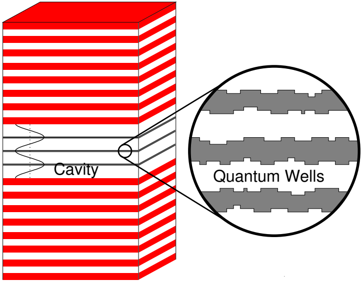

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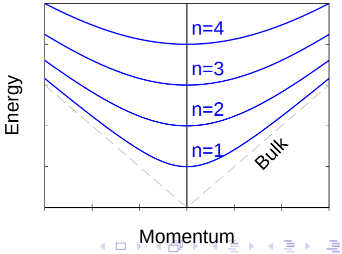

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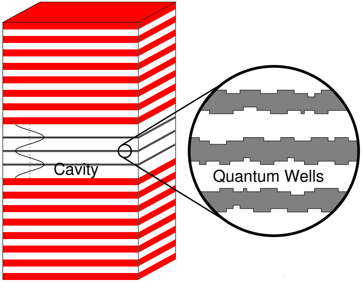

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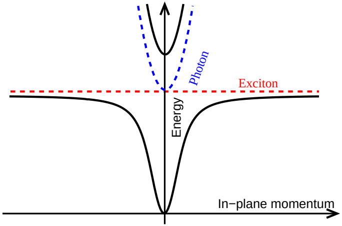

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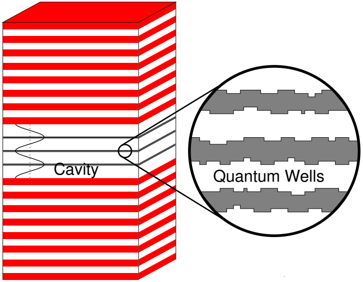

Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
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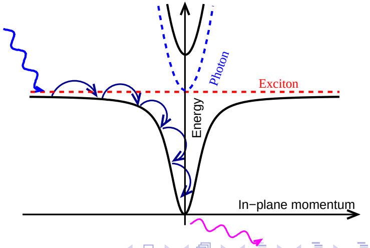

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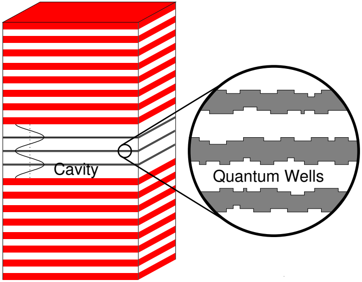

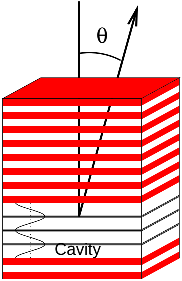

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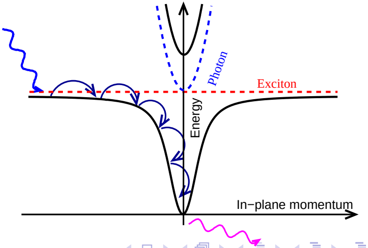

Lasing-condensation crossover model

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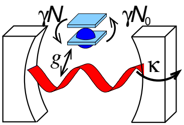
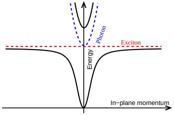


$\Leftrightarrow$



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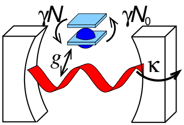

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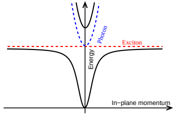

Dicke model:

$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon S^{z}_{\alpha} + \frac{\left(g_{\alpha,\mathbf{k}} \psi_{\mathbf{k}} S^{+}_{\alpha} + \text{H.c.}\right)}{\sqrt{A}} \right]$$

Polariton model and equilibrium results

Localised excitons, propagating photons

$$H - \mu N = \sum_{\mathbf{k}} (\omega_{\mathbf{k}} - \mu)\psi^\dagger_{\mathbf{k}}\psi_{\mathbf{k}} + \sum_{\alpha} (\epsilon_\alpha - \mu) S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} S^+_\alpha + \text{H.c.}$$

Self-consistent polarisation and field

$$(\omega - \mu)\,\psi = \frac{1}{A}\sum_{\alpha} \frac{g^2_\alpha \psi}{2E_\alpha} \tanh(\beta E_\alpha)\,, \quad {E_\alpha}^2 = \left(\frac{\epsilon_\alpha - \mu}{2}\right)^2 + g^2_\alpha |\psi|^2$$

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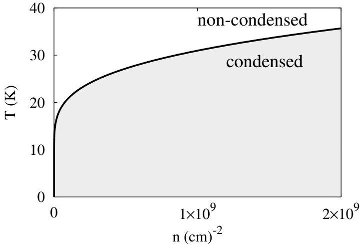

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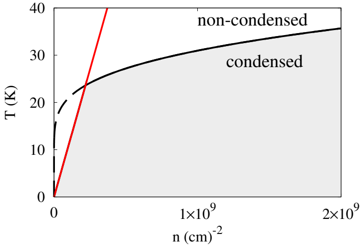

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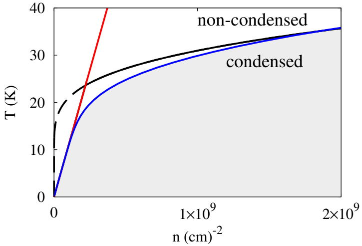

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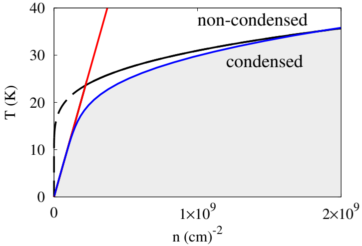
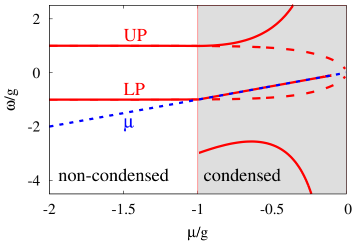
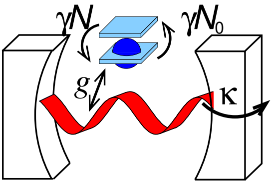


Modes (at $k = 0$)



Simple Laser: Maxwell Bloch equations

$$H = \omega\psi^\dagger\psi + \sum_\alpha \epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}}\psi S^+_\alpha + \text{H.c.}$$



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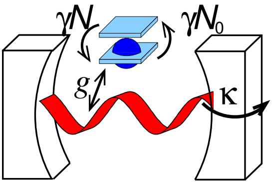

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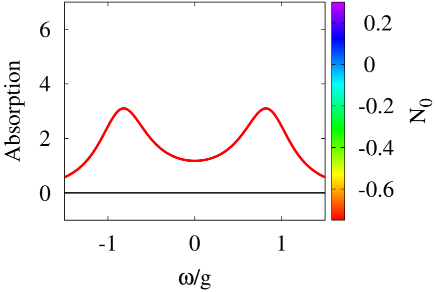

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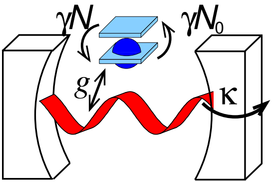

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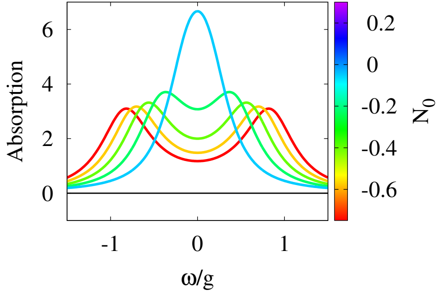

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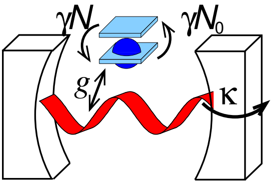

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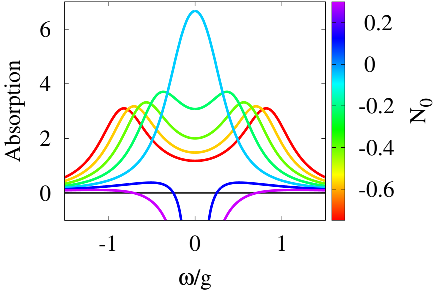

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Poles of Retarded Green's function and gain

$$\left[D^R(\nu)\right]^{-1} = \nu - \omega_k + i\kappa + \frac{g^2 N_0}{\nu - 2\epsilon + i2\gamma}$$

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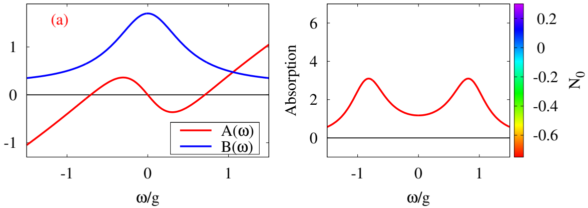

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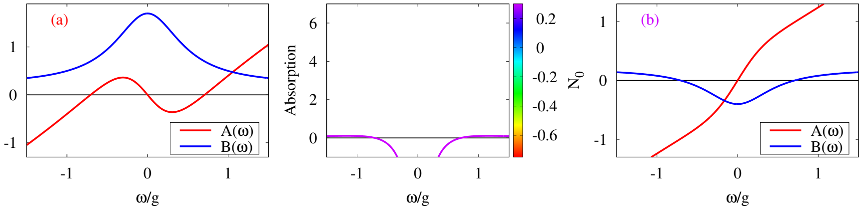

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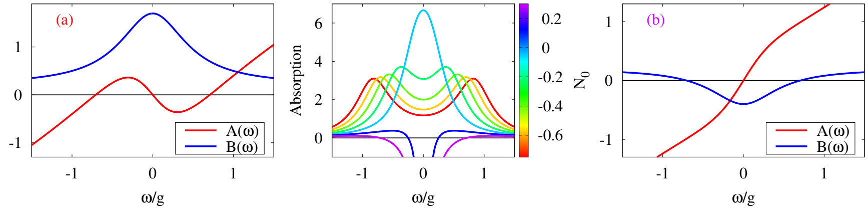
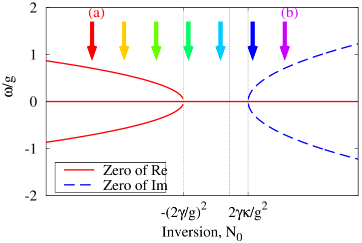


Laser:



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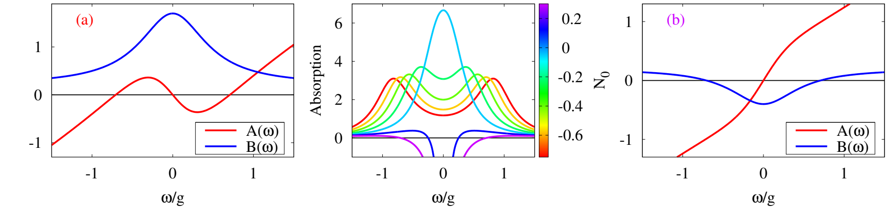

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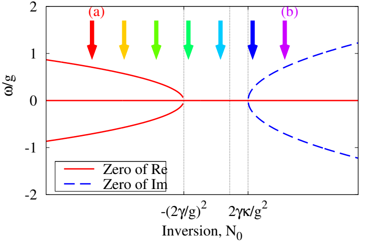
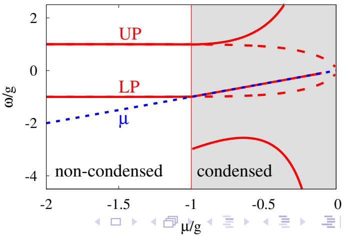
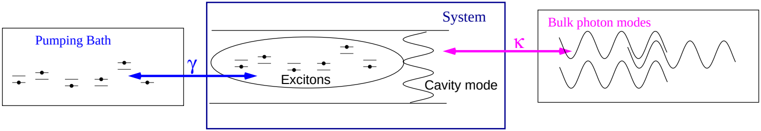
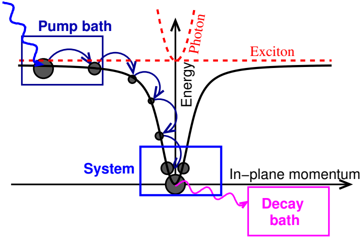


Equilibrium:



Non-equilibrium description: baths





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- Pump bath: Thermal μ_B, T_B

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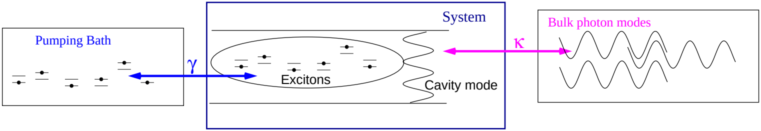

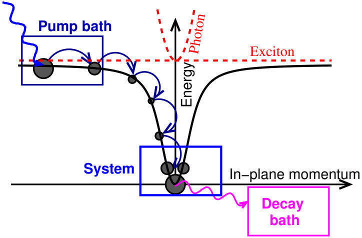

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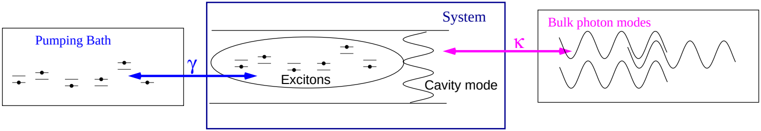

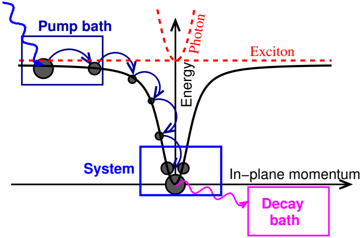

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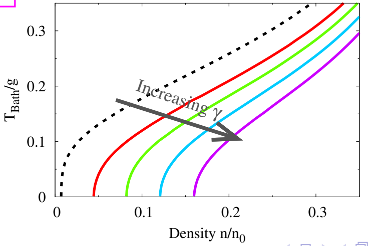
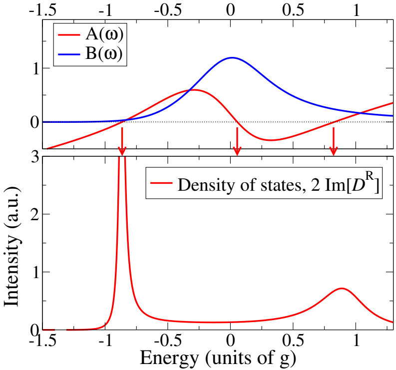
Mean field theory



Stability and evolution with pumping



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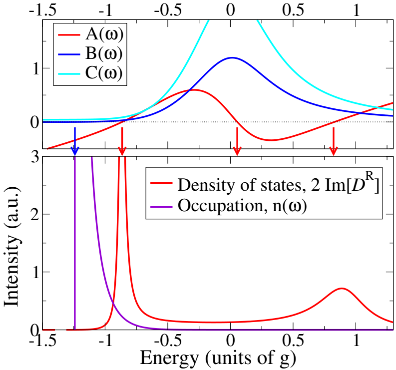

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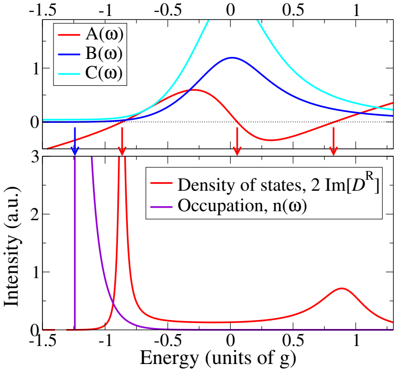

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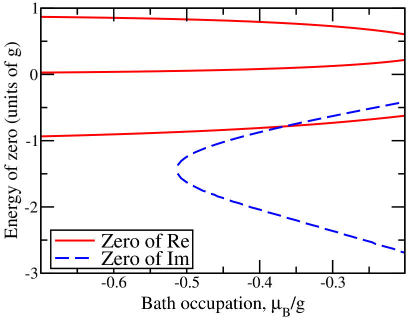
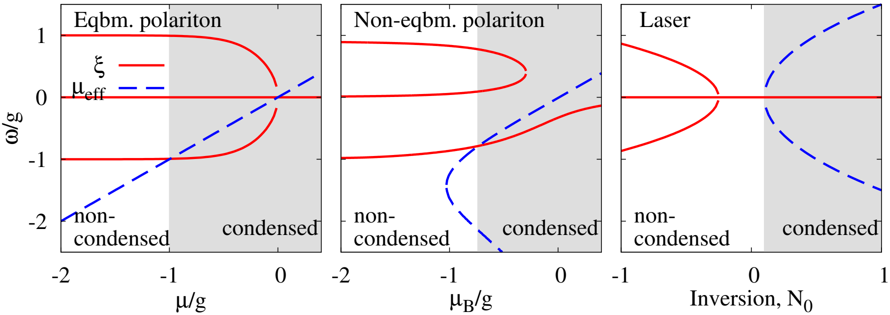
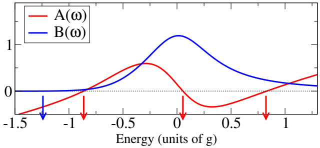


Strong coupling and lasing — low temperature phenomenon



- Laser: Uniformly invert TLS
- Non-equilibrium polaritons: Cold bath
- If $T_B \gg \gamma \rightarrow$ Laser limit



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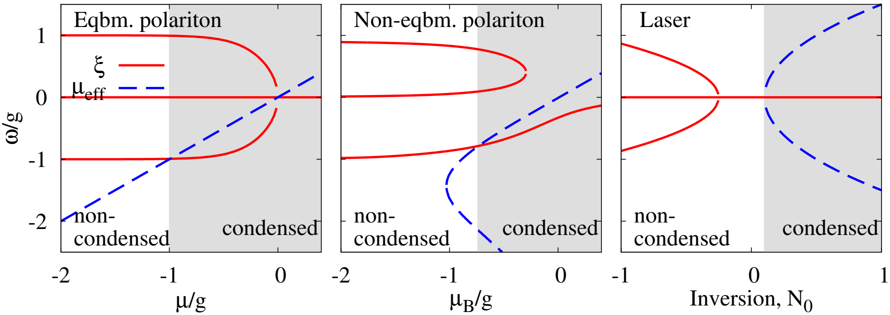

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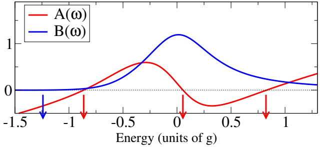

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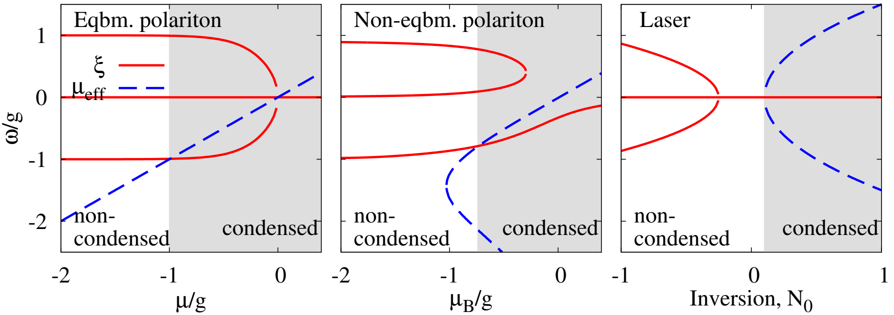

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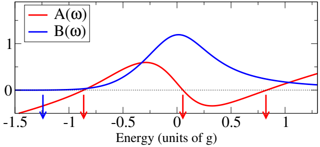
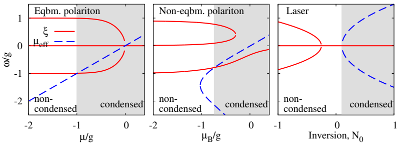


Coherence, inversion, strong-coupling

Polariton condensation:

- Inversionless
- **allows** strong coupling
- **requires** low $T \leftrightarrow$ condensation
- NB **NOT** thresholdless/single atom lasing.

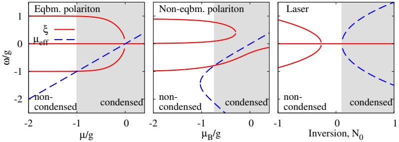


- Circuit QED [Marthaler *et al.* PRL '11]
  - Noise-assisted
  - Off-resonant cavity
  - Emission/absorption $\Gamma^{\pm} \sim 2n_B(\pm\delta\omega) + 1$
  - Low $T \rightarrow$ inversionless threshold

Coherence, inversion, strong-coupling

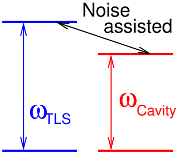
Polariton condensation:

- Inversionless
- **allows** strong coupling
- **requires** low $T \leftrightarrow$ condensation
- NB **NOT** thresholdless/single atom lasing.



Related *weak-coupling* **inversionless** lasing:

- Circuit QED [Marthaler *et al.* PRL '11]



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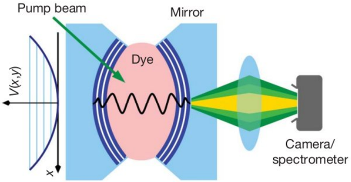
Polariton and photon Condensation

Photon BEC experiments



- Dye filled microcavity
- Pump at angle
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- Condensation:
  - Far below inversion
  - Thermalised emission spectrum

[Klaers et al, Nature, 2010]

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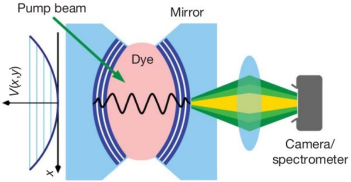

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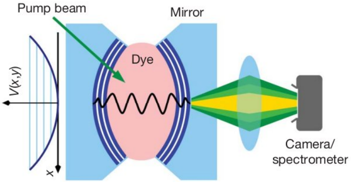

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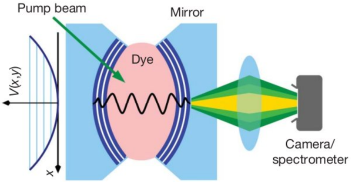

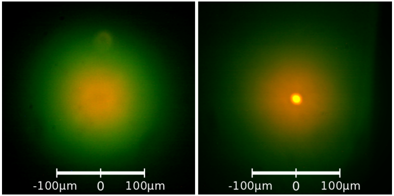

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Modelling

$$H_{\text{sys}} = \sum_m \omega_m \psi_m^\dagger \psi_m + \sum_\alpha \left[\epsilon S_\alpha^z + g \left(\psi_m S_\alpha^+ + \text{H.c.} \right) \right.$$

$$\left. \right]$$

- Consider harmonic cavity modes
  $\omega_m = \omega_{\text{cutoff}} + m\omega_{H.O.}$
- Add local vibrational mode
- Integrate out phonon effects
  - Polaron transform
  - Perturbation theory in g

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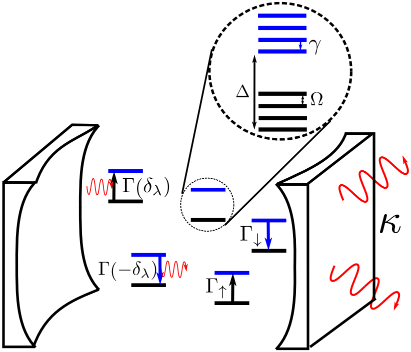

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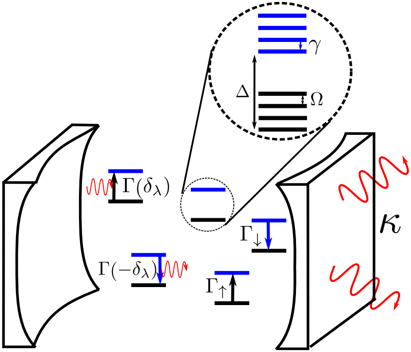

Modelling

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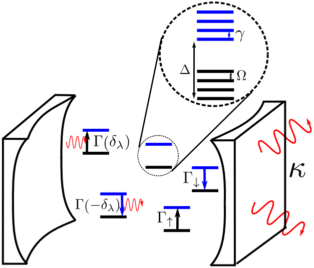

- $\Gamma(+\delta) \simeq \Gamma(-\delta)e^{-\beta\delta}$
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[Marthaler et al PRL '11, Kirton & JK arXiv:1303.3459]

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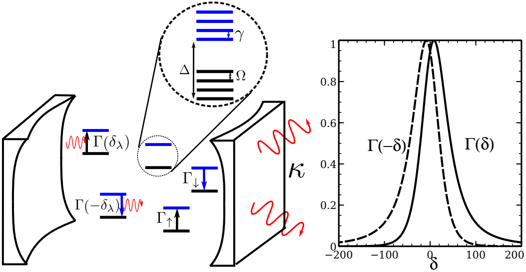

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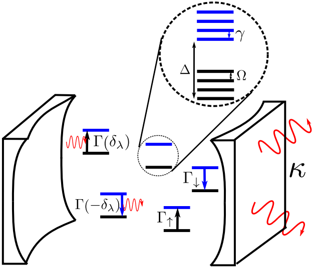

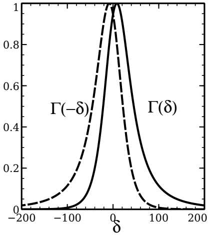

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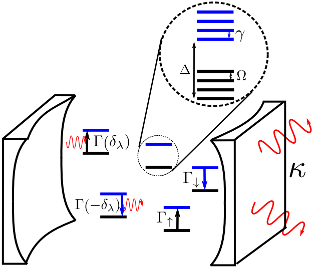

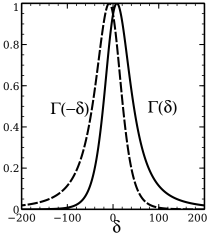

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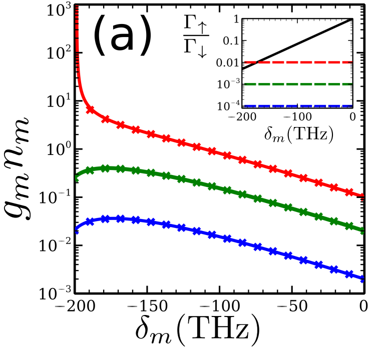

Low loss: Thermal

[Kirton & JK arXiv:1303.3459]

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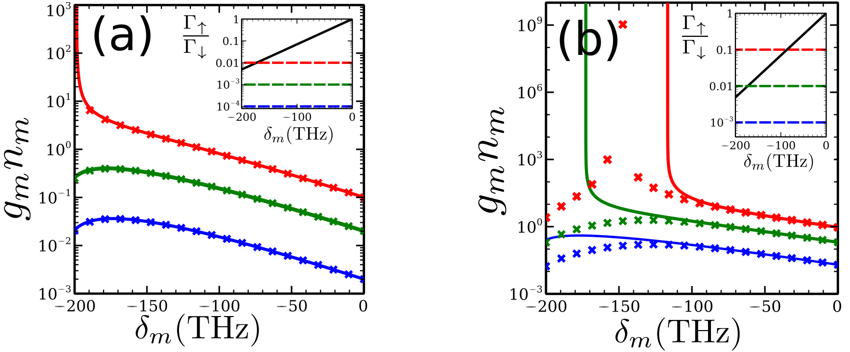
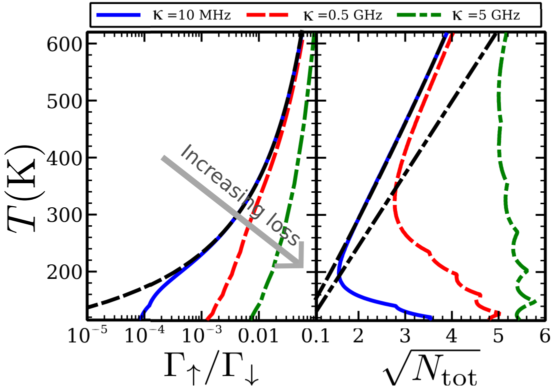


Low loss: Thermal

High loss → Laser

[Kirton & JK arXiv:1303.3459]

Threshold condition



Compare threshold:

- Pump rate (Laser)
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- Thermal at low κ/high temperature
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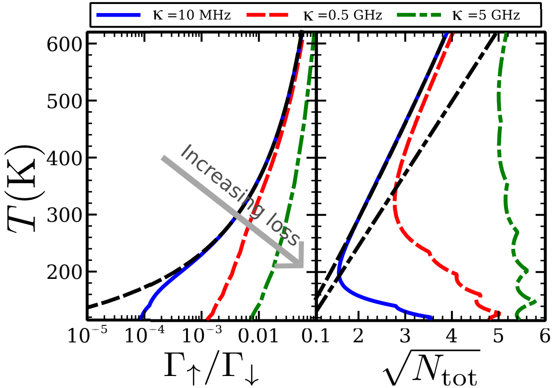

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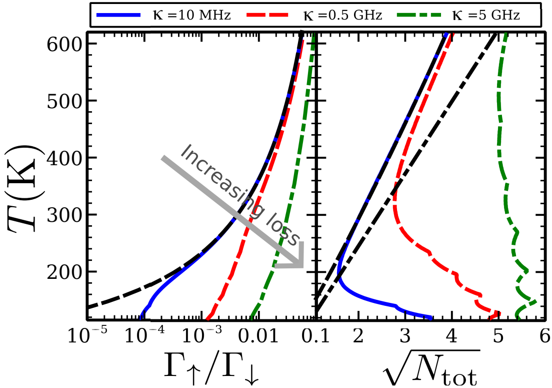

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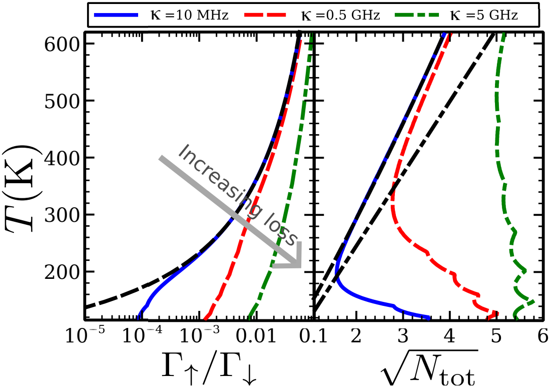

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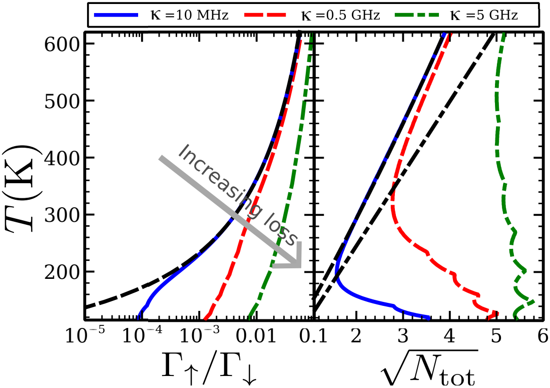

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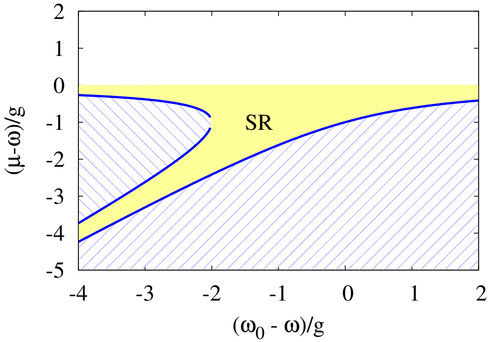
Jaynes Cummings Hubbard model

Equilibrium: Dicke model with chemical potential

$$H - \mu N = (\omega - \mu)\psi^\dagger\psi + (\omega_0 - \mu)S^z + g\left(\psi^\dagger S^- + \psi S^+\right)$$



- Transition at: $g^2 N > (\omega - \mu)(\omega_0 - \mu)$
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[Eastham and Littlewood, PRB ’01]

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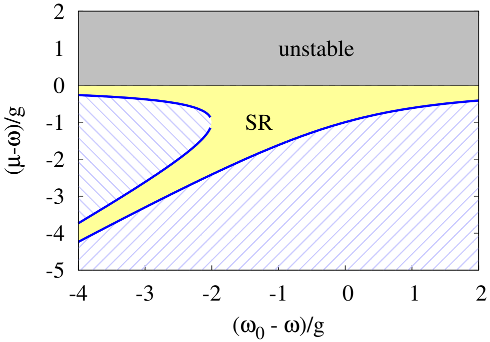

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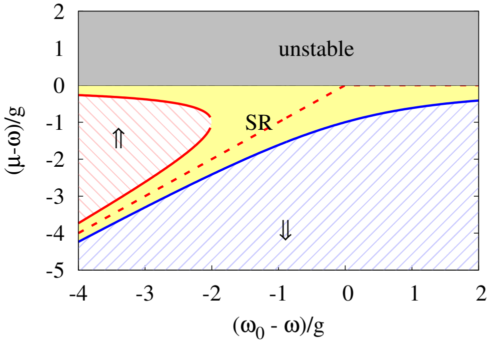

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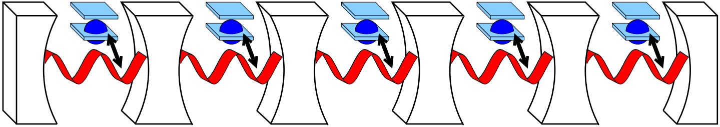

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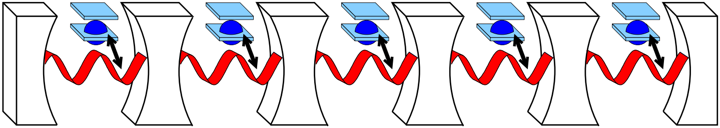

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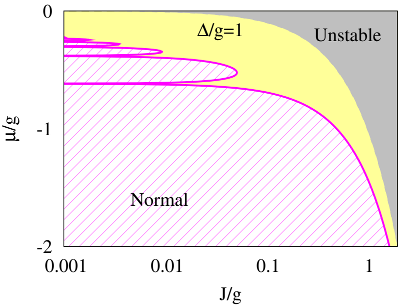

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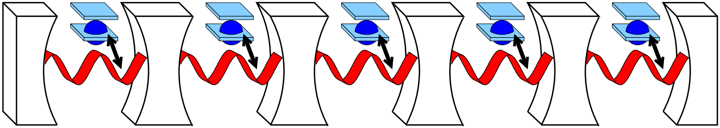

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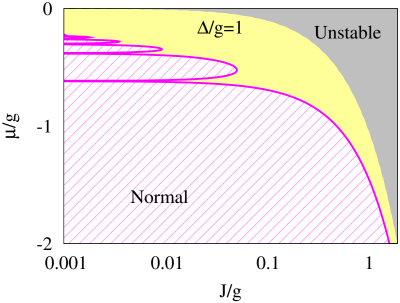
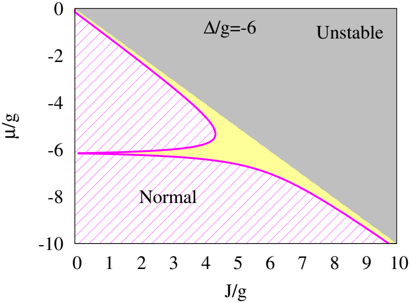
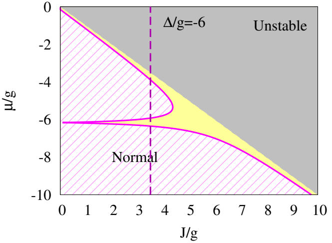


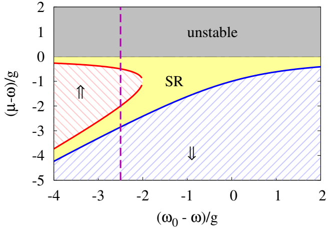


Dicke vs JCHM

JCHM



Dicke



- $k = 0$ mode of JCHM ↔ Dicke photon mode
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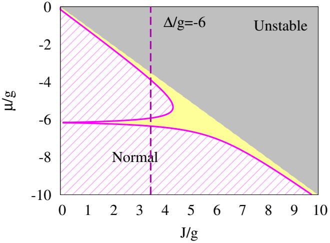

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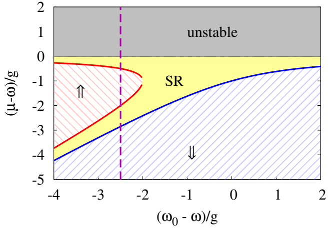

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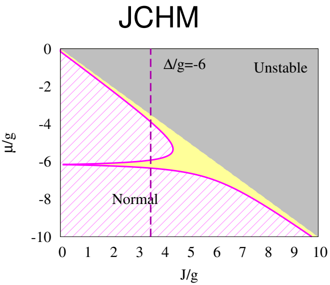

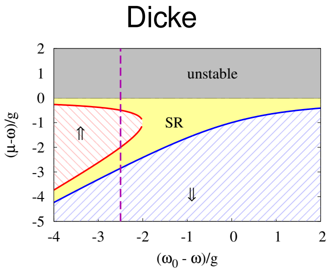

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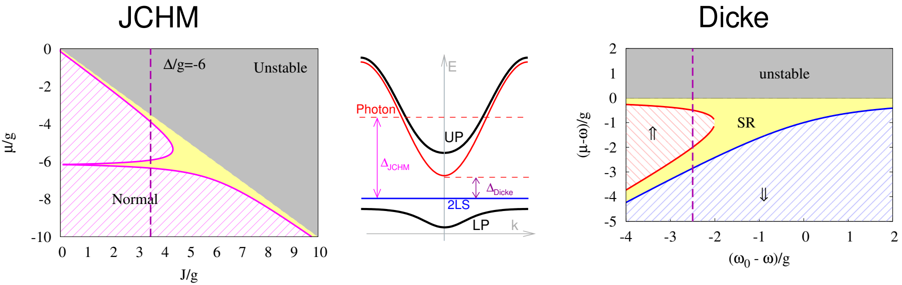

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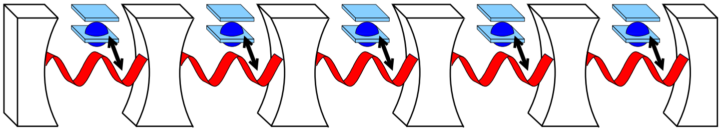
Jaynes Cummings Hubbard model

1. Dicke model and superradiance
2. Polariton and photon condensation
   - Polaritons
   - Non-equilibrium condensation vs lasing
   - Photon condensation
3. Jaynes Cummings Hubbard model
   - JCHM vv Dicke
   - **Coherently driven array**
   - Disorder
4. Phase transitions with SC qubits
   - Pumping without symmetry breaking
   - Collective dephasing

Coherently pumped JCHM



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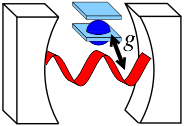
$$\partial_t\rho = -i[H,\rho] - \frac{\kappa}{2}L_\psi[\rho] - \frac{\gamma}{2}L_{\sigma^-}[\rho]$$

Coherently pumped single cavity [Bishop *et al.* Nat. Phys '09]



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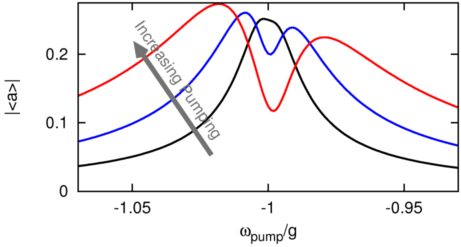
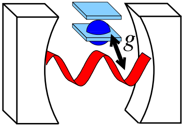
- Anti-resonance in $|\langle\psi\rangle|$.
- Effective 2LS: $|\text{Empty}\rangle, |1 \text{ polariton}\rangle$

- Mollow triplet fluorescence



[Lang *et al.* PRL '11]

Coherently pumped single cavity [Bishop *et al.* Nat. Phys '09]



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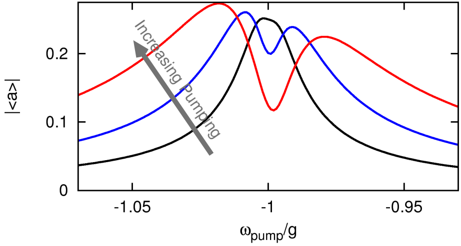

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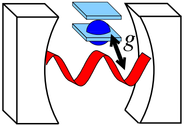

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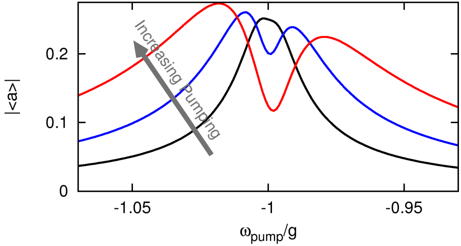

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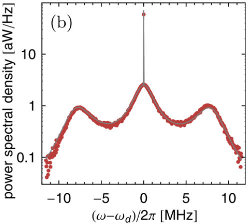

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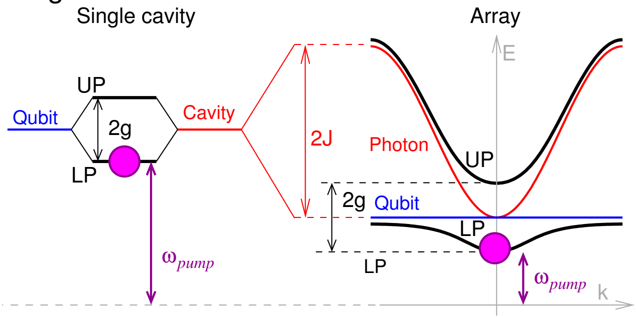
Coherently pumped dimer & array

Chose detuning *a la* Dicke model



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[Nissen *et al.* PRL '12]

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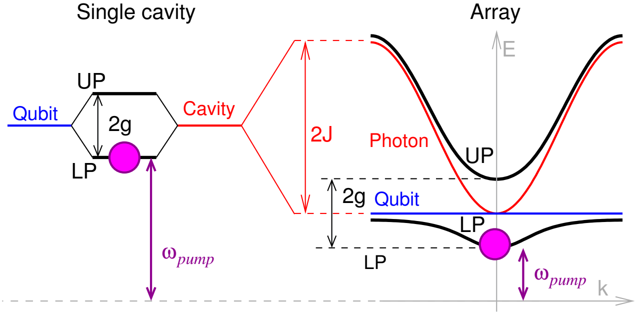

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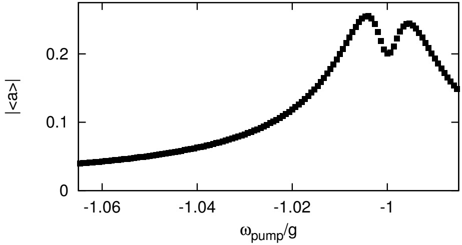

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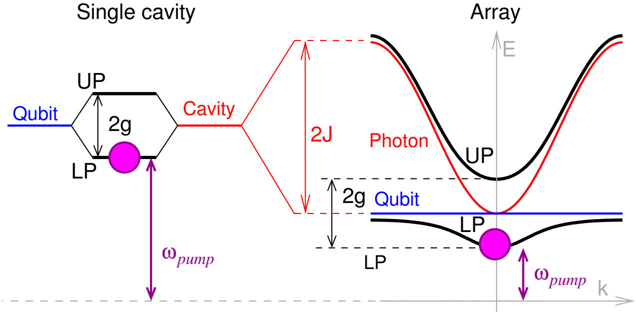

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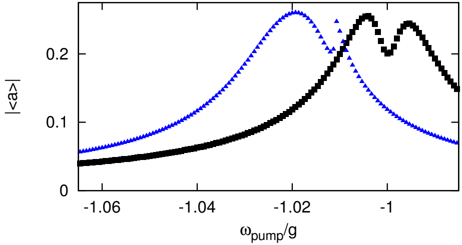

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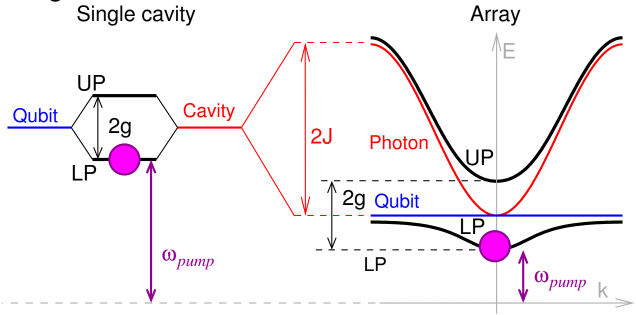

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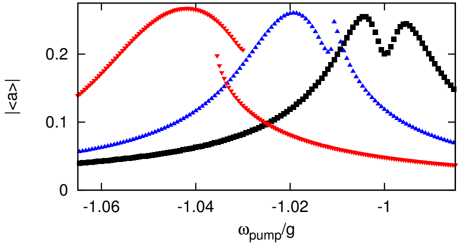

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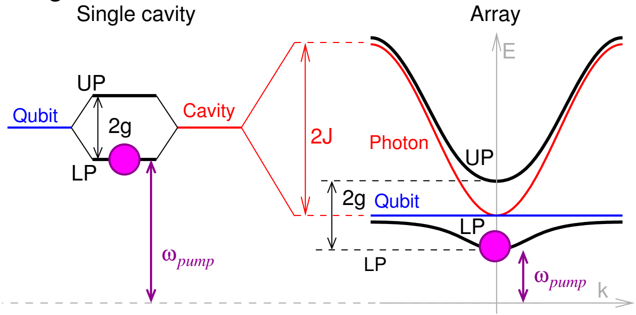

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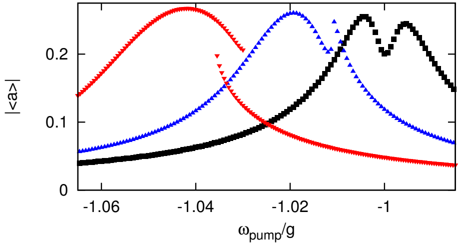

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Photon blockade picture $J \lesssim g$

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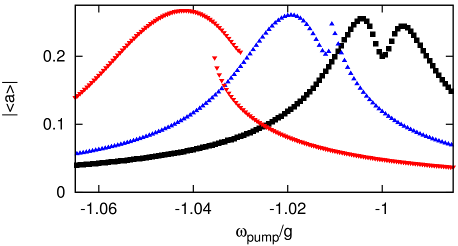

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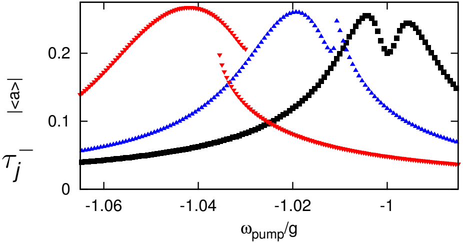

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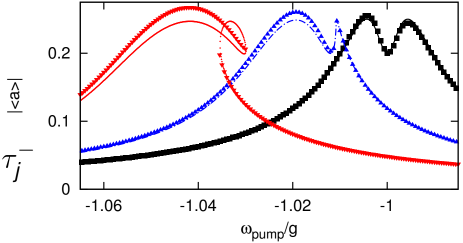

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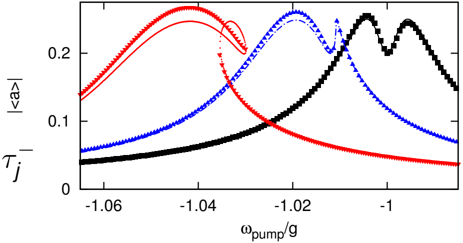

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Coherent pumped array – disorder

- Effect of disorder, $\Delta \to \Delta_i$
  - Distribution of ψ – Washes out bistable jump
- Bistability near resonance — phase of ψ depends on Δ_i
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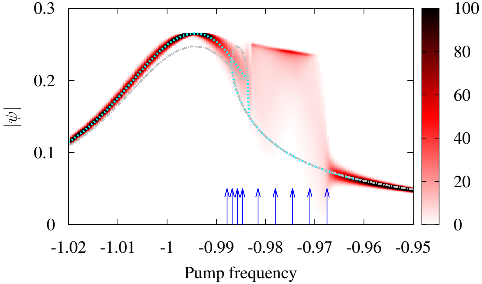
- Superfluid phases in driven system?



[Kulaitis *et al.* PRA, ’13]

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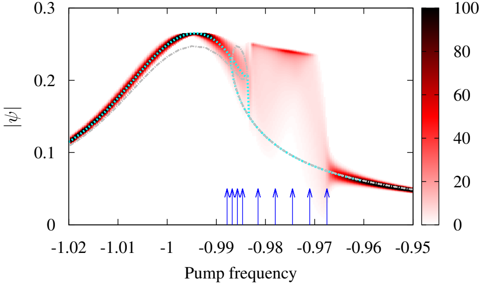

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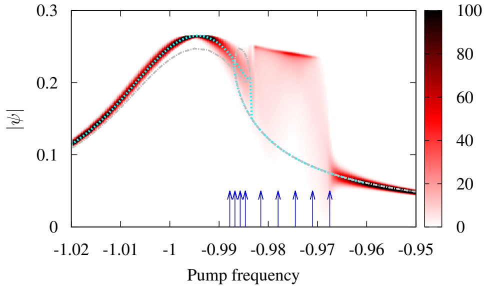

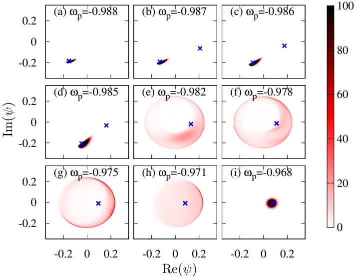

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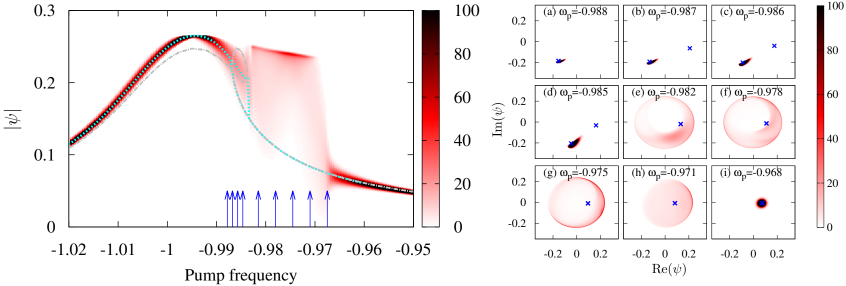

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Phase transitions with SC qubits

Raman pumping

- How to pump without breaking symmetry
- Counter-rotating terms — Raman pumping
  - Atom proposal [Dimer *et al.* PRA '07]
  - Atom experiment [Baumann *et al.* Nature '10]
- Qubit — allowed transitions $\Delta n = 1$
- Qubit dephasing much bigger than atom

JK, Türeci, Houck in progress

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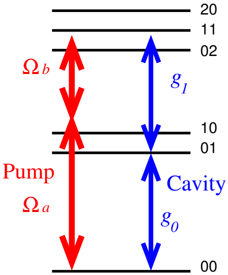

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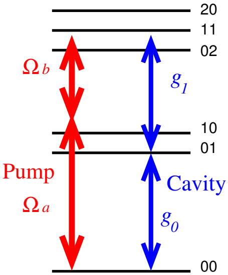

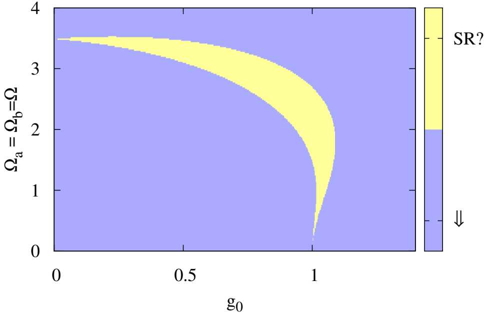

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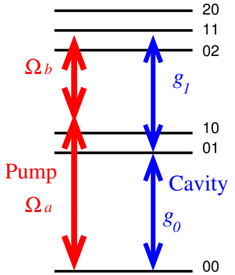

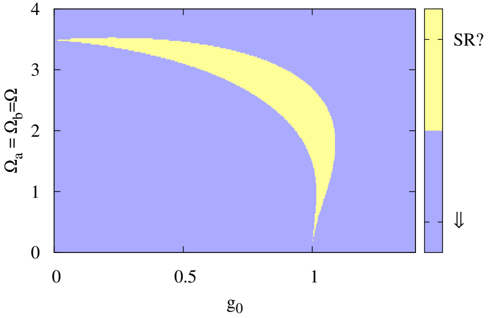

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Collective dephasing

- Real environment is not Markovian
  - [Carmichael & Walls JPA '73] Requirements for correct equilibrium
  - [Ciuti & Carusotto PRA '09] Dicke SR and emission
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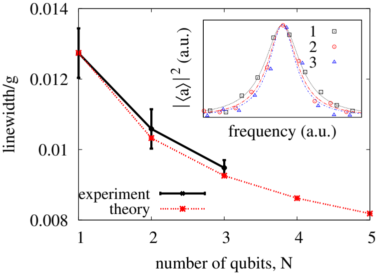

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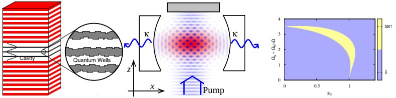
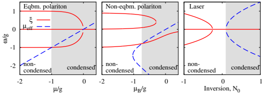
Summary

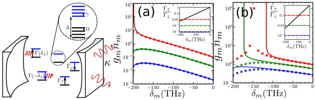
- Non-equilibrium Dicke relevant to increasing number of systems



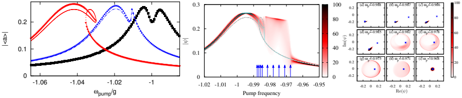
- Polariton condensation vs lasing



- Photon condensation and thermalisation



- Pumped coupled cavity array — bistability and disorder



- Future prospects – SC cavity array transitions

Extra slides

Ferroelectric transition

Atoms in Coulomb gauge

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Two-level systems — dipole-dipole coupling

$$H = \omega_0 S^z + \omega\psi^\dagger\psi + g(S^+ + S^-)(\psi + \psi^\dagger) + N\zeta(\psi + \psi^\dagger)^2 - \eta(S^+ - S^-)^2$$

(nb $g^2, \zeta, \eta \propto 1/V$).

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Gauge transform to dipole gauge $\mathbf{D} \cdot \mathbf{r}$

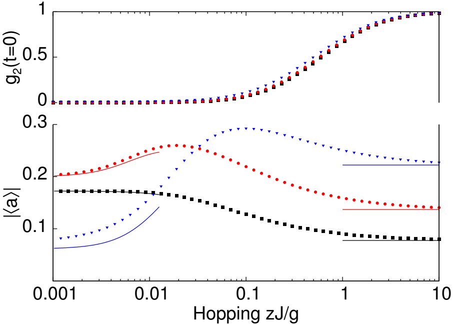
$$H = \omega_0 S^z + \omega\psi^\dagger\psi + \bar{g}(S^+ - S^-)(\psi - \psi^\dagger)$$

"Dicke" transition at $\omega_0 < N\bar{g}^2/\omega \equiv 2\eta N$

But, ψ describes **electric displacement**

Extra slides

Coherently pumped array: correlations & fluorescence



Correlations

- $g_2 : 0 \rightarrow 1$ crossover.

- Small J: Mollow triplet
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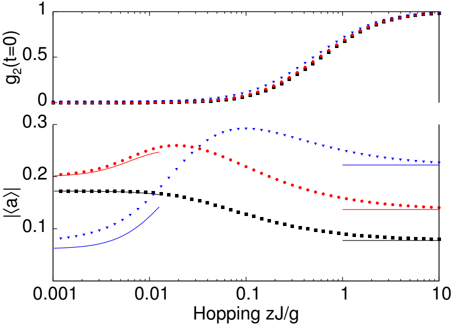

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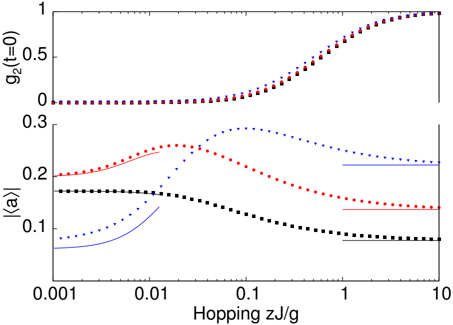

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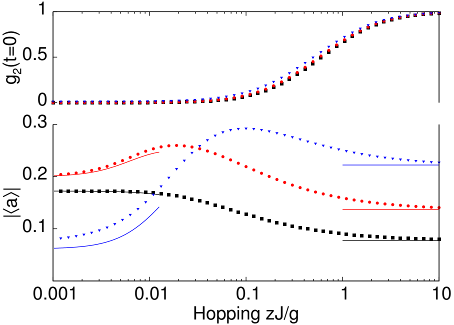

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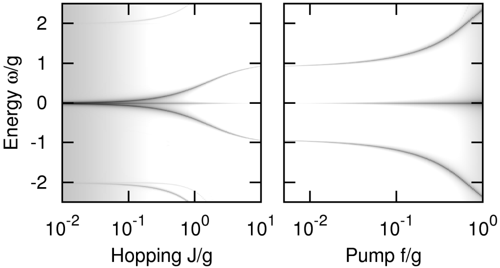

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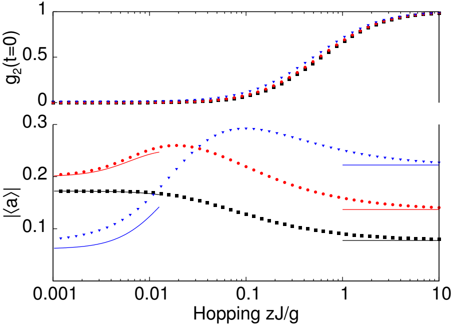

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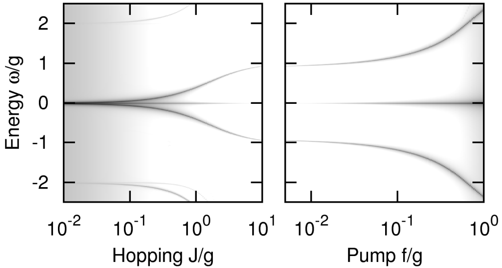
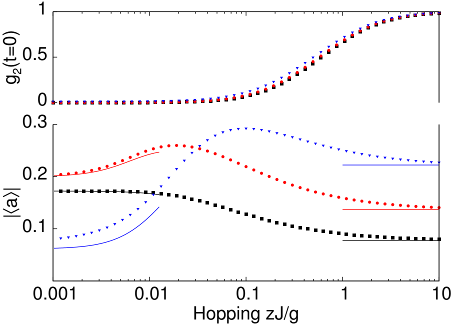
- Small J: Mollow triplet



Coherently pumped array: correlations & fluorescence

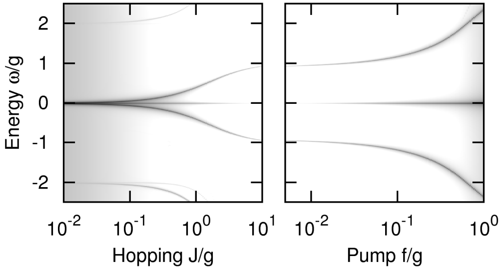


Correlations

- $g_2 : 0 \to 1$ crossover.

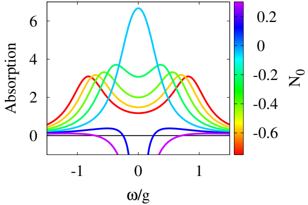
Fluorescence

- Small J: Mollow triplet
- Large J: Off resonance fluorescence
  - Pump at collective resonance
  - Mismatch if $J \neq 0$.



Extra slides

Maxwell-Bloch Equations: Retarded Green's function



- Introduce $D^R(\omega)$:
  Response to perturbation
- Absorption $= -2\Im[D^R(\omega)]$

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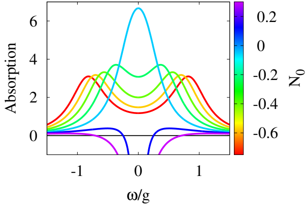

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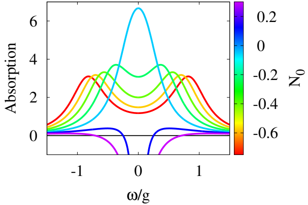

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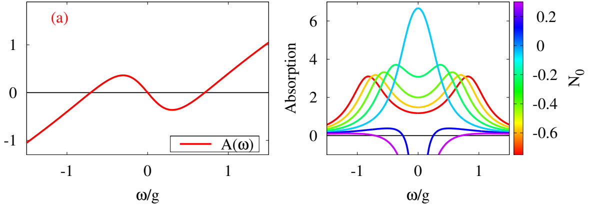

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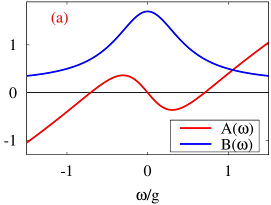

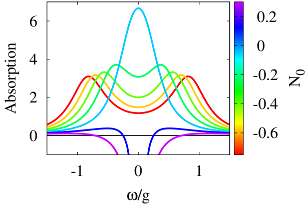

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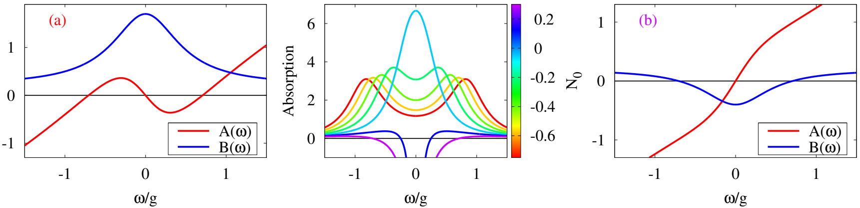

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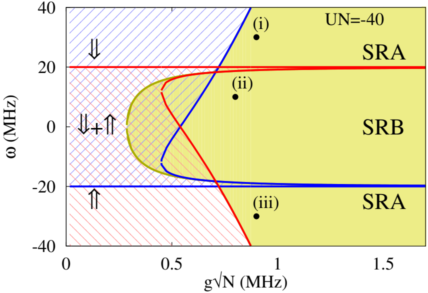
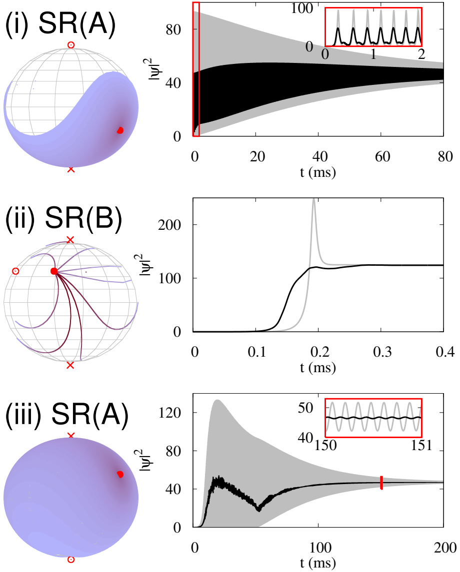
Extra slides

Dynamics: Evolution from normal state

Gray: $\mathbf{S} = (\sqrt{N}, \sqrt{N}, -N/2)$
Black: Wigner distribution of $\mathbf{S}$, ψ



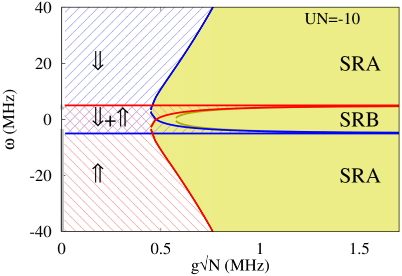
Oscillations: ~ 0.1ms
Decay: 20ms, 0.1ms, 20ms



Asymptotic state: Evolution from normal state

(Near to experimental $UN = -13$MHz).

All stable attractors:



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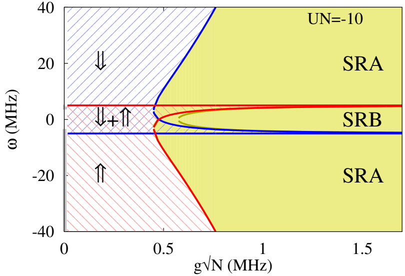
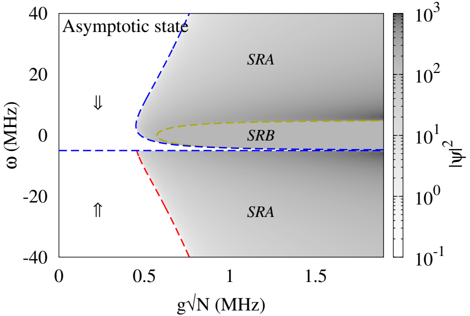
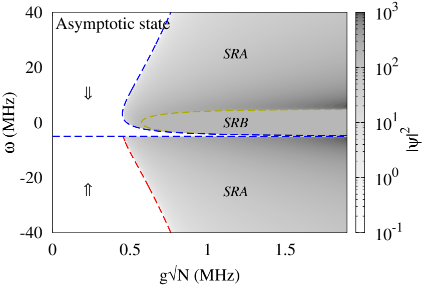
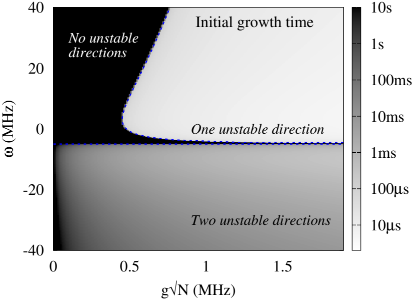


Starting from ⇓



Timescales for dynamics: What are they?





Growth Most unstable eigenvalues near $\mathbf{S} = (0, 0, -N/2)$

Decay Slowest stable eigenvalues near final state

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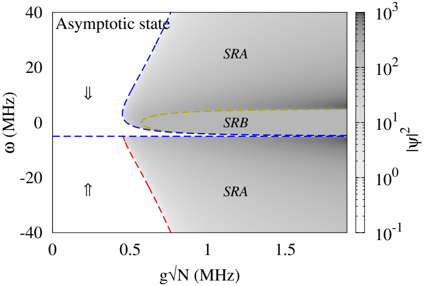

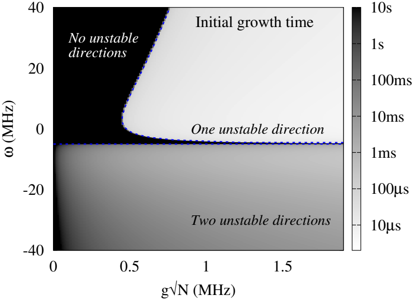

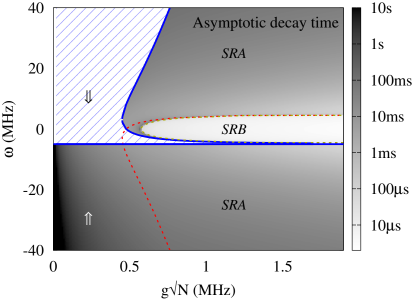

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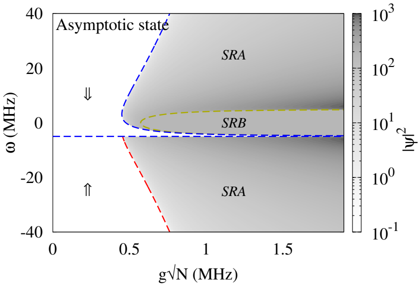
Timescales for dynamics: Consequences for experiment



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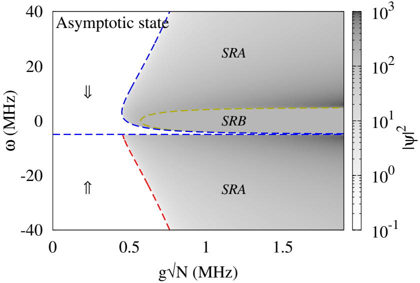

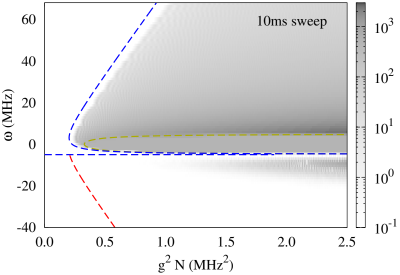

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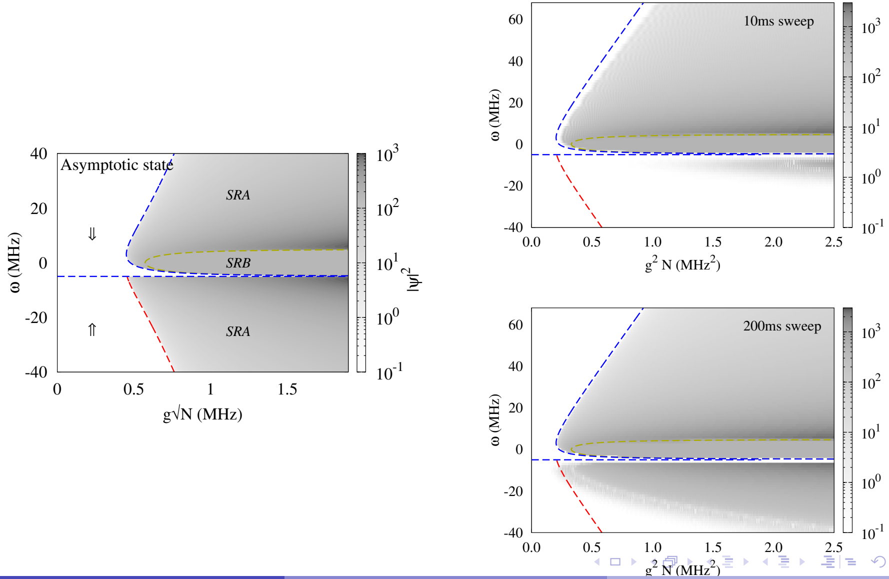
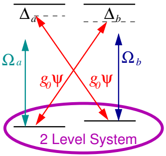


Timescales for dynamics: Why so slow and varied?

Suppose co- and counter-rotating terms differ



$$H = \ldots + g(\psi^\dagger S^- + \psi S^+) + g'(\psi^\dagger S^+ + \psi S^-) + \ldots$$

- SR(A) near phase boundary at small δg → Critical slowing down
- SR(A), SR(B) continuously connect

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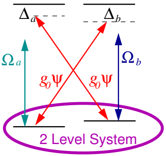

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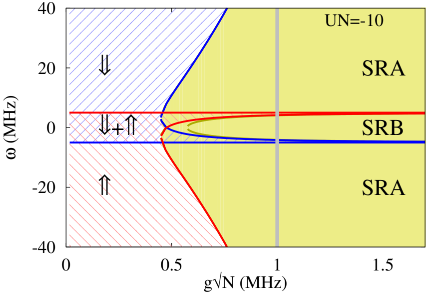

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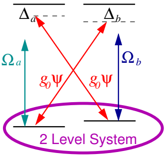

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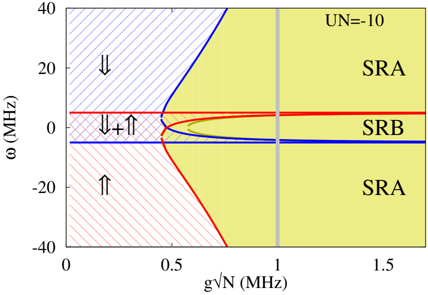

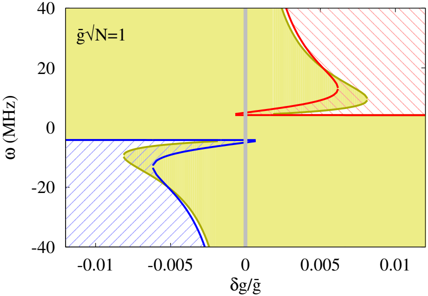

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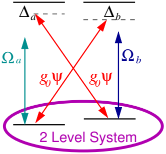

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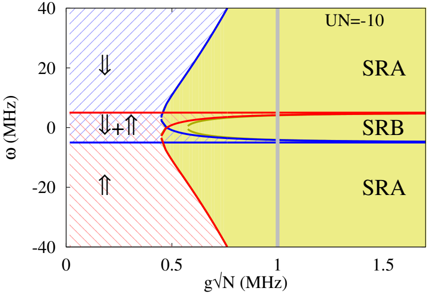

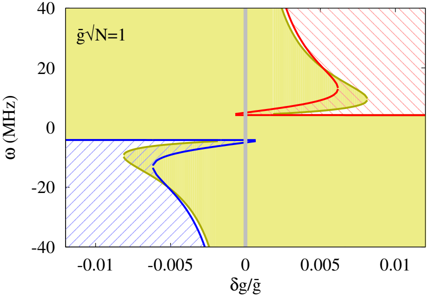

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