Non-equilibrium coherence in light-matter systems

Condensation, lasing and the superradiance transiton

Jonathan Keeling

University of St Andrews 600 YEARS

SUPA

PCTP, November 2012

Acknowledgements

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Coupling many atoms to light

Old question: *What happens to radiation when many atoms interact "collectively" with light.*

Superradiance — dynamical and steady state.

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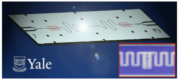
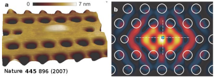
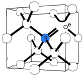
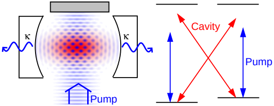
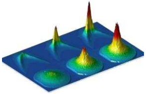
New relevance

- Superconducting qubits 

- Quantum dots 

- Nitrogen-vacancies in diamond 
- Ultra-cold atoms 

- Rydberg atoms
- Microcavity Polaritons 

Dicke effect: Enhanced emission

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

R. H. DICKE
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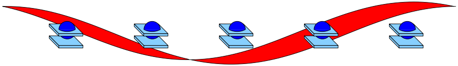

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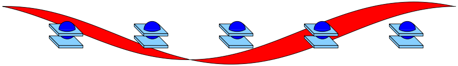

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Collective decay:

$$\frac{d\rho}{dt} = -\frac{\Gamma}{2}\left[S^+S^-\rho - S^-\rho S^+ + \rho S^+S^- \right]$$

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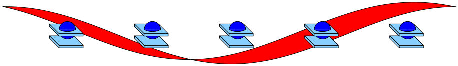

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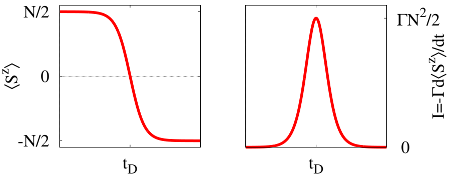

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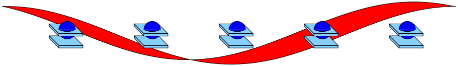

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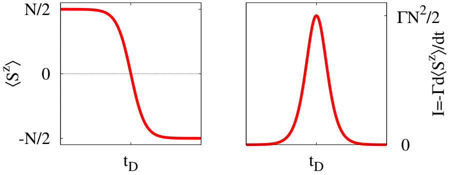
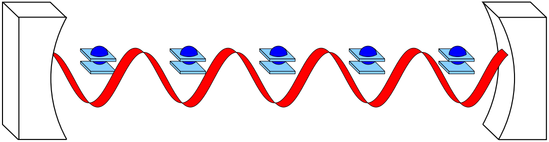


Problem: dipole interactions dephase. [Friedberg et al, Phys. Lett. 1972]

Collective radiation **with a cavity**: Dynamics



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Single cavity mode: oscillations

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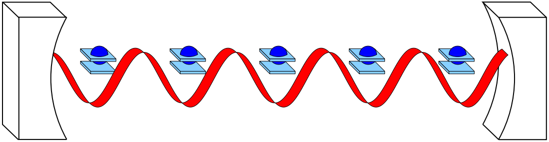

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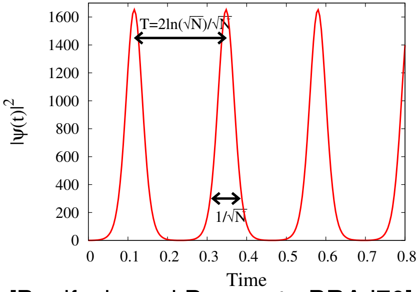

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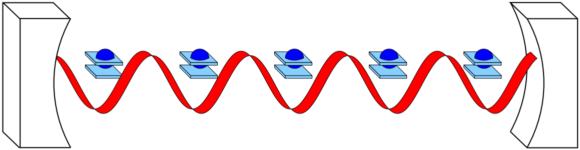
Dicke model: Equilibrium superradiance transition



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- Small g, min at $\lambda, \eta = 0$

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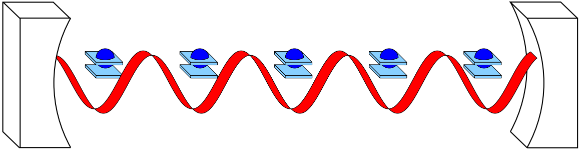

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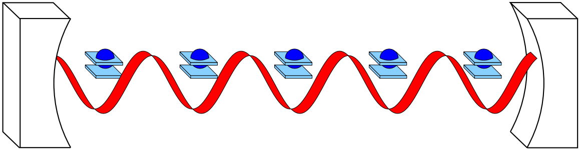

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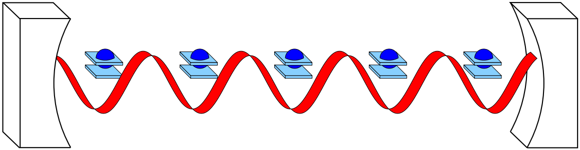

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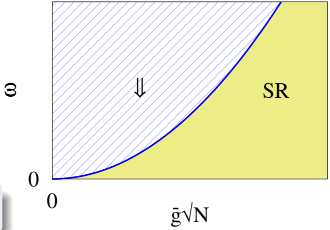

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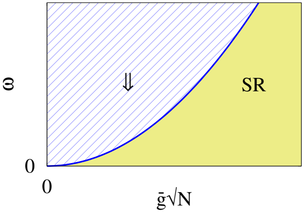

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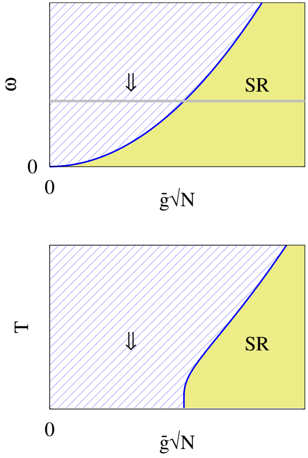

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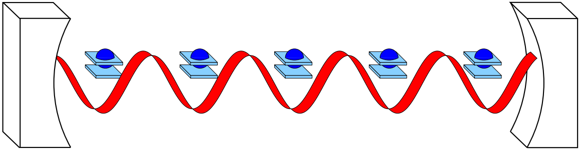

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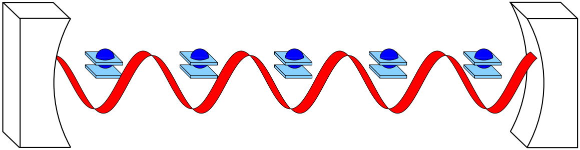

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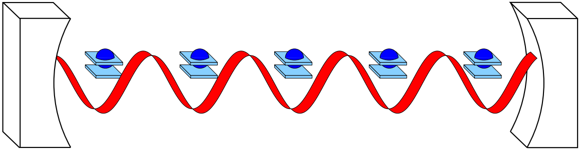

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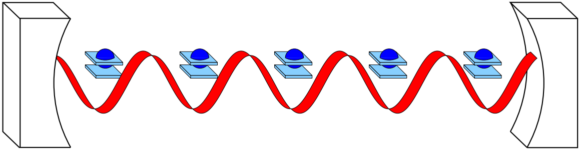

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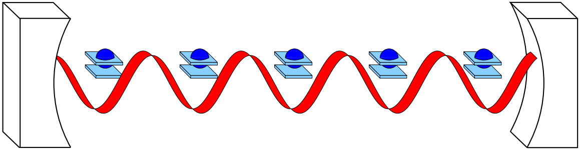

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But Thomas-Reiche-Kuhn sum rule states: $g^2/\omega_0 < 2\zeta$. **No transition**

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Dicke phase transition: ways out

Problem: $g^2/\omega_0 < 2\zeta$ for intrinsic parameters. **Solutions**:

- **Interpretation**
  Ferroelectric transition in $\mathbf{D}\cdot\mathbf{r}$ gauge.
  [JK JPCM '07, Vukics & Domokos arXiv:1206.0752]
- Circuit QED [Nataf and Ciuti, Nat. Comm. '10; Viehmann *et al.* PRL '11]

- Grand canonical ensemble:
  - If $H \to H - \mu(S^z + \psi^\dagger\psi)$, need only:
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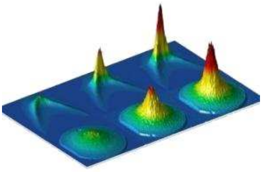

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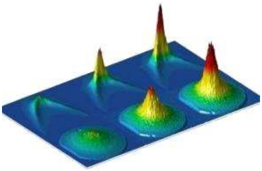

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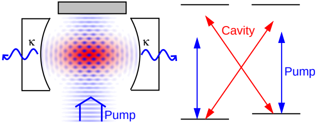

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2. Microcavity Polariton condensation
   - Polariton Introduction
   - Non-equilibrium condensation vs lasing

3. Raman pumped atoms
   - Raman pumped atoms – Introduction
   - Attractors of dynamics (fixed points)
   - Attractors of dynamics (oscillations)

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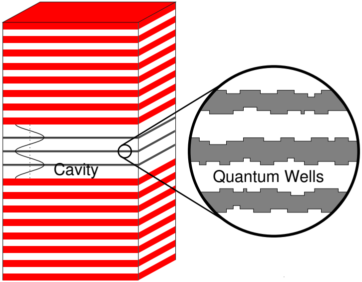
Microcavity polaritons



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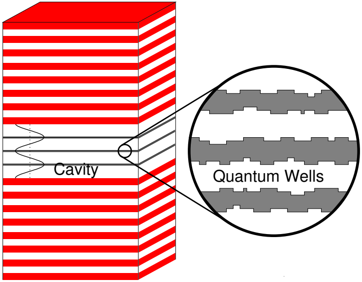

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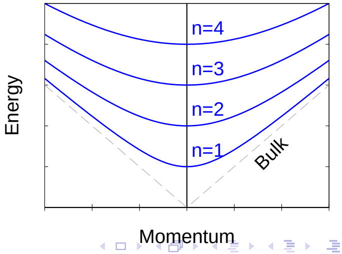

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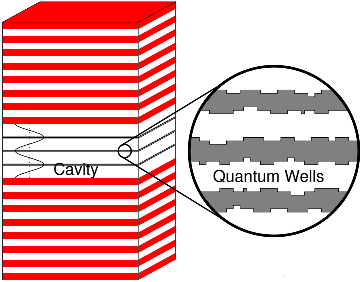

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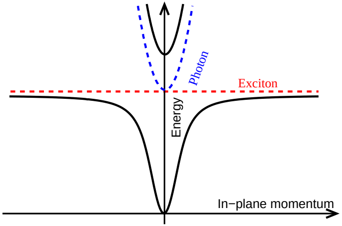

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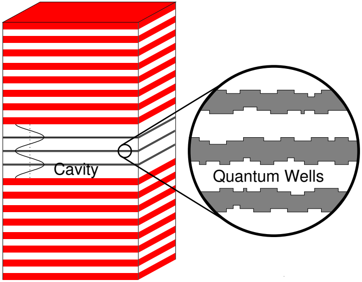

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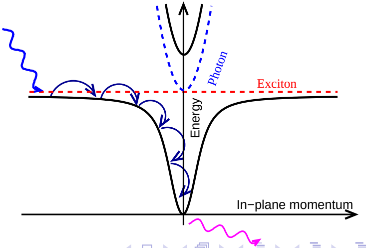
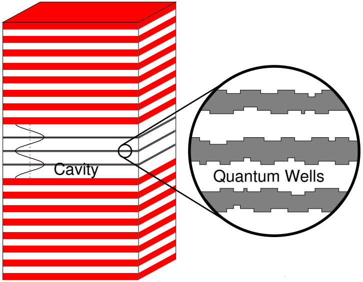
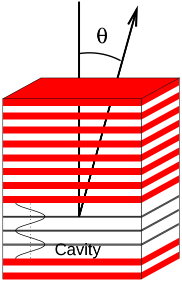


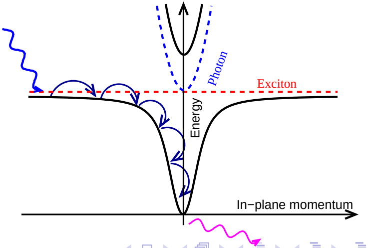
Microcavity polaritons



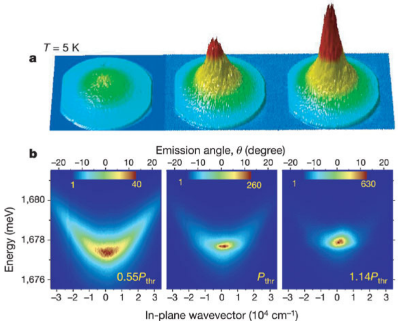


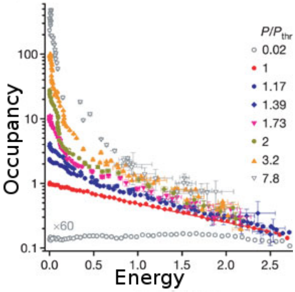
Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$



Polariton experiments: occupation and coherence





[Kasprzak, *et al.* Nature, '06]

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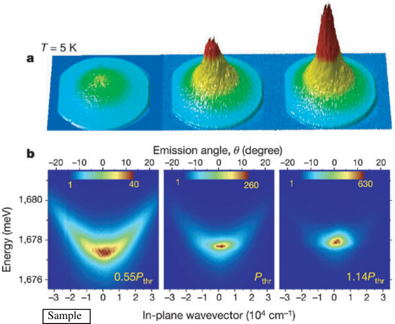
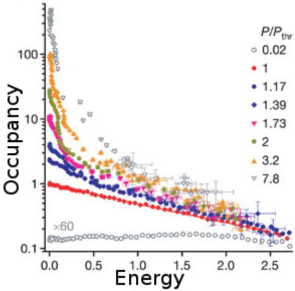
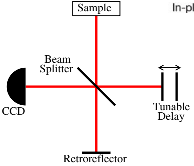
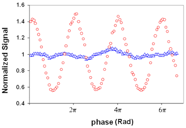
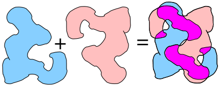


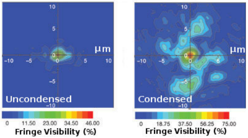






Coherence map:

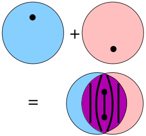


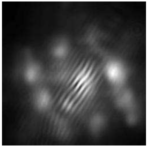


[Kasprzak, *et al.* Nature, ’06]

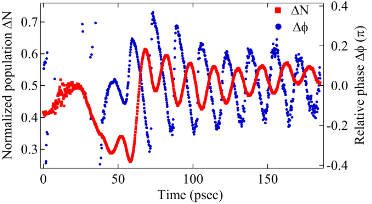
(Some) other polariton condensation experiments

- Quantised vortices
  [**Lagoudakis *et al.* Nat. Phys. '08**. Science '09, PRL '10; Sanvitto *et al.* Nat. Phys. '10; Roumpos *et al.* Nat. Phys. '10]

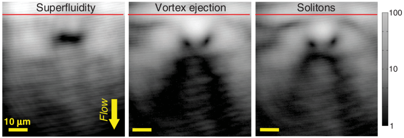




- Josephson oscillations
  [**Lagoudakis *et al.* PRL '10**]

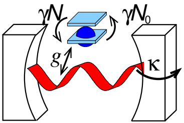


- Pattern formation/Hydrodynamics
  [**Amo *et al.* Science '11**, Nature '09; Wertz *et al.* Nat. Phys '10]

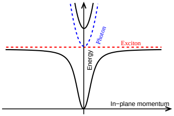


Lasing-condensation crossover model

- Use model that can show lasing and condensation:



$\Leftrightarrow$



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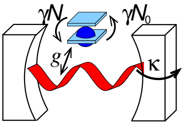

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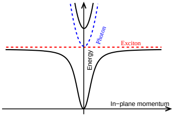

Dicke model:

$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_{\alpha} S^{z}_{\alpha} + \frac{1}{\sqrt{A}} g_{\alpha,\mathbf{k}} \psi_{\mathbf{k}} S^{+}_{\alpha} + \text{H.c.} \right]$$

Polariton model and equilibrium results

Localised excitons, propagating photons

$$H - \mu N = \sum_{\mathbf{k}} (\omega_{\mathbf{k}} - \mu)\psi^\dagger_{\mathbf{k}}\psi_{\mathbf{k}} + \sum_{\alpha} (\epsilon_\alpha - \mu) S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} S^+_\alpha + \text{H.c.}$$

Self-consistent polarisation and field

$$(\omega - \mu)\,\psi = \frac{1}{A}\sum_\alpha \frac{g_\alpha^2 \psi}{2E_\alpha} \tanh\left(\beta E_\alpha\right), \quad {E_\alpha}^2 = \left(\frac{\epsilon_\alpha - \mu}{2}\right)^2 + g_\alpha^2 |\psi|^2$$

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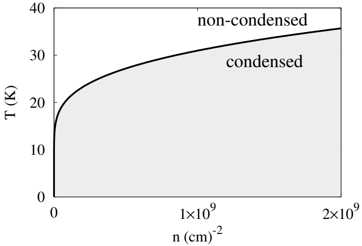

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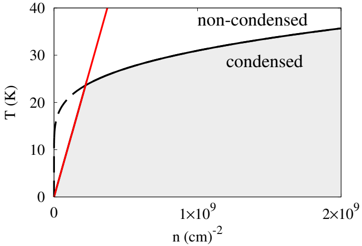

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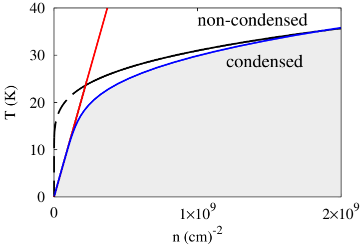

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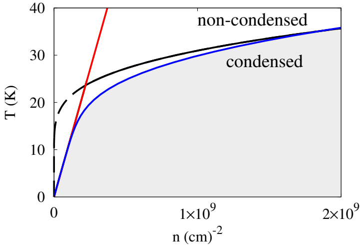
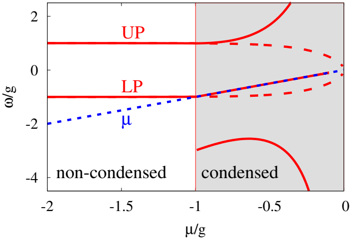
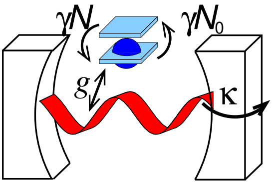


Modes (at $k = 0$)



Simple Laser: Maxwell Bloch equations

$$H = \omega\psi^\dagger\psi + \sum_\alpha \epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}}\psi S^+_\alpha + \text{H.c.}$$



Maxwell-Bloch eqns: $P = -i\langle S^- \rangle, N = 2\langle S^z \rangle$

$$\partial_t \psi = -i\omega\psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$

$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$

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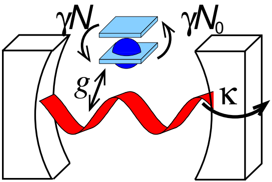

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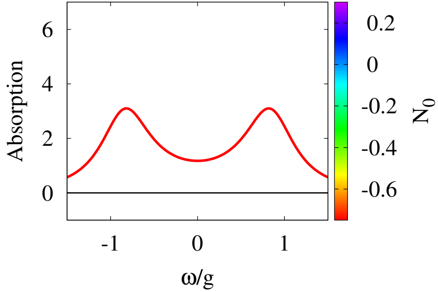

- Strong coupling. $\kappa, \gamma < g\sqrt{n}$
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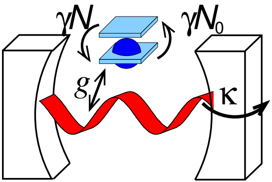

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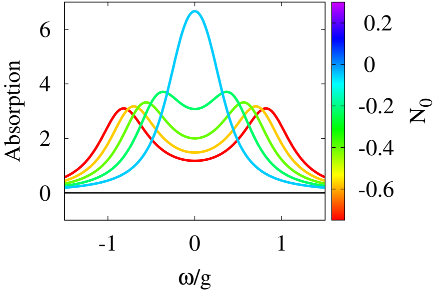

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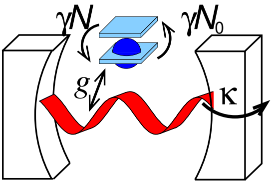

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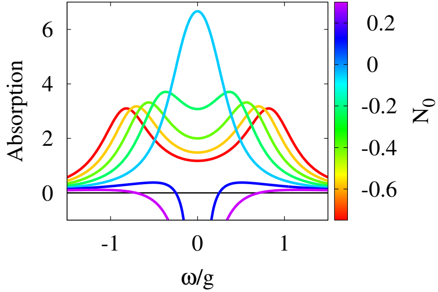

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Poles of Retarded Green's function and gain

$$\left[D^R(\nu)\right]^{-1} = \nu - \omega_k + i\kappa + \frac{g^2 N_0}{\nu - 2\epsilon + i2\gamma}$$

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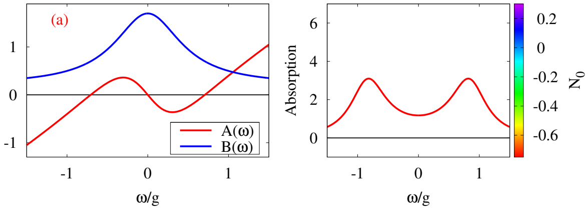

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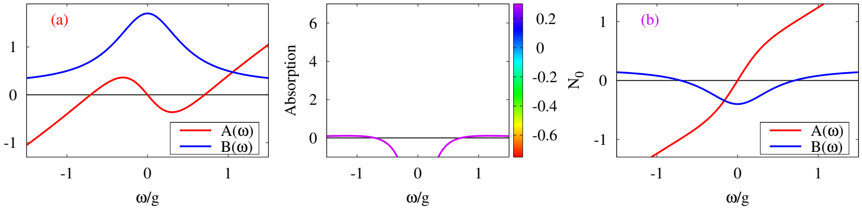

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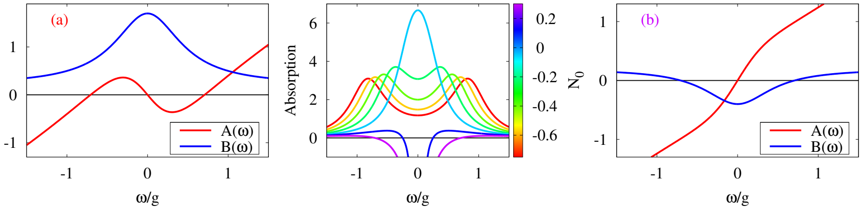
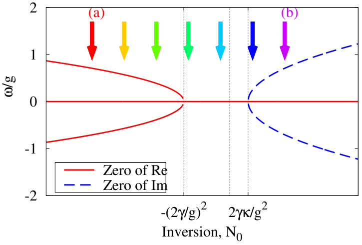


Laser:



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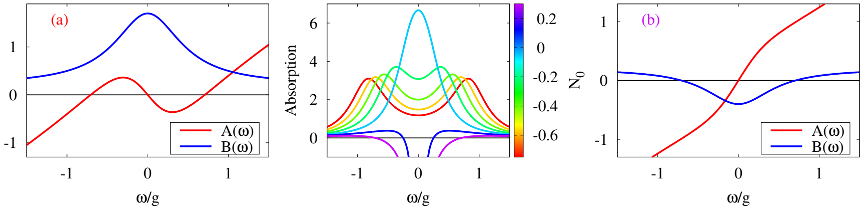

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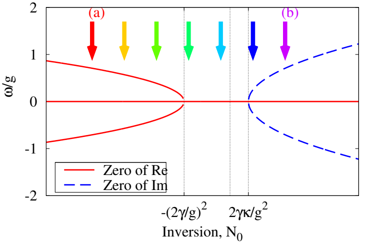
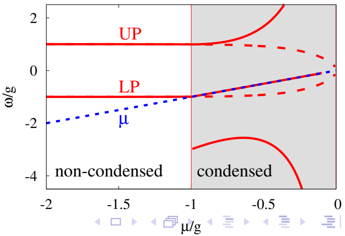
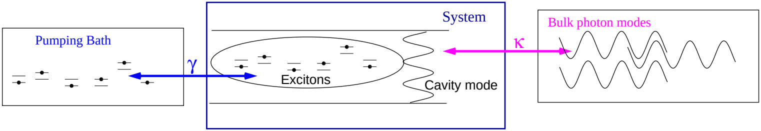
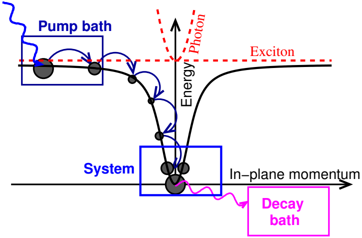


Equilibrium:



Non-equilibrium description: baths





$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

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- Pump bath: Thermal μ_B, T_B

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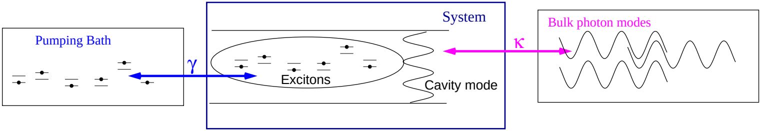

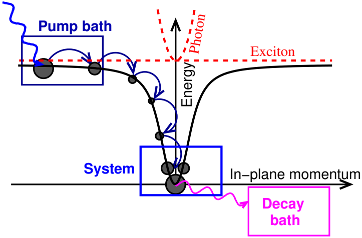

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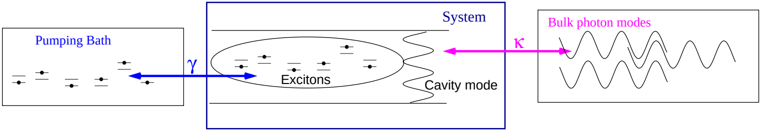

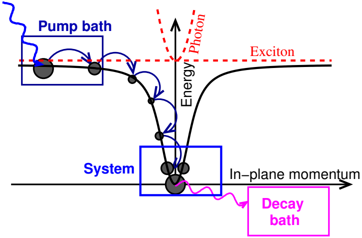

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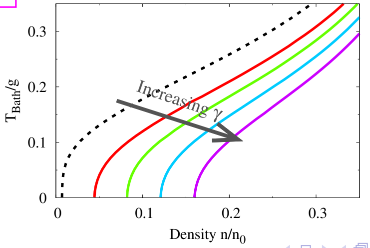
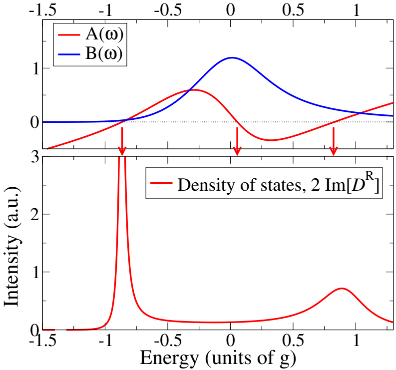
Mean field theory



Stability and evolution with pumping



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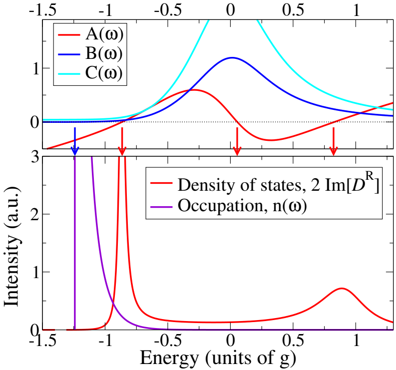
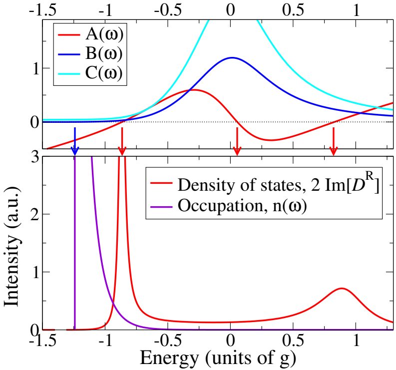
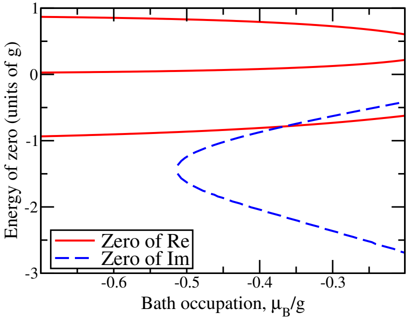


$$\left[D^R(\nu)\right]^{-1} = A(\nu) + iB(\nu)$$

$$2n(\nu) + 1 = \frac{iD^K(\nu)}{-2\Im[D^R(\nu)]} = \frac{C(\nu)}{2B(\nu)}$$

Stability and evolution with pumping

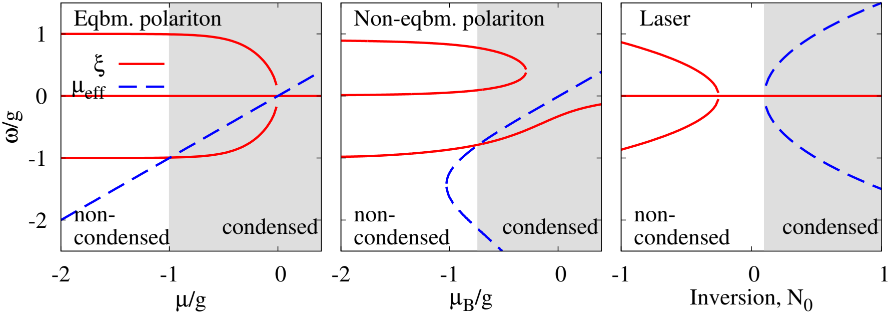




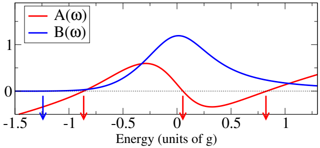
$$\left[D^R(\nu)\right]^{-1} = A(\nu) + iB(\nu)$$

$$2n(\nu) + 1 = \frac{iD^K(\nu)}{-2\Im[D^R(\nu)]} = \frac{C(\nu)}{2B(\nu)}$$

Strong coupling and lasing — low temperature phenomenon



- Laser: Uniformly invert TLS
- Non-equilibrium polaritons: Cold bath
- If $T_B \gg \gamma \rightarrow$ Laser limit



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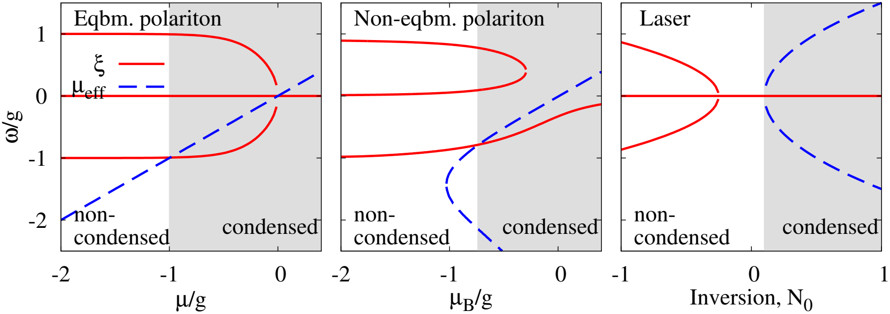

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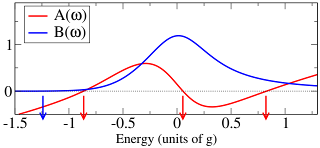

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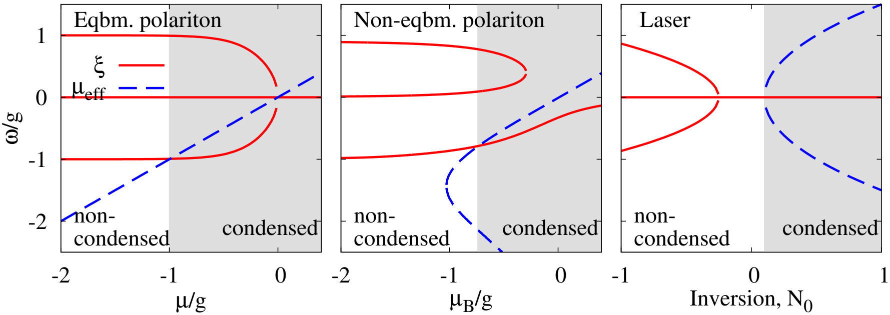

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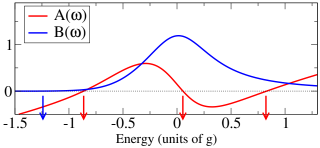
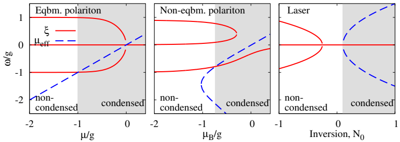


Coherence, inversion, strong-coupling

Polariton condensation:



- Inversionless
- **allows** strong coupling
- **requires** low $T \leftrightarrow$ condensation
- NB **NOT** thresholdless/single atom lasing.

- Circuit QED [Marthaler *et al.* PRL '11]
  - Noise-assisted
  - Off-resonant cavity
  - Emission/absorption $\Gamma^{\pm} \sim 2n_B(\pm\delta\omega) + 1$
  - Low $T \rightarrow$ inversionless threshold
- Photon condensation [Klaers *et al.* Nature '10]
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  - Inversionless weak coupling lasing

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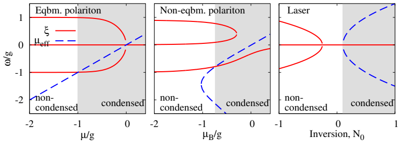

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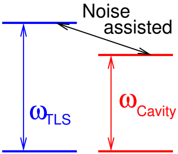

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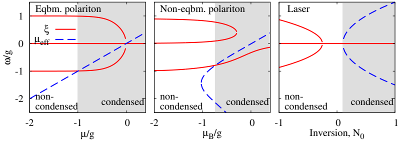

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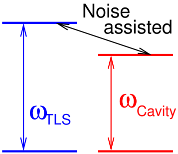

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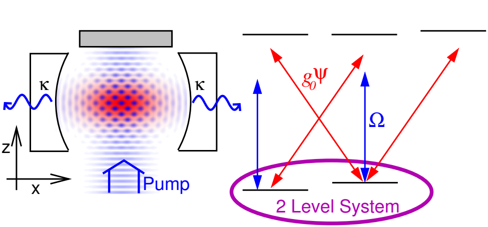
Raman pumped Dicke model (atoms)

1 Dicke model and superradiance

2 Microcavity Polariton condensation
- Polariton Introduction
- Non-equilibrium condensation vs lasing

3 Raman pumped atoms
- Raman pumped atoms – Introduction
- Attractors of dynamics (fixed points)
- Attractors of dynamics (oscillations)

Extended Dicke model [Baumann *et al.* Nature 2010]



2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$:

$\Downarrow$: $\Psi(x,z) = 1$

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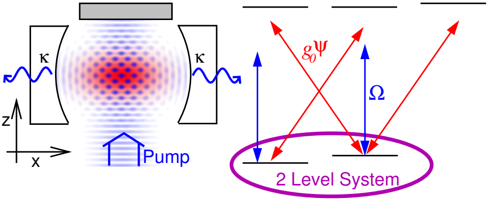
$\omega_0 = 2\omega_{\text{recoil}}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+) + U S_z \psi^\dagger\psi$$

$$\partial_t \rho = -i[H,\rho] - \kappa(\psi^\dagger\psi\rho - 2\psi\rho\psi^\dagger + \rho\psi^\dagger\psi)$$

Extended Dicke model

[Baumann *et al.* Nature 2010]



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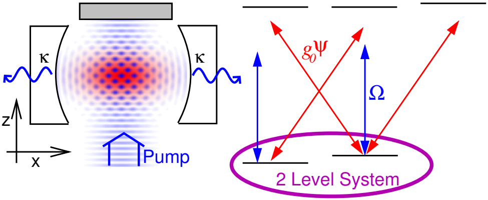

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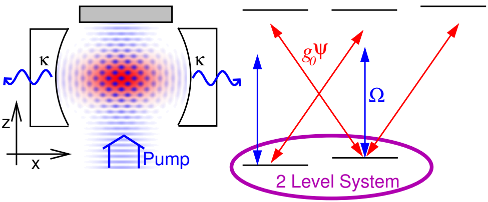

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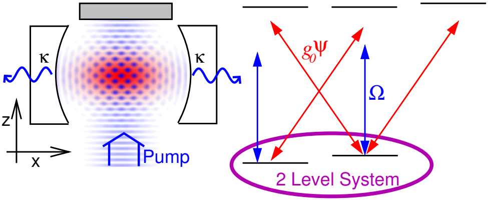

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Classical EOM ($|\mathbf{S}| = N/2 \gg 1$)

$$\dot{S}^- = -i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$

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$$\dot{\psi} = -[\kappa + i(\omega + US^z)]\psi - ig(S^- + S^+)$$

Fixed points (steady states)

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- $\psi = 0, \mathbf{S} = (0, 0, \pm N/2)$ always a solution.
- If $g > g_c$, $\psi \neq 0$ too
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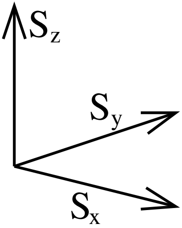

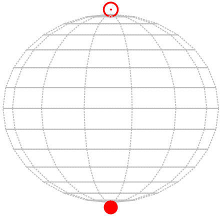

Small g: ⇑, ⇓ only.
($\omega = 30$MHz, $UN = -40$MHz)

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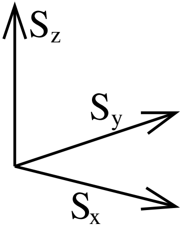

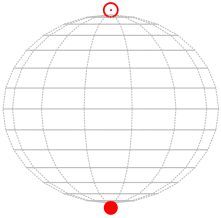

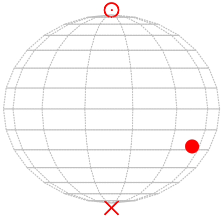

Small g: $\Uparrow, \Downarrow$ only. Larger g: SR too.
($\omega = 30$MHz, $UN = -40$MHz)

Steady state phase diagram

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
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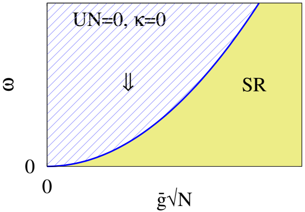

See also Domokos and Ritsch PRL '02, Domokos *et al.* PRL '10

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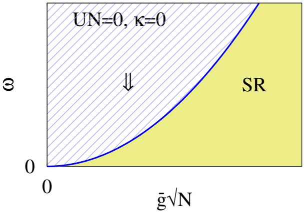

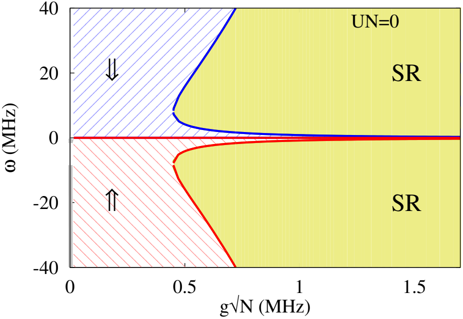

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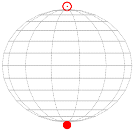
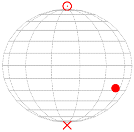


SR(A): $S_y = 0$



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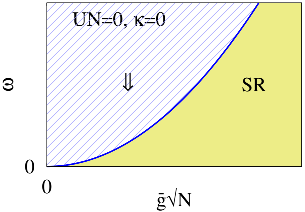
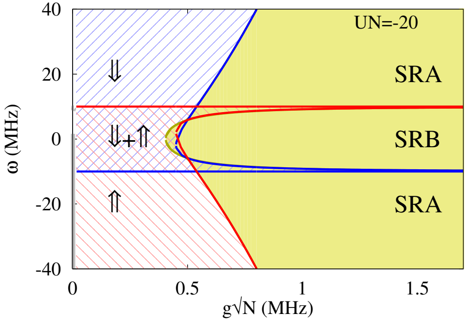
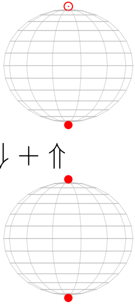




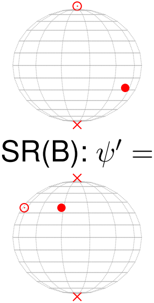
$\Downarrow$

$\Downarrow + \Uparrow$



SR(A): $S_y = 0$

SR(B): $\psi' = 0$



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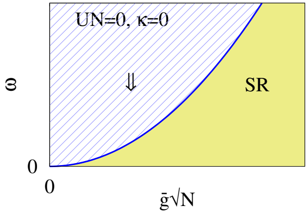

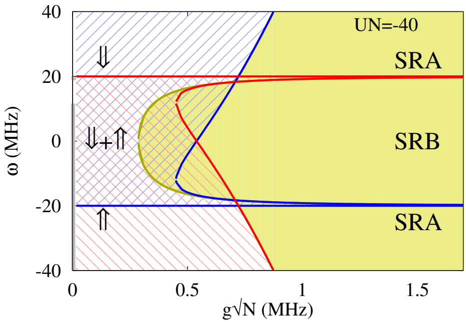

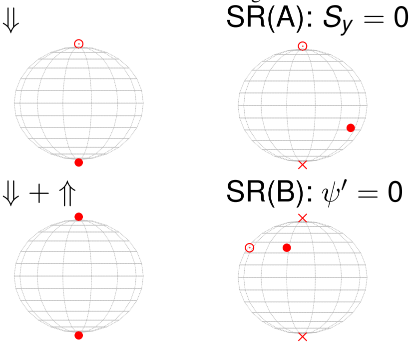

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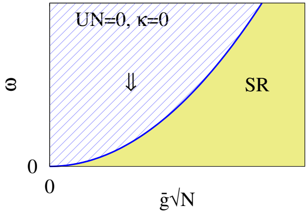

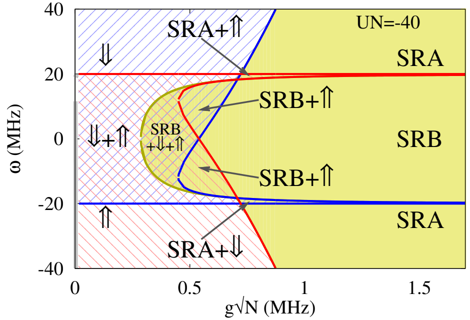

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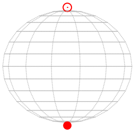

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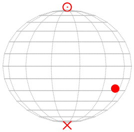
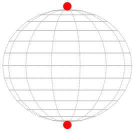
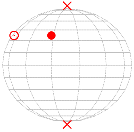


$\Downarrow + \Uparrow$



SR(B): $\psi' = 0$



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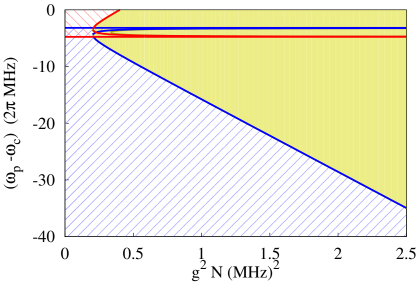
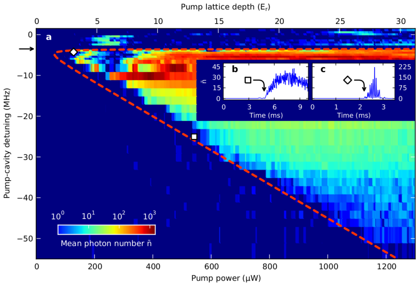
Comparison to experiment





$$UN = -10\text{MHz}$$

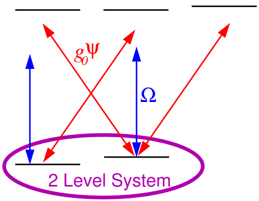
Adapted from: [Bhaseen *et al.* PRA '12]

[Baumann *et al* Nature '10]

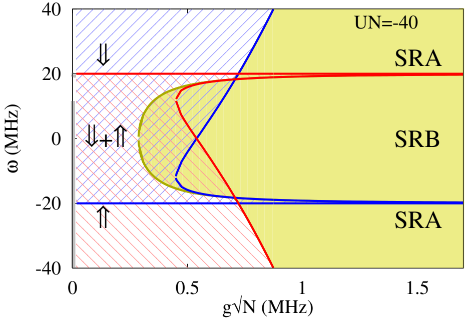
$$\omega = \omega_c - \omega_p + \frac{5}{2} UN, \qquad UN = -\frac{g_0^2}{4(\omega_a - \omega_c)}$$

Regions without fixed points

Changing U:



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



Regions without fixed points

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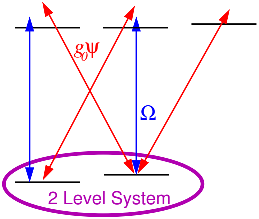
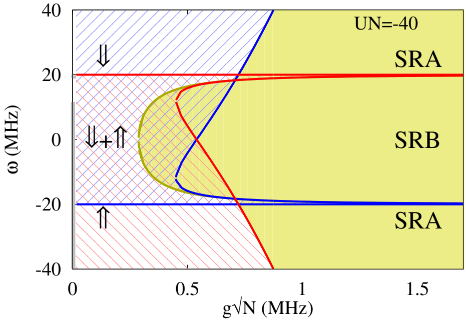


$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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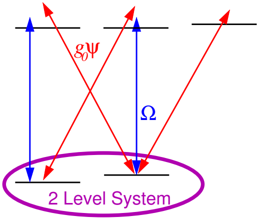

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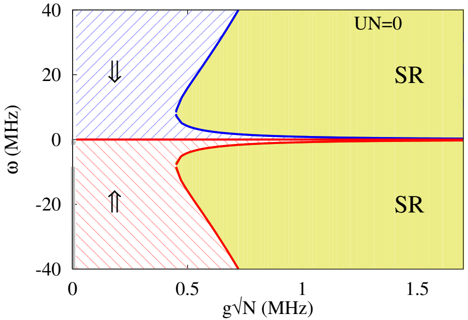

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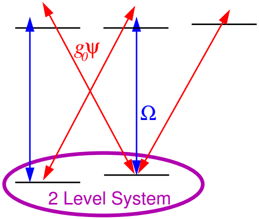

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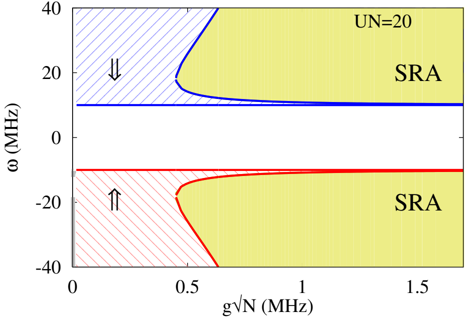

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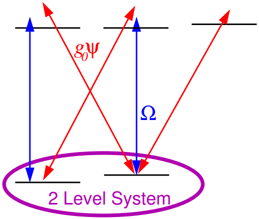

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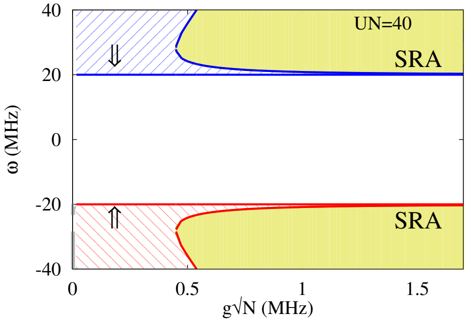

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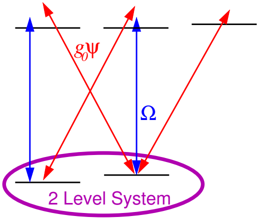

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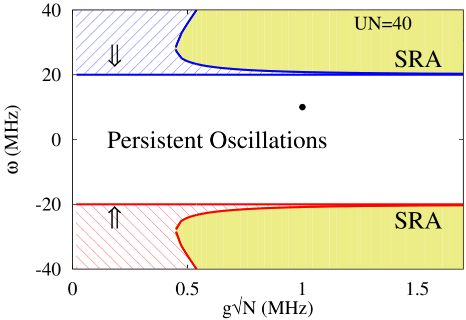
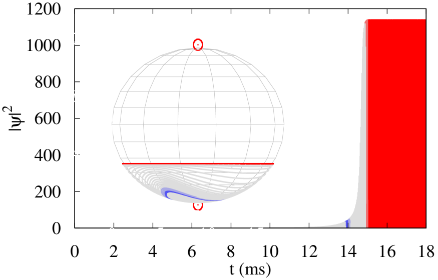
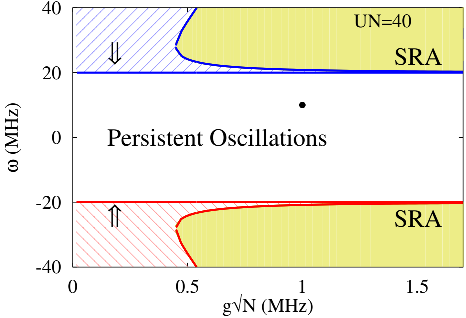
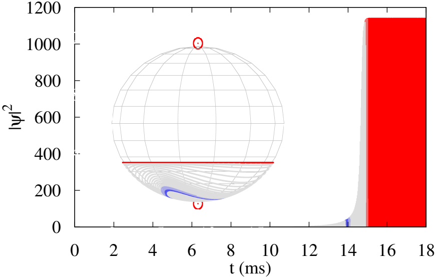
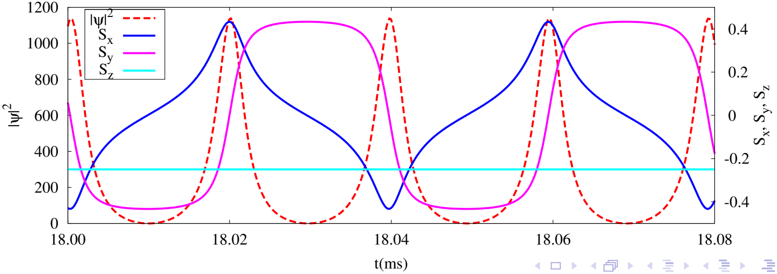




Persistent (optomechanical) oscillations

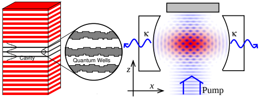




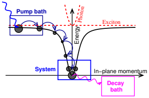


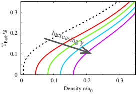
Summary

- Non-equilibrium Dicke relevant to increasing number of systems

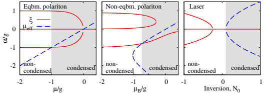


- Effects of pumping on mean-field theory

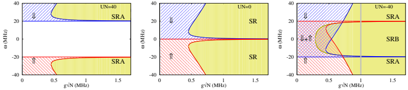




- Polariton condensation vs lasing



- Dynamical phases of Raman pumped scheme



Extra slides

4. Ferroelectric transition
5. Dicke vs JCHM
6. Retarded Green's function for laser
7. Timescales for Raman pumped experiment

Ferroelectric transition

Atoms in Coulomb gauge

$$H = \sum \omega_k a_k^\dagger a_k + \sum_i [p_i - eA(r_i)]^2 + V_{\text{coul}}$$

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Two-level systems — dipole-dipole coupling

$$H = \omega_0 S^z + \omega \psi^\dagger \psi + g(S^+ + S^-)(\psi + \psi^\dagger) + N\zeta(\psi + \psi^\dagger)^2 - \eta(S^+ - S^-)^2$$

(nb $g^2, \zeta, \eta \propto 1/V$).

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Ferroelectric polarisation if $\omega_0 < 2\eta N$

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Ferroelectric polarisation if $\omega_0 < 2\eta N$

Gauge transform to dipole gauge $\mathbf{D} \cdot \mathbf{r}$

$$H = \omega_0 S^z + \omega\psi^\dagger\psi + \bar{g}(S^+ - S^-)(\psi - \psi^\dagger)$$

"Dicke" transition at $\omega_0 < N\bar{g}^2/\omega \equiv 2\eta N$

But, ψ describes **electric displacement**

Extra slides

4 Ferroelectric transition

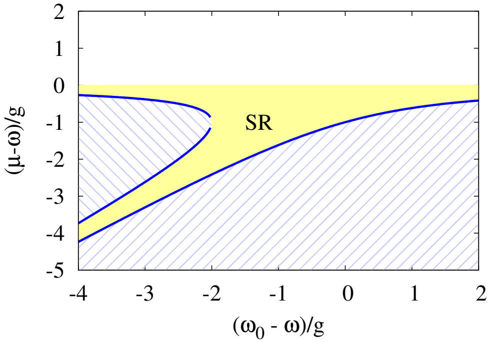
5 Dicke vs JCHM

6 Retarded Green's function for laser

7 Timescales for Raman pumped experiment

Equilibrium: Dicke model with chemical potential

$$H - \mu N = (\omega - \mu)\psi^\dagger\psi + (\omega_0 - \mu)S^z + g\left(\psi^\dagger S^- + \psi S^+\right)$$



- Transition at: $g^2 N > (\omega - \mu)(\omega_0 - \mu)$
- Reduce critical g
- Unstable if $\mu > \omega$
- Inverted if $\mu > \omega_0$

[Eastham and Littlewood, PRB '01]

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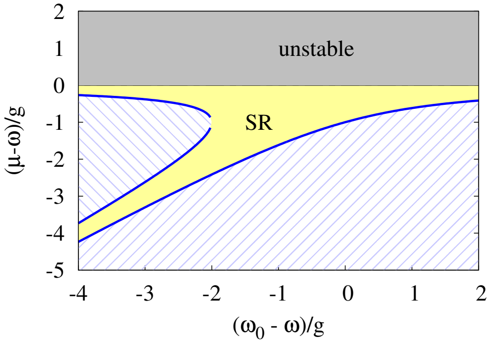

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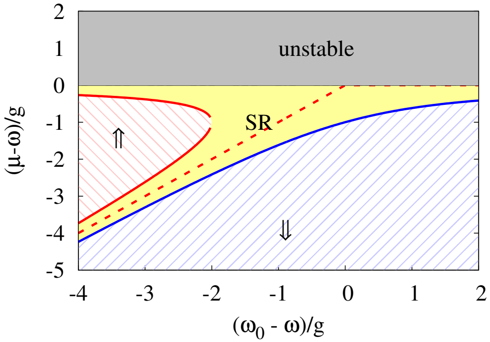

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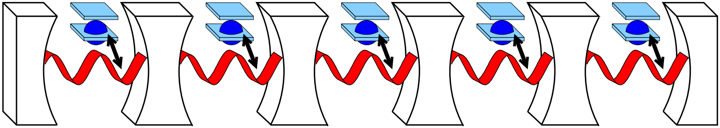
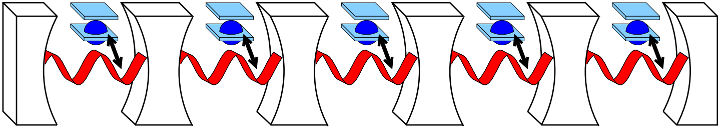
Jaynes-Cummings Hubbard model



$$H = -\frac{J}{z}\sum_{ij}\psi_i^\dagger\psi_j + \sum_i \frac{\Delta}{2}\sigma_i^z + g(\psi_i^\dagger\sigma_i^- + \text{H.c.})$$

Jaynes-Cummings Hubbard model



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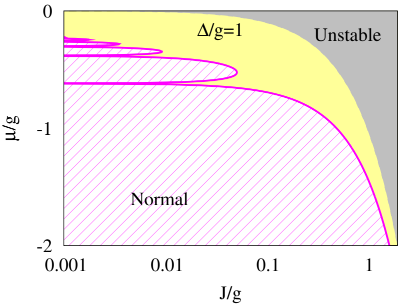

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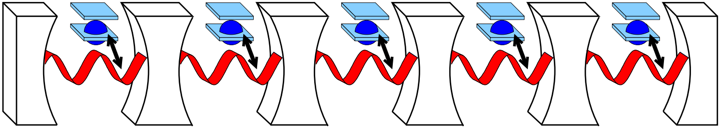

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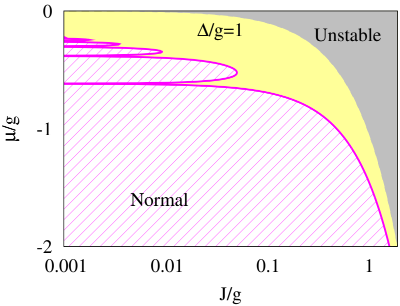
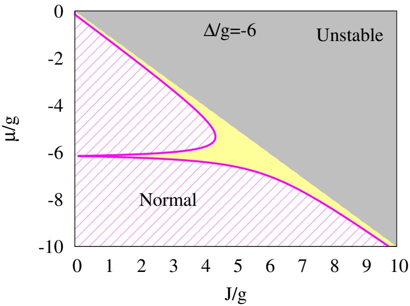
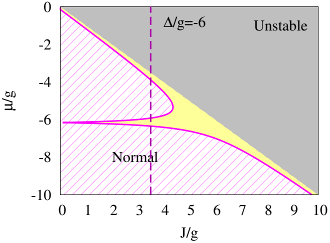


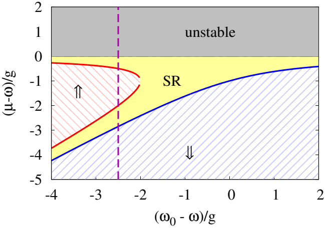


Dicke vs JCHM

JCHM



Dicke



- $k = 0$ mode of JCHM $\leftrightarrow$ Dicke photon mode
- $\Uparrow$ $\leftrightarrow$ $n = 1$ Mott lobe

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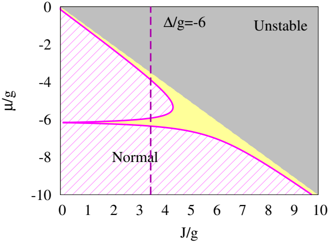
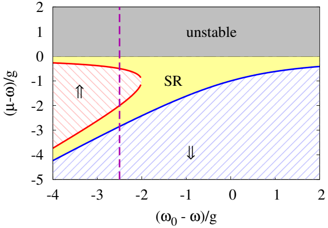


Dicke



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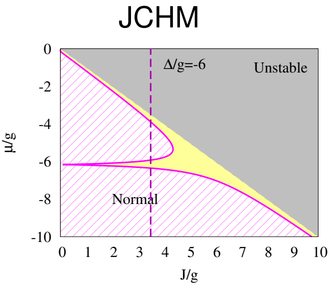

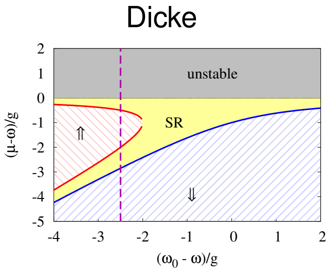

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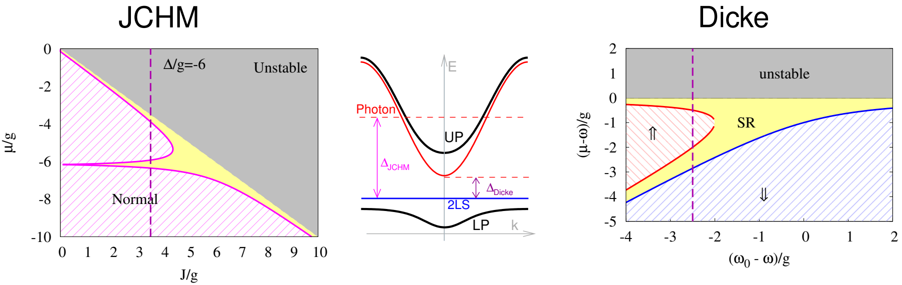

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Extra slides

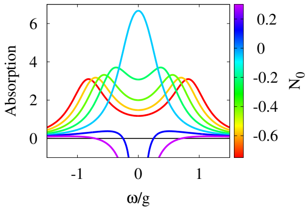
4 Ferroelectric transition

5 Dicke vs JCHM

6 Retarded Green's function for laser

7 Timescales for Raman pumped experiment

Maxwell-Bloch Equations: Retarded Green's function



- Introduce $D^R(\omega)$:
  Response to perturbation

- Absorption $= -2\Im[D^R(\omega)]$

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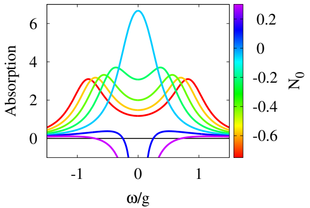

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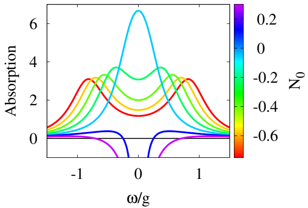

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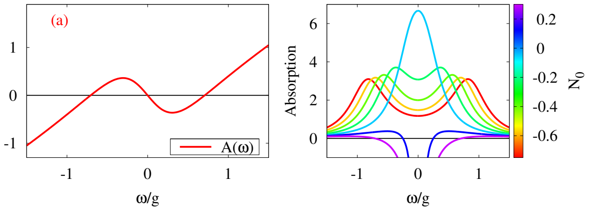

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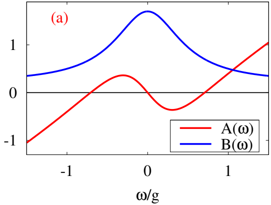

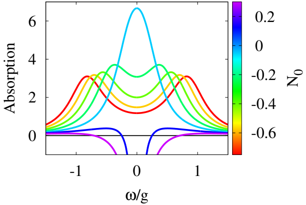

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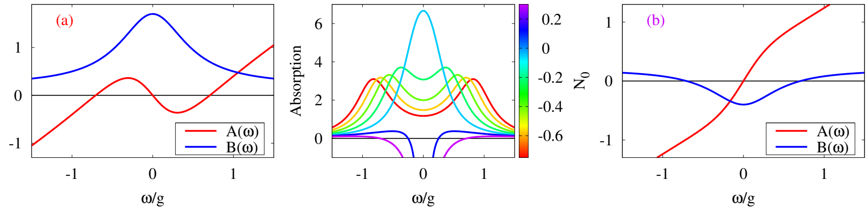

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Extra slides

4 Ferroelectric transition

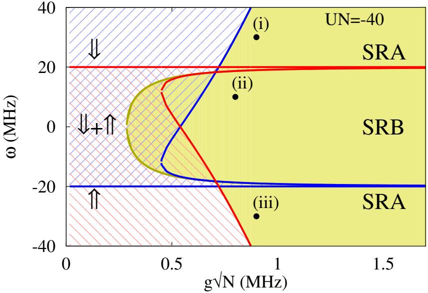
5 Dicke vs JCHM

6 Retarded Green's function for laser

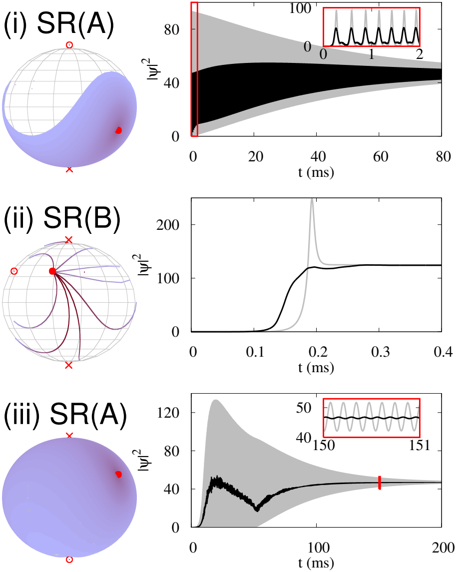
7 Timescales for Raman pumped experiment

Dynamics: Evolution from normal state

Gray: $\mathbf{S} = (\sqrt{N}, \sqrt{N}, -N/2)$
Black: Wigner distribution of $\mathbf{S}$, ψ



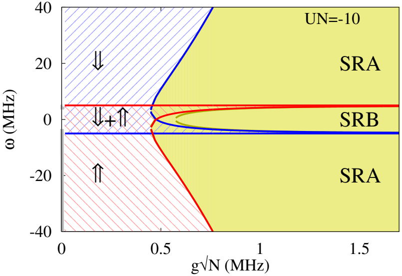
Oscillations: ~ 0.1ms
Decay: 20ms, 0.1ms, 20ms



Asymptotic state: Evolution from normal state

(Near to experimental $UN = -13$MHz).

All stable attractors:



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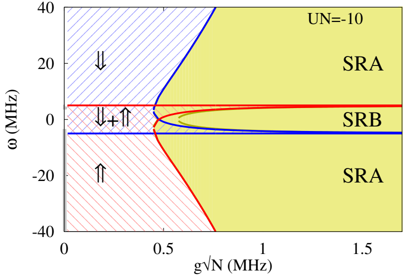
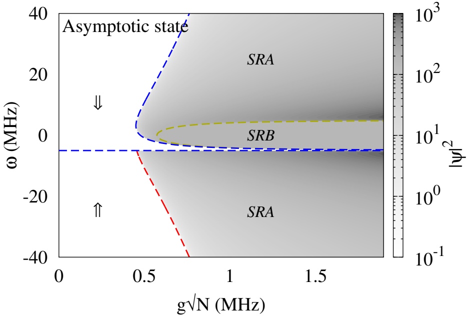
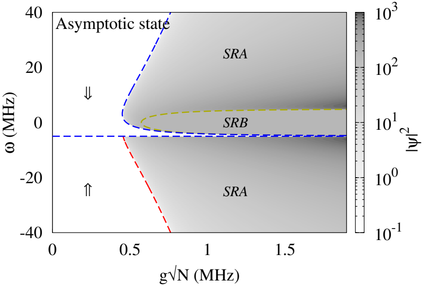
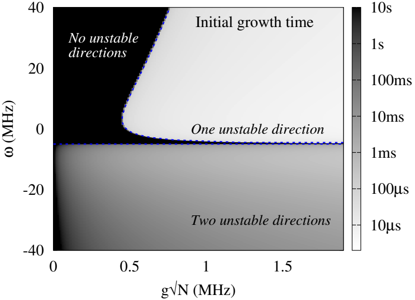


Starting from ⇓



Timescales for dynamics: What are they?





Growth Most unstable eigenvalues near $\mathbf{S} = (0, 0, -N/2)$

Decay Slowest stable eigenvalues near final state

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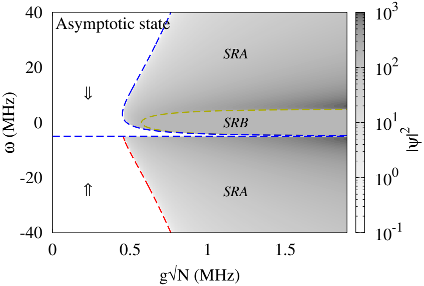

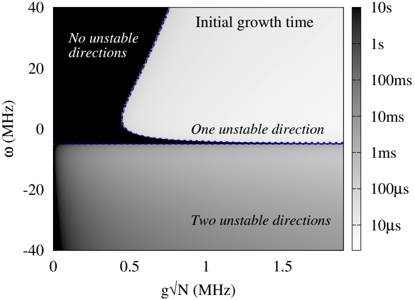

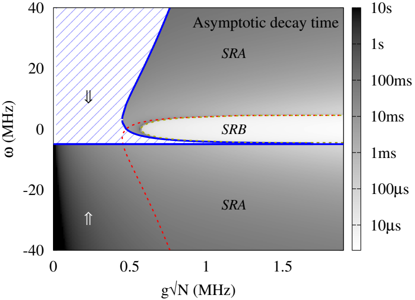

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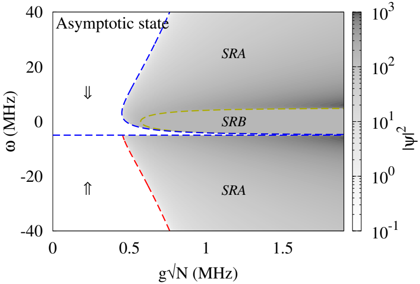
Timescales for dynamics: Consequences for experiment



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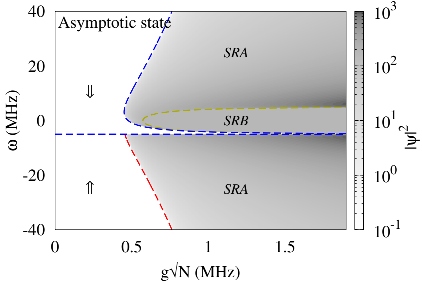

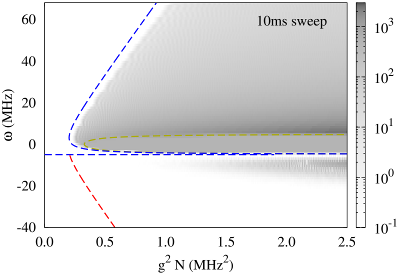

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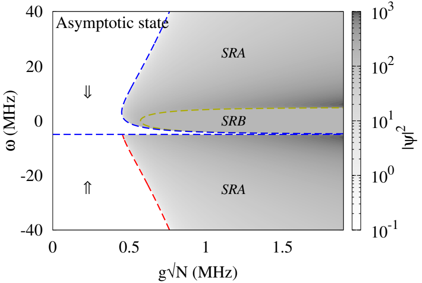
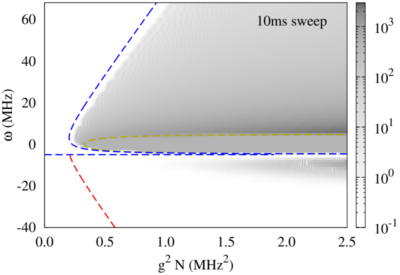
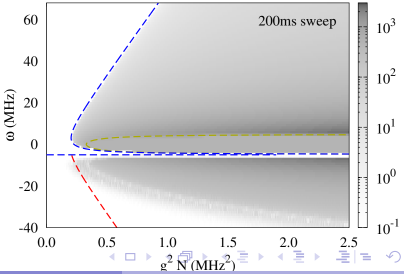
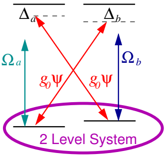






Timescales for dynamics: Why so slow and varied?

Suppose co- and counter-rotating terms differ



$$H = \ldots + g(\psi^\dagger S^- + \psi S^+) + g'(\psi^\dagger S^+ + \psi S^-) + \ldots$$

- SR(A) near phase boundary at small $\delta g \rightarrow$ Critical slowing down
- SR(A), SR(B) continuously connect

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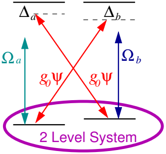

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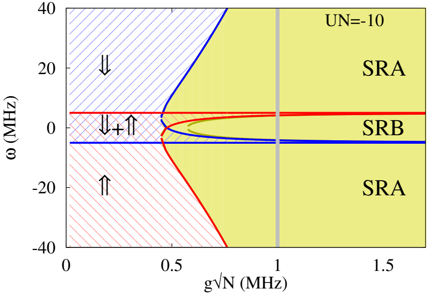

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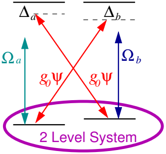

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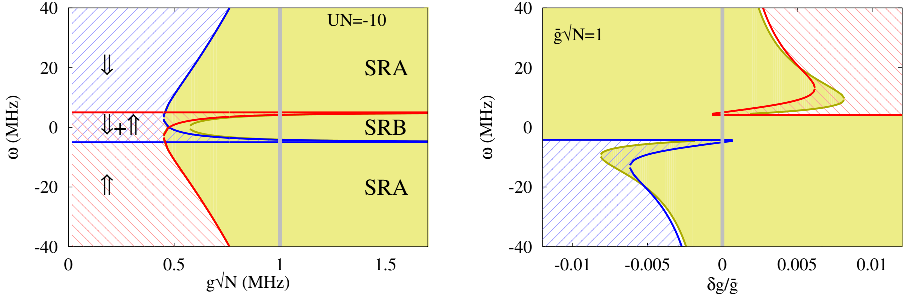

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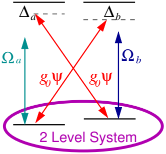

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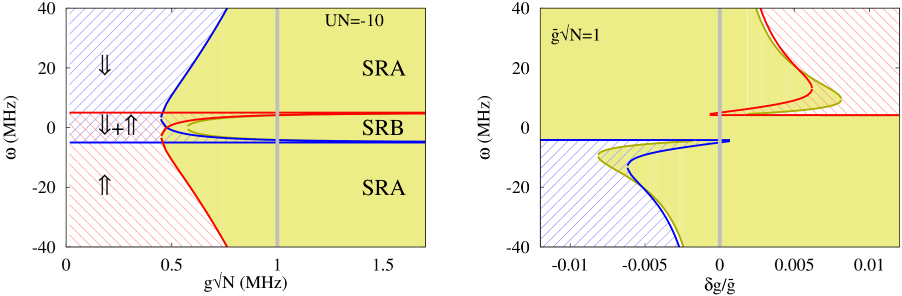

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