Condensation vs lasing, and superfluidity of coupled light-matter systems.

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University of St Andrews 600 YEARS

SUPA®

ICSCE6, August 2012

Acknowledgements

People:

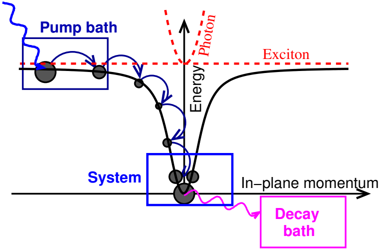
Funding:

EPSRC
Engineering and Physical Sciences
Research Council

Non-equilibrium approach: Steady state, and fluctuations

$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}},$$

$$H_{\text{sys}} = \sum_k \omega_k \psi_k \psi_k^\dagger + \sum_\alpha g_\alpha (\phi_\alpha^\dagger \psi_k + \text{H.c.}) + H_{\text{ex}}[\phi_\alpha, \phi_\alpha^\dagger]$$



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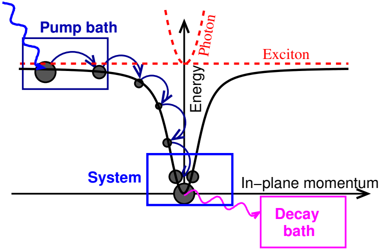

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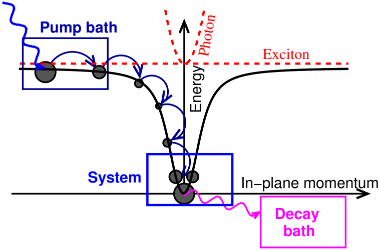

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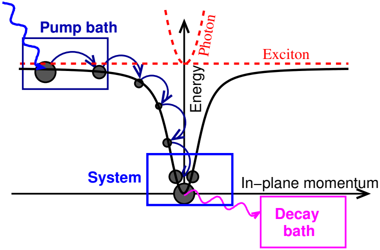

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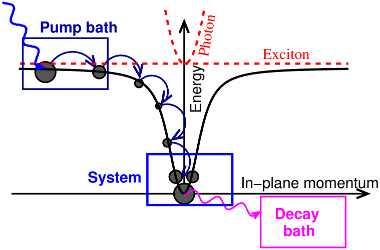

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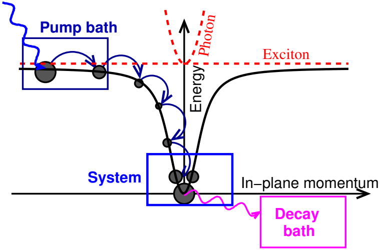

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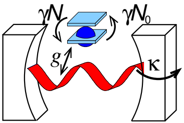
Outline

Lasing-condensation crossover model

- Use model that can show lasing and condensation:



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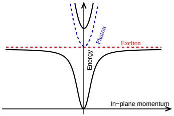

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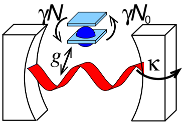

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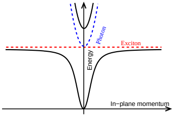
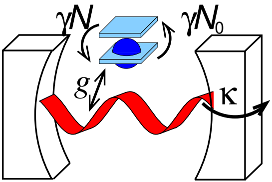


Dicke model:

$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_{\alpha} S^{z}_{\alpha} + \frac{1}{\sqrt{A}} g_{\alpha,\mathbf{k}} \psi_{\mathbf{k}} S^{+}_{\alpha} + \text{H.c.} \right]$$

Simple Laser: Maxwell Bloch equations

$$H = \omega_0 \psi^\dagger \psi + \sum_\alpha \epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi S^+_\alpha + \text{H.c.}$$



Maxwell-Bloch eqns: $P = -i\langle S^- \rangle, N = 2\langle S^z \rangle$

$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$

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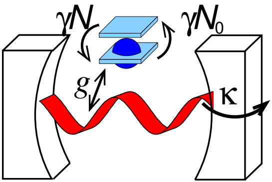

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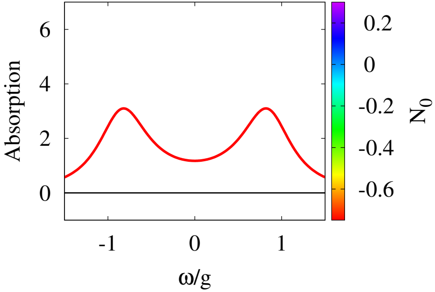

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- Inversion causes collapse before lasing

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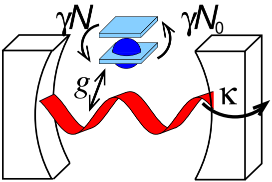

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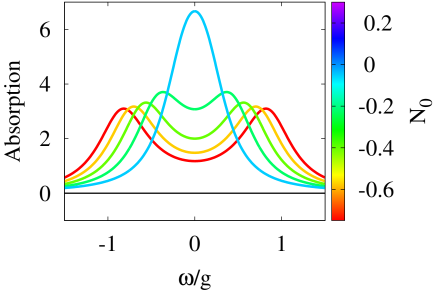

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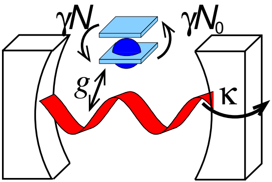

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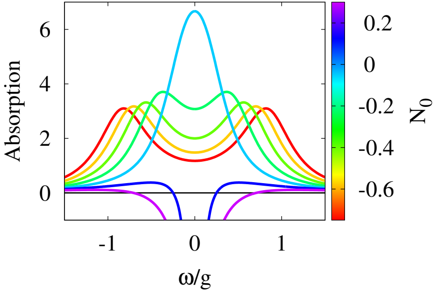

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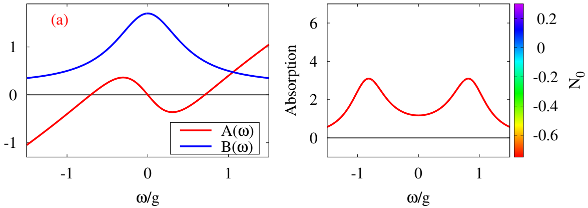

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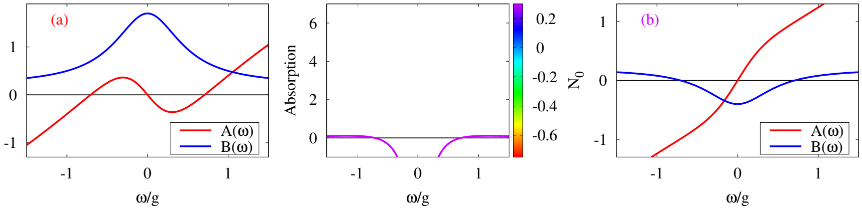

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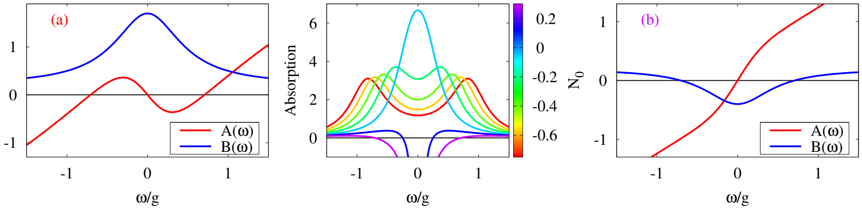

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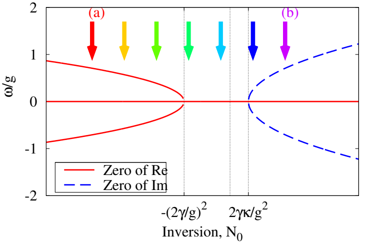

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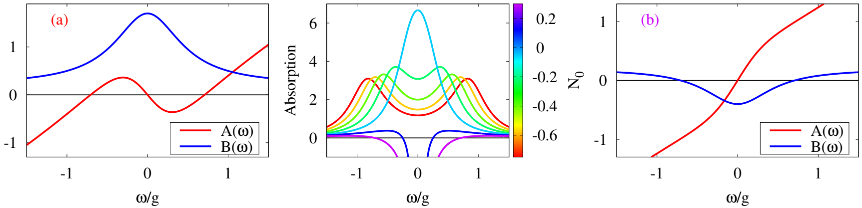

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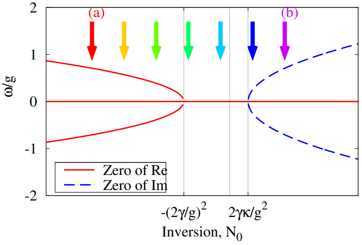
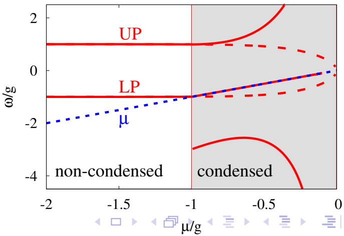
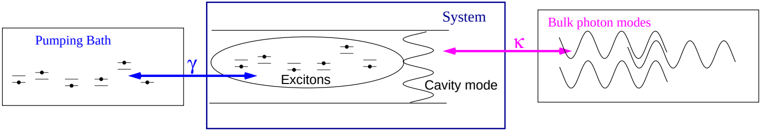
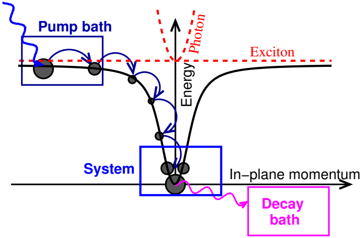


Equilibrium:



Non-equilibrium description: baths





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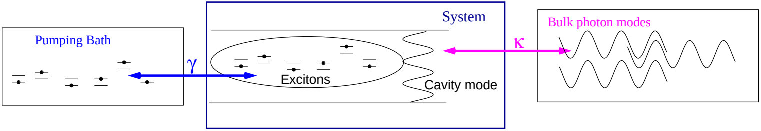

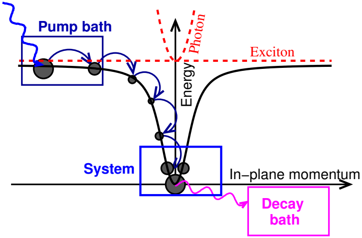

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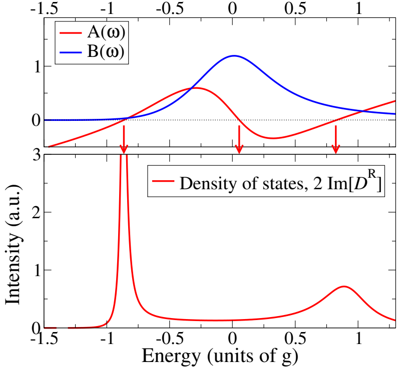

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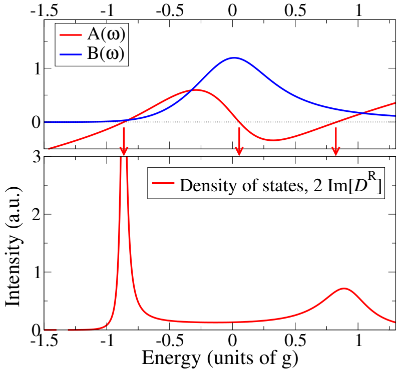

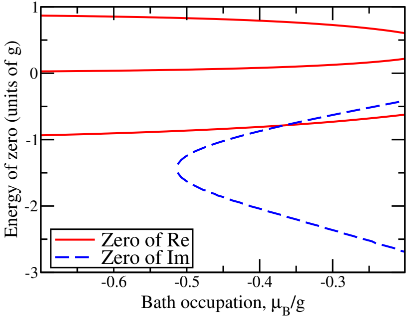

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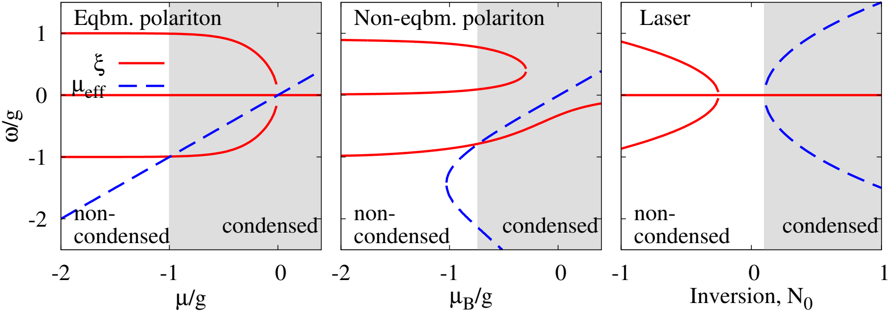
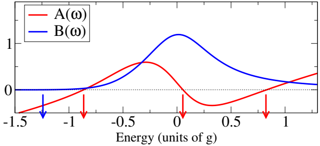
Strong coupling and lasing — low temperature phenomenon



- Non-equilibrium polaritons: Cold bath
- If $T_B \gg \gamma \to$ Laser limit (uniform inversion)



Reviews: JK, MHS, PBL arXiv:1001.3338, MHS JK PBL 1206.1784

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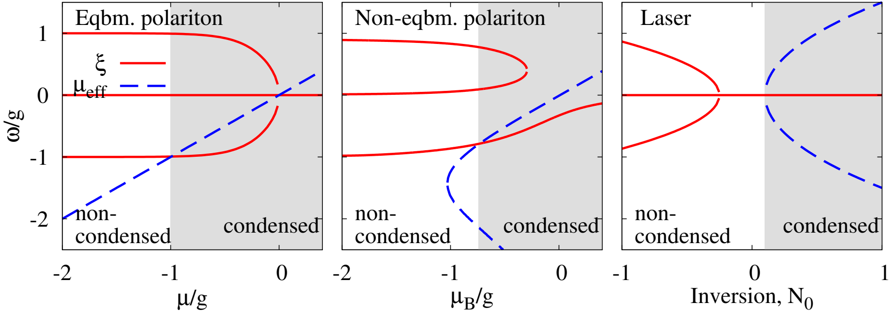

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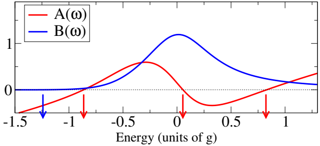

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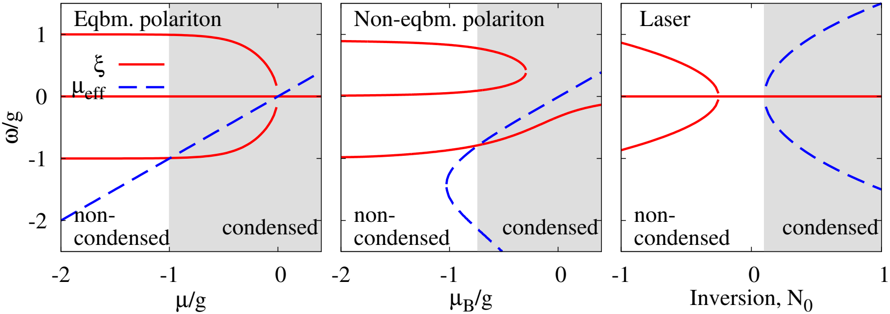

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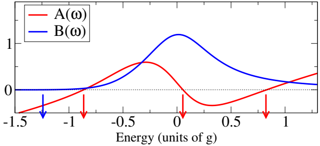

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Coherence, inversion, strong-coupling

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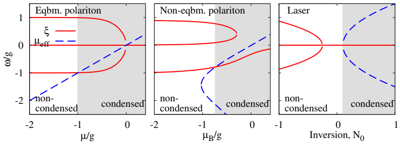

- Inversionless
- **allows** strong coupling
- **requires** low $T \leftrightarrow$ condensation
- NB **NOT** thresholdless/single atom lasing.

- Circuit QED [Marthaler *et al.* PRL '11]
  - Noise-assisted
  - Off-resonant cavity
  - Emission/absorption $\Gamma^{\pm} \sim 2n_B(\pm\delta\omega) + 1$
  - Low $T \rightarrow$ inversionless threshold
- Photon condensation [Klaers *et al.* Nature '10]
  - Vibrational modes $\rightarrow$ thermalisation
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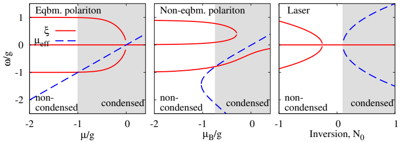

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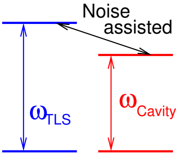

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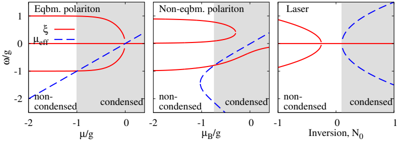

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Related *weak-coupling* **inversionless** lasing:

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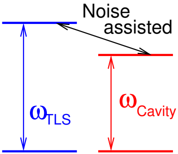


  - Noise-assisted
  - Off-resonant cavity
  - Emission/absorption $\Gamma^{\pm} \sim 2n_B(\pm\delta\omega) + 1$
  - Low $T \rightarrow$ inversionless threshold
- Photon condensation [Klaers *et al.* Nature '10]
  - Vibrational modes $\rightarrow$ thermalisation
  - Inversionless weak coupling lasing

Superfluidity

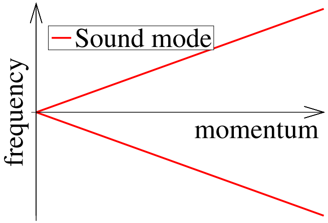
Fluctuations above transition

When condensed

$$\text{Det}\left[D^R(\omega, k)\right]^{-1} = \omega^2 - \xi_k^2$$

With $\xi_k \simeq ck$
Poles:

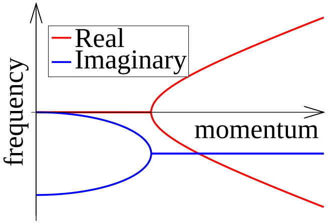
$$\omega^* = \xi_k$$



Fluctuations above transition

When condensed

$$\text{Det}\left[D^R(\omega,k)\right]^{-1} = (\omega+i\gamma_{\text{net}})^2+\gamma_{\text{net}}^2-\xi_k^2$$

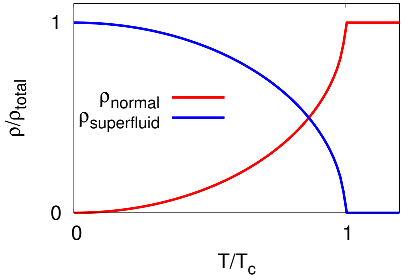


With $\xi_k \simeq ck$
Poles:

$$\omega^* = -i\gamma_{\text{net}} \pm \sqrt{\xi_k^2 - \gamma_{\text{net}}^2}$$

Superfluid density

- Two-fluid hydrodynamics



- ρ_s, ρ_n distinguished by slow rotation

- Experimentally, rotation:

- To calculate, transverse/longitudinal:

Superfluid density

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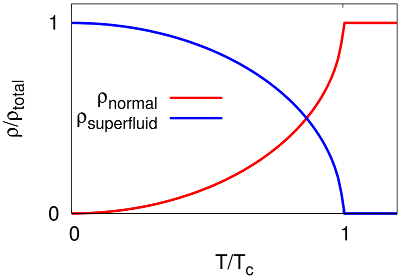

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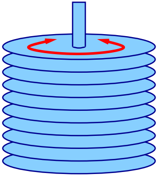

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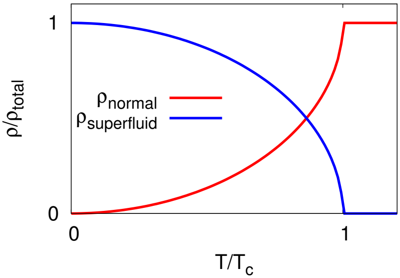

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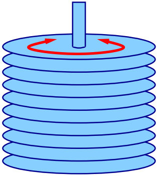

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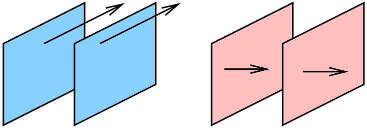

Superfluid density

- Currrent:

$$\mathbf{J}(r) = \psi^\dagger i \nabla \psi$$

$$J_i(\mathbf{q}) = \psi^\dagger_{\mathbf{k}+\mathbf{q}} \frac{2k_i + q_i}{2m} \psi_{\mathbf{k}}$$

- Response function:

$$H \to H - \sum_q \mathbf{f}(\mathbf{q}) \cdot \mathbf{J}_i(\mathbf{q}) \qquad J_i(\mathbf{q}) = \chi_{ij}(\mathbf{q}) f_j(\mathbf{q})$$

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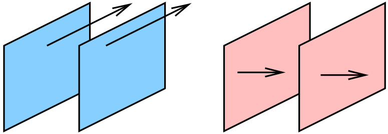

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$$\chi_{ij}(\omega = 0, \mathbf{q} \to 0) = \langle [J_i(\mathbf{q}), J_j(-\mathbf{q})] \rangle = \frac{\rho_S}{m} \frac{q_i q_j}{q^2} + \frac{\rho_N}{m} \delta_{ij}$$

Calculating superfluid response function

- Use WIDBG model
- Require vertex corrections
- Saddle point + fluctuations:

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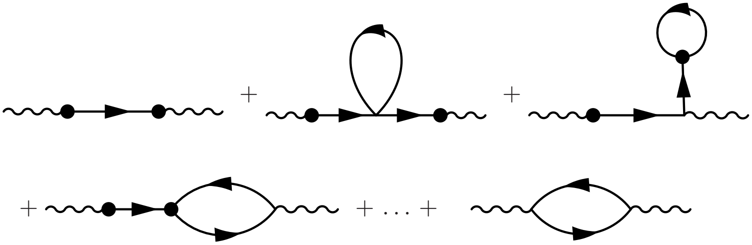
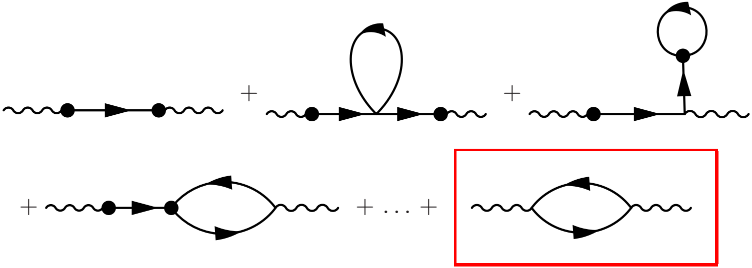


Calculating superfluid response function

- Use WIDBG model
- Require vertex corrections
- Saddle point + fluctuations: Only one diagram for χ_N



Non-equilibrium superfluid response

- Superfluid response exists because:

 $$= \frac{i\psi_0 q_i}{2m} (1, -1)\, D^R(q, \omega = 0) \begin{pmatrix} 1 \\ -1 \end{pmatrix} \frac{i\psi_0 q_j}{2m}$$

- $D^R(\omega = 0) \propto 1/\epsilon_q$ despite pumping/decay — superfluid response exists.
- Normal density:

$$\rho_N = \int d^d k \epsilon_k \int \frac{d\omega}{2\pi} \mathrm{Tr} \left[\sigma_z D^K \sigma_z (D^R + D^A) \right]$$

- Is affected by pump/decay:
  Does not vanish at $T \to 0$.

[JK PRL '11]

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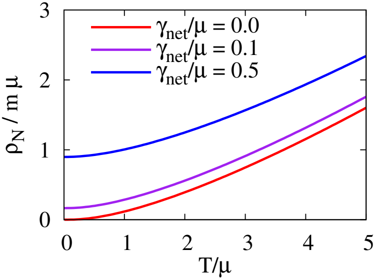


[JK PRL '11]

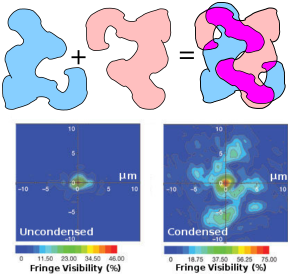
Superfluidity

1. Condensation vs lasing
   - Model Hamiltonian
   - Maxwell-Bloch
   - Low temperature non-equilibrium state
2. Superfluidity
   - Spectrum
   - Superfluid density
   - Response function
3. Power-law decay of coherence

Coherence in a 2D Gas

Correlations:

$$g_1(\mathbf{r}, \mathbf{r}', t) = \left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0) \right\rangle$$



- $D^< = D^K - D^R + D^A$
- Generally, get:

$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0,0) \right\rangle \simeq |\psi_0|^2 \exp\left[-a_p \begin{cases} \ln(r/r_0) & t \simeq 0 \\ \frac{1}{2}\ln(c^2 t/\gamma_{net} r_0^2) & r \simeq 0 \end{cases} \right]$$

[Szymańska *et al.* PRL ’06; PRB ’07] [Wouters and Savona PRB ’09]

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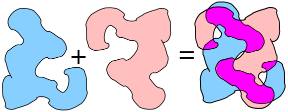

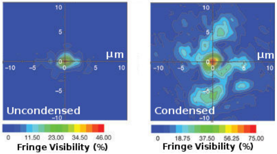

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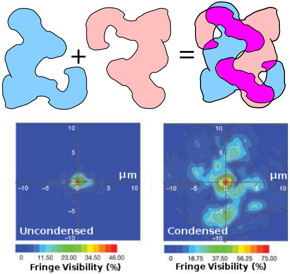

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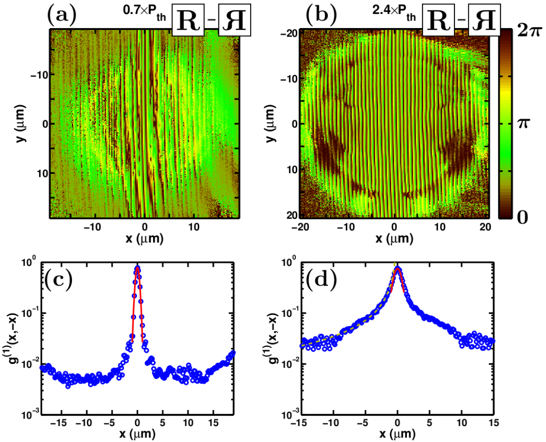
Experimental observation of power-law decay



G. Rompos, *et al.* PNAS '12

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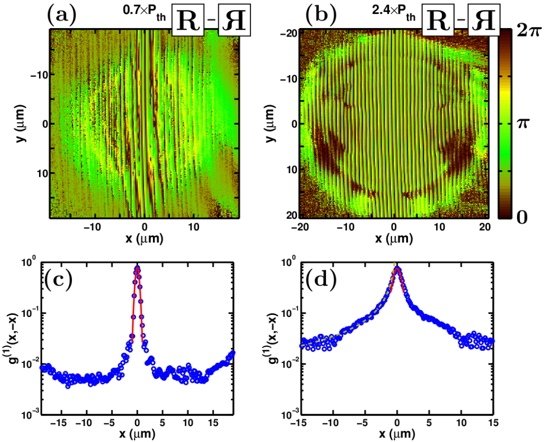
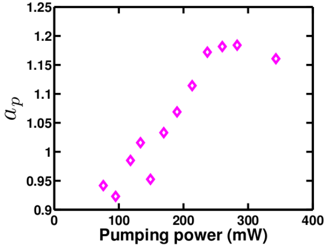


G. Rompos, *et al.* PNAS ’12

$$g_1(\mathbf{r}, -\mathbf{r}) \propto \left(\frac{r}{r_0}\right)^{-a_p}$$



Exponent in a non-equilibrium 2D gas

$$\lim_{r\to\infty}\left\langle \psi^\dagger(\mathbf{r},0)\psi(-\mathbf{r},0)\right\rangle = |\psi_0|^2 \exp\left[-D^{<}_{\phi\phi}(\mathbf{r},-\mathbf{r})\right] \propto \exp\left[-a_p \ln\left(\frac{2r}{r_0}\right)\right]$$

- Experimentally, $a_P \simeq 1.2$
- In equilibrium $a_p = \dfrac{mk_BT}{2\pi\hbar^2 n_s} < \dfrac{1}{4}$ (BKT transition)
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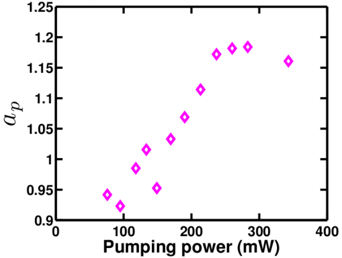

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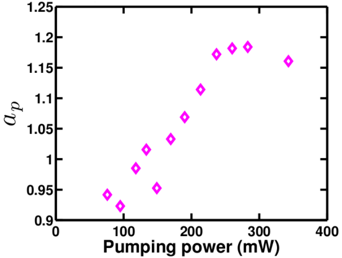

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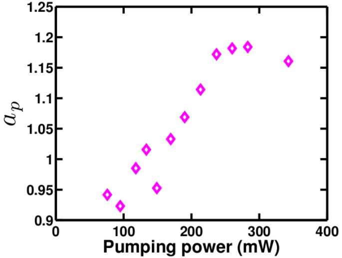

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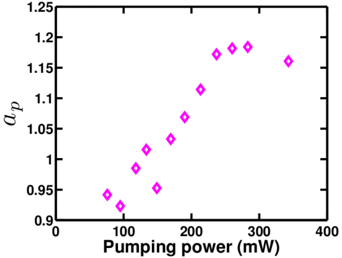

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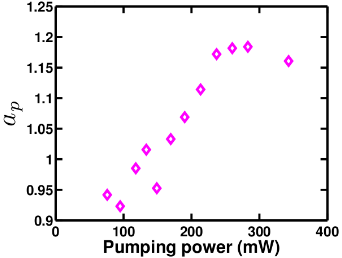
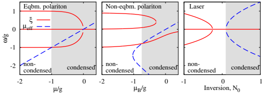
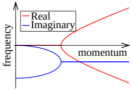


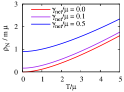
Conclusions

- Polariton condensation vs lasing

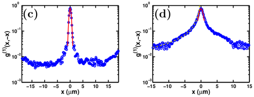


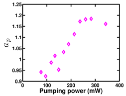
- Survival of superfluid response



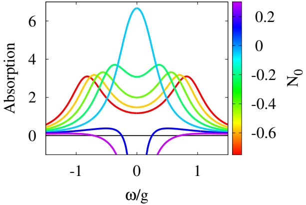


- Power law decay of correlations





Maxwell-Bloch Equations: Retarded Green's function



- Introduce $D^R(\omega)$:
  Response to perturbation

- Absorption $= -2\Im[D^R(\omega)]$

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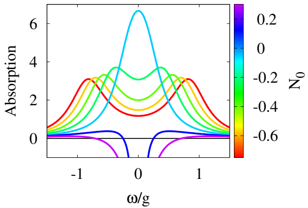

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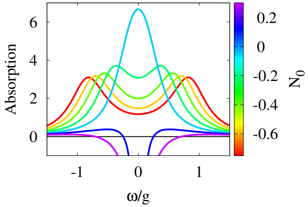

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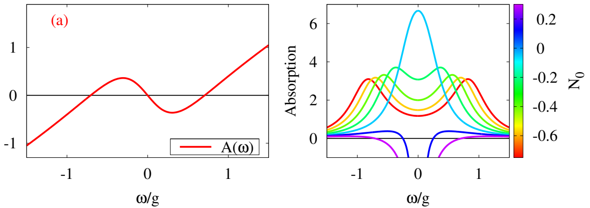

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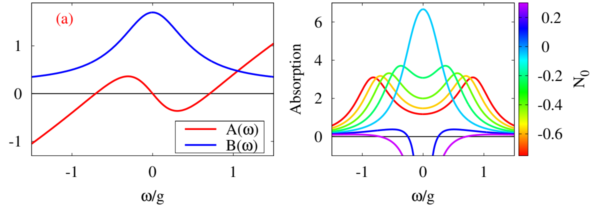
Maxwell-Bloch Equations: Retarded Green's function



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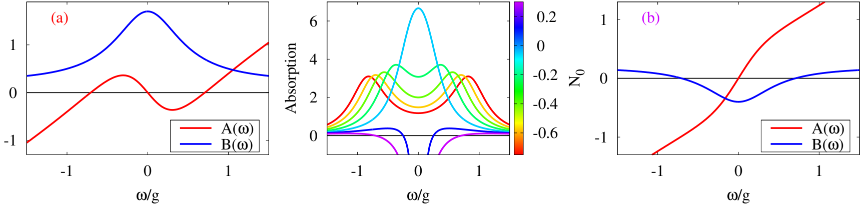

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- Generic structure of Green's function:

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$$\chi_{ij}(q) = -\frac{i}{2}\frac{d^2 \mathcal{Z}[f,\theta]}{df_i(q)d\theta_j(-q)}, \qquad \mathcal{Z}[f,\theta] = \int \mathcal{D}\psi \exp(iS[f,\theta])$$

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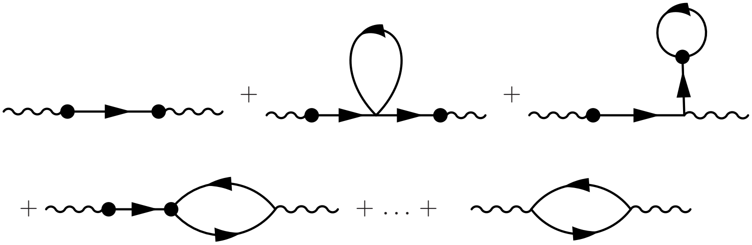
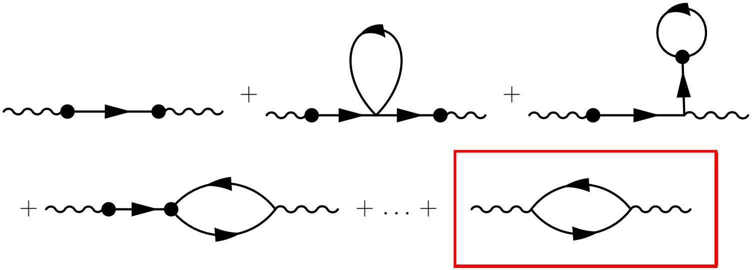


Calculating superfluid response function

- Use WIDBG model
- Require vertex corrections
- Saddle point + fluctuations: Only one diagram for χ_N

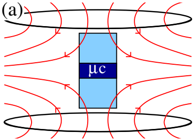


Measuring superfluid density

1. Effect rotating frame

Polariton polarization: $(\psi_{\circlearrowleft}, \psi_{\circlearrowright})$

$$H = \lambda \begin{pmatrix} \ell^2 & r^2 e^{2i\phi} \\ r^2 e^{-2i\phi} & -\ell^2 \end{pmatrix}$$



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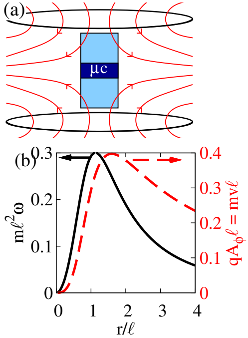
$$q\mathbf{A}_{\text{eff}} = m\omega \times \mathbf{r} = \frac{\hat{\phi}}{r}\left[1 - \frac{\ell^2}{\sqrt{r^4 + \ell^4}}\right]$$



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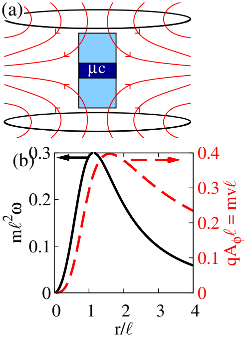
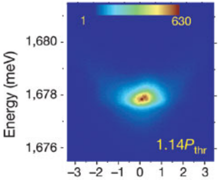


2. Measure resulting current

Energy shift of normal state:

$\Delta E = (1/2)mv^2 = 0.08/m\ell^2 \simeq 0.1\text{meV}$



Finite size effects: Single mode vs many mode

$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0)\right\rangle \simeq |\psi_0|^2 \exp\left[-D^<_{\phi\phi}(\mathbf{r}, \mathbf{r}', t)\right]$$

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$$D^<_{\phi\phi}(\mathbf{r},\mathbf{r},t) \propto \sum_n^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(\mathbf{r})|^2(1-e^{i\omega t})}{\left|(\omega + i\gamma_{\mathrm{net}})^2 + \gamma_{\mathrm{net}}^2 - \xi_n^2\right|^2}$$

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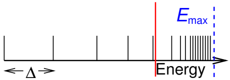
$$\Delta\xi \ll \sqrt{\frac{\gamma_{\text{net}}}{t}} \ll E_{\text{max}}$$



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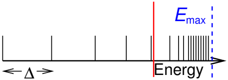

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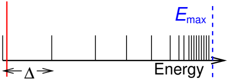

$$D^<_{\phi\phi} \sim \left(\frac{\pi C}{2\gamma_{\text{net}}}\right)\left(\frac{t}{2\gamma_{\text{net}}}\right)$$

(Recovers Schawlow-Townes laser linewidth)