Non-equilibrium phases of coupled matter-light systems

Jonathan Keeling

University of St Andrews 600 YEARS

SUPA

Oxford, May 2012

Coupling many atoms to light

Old question: *What happens to radiation when many atoms interact "collectively" with light.*

Superradiance — dynamical and steady state.

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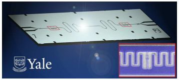
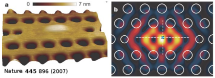
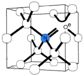
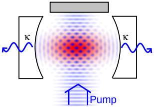
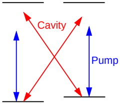
New relevance

- Superconducting qubits 

- Quantum dots 

- Nitrogen-vacancies in diamond 
- Ultra-cold atoms 



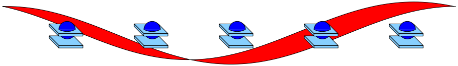
- Rydberg atoms

Dicke effect: Enhanced emission

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

R. H. Dicke
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey



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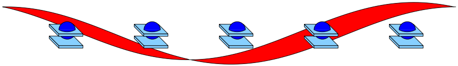

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$$\frac{d\rho}{dt} = -\frac{\Gamma}{2} \left[S^+ S^- \rho - S^- \rho S^+ + \rho S^+ S^- \right]$$

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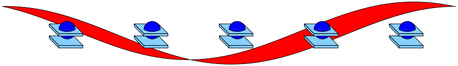

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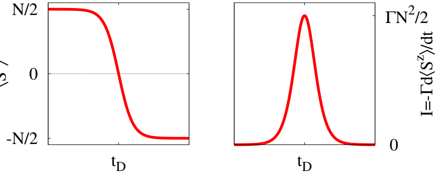

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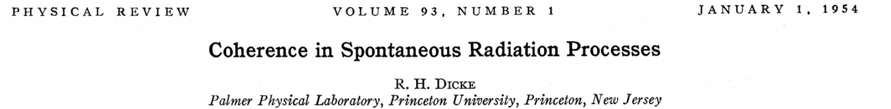

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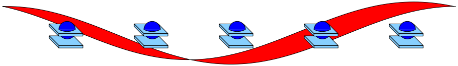

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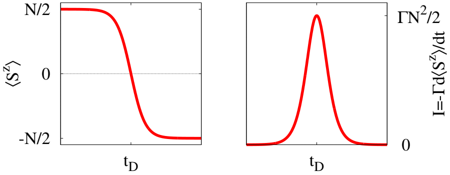
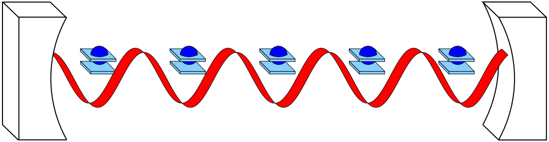


Problem: dipole interactions dephase. [Friedberg et al, Phys. Lett. 1972]

Collective radiation **with a cavity**: Dynamics



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Single cavity mode: oscillations

[Bonifacio and Preparata PRA ’70; **JK** PRA ’09]

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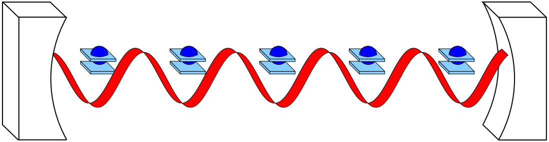

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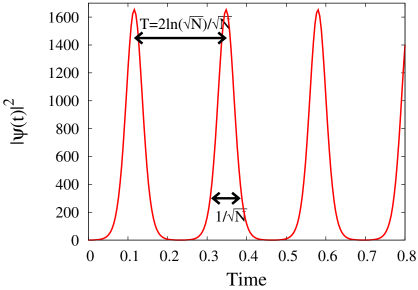

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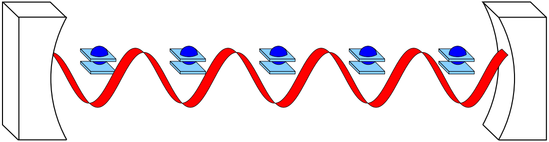

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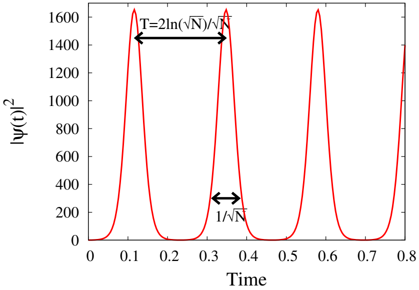

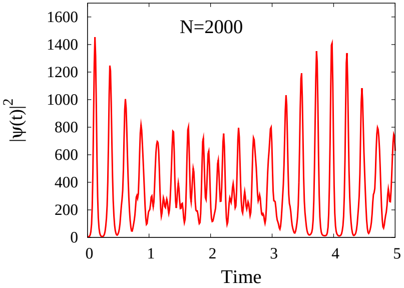

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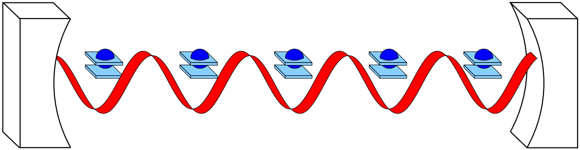
Dicke model: Equilibrium superradiance transition



$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g\left(\psi^\dagger S^- + \psi S^+\right).$$

- Coherent state: $|\Psi\rangle \to e^{\lambda\psi^\dagger + \eta S^+}|\Omega\rangle$
- Small g, min at $\lambda, \eta = 0$

[Hepp, Lieb, Ann. Phys. '73]

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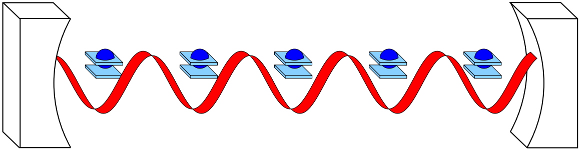

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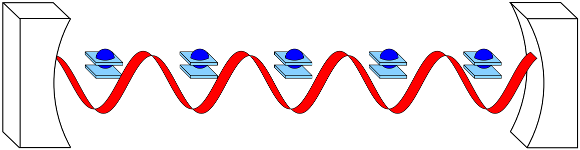

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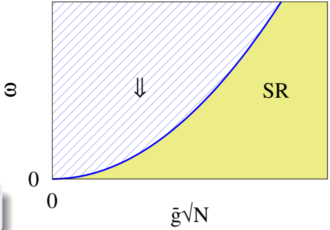
Spontaneous polarisation if: $Ng^2 > \omega\omega_0$



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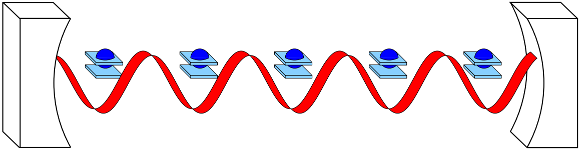
No go theorem and transition



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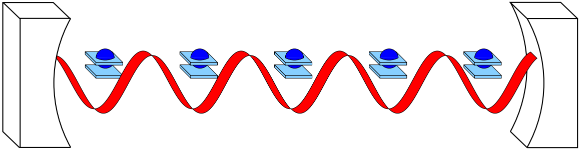

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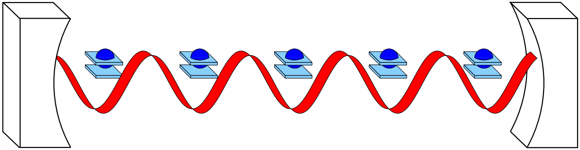

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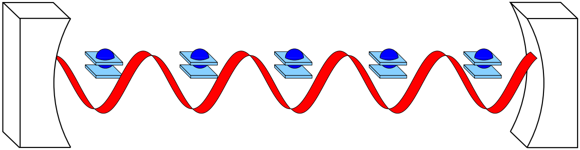

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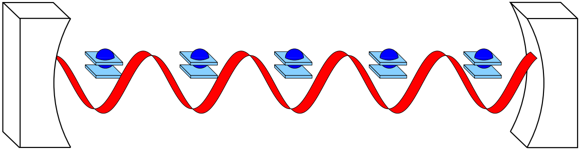

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But Thomas-Reiche-Kuhn sum rule states: $g^2/\omega_0 < 2\zeta$. **No transition**
[Rzazewski *et al* PRL '75]

Dicke phase transition: ways out

Problem: $g^2/\omega_0 < 2\zeta$ for intrinsic parameters. **Solutions**:

- **Interpretation**
  Ferroelectric transition in $\mathbf{D} \cdot \mathbf{r}$ gauge.
  [JK JPCM '07]
- Circuit QED [Nataf and Ciuti, Nat. Comm. '10; Viehmann *et al.* PRL '11]

- Grand canonical ensemble:
  - If $H \to H - \mu(S^z + \psi^\dagger \psi)$, need only:
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- Dissociate g, ω_0,
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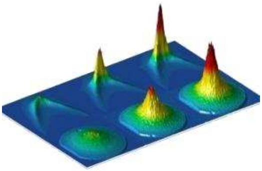

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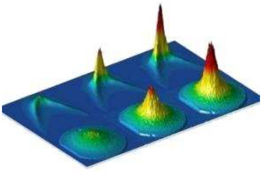

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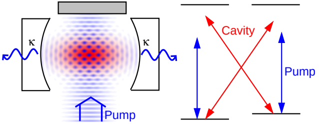

Outline

1. Introduction: Dicke model and superradiance
2. Dynamics of generalized Dicke model
   - Summary of experiment and classical dynamcs
   - Fixed points and dynamical phases
   - Timescales and consequences for experiment
   - Persistent oscillating phases
3. Non-equilibrium states of Jaynes-Cummings-Hubbard Model
   - Relating equilibrium JCHM & Dicke model
   - Coherently pumped JCHM

Acknowledgements

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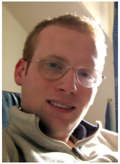
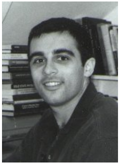


J.Mayoh



M.J.Bhaseen

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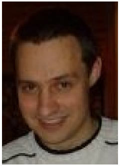

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M.Biondi

S.Schmidt

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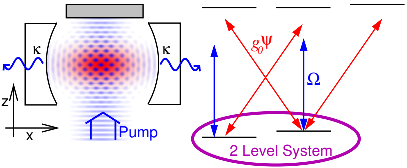
Funding: EPSRC Engineering and Physical Sciences Research Council

Reminder of cold-atom extended Dicke model



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$\Downarrow$: $\Psi(x, z) = 1$

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$$H = \omega \psi^\dagger \psi + \omega_0 S^z$$

[Baumann *et al* Nature '10]

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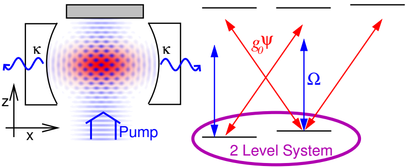

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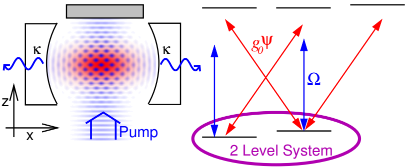

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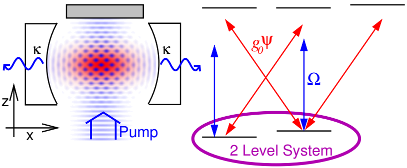
$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+) + U S_z \psi^\dagger \psi.$$

$$\partial_t \rho = -i[H,\rho] - \kappa(\psi^\dagger\psi\rho - 2\psi\rho\psi^\dagger + \rho\psi^\dagger\psi)$$

[Baumann *et al* Nature '10]

Reminder of cold-atom extended Dicke model



2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$:

$\Downarrow$: $\Psi(x,z) = 1$

$\Uparrow$: $\Psi(x,z) = \sum_{\sigma,\sigma'=\pm} e^{ik(\sigma x + \sigma' z)}$

Feedback: $U \propto \dfrac{g_0^2}{\omega_c - \omega_a}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+) + U S_z \psi^\dagger\psi.$$

$$\partial_t \rho = -i[H,\rho] - \kappa(\psi^\dagger\psi\rho - 2\psi\rho\psi^\dagger + \rho\psi^\dagger\psi)$$

$\omega_0 \sim \text{kHz} \ll \omega, \kappa, g\sqrt{N} \sim \text{MHz}.$

[Baumann *et al* Nature '10]

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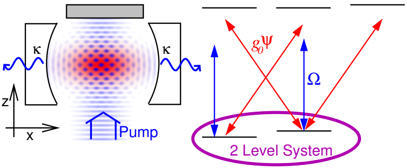

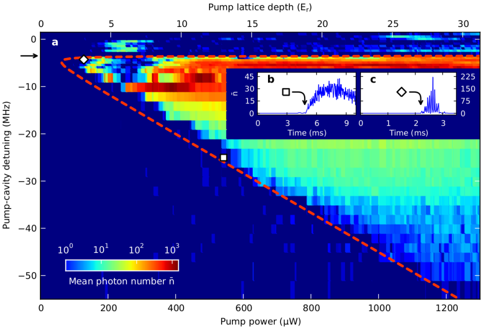

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$\omega_0 \sim$ kHz $\ll \omega, \kappa, g\sqrt{N} \sim$ MHz.

[Baumann *et al* Nature '10]

Classical dynamics of the extended Dicke model

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  - Recover Retarded Green's function (spectrum)
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- $\psi = 0, \mathbf{S} = (0, 0, \pm N/2)$ always a solution.
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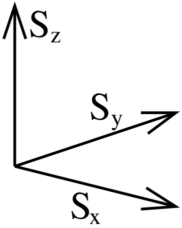

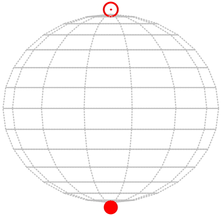

Small g: $\Uparrow, \Downarrow$ only.
($\omega = 30$MHz, $UN = -40$MHz)

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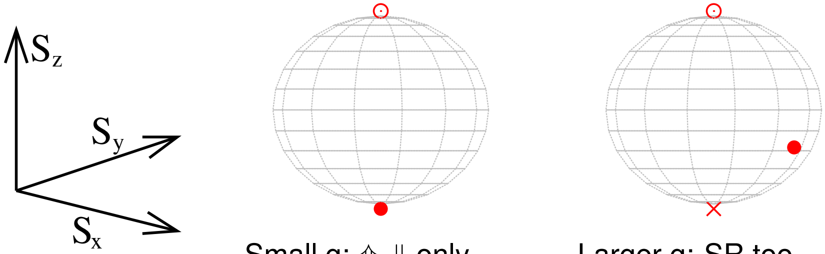

Small g: $\Uparrow, \Downarrow$ only. Larger g: SR too.
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Steady state phase diagram

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See also Domokos and Ritsch PRL '02, Domokos *et al.* PRL '10

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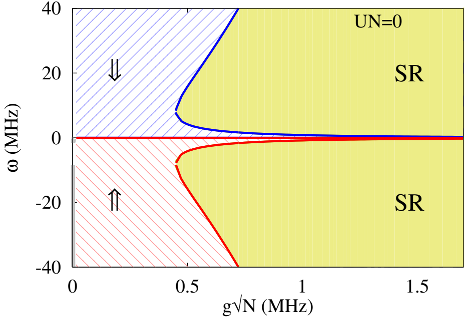

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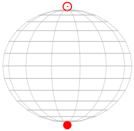
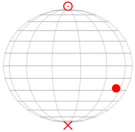


SR(A): $S_y = 0$



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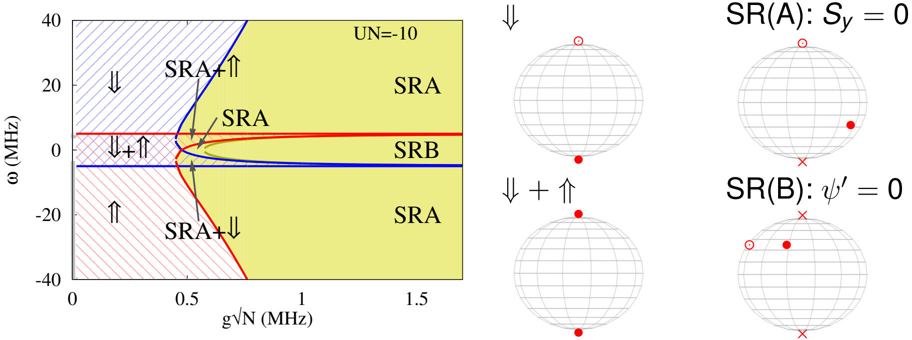

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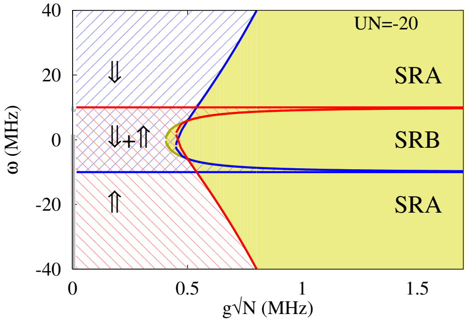

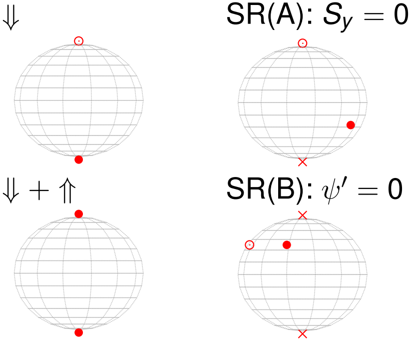

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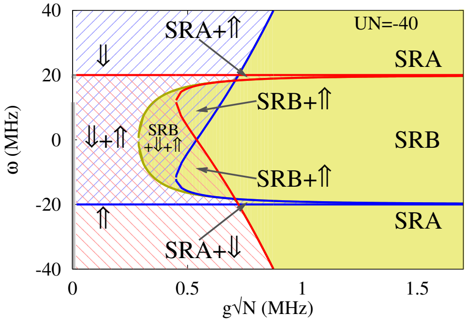
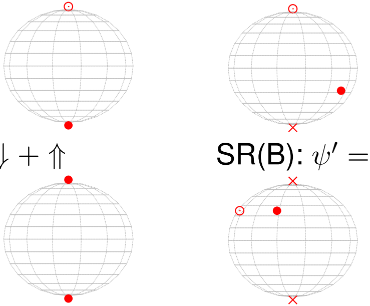


$\Downarrow$ SR(A): $S_y = 0$

$\Downarrow + \Uparrow$ SR(B): $\psi' = 0$



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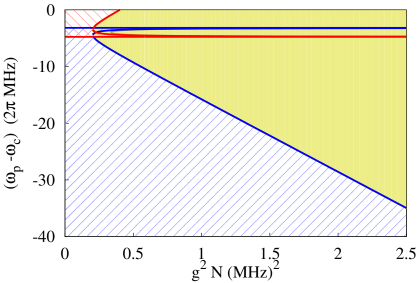
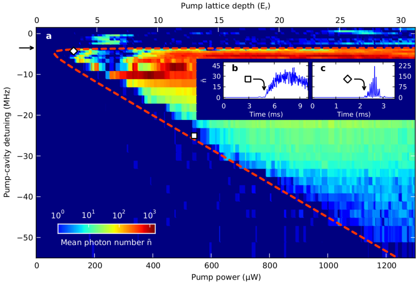
Comparison to experiment





$UN = -10\text{MHz}$

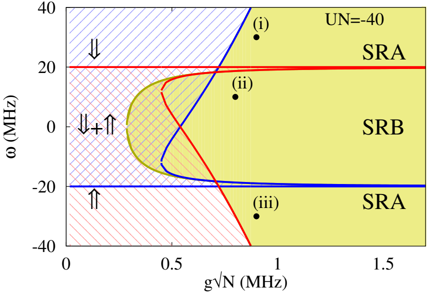
Adapted from: [Bhaseen *et al.* PRA '12]

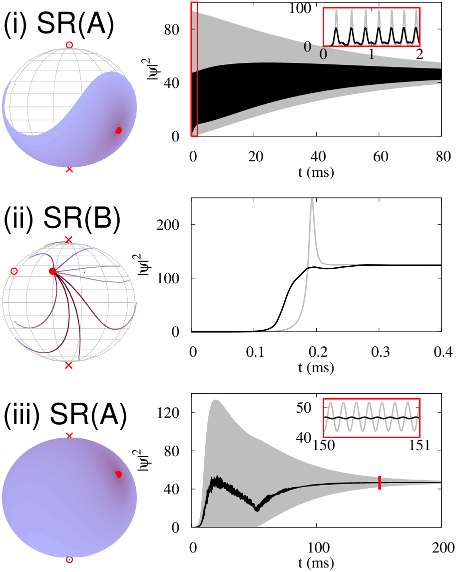
[Baumann *et al* Nature '10]

$$\omega = \omega_c - \omega_p + \frac{5}{2} UN, \qquad UN = -\frac{g_0^2}{4(\omega_a - \omega_c)}$$

Dynamics: Evolution from normal state

Gray: $\mathbf{S} = (\sqrt{N}, \sqrt{N}, -N/2)$
Black: Wigner distribution of $\mathbf{S}$, ψ



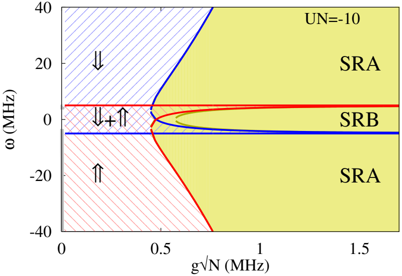


Oscillations: $\sim$ 0.1ms
Decay: 20ms, 0.1ms, 20ms

Asymptotic state: Evolution from normal state

(Near to experimental $UN = -13$MHz).

All stable attractors:



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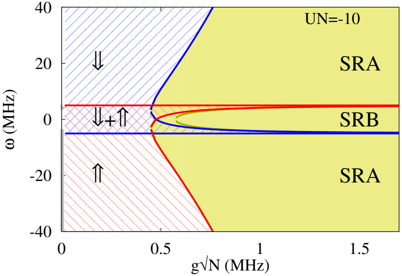
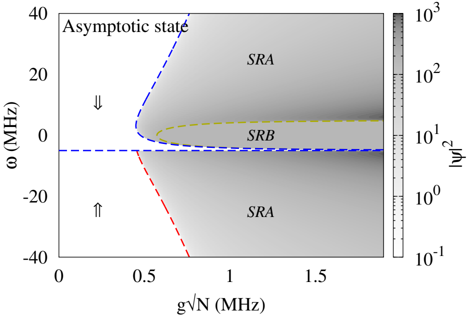
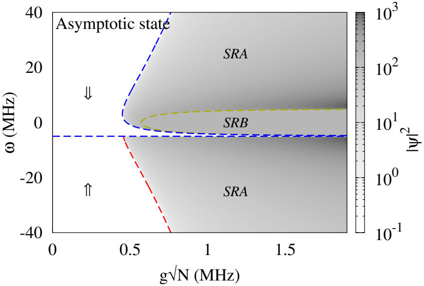
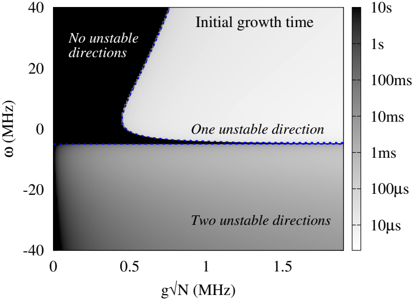


Starting from ⇓



Timescales for dynamics: What are they?





Growth Most unstable eigenvalues near $\mathbf{S} = (0, 0, -N/2)$

Decay Slowest stable eigenvalues near final state

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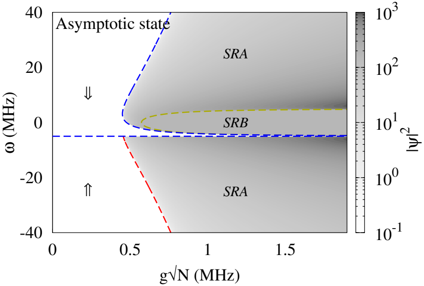

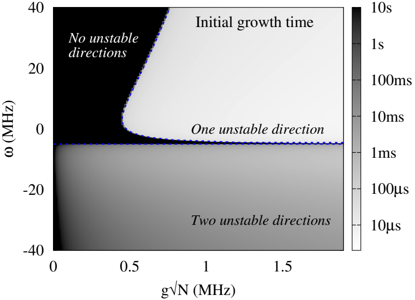

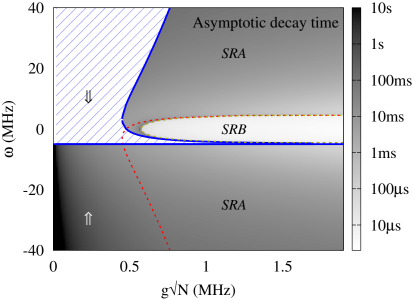

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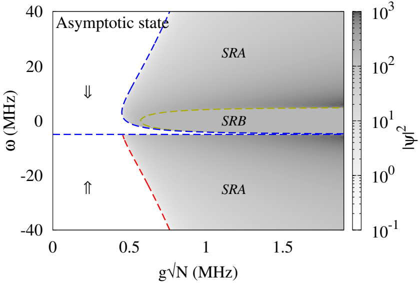
Timescales for dynamics: Consequences for experiment



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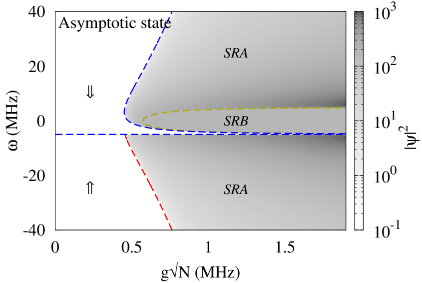

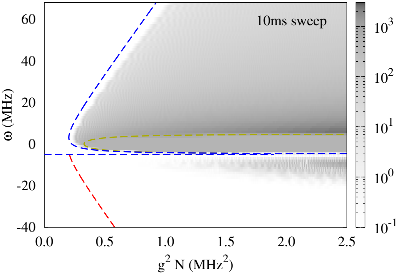

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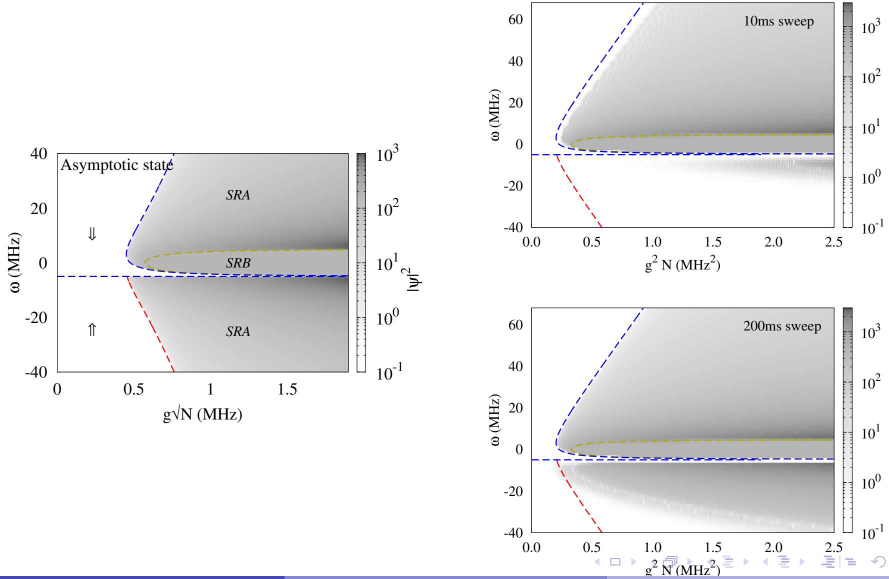
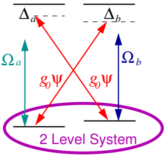


Timescales for dynamics: Why so slow and varied?

Suppose co- and counter-rotating terms differ



$$H = \ldots + g(\psi^\dagger S^- + \psi S^+) + g'(\psi^\dagger S^+ + \psi S^-) + \ldots$$

- SR(A) near phase boundary at small $\delta g \to$ Critical slowing down
- SR(A), SR(B) continuously connect

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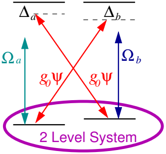

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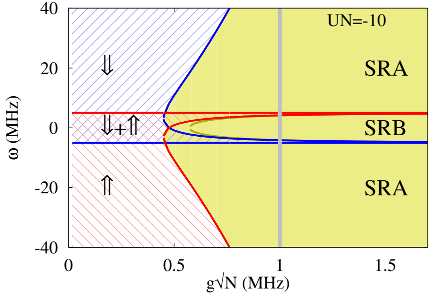

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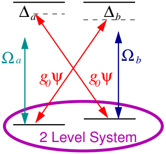

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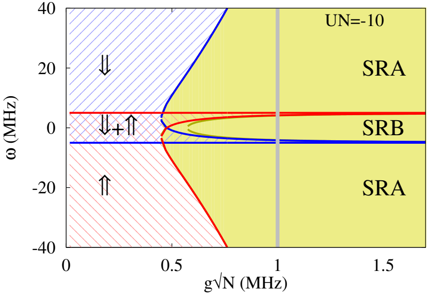

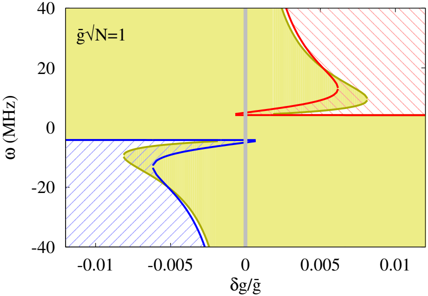

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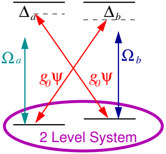

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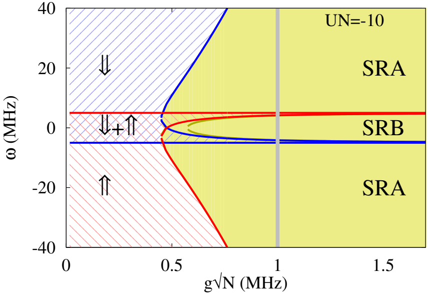

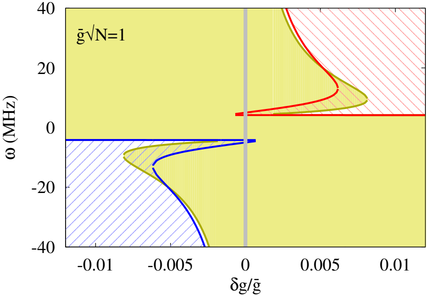

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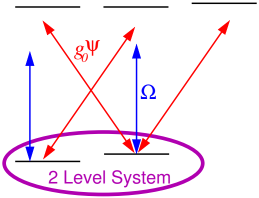
Regions without fixed points

Changing U:



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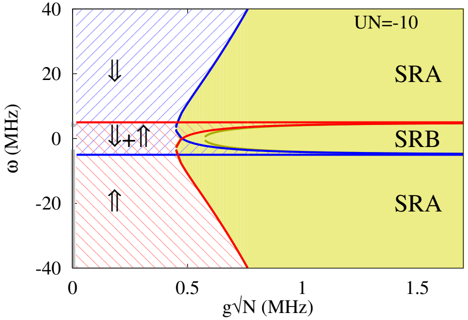

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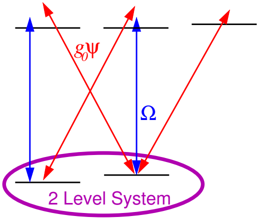

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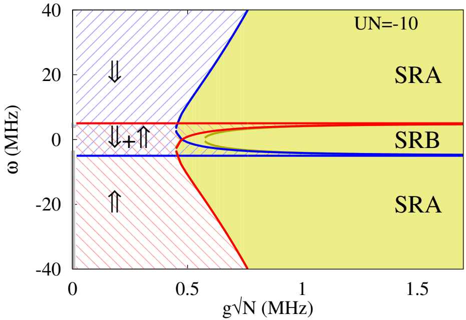

Regions without fixed points

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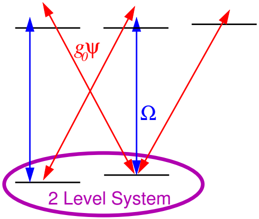
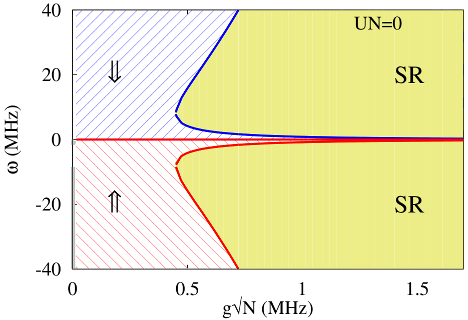


$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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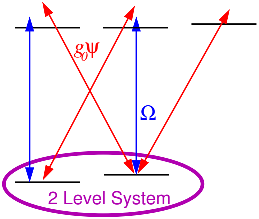

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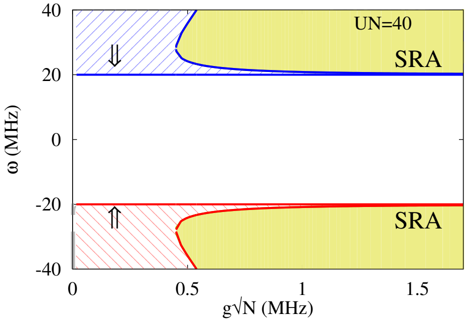

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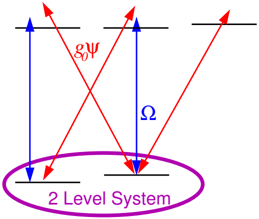

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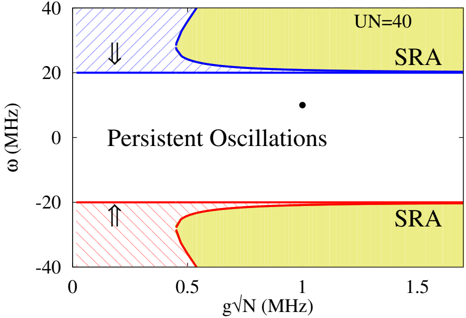
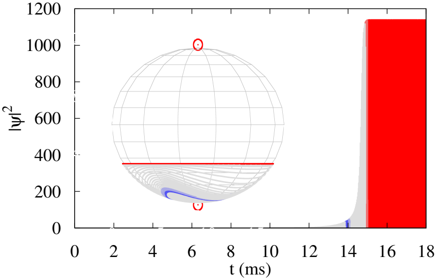
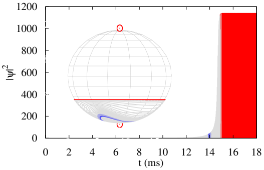




Persistent (optomechanical) oscillations



$$\dot{S}^- = -i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
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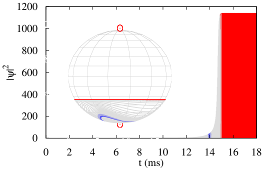

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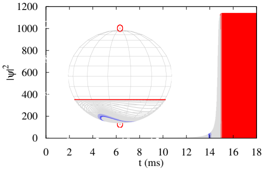

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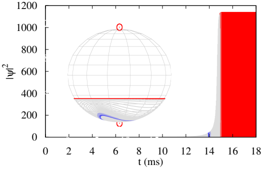

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Get:

Fix $\omega + US^z = 0$ **if** $\psi' = 0$.

$$S^- = re^{-i\theta} \quad r = \sqrt{\frac{N^2}{4} - (S^z)^2}$$

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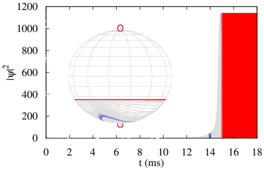

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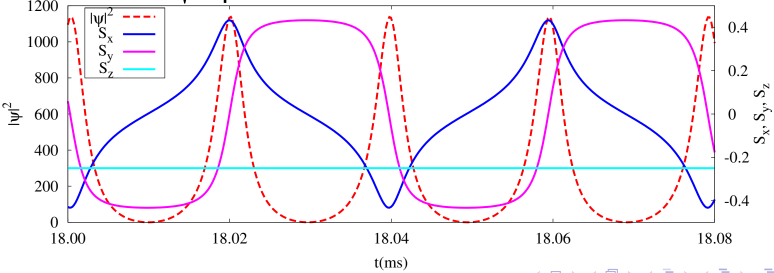
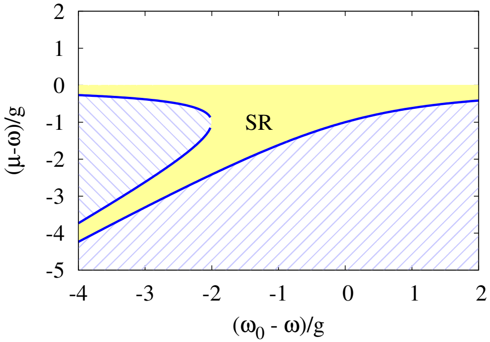


Outline

Equilibrium: Dicke model with chemical potential

$$H - \mu N = (\omega - \mu)\psi^\dagger\psi + (\omega_0 - \mu)S^z + g\left(\psi^\dagger S^- + \psi S^+\right)$$



- Transition at: $g^2 N > (\omega - \mu)(\omega_0 - \mu)$
- Reduce critical g
- Unstable if $\mu > \omega$
- Inverted if $\mu > \omega_0$

[Eastham and Littlewood, PRB '01]

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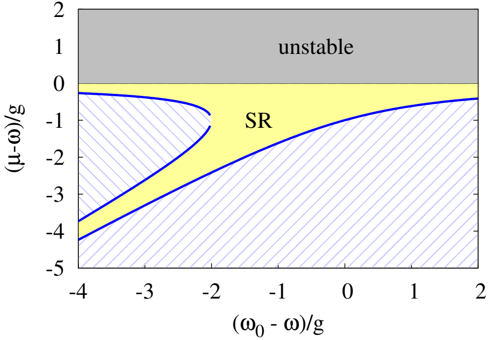

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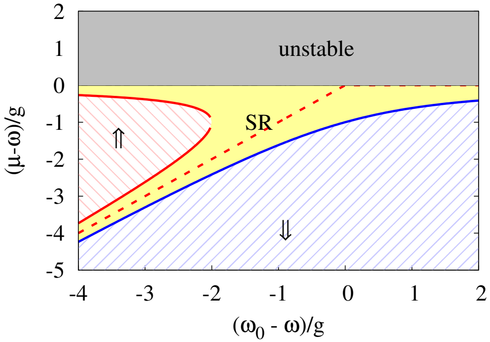

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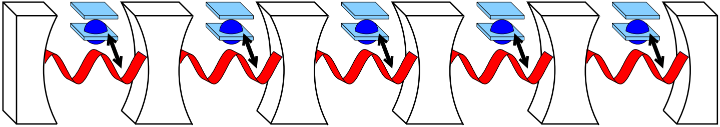
Jaynes-Cummings Hubbard model



$$H = -\frac{J}{z}\sum_{ij}\psi_i^\dagger\psi_j + \sum_i \frac{\Delta}{2}\sigma_i^z + g(\psi_i^\dagger\sigma_i^- + \text{H.c.})$$

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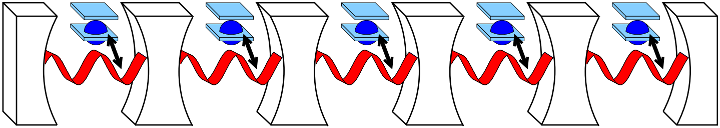

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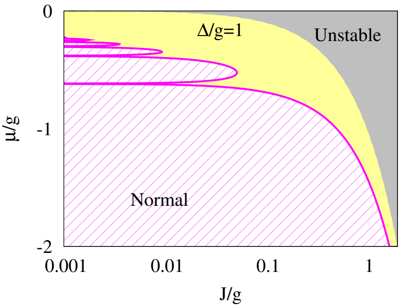

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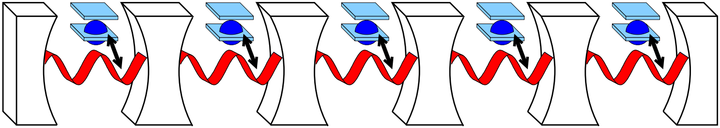

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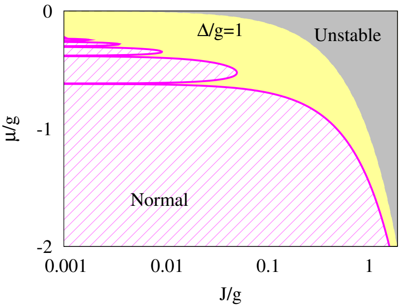
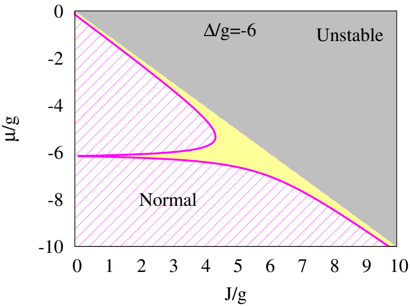
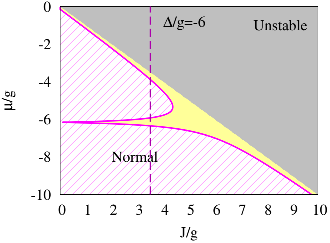


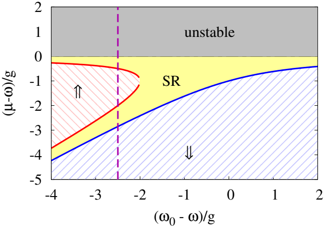


Dicke vs JCHM

JCHM



Dicke



- $k = 0$ mode of JCHM $\leftrightarrow$ Dicke photon mode
- $\Uparrow$ $\leftrightarrow$ $n = 1$ Mott lobe

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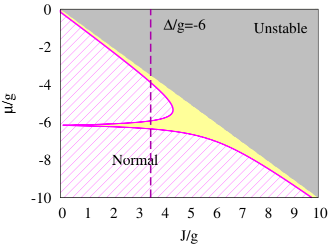
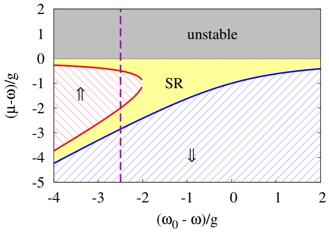


Dicke



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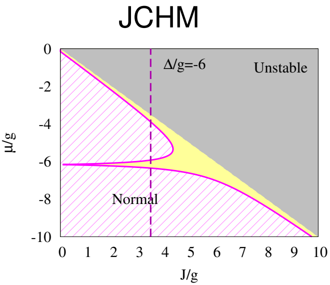

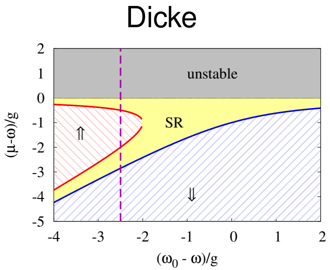

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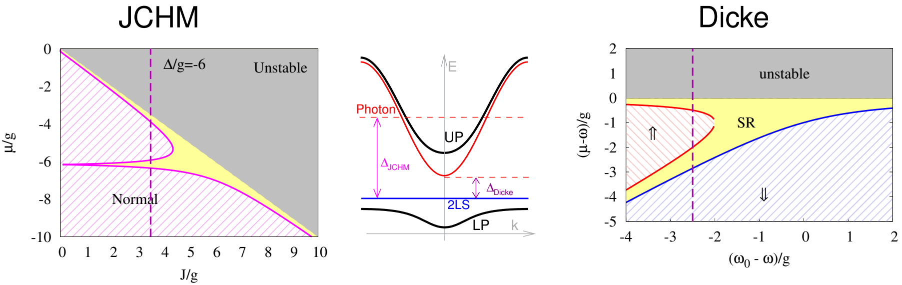

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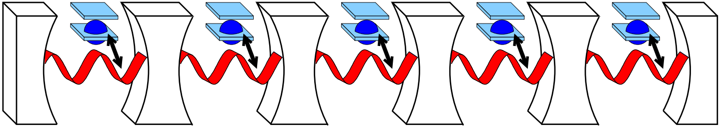
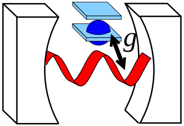
Coherently pumped JCHM



$$H = -\frac{J}{z}\sum_{ij}\psi_i^\dagger\psi_j + \sum_i \frac{\Delta}{2}\sigma_i^z + g(\psi_i^\dagger\sigma_i^- + \text{H.c.}) + f(\psi_i e^{i\omega_L t} + \text{H.c.})$$

$$\partial_t\rho = -i[H,\rho] - \frac{\kappa}{2}L_\psi[\rho] - \frac{\gamma}{2}L_{\sigma^-}[\rho]$$

Coherently pumped single cavity [Bishop *et al.* Nat. Phys '09]



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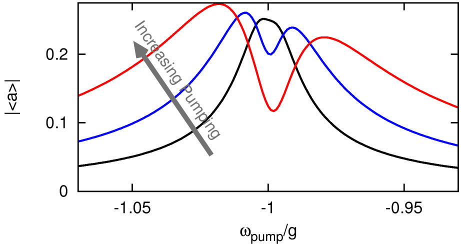

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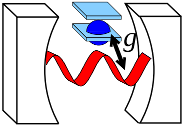

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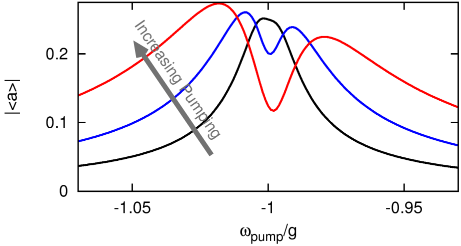
- Effective 2LS: $|\text{Empty}\rangle, |1 \text{ polariton}\rangle$



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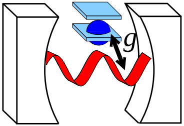

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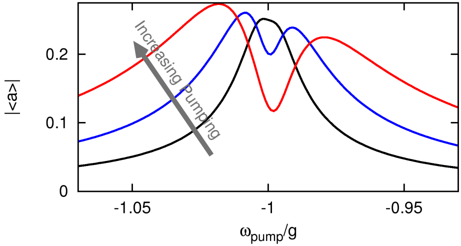
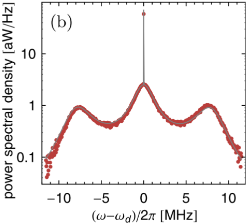


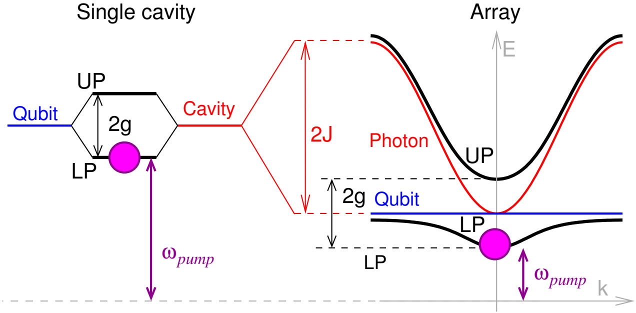
- Mollow triplet fluorescence



[Lang *et al.* PRL '11]

Coherently pumped dimer & array

Chose detuning *a la* Dicke model



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  - More/less localised states
  - Connects to Dicke limit

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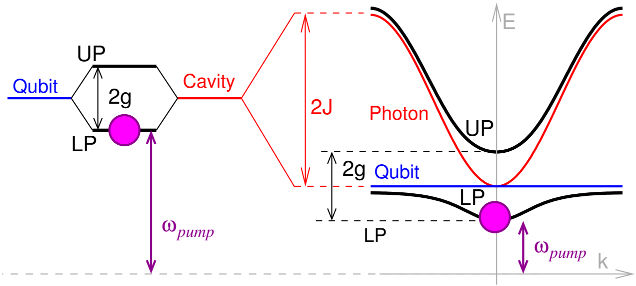
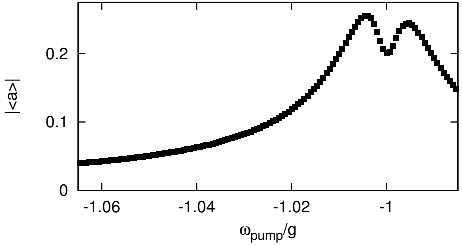


Evolution of anti-resonance vs J.



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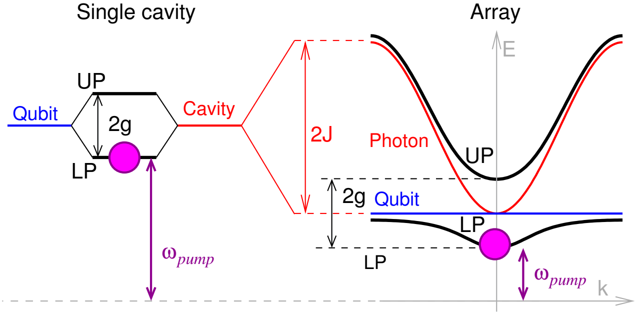

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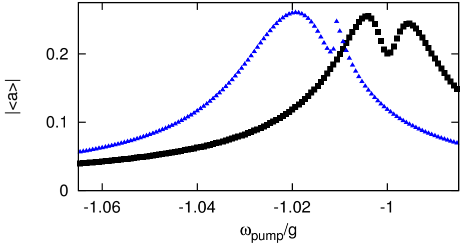

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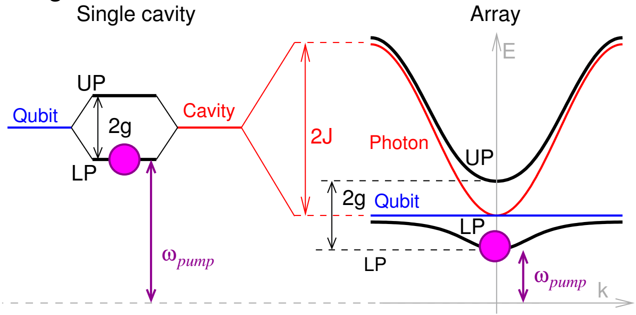

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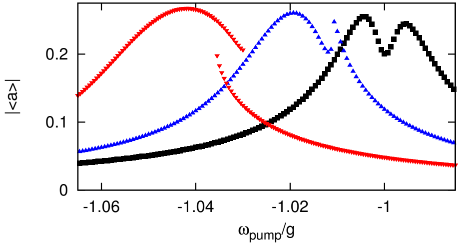

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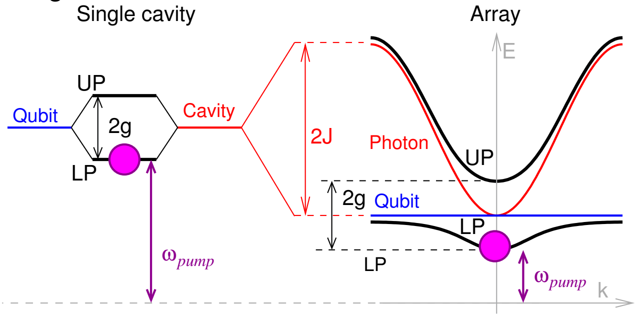

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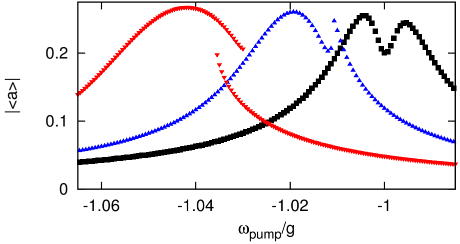

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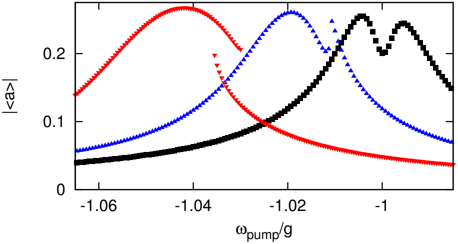
Photon blockade picture $J \lesssim g$

- Polariton basis
- Nonlinearity $|\epsilon_2 - 2\epsilon_1| \propto g$.

$$H = \sum_i \left(\frac{\epsilon}{2}\tau_i^z + \tilde{f}\tau_i^x \right)$$



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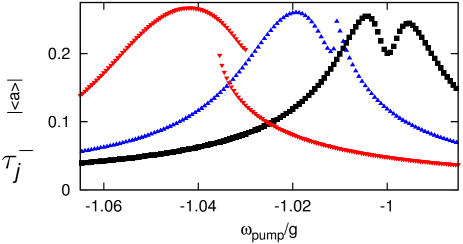

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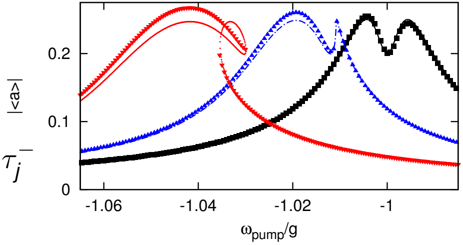

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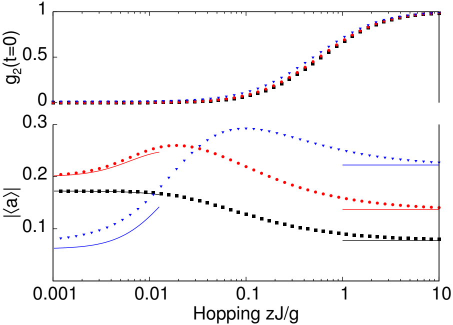
Coherently pumped array: correlations & fluorescence



Correlations

- g_2 : $0 \rightarrow 1$ crossover.

- Small J: Mollow triplet
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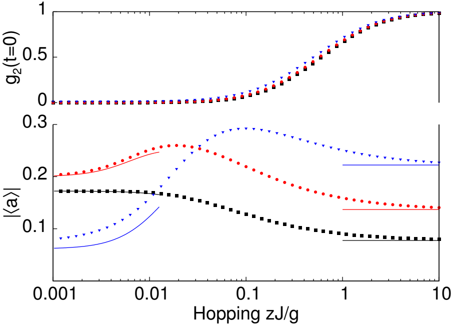

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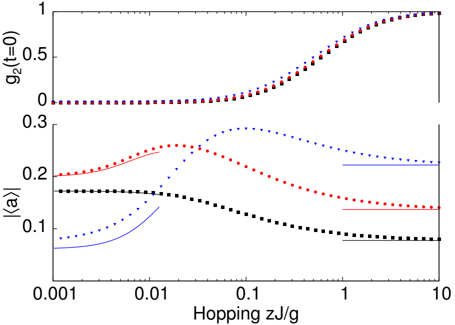

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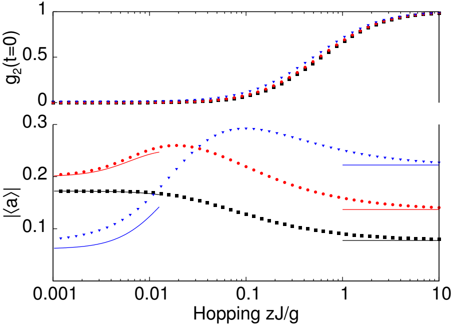

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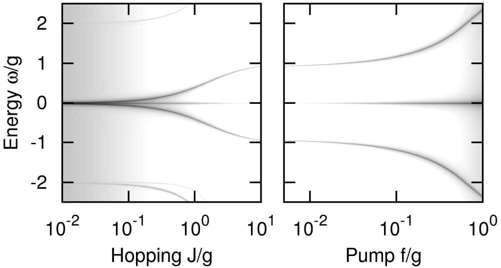

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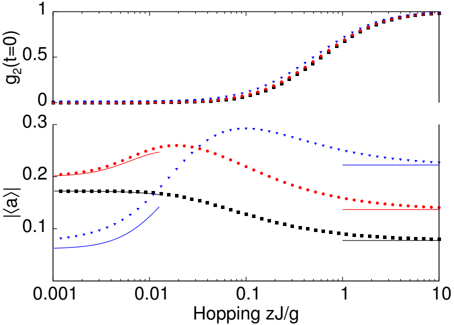

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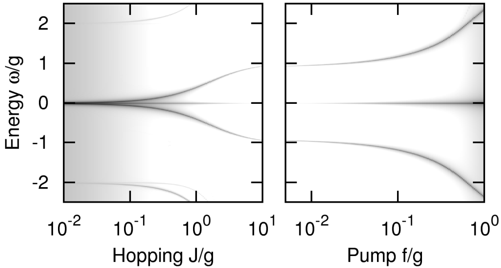

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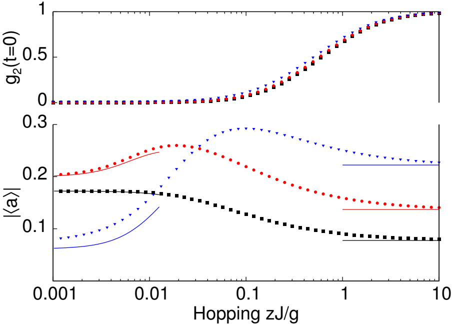

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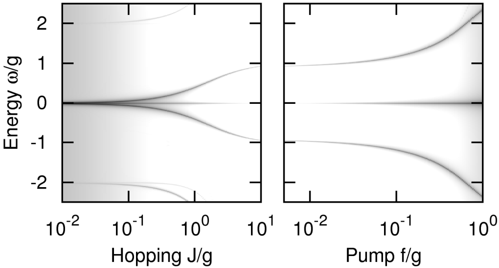
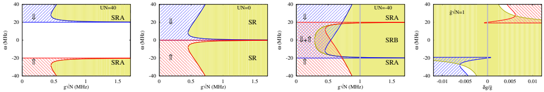
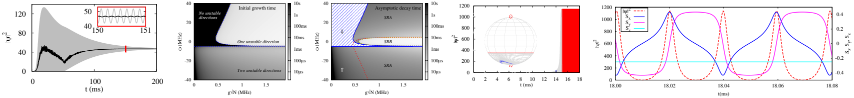


Summary

- Wide variety of dynamical phases

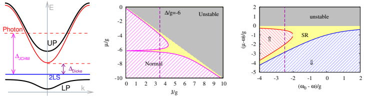


- Slow dynamics for $U < 0$ & Persistent oscillations for $U > 0$

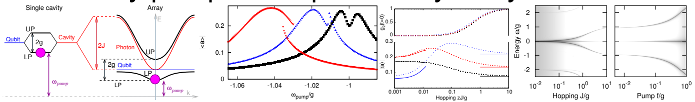


JK *et al.* PRL '10, Bhaseen *et al.* PRA '12

- Dicke model and JCHM: connection at $J \to \infty$



- Coherently pumped coupled cavity array



Nissen *et al.* PRL in press '12

Ferroelectric transition

Atoms in Coulomb gauge

$$H = \sum \omega_k a_k^\dagger a_k + \sum_i [p_i - eA(r_i)]^2 + V_{\text{coul}}$$

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Gauge transform to dipole gauge $\mathbf{D} \cdot \mathbf{r}$

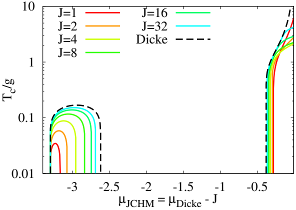
$$H = \omega_0 S^z + \omega\psi^\dagger\psi + \bar{g}(S^+ - S^-)(\psi - \psi^\dagger)$$

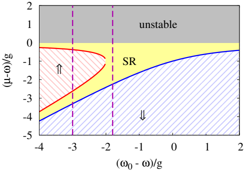
"Dicke" transition at $\omega_0 < N\bar{g}^2/\omega \equiv 2\eta N$

But, ψ describes **electric displacement**

Dicke vs JCHM, $T \neq 0$

$\Delta_{Dicke} = -3$





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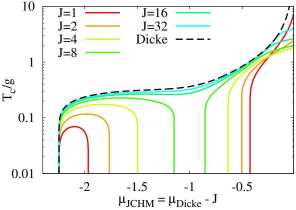

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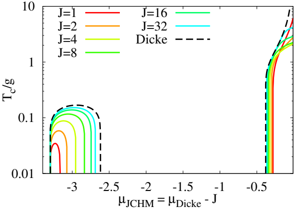

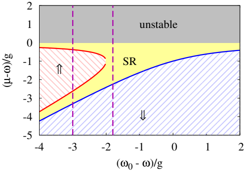

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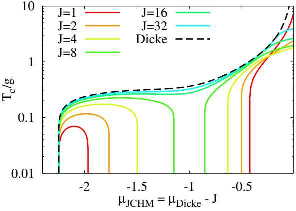

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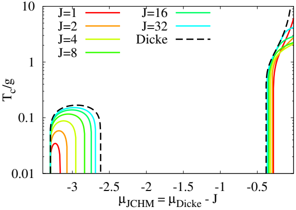

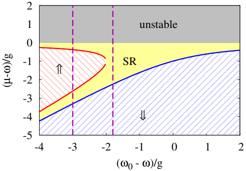

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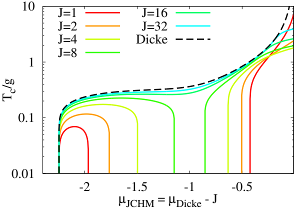

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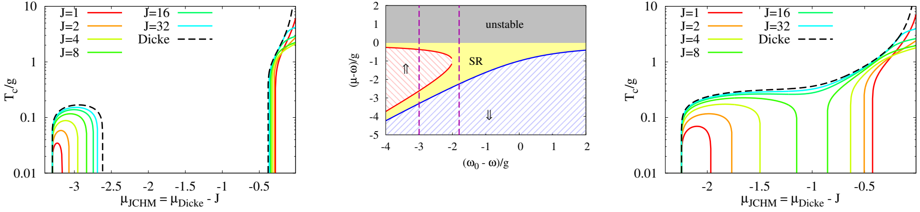

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