Non-equilibrium phases of coupled matter-light systems

Jonathan Keeling

University of St Andrews 600 YEARS

SUPA

Mátraháza, May 2012

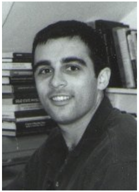
Outline

1. Dynamics of generalized Dicke model
   - Summary of experiment and classical dynamcs
   - Fixed points and dynamical phases
   - Timescales and consequences for experiment
   - Persistent oscillating phases
2. Non-equilibrium states of Jaynes-Cummings-Hubbard Model
   - Relating equilibrium JCHM & Dicke model
   - Coherently pumped JCHM

Acknowledgements

Part 1:

J.Mayoh



M.J.Bhaseen

B.D.Simons

Part 2:

F.Nissen

G.Blatter

M.Biondi

S.Schmidt

H.Türeci

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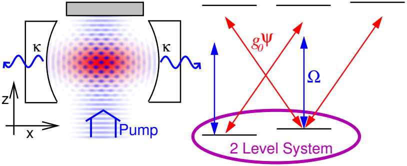
Reminder of cold-atom extended Dicke model



2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$:

$\Downarrow$: $\Psi(x, z) = 1$

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$$H = \omega\psi^\dagger\psi + \omega_0 S^z$$

[Baumann *et al* Nature '10]

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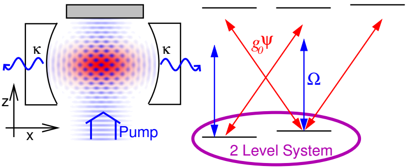

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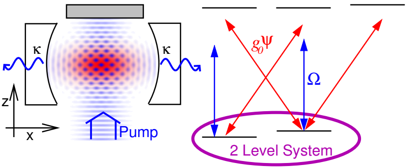

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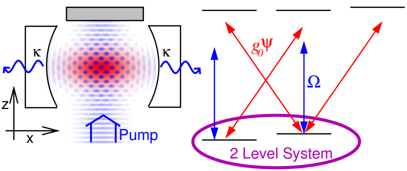

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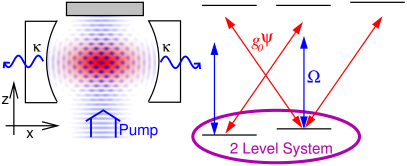

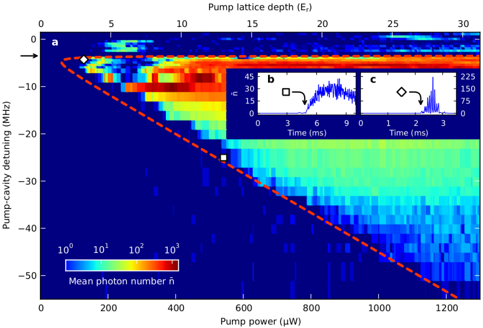

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Classical dynamics of the extended Dicke model

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  - Recover Retarded Green's function (spectrum)
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($|\mathbf{S}| = N/2 \gg 1$)

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- If $g > g_c$, $\psi \neq 0$ too
  - A $S^y = -\Im[S^-] = 0$
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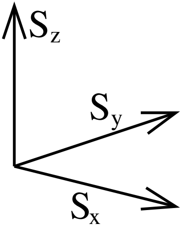

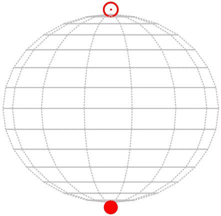

Small g: $\Uparrow, \Downarrow$ only.
($\omega = 30$MHz, $UN = -40$MHz)

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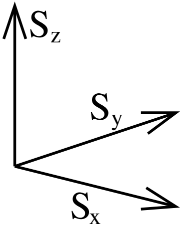

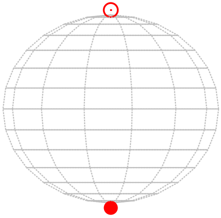

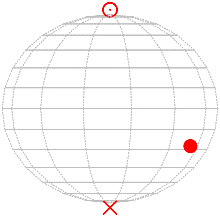

Small g: $\Uparrow, \Downarrow$ only. Larger g: SR too.
($\omega = 30$MHz, $UN = -40$MHz)

Steady state phase diagram

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See also Domokos and Ritsch PRL '02, Domokos *et al.* PRL '10

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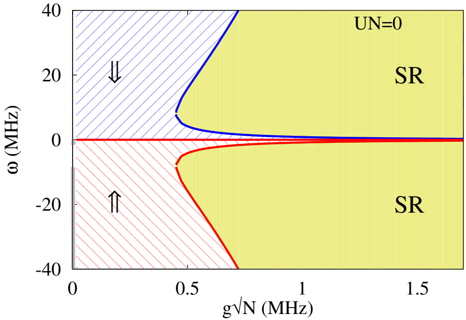
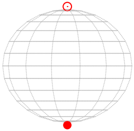
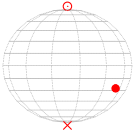


$\Downarrow$



SR(A): $S_y = 0$



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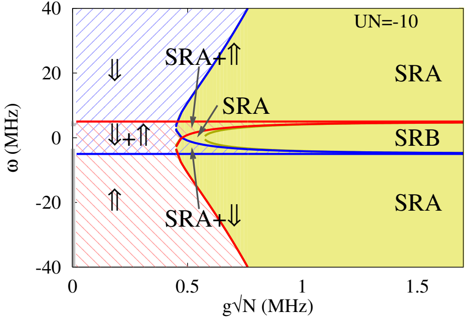

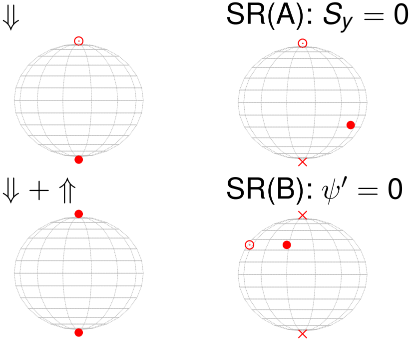

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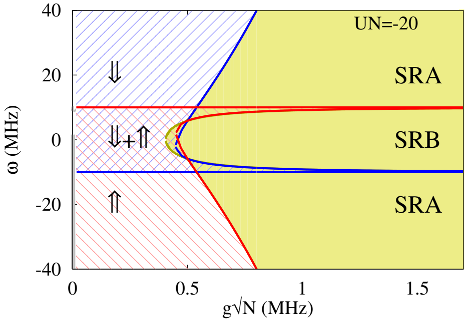

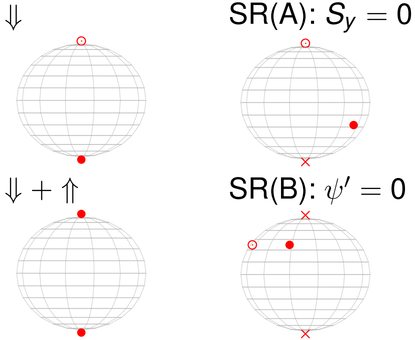

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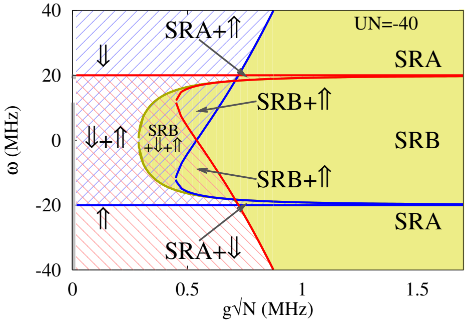

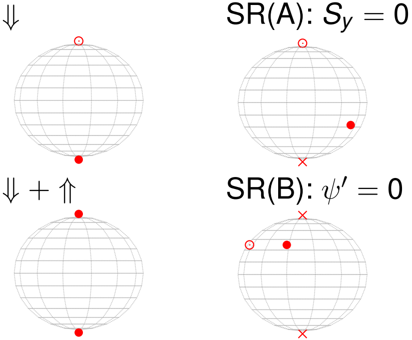

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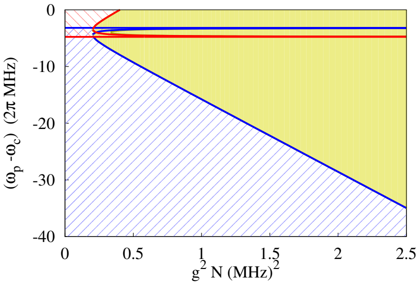
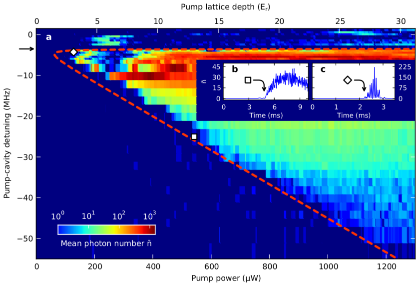
Comparison to experiment





$UN = -10\text{MHz}$

Adapted from: [Bhaseen *et al.* PRA '12]

[Baumann *et al* Nature '10]

$$\omega = \omega_c - \omega_p + \frac{5}{2} UN, \qquad UN = -\frac{g_0^2}{4(\omega_a - \omega_c)}$$

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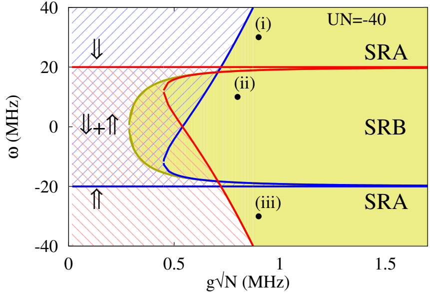
Dynamics: Evolution from normal state

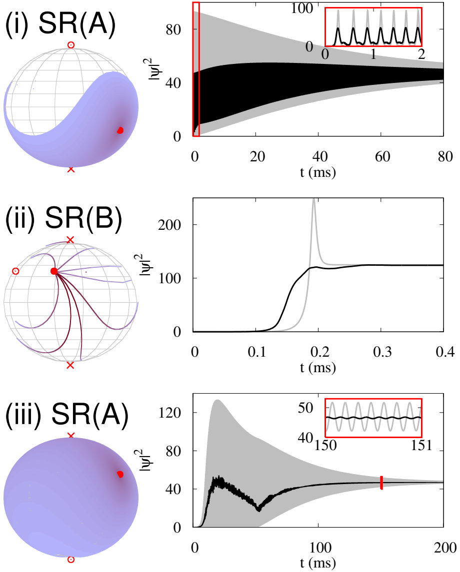
Gray: $\mathbf{S} = (\sqrt{N}, \sqrt{N}, -N/2)$

Black: Wigner distribution of $\mathbf{S}$, ψ



Oscillations: ~ 0.1ms

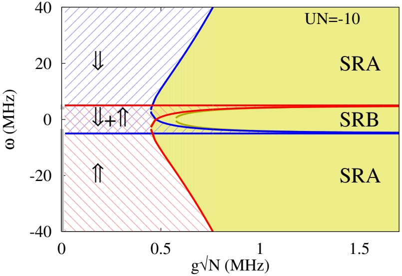
Decay: 20ms, 0.1ms, 20ms



Asymptotic state: Evolution from normal state

(Near to experimental $UN = -13$MHz).

All stable attractors:



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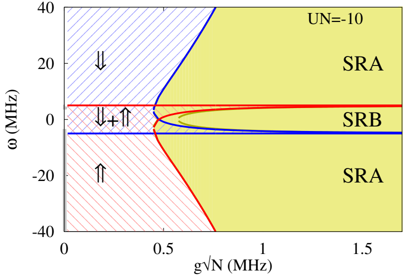
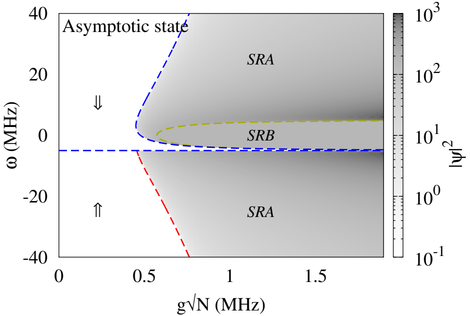
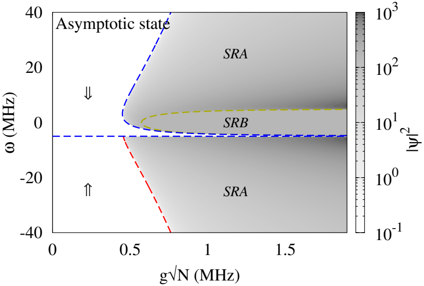
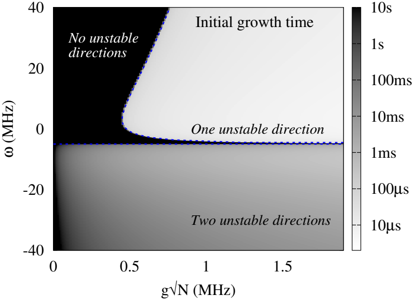


Starting from ⇓



Timescales for dynamics: What are they?





Growth Most unstable eigenvalues near $\mathbf{S} = (0, 0, -N/2)$

Decay Slowest stable eigenvalues near final state

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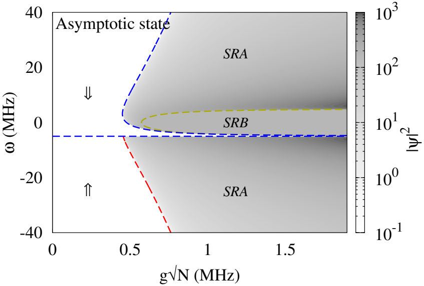

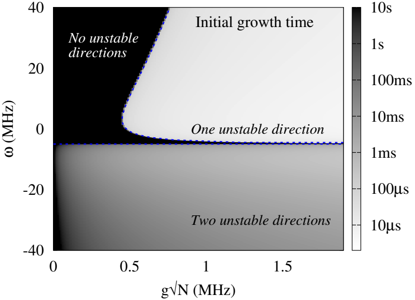

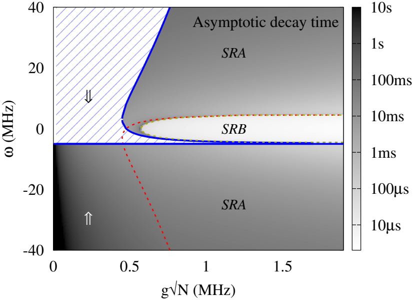

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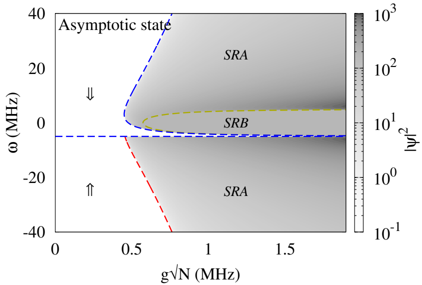
Timescales for dynamics: Consequences for experiment



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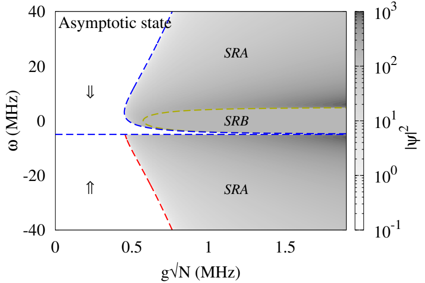

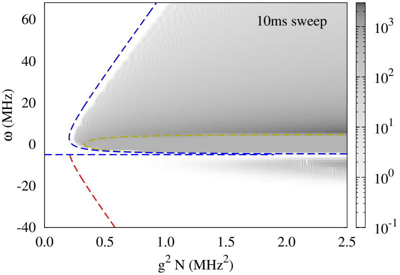

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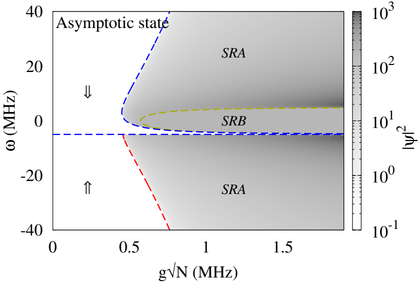
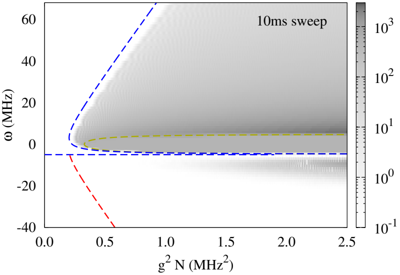
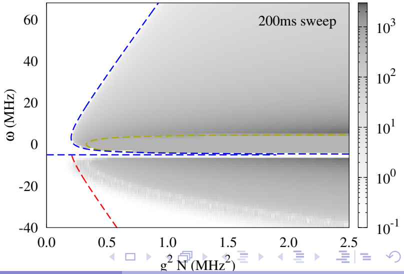
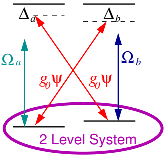






Timescales for dynamics: Why so slow and varied?

Suppose co- and counter-rotating terms differ



$$H = \ldots + g(\psi^\dagger S^- + \psi S^+) + g'(\psi^\dagger S^+ + \psi S^-) + \ldots$$

- SR(A) near phase boundary at small δg → Critical slowing down
- SR(A), SR(B) continuously connect

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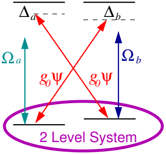

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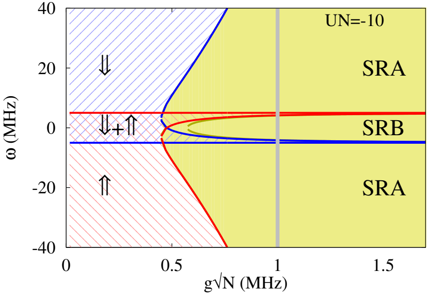
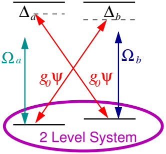


- SR(A) near phase boundary at small $\delta g \to$ Critical slowing down
- SR(A), SR(B) continuously connect

Timescales for dynamics: Why so slow and varied?

Suppose co- and counter-rotating terms differ



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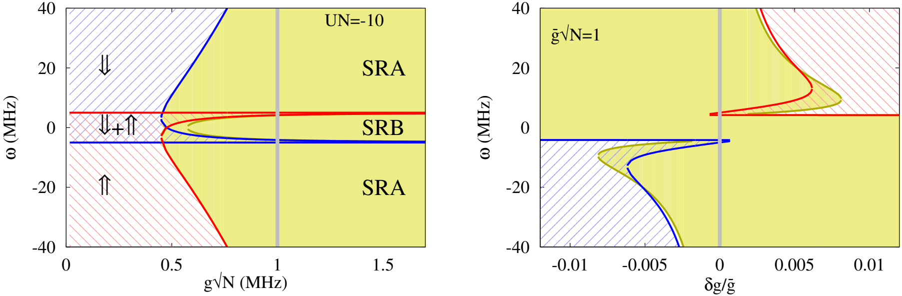

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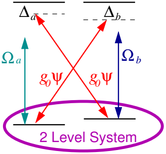

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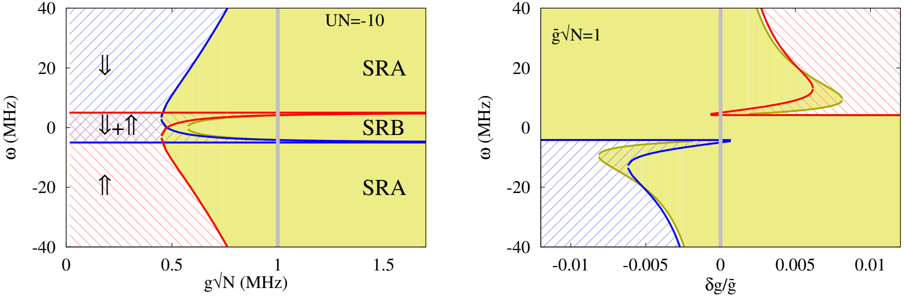

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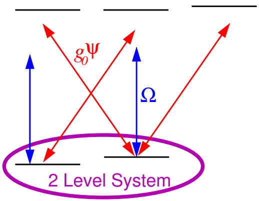
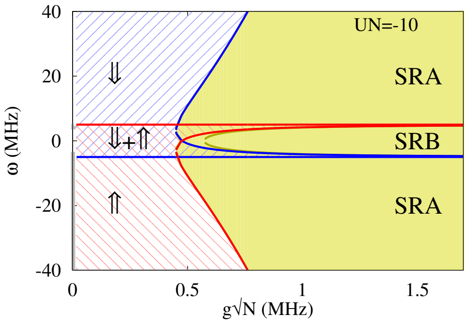
Regions without fixed points

Changing U:



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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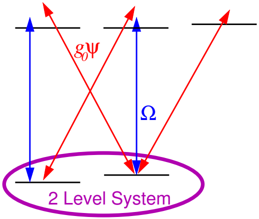

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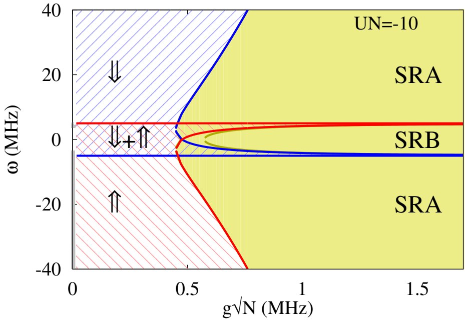

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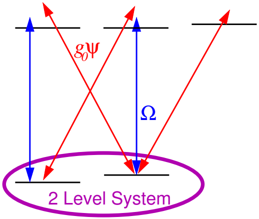

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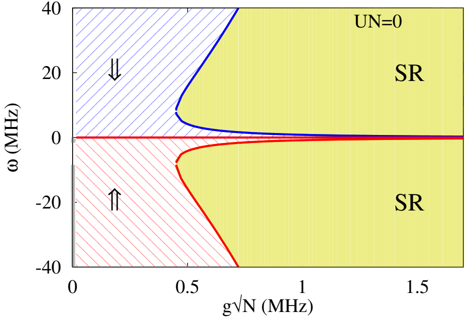

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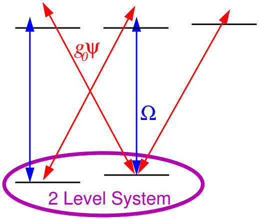

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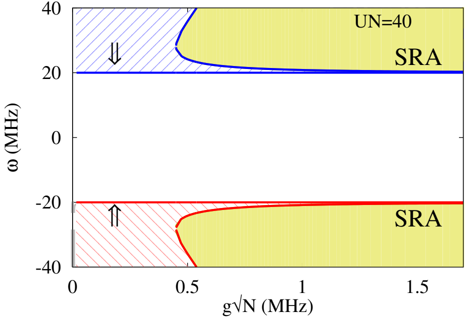

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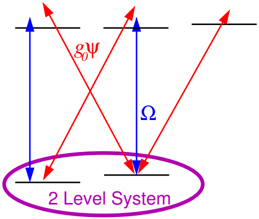

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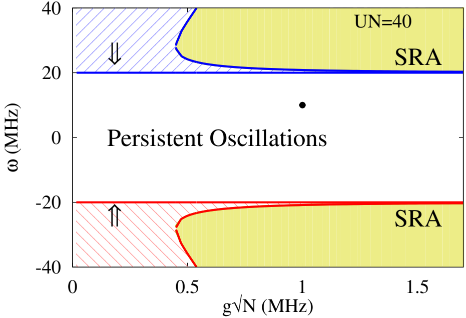
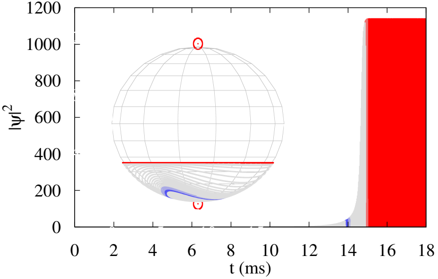
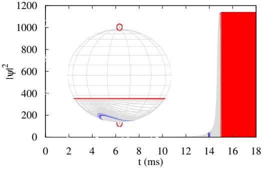




Persistent (optomechanical) oscillations



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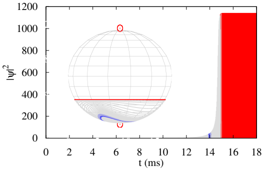

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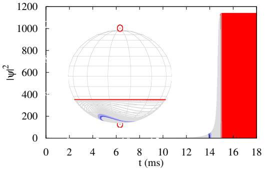

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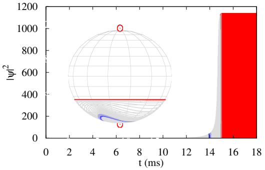

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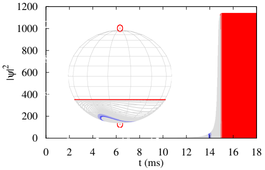

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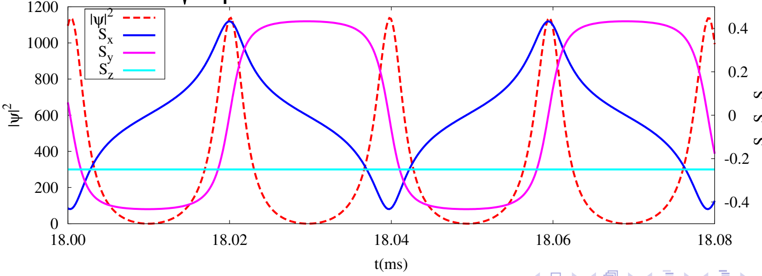

Outline

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Equilibrium: Dicke model with chemical potential

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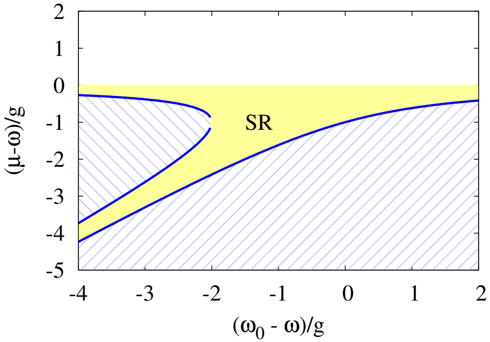

- Transition at: $g^2 N > (\omega - \mu)(\omega_0 - \mu)$
- Reduce critical g
- Unstable if $\mu > \omega$
- Inverted if $\mu > \omega_0$

[Eastham and Littlewood, PRB '01]

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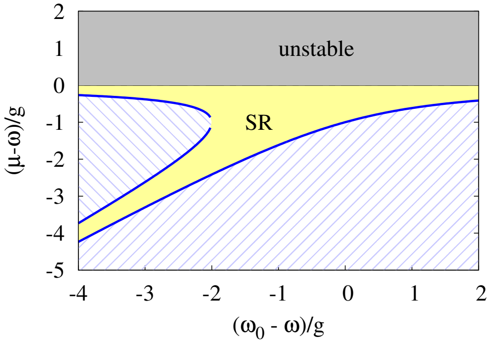

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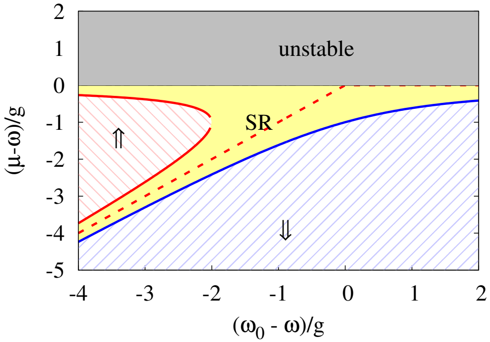

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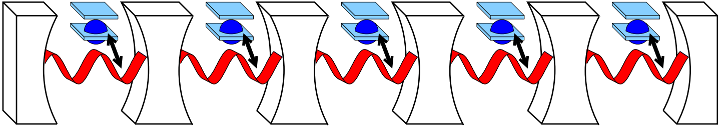
Jaynes-Cummings Hubbard model



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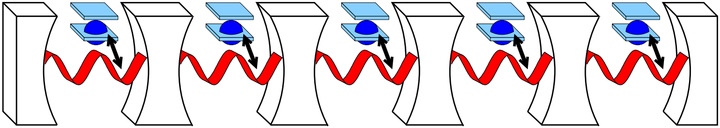

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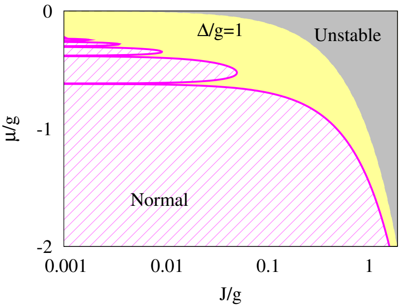

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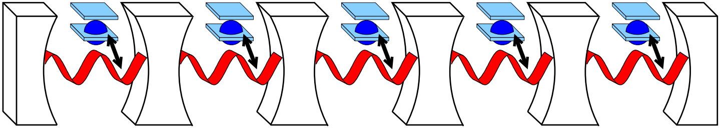

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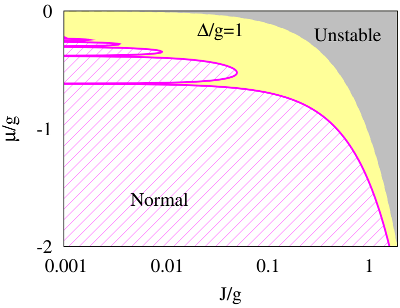
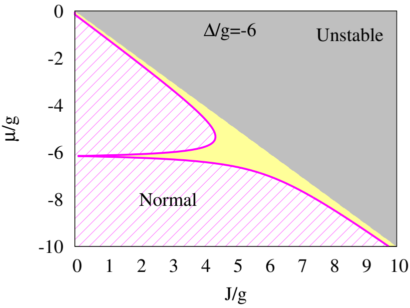
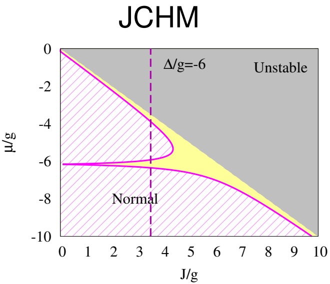
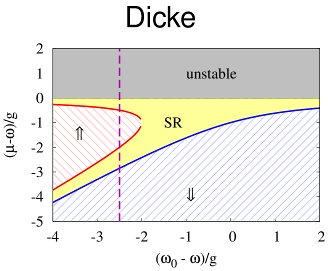




Dicke vs JCHM





- $k = 0$ mode of JCHM $\leftrightarrow$ Dicke photon mode
- $\Uparrow$ $\leftrightarrow$ $n = 1$ Mott lobe

Dicke vs JCHM

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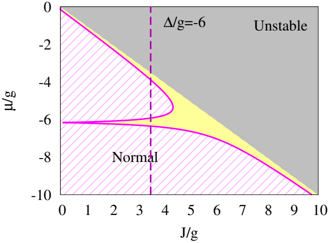
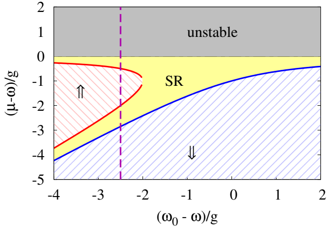


Dicke



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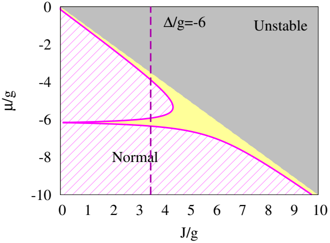

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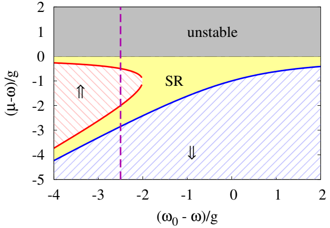

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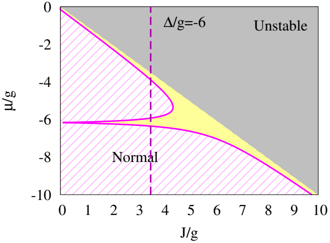

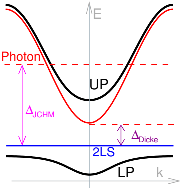

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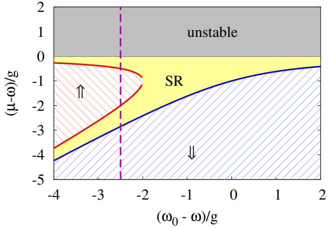

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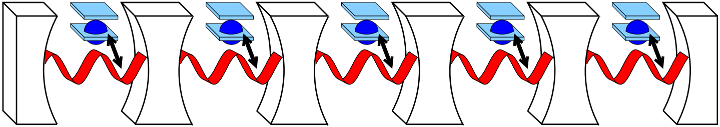
Coherently pumped JCHM



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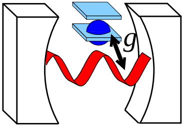
$$\partial_t\rho = -i[H,\rho] - \frac{\kappa}{2}L_\psi[\rho] - \frac{\gamma}{2}L_{\sigma^-}[\rho]$$

Coherently pumped single cavity [Bishop *et al.* Nat. Phys '09]



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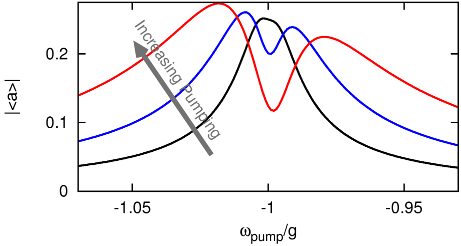

[Lang *et al.* PRL '11]

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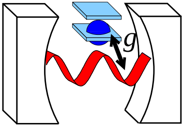

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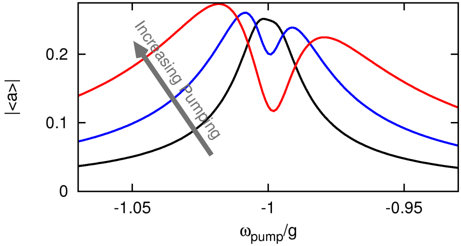

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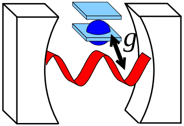

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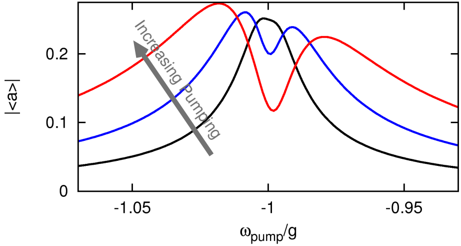

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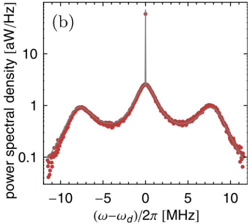
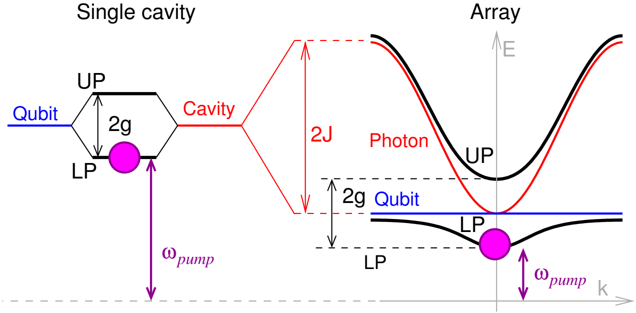


[Lang *et al.* PRL '11]

Coherently pumped dimer & array

Chose detuning *a la* Dicke model



- Bistability at intermediate J
  - More/less localised states
  - Connects to Dicke limit
- Lines — effective TLS

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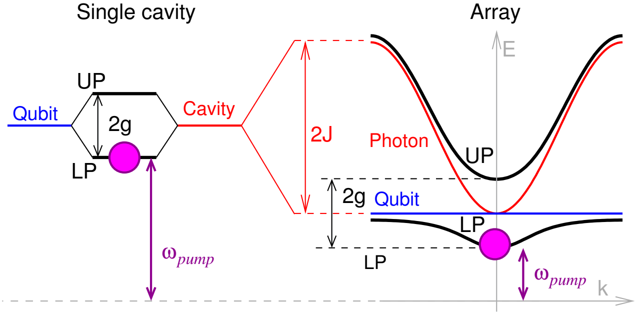
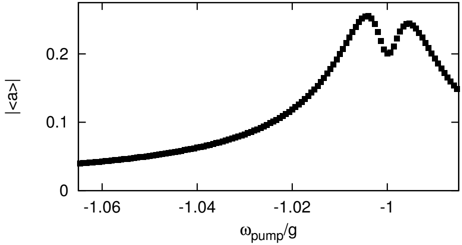


Evolution of anti-resonance vs J.



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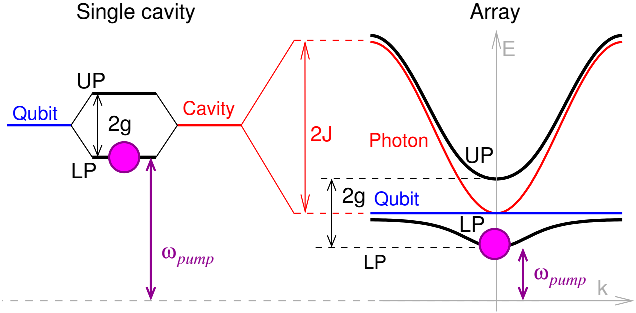

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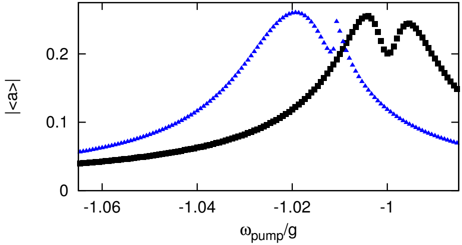

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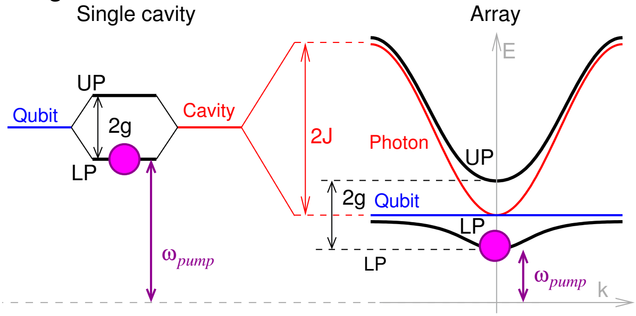

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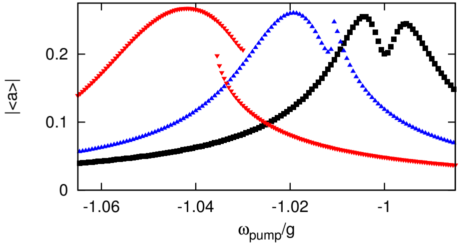

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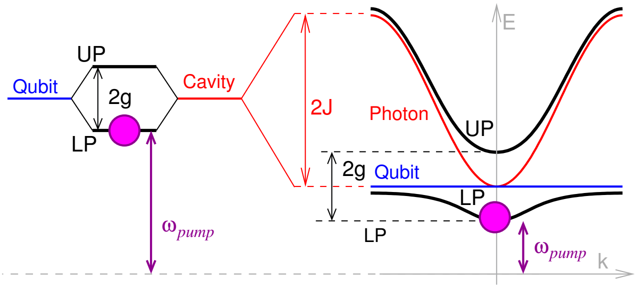

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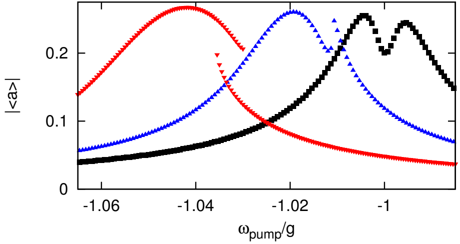

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Coherently pumped dimer & array

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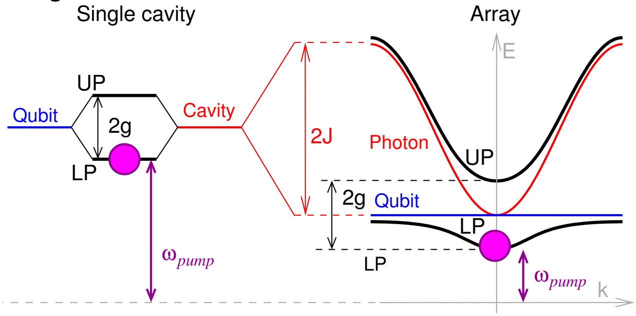

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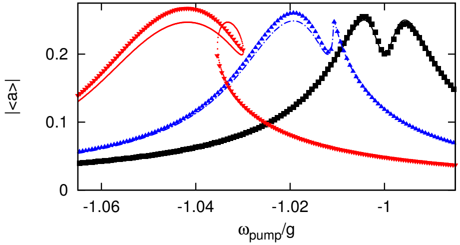

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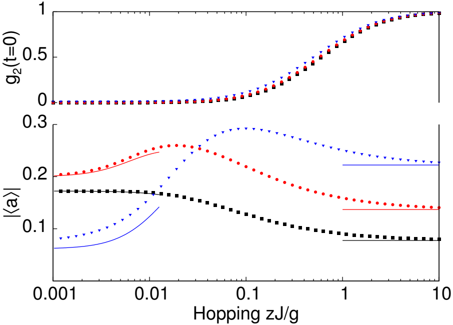
Coherently pumped array: correlations & fluorescence

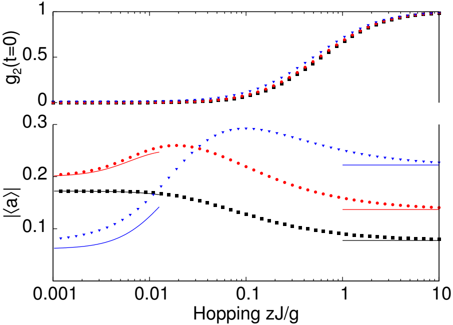


Correlations

- g_2 : $0 \to 1$ crossover.

- Small J: Mollow triplet
- Large J: Off resonance fluorescence
  - Pump at collective resonance
  - Mismatch if $J \neq 0$.

Coherently pumped array: correlations & fluorescence



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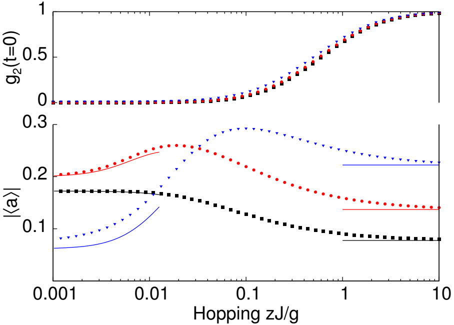

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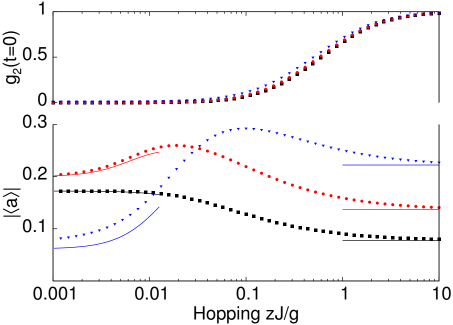

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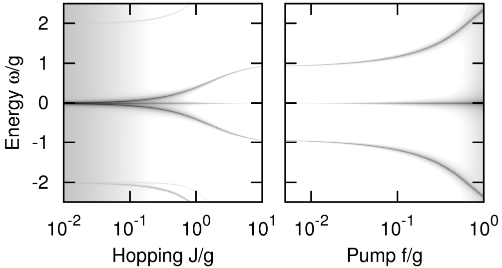

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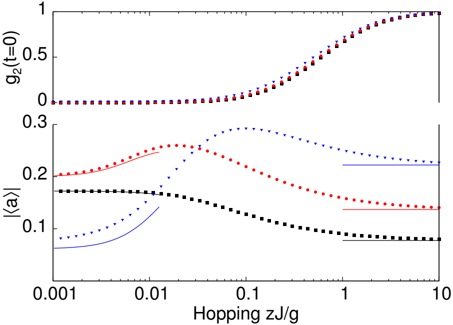

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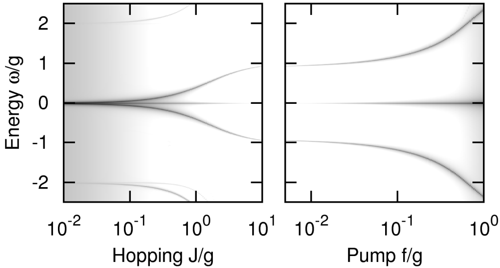
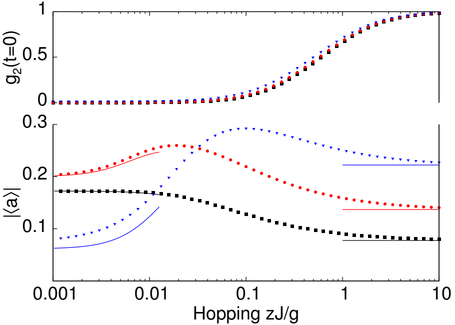


Coherently pumped array: correlations & fluorescence



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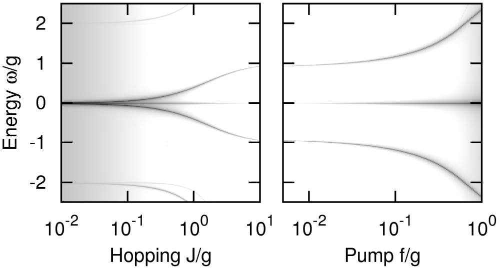
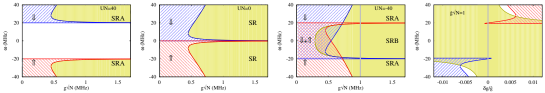
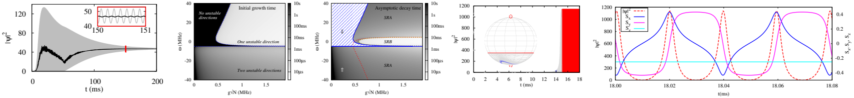


Summary

- Wide variety of dynamical phases

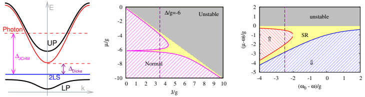


- Slow dynamics for $U < 0$ & Persistent oscillations for $U > 0$

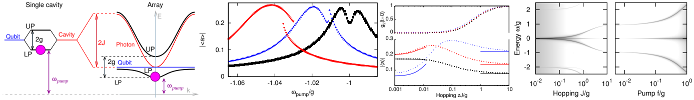


JK *et al.* PRL '10, Bhaseen *et al.* PRA '12

- Dicke model and JCHM: connection at $J \to \infty$



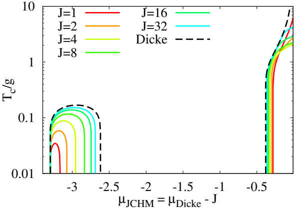
- Coherently pumped coupled cavity array

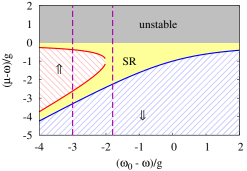


Nissen *et al.* PRL in press '12

Dicke vs JCHM, $T \neq 0$

$\Delta_{Dicke} = -3$





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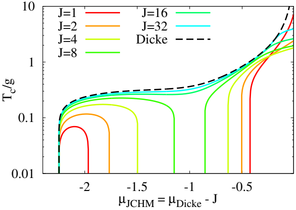

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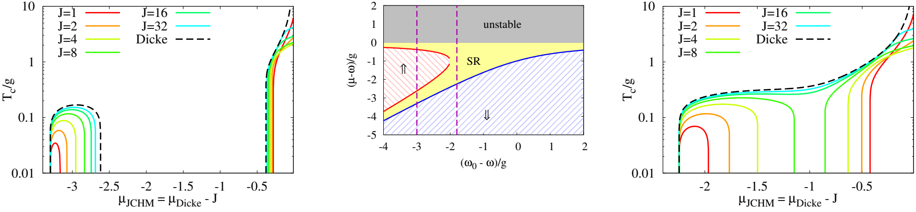

- Match improves as $J \to \infty$
- Finite bandwidth: fluctuations suppresses T_c.
  - Fluctuation mass $m \sim 1/J$, fluctuations suppress $T_c < \rho/m$
  - Fluctuations can induce re-entrance [JK *et al.* PRB '05]

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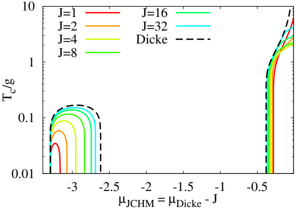

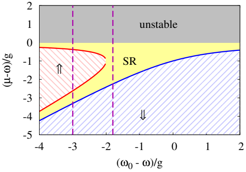

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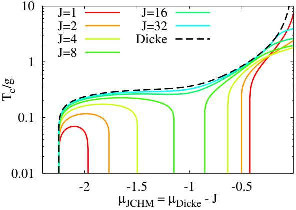

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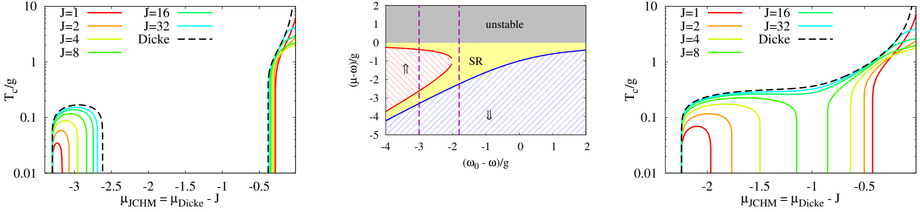

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