Collective Dynamics of Generalized Dicke Models

J. Keeling, J. A. Mayoh, M. J. Bhaseen, B. D. Simons

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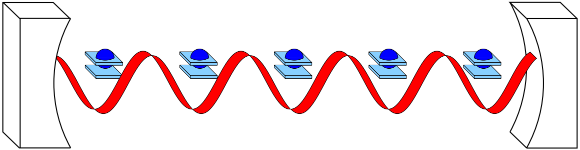
Engineering and Physical Sciences Research Council

Dicke model: Superradiance transition



[Dicke, Phys. Rev. '54]
Many 2 level atoms
Use $\sum_i \mathbf{S}_i \to \mathbf{S}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g\left(\psi^\dagger S^- + \psi S^+\right).$$

- Coherent state: $|\Psi\rangle \to e^{\lambda\psi^\dagger + \eta S^+}|\Omega\rangle$
- Small g, min at $\lambda, \eta = 0$

[Hepp, Lieb, Ann. Phys. '73]

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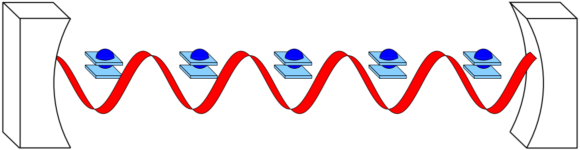

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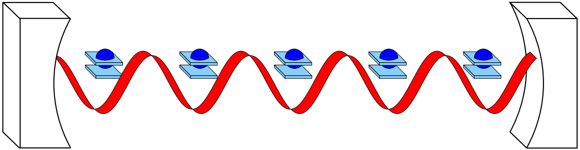

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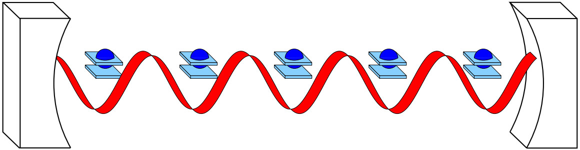

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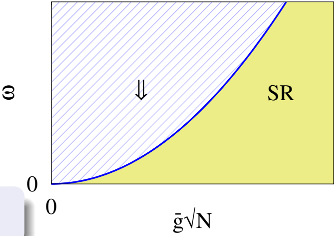

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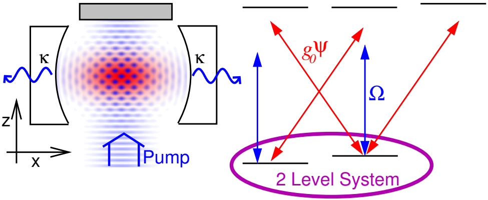
Outline

Extended Dicke model

[Baumann *et al.* Nature '10]



2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$:

$\Downarrow$: $\Psi(x,z) = 1$

$\Uparrow$: $\Psi(x,z) = \sum_{\sigma,\sigma'=\pm} e^{ik(\sigma x + \sigma' z)}$

$\omega_0 = 2\omega_{\text{recoil}}$

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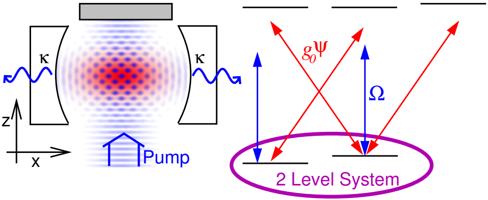

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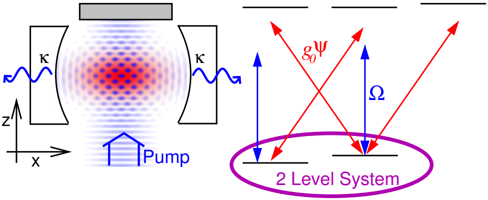

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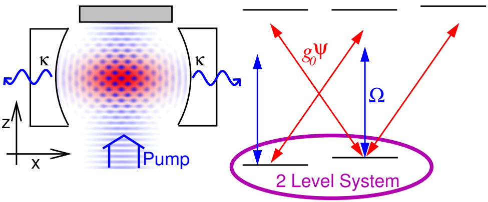

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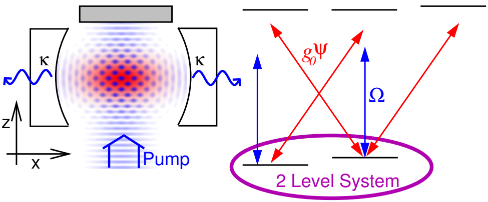

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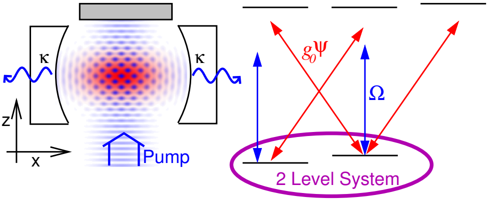

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$\omega_0 \sim$ kHz $\ll \omega, \kappa, g\sqrt{N} \sim$ MHz.

Fixed points (steady states)

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- $\psi = 0, \mathbf{S} = (0, 0, \pm N/2)$ always a solution.
- If $g > g_c$, $\psi \neq 0$ too
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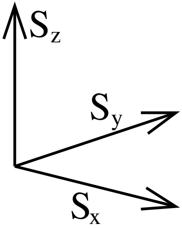

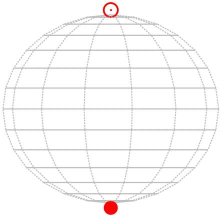

Small g: ⇑, ⇓ only.
($\omega = 30$MHz, $UN = -40$MHz)

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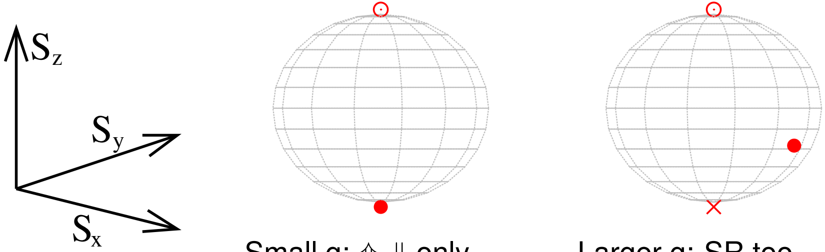

Small g: $\Uparrow, \Downarrow$ only. Larger g: SR too.
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Steady state phase diagram

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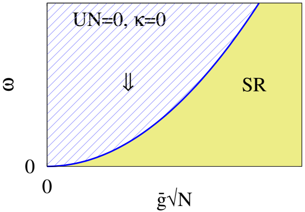

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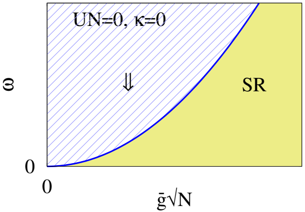
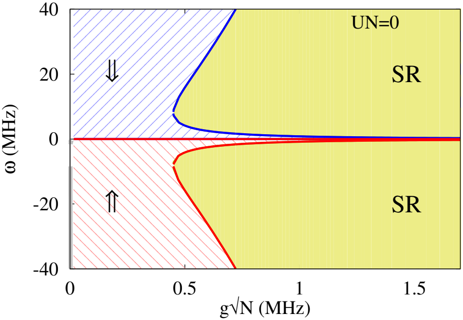
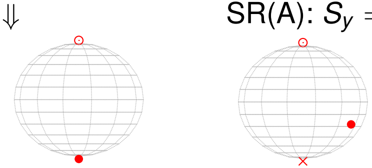




SR(A): $S_y = 0$



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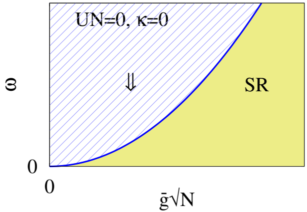

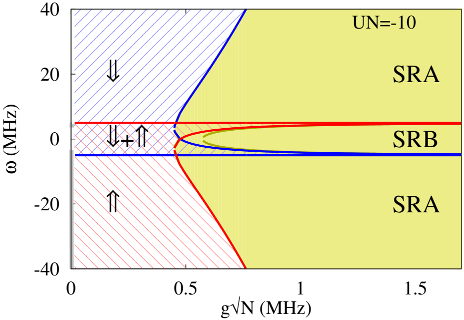

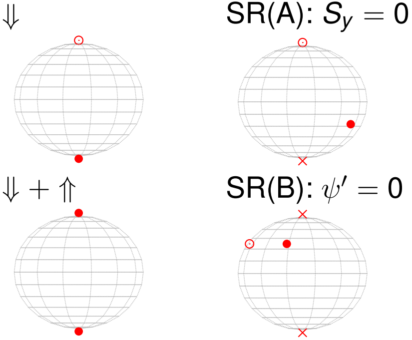

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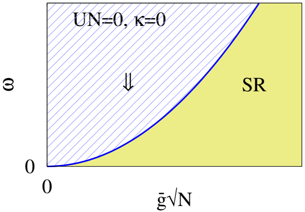

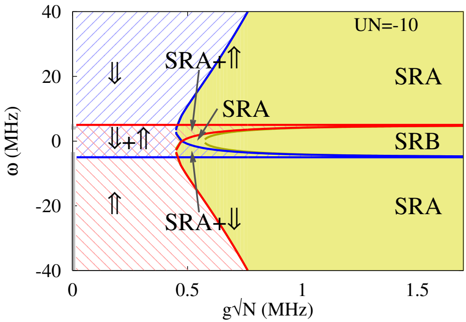

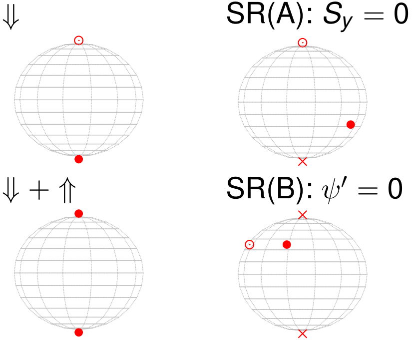

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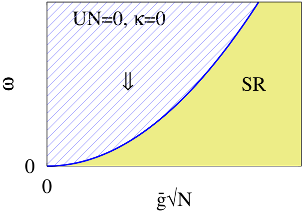

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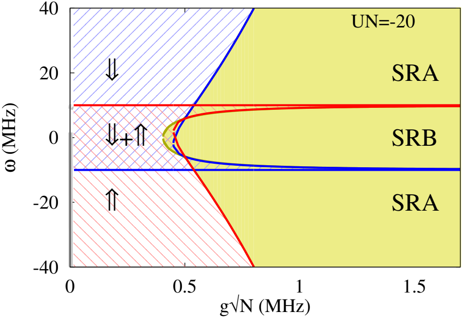

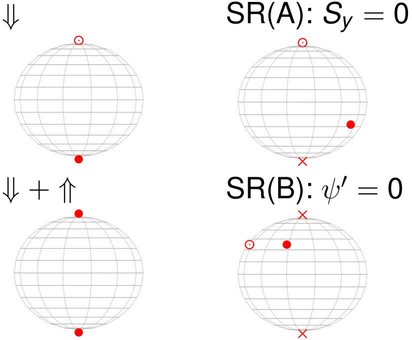

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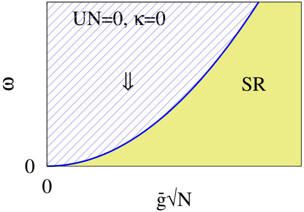
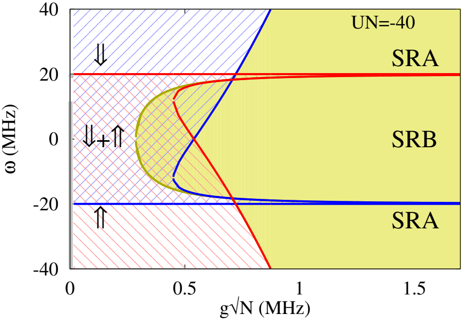
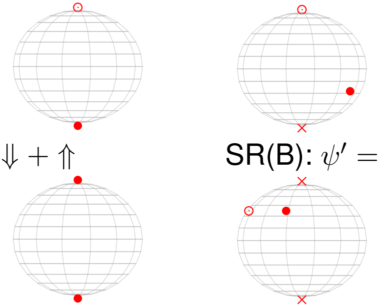




$\Downarrow$ SR(A): $S_y = 0$

$\Downarrow + \Uparrow$ SR(B): $\psi' = 0$



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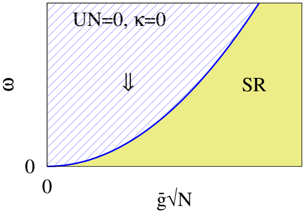

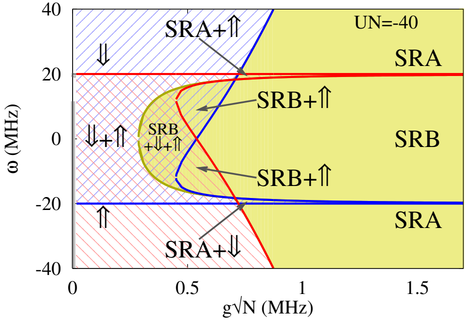

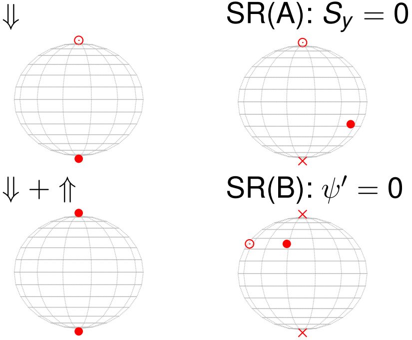

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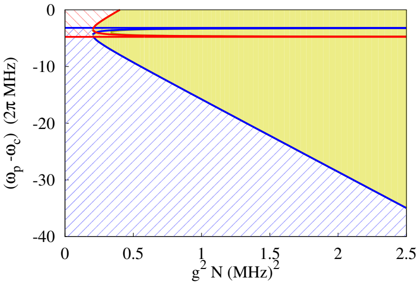
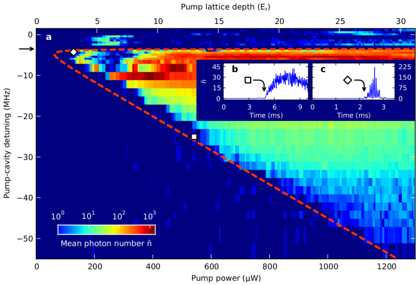
Comparison to experiment





$UN = -10\text{MHz}$

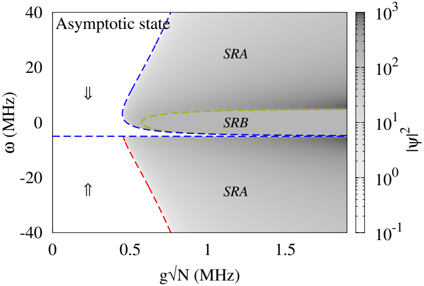
Adapted from: [Bhaseen *et al.* PRA '12]

[Baumann *et al* Nature '10]

$$\omega = \omega_c - \omega_p + \frac{5}{2} UN, \qquad UN = -\frac{g_0^2}{4(\omega_a - \omega_c)}$$

Outline

Timescales for dynamics



Growth Most unstable eigenvalues near $\mathbf{S} = (0, 0, -N/2)$

Decay Slowest stable eigenvalues near final state

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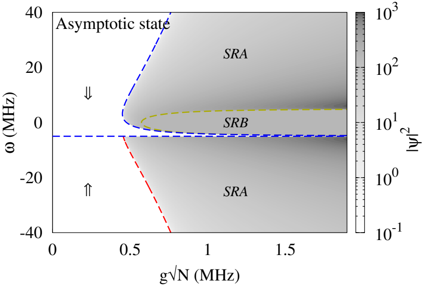

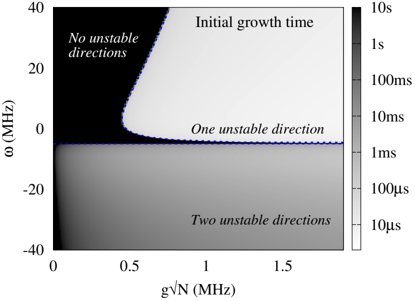

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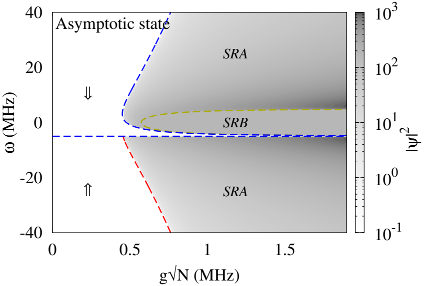

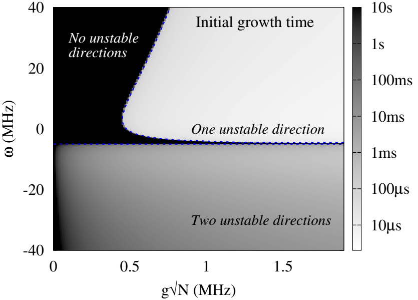

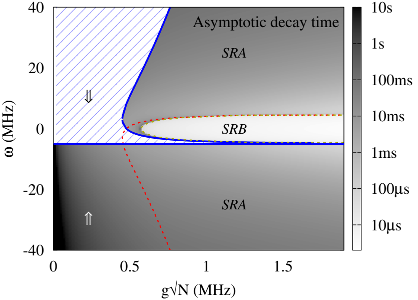

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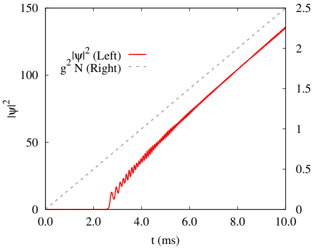
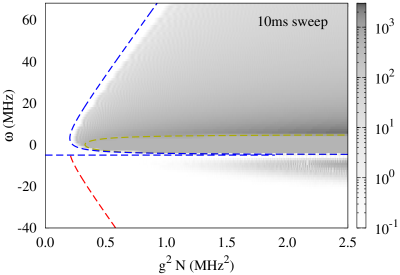
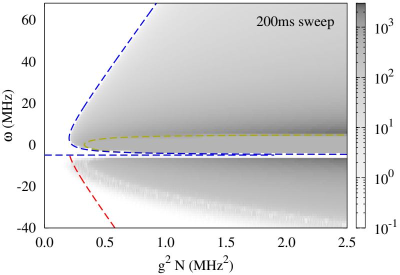
Decay Slowest stable eigenvalues near final state

Timescales and sweeps



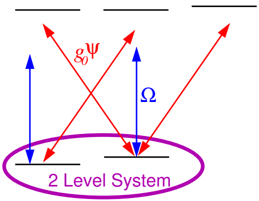




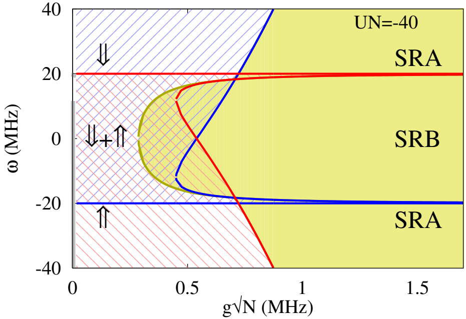
Outline

Regions without fixed points

Changing U:



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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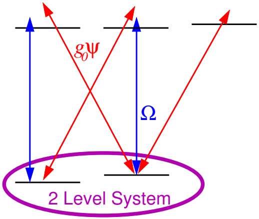

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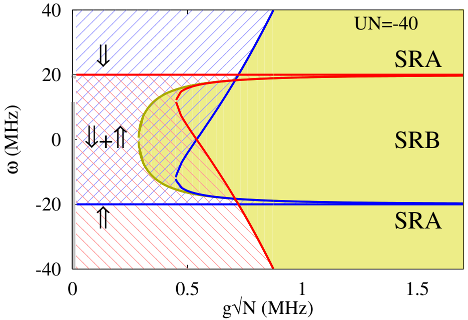
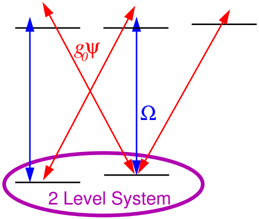


Regions without fixed points

Changing *U*:



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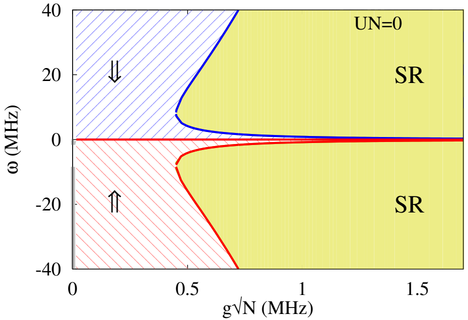

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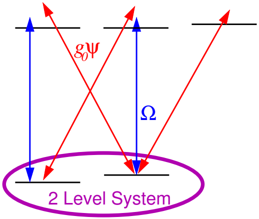

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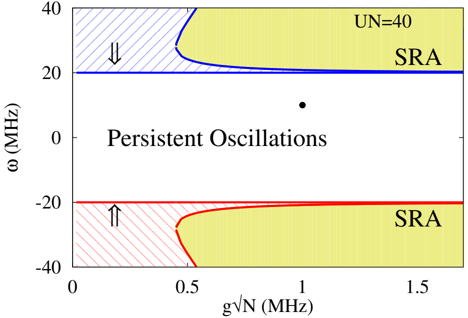

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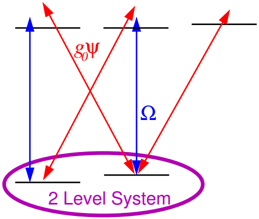

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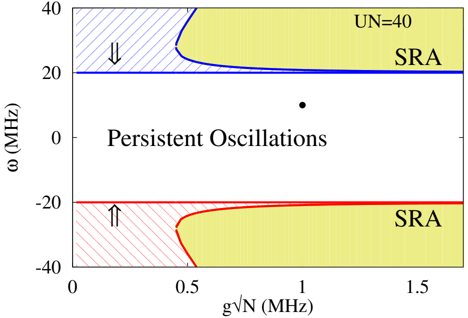
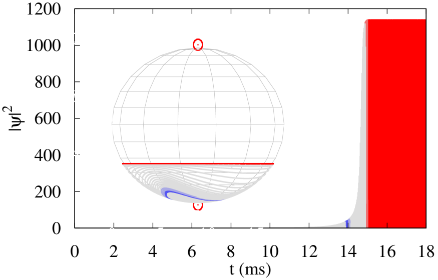
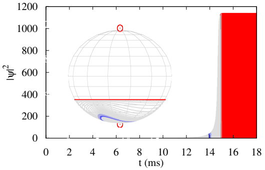




Persistent (optomechanical) oscillations



$$\dot{S}^- = -i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$

$$\dot{S}^z = ig(\psi + \psi^*)(S^- - S^+)$$

$$\dot{\psi} = -[\kappa + i(\omega + US^z)]\psi - ig(S^- + S^+)$$

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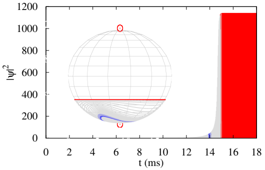

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Fix $\omega + US^z = 0$ **if** $\psi' = 0$.

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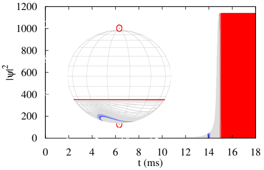

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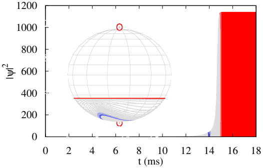

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Get:

Fix $\omega + US^z = 0$ **if** $\psi' = 0$.

$$S^- = re^{-i\theta} \quad r = \sqrt{\frac{N^2}{4} - (S^z)^2}$$

$$\dot{\theta} = \omega_0 + U|\psi|^2$$
$$\dot{\psi} + \kappa\psi = -2igr\cos(\theta)$$

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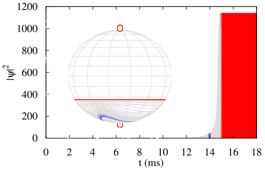

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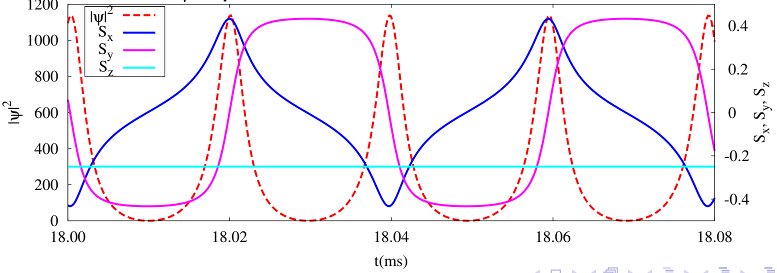
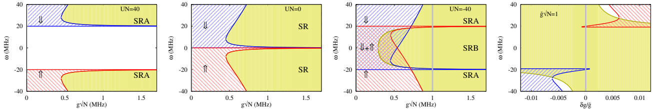
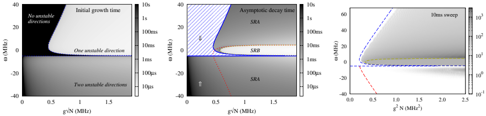


Summary

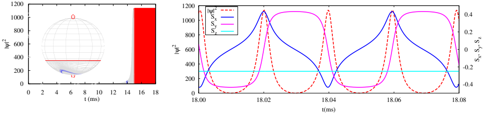
- Wide variety of dynamical phases



- Slow dynamics



- Persistent oscillations if $U > 0$



JK *et al.* PRL '10, Bhaseen *et al.* PRA '12

Extra slides

No go theorem and transition

Spontaneous polarisation if: $Ng^2 > \omega\omega_0$

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No go theorem:. Minimal coupling $(p - eA)^2/2m$

$$-\sum_i \frac{e}{m} A \cdot p_i \Leftrightarrow g(\psi^\dagger S^- + \psi S^+), \qquad \sum_i \frac{A^2}{2m} \Leftrightarrow N\zeta(\psi + \psi^\dagger)^2$$

[Rzazewski *et al* PRL '75]

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For large N, $\omega \to \omega + 4N\zeta$.

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But $g^2 < \omega_0 4\zeta$. **No transition** [Rzazewski *et al* PRL '75]

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Solutions:

- Fixed excitation density
  (Grand canonical ensemble)
- Dissociate g, ω_0, e.g. Raman Scheme: $\omega_0 \ll \omega$.
  [Dimer *et al* PRA '07; Baumann *et al* Nature '10]

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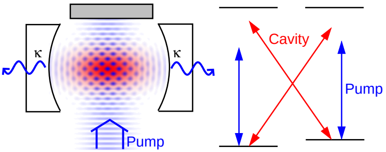
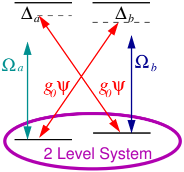


Tuning g, g', U

[Dimer *et al.* Phys. Rev. A. (2007)]



- Separate pump strength/detuning
- $g \sim \frac{g_0 \Omega_b}{\Delta_b}, g' \sim \frac{g_0 \Omega_a}{\Delta_a}, U \sim \frac{g_0{}^2}{\Delta_a} - \frac{g_0{}^2}{\Delta_b}$

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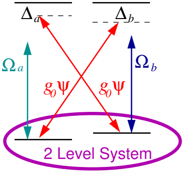

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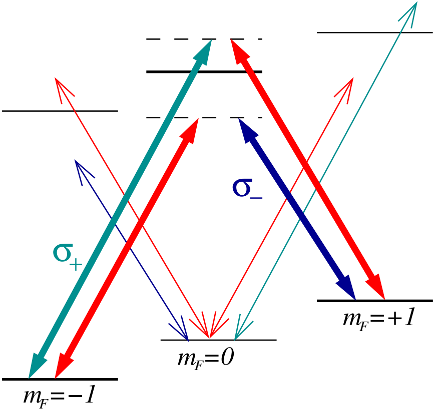
Possible realization: Hyperfine levels



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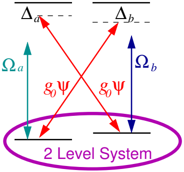

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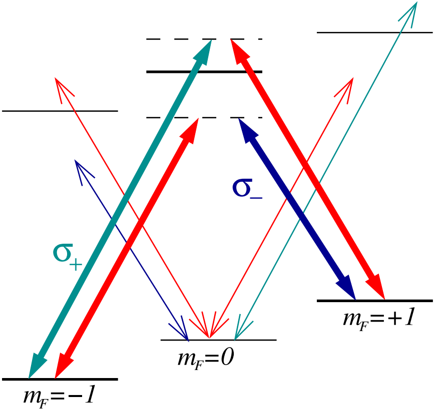
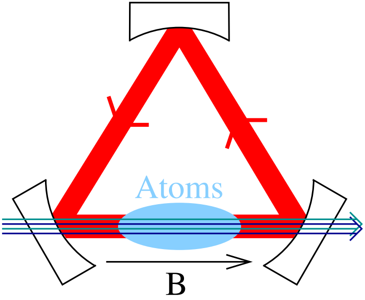
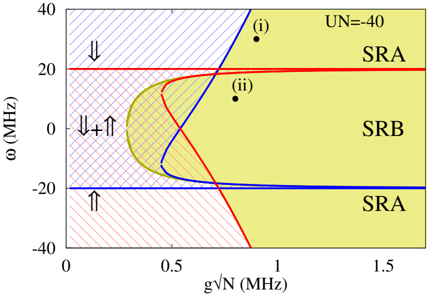




Dynamics: Evolution from normal state



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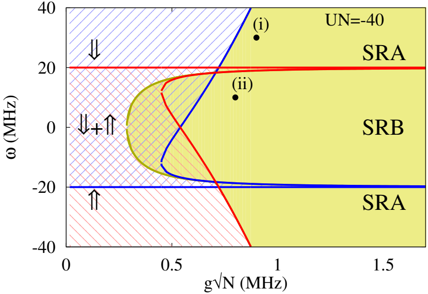
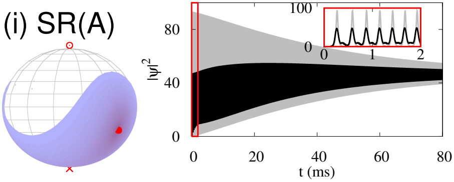


Gray: $\mathbf{S} = (\sqrt{N}, \sqrt{N}, -N/2)$
Black: Wigner distribution of $\mathbf{S}$, ψ



Oscillations: $\sim$ 0.1ms
Decay: 20ms

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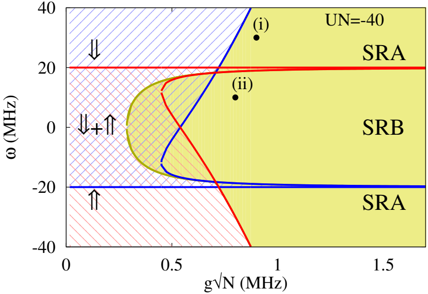

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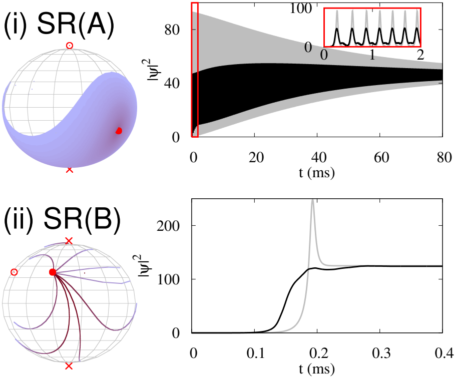
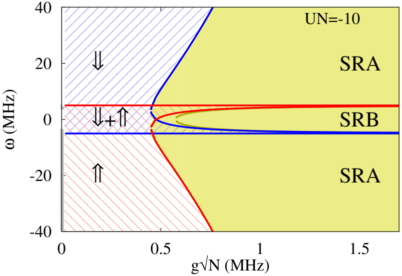


Oscillations: ~ 0.1ms
Decay: 20ms, 0.1ms

Asymptotic state: Evolution from normal state

(Near to experimental $UN = -13\text{MHz}$).

All stable attractors:



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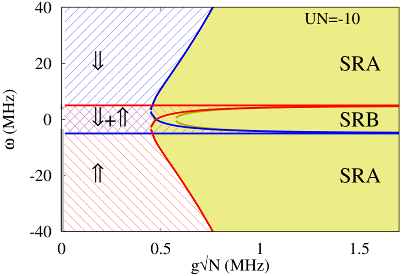
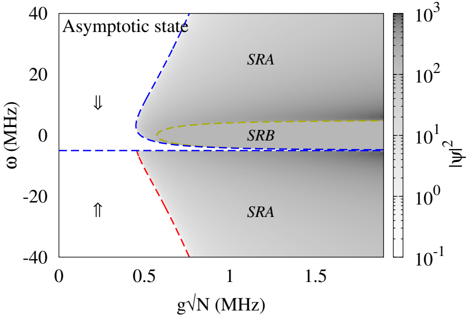
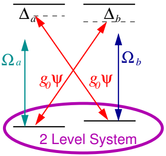


Starting from ⇓



Timescales for dynamics: Why so slow and varied?

Suppose co- and counter-rotating terms differ



$$H = \ldots + g(\psi^\dagger S^- + \psi S^+) + g'(\psi^\dagger S^+ + \psi S^-) + \ldots$$

- SR(A) near phase boundary at small $\delta g \rightarrow$ Critical slowing down
- SR(A), SR(B) continuously connect

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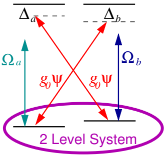

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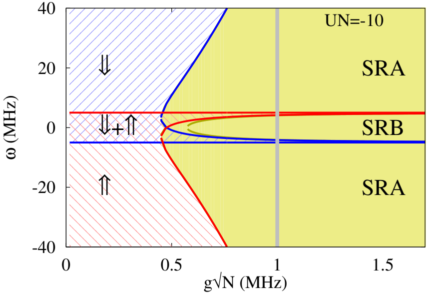

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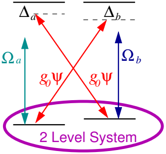

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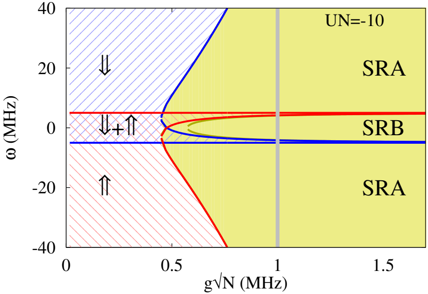

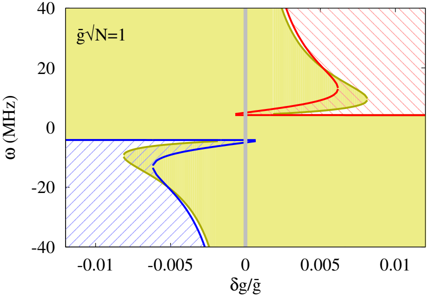

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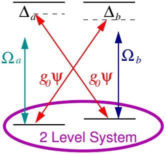

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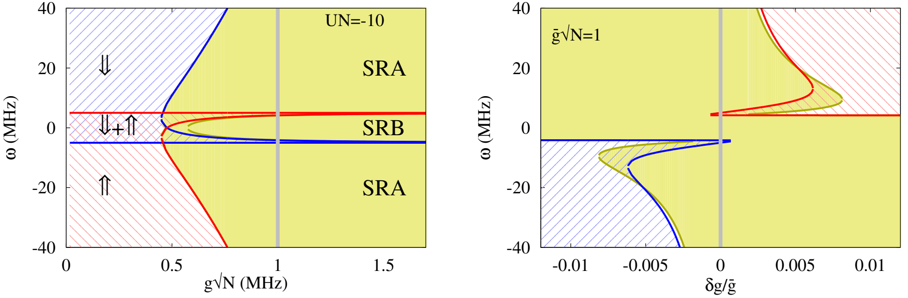

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