# Non-equilibrium coherence in light-matter systems

## Condensation, lasing and the superradiance transiton

**J. Keeling**, P. B. Littlewood, M. H. Szymanska.
J. A. Mayoh, M. J. Bhaseen, B. D. Simons.

IAP, Universität Bonn, January 2012

Funding: EPSRC Engineering and Physical Sciences Research Council

# Coupling many atoms to light

**Old question:** *What happens to radiation when many atoms interact "collectively" with light.*

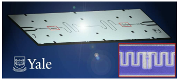
**Superradiance** — dynamical and steady state.

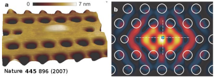
# Coupling many atoms to light

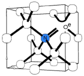
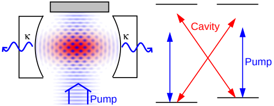
**Old question:** *What happens to radiation when many atoms interact "collectively" with light.*

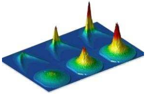
**Superradiance** — dynamical and steady state.

**New relevance**

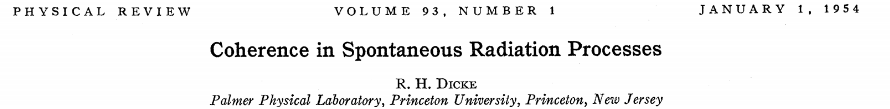
- Superconducting qubits 

- Quantum dots 

- Nitrogen-vacancies in diamond 
- Ultra-cold atoms 

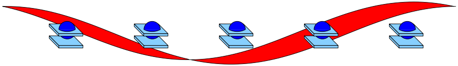
- Rydberg atoms
- Microcavity Polaritons 

# Dicke effect: Enhanced emission


PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

**Coherence in Spontaneous Radiation Processes**

R. H. Dicke
*Palmer Physical Laboratory, Princeton University, Princeton, New Jersey*



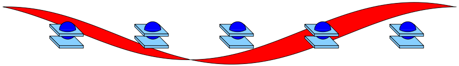
$$H_{\rm int} = \sum_{k,i} g_k \left( \psi_k^\dagger S_i^- e^{-i\mathbf{k}\cdot\mathbf{r}_i} + \mathrm{H.c.} \right)$$

# Dicke effect: Enhanced emission

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

**Coherence in Spontaneous Radiation Processes**

R. H. DICKE
*Palmer Physical Laboratory, Princeton University, Princeton, New Jersey*



$$H_{\text{int}} = \sum_{k,i} g_k \left( \psi_k^\dagger S_i^- e^{-i\mathbf{k}\cdot\mathbf{r}_i} + \text{H.c.} \right)$$

If $|\mathbf{r}_i - \mathbf{r}_j| \ll \lambda$, use $\sum_i \mathbf{S}_i \to \mathbf{S}$
Collective decay:

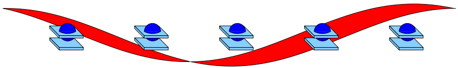
$$\frac{d\rho}{dt} = -\frac{\Gamma}{2} \left[ S^+ S^- \rho - S^- \rho S^+ + \rho S^+ S^- \right]$$

# Dicke effect: Enhanced emission

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

**Coherence in Spontaneous Radiation Processes**

R. H. DICKE
*Palmer Physical Laboratory, Princeton University, Princeton, New Jersey*



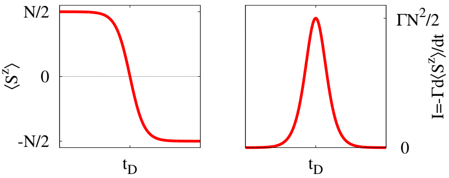
$$H_{\rm int} = \sum_{k,i} g_k \left( \psi_k^\dagger S_i^- e^{-i\mathbf{k}\cdot\mathbf{r}_i} + \text{H.c.} \right)$$

If $|\mathbf{r}_i - \mathbf{r}_j| \ll \lambda$, use $\sum_i \mathbf{S}_i \to \mathbf{S}$
Collective decay:

$$\frac{d\rho}{dt} = -\frac{\Gamma}{2}\left[ S^+S^-\rho - S^-\rho S^+ + \rho S^+ S^- \right]$$

If $S^z = |S| = N/2$ initially:

$$I \propto -\Gamma \frac{d\langle S^z \rangle}{dt} = \frac{\Gamma N^2}{4} \text{sech}^2\left[\frac{\Gamma N}{2} t\right]$$

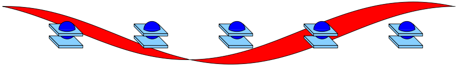


# Dicke effect: Enhanced emission

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

**Coherence in Spontaneous Radiation Processes**

R. H. DICKE
*Palmer Physical Laboratory, Princeton University, Princeton, New Jersey*



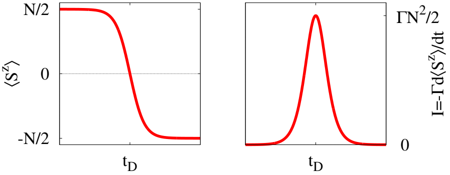
$$H_{\text{int}} = \sum_{k,i} g_k \left( \psi_k^\dagger S_i^- e^{-i\mathbf{k}\cdot\mathbf{r}_i} + \text{H.c.} \right)$$

If $|\mathbf{r}_i - \mathbf{r}_j| \ll \lambda$, use $\sum_i \mathbf{S}_i \to \mathbf{S}$
Collective decay:

$$\frac{d\rho}{dt} = -\frac{\Gamma}{2}\left[S^+S^-\rho - S^-\rho S^+ + \rho S^+S^-\right]$$

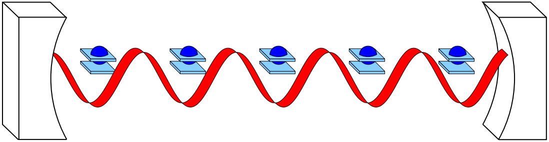
If $S^z = |S| = N/2$ initially:

$$I \propto -\Gamma \frac{d\langle S^z \rangle}{dt} = \frac{\Gamma N^2}{4} \text{sech}^2\left[\frac{\Gamma N}{2} t\right]$$




**Problem:** dipole interactions dephase. [Friedberg et al, Phys. Lett. 1972]

# Collective radiation **with a cavity**: Dynamics

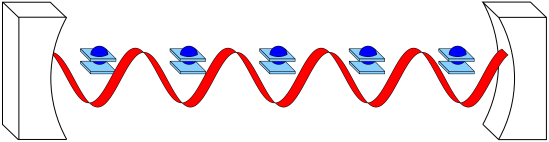


$$H_{\text{int}} = \sum_i \left( \psi^\dagger S_i^- + \psi S_i^+ \right)$$

Single cavity mode: oscillations

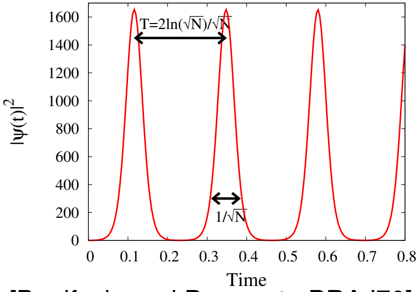
[Bonifacio and Preparata PRA '70]

# Collective radiation **with a cavity**: Dynamics



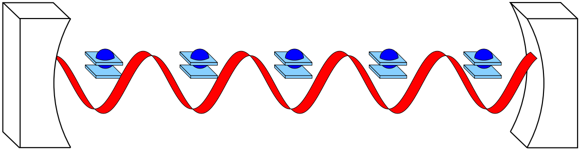
$$H_{\text{int}} = \sum_i \left( \psi^\dagger S_i^- + \psi S_i^+ \right)$$

Single cavity mode: oscillations
If $S^z = |S| = N/2$ initially




[Bonifacio and Preparata PRA '70]

# Dicke model: Equilibrium superradiance transition

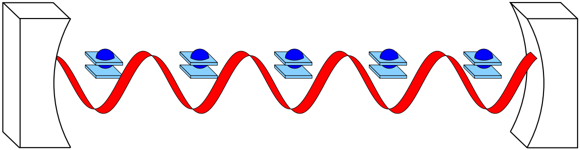


$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g\left(\psi^\dagger S^- + \psi S^+\right).$$

- Coherent state: $|\Psi\rangle \to e^{\lambda\psi^\dagger + \eta S^+}|\Omega\rangle$
- Small $g$, min at $\lambda, \eta = 0$

[Hepp, Lieb, Ann. Phys. '73]

# Dicke model: Equilibrium superradiance transition

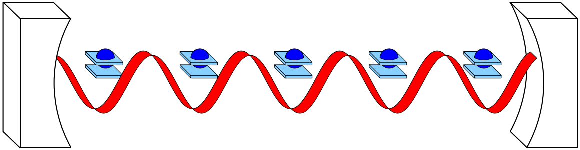


$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g\left(\psi^\dagger S^- + \psi S^+\right).$$

- Coherent state: $|\Psi\rangle \to e^{\lambda\psi^\dagger + \eta S^+}|\Omega\rangle$
- Small $g$, min at $\lambda, \eta = 0$

[Hepp, Lieb, Ann. Phys. ’73]

# Dicke model: Equilibrium superradiance transition



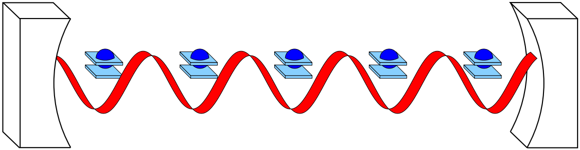
$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g\left(\psi^\dagger S^- + \psi S^+\right).$$

- Coherent state: $|\Psi\rangle \to e^{\lambda\psi^\dagger + \eta S^+}|\Omega\rangle$
- Small $g$, min at $\lambda, \eta = 0$

Spontaneous polarisation if: $Ng^2 > \omega\omega_0$

[Hepp, Lieb, Ann. Phys. '73]

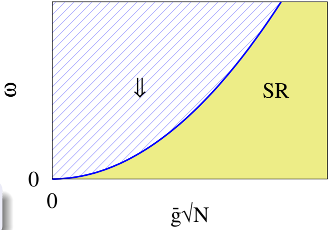
# Dicke model: Equilibrium superradiance transition



$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g\left(\psi^\dagger S^- + \psi S^+\right).$$

- Coherent state: $|\Psi\rangle \to e^{\lambda\psi^\dagger + \eta S^+}|\Omega\rangle$
- Small $g$, min at $\lambda, \eta = 0$

Spontaneous polarisation if: $Ng^2 > \omega\omega_0$




[Hepp, Lieb, Ann. Phys. '73]

# Dicke model: Superradiance at $T \neq 0$

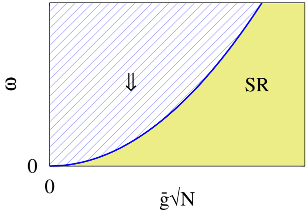
$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g\left(\psi^\dagger S^- + \psi S^+\right).$$

- $T = 0$ ground state if:

$$Ng^2 > \omega\omega_0$$

- $T > 0$, minimum free energy if

$$Ng^2 \frac{\tanh(\beta\omega_0)}{\omega_0} > \omega$$




[Hepp, Lieb, Ann. Phys. ’73]

# Dicke model: Superradiance at $T \neq 0$

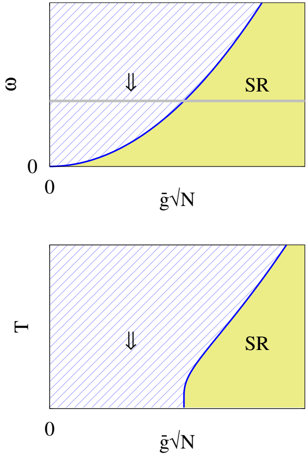
$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g\left(\psi^\dagger S^- + \psi S^+\right).$$

- $T = 0$ ground state if:

$$Ng^2 > \omega\omega_0$$

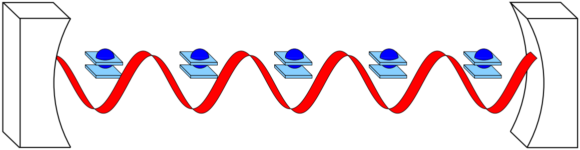
- $T > 0$, minimum free energy if

$$Ng^2 \frac{\tanh(\beta\omega_0)}{\omega_0} > \omega$$




[Hepp, Lieb, Ann. Phys. ’73]

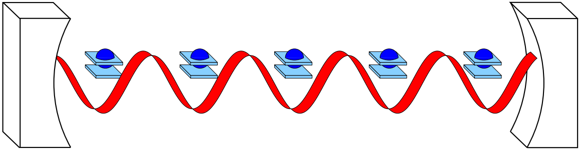
# No go theorem and transition



Spontaneous polarisation if: $Ng^2 > \omega\omega_0$

[Rzazewski *et al* PRL '75]

# No go theorem and transition



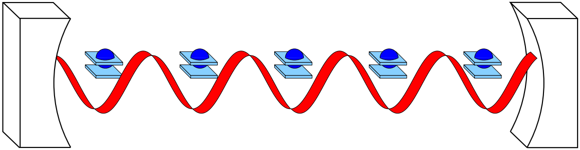
Spontaneous polarisation if: $Ng^2 > \omega\omega_0$

**No go theorem:.** Minimal coupling $(p - eA)^2/2m$

$$-\sum_i \frac{e}{m} A \cdot p_i \Leftrightarrow g(\psi^\dagger S^- + \psi S^+), \qquad \sum_i \frac{A^2}{2m} \Leftrightarrow N\zeta(\psi + \psi^\dagger)^2$$

[Rzazewski *et al* PRL '75]

# No go theorem and transition



Spontaneous polarisation if: $Ng^2 > \omega\omega_0$

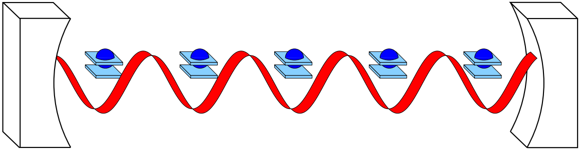
**No go theorem:.** Minimal coupling $(p - eA)^2/2m$

$$-\sum_i \frac{e}{m} A \cdot p_i \Leftrightarrow g(\psi^\dagger S^- + \psi S^+), \qquad \sum_i \frac{A^2}{2m} \Leftrightarrow N\zeta(\psi + \psi^\dagger)^2$$

For large $N$, $\omega \to \omega + 2N\zeta$. (RWA)

[Rzazewski *et al* PRL '75]

# No go theorem and transition



Spontaneous polarisation if: $Ng^2 > \omega\omega_0$

**No go theorem:**. Minimal coupling $(p - eA)^2/2m$

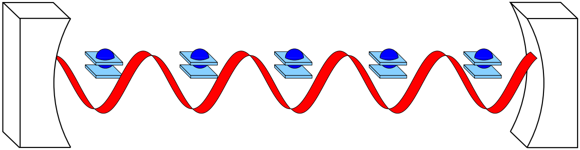
$$-\sum_i \frac{e}{m} A \cdot p_i \Leftrightarrow g(\psi^\dagger S^- + \psi S^+), \qquad \sum_i \frac{A^2}{2m} \Leftrightarrow N\zeta(\psi + \psi^\dagger)^2$$

For large $N$, $\omega \to \omega + 2N\zeta$. (RWA)

Need $Ng^2 > \omega_0(\omega + 2N\zeta)$.

[Rzazewski *et al* PRL '75]

# No go theorem and transition



Spontaneous polarisation if: $Ng^2 > \omega\omega_0$

**No go theorem:.** Minimal coupling $(p - eA)^2/2m$

$$-\sum_i \frac{e}{m} A \cdot p_i \Leftrightarrow g(\psi^\dagger S^- + \psi S^+), \qquad \sum_i \frac{A^2}{2m} \Leftrightarrow N\zeta(\psi + \psi^\dagger)^2$$

For large $N$, $\omega \to \omega + 2N\zeta$. (RWA)

Need $Ng^2 > \omega_0(\omega + 2N\zeta)$.

But Thomas-Reiche-Kuhn sum rule states: $g^2/\omega_0 < 2\zeta$. **No transition**

[Rzazewski *et al* PRL '75]

# Dicke phase transition: ways out

**Problem**: $g^2/\omega_0 < 2\zeta$ for intrinsic parameters. **Solutions**:

- **Non-solution**
  Ferroelectric transition in $\mathbf{D}\cdot\mathbf{r}$ gauge.
  [JK JPCM '07 ]
- See also [Nataf and Ciuti, Nat. Comm. '10; Viehmann *et al.* PRL '11]

- Grand canonical ensemble:
  - If $H \to H - \mu(S^z + \psi^\dagger\psi)$, need only: $g^2N > (\omega - \mu)(\omega_0 - \mu)$
  - Incoherent pumping — polariton condensation.

- Dissociate $g, \omega_0$.
  e.g. Raman scheme: $\omega_0 \ll \omega$.
  [Dimer *et al.* PRA '07; Baumann *et al.* Nature '10. Also, Black *et al.* PRL '03 ]

# Dicke phase transition: ways out

**Problem**: $g^2/\omega_0 < 2\zeta$ for intrinsic parameters. **Solutions**:

- **Non-solution**
  Ferroelectric transition in $\mathbf{D} \cdot \mathbf{r}$ gauge.
  [**JK** JPCM '07 ]
- See also [Nataf and Ciuti, Nat. Comm. '10; Viehmann *et al.* PRL '11]

- Grand canonical ensemble:
  - If $H \to H - \mu(S^z + \psi^\dagger\psi)$, need only:
    $g^2 N > (\omega - \mu)(\omega_0 - \mu)$
  - Incoherent pumping — polariton condensation.

- Dissociate $g, \omega_0$.
  e.g. Raman scheme: $\omega_0 \ll \omega$.
  [Dimer *et al.* PRA '07; Baumann *et al.* Nature '10. Also, Black *et al.* PRL '03 ]

# Dicke phase transition: ways out

**Problem**: $g^2/\omega_0 < 2\zeta$ for intrinsic parameters. **Solutions**:

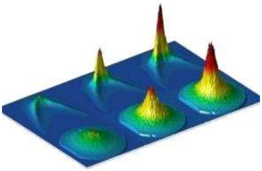
- **Non-solution**
  Ferroelectric transition in $\mathbf{D} \cdot \mathbf{r}$ gauge.
  [**JK** JPCM '07 ]
- See also [Nataf and Ciuti, Nat. Comm. '10; Viehmann *et al.* PRL '11]
- Grand canonical ensemble:
  - If $H \to H - \mu(S^z + \psi^\dagger\psi)$, need only:
    $g^2 N > (\omega - \mu)(\omega_0 - \mu)$
  - Incoherent pumping — polariton condensation.
- Dissociate $g, \omega_0$,
  e.g. Raman scheme: $\omega_0 \ll \omega$.
  [Dimer *et al.* PRA '07; Baumann *et al.* Nature '10. Also, Black *et al.* PRL '03 ]

# Dicke phase transition: ways out

**Problem**: $g^2/\omega_0 < 2\zeta$ for intrinsic parameters. **Solutions**:

- **Non-solution**
  Ferroelectric transition in $\mathbf{D} \cdot \mathbf{r}$ gauge.
  [**JK** JPCM '07 ]
- See also [Nataf and Ciuti, Nat. Comm. '10; Viehmann *et al.* PRL '11]

- Grand canonical ensemble:
  - If $H \to H - \mu(S^z + \psi^\dagger\psi)$, need only: $g^2 N > (\omega - \mu)(\omega_0 - \mu)$
  - Incoherent pumping — polariton condensation.



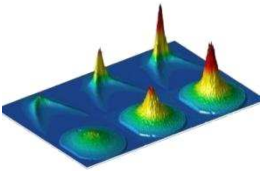
- Dissociate $g, \omega_0$.
  e.g. Raman scheme: $\omega_0 \ll \omega$.
  [Dimer *et al.* PRA '07; Baumann *et al.* Nature '10. Also, Black *et al.* PRL '03 ]

# Dicke phase transition: ways out

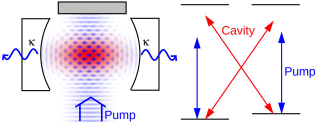
**Problem**: $g^2/\omega_0 < 2\zeta$ for intrinsic parameters. **Solutions**:

- **Non-solution**
  Ferroelectric transition in $\mathbf{D} \cdot \mathbf{r}$ gauge.
  [**JK** JPCM '07 ]
- See also [Nataf and Ciuti, Nat. Comm. '10; Viehmann *et al.* PRL '11]

- Grand canonical ensemble:
  - If $H \to H - \mu(S^z + \psi^\dagger\psi)$, need only: $g^2 N > (\omega - \mu)(\omega_0 - \mu)$
  - Incoherent pumping — polariton condensation.



- Dissociate $g, \omega_0$,
  e.g. Raman scheme: $\omega_0 \ll \omega$.
  [Dimer *et al.* PRA '07; Baumann *et al.* Nature '10. Also, Black *et al.* PRL '03 ]



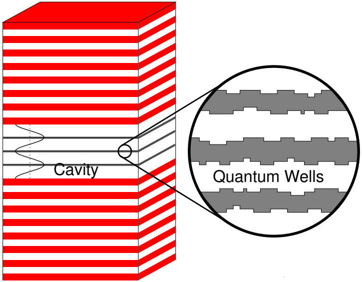
1 Dicke model and superradiance

2 Microcavity Polariton condensation
- Polariton Introduction
- Non-equilibrium condensation vs lasing

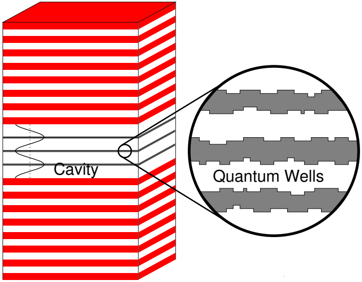
3 Raman pumped atoms
- Raman pumped atoms – Introduction
- Attractors of dynamics (fixed points)
- Attractors of dynamics (oscillations)

# Microcavity Polariton Condensation

# Microcavity polaritons

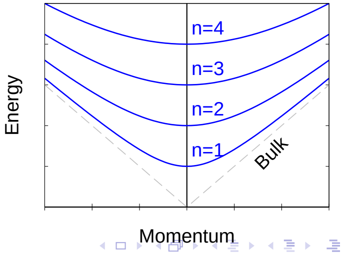


# Microcavity polaritons

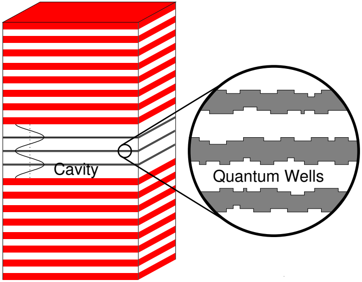



Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$

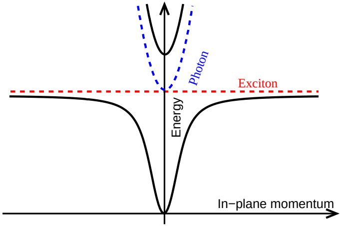


# Microcavity polaritons

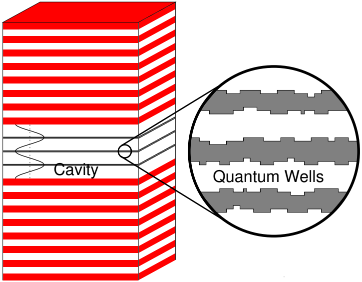



Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$

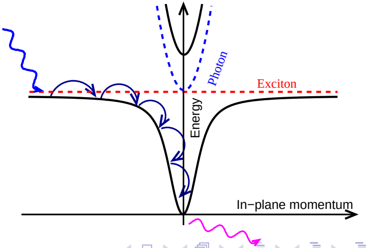


# Microcavity polaritons

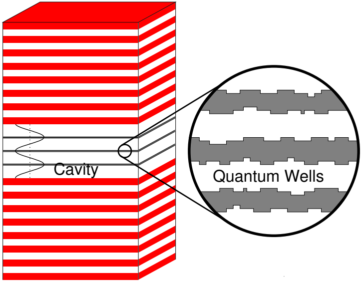


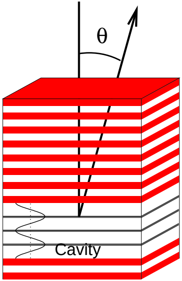

Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$



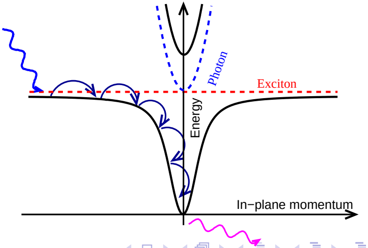
# Microcavity polaritons



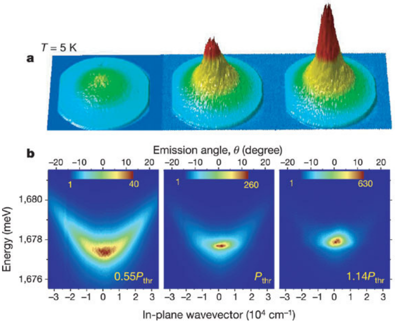



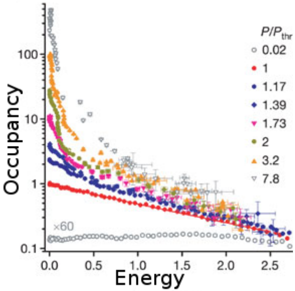

Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$



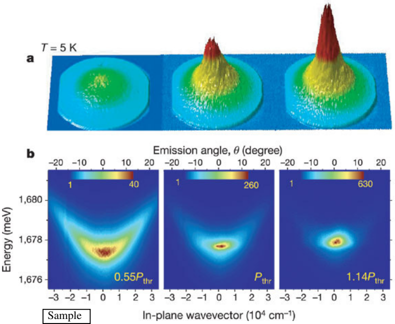
# Polariton experiments: occupation and coherence

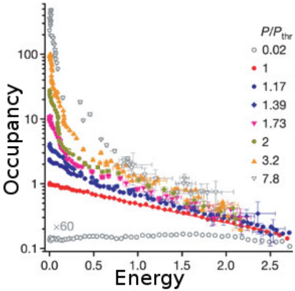


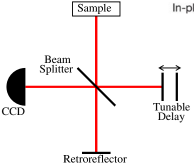



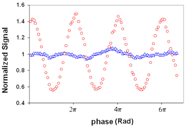

[Kasprzak, *et al.* Nature, ’06]

# Polariton experiments: occupation and coherence

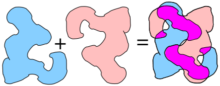


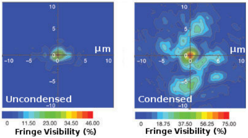




Coherence map:



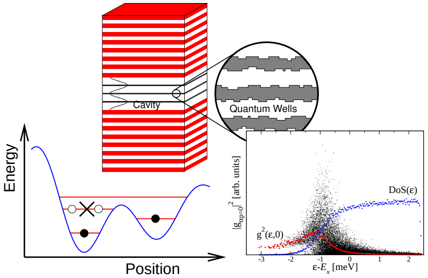




[Kasprzak, *et al.* Nature, ’06]

# Polariton system model

Polariton model

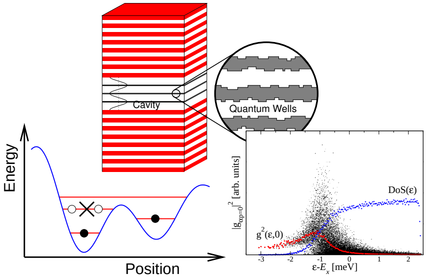
- Disorder-localised excitons
- Treat disorder sites as two-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling *g*.



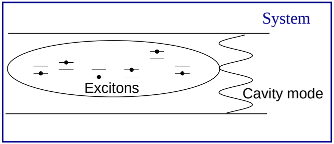
# Polariton system model

Polariton model

- Disorder-localised excitons
- Treat disorder sites as two-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling $g$.




$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[ \epsilon_\alpha S^z_\alpha + \frac{1}{\sqrt{A}} g_{\alpha,\mathbf{k}} \psi_{\mathbf{k}} S^+_\alpha + \text{H.c.} \right]$$



# Polariton model and equilibrium results

Localised excitons, propagating photons

$$H - \mu N = \sum_{\mathbf{k}} (\omega_{\mathbf{k}} - \mu) \psi^{\dagger}_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} (\epsilon_\alpha - \mu) S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} S^+_\alpha + \text{H.c.}$$

Self-consistent polarisation and field

$$(\omega_{\mathbf{k}=0} - \mu) \psi = \frac{1}{A} \sum_{\alpha} \frac{g_\alpha^2 \psi}{2 E_\alpha} \tanh(\beta E_\alpha), \quad {E_\alpha}^2 = \left( \frac{\epsilon_\alpha - \mu}{2} \right)^2 + g_\alpha^2 |\psi|^2$$

# Polariton model and equilibrium results

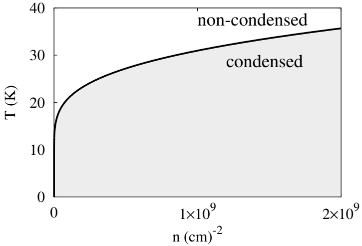
Localised excitons, propagating photons

$$H - \mu N = \sum_{\mathbf{k}} (\omega_{\mathbf{k}} - \mu)\psi^\dagger_{\mathbf{k}}\psi_{\mathbf{k}} + \sum_{\alpha} (\epsilon_\alpha - \mu) S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} S^+_\alpha + \text{H.c.}$$

Self-consistent polarisation and field

$$(\omega_{\mathbf{k}=0} - \mu)\,\psi = \frac{1}{A}\sum_\alpha \frac{g_\alpha^2 \psi}{2E_\alpha} \tanh(\beta E_\alpha), \qquad {E_\alpha}^2 = \left(\frac{\epsilon_\alpha - \mu}{2}\right)^2 + g_\alpha^2 |\psi|^2$$

Phase diagram:



# Polariton model and equilibrium results

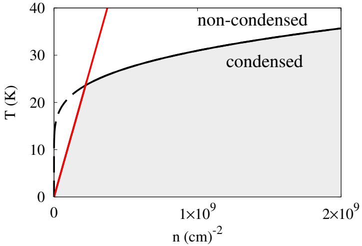
Localised excitons, propagating photons

$$H - \mu N = \sum_{\mathbf{k}} (\omega_{\mathbf{k}} - \mu) \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} (\epsilon_\alpha - \mu) S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} S^+_\alpha + \text{H.c.}$$

Self-consistent polarisation and field

$$(\omega_{\mathbf{k}=0} - \mu)\,\psi = \frac{1}{A} \sum_\alpha \frac{g_\alpha^2 \psi}{2 E_\alpha} \tanh(\beta E_\alpha), \qquad {E_\alpha}^2 = \left(\frac{\epsilon_\alpha - \mu}{2}\right)^2 + g_\alpha^2 |\psi|^2$$

Phase diagram:



# Polariton model and equilibrium results

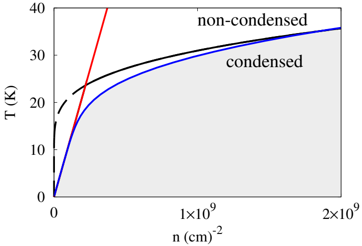
Localised excitons, propagating photons

$$H - \mu N = \sum_{\mathbf{k}} (\omega_{\mathbf{k}} - \mu)\psi^{\dagger}_{\mathbf{k}}\psi_{\mathbf{k}} + \sum_{\alpha} (\epsilon_{\alpha} - \mu) S^{z}_{\alpha} + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}}\psi_{\mathbf{k}} S^{+}_{\alpha} + \text{H.c.}$$

Self-consistent polarisation and field

$$(\omega_{\mathbf{k}=0} - \mu)\,\psi = \frac{1}{A}\sum_{\alpha} \frac{g^2_{\alpha}\psi}{2E_{\alpha}} \tanh(\beta E_{\alpha}), \qquad E_{\alpha}{}^2 = \left(\frac{\epsilon_{\alpha} - \mu}{2}\right)^2 + g^2_{\alpha}|\psi|^2$$

Phase diagram:



# Polariton model and equilibrium results

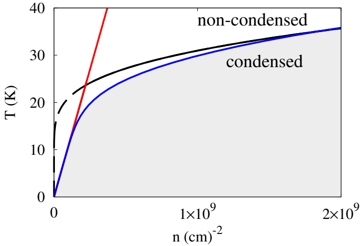
Localised excitons, propagating photons

$$H - \mu N = \sum_{\mathbf{k}} (\omega_{\mathbf{k}} - \mu)\psi^\dagger_{\mathbf{k}}\psi_{\mathbf{k}} + \sum_\alpha (\epsilon_\alpha - \mu) S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}}\psi_{\mathbf{k}} S^+_\alpha + \text{H.c.}$$

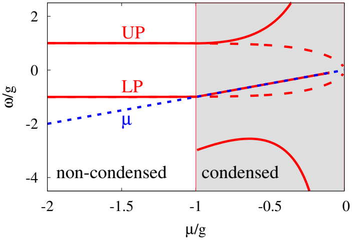
Self-consistent polarisation and field

$$(\omega_{\mathbf{k}=0} - \mu)\,\psi = \frac{1}{A}\sum_\alpha \frac{g_\alpha^2 \psi}{2E_\alpha}\tanh(\beta E_\alpha), \qquad {E_\alpha}^2 = \left(\frac{\epsilon_\alpha - \mu}{2}\right)^2 + g_\alpha^2 |\psi|^2$$

Phase diagram:

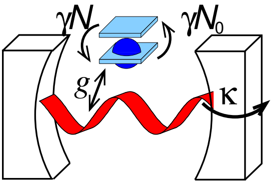



Modes (at $k = 0$)



# Simple Laser: Maxwell Bloch equations

$$H = \omega_0 \psi^\dagger \psi + \sum_\alpha \epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi S^+_\alpha + \text{H.c.}$$




Maxwell-Bloch eqns: $P = -i\langle S^- \rangle, N = 2\langle S^z \rangle$

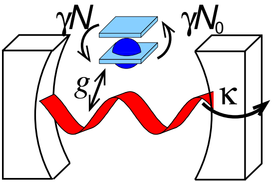
$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$

$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$

$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P^*_\alpha \psi)$$

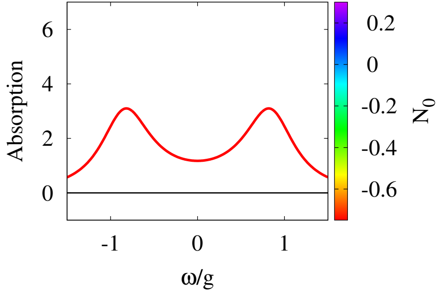
# Simple Laser: Maxwell Bloch equations

$$H = \omega_0 \psi^\dagger \psi + \sum_\alpha \epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi S^+_\alpha + \text{H.c.}$$




Maxwell-Bloch eqns: $P = -i\langle S^- \rangle, N = 2\langle S^z \rangle$

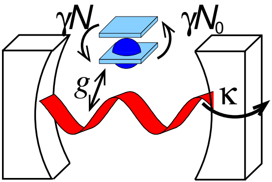
$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \textstyle\sum_\alpha g_\alpha P_\alpha$$
$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$
$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P^*_\alpha \psi)$$




- Strong coupling. $\kappa, \gamma < g\sqrt{n}$
- Inversion causes collapse before lasing

# Simple Laser: Maxwell Bloch equations

$$H = \omega_0 \psi^\dagger \psi + \sum_\alpha \epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{\mathrm{A}}} \psi S^+_\alpha + \text{H.c.}$$

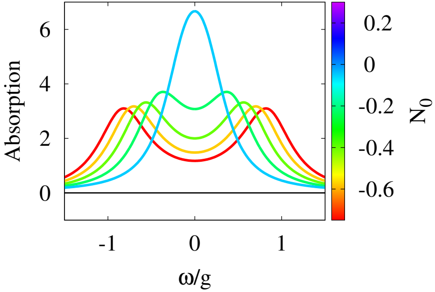



Maxwell-Bloch eqns: $P = -i\langle S^- \rangle, N = 2\langle S^z \rangle$

$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$

$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$

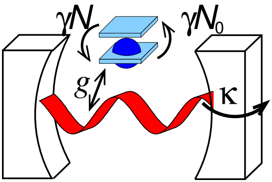
$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P^*_\alpha \psi)$$




- Strong coupling. $\kappa, \gamma < g\sqrt{n}$
- Inversion causes collapse before lasing

# Simple Laser: Maxwell Bloch equations

$$H = \omega_0 \psi^\dagger \psi + \sum_\alpha \epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi S^+_\alpha + \text{H.c.}$$

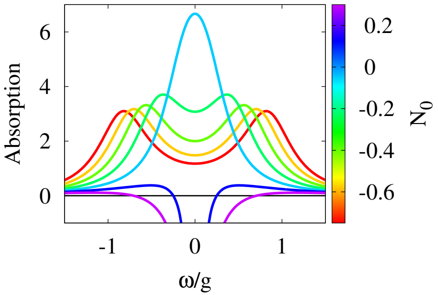



Maxwell-Bloch eqns: $P = -i\langle S^- \rangle, N = 2\langle S^z \rangle$

$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$

$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$

$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P^*_\alpha \psi)$$



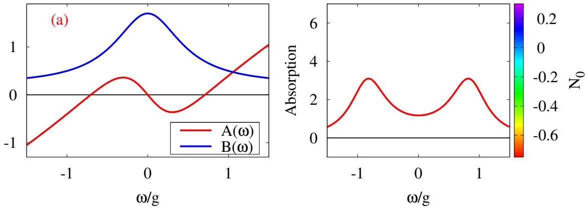

- Strong coupling. $\kappa, \gamma < g\sqrt{n}$
- Inversion causes collapse before lasing

# Poles of Retarded Green's function and gain

$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma}$$

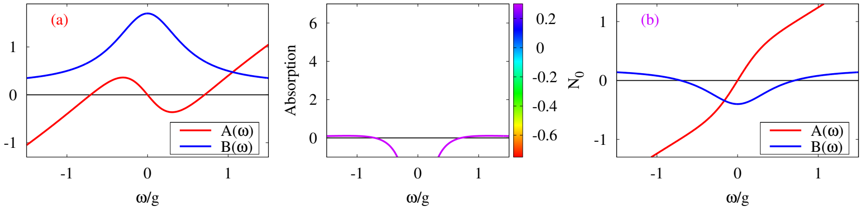
# Poles of Retarded Green's function and gain

$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma} = A(\omega) + iB(\omega)$$



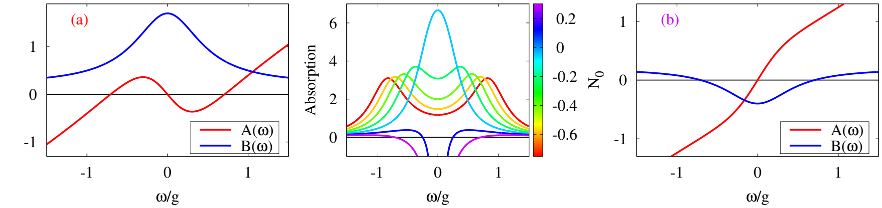
# Poles of Retarded Green's function and gain

$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma} = A(\omega) + iB(\omega)$$

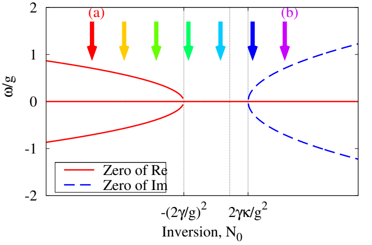


# Poles of Retarded Green's function and gain

$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma} = A(\omega) + iB(\omega)$$

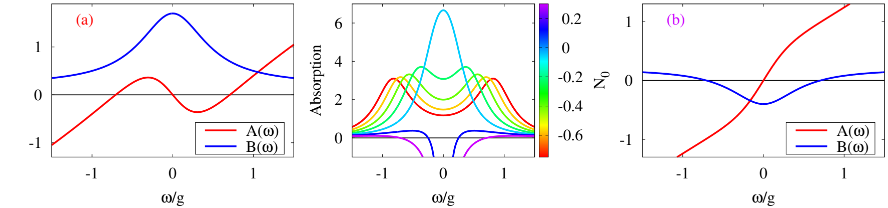



Laser:

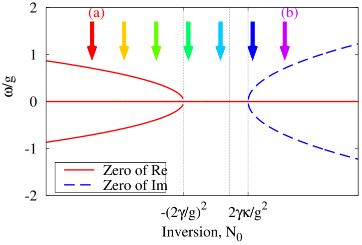


# Poles of Retarded Green's function and gain

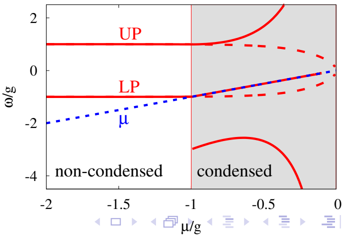
$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma} = A(\omega) + iB(\omega)$$



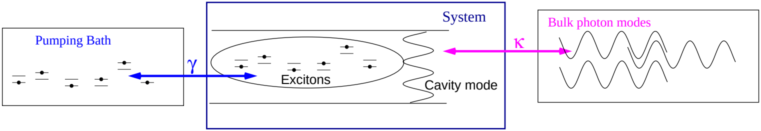

Laser:

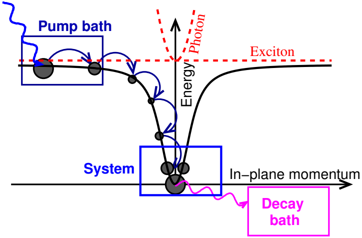



Equilibrium:



# Non-equilibrium description: baths

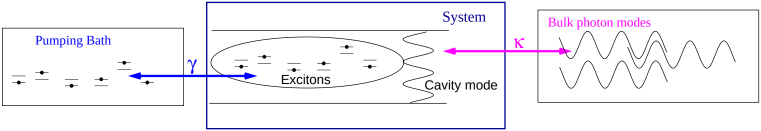


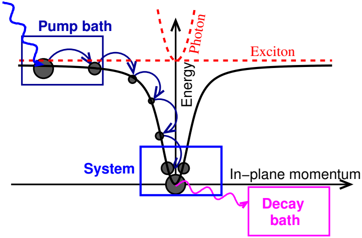




$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

- Decay bath: Empty ($\mu \to -\infty$)
- Pump bath: Thermal $\mu_B$, $T_B$

# Non-equilibrium description: baths

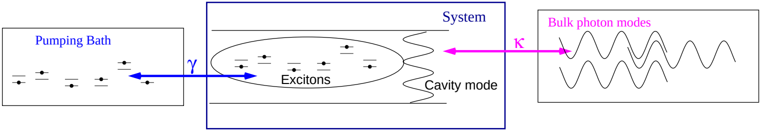


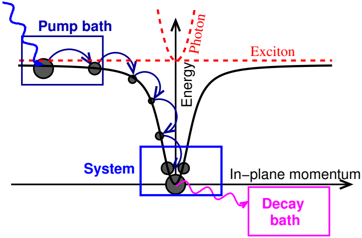




$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

- Decay bath: Empty ($\mu \to -\infty$)
- Pump bath: Thermal $\mu_B, T_B$

# Non-equilibrium description: baths

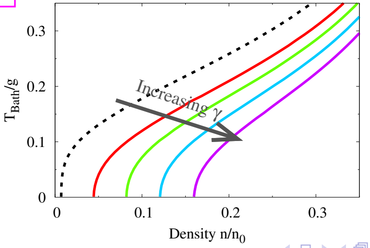


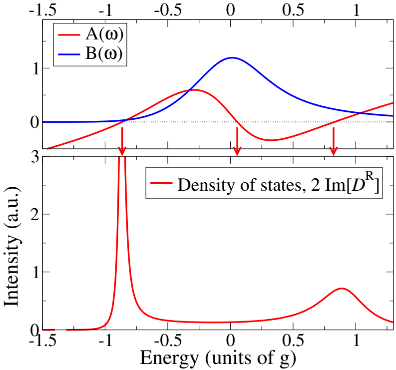

$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

- Decay bath: Empty ($\mu \to -\infty$)
- Pump bath: Thermal $\mu_B$, $T_B$

Mean field theory

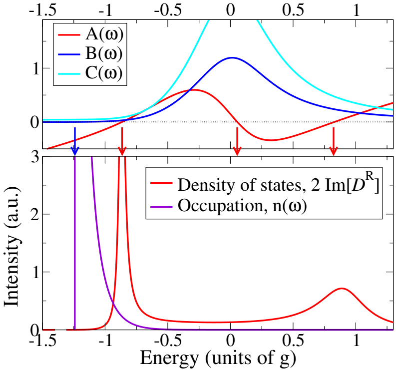


# Stability and evolution with pumping




$$\left[D^R(\omega)\right]^{-1} = A(\omega) + iB(\omega)$$

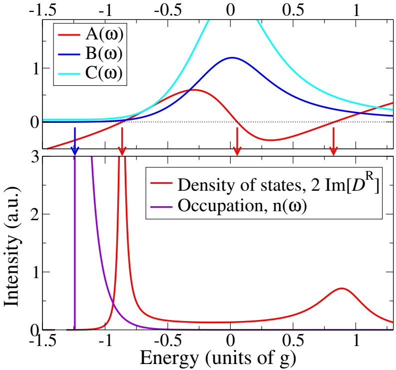
# Stability and evolution with pumping

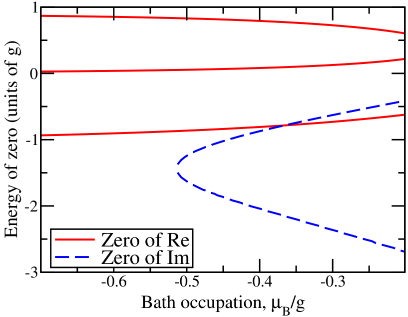



$$\left[D^R(\omega)\right]^{-1} = A(\omega) + iB(\omega)$$

$$2n(\omega)+1 = \frac{iD^K(\omega)}{-2\Im[D^R(\omega)]} = \frac{C(\omega)}{2B(\omega)}$$

# Stability and evolution with pumping

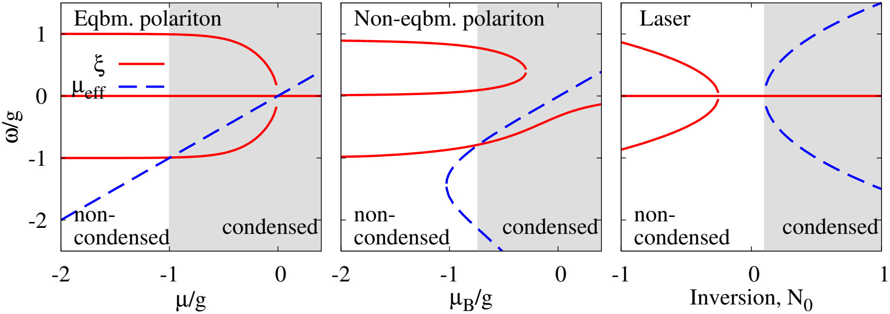


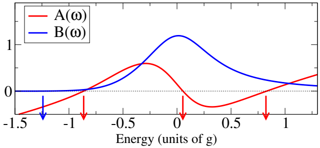

$$\left[D^R(\omega)\right]^{-1} = A(\omega) + iB(\omega)$$

$$2n(\omega)+1 = \frac{iD^K(\omega)}{-2\Im[D^R(\omega)]} = \frac{C(\omega)}{2B(\omega)}$$

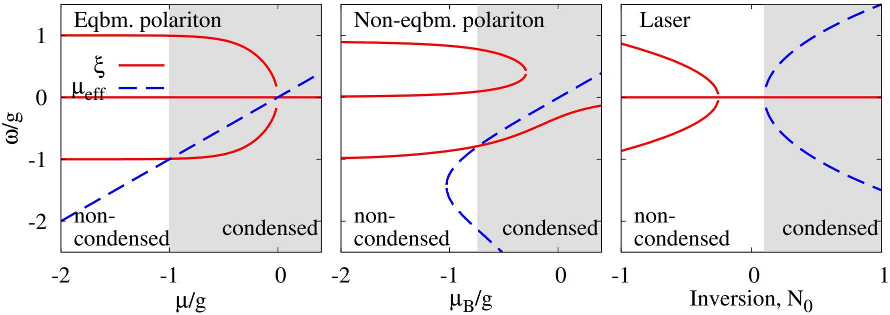
# Strong coupling and lasing — low temperature phenomenon



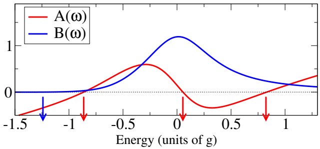

- Laser: Uniformly invert TLS
- Non-equilibrium polaritons: Cold bath
- If $T_B \gg \gamma \to$ Laser limit



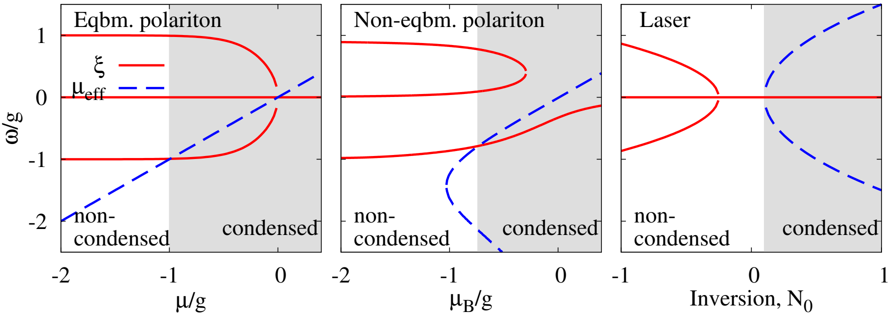
# Strong coupling and lasing — low temperature phenomenon



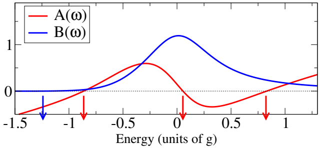

- Laser: Uniformly invert TLS
- Non-equilibrium polaritons: Cold bath
- If $T_B \gg \gamma \to$ Laser limit



# Strong coupling and lasing — low temperature phenomenon



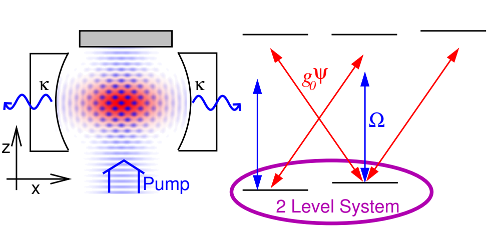

- Laser: Uniformly invert TLS
- Non-equilibrium polaritons: Cold bath
- If $T_B \gg \gamma \to$ Laser limit



# Raman pumped Dicke model (atoms)

# Extended Dicke model

[Baumann *et al.* Nature 2010]




2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$:

$\Downarrow$: $\Psi(x,z) = 1$

$\Uparrow$: $\Psi(x,z) = \sum_{\sigma,\sigma'=\pm} e^{ik(\sigma x + \sigma' z)}$

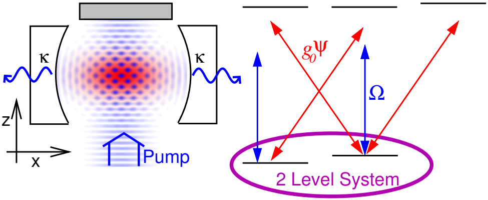
$\omega_0 = 2\omega_{\text{recoil}}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+) + U S_z \psi^\dagger\psi$$

$$\partial_t\rho = -i[H,\rho] - \kappa(\psi^\dagger\psi\rho - 2\psi\rho\psi^\dagger + \rho\psi^\dagger\psi)$$

# Extended Dicke model

[Baumann *et al.* Nature 2010]




2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$:

$\Downarrow$: $\Psi(x,z) = 1$

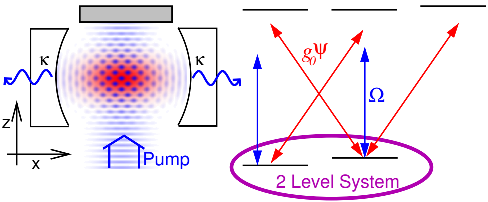
$\Uparrow$: $\Psi(x,z) = \sum_{\sigma,\sigma'=\pm} e^{ik(\sigma x + \sigma' z)}$

$\omega_0 = 2\omega_{\text{recoil}}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+)$$

# Extended Dicke model

[Baumann *et al.* Nature 2010]




2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$:

$\Downarrow$: $\Psi(x,z) = 1$

$\Uparrow$: $\Psi(x,z) = \sum_{\sigma,\sigma'=\pm} e^{ik(\sigma x + \sigma' z)}$

$\omega_0 = 2\omega_{\text{recoil}}$

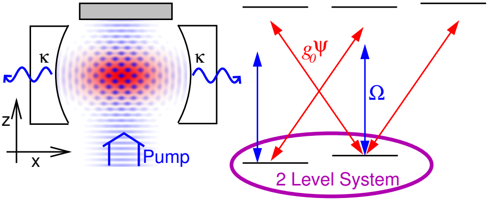
Feedback: $U \propto \frac{g_0^2}{\omega_c - \omega_a}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+) + U S_z \psi^\dagger\psi.$$

$$\partial_t\rho = -i[H,\rho] - \kappa(\psi^\dagger\psi\rho - 2\psi\rho\psi^\dagger + \rho\psi^\dagger\psi)$$

# Extended Dicke model

[Baumann *et al.* Nature 2010]




2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$:

$\Downarrow$: $\Psi(x,z) = 1$

$\Uparrow$: $\Psi(x,z) = \sum_{\sigma,\sigma'=\pm} e^{ik(\sigma x + \sigma' z)}$

$\omega_0 = 2\omega_{\text{recoil}}$

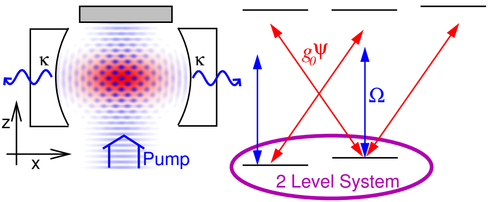
Feedback: $U \propto \frac{g_0^2}{\omega_c - \omega_a}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+) + U S_z \psi^\dagger\psi.$$

$$\partial_t \rho = -i[H,\rho] - \kappa(\psi^\dagger\psi\rho - 2\psi\rho\psi^\dagger + \rho\psi^\dagger\psi)$$

# Extended Dicke model

[Baumann *et al.* Nature 2010]




2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$:

$\Downarrow$: $\Psi(x,z) = 1$

$\Uparrow$: $\Psi(x,z) = \sum_{\sigma,\sigma'=\pm} e^{ik(\sigma x + \sigma' z)}$

$\omega_0 = 2\omega_{\text{recoil}}$

Feedback: $U \propto \dfrac{g_0^2}{\omega_c - \omega_a}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+) + U S_z \psi^\dagger\psi.$$

$$\partial_t \rho = -i[H,\rho] - \kappa(\psi^\dagger\psi\rho - 2\psi\rho\psi^\dagger + \rho\psi^\dagger\psi)$$

Semiclassical EOM ($|\mathbf{S}| = N/2 \gg 1$)

$$\dot{S}^- = -i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$

$$\dot{S}^z = ig(\psi + \psi^*)(S^- - S^+)$$

$$\dot{\psi} = -[\kappa + i(\omega + US^z)]\psi - ig(S^- + S^+)$$

# Fixed points (steady states)

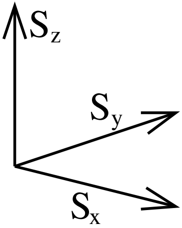
$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
$$0 = ig(\psi + \psi^*)(S^- - S^+)$$
$$0 = -\left[\kappa + i(\omega + US^z)\right]\psi - ig(S^- + S^+)$$

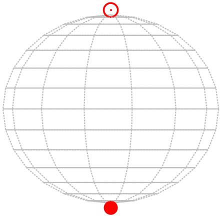
- $\psi = 0, \mathbf{S} = (0, 0, \pm N/2)$ always a solution.
- If $g > g_c$, $\psi \neq 0$ too
  - A $S^y = -\Im[S^-] = 0$
  - B $\psi' = \Re[\psi] = 0$

# Fixed points (steady states)

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
$$0 = ig(\psi + \psi^*)(S^- - S^+)$$
$$0 = -[\kappa + i(\omega + US^z)]\psi - ig(S^- + S^+)$$

- $\psi = 0, \mathbf{S} = (0, 0, \pm N/2)$ always a solution.
- If $g > g_c$, $\psi \neq 0$ too
  - A $S^y = -\Im[S^-] = 0$
  - B $\psi' = \Re[\psi] = 0$



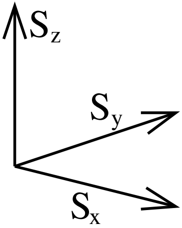



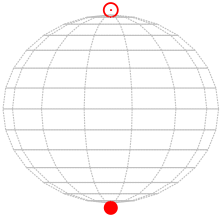
Small g: ⇑, ⇓ only.
($\omega = 30\text{MHz}$, $UN = -40\text{MHz}$)

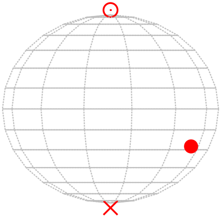
# Fixed points (steady states)

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
$$0 = ig(\psi + \psi^*)(S^- - S^+)$$
$$0 = -\left[\kappa + i(\omega + US^z)\right]\psi - ig(S^- + S^+)$$

- $\psi = 0, \mathbf{S} = (0, 0, \pm N/2)$ always a solution.
- If $g > g_c, \psi \neq 0$ too
  - A $S^y = -\Im[S^-] = 0$
  - B $\psi' = \Re[\psi] = 0$



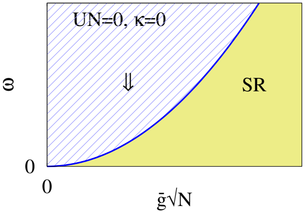


Small g: $\Uparrow, \Downarrow$ only. Larger g: SR too.

($\omega = 30$MHz, $UN = -40$MHz)

# Steady state phase diagram

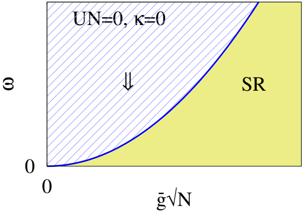
$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
$$0 = ig(\psi + \psi^*)(S^- - S^+)$$
$$0 = -\left[\kappa + i(\omega + US^z)\right]\psi - ig(S^- + S^+)$$

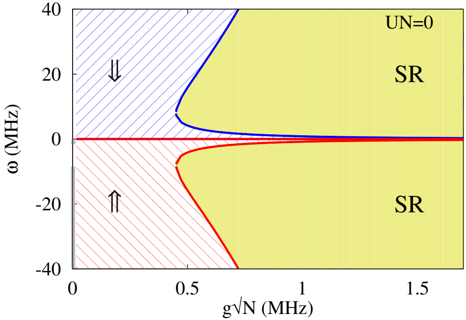



See also Domokos and Ritsch PRL ’02, Domokos *et al.* PRL ’10

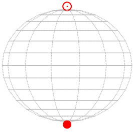
# Steady state phase diagram

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
$$0 = ig(\psi + \psi^*)(S^- - S^+)$$
$$0 = -[\kappa + i(\omega + US^z)]\psi - ig(S^- + S^+)$$

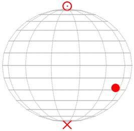



⇓



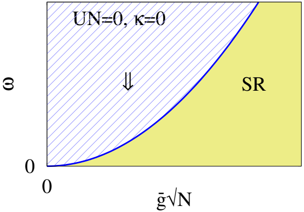
SR(A): $S_y = 0$

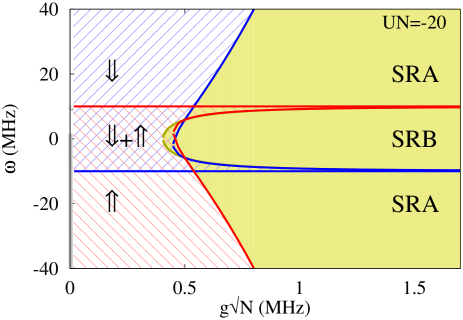


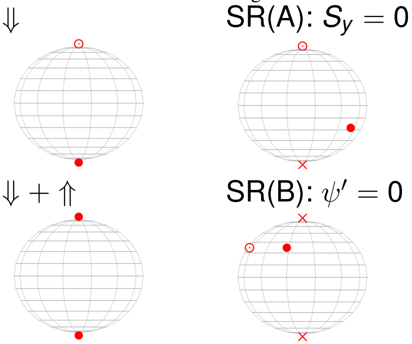
See also Domokos and Ritsch PRL '02, Domokos *et al.* PRL '10

# Steady state phase diagram

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
$$0 = ig(\psi + \psi^*)(S^- - S^+)$$
$$0 = -[\kappa + i(\omega + US^z)]\psi - ig(S^- + S^+)$$



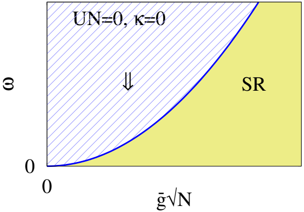



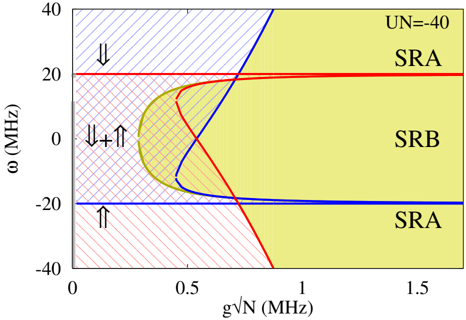



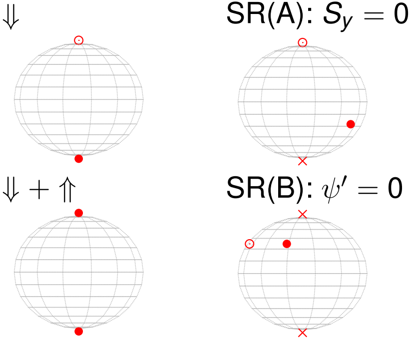

See also Domokos and Ritsch PRL '02, Domokos *et al.* PRL '10

# Steady state phase diagram

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
$$0 = ig(\psi + \psi^*)(S^- - S^+)$$
$$0 = -[\kappa + i(\omega + US^z)]\psi - ig(S^- + S^+)$$



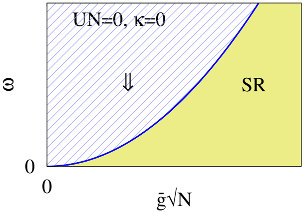



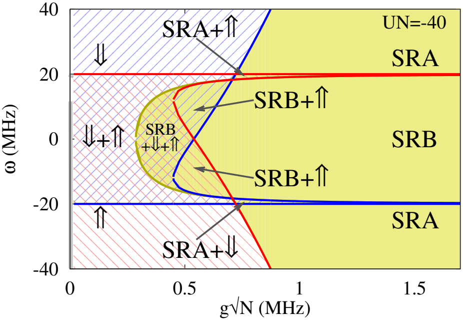



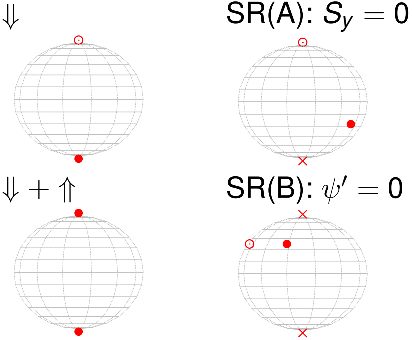

See also Domokos and Ritsch PRL ’02, Domokos *et al.* PRL ’10

# Steady state phase diagram

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
$$0 = ig(\psi + \psi^*)(S^- - S^+)$$
$$0 = -[\kappa + i(\omega + US^z)]\psi - ig(S^- + S^+)$$

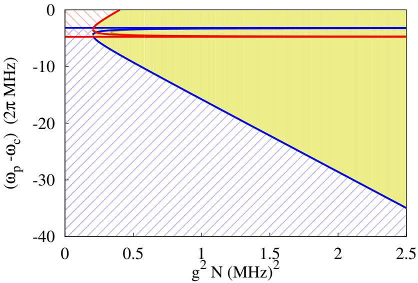


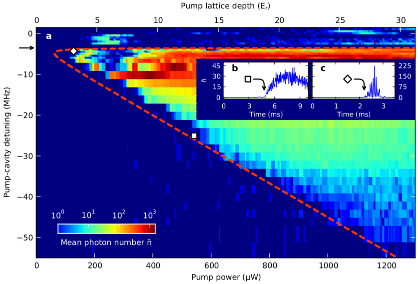




See also Domokos and Ritsch PRL ’02, Domokos *et al.* PRL ’10

# Comparison to experiment




$UN = -10\text{MHz}$

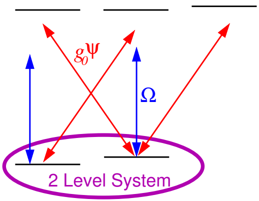
Adapted from: [Bhaseen *et al.* PRA '12]

[Baumann *et al* Nature '10 ]

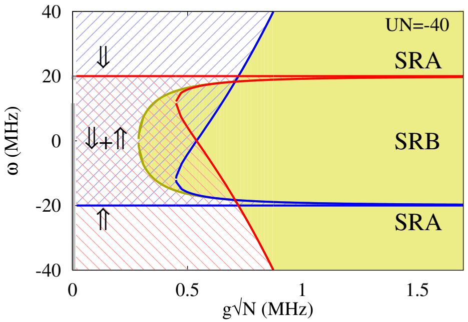
$$\omega = \omega_c - \omega_p + \frac{5}{2} UN, \qquad UN = -\frac{g_0^2}{4(\omega_a - \omega_c)}$$

# Regions without fixed points

Changing $U$:

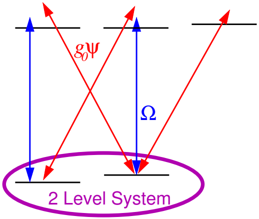



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$

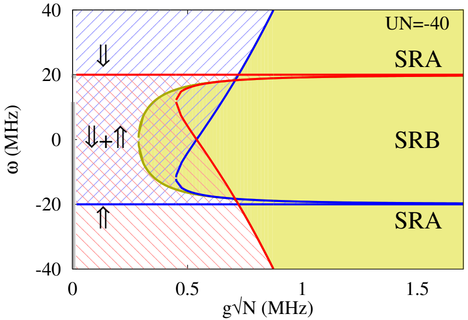


# Regions without fixed points

Changing $U$:

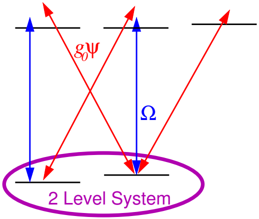



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$

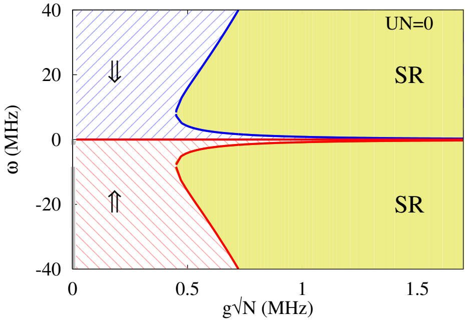


# Regions without fixed points

Changing $U$:

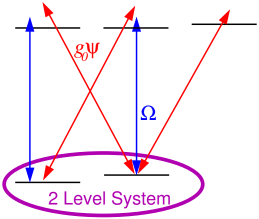



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$

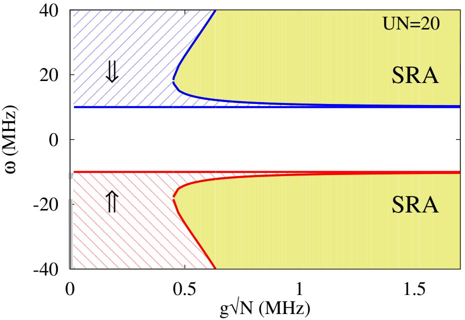


# Regions without fixed points

Changing $U$:

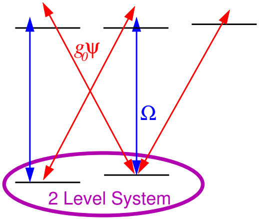



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$

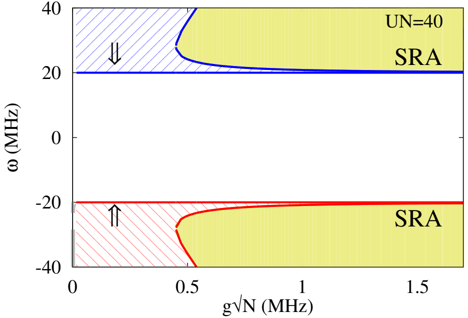


# Regions without fixed points

Changing *U*:

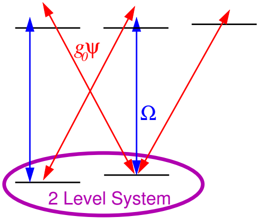



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$

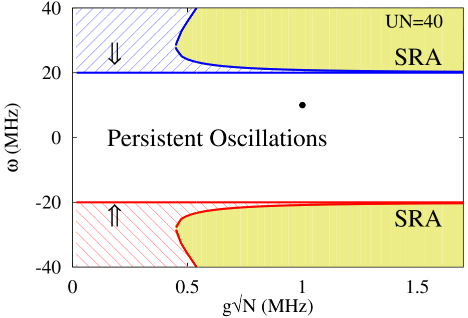


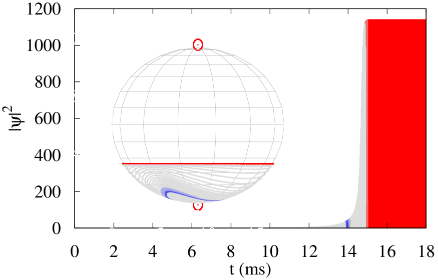
# Regions without fixed points

Changing $U$:

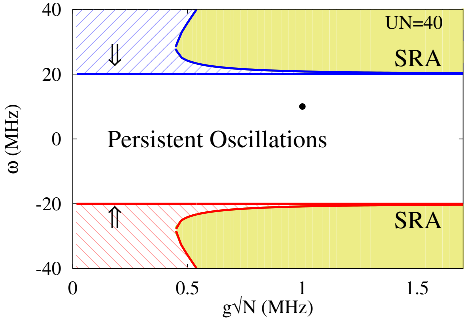


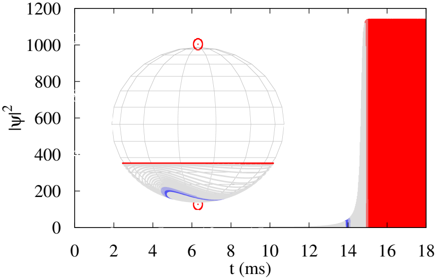

$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$

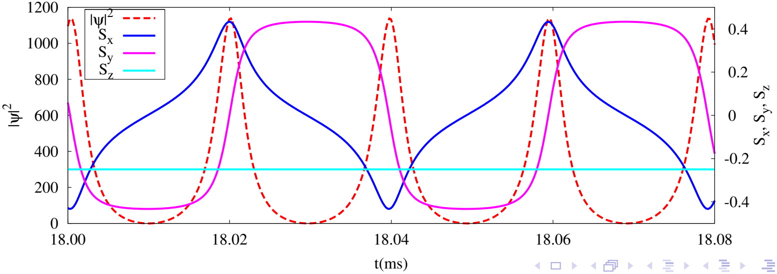


# Persistent (optomechanical) oscillations

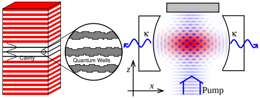


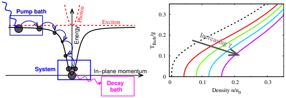



# Summary

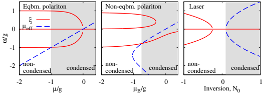
- Non-equilibrium Dicke relevant to increasing number of systems



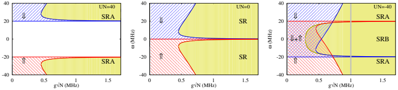

- Effects of pumping on mean-field theory




- Polariton condensation vs lasing



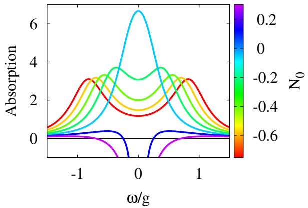

- Dynamical phases of Raman pumped scheme



# Extra slides

4 Retarded Green's function for laser

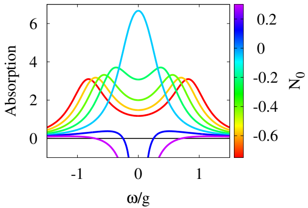
# Maxwell-Bloch Equations: Retarded Green's function




- Introduce $D^R(\omega)$:
  Response to perturbation

- Absorption $= -2\Im[D^R(\omega)]$

# Maxwell-Bloch Equations: Retarded Green's function



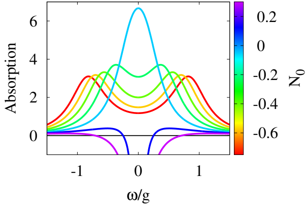

- Introduce $D^R(\omega)$:
  Response to perturbation

$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$
$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$
$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P^*_\alpha \psi)$$

- Absorption $= -2\Im[D^R(\omega)]$

$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma}$$

# Maxwell-Bloch Equations: Retarded Green's function



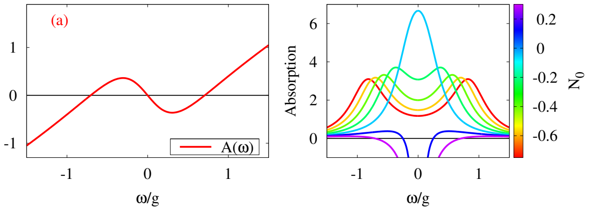

- Introduce $D^R(\omega)$: Response to perturbation

$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$
$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$
$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P_\alpha^* \psi)$$

- Absorption $= -2\Im[D^R(\omega)] = \dfrac{2B(\omega)}{A(\omega)^2 + B(\omega)^2}$

$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma} = A(\omega) + iB(\omega)$$

# Maxwell-Bloch Equations: Retarded Green's function



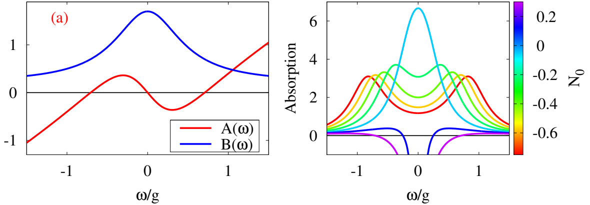

- Introduce $D^R(\omega)$:
  Response to perturbation

$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$
$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$
$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P_\alpha^* \psi)$$

- Absorption $= -2\Im[D^R(\omega)] = \dfrac{2B(\omega)}{A(\omega)^2 + B(\omega)^2}$

$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma} = A(\omega) + iB(\omega)$$

# Maxwell-Bloch Equations: Retarded Green's function



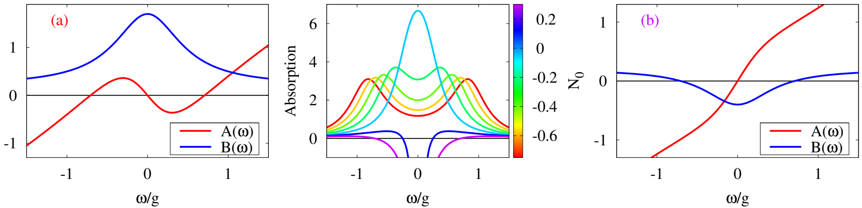

- Introduce $D^R(\omega)$:
  Response to perturbation

$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$
$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$
$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P_\alpha^* \psi)$$

- Absorption $= -2\Im[D^R(\omega)] = \dfrac{2B(\omega)}{A(\omega)^2 + B(\omega)^2}$

$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma} = A(\omega) + iB(\omega)$$

# Maxwell-Bloch Equations: Retarded Green's function




- Introduce $D^R(\omega)$:
  Response to perturbation

$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$
$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$
$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P^*_\alpha \psi)$$

- Absorption $= -2\Im[D^R(\omega)] = \dfrac{2B(\omega)}{A(\omega)^2 + B(\omega)^2}$

$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma} = A(\omega) + iB(\omega)$$