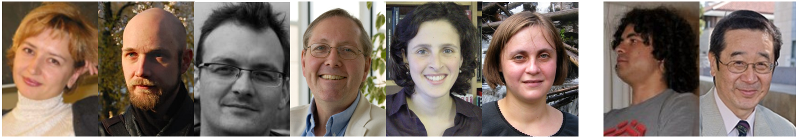
Condensation, superfluidity and lasing of coupled light-matter systems.

Jonathan Keeling

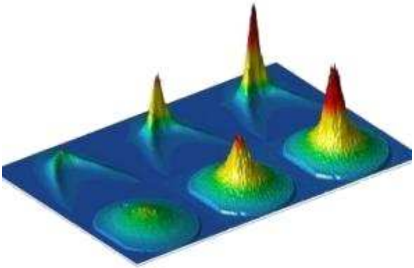


ANL, January 2012

Funding: EPSRC
Engineering and Physical Sciences Research Council

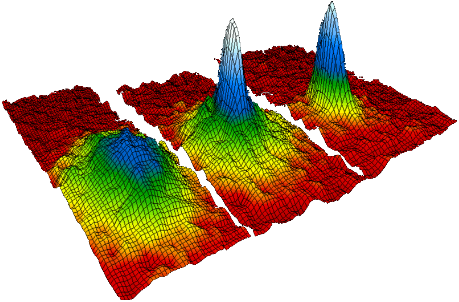
Bose-Einstein condensation: macroscopic occupation

Polaritons. $\sim$ 20K



[Kasprzak *et al.* Nature, '06]

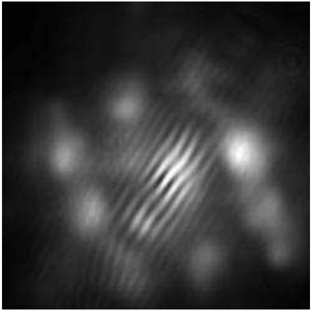
Atoms. $\sim 10^{-7}$K



[Anderson *et al.* Science '95]

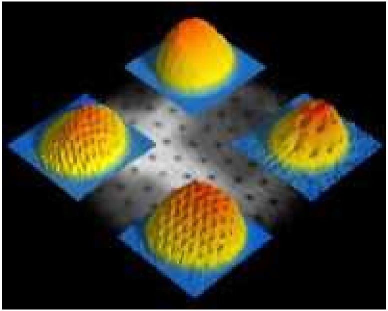
Macroscopic coherence: vortices

Polaritons:



[Lagoudakis *et al.* Nat. Phys. '08]

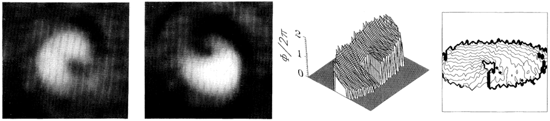
Atoms:



[Abo-Shaeer *et al.* Science '01]

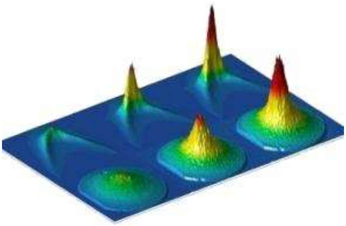
But also, nonlinear optics:

[Arecchi *et al.* PRL '91]

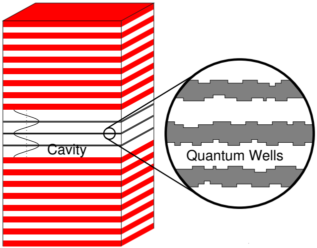


Polariton condensate and photon condensate

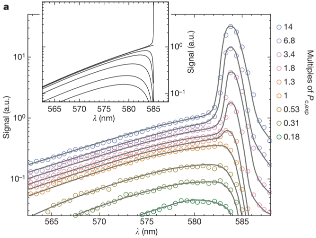
Polaritons:



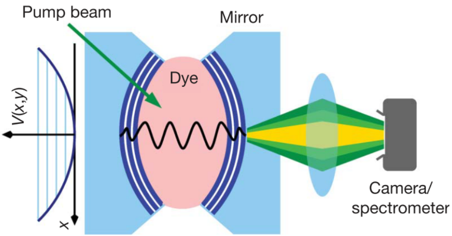
[Kasprzak *et al.* Nature, '06]



Photons:



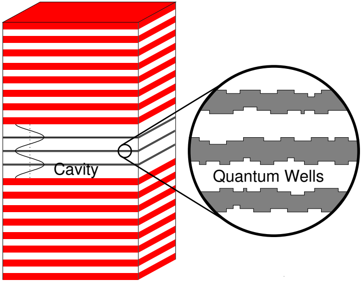
[Klaers *et al.* Nature, '10]



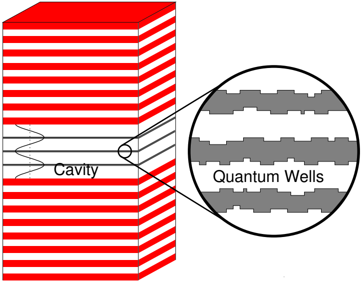
Questions

- What are polaritons?
- How can photon-like objects form a BEC?
- Is this “just a laser”?
- How to model a non-equilibrium condensate?
- Effects of non-equilibrium nature on:
  - Steady states
  - Coherence
  - Superfluidity
  - ...

Microcavity polaritons

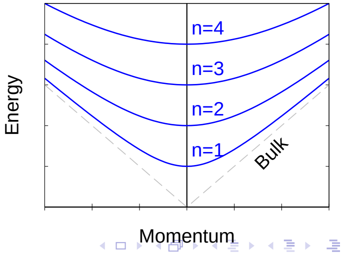


Microcavity polaritons



Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$



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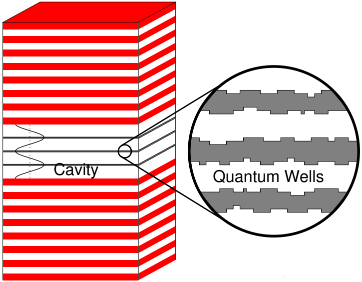

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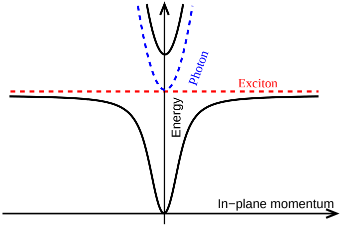
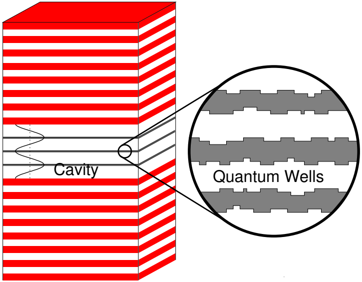


Microcavity polaritons



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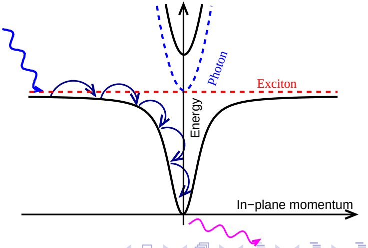

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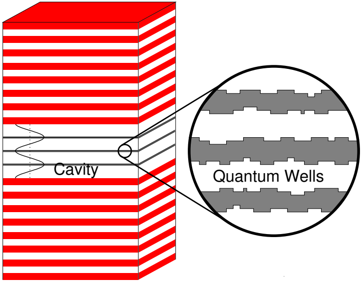

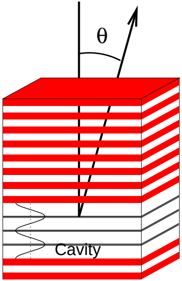

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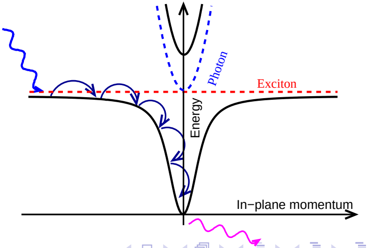
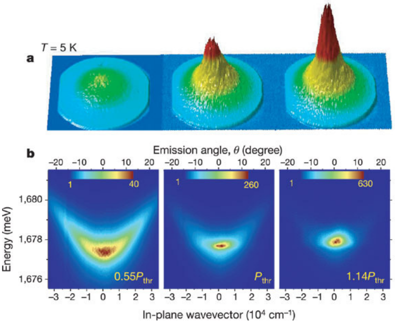
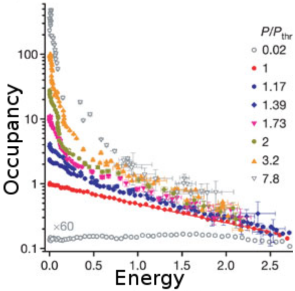


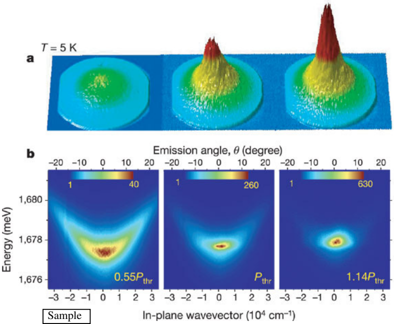
Polariton experiments: occupation and coherence

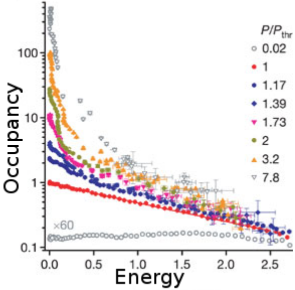


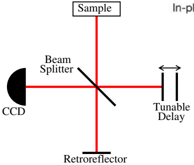


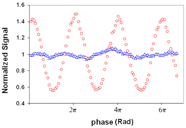
[Kasprzak, *et al.* Nature, 2006]

Polariton experiments: occupation and coherence

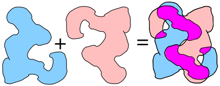


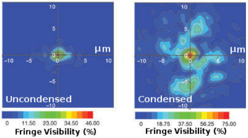






Coherence map:

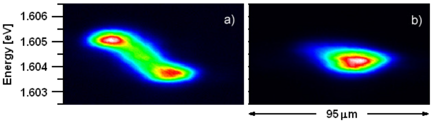




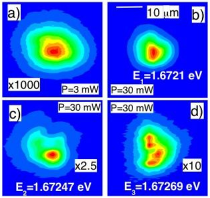
[Kasprzak, *et al.* Nature, 2006]

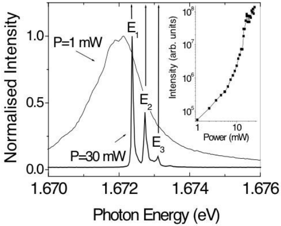
(Some) other polariton condensation experiments

- Polariton traps
  [**Balili *et al.* Science ’07)**]

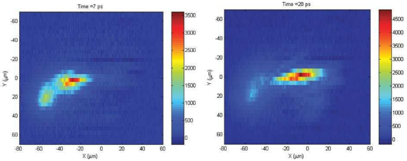


- Multimode condensate and sharp lines
  [**Love *et al.* PRL ’08**]



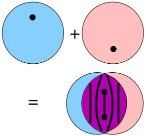


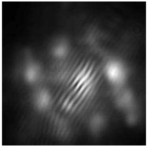
- Wavepacket propagation
  [**Amo *et al.* Nature 457 291 (2009)**]



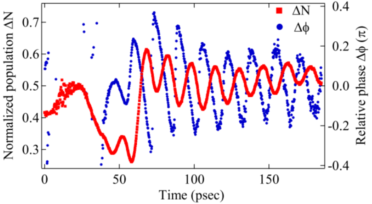
(Some) other polariton condensation experiments

- Quantised vortices
  [**Lagoudakis *et al.* Nat. Phys. ’08**. Science ’09, PRL ’10; Sanvitto *et al.* Nat. Phys. ’10; Roumpos *et al.* Nat. Phys. ’10]

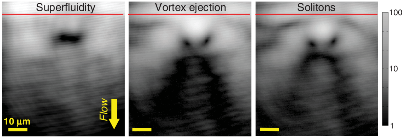




- Josephson oscillations
  [**Lagoudakis *et al.* PRL ’10**]



- Pattern formation/Hydrodynamics
  [**Amo *et al.* Science ’11**, Nature ’09; Wertz *et al.* Nat. Phys ’10]



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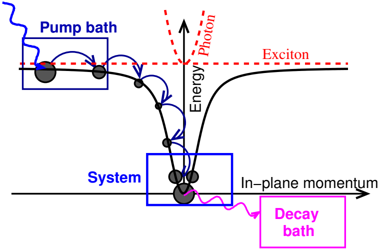
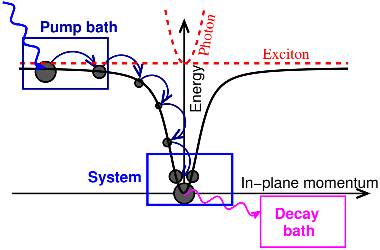
Non-equilibrium approach: Steady state, and fluctuations

$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}},$$

$$H_{\text{sys}} = \sum_k \omega_k \psi_k \psi_k^\dagger + \sum_\alpha g_\alpha(\phi_\alpha^\dagger \psi_k + \text{H.c.}) + H_{\text{ex}}[\phi_\alpha, \phi_\alpha^\dagger]$$



Non-equilibrium approach: Steady state, and fluctuations



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Steady state, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$.

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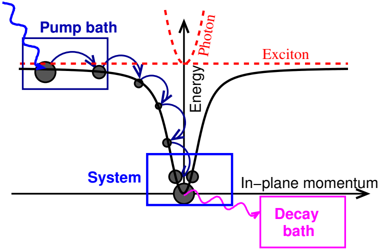

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Self-consistent equation: $(i\partial_t - \omega_0 + i\kappa)\,\psi = \sum_\alpha g_\alpha \langle \phi_\alpha \rangle$

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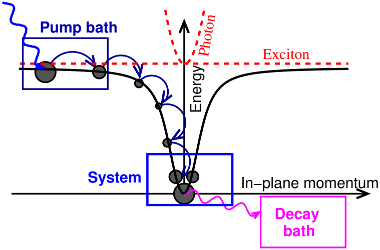

Steady state, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$.

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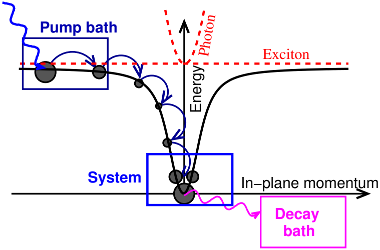

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Fluctuations

$$[D^R - D^A](t, t') = -i \left\langle \left[\psi(t), \psi^\dagger(t')\right]_- \right\rangle$$

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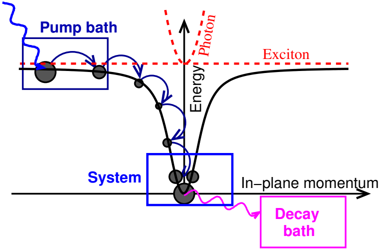

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$$[D^R - D^A](t, t') = -i\left\langle \left[\psi(t), \psi^\dagger(t')\right]_- \right\rangle \qquad [D^R - D^A](\omega) = \text{DoS}(\omega)$$

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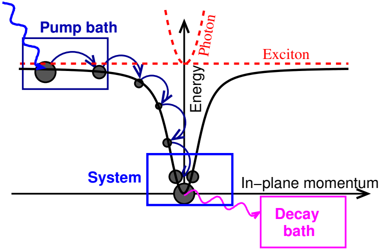

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$$[D^R - D^A](t, t') = -i \left\langle \left[\psi(t), \psi^\dagger(t')\right]_- \right\rangle \qquad [D^R - D^A](\omega) = \text{DoS}(\omega)$$

$$D^K(t, t') = -i \left\langle \left[\psi(t), \psi^\dagger(t')\right]_+ \right\rangle$$

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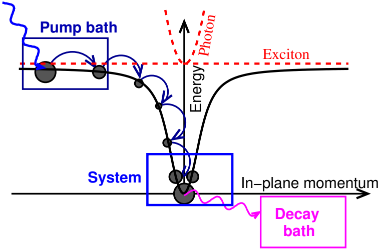

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Fluctuations

$$[D^R - D^A](t, t') = -i \left\langle \left[\psi(t), \psi^\dagger(t')\right]_- \right\rangle \qquad [D^R - D^A](\omega) = \text{DoS}(\omega)$$
$$D^K(t, t') = -i \left\langle \left[\psi(t), \psi^\dagger(t')\right]_+ \right\rangle \qquad D^K(\omega) = (2n(\omega) + 1)\text{DoS}(\omega)$$

Pattern formation:

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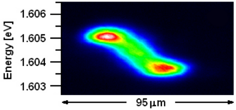
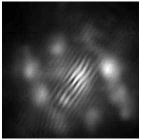
Pattern formation in experiments

Polariton Traps



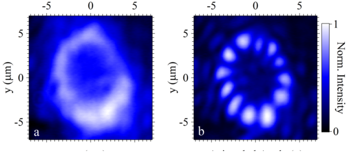
[Balili *et al.* Science ’07]

Vortex formation



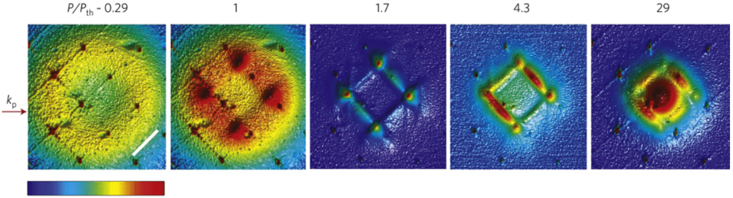
[Lagoudakis *et al.* Nat. Phys ’08]

Elliptical ring pump



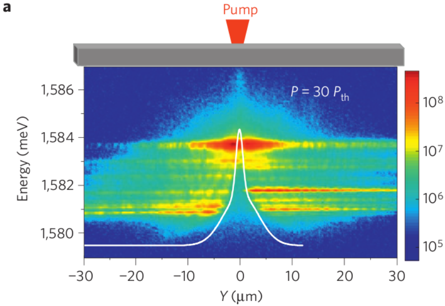
[Manni *et al.* PRL ’11]

Patterned lattice: Momentum space image

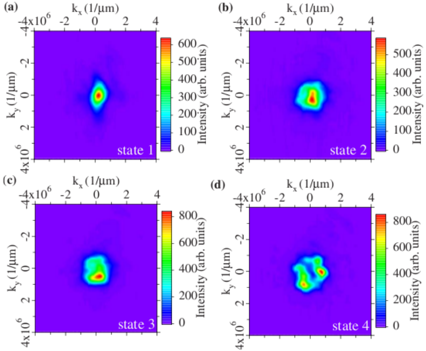


[Kim *et al.* Nat. Phys ’11]

Non-equilibrium features in experiment



Flow from pumping spot
[Wertz *et al.* Nat. Phys. (2010)]



$|\psi(\mathbf{k})|^2 \neq |\psi(-\mathbf{k})|^2$:
Broken time-reversal symmetry.
[Krizhanovskii *et al.* PRB (2009)]

Complex Gross-Pitaevskii equation

Steady state equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit:

Complex Gross-Pitaevskii equation

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$$\left(i\partial_t + i\kappa - \left[V(r) - \frac{\nabla^2}{2m}\right]\right)\psi(r) = \chi(\psi(r,t))\psi(r,t)$$

Nonlinear, complex susceptibility

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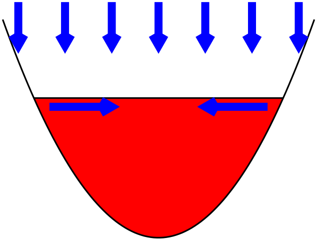
Gross-Pitaevskii equation: Harmonic trap

$$i\partial_t\psi = \left[-\frac{\nabla^2}{2m} + \frac{m\omega^2}{2}r^2 + U|\psi|^2 + i\left(\gamma_{\text{eff}} - \kappa - \Gamma|\psi|^2\right)\right]\psi$$



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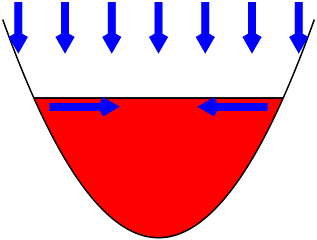
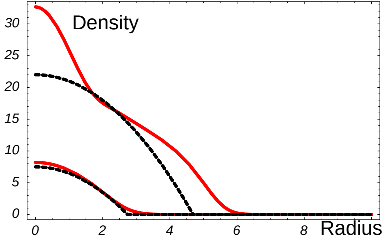
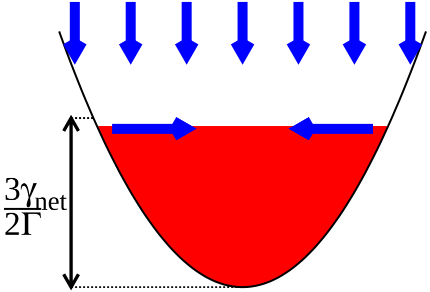




Stability of Thomas-Fermi solution

$$\frac{1}{2}\partial_t \rho + \nabla\cdot(\rho\mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$



Stability of Thomas-Fermi solution

High m modes: $\delta\rho_{n,m} \simeq e^{im\theta} r^m \ldots$

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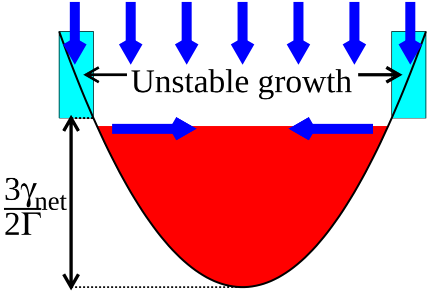

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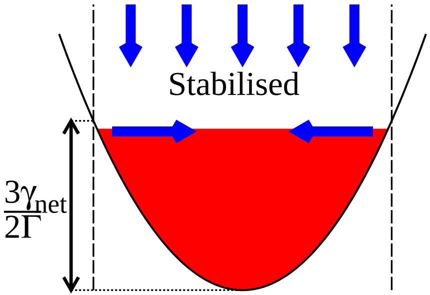

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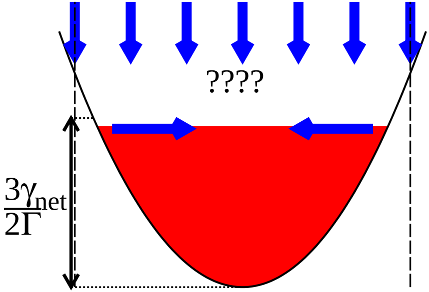
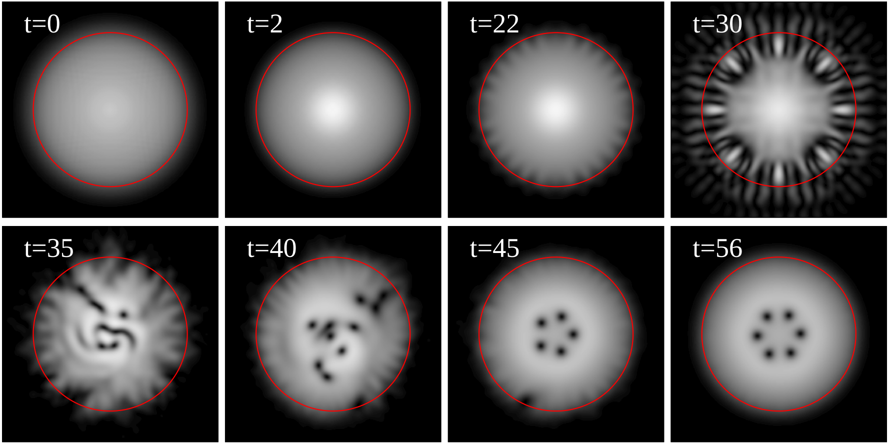


Time evolution:



[Keeling & Berloff PRL '08]

Superfluidity

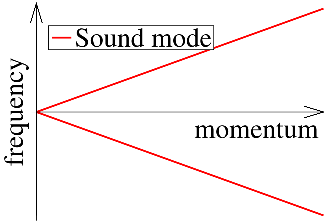
Spectrum above transition

When condensed

$$\text{Det}\left[D^R(\omega, k)\right]^{-1} = \omega^2 - \xi_k^2$$

With $\xi_k \simeq ck$
Poles:

$$\omega^* = \xi_k$$



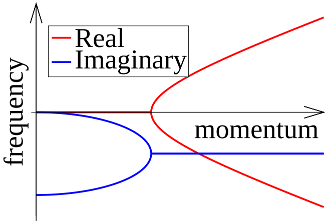
Spectrum above transition

When condensed

$$\text{Det}\left[D^R(\omega,k)\right]^{-1} = (\omega + i\gamma_{\text{net}})^2 + \gamma_{\text{net}}^2 - \xi_k^2$$

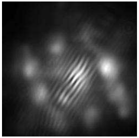
With $\xi_k \simeq ck$
Poles:

$$\omega^* = -i\gamma_{\text{net}} \pm \sqrt{\xi_k^2 - \gamma_{\text{net}}^2}$$

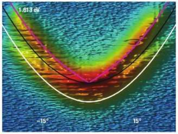


Polariton “superfluidity” experiments

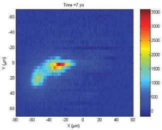
- Quantised vortices in disorder potential
  [Lagoudakis *et al.* Nature Phys. 4, 706 (2008)]

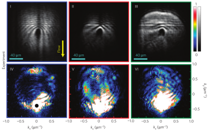


- Changes to excitation spectrum
  [Utsunomiya *et al.* Nature Phys. 4 700 (2008)]



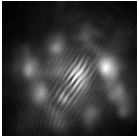
- Wavepacket propagation
  [Amo *et al.* Nature 457 291 (2009)]
- Driven superfluidity
  [Amo *et al.* Nature Phys. (2009)]

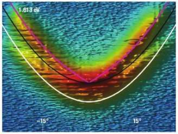


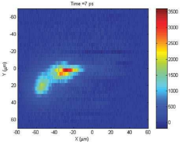


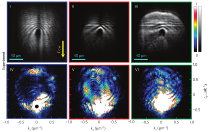
Aspects of superfluidity

	Quantised vortices	Landau critical velocity	Metastable persistent flow	Two-fluid hydrody-namics	Local thermal equilib-rium	Solitary waves
Superfluid ^{4}He/cold atom Bose-Einstein condensate	✓	✓	✓	✓	✓	✓
Non-interacting Bose-Einstein condensate	✓	✗	✗	✓	✓	✗
Classical irrotational fluid	✗	✓	✗	✓	✓	✓
Incoherently pumped polariton condensates	✓	✗	?	?	✗	?









Lagoudakis *et al.* Nat. Phys. '08. Utsunomiya *et al.* Nat. Phys. '08. Amo *et al.* Nature '09; Nat. Phys. '09

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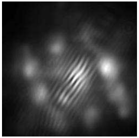
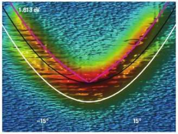
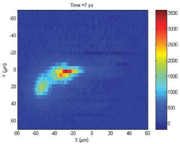
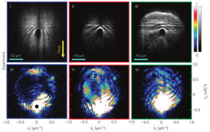




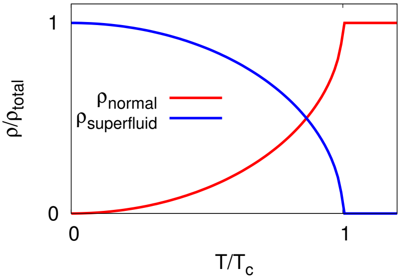




Lagoudakis *et al.* Nat. Phys. '08. Utsunomiya *et al.* Nat. Phys. '08. Amo *et al.* Nature '09; Nat. Phys. '09

Superfluid density

- Two-fluid hydrodynamics



- ρ_s, ρ_n distinguished by slow rotation

- Experimentally, rotation:

- To calculate, transverse/longitudinal:

Superfluid density

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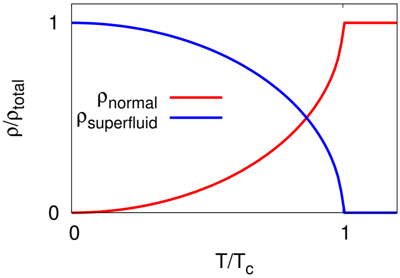

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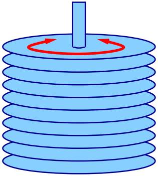

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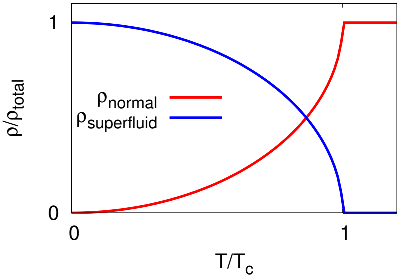

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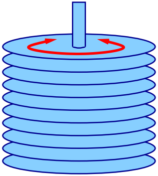

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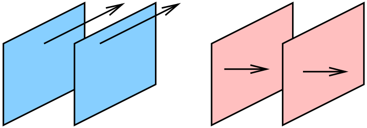

Superfluid density

- Currrent:

$$\mathbf{J} = \rho\mathbf{v} = \psi^\dagger i\nabla\psi = |\psi|^2\nabla\phi$$

$$J_i(\mathbf{q}) = \psi^\dagger_{\mathbf{k}+\mathbf{q}}\frac{2k_i + q_i}{2m}\psi_{\mathbf{k}}$$

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$$H \to H - \sum_q \mathbf{f}(\mathbf{q})\cdot\mathbf{J}_i(\mathbf{q}) \qquad J_i(\mathbf{q}) = \chi_{ij}(\mathbf{q})f_j(\mathbf{q})$$

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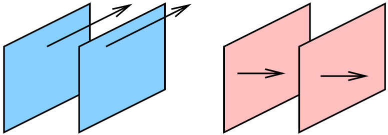

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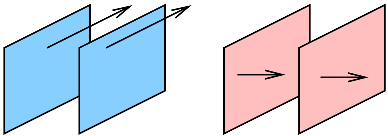

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Non-equilibrium current response functions

- Superfluid response exists because:

$$= \left(\frac{i\psi_0 q_i}{2m}\right) D^R(q, \omega = 0) \left(\frac{i\psi_0 q_j}{2m}\right)$$

- $D^R(\omega = 0) \propto 1/q^2$ despite pumping/decay — superfluid response exists.
- Normal density:

$$\rho_N = \int d^d k \epsilon_k \int \frac{d\omega}{2\pi} \mathrm{Tr}\left[\sigma_z D^K \sigma_z (D^R + D^A)\right]$$

- Is affected by pump/decay:
  Does not vanish at $T \to 0$.

[Keeling PRL '11]

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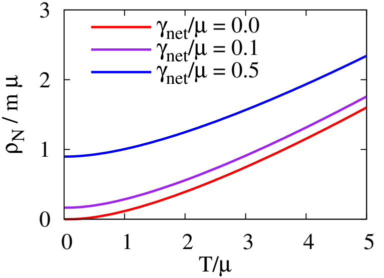


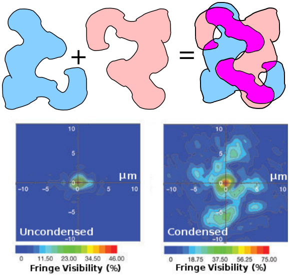
[Keeling PRL '11]

Coherence:


Correlations in a 2D Gas

Correlations:

$$g_1(\mathbf{r}, \mathbf{r}', t) = \left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0) \right\rangle$$

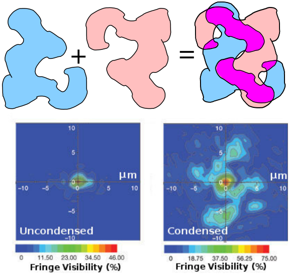


[Szymańska *et al.* PRL '06; PRB '07]

Correlations in a 2D Gas

Correlations: (in 2D)

$$g_1(\mathbf{r}, \mathbf{r}', t) = \left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0) \right\rangle \simeq |\psi_0|^2 \exp\left[-D^<_{\phi\phi}(\mathbf{r}, \mathbf{r}', t) \right]$$



- $D^< = D^K - D^R + D^A$
- Generally, get:

$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, 0) \right\rangle \simeq |\psi_0|^2 \exp\left[-a_p \begin{cases} \ln(r/r_0) & t \simeq 0 \\ \frac{1}{2}\ln(c^2 t/\gamma_{\text{net}} r_0^2) & r \simeq 0 \end{cases} \right]$$

[Szymańska *et al.* PRL '06; PRB '07]

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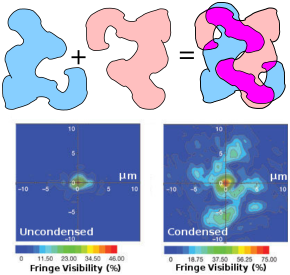
$$g_1(\mathbf{r}, \mathbf{r}', t) = \left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0) \right\rangle$$
$$\simeq |\psi_0|^2 \exp\left[-D^<_{\phi\phi}(\mathbf{r}, \mathbf{r}', t) \right]$$

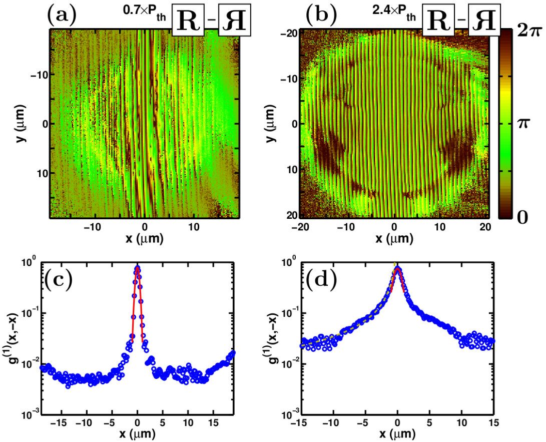


- $D^< = D^K - D^R + D^A$
- Generally, get:

$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, 0) \right\rangle \simeq |\psi_0|^2 \exp\left[-a_p \begin{cases} \ln(r/r_0) & t \simeq 0 \\ \frac{1}{2}\ln(c^2 t/\gamma_{\text{net}} r_0^2) & r \simeq 0 \end{cases} \right]$$

[Szymańska *et al.* PRL '06; PRB '07]

Experimental observation of power-law decay



G. Rompos, Y. Yamamoto *et al.* submitted

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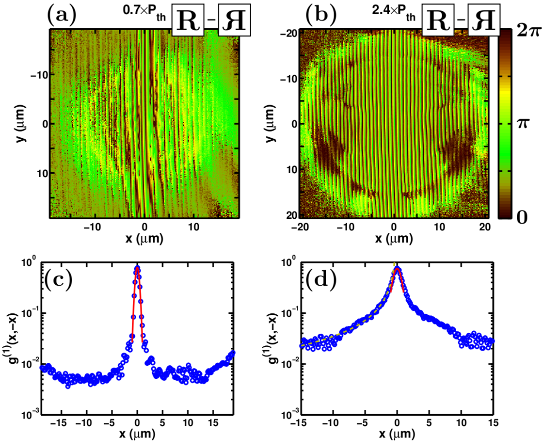
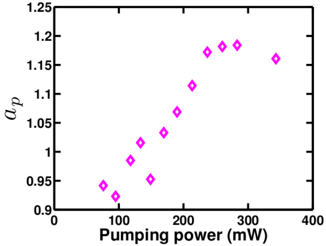


$$g_1(\mathbf{r}, -\mathbf{r}) \propto \left(\frac{r}{r_0}\right)^{-a_p}$$



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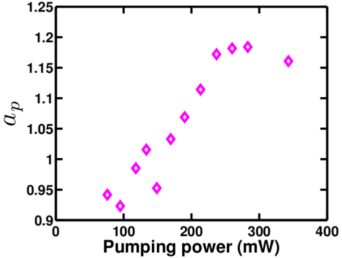
Exponent in a non-equilibrium 2D gas

$$\lim_{r\to\infty} \left\langle \psi^\dagger(\mathbf{r},0)\psi(-\mathbf{r},0) \right\rangle = |\psi_0|^2 \exp\left[-D^<_{\phi\phi}(\mathbf{r},-\mathbf{r})\right] \propto \exp\left[-a_p \ln\left(\frac{2r}{r_0}\right)\right]$$

- Experimentally, $a_P \simeq 1.2$
- In equilibrium $a_p = \frac{mk_BT}{2\pi\hbar^2 n_s} < \frac{1}{4}$ (BKT transition)
- Non-equilibrium theory depends on thermalisation.



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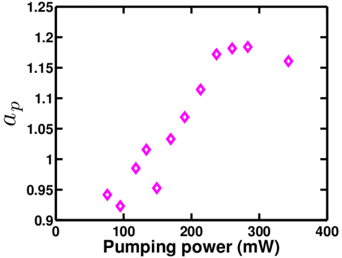

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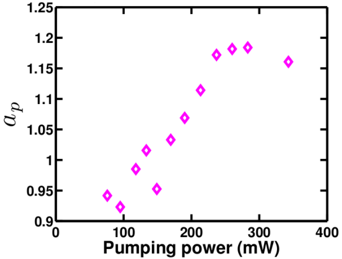

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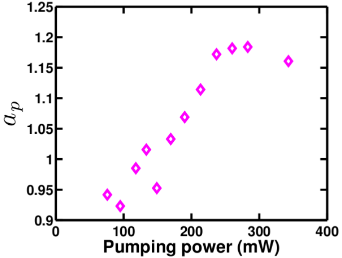

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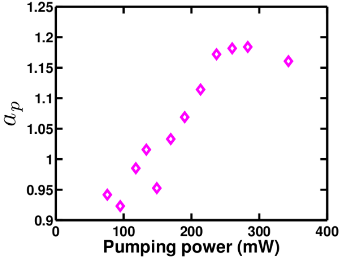


Questions

- What are polaritons?
- How can photon-like objects form a BEC?
- Is this “just a laser”?
- How to model a non-equilibrium condensate?
- Effects of non-equilibrium nature on:
  - Steady states
  - Coherence
  - Superfluidity
  - …

Extra slides

5 Microscopic model: lasing vs condensation
- Model: localised excitons, propagating photons
- Simple laser: Maxwell-Bloch
- Non-equilibrium polaritons: coherence and strong coupling

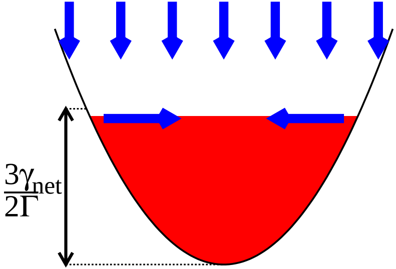
6 Measuring superfluid density

7 Coherence Finite size and Schawlow-Townes

Instability of Thomas-Fermi: details

$$\frac{1}{2}\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$

$$\partial_t \mathbf{v} + \nabla(U\rho + \frac{m\omega^2}{2} r^2 + \frac{m}{2}|\mathbf{v}|^2) = 0$$



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$$\delta\rho_{n,m}(r,\theta,t) = e^{im\theta}h_{n,m}(r)e^{i\omega_{n,m}t}$$

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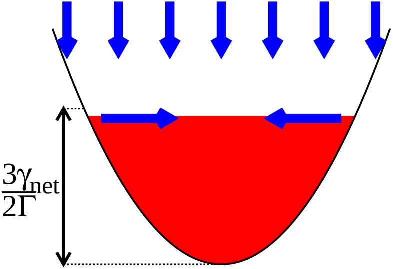

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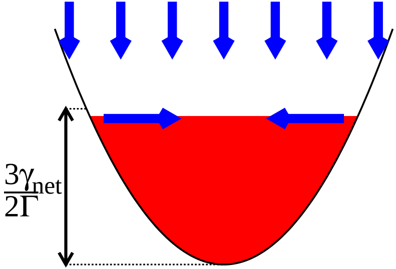

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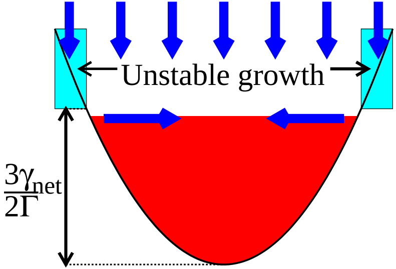

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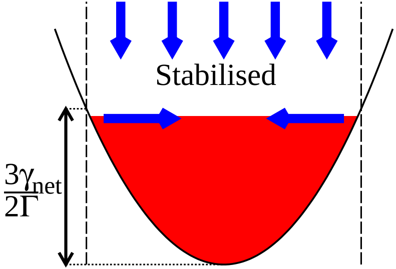

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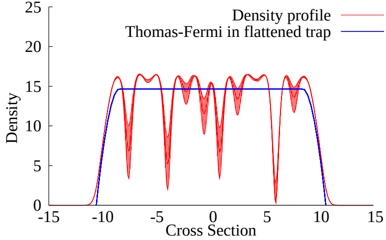
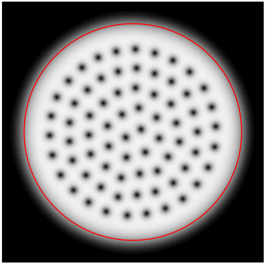
Why vortices





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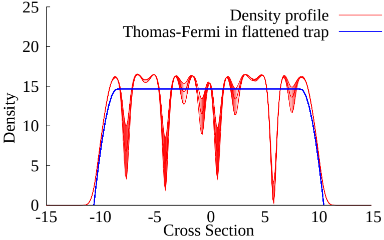

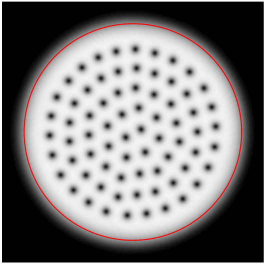

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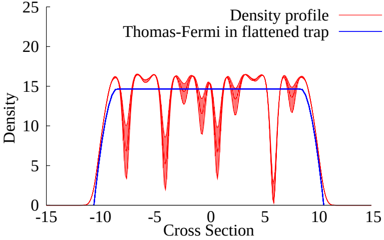

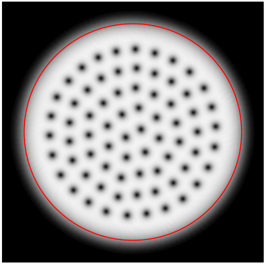

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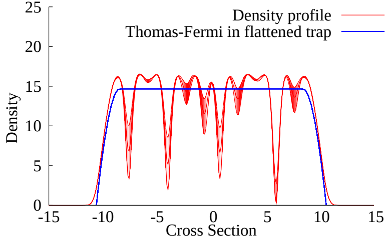

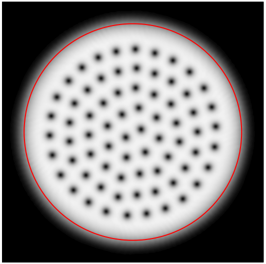

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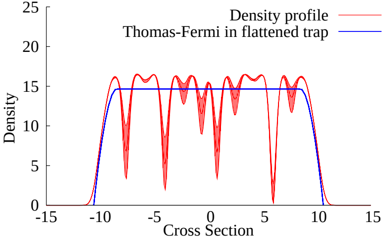

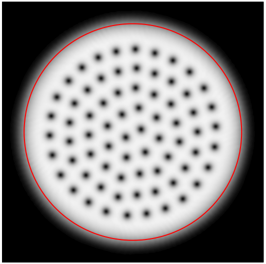

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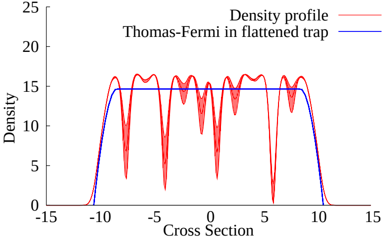

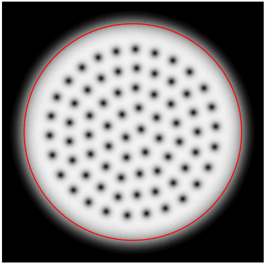

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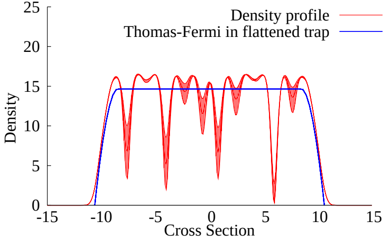

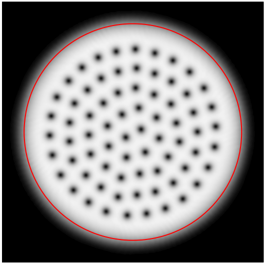

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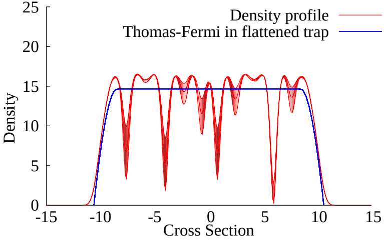

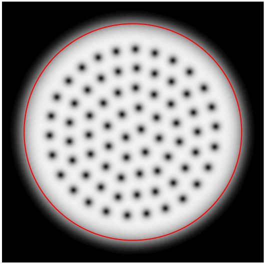

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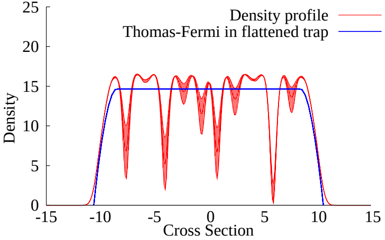

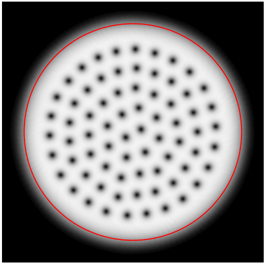

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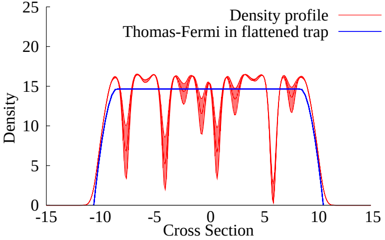

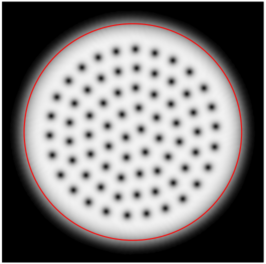

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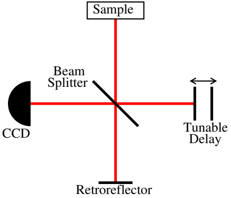
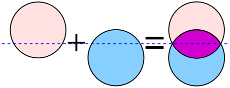
$$\mu = \frac{m}{2}|\mathbf{v} - \Omega \times \mathbf{r}|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{net}}{\Gamma}\Theta(r_0 - r) = \frac{\mu}{U}$$

Observing vortices: fringe pattern





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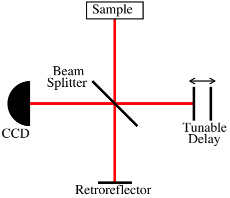
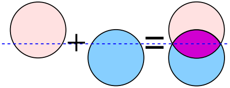
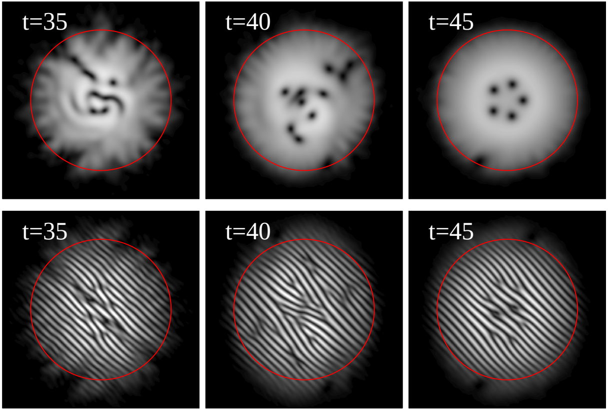
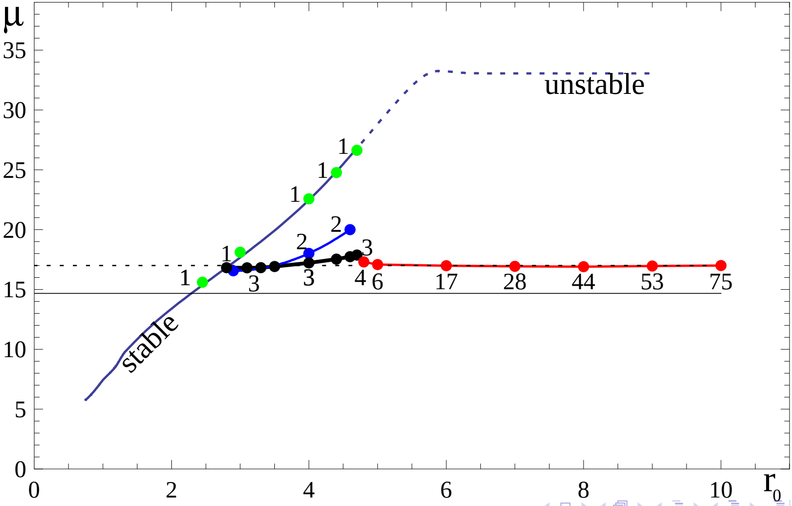






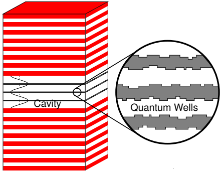
Why vortices: chemical potential vs size

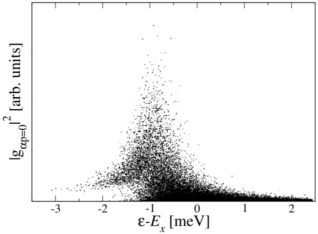
$$\text{Thomas-Fermi}: \mu = f(r_0) \quad \text{Vortex}: \mu = \frac{U\gamma_{net}}{\Gamma}$$



Polariton system model

- Disorder-localised excitons
- Treat disorder sites as 2-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling g.





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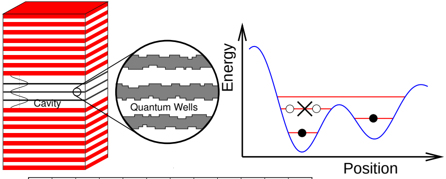

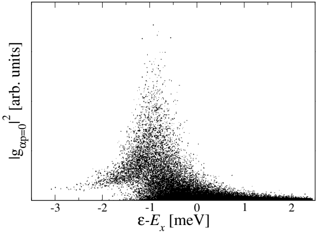

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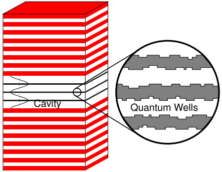

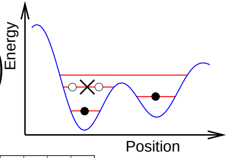

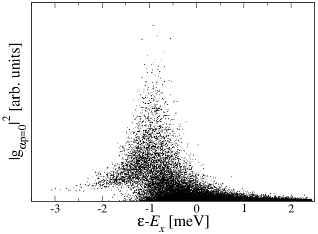

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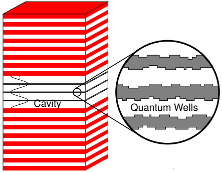
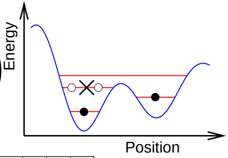
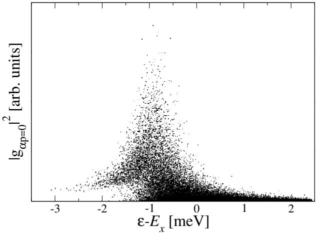
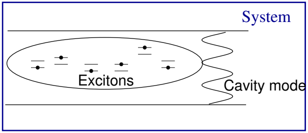






$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \epsilon_{\alpha} S^{z}_{\alpha} + \sum_{\alpha} \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} S^{+}_{\alpha} \quad + \text{H.c.}$$



Equilibrium: Mean-field theory

Self-consistent polarisation and field

$$(-i\partial_t - \omega_0)\,\psi = -\sum_{\alpha} \frac{g_\alpha}{\sqrt{A}} S^-_\alpha$$

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$$(\quad \mu - \omega_0)\,\psi = -\sum_\alpha \frac{g_\alpha}{\sqrt{A}} \frac{g_\alpha \psi}{2E_\alpha} \tanh(\beta E_\alpha), \qquad {E_\alpha}^2 = \left(\frac{\epsilon_\alpha - \mu}{2}\right)^2 + g_\alpha^2 |\psi|^2$$

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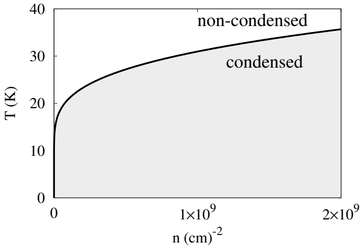
Phase diagram:



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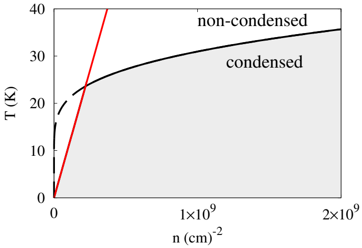

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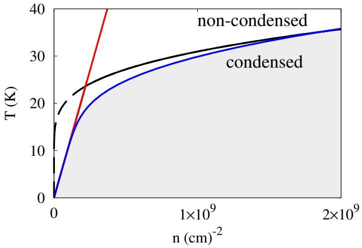

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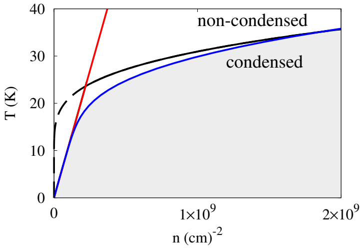
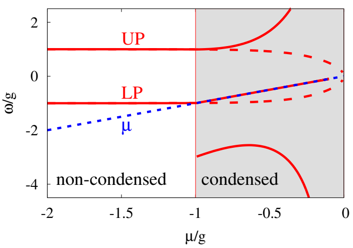
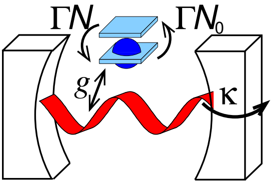


Modes (at $k = 0$)



Simple Laser: Maxwell Bloch equations

$$H = \omega_0 \psi^\dagger \psi + \sum_\alpha \epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi S^+_\alpha + \text{H.c.}$$



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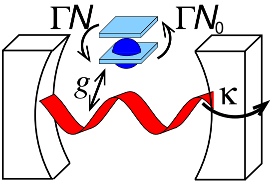

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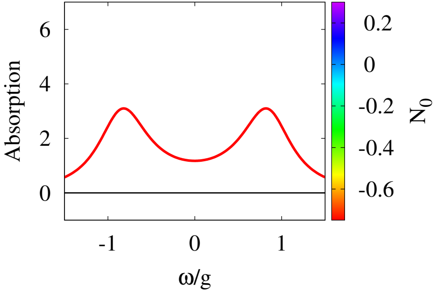

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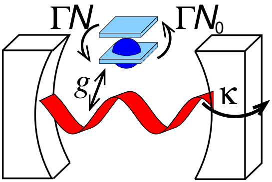

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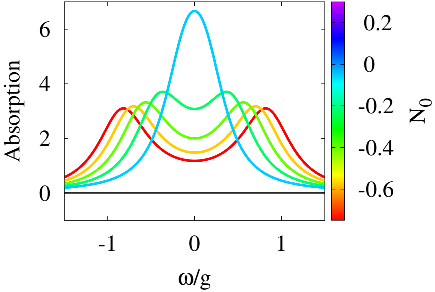

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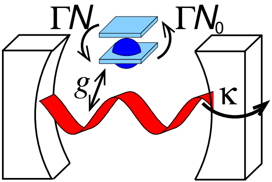

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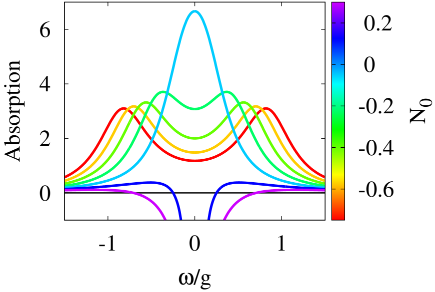

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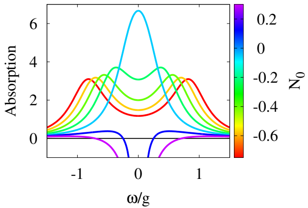
Maxwell-Bloch Equations: Retarded Green's function



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  Response to perturbation

- Absorption $= -2\Im[D^R(\omega)]$

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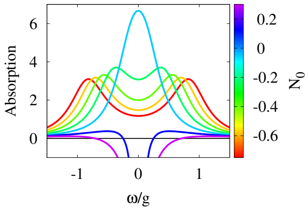

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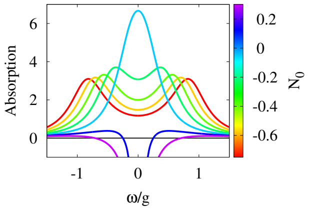

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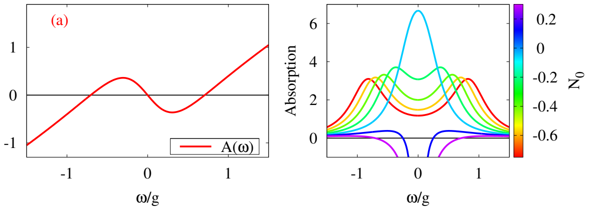

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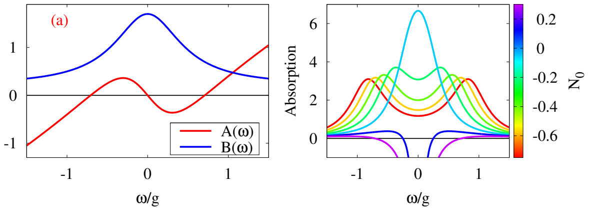

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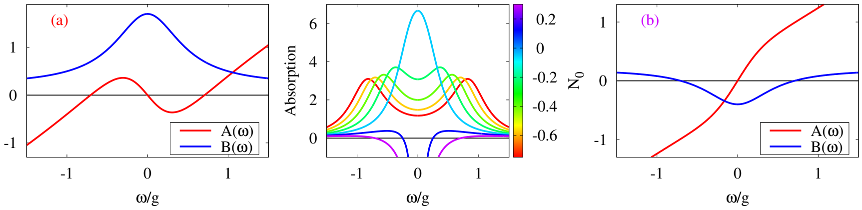

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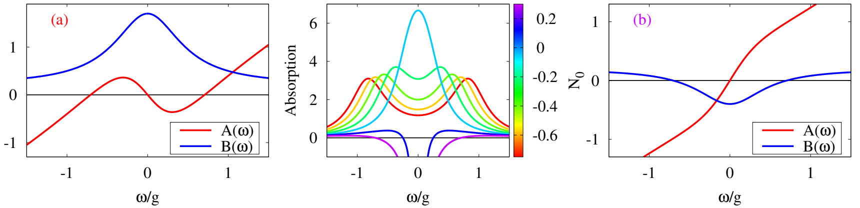
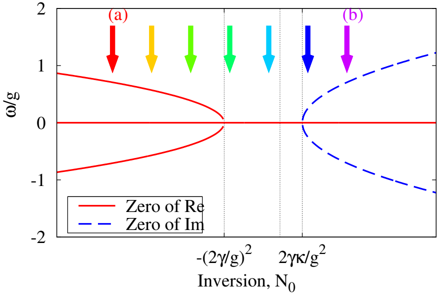
Evolution of poles with Inversion





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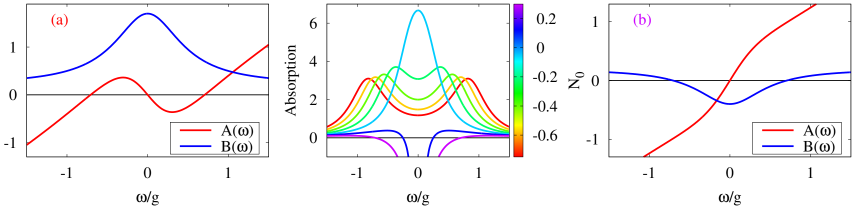
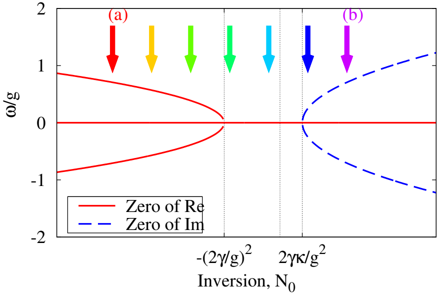
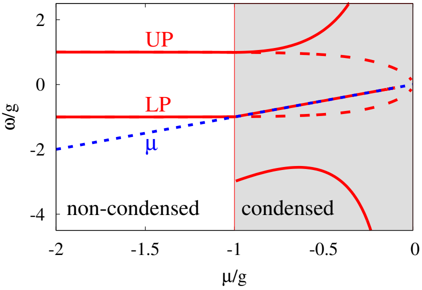


Laser:



Equilibrium:



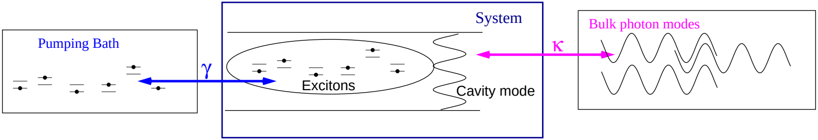
5 Microscopic model: lasing vs condensation

- Model: localised excitons, propagating photons
- Simple laser: Maxwell-Bloch
- Non-equilibrium polaritons: coherence and strong coupling

6 Measuring superfluid density

7 Coherence Finite size and Schawlow-Townes

Non-equilibrium description: baths



$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

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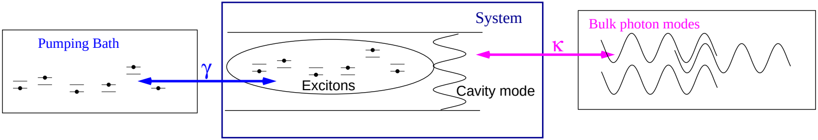

$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

Schematically: pump γ, decay κ

$$H_{\text{sys,bath}} \simeq \sum_{\mathbf{p},\mathbf{k}} \sqrt{\kappa}\psi_{\mathbf{k}}\Psi^{\dagger}_{\mathbf{p}} + \sum_{\alpha,\beta} \sqrt{\gamma}\left(a^{\dagger}_{\alpha}A_{\beta} + b^{\dagger}_{\alpha}B_{\beta}\right) + \text{H.c.}$$

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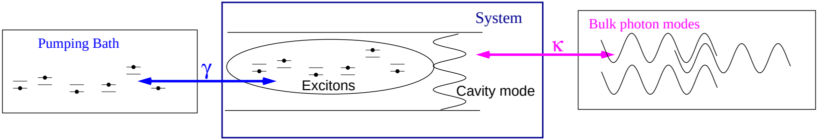

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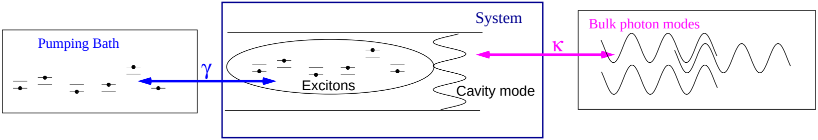
$$H_{\text{sys,bath}} \simeq \sum_{\mathbf{p},\mathbf{k}} \sqrt{\kappa}\psi_{\mathbf{k}}\Psi^{\dagger}_{\mathbf{p}} + \sum_{\alpha,\beta} \sqrt{\gamma}\left(a^{\dagger}_{\alpha}A_{\beta} + b^{\dagger}_{\alpha}B_{\beta}\right) + \text{H.c.}$$

Bath correlations, $\langle\Psi^{\dagger}\Psi\rangle$, $\langle A^{\dagger}A\rangle$, $\langle B^{\dagger}B\rangle$ fixed:

Non-equilibrium description: baths



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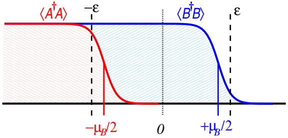
Bath correlations, $\langle \Psi^\dagger \Psi \rangle$, $\langle A^\dagger A \rangle$, $\langle B^\dagger B \rangle$ fixed:
Ψ bath is empty. Pumping bath thermal, μ_B, T_B:



Non-equilibrium mean-field theory

Look for mean-field solution, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$. Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi_0 = \chi(\psi_0, \mu_s)\psi_0$$

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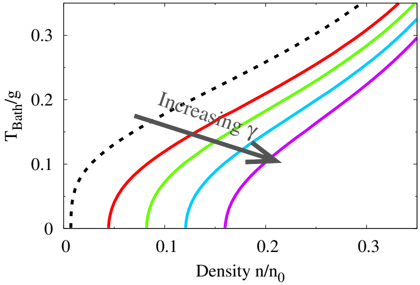

Luminescence spectrum and Green's functions

$$-2\Im[D^R(\omega)] = \mathrm{DoS}(\omega)$$

$$\left[D^R(\omega)\right]^{-1} = A(\omega) + iB(\omega)$$

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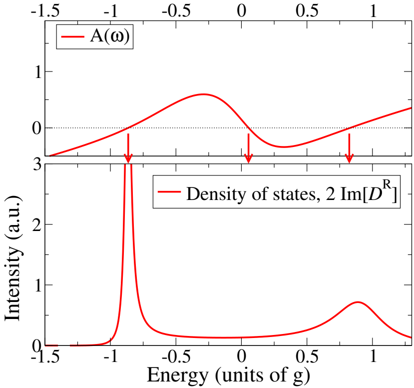

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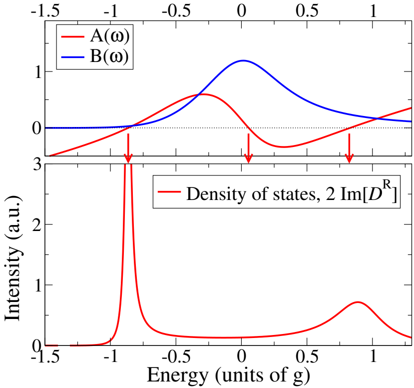

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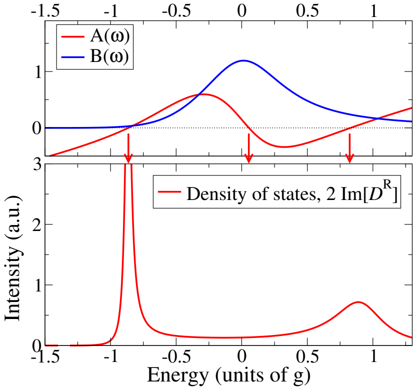

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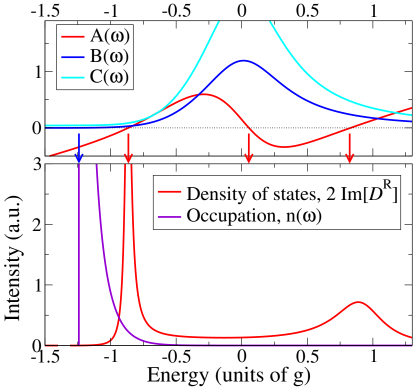
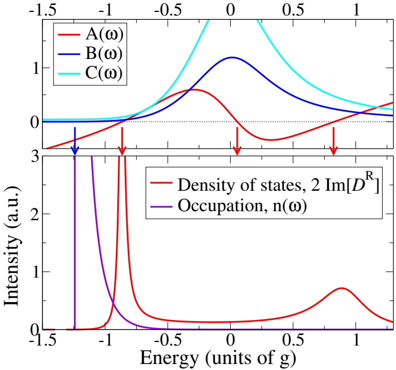


Stability and evolution with pumping



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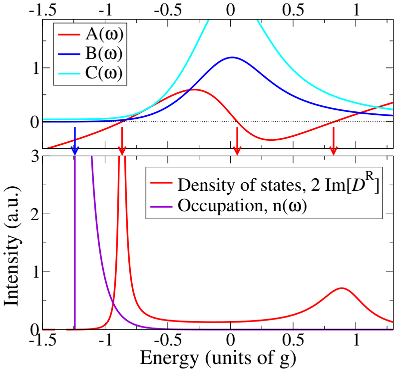

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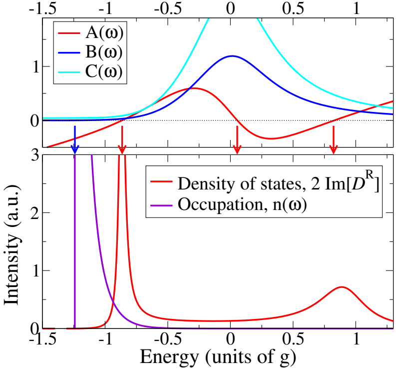

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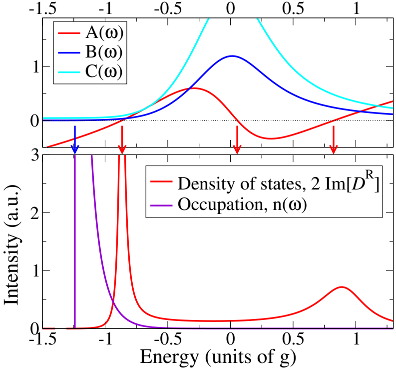

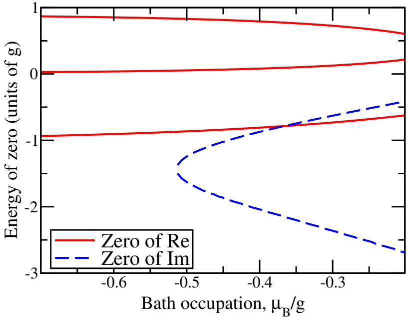

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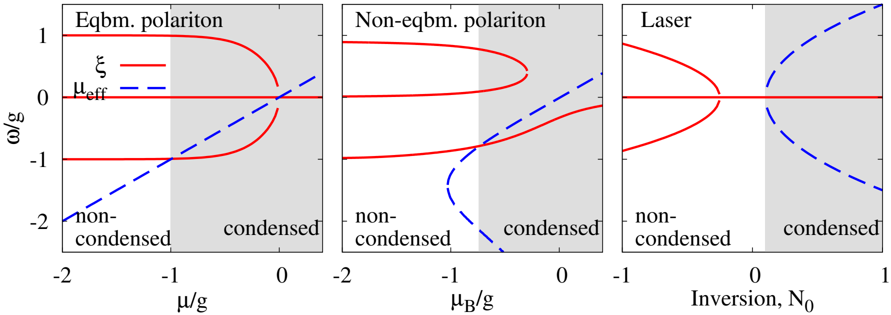
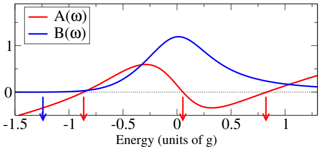
Strong coupling and lasing — low temperature phenomenon



- Laser: Uniformly invert TLS
- Non-equilibrium polaritons: Cold bath
- If $T_B \gg \gamma \rightarrow$ Laser limit



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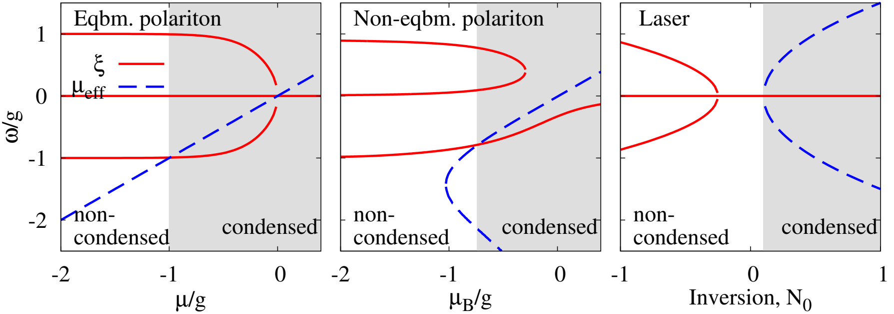

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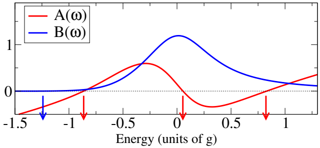

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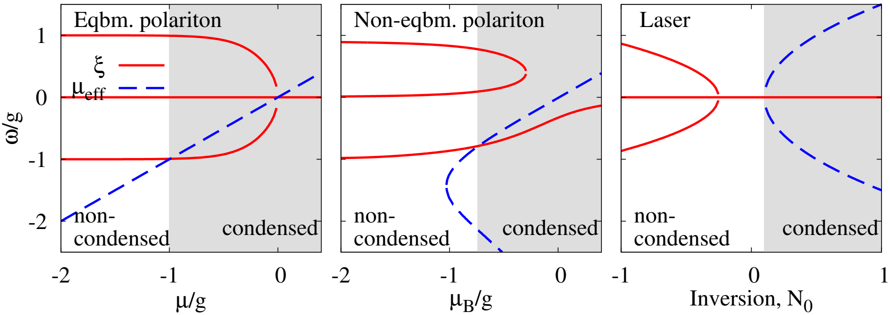

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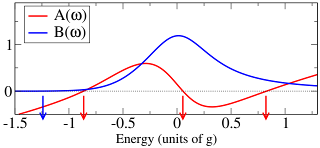
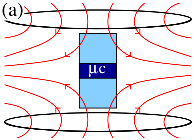


Measuring superfluid density

1. Effect rotating frame

Polariton polarization: $(\psi_{\circlearrowleft}, \psi_{\circlearrowright})$

$$H = \lambda \begin{pmatrix} \ell^2 & r^2 e^{2i\phi} \\ r^2 e^{-2i\phi} & -\ell^2 \end{pmatrix}$$



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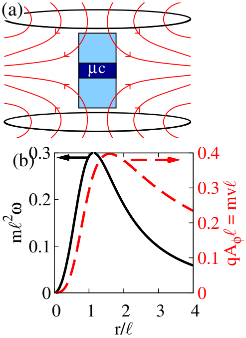
Ground state Berry phase:

$$q\mathbf{A}_{\text{eff}} = m\omega \times \mathbf{r} = \frac{\hat{\phi}}{r}\left[1 - \frac{\ell^2}{\sqrt{r^4 + \ell^4}}\right]$$



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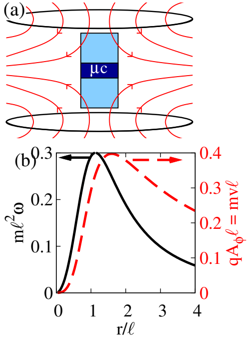
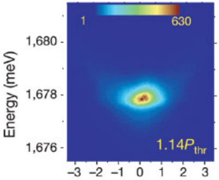


2. Measure resulting current

Energy shift of normal state:

$\Delta E = (1/2)mv^2 = 0.08/m\ell^2 \simeq 0.1$meV



5 Microscopic model: lasing vs condensation

- Model: localised excitons, propagating photons
- Simple laser: Maxwell-Bloch
- Non-equilibrium polaritons: coherence and strong coupling

6 Measuring superfluid density

7 Coherence Finite size and Schawlow-Townes

Finite size effects: Single mode vs many mode

$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0) \right\rangle \simeq |\psi_0|^2 \exp\left[-D^{<}_{\phi\phi}(\mathbf{r}, \mathbf{r}', t) \right]$$

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$D^<_{\phi\phi}(\mathbf{r},\mathbf{r}',t)$ from sum of phase modes. Study $ct \gg r$ limit:

$$D^<_{\phi\phi}(\mathbf{r},\mathbf{r},t) \propto \sum_n^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(\mathbf{r})|^2(1-e^{i\omega t})}{\left|(\omega + i\gamma_{\text{net}})^2 + \gamma_{\text{net}}^2 - \xi_n^2\right|^2}$$

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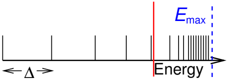
$$\Delta\xi \ll \sqrt{\frac{\gamma_{\text{net}}}{t}} \ll E_{\text{max}}$$



$$D^<_{\phi\phi} \sim 1 + \ln\left(E_{\text{max}}\sqrt{\frac{t}{\gamma_{\text{net}}}}\right)$$

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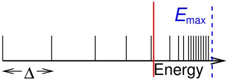

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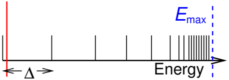

$$D^<_{\phi\phi} \sim \left(\frac{\pi C}{2\gamma_{\text{net}}}\right)\left(\frac{t}{2\gamma_{\text{net}}}\right)$$

(Recovers Schawlow-Townes laser linewidth)