Collective Dynamics of a Generalized Dicke Model

J. Keeling, J. A. Mayoh, M. J. Bhaseen, B. D. Simons

Harvard, January 2012

Funding: EPSRC
Engineering and Physical Sciences
Research Council

Coupling many atoms to light

Old question: *What happens to radiation when many atoms interact "collectively" with light.*

Superradiance — dynamical and steady state.

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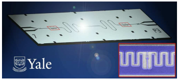
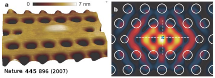
New relevance

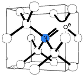
- Superconducting qubits



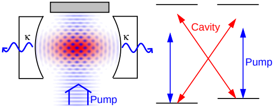
- Quantum dots



- Nitrogen-vacancies in diamond



- Ultra-cold atoms



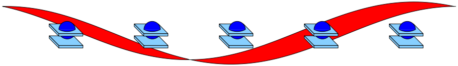
- Rydberg atoms

Dicke effect: Enhanced emission

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

R. H. Dicke
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey



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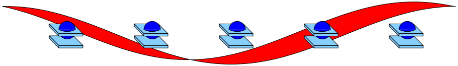

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Collective decay:

$$\frac{d\rho}{dt} = -\frac{\Gamma}{2}\left[S^+S^-\rho - S^-\rho S^+ + \rho S^+ S^-\right]$$

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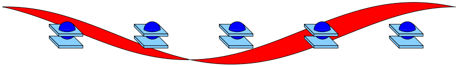

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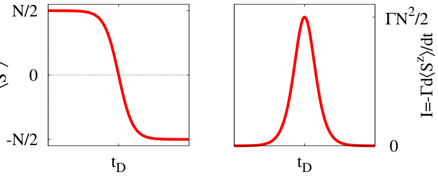
If $S^z = |S| = N/2$ initially:

$$I \propto -\Gamma \frac{d\langle S^z \rangle}{dt} = \frac{\Gamma N^2}{4} \text{sech}^2 \left[\frac{\Gamma N}{2} t \right]$$



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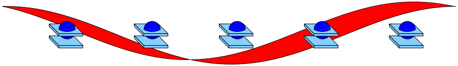

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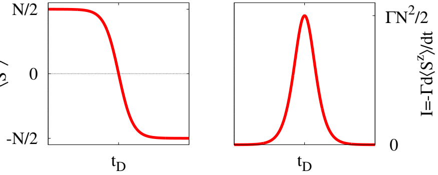
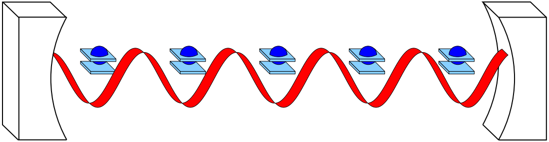


Problem: dipole interactions dephase. [Friedberg et al, Phys. Lett. 1972]

Collective radiation **with a cavity**: Dynamics



$$H_{\text{int}} = \sum_i \left(\psi^\dagger S_i^- + \psi S_i^+ \right)$$

Single cavity mode: oscillations

[Bonifacio and Preparata PRA '70; **JK** PRA '09]

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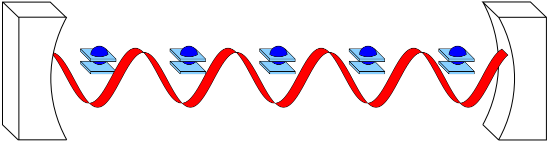

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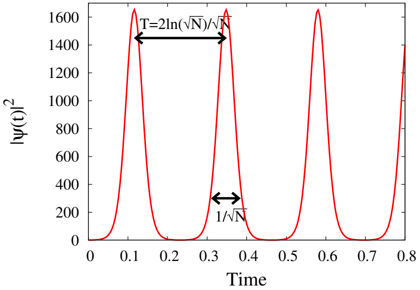

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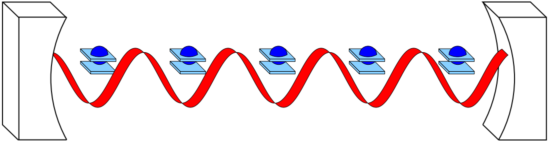

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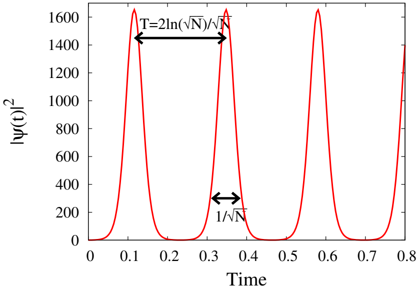

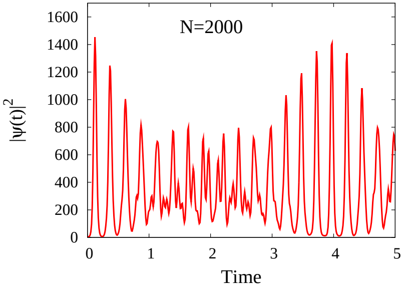

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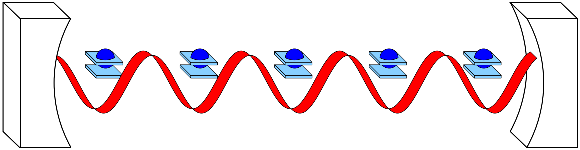
Dicke model: Equilibrium superradiance transition



$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g\left(\psi^\dagger S^- + \psi S^+\right).$$

- Coherent state: $|\Psi\rangle \rightarrow e^{\lambda\psi^\dagger + \eta S^+}|\Omega\rangle$
- Energy:

$$E = \omega|\lambda|^2 + \frac{\omega_0 N}{2}\frac{|\eta|^2 - 1}{|\eta|^2 + 1} + gN\frac{\eta^*\lambda + \lambda^*\eta}{1 + |\eta|^2}$$

- Small g, min at $\lambda, \eta = 0$

[Hepp, Lieb, Ann. Phys. '73]

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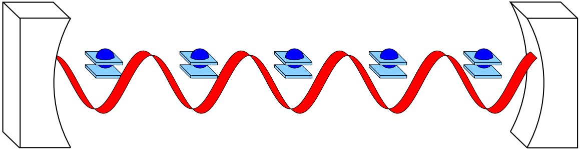

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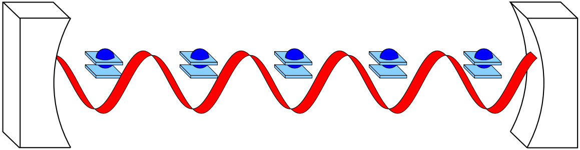

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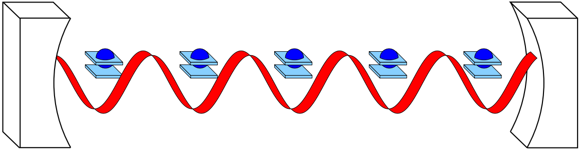

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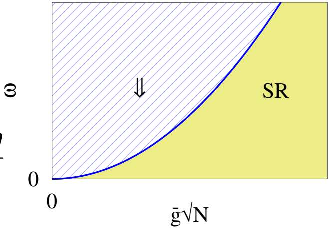
Spontaneous polarisation if: $Ng^2 > \omega\omega_0$



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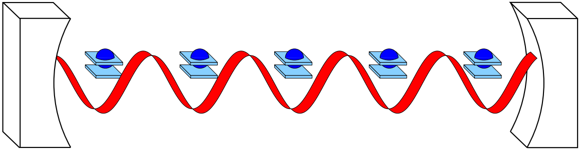
No go theorem and transition



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[Rzazewski *et al* PRL '75]

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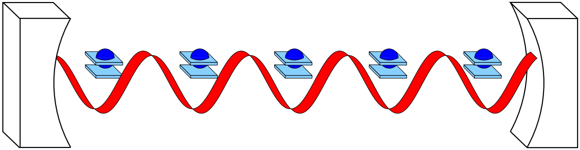

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$$-\sum_i \frac{e}{m} A \cdot p_i \Leftrightarrow g(\psi^\dagger S^- + \psi S^+), \qquad \sum_i \frac{A^2}{2m} \Leftrightarrow N\zeta(\psi + \psi^\dagger)^2$$

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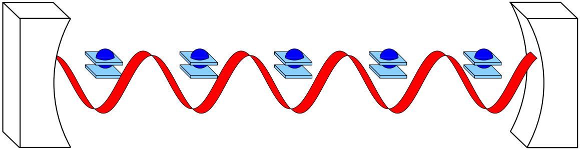

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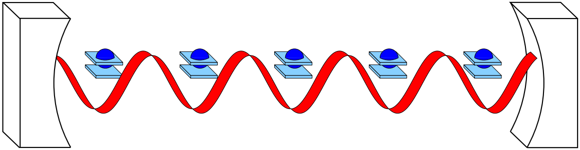

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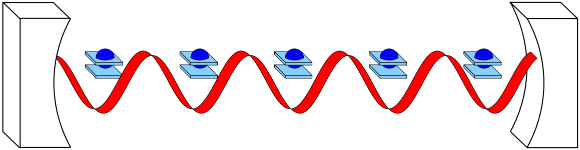

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But Thomas-Reiche-Kuhn sum rule states: $g^2/\omega_0 < 2\zeta$. **No transition**

[Rzazewski *et al* PRL '75]

Dicke phase transition: ways out

Problem: $g^2/\omega_0 < 2\zeta$ for intrinsic parameters. **Solutions**:

- **Non-solution**
  Ferroelectric transition in $\mathbf{D} \cdot \mathbf{r}$ gauge.
  [JK JPCM '07]
- See also [Nataf and Ciuti, Nat. Comm. '10; Viehmann *et al.* PRL '11]

- Grand canonical ensemble:
  - If $H \to H - \mu(S^z + \psi^\dagger\psi)$, need only: $g^2 N > (\omega - \mu)(\omega_0 - \mu)$
  - Incoherent pumping — polariton condensation.
- Dissociate g, ω_0.
  e.g. Raman scheme: $\omega_0 \ll \omega$.
  [Dimer *et al.* PRA '07; Baumann *et al.* Nature '10. Also, Black *et al.* PRL '03]

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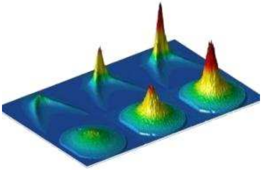

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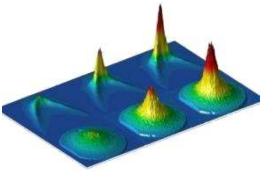

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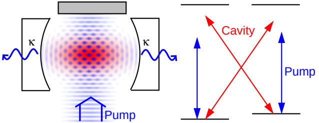


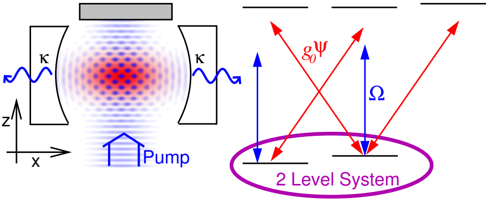
Outline

1. Introduction: Dicke model and superradiance
   - Rayleigh scheme: Generalised Dicke model
2. Attractors of dynamics (fixed points)
   - Phase diagram: comparison to experiment
3. Approach to attractors: timescales
   - Why slow timescales emerge
4. Attractors of dynamics (oscillations)
   - Reaching other parameter ranges

Outline

Extended Dicke model

[Baumann *et al.* Nature ’10]



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$\Uparrow$: $\Psi(x,z) = \sum_{\sigma,\sigma'=\pm} e^{ik(\sigma x + \sigma' z)}$

$\omega_0 = 2\omega_{\text{recoil}}$

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$$\partial_t\rho = -i[H,\rho] - \kappa(\psi^\dagger\psi\rho - 2\psi\rho\psi^\dagger + \rho\psi^\dagger\psi)$$

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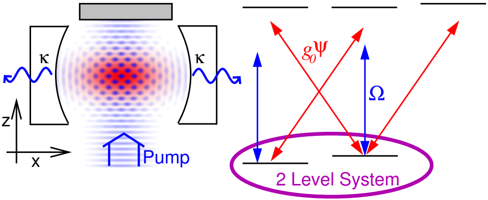

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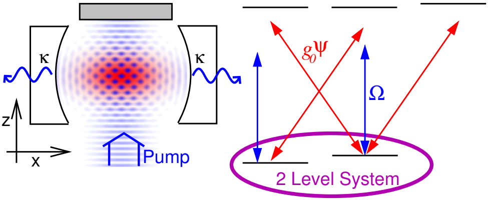

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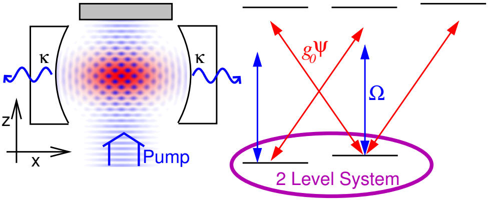
[Baumann *et al.* Nature '10]



2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$:

$\Downarrow$: $\Psi(x,z) = 1$

$\Uparrow$: $\Psi(x,z) = \sum_{\sigma,\sigma'=\pm} e^{ik(\sigma x + \sigma' z)}$

$\omega_0 = 2\omega_{\text{recoil}}$

Feedback: $U \propto \frac{g_0^2}{\omega_c - \omega_a}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+) + U S_z \psi^\dagger \psi.$$

$$\partial_t \rho = -i[H,\rho] - \kappa(\psi^\dagger\psi\rho - 2\psi\rho\psi^\dagger + \rho\psi^\dagger\psi)$$

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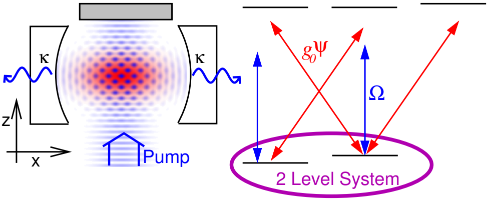

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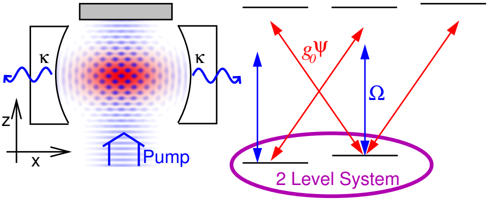

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$\omega_0 \sim \text{kHz} \ll \omega, \kappa, g\sqrt{N} \sim \text{MHz}.$

Outline

Fixed points (steady states)

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
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- $\psi = 0, \mathbf{S} = (0, 0, \pm N/2)$ always a solution.
- If $g > g_c$, $\psi \neq 0$ too
  - A $S^y = -\Im[S^-] = 0$
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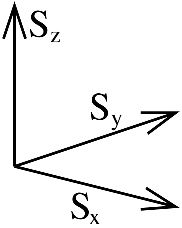

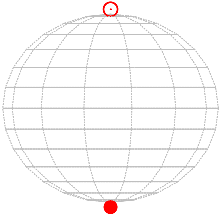

Small g: ⇑, ⇓ only.
($\omega = 30$MHz, $UN = -40$MHz)

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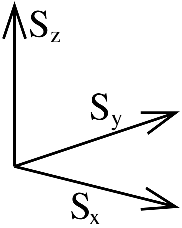
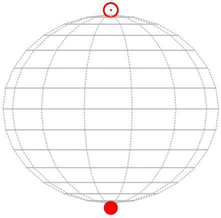
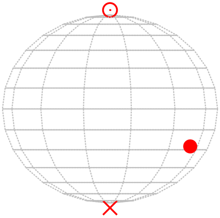




Small g: $\Uparrow, \Downarrow$ only.



Larger g: SR too.

($\omega = 30$MHz, $UN = -40$MHz)

Steady state phase diagram

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
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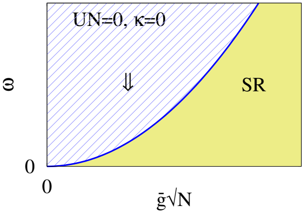

See also Domokos and Ritsch PRL ’02, Domokos *et al.* PRL ’10

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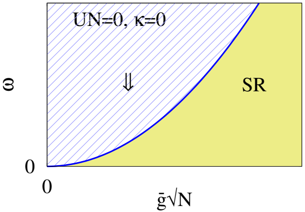

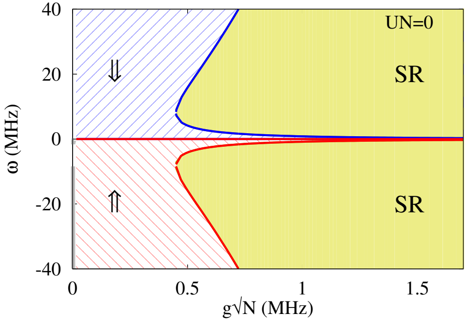

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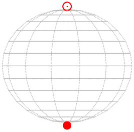
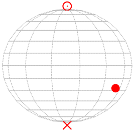


SR(A): $S_y = 0$



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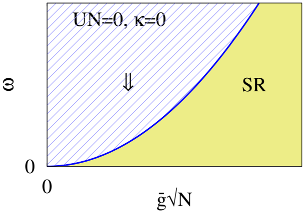

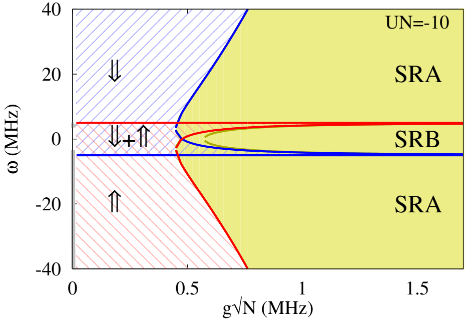

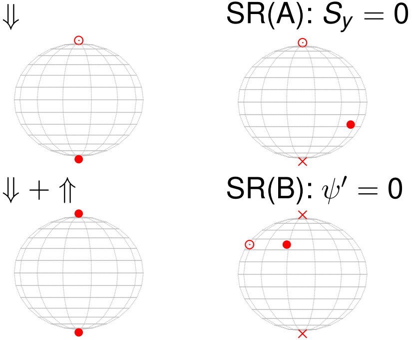

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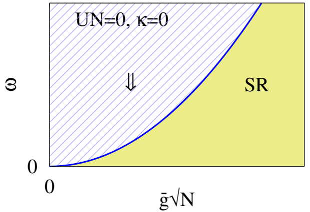

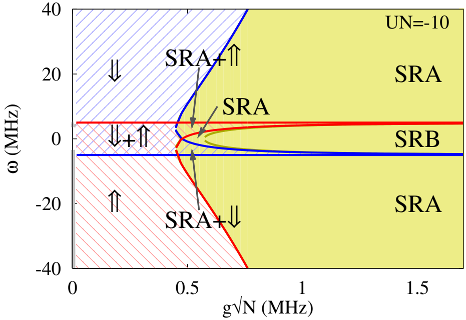

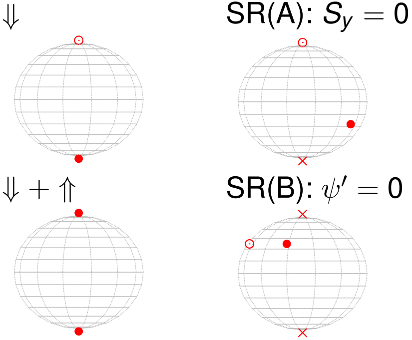

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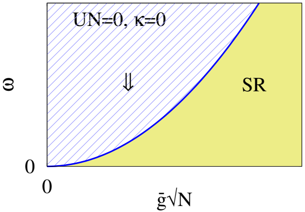

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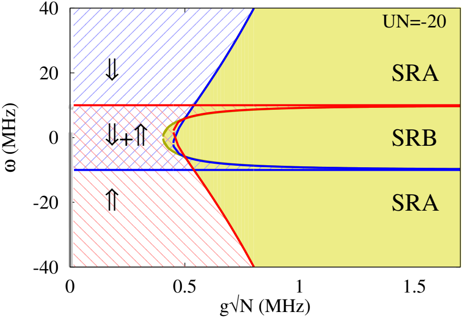

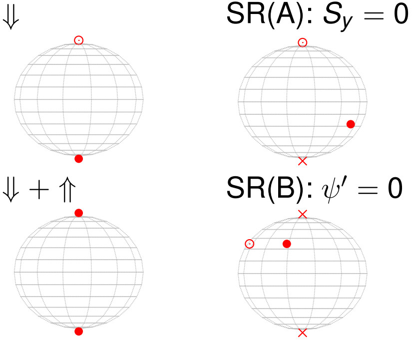

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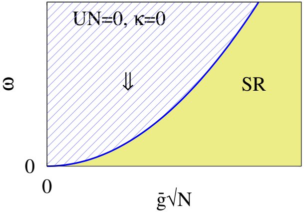

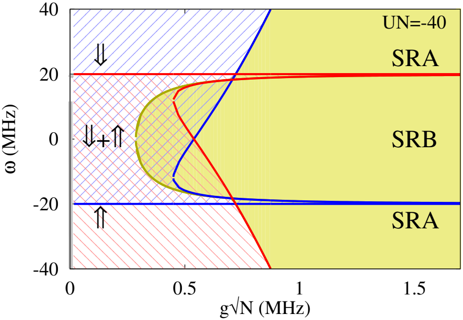

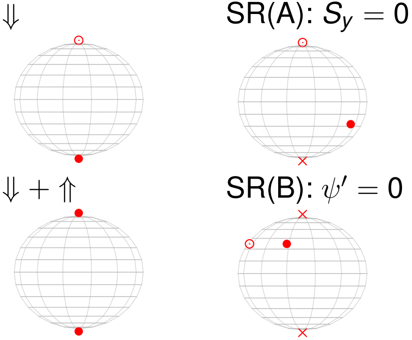

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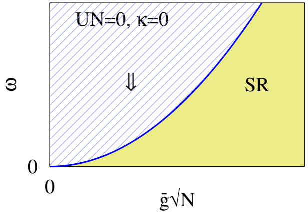

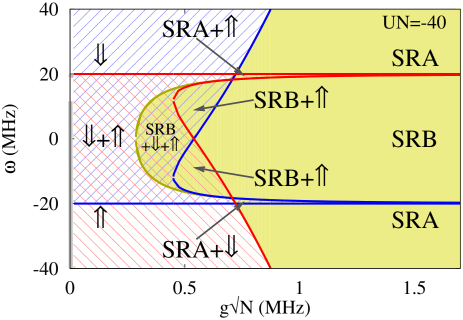

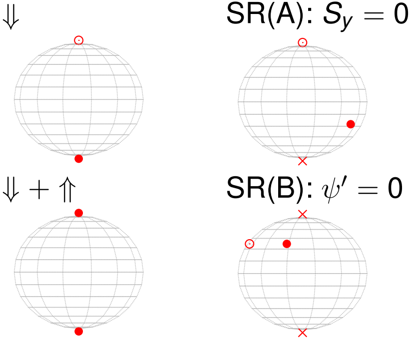

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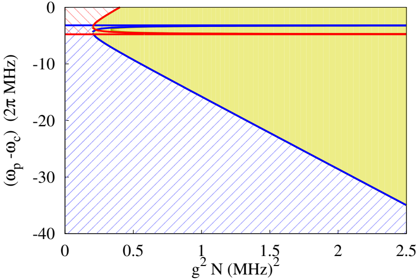
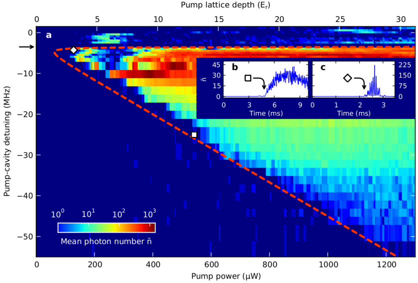
Comparison to experiment





$$UN = -10\text{MHz}$$

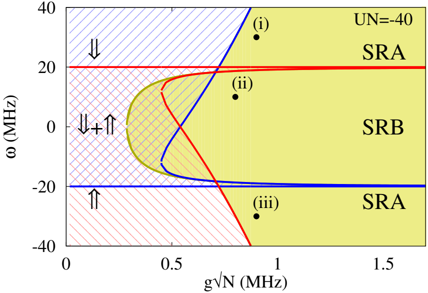
Adapted from: [Bhaseen *et al.* PRA '12]

[Baumann *et al* Nature '10]

$$\omega = \omega_c - \omega_p + \frac{5}{2} UN, \qquad UN = -\frac{g_0^2}{4(\omega_a - \omega_c)}$$

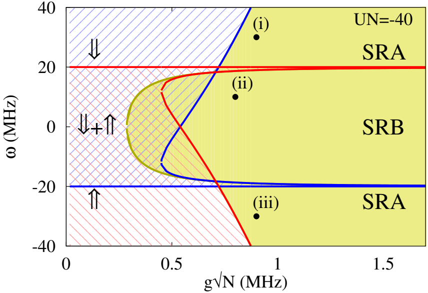
Outline

Dynamics: Evolution from normal state

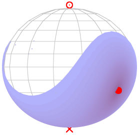


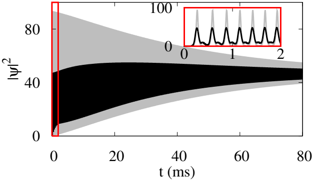
Dynamics: Evolution from normal state

Gray: $\mathbf{S} = (\sqrt{N}, \sqrt{N}, -N/2)$
Black: Wigner distribution of $\mathbf{S}$, ψ



(i) SR(A)





Oscillations: ~ 0.1ms
Decay: 20ms

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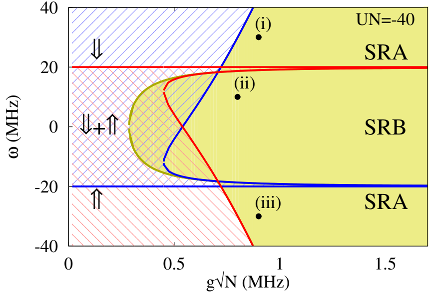

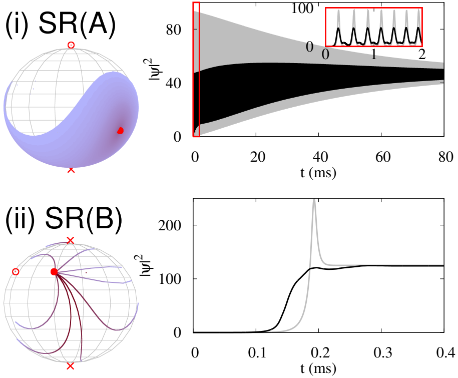

Oscillations: $\sim$ 0.1ms
Decay: 20ms, 0.1ms

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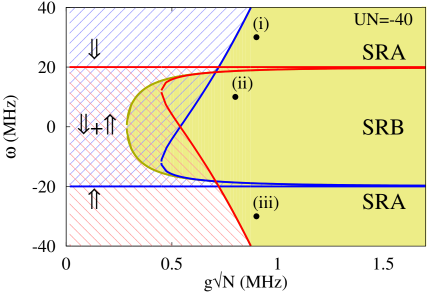
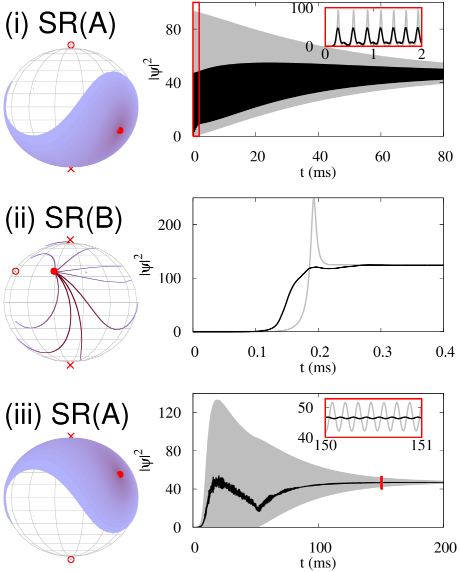


Oscillations: $\sim$ 0.1ms
Decay: 20ms, 0.1ms, 20ms



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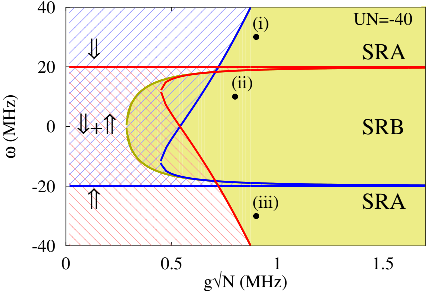
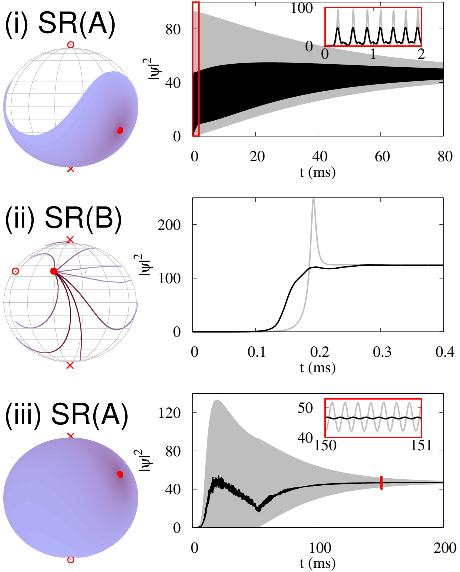


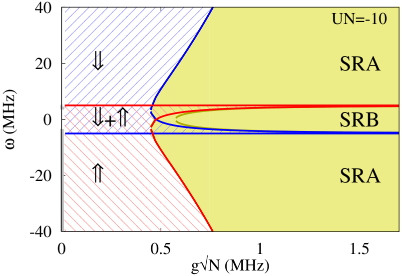
Oscillations: $\sim$ 0.1ms
Decay: 20ms, 0.1ms, 20ms



Asymptotic state: Evolution from normal state

(Near to experimental $UN = -13$MHz).

All stable attractors:



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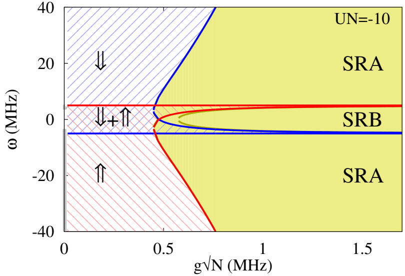
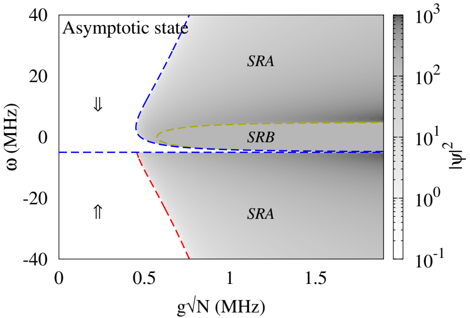
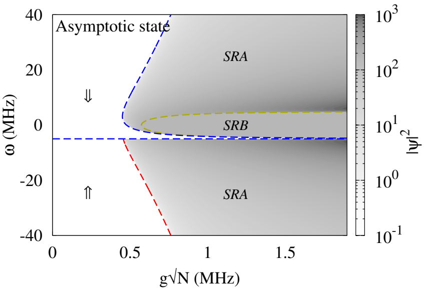
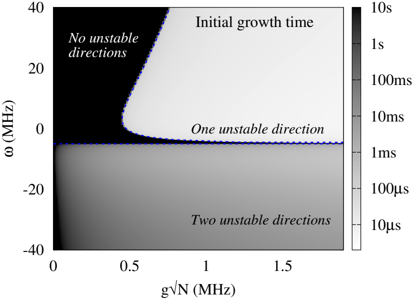


Starting from ⇓



Timescales for dynamics: What are they?





Growth Most unstable eigenvalues near $\mathbf{S} = (0, 0, -N/2)$

Decay Slowest stable eigenvalues near final state

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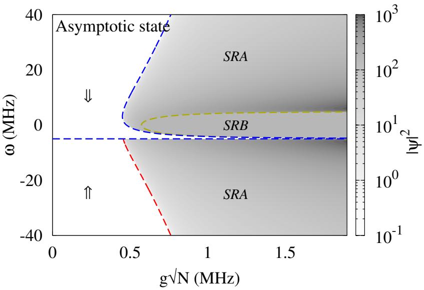

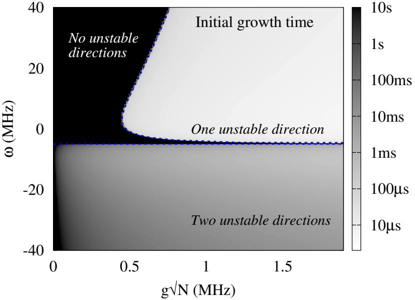

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Expand in ω_0/κ:
Oscillations: $\sim \omega_0$,
Decay: $\sim \omega_0$ **or** ω_0^2/κ

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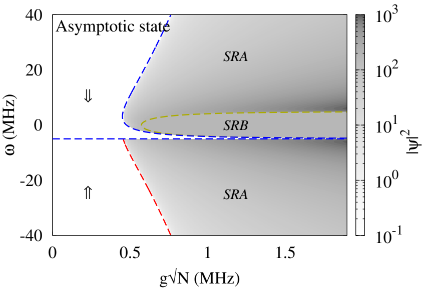
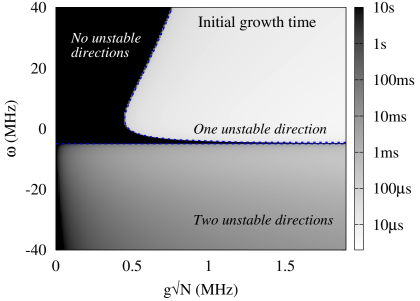
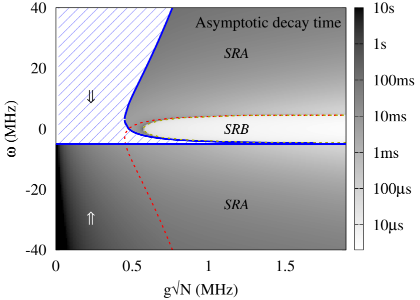




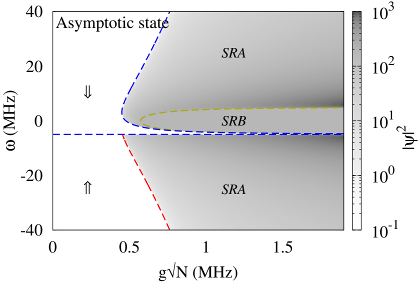


Growth Most unstable eigenvalues near $\mathbf{S} = (0, 0, -N/2)$

Decay Slowest stable eigenvalues near final state

Expand in ω_0/κ:
Oscillations: $\sim \omega_0$,
Decay: $\sim \omega_0$ **or** ω_0^2/κ

Timescales for dynamics: Consequences for experiment



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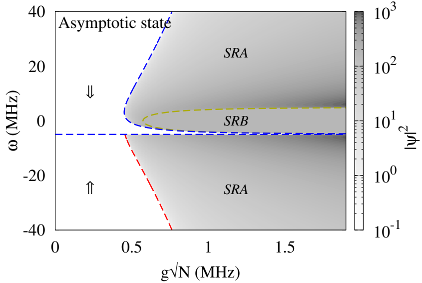

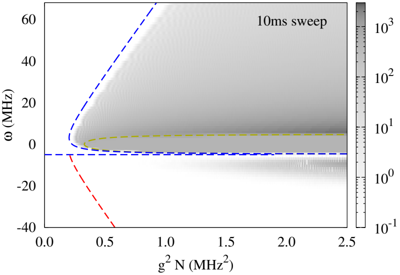

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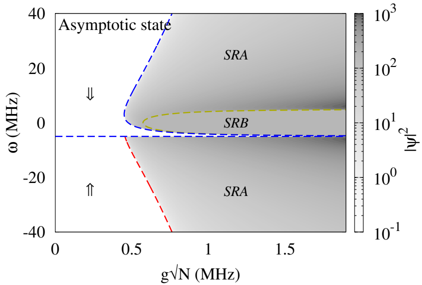
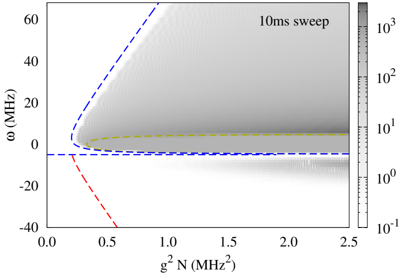
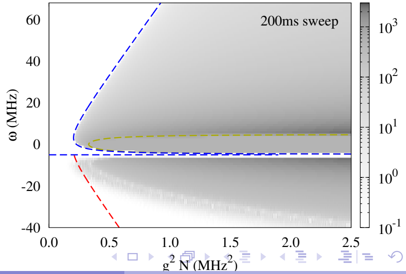
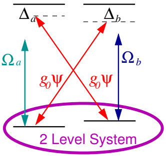






Timescales for dynamics: Why so slow and varied?

Suppose co- and counter-rotating terms differ



$$H = \ldots + g(\psi^\dagger S^- + \psi S^+) + g'(\psi^\dagger S^+ + \psi S^-) + \ldots$$

- SR(A) near phase boundary at small δg → Critical slowing down
- SR(A), SR(B) continuously connect

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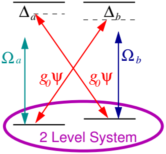

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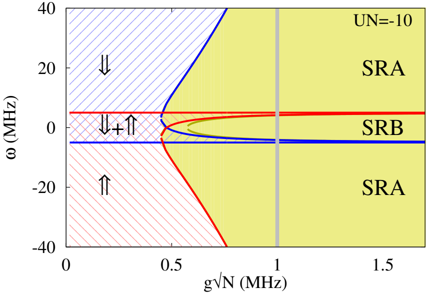

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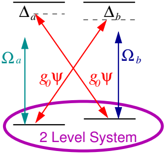

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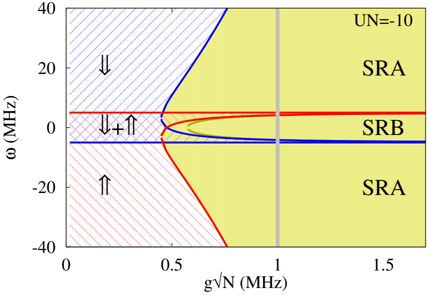
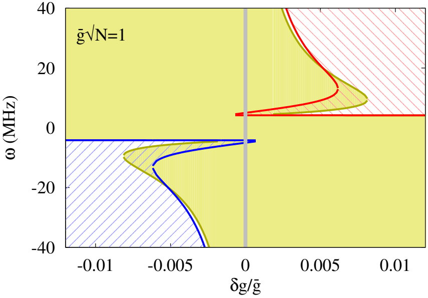
$$\delta g = g' - g, \quad 2\bar{g} = g' + g$$





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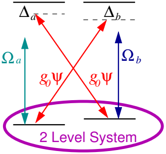

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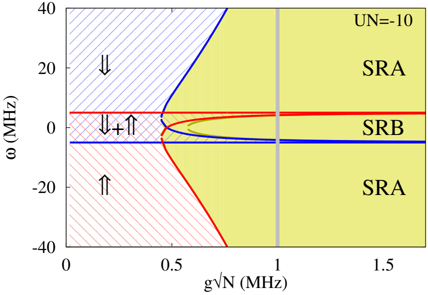

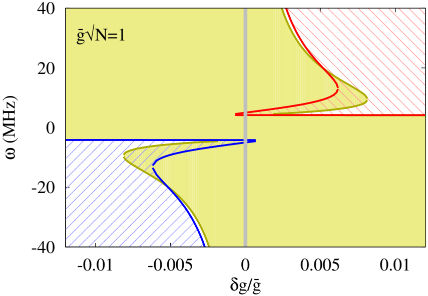

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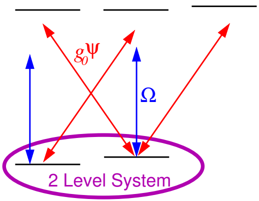
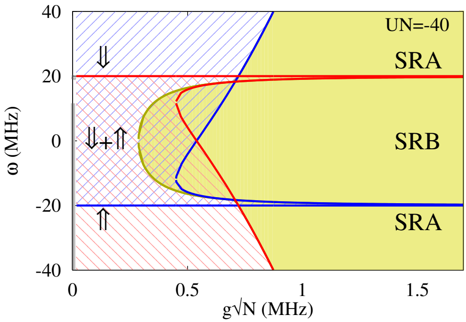
Outline

Regions without fixed points

Changing U:



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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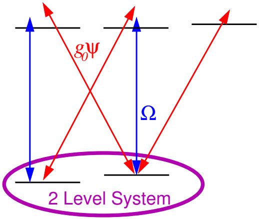

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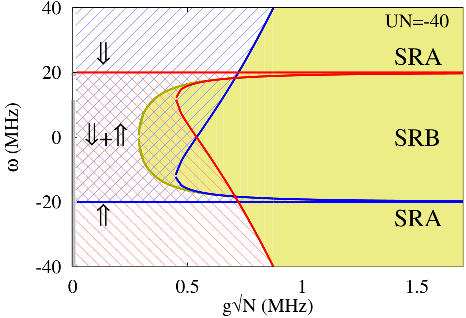

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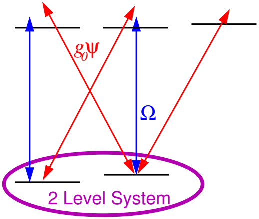

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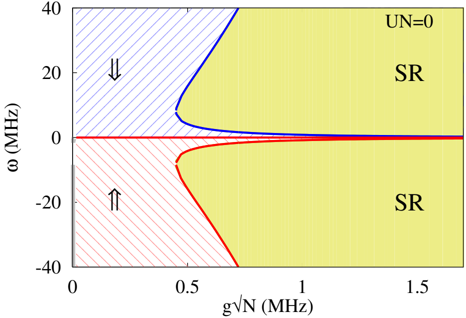

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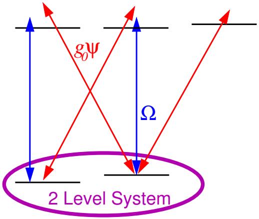

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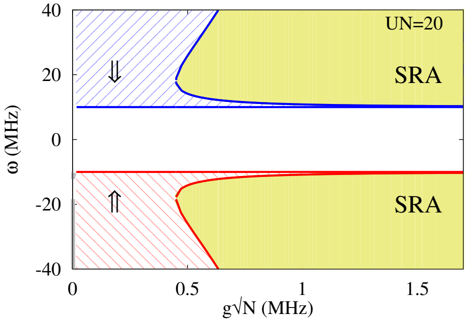

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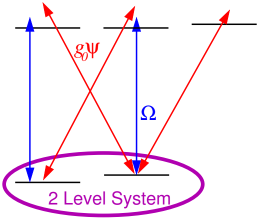

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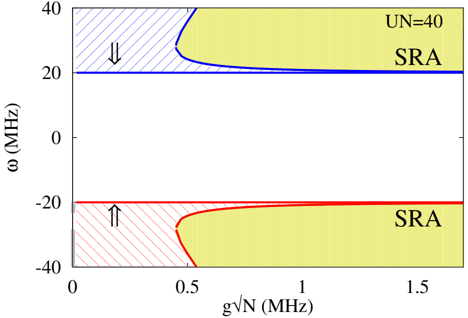

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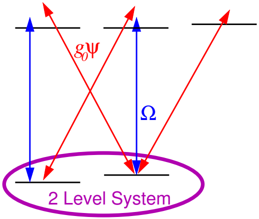

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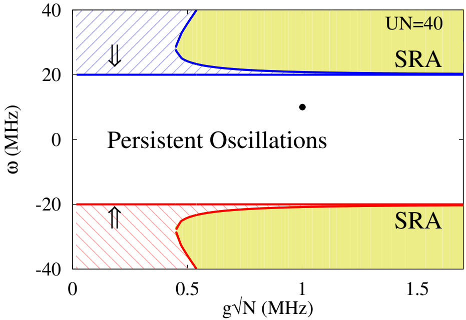
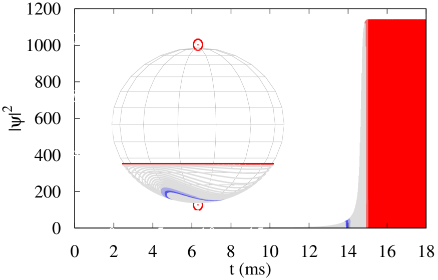
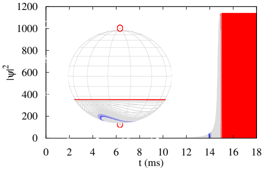




Persistent (optomechanical) oscillations



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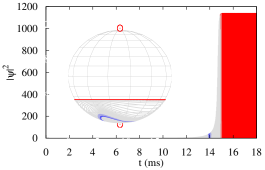

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Fix $\omega + US^z = 0$ **if** $\psi' = 0$.

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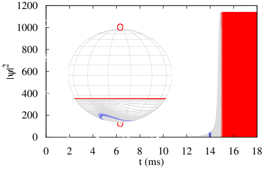

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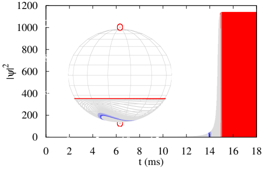

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Get:

Fix $\omega + US^z = 0$ **if** $\psi' = 0$.

$$S^- = re^{-i\theta} \quad r = \sqrt{\frac{N^2}{4} - (S^z)^2}$$

$$\dot{\theta} = \omega_0 + U|\psi|^2$$
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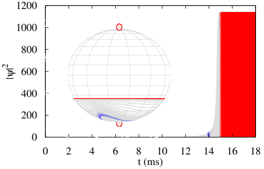

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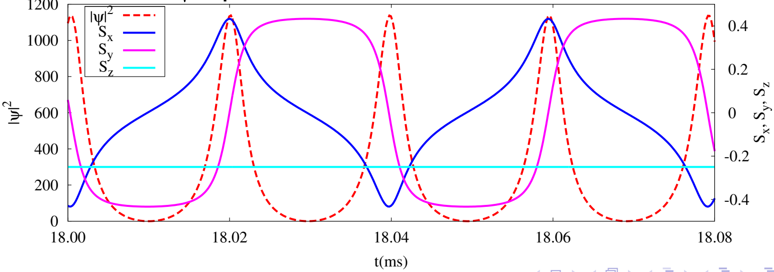
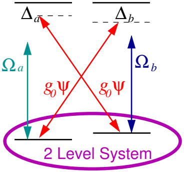


Tuning g, g', U

[Dimer *et al.* Phys. Rev. A. (2007)]



- Separate pump strength/detuning
- $g \sim \frac{g_0 \Omega_b}{\Delta_b}, g' \sim \frac{g_0 \Omega_a}{\Delta_a}, U \sim \frac{{g_0}^2}{\Delta_a} - \frac{{g_0}^2}{\Delta_b}$

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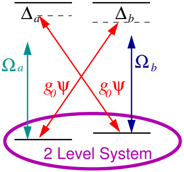

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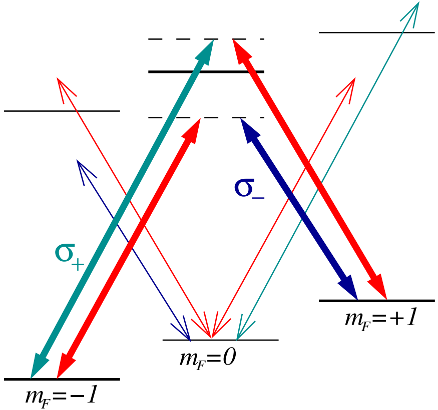
Possible realization: Hyperfine levels



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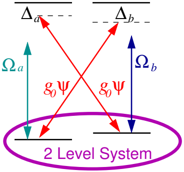

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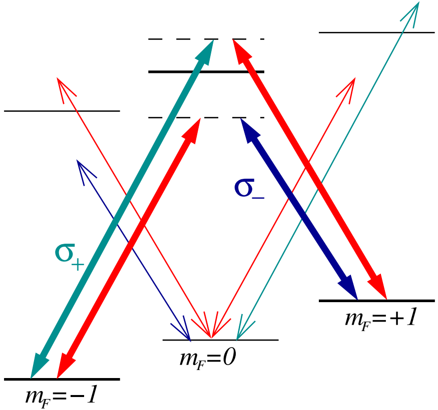
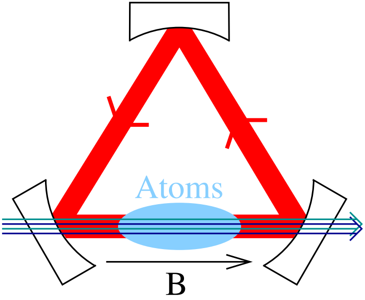
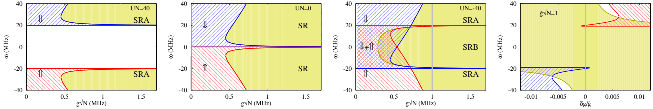


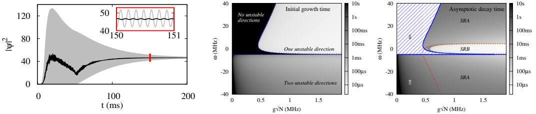


Summary

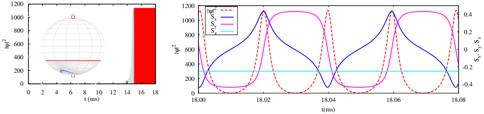
- Wide variety of dynamical phases



- Slow dynamics



- Persistent oscillations if $U > 0$



JK *et al.* PRL '10, Bhaseen *et al.* PRA '12

Extra slides

5 Ferroelectric phase transition

6 Lipkin-Meshkov-Glick

7 Tricritical point

Ferroelectric transition

Atoms in Coulomb gauge

$$H = \sum \omega_k a_k^\dagger a_k + \sum_i [p_i - eA(r_i)]^2 + V_{\text{coul}}$$

Ferroelectric transition

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Two-level systems — dipole-dipole coupling

$$H = \omega_0 S^z + \omega\psi^\dagger\psi + g(S^+ + S^-)(\psi + \psi^\dagger) + N\zeta(\psi + \psi^\dagger)^2 - \eta(S^+ - S^-)^2$$

(nb $g^2, \zeta, \eta \propto 1/V$).

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Ferroelectric polarisation if $\omega_0 < 2\eta N$

Gauge transform to dipole gauge $\mathbf{D} \cdot \mathbf{r}$

$$H = \omega_0 S^z + \omega\psi^\dagger\psi + \bar{g}(S^+ - S^-)(\psi - \psi^\dagger)$$

"Dicke" transition at $\omega_0 < N\bar{g}^2/\omega \equiv 2\eta N$

But, ψ describes **electric displacement**

Timescales for dynamics: $U = 0$ and Lipkin-Meshkov-Glick

- Since $\kappa \gg \omega_0$, can consider eliminating ψ

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- If $U = 0$, simple result:

$$\partial_t \mathbf{S} = \{\mathbf{S}, H\} - \Gamma \mathbf{S} \times (\mathbf{S} \times \hat{z}), \quad H = \omega_0 S_z - \Lambda_+ S_x^2 - \Lambda_- S_y^2$$

with: $\Lambda_\pm = \frac{\omega}{\kappa^2+\omega^2}(g \pm g')^2$, $\Gamma = \frac{2\kappa}{\kappa^2+\omega^2}(g'^2 - g^2)$

- NB, $g' = g$, no dissipation. Dissipation restored by finite κ.

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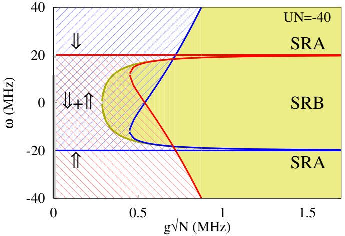
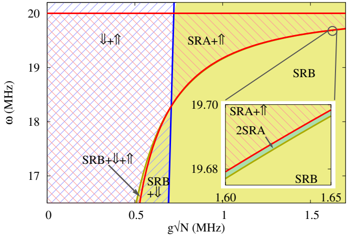
Tricritical point





If $-\omega_u = -UN/2 > \kappa$, crossing of boundaries at:

$$\omega^* = \sqrt{\omega_u^2 - \kappa^2}$$

$$g^*\sqrt{N} = \sqrt{\frac{-\omega_0 UN}{4}}$$

For $g > g^*$, 2SRA region exists at edge of SRB.