Condensation, superfluidity and lasing of coupled light-matter systems.

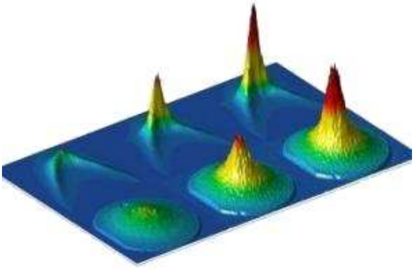
Jonathan Keeling

Newcastle, December 2011

Funding: EPSRC
Engineering and Physical Sciences Research Council

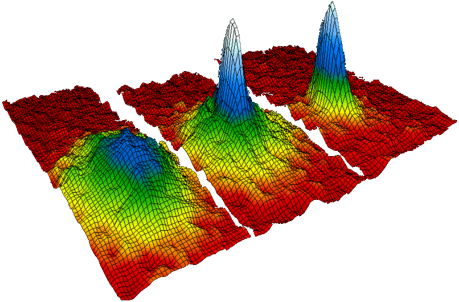
Bose-Einstein condensation: macroscopic occupation

Polaritons. $\sim$ 20K



[Kasprzak *et al.* Nature, '06]

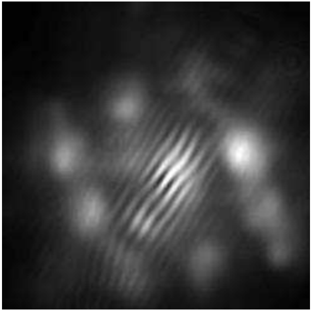
Atoms. $\sim 10^{-7}$K



[Anderson *et al.* Science '95]

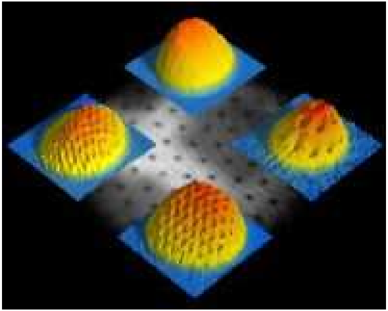
Macroscopic coherence: vortices

Polaritons:



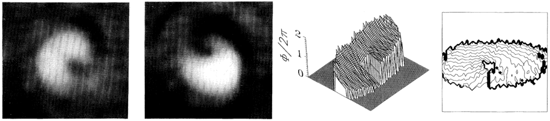
[Lagoudakis *et al.* Nat. Phys. '08]

Atoms:

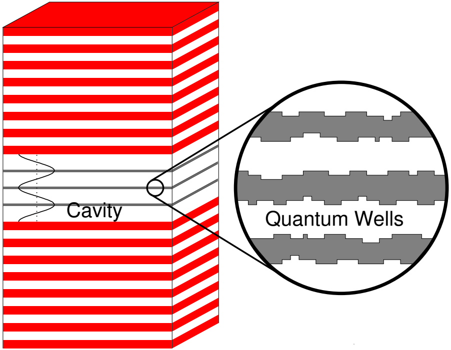


[Abo-Shaeer *et al.* Science '01]

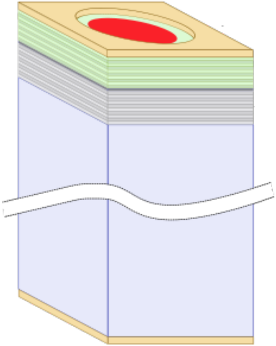
But also, nonlinear optics:
[Arecchi *et al.* PRL '91]



Polariton devices and VCSEL



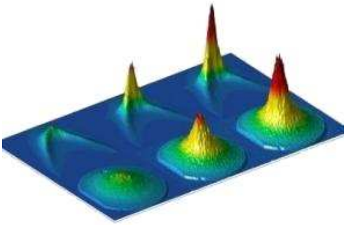
Strong exciton-photon coupling



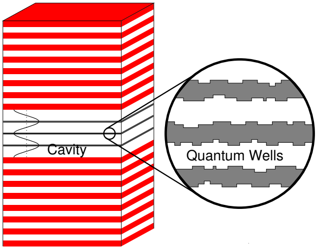
Vertical **C**avity **S**urface **E**mitting **L**aser — electron-hole gain medium

Polariton condensate and photon condensate

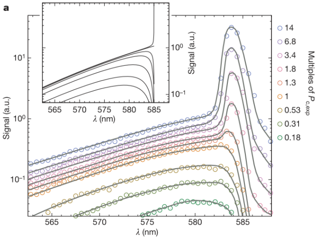
Polaritons:



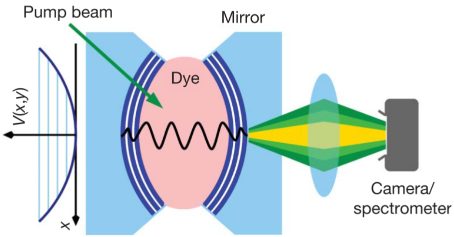
[Kasprzak *et al.* Nature, ’06]



Photons:



[Klaers *et al.* Nature, ’10]

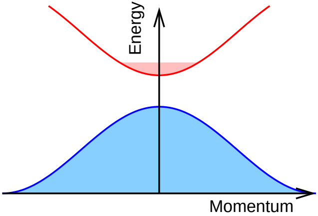


Questions

- What are polaritons?
- How can photon-like objects form a BEC?
- Is this “just a laser”?
- How to model a non-equilibrium condensate?
- Effects of non-equilibrium nature on:
  - Steady states
  - Coherence
  - Superfluidity
  - ...

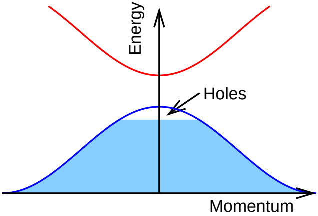
Excitons

Electronic spectrum:



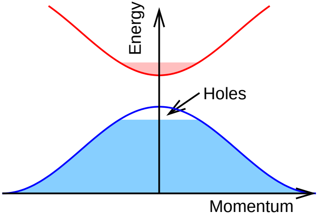
Excitons

Electronic spectrum:



Excitons

Electronic spectrum:

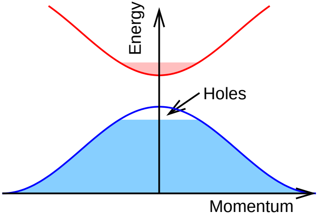


$$H = \sum_i T_i^e + T_i^h + \sum_{ij} V_{ij}^{ee} + V_{ij}^{hh} - V_{ij}^{eh}$$

$$T_i = \frac{p_i^2}{2m_j} \qquad V_{ij} = \frac{e^2}{\epsilon_r |r_i - r_j|}$$

Excitons

Electronic spectrum:



Bound state: Exciton,
$M \sim m_e + m_h$

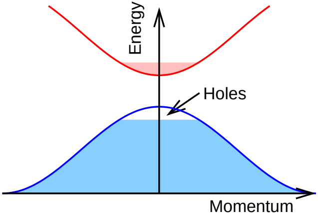
Approximate Bose statistics:
$[c_{\text{exciton},k}, c^\dagger_{\text{exciton},k'}] \simeq \delta_{k,k'}$
If $\rho (a_{B,\text{exciton}})^D \ll 1$

$$H = \sum_i T_i^e + T_i^h + \sum_{ij} V_{ij}^{ee} + V_{ij}^{hh} - V_{ij}^{eh}$$

$$T_i = \frac{p_i^2}{2m_i} \qquad V_{ij} = \frac{e^2}{\epsilon_r |r_i - r_j|}$$

Excitons

Electronic spectrum:



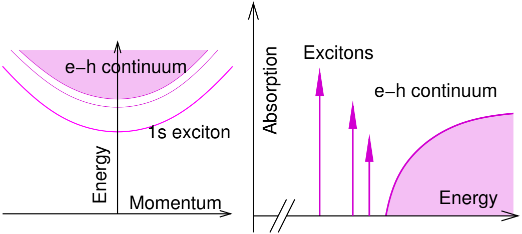
$$H = \sum_i T_i^e + T_i^h + \sum_{ij} V_{ij}^{ee} + V_{ij}^{hh} - V_{ij}^{eh}$$

$$T_i = \frac{p_i^2}{2m_i} \qquad V_{ij} = \frac{e^2}{\epsilon_r |r_i - r_j|}$$

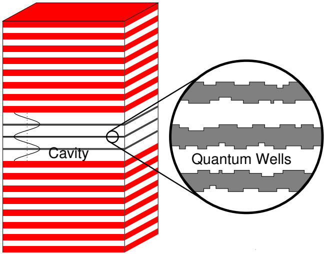
Bound state: Exciton,
$M \sim m_e + m_h$

Approximate Bose statistics:
$[c_{\text{exciton},k}, c^\dagger_{\text{exciton},k'}] \simeq \delta_{k,k'}$
If $\rho(a_{B,\text{exciton}})^D \ll 1$

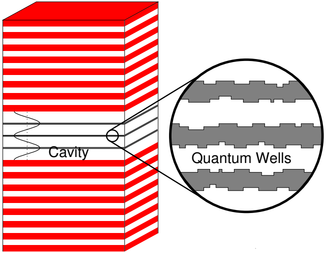
Optical spectrum

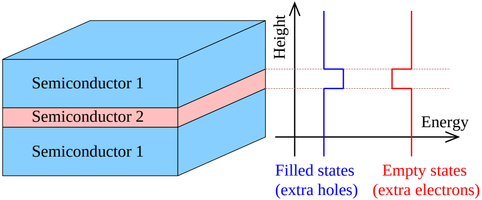


Microcavity polaritons: Excitons + photons

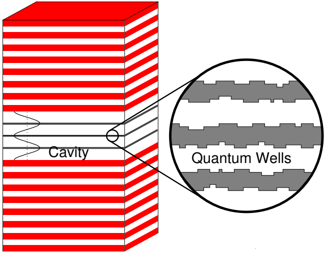


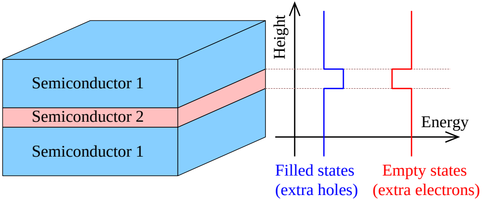
Microcavity polaritons: Excitons + photons





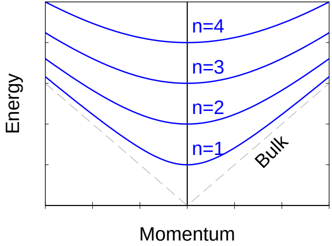
Microcavity polaritons: Excitons + photons



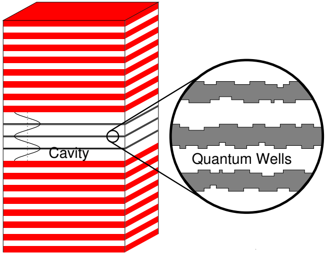


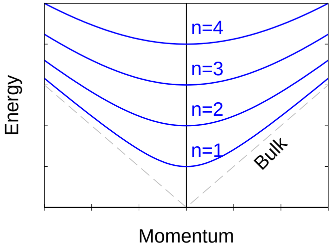
Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$

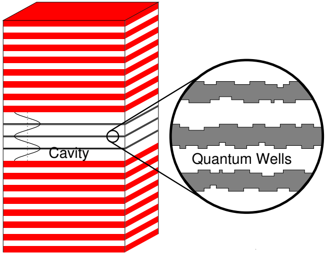


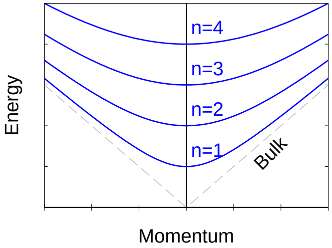
Microcavity polaritons: strong coupling

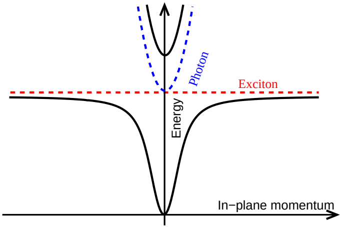




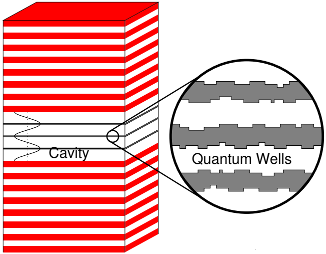
Microcavity polaritons: strong coupling

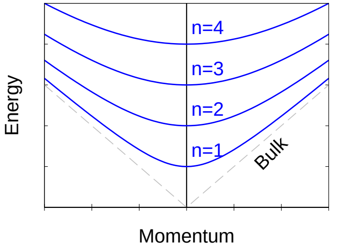


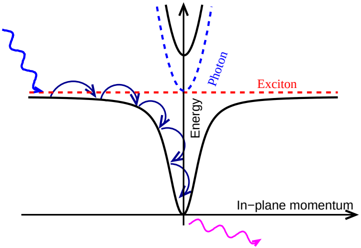




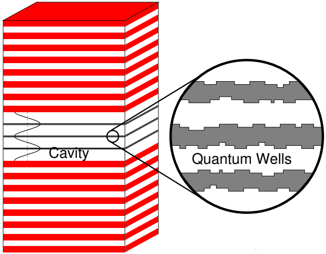
Microcavity polaritons: strong coupling

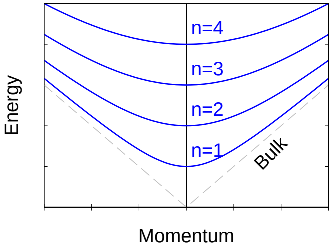


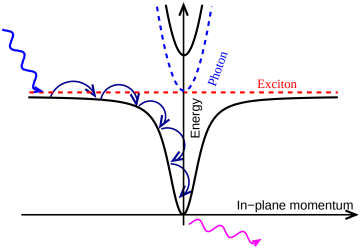


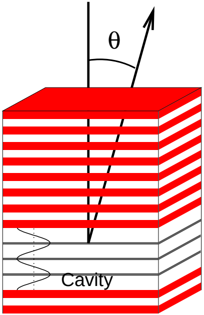


Microcavity polaritons: strong coupling

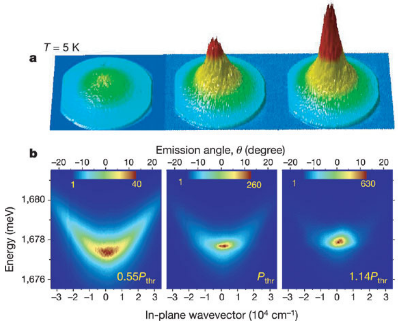


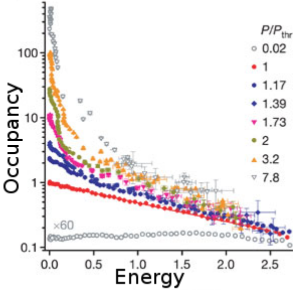






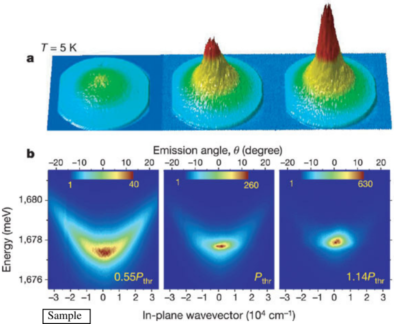
Polariton experiments: occupation and coherence

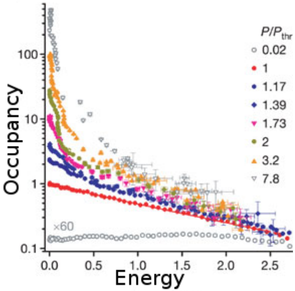


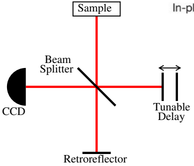


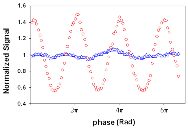
[Kasprzak, *et al.* Nature, 2006]

Polariton experiments: occupation and coherence

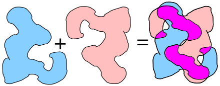


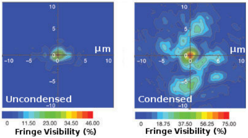






Coherence map:

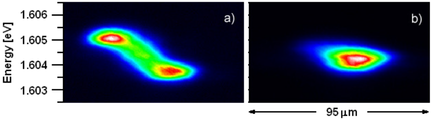




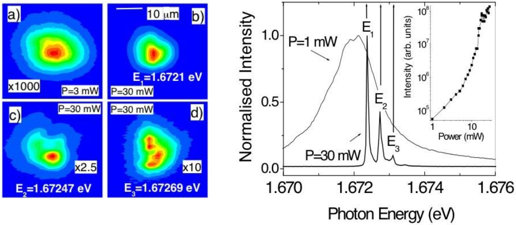
[Kasprzak, *et al.* Nature, 2006]

(Some) other polariton condensation experiments

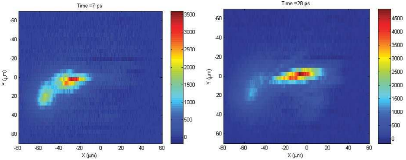
- Polariton traps
  [**Balili *et al.* Science '07)**]



- Multimode condensate and sharp lines
  [**Love *et al.* PRL '08**]

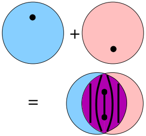


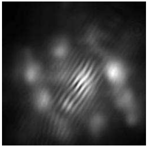
- Wavepacket propagation
  [**Amo *et al.* Nature 457 291 (2009)**]



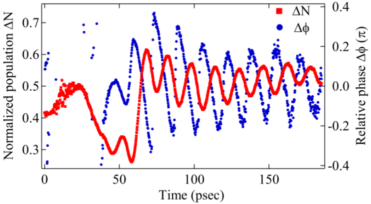
(Some) other polariton condensation experiments

- Quantised vortices
  [**Lagoudakis *et al.* Nat. Phys. ’08**. Science ’09, PRL ’10; Sanvitto *et al.* Nat. Phys. ’10; Roumpos *et al.* Nat. Phys. ’10]

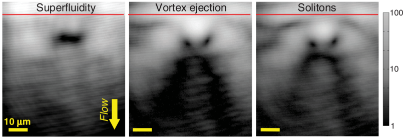




- Josephson oscillations
  [**Lagoudakis *et al.* PRL ’10**]



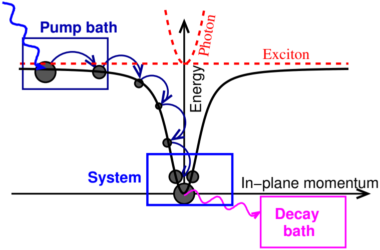
- Pattern formation/Hydrodynamics
  [**Amo *et al.* Science ’11**, Nature ’09; Wertz *et al.* Nat. Phys ’10]



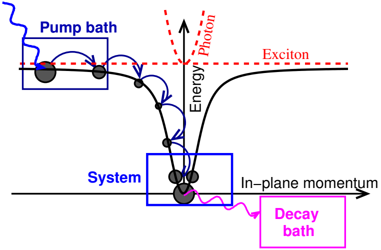
Non-equilibrium approach: Steady state, and fluctuations

$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}},$$

$$H_{\text{sys}} = \sum_k \omega_k \psi_k \psi_k^\dagger + \sum_\alpha g_\alpha (\phi_\alpha^\dagger \psi_k + \text{H.c.}) + H_{\text{ex}}[\phi_\alpha, \phi_\alpha^\dagger]$$



Non-equilibrium approach: Steady state, and fluctuations

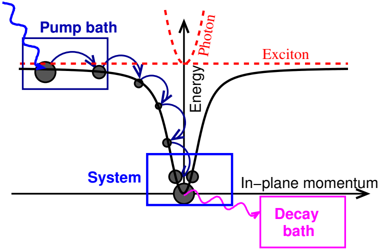


$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}},$$

$$H_{\text{sys}} = \sum_k \omega_k \psi_k \psi_k^\dagger + \sum_\alpha g_\alpha (\phi_\alpha^\dagger \psi_k + \text{H.c.}) + H_{\text{ex}}[\phi_\alpha, \phi_\alpha^\dagger]$$

Steady state, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$.

Non-equilibrium approach: Steady state, and fluctuations



$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}},$$

$$H_{\text{sys}} = \sum_k \omega_k \psi_k \psi_k^\dagger + \sum_\alpha g_\alpha (\phi_\alpha^\dagger \psi_k + \text{H.c.}) + H_{\text{ex}}[\phi_\alpha, \phi_\alpha^\dagger]$$

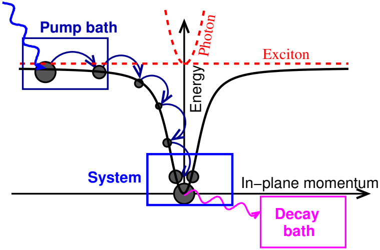
Steady state, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$.

Self-consistent equation: $(i\partial_t - \omega_0 + i\kappa)\,\psi = \sum_\alpha g_\alpha \langle \phi_\alpha \rangle$

Non-equilibrium approach: Steady state, and fluctuations

$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}},$$

$$H_{\text{sys}} = \sum_k \omega_k \psi_k \psi_k^\dagger + \sum_\alpha g_\alpha (\phi_\alpha^\dagger \psi_k + \text{H.c.}) + H_{\text{ex}}[\phi_\alpha, \phi_\alpha^\dagger]$$



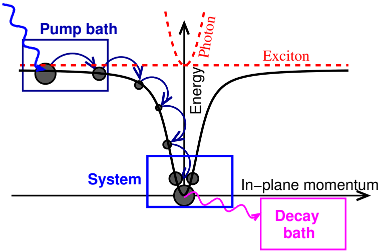
Steady state, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$.

Self-consistent equation: $(\mu_s - \omega_0 + i\kappa)\,\psi_0 = \chi(\psi_0, \mu_s)\psi_0$

Non-equilibrium approach: Steady state, and fluctuations

$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}},$$

$$H_{\text{sys}} = \sum_k \omega_k \psi_k \psi_k^\dagger + \sum_\alpha g_\alpha (\phi_\alpha^\dagger \psi_k + \text{H.c.}) + H_{\text{ex}}[\phi_\alpha, \phi_\alpha^\dagger]$$



Steady state, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_s t}$.

Self-consistent equation: $(\mu_s - \omega_0 + i\kappa)\,\psi_0 = \chi(\psi_0, \mu_s)\psi_0$

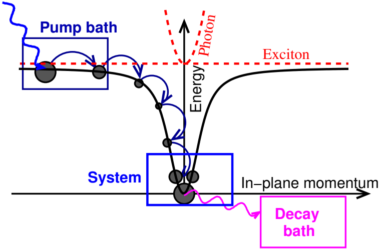
Fluctuations

$$[D^R - D^A](t, t') = -i \left\langle \left[\psi(t), \psi^\dagger(t')\right]_- \right\rangle$$

Non-equilibrium approach: Steady state, and fluctuations

$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}},$$

$$H_{\text{sys}} = \sum_k \omega_k \psi_k \psi_k^\dagger + \sum_\alpha g_\alpha (\phi_\alpha^\dagger \psi_k + \text{H.c.}) + H_{\text{ex}}[\phi_\alpha, \phi_\alpha^\dagger]$$



Steady state, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_s t}$.
Self-consistent equation: $(\mu_s - \omega_0 + i\kappa)\,\psi_0 = \chi(\psi_0, \mu_s)\psi_0$

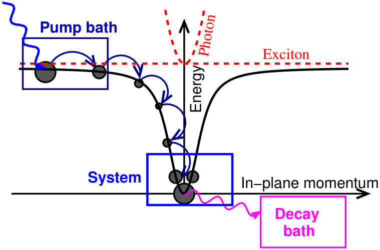
Fluctuations

$$[D^R - D^A](t, t') = -i \left\langle \left[\psi(t), \psi^\dagger(t')\right]_- \right\rangle \qquad [D^R - D^A](\omega) = \text{DoS}(\omega)$$

Non-equilibrium approach: Steady state, and fluctuations

$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}},$$

$$H_{\text{sys}} = \sum_k \omega_k \psi_k \psi_k^\dagger + \sum_\alpha g_\alpha (\phi_\alpha^\dagger \psi_k + \text{H.c.}) + H_{\text{ex}}[\phi_\alpha, \phi_\alpha^\dagger]$$



Steady state, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_s t}$.

Self-consistent equation: $(\mu_s - \omega_0 + i\kappa)\,\psi_0 = \chi(\psi_0, \mu_s)\psi_0$

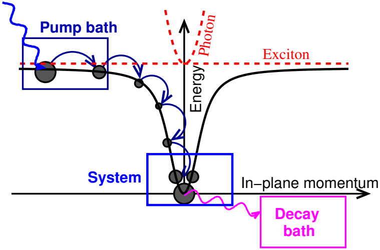
Fluctuations

$$[D^R - D^A](t, t') = -i \left\langle \left[\psi(t), \psi^\dagger(t') \right]_- \right\rangle \qquad [D^R - D^A](\omega) = \text{DoS}(\omega)$$

$$D^K(t, t') = -i \left\langle \left[\psi(t), \psi^\dagger(t') \right]_+ \right\rangle$$

Non-equilibrium approach: Steady state, and fluctuations

$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}},$$
$$H_{\text{sys}} = \sum_k \omega_k \psi_k \psi_k^\dagger + \sum_\alpha g_\alpha (\phi_\alpha^\dagger \psi_k + \text{H.c.}) + H_{\text{ex}}[\phi_\alpha, \phi_\alpha^\dagger]$$



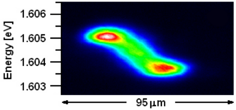
Steady state, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_s t}$.
Self-consistent equation: $(\mu_s - \omega_0 + i\kappa)\,\psi_0 = \chi(\psi_0, \mu_s)\psi_0$

Fluctuations

$$[D^R - D^A](t, t') = -i \left\langle \left[\psi(t), \psi^\dagger(t')\right]_- \right\rangle \qquad [D^R - D^A](\omega) = \text{DoS}(\omega)$$
$$D^K(t, t') = -i \left\langle \left[\psi(t), \psi^\dagger(t')\right]_+ \right\rangle \qquad D^K(\omega) = (2n(\omega) + 1)\text{DoS}(\omega)$$

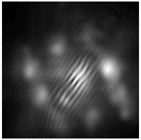
Pattern formation in experiments

Polariton Traps



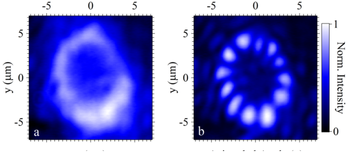
[Balili *et al.* Science '07]

Vortex formation



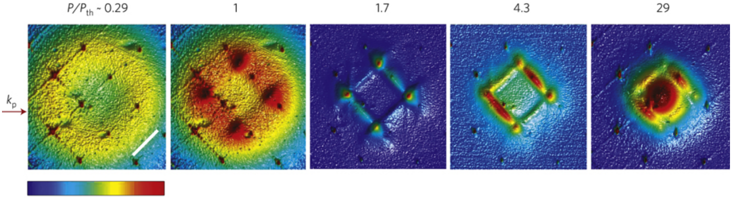
[Lagoudakis *et al.* Nat. Phys '08]

Elliptical ring pump



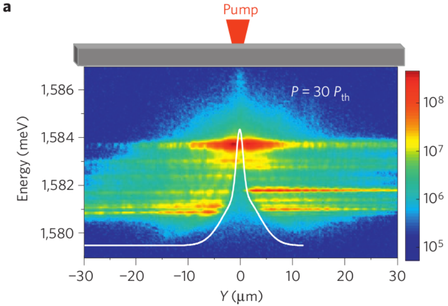
[Manni *et al.* PRL '11]

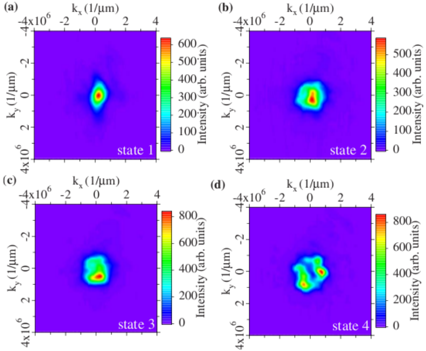
Patterned lattice: Momentum space image



[Kim *et al.* Nat. Phys '11]

Non-equilibrium features in experiment





Flow from pumping spot
[Wertz *et al.* Nat. Phys. (2010)]

$|\psi(\mathbf{k})|^2 \neq |\psi(-\mathbf{k})|^2$:
Broken time-reversal symmetry.
[Krizhanovskii *et al.* PRB (2009)]

Complex Gross-Pitaevskii equation

Steady state equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit:

Complex Gross-Pitaevskii equation

Steady state equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit: Gross-Pitaevskii equation

$$\left(i\partial_t + i\kappa - \left[V(r) - \frac{\nabla^2}{2m}\right]\right)\psi(r) = \chi(\psi(r,t))\psi(r,t)$$

Nonlinear, complex susceptibility

Complex Gross-Pitaevskii equation

Steady state equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit: Gross-Pitaevskii equation

$$\left(i\partial_t + i\kappa - \left[V(r) - \frac{\nabla^2}{2m}\right]\right)\psi(r) = \chi(\psi(r,t))\psi(r,t)$$

Nonlinear, complex susceptibility

$$i\partial_t\psi|_{\rm nlin} = U|\psi|^2\psi$$

Complex Gross-Pitaevskii equation

Steady state equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit: Gross-Pitaevskii equation

$$\left(i\partial_t + i\kappa - \left[V(r) - \frac{\nabla^2}{2m}\right]\right)\psi(r) = \chi(\psi(r,t))\psi(r,t)$$

Nonlinear, complex susceptibility

$$i\partial_t\psi|_{\text{nlin}} = U|\psi|^2\psi$$

$$i\partial_t\psi|_{\text{loss}} = -i\kappa\psi \qquad i\partial_t\psi|_{\text{gain}} = i\gamma_{\text{eff}}\psi$$

Complex Gross-Pitaevskii equation

Steady state equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit: Gross-Pitaevskii equation

$$\left(i\partial_t + i\kappa - \left[V(r) - \frac{\nabla^2}{2m}\right]\right)\psi(r) = \chi(\psi(r,t))\psi(r,t)$$

Nonlinear, complex susceptibility

$$i\partial_t\psi|_{\text{nlin}} = U|\psi|^2\psi$$

$$i\partial_t\psi|_{\text{loss}} = -i\kappa\psi \qquad i\partial_t\psi|_{\text{gain}} = i\gamma_{\text{eff}}\psi - i\Gamma|\psi|^2\psi$$

Complex Gross-Pitaevskii equation

Steady state equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit: Gross-Pitaevskii equation

$$\left(i\partial_t + i\kappa - \left[V(r) - \frac{\nabla^2}{2m}\right]\right)\psi(r) = \chi(\psi(r,t))\psi(r,t)$$

Nonlinear, complex susceptibility

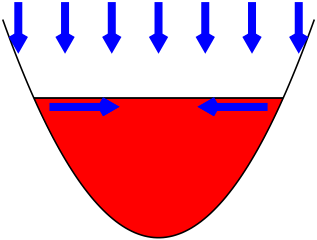
$$i\partial_t\psi|_{\text{nlin}} = U|\psi|^2\psi$$

$$i\partial_t\psi|_{\text{loss}} = -i\kappa\psi \qquad i\partial_t\psi|_{\text{gain}} = i\gamma_{\text{eff}}\psi - i\Gamma|\psi|^2\psi$$

$$i\partial_t\psi = \left[-\frac{\nabla^2}{2m} + V(r) + U|\psi|^2 + i\left(\gamma_{\text{eff}} - \kappa - \Gamma|\psi|^2\right)\right]\psi$$

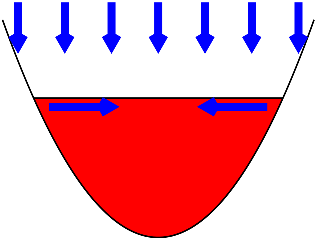
Gross-Pitaevskii equation: Harmonic trap

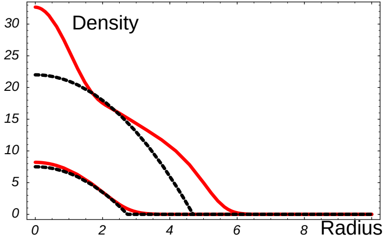
$$i\partial_t\psi = \left[-\frac{\nabla^2}{2m} + \frac{m\omega^2}{2}r^2 + U|\psi|^2 + i\left(\gamma_{\text{eff}} - \kappa - \Gamma|\psi|^2\right)\right]\psi$$



Gross-Pitaevskii equation: Harmonic trap

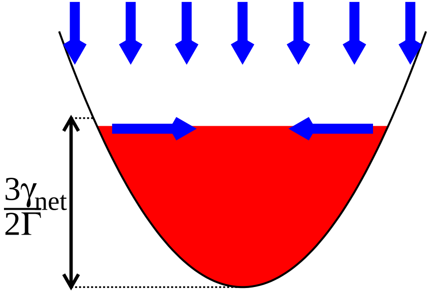
$$i\partial_t\psi = \left[-\frac{\nabla^2}{2m} + \frac{m\omega^2}{2}r^2 + U|\psi|^2 + i\left(\gamma_{\text{eff}} - \kappa - \Gamma|\psi|^2\right)\right]\psi$$





Stability of Thomas-Fermi solution

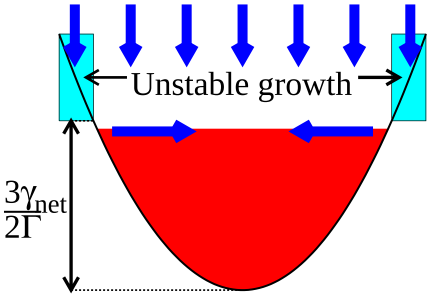
$$\frac{1}{2}\partial_t \rho + \nabla\cdot(\rho \mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$



Stability of Thomas-Fermi solution

High m modes: $\delta\rho_{n,m} \simeq e^{im\theta} r^m \ldots$

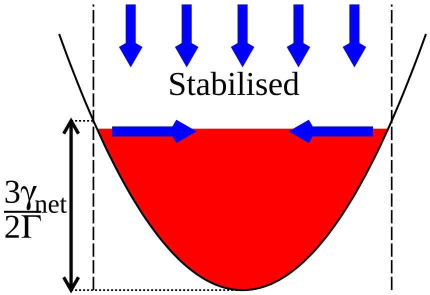
$$\frac{1}{2}\partial_t\rho + \nabla\cdot(\rho\mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$



Stability of Thomas-Fermi solution

High m modes: $\delta\rho_{n,m} \simeq e^{im\theta} r^m \ldots$

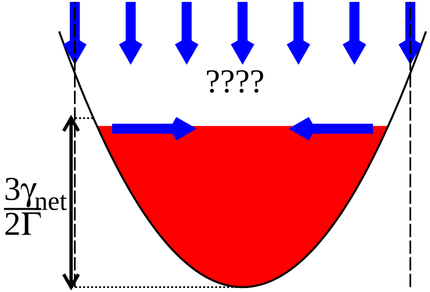
$$\frac{1}{2}\partial_t\rho + \nabla\cdot(\rho\mathbf{v}) = (\gamma_{\rm net}\Theta(r_0 - r) - \Gamma\rho)\rho$$



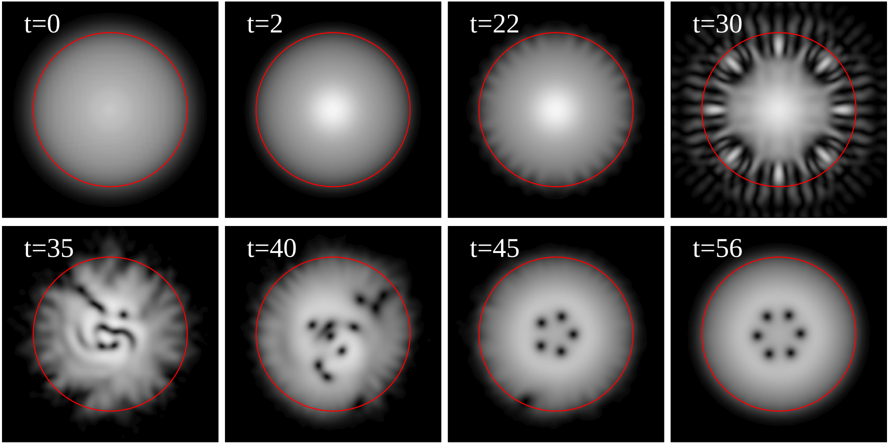
Stability of Thomas-Fermi solution

High m modes: $\delta\rho_{n,m} \simeq e^{im\theta} r^m \ldots$

$$\frac{1}{2}\partial_t\rho + \nabla\cdot(\rho\mathbf{v}) = (\gamma_{\rm net}\Theta(r_0 - r) - \Gamma\rho)\rho$$

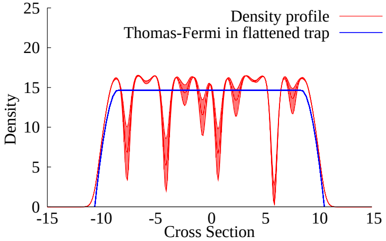


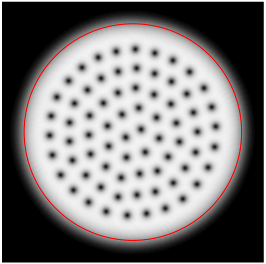
Time evolution:



[Keeling & Berloff PRL '08]

Why vortices





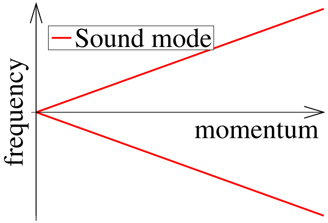
Spectrum above transition

When condensed

$$\text{Det}\left[D^R(\omega, k)\right]^{-1} = \omega^2 - \xi_k^2$$

With $\xi_k \simeq ck$
Poles:

$$\omega^* = \xi_k$$



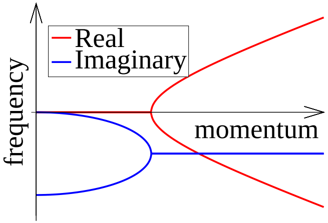
Spectrum above transition

When condensed

$$\text{Det}\left[D^R(\omega,k)\right]^{-1} = (\omega+i\gamma_{\text{net}})^2+\gamma_{\text{net}}^2-\xi_k^2$$

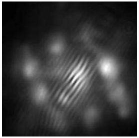
With $\xi_k \simeq ck$
Poles:

$$\omega^* = -i\gamma_{\text{net}} \pm \sqrt{\xi_k^2-\gamma_{\text{net}}^2}$$

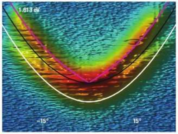


Polariton "superfluidity" experiments

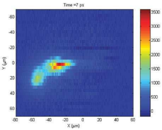
- Quantised vortices in disorder potential
  [Lagoudakis *et al.* Nature Phys. 4, 706 (2008)]

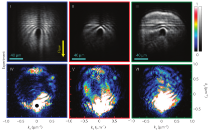


- Changes to excitation spectrum
  [Utsunomiya *et al.* Nature Phys. 4 700 (2008)]



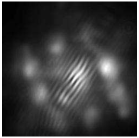
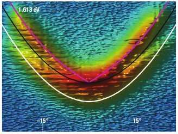
- Wavepacket propagation
  [Amo *et al.* Nature 457 291 (2009)]
- Driven superfluidity
  [Amo *et al.* Nature Phys. (2009)]

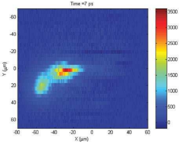


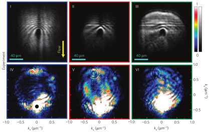


Aspects of superfluidity

	Quantised vortices	Landau critical velocity	Metastable persistent flow	Two-fluid hydrody-namics	Local thermal equilib-rium	Solitary waves
Superfluid ^{4}He/cold atom Bose-Einstein condensate	✔	✔	✔	✔	✔	✔
Non-interacting Bose-Einstein condensate	✔	✗	✗	✔	✔	✗
Classical irrotational fluid	✗	✔	✗	✔	✔	✔
Incoherently pumped polariton condensates	✔	✗	?	?	✗	?

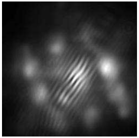


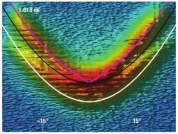


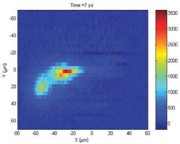
Lagoudakis *et al.* Nat. Phys. ’08. Utsunomiya *et al.* Nat. Phys. ’08. Amo *et al.* Nature ’09; Nat. Phys. ’09

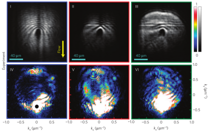
Aspects of superfluidity

	Quantised vortices	Landau critical velocity	Metastable persistent flow	Two-fluid hydrody-namics	Local thermal equilib-rium	Solitary waves
Superfluid ^{4}He/cold atom Bose-Einstein condensate	✔	✔	✔	✔	✔	✔
Non-interacting Bose-Einstein condensate	✔	✗	✗	✔	✔	✗
Classical irrotational fluid	✗	✔	✗	✔	✔	✔
Incoherently pumped polariton condensates	✔	✗	?	?	✗	?





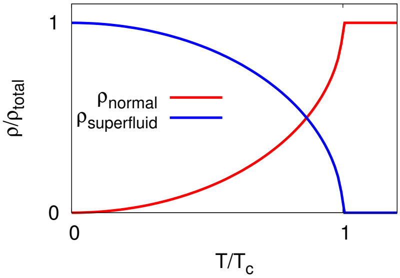




Lagoudakis *et al.* Nat. Phys. '08. Utsunomiya *et al.* Nat. Phys. '08. Amo *et al.* Nature '09; Nat. Phys. '09

Superfluid density

- Two-fluid hydrodynamics



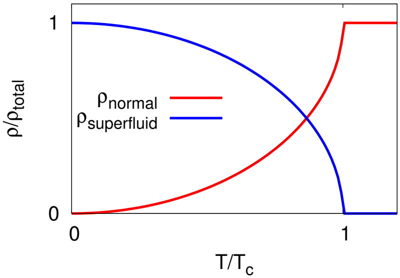
- ρ_s, ρ_n distinguished by slow rotation

- Experimentally, rotation:

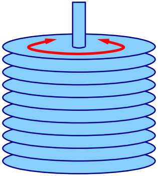
- To calculate, transverse/longitudinal:

Superfluid density

- Two-fluid hydrodynamics



- Experimentally, rotation:

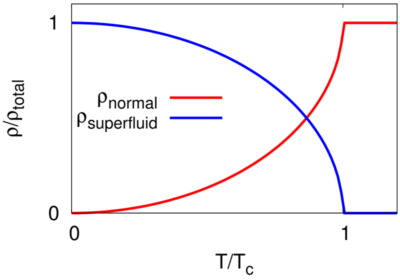


- To calculate, transverse/longitudinal:

- ρ_s, ρ_n distinguished by slow rotation

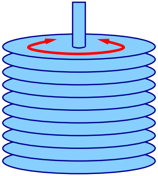
Superfluid density

- Two-fluid hydrodynamics

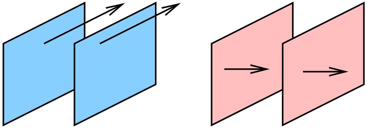


- ρ_s, ρ_n distinguished by slow rotation

- Experimentally, rotation:



- To calculate, transverse/longitudinal:



Superfluid density

- Currrent:

$$\mathbf{J} = \rho \mathbf{v} = \psi^\dagger i \nabla \psi = |\psi|^2 \nabla \phi$$

$$J_i(\mathbf{q}) = \psi^\dagger_{\mathbf{k}+\mathbf{q}} \frac{2k_i + q_i}{2m} \psi_{\mathbf{k}}$$

- Response function:

$$H \to H - \sum_q \mathbf{f}(\mathbf{q}) \cdot \mathbf{J}_i(\mathbf{q}) \qquad J_i(\mathbf{q}) = \chi_{ij}(\mathbf{q}) f_j(\mathbf{q})$$

- Vertex corrections essential for superfluid part.

Superfluid density

- Currrent:

$$\mathbf{J} = \rho \mathbf{v} = \psi^\dagger i \nabla \psi = |\psi|^2 \nabla \phi$$

$$J_i(\mathbf{q}) = \psi^\dagger_{\mathbf{k}+\mathbf{q}} \frac{2k_i + q_i}{2m} \psi_{\mathbf{k}}$$

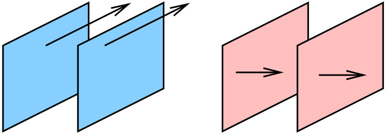
- Response function:

$$H \to H - \sum_q \mathbf{f}(\mathbf{q}) \cdot \mathbf{J}_i(\mathbf{q}) \quad J_i(\mathbf{q}) = \chi_{ij}(\mathbf{q}) f_j(\mathbf{q})$$

- Vertex corrections essential for superfluid part.

Superfluid density

- Currrent:



$$\mathbf{J} = \rho\mathbf{v} = \psi^\dagger i\nabla\psi = |\psi|^2\nabla\phi$$

$$J_i(\mathbf{q}) = \psi^\dagger_{\mathbf{k}+\mathbf{q}}\frac{2k_i + q_i}{2m}\psi_{\mathbf{k}}$$

- Response function:

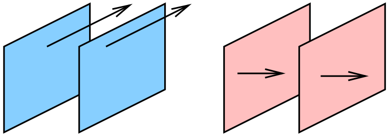
$$H \rightarrow H - \sum_q \mathbf{f}(\mathbf{q})\cdot\mathbf{J}_i(\mathbf{q}) \quad J_i(\mathbf{q}) = \chi_{ij}(\mathbf{q})f_j(\mathbf{q})$$

$$\chi_{ij}(\omega = 0, \mathbf{q}\rightarrow 0) = \langle[J_i(\mathbf{q}), J_j(-\mathbf{q})]\rangle = \frac{\rho_S}{m}\frac{q_i q_j}{q^2} + \frac{\rho_N}{m}\delta_{ij}$$

- Vertex corrections essential for superfluid part.

Superfluid density

- Currrent:



$$\mathbf{J} = \rho \mathbf{v} = \psi^\dagger i \nabla \psi = |\psi|^2 \nabla \phi$$

$$J_i(\mathbf{q}) = \psi^\dagger_{\mathbf{k}+\mathbf{q}} \frac{2k_i + q_i}{2m} \psi_{\mathbf{k}}$$

- Response function:

$$H \to H - \sum_q \mathbf{f}(\mathbf{q}) \cdot \mathbf{J}_i(\mathbf{q}) \quad J_i(\mathbf{q}) = \chi_{ij}(\mathbf{q}) f_j(\mathbf{q})$$

$$\chi_{ij}(\omega = 0, \mathbf{q} \to 0) = \langle [J_i(\mathbf{q}), J_j(-\mathbf{q})] \rangle = \frac{\rho_S}{m} \frac{q_i q_j}{q^2} + \frac{\rho_N}{m} \delta_{ij}$$

- Vertex corrections essential for superfluid part.

Non-equilibrium current response functions

- Superfluid response exists because:

$$= \left(\frac{i\psi_0 q_i}{2m}\right) D^R(q, \omega = 0) \left(\frac{i\psi_0 q_j}{2m}\right)$$

- $D^R(\omega = 0) \propto 1/q^2$ despite pumping/decay — superfluid response exists.
- Normal density:

$$\rho_N = \int d^d k \epsilon_k \int \frac{d\omega}{2\pi} \mathrm{Tr}\left[\sigma_z D^K \sigma_z (D^R + D^A)\right]$$

- Is affected by pump/decay:
  Does not vanish at $T \to 0$.

[Keeling PRL '11]

Non-equilibrium current response functions

- Superfluid response exists because:

$$= \left(\frac{i\psi_0 q_i}{2m}\right) D^R(q, \omega = 0) \left(\frac{i\psi_0 q_j}{2m}\right)$$

- $D^R(\omega = 0) \propto 1/q^2$ despite pumping/decay — superfluid response exists.
- Normal density:

$$\rho_N = \int d^d k \epsilon_k \int \frac{d\omega}{2\pi} \mathrm{Tr}\left[\sigma_z D^K \sigma_z (D^R + D^A)\right]$$

- Is affected by pump/decay:
  Does not vanish at $T \to 0$.

[Keeling PRL '11]

Non-equilibrium current response functions

- Superfluid response exists because:

 $$= \left(\frac{i\psi_0 q_i}{2m}\right) D^R(q,\omega=0) \left(\frac{i\psi_0 q_j}{2m}\right)$$

- $D^R(\omega=0) \propto 1/q^2$ despite pumping/decay — superfluid response exists.
- Normal density:

$$\rho_N = \int d^d k \epsilon_k \int \frac{d\omega}{2\pi} \mathrm{Tr}\left[\sigma_z D^K \sigma_z (D^R + D^A)\right]$$

- Is affected by pump/decay:
  Does not vanish at $T \to 0$.

[Keeling PRL '11]

Non-equilibrium current response functions

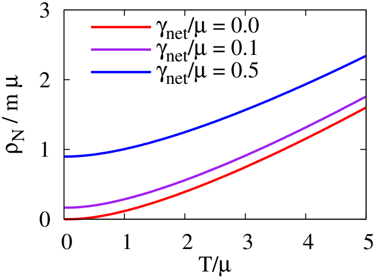
- Superfluid response exists because:

 $$= \left(\frac{i\psi_0 q_i}{2m}\right) D^R(q, \omega = 0) \left(\frac{i\psi_0 q_j}{2m}\right)$$

- $D^R(\omega = 0) \propto 1/q^2$ despite pumping/decay — superfluid response exists.
- Normal density:

$$\rho_N = \int d^d k \epsilon_k \int \frac{d\omega}{2\pi} \mathrm{Tr}\left[\sigma_z D^K \sigma_z (D^R + D^A)\right]$$

- Is affected by pump/decay: Does not vanish at $T \to 0$.



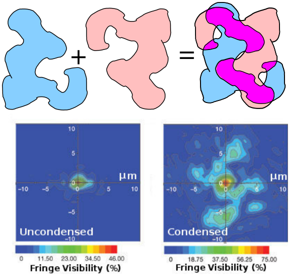
[Keeling PRL '11]

1. Introduction to polariton condensation
   - What are polaritons
   - Experimental features
   - Approaches to modelling
2. Non-equilibrium pattern formation
   - Experiments
   - Modelling pattern formation
3. Superfluidity
   - Non-equilibrium condensate spectrum
   - Experiments
   - Aspects of superfluidity
   - Superfluid response function
4. Coherence
   - Experiments
   - Power law decay of coherence

Correlations in a 2D Gas

Correlations:

$$g_1(\mathbf{r}, \mathbf{r}', t) = \left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0) \right\rangle$$



- $D^< = D^K - D^R + D^A$
- Generally, get:

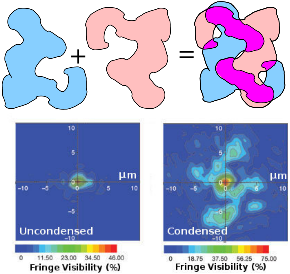
$$\left\langle \psi^\dagger(r, t)\psi(0, 0) \right\rangle \simeq |\psi_0|^2 \exp\left[-a_p \begin{cases} \ln(r/r_0) & t \simeq 0 \\ \frac{1}{2}\ln(c^2 t/\gamma_{net} r_0^2) & r \simeq 0 \end{cases} \right]$$

[Szymańska *et al.* PRL '06; PRB '07]

Correlations in a 2D Gas

Correlations: (in 2D)

$$g_1(\mathbf{r}, \mathbf{r}', t) = \left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0) \right\rangle$$
$$\simeq |\psi_0|^2 \exp\left[-D^<_{\phi\phi}(\mathbf{r}, \mathbf{r}', t) \right]$$

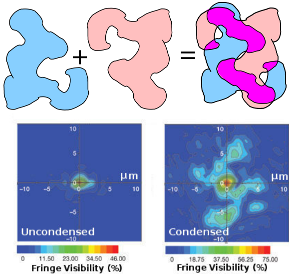


- $D^< = D^K - D^R + D^A$
- Generally, get:

$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, 0) \right\rangle \simeq |\psi_0|^2 \exp\left[-a_p \begin{cases} \ln(r/r_0) & t \simeq 0 \\ \frac{1}{2}\ln(c^2 t/\gamma_{net} r_0^2) & r \simeq 0 \end{cases} \right]$$

[Szymańska *et al.* PRL '06; PRB '07]

Correlations in a 2D Gas



Correlations: (in 2D)

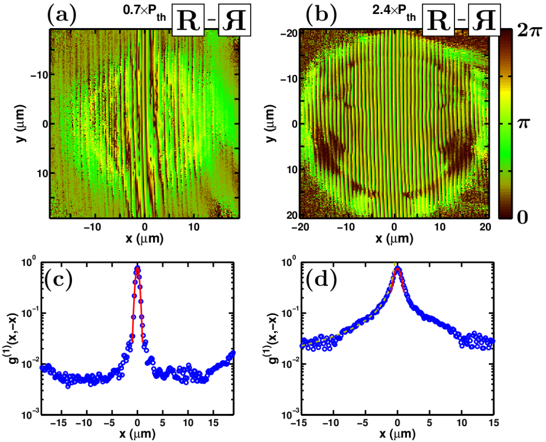
$$g_1(\mathbf{r}, \mathbf{r}', t) = \left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0) \right\rangle \simeq |\psi_0|^2 \exp\left[-D^<_{\phi\phi}(\mathbf{r}, \mathbf{r}', t)\right]$$

- $D^< = D^K - D^R + D^A$
- Generally, get:

$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, 0) \right\rangle \simeq |\psi_0|^2 \exp\left[-a_p \begin{cases} \ln(r/r_0) & t \simeq 0 \\ \frac{1}{2}\ln(c^2 t/\gamma_{\text{net}} r_0^2) & r \simeq 0 \end{cases}\right]$$

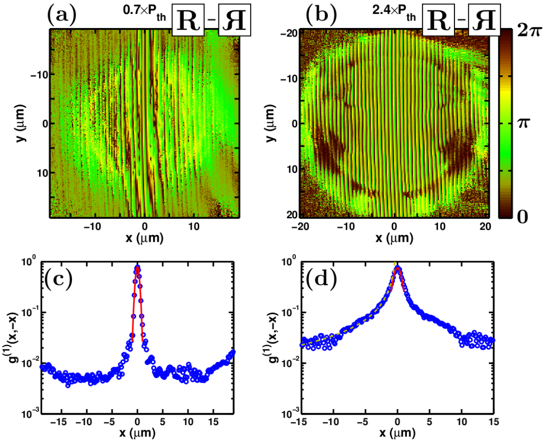
[Szymańska *et al.* PRL '06; PRB '07]

Experimental observation of power-law decay

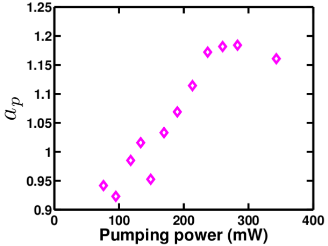


G. Rompos, Y. Yamamoto *et al.* submitted

Experimental observation of power-law decay



$$g_1(\mathbf{r}, -\mathbf{r}) \propto \left(\frac{r}{r_0}\right)^{-a_p}$$

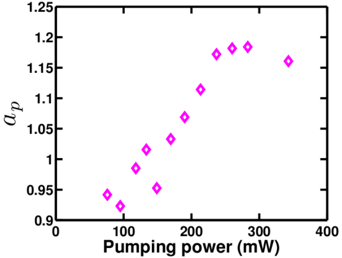


G. Rompos, Y. Yamamoto *et al.* submitted

Exponent in a non-equilibrium 2D gas

$$\lim_{r\to\infty} \left\langle \psi^\dagger(\mathbf{r},0)\psi(-\mathbf{r},0) \right\rangle = |\psi_0|^2 \exp\left[-D^{<}_{\phi\phi}(\mathbf{r},-\mathbf{r})\right] \propto \exp\left[-a_p \ln\left(\frac{2r}{r_0}\right)\right]$$

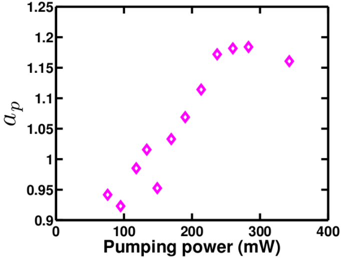
- Experimentally, $a_P \simeq 1.2$
- In equilibrium $a_p = \dfrac{mk_BT}{2\pi\hbar^2 n_s} < \dfrac{1}{4}$ (BKT transition)
- Non-equilibrium theory depends on thermalisation.



Exponent in a non-equilibrium 2D gas

$$\lim_{r\to\infty} \left\langle \psi^\dagger(\mathbf{r},0)\psi(-\mathbf{r},0)\right\rangle = |\psi_0|^2 \exp\left[-D^{<}_{\phi\phi}(\mathbf{r},-\mathbf{r})\right] \propto \exp\left[-a_p \ln\left(\frac{2r}{r_0}\right)\right]$$

- Experimentally, $a_P \simeq 1.2$
- In equilibrium $a_p = \dfrac{mk_B T}{2\pi\hbar^2 n_s} < \dfrac{1}{4}$ (BKT transition)
- Non-equilibrium theory depends on thermalisation.



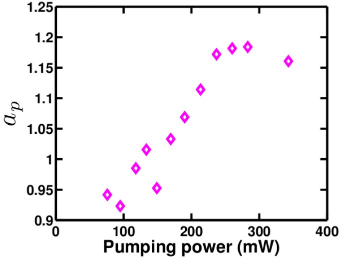
Exponent in a non-equilibrium 2D gas

$$\lim_{r\to\infty}\left\langle \psi^\dagger(\mathbf{r},0)\psi(-\mathbf{r},0)\right\rangle = |\psi_0|^2 \exp\left[-D^<_{\phi\phi}(\mathbf{r},-\mathbf{r})\right] \propto \exp\left[-a_p \ln\left(\frac{2r}{r_0}\right)\right]$$

- Experimentally, $a_P \simeq 1.2$
- In equilibrium $a_p = \dfrac{mk_BT}{2\pi\hbar^2 n_s} < \dfrac{1}{4}$ (BKT transition)
- Non-equilibrium theory depends on thermalisation.
  - Thermalised (yet diffusive modes)

    $$a_p = \frac{mk_BT}{2\pi\hbar^2 n_s}$$
  - Non-thermalised,

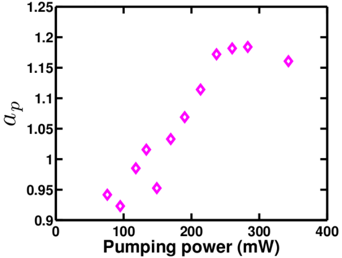
    $$a_p \propto \frac{\text{Pumping noise}}{n_s}$$



Exponent in a non-equilibrium 2D gas

$$\lim_{r\to\infty}\left\langle \psi^\dagger(\mathbf{r},0)\psi(-\mathbf{r},0)\right\rangle = |\psi_0|^2 \exp\left[-D^<_{\phi\phi}(\mathbf{r},-\mathbf{r})\right] \propto \exp\left[-a_p \ln\left(\frac{2r}{r_0}\right)\right]$$

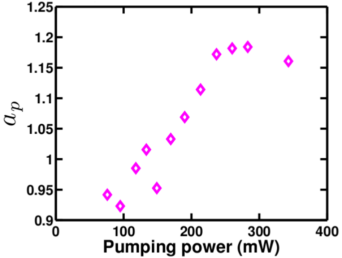
- Experimentally, $a_P \simeq 1.2$
- In equilibrium $a_p = \dfrac{mk_BT}{2\pi\hbar^2 n_s} < \dfrac{1}{4}$ (BKT transition)
- Non-equilibrium theory depends on thermalisation.
  - Thermalised (yet diffusive modes) $a_p = \dfrac{mk_BT}{2\pi\hbar^2 n_s}$
  - Non-thermalised, $a_p \propto \dfrac{\text{Pumping noise}}{n_s}$



Exponent in a non-equilibrium 2D gas

$$\lim_{r\to\infty}\left\langle \psi^\dagger(\mathbf{r},0)\psi(-\mathbf{r},0)\right\rangle = |\psi_0|^2 \exp\left[-D^<_{\phi\phi}(\mathbf{r},-\mathbf{r})\right] \propto \exp\left[-a_p \ln\left(\frac{2r}{r_0}\right)\right]$$

- Experimentally, $a_P \simeq 1.2$
- In equilibrium $a_p = \dfrac{mk_BT}{2\pi\hbar^2 n_s} < \dfrac{1}{4}$ (BKT transition)
- Non-equilibrium theory depends on thermalisation.
  - Thermalised (yet diffusive modes) $a_p = \dfrac{mk_BT}{2\pi\hbar^2 n_s}$
  - Non-thermalised, $a_P \propto \dfrac{\text{Pumping noise}}{n_s}$.



Questions

- What are polaritons?
- How can photon-like objects form a BEC?
- Is this “just a laser”?
- How to model a non-equilibrium condensate?
- Effects of non-equilibrium nature on:
  - Steady states
  - Coherence
  - Superfluidity
  - . . .

Extra slides

5. Microscopic model: lasing vs condensation
   - Model: localised excitons, propagating photons
   - Simple laser: Maxwell-Bloch
   - Non-equilibrium polaritons: coherence and strong coupling

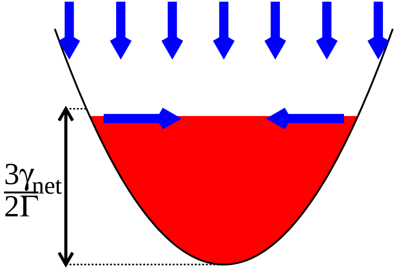
6. Measuring superfluid density

7. Coherence Finite size and Schawlow-Townes

Instability of Thomas-Fermi: details

$$\frac{1}{2}\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = (\gamma_{\text{net}} - \Gamma \rho)\rho$$

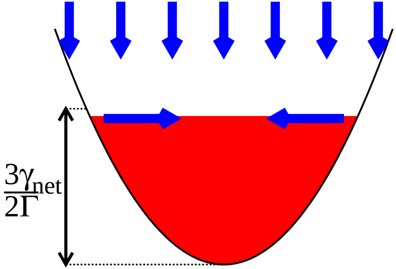
$$\partial_t \mathbf{v} + \nabla\left(U\rho + \frac{m\omega^2}{2} r^2 + \frac{m}{2}|\mathbf{v}|^2\right) = 0$$



Instability of Thomas-Fermi: details

$$\frac{1}{2}\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$

$$\partial_t \mathbf{v} + \nabla\left(U\rho + \frac{m\omega^2}{2} r^2 + \frac{m}{2}|\mathbf{v}|^2\right) = 0$$



Normal modes for $\gamma_{\text{net}}, \Gamma \to 0$:

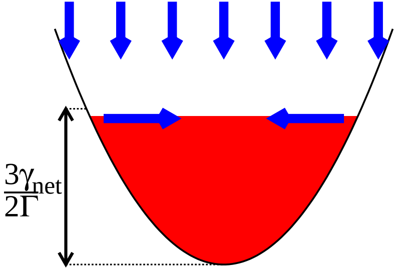
$$\delta\rho_{n,m}(r,\theta,t) = e^{im\theta} h_{n,m}(r) e^{i\omega_{n,m}t}$$

$$\omega_{n,m} = \omega 2\sqrt{m(1+2n) + 2n(n+1)}$$

Instability of Thomas-Fermi: details

$$\frac{1}{2}\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$

$$\partial_t \mathbf{v} + \nabla\left(U\rho + \frac{m\omega^2}{2} r^2 + \frac{m}{2}|\mathbf{v}|^2\right) = 0$$



Normal modes for $\gamma_{\text{net}}, \Gamma \to 0$:

$$\delta\rho_{n,m}(r,\theta,t) = e^{im\theta} h_{n,m}(r) e^{i\omega_{n,m} t}$$

$$\omega_{n,m} = \omega 2\sqrt{m(1+2n) + 2n(n+1)}$$

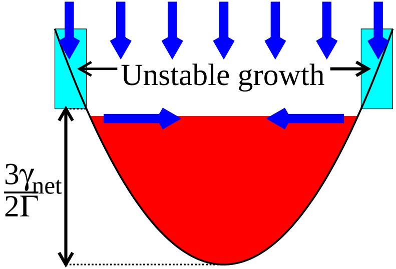
Add weak pumping/decay:

$$\omega_{n,n} \to \omega_{n,m} + i\gamma_{\text{net}}\left[\frac{m(1+2n) + 2n(n+1) - m^2}{2m(1+2n) + 4n(n+1) + m^2}\right]$$

Instability of Thomas-Fermi: details

$$\frac{1}{2}\partial_t\rho + \nabla\cdot(\rho\mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$

$$\partial_t\mathbf{v} + \nabla\left(U\rho + \frac{m\omega^2}{2}r^2 + \frac{m}{2}|\mathbf{v}|^2\right) = 0$$



Normal modes for $\gamma_{\text{net}}, \Gamma \to 0$:

$$\delta\rho_{n,m}(r,\theta,t) = e^{im\theta}h_{n,m}(r)e^{i\omega_{n,m}t}$$

$$\omega_{n,m} = \omega 2\sqrt{m(1+2n) + 2n(n+1)}$$

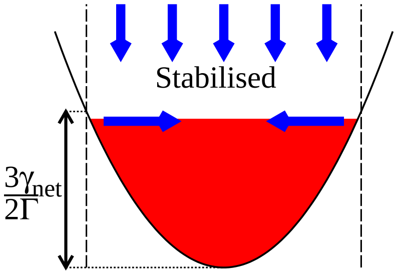
Add weak pumping/decay:

$$\omega_{n,n} \to \omega_{n,m} + i\gamma_{\text{net}}\left[\frac{m(1+2n) + 2n(n+1) - m^2}{2m(1+2n) + 4n(n+1) + m^2}\right]$$

Instability of Thomas-Fermi: details

$$\frac{1}{2}\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$

$$\partial_t \mathbf{v} + \nabla\left(U\rho + \frac{m\omega^2}{2}r^2 + \frac{m}{2}|\mathbf{v}|^2\right) = 0$$



Normal modes for $\gamma_{\text{net}}, \Gamma \to 0$:

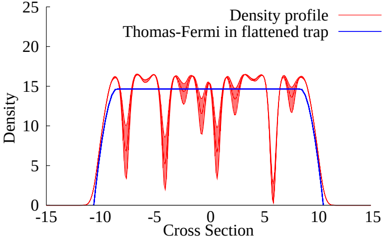
$$\delta\rho_{n,m}(r,\theta,t) = e^{im\theta} h_{n,m}(r) e^{i\omega_{n,m}t}$$

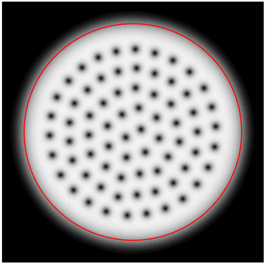
$$\omega_{n,m} = \omega 2\sqrt{m(1+2n) + 2n(n+1)}$$

Add weak pumping/decay:

$$\omega_{n,n} \to \omega_{n,m} + i\gamma_{\text{net}}\left[\frac{m(1+2n) + 2n(n+1) - m^2}{2m(1+2n) + 4n(n+1) + m^2}\right]$$

Why vortices



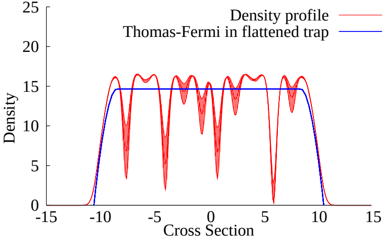


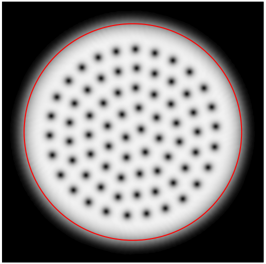
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{net}\Theta(r_0 - r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}|\mathbf{v} - \Omega \times \mathbf{r}|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{net}}{\Gamma}\Theta(r_0 - r) = \frac{\mu}{U}$$

Why vortices





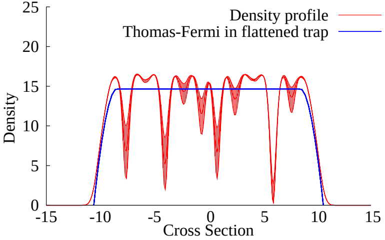
Rotating solution: $i\partial_t\psi = (\mu - 2\Omega L_z)\psi$

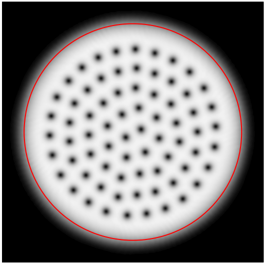
$$\nabla\cdot[\rho(\mathbf{v} - \Omega\times\mathbf{r})] = (\gamma_{net}\Theta(r_0 - r) - \Gamma\rho)\rho,$$

$$\mu = \frac{m}{2}|\mathbf{v} - \Omega\times\mathbf{r}|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega\times\mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{net}}{\Gamma}\Theta(r_0 - r) = \frac{\mu}{U}$$

Why vortices



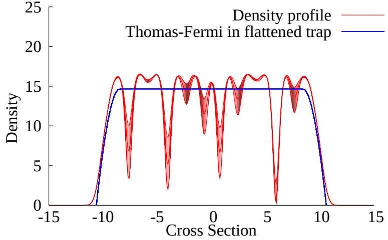


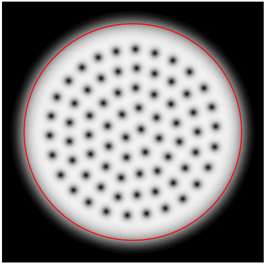
Rotating solution: $i\partial_t\psi = (\mu - 2\Omega L_z)\psi$

$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = \left(\gamma_{\text{net}}\Theta(r_0 - r) - \Gamma\rho\right)\rho,$$

$$\mu = \frac{m}{2}\left|\mathbf{v} - \Omega \times \mathbf{r}\right|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

Why vortices





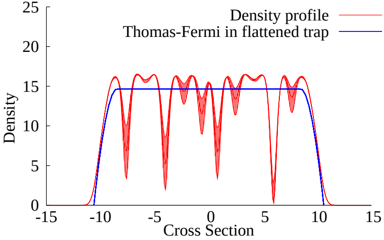
Rotating solution: $i\partial_t\psi = (\mu - 2\Omega L_z)\psi$

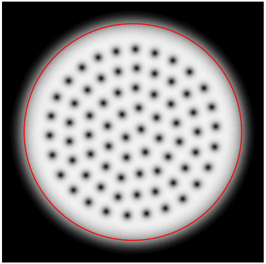
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\mathrm{net}}\Theta(r_0 - r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}\left|\mathbf{v} - \Omega \times \mathbf{r}\right|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r},$$

Why vortices





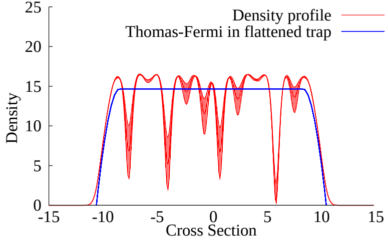
Rotating solution: $i\partial_t\psi = (\mu - 2\Omega L_z)\psi$

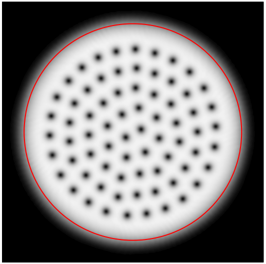
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\text{net}}\Theta(r_0 - r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}|\mathbf{v} - \Omega \times \mathbf{r}|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{\text{net}}}{\Gamma}\Theta(r_0 - r) = \frac{\mu}{U}$$

Why vortices





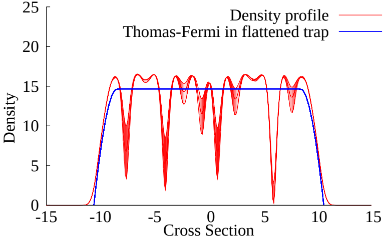
Rotating solution: $i\partial_t \psi = (\mu - 2\Omega L_z)\psi$

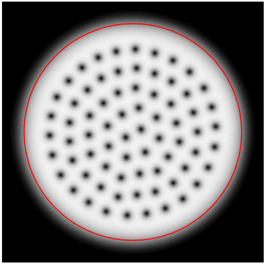
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{net}\Theta(r_0 - r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}|\mathbf{v} - \Omega \times \mathbf{r}|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{net}}{\Gamma}\Theta(r_0 - r) = \frac{\mu}{U}$$

Why vortices





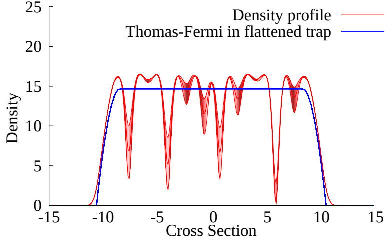
Rotating solution: $i\partial_t\psi = (\mu - 2\Omega L_z)\psi$

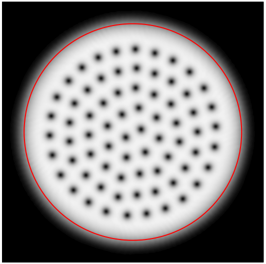
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\text{net}}\Theta(r_0 - r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}|\mathbf{v} - \Omega \times \mathbf{r}|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{\text{net}}}{\Gamma}\Theta(r_0 - r) = \frac{\mu}{U}$$

Why vortices





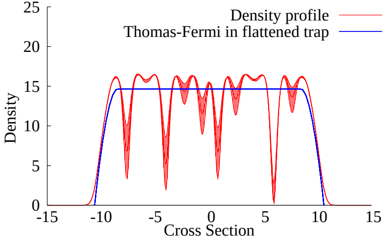
Rotating solution: $i\partial_t \psi = (\mu - 2\Omega L_z)\psi$

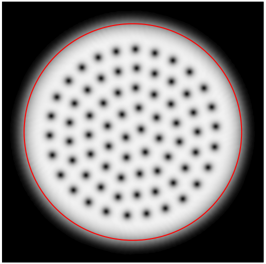
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\text{net}} \Theta(r_0 - r) - \Gamma \rho)\, \rho,$$

$$\mu = \frac{m}{2} |\mathbf{v} - \Omega \times \mathbf{r}|^2 + \frac{m}{2} r^2 (\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2 \nabla^2 \sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{\text{net}}}{\Gamma} \Theta(r_0 - r) = \frac{\mu}{U}$$

Why vortices





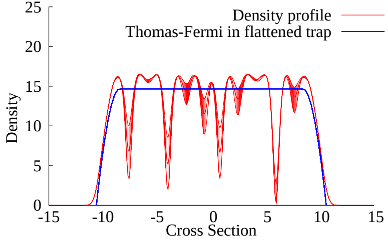
Rotating solution: $i\partial_t \psi = (\mu - 2\Omega L_z)\psi$

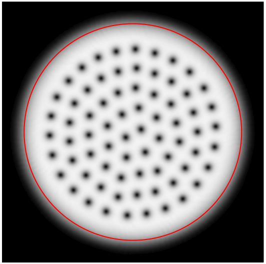
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\rm net}\Theta(r_0 - r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}|\mathbf{v} - \Omega \times \mathbf{r}|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{\rm net}}{\Gamma}\Theta(r_0 - r) = \frac{\mu}{U}$$

Why vortices





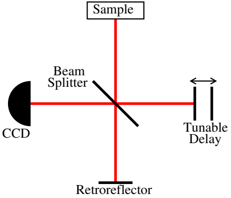
Rotating solution: $i\partial_t\psi = (\mu - 2\Omega L_z)\psi$

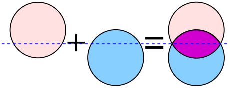
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\text{net}}\Theta(r_0 - r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}|\mathbf{v} - \Omega \times \mathbf{r}|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

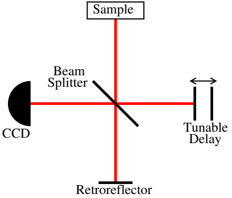
$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{\text{net}}}{\Gamma}\Theta(r_0 - r) = \frac{\mu}{U}$$

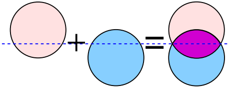
Observing vortices: fringe pattern

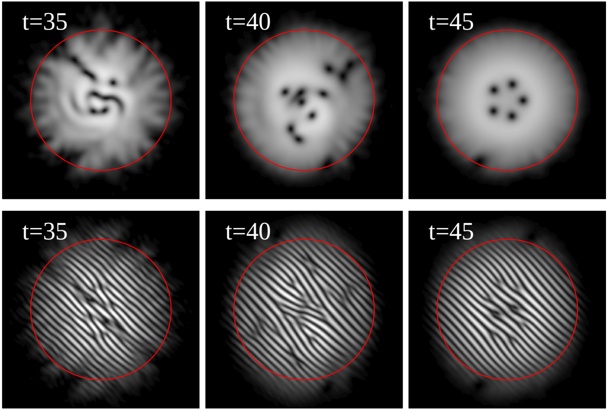




Observing vortices: fringe pattern

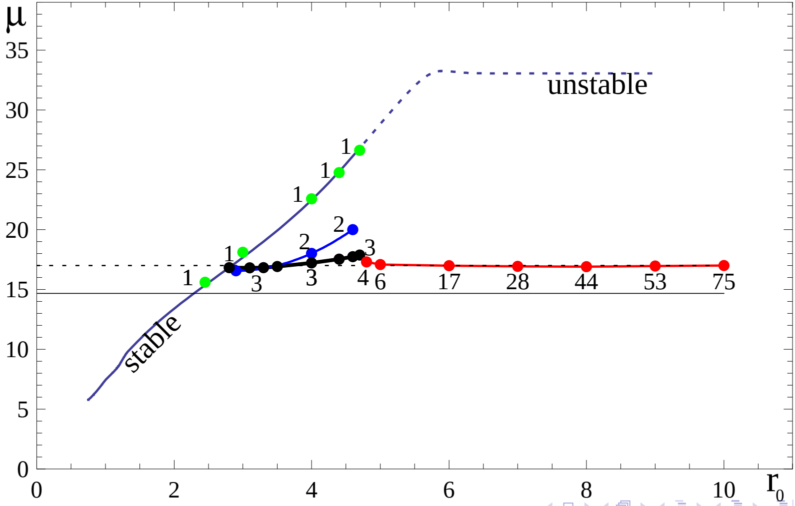






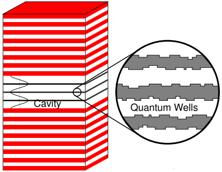
Why vortices: chemical potential vs size

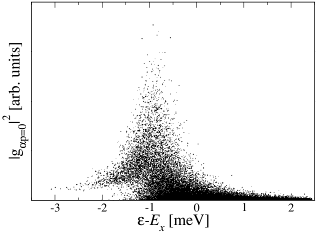
$$\text{Thomas-Fermi} : \mu = f(r_0) \quad \text{Vortex} : \mu = \frac{U\gamma_{\rm net}}{\Gamma}$$



Polariton system model

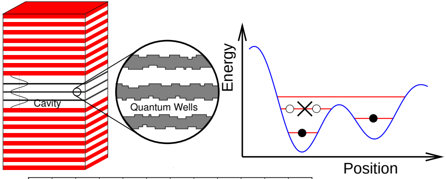
- Disorder-localised excitons
- Treat disorder sites as 2-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling g.

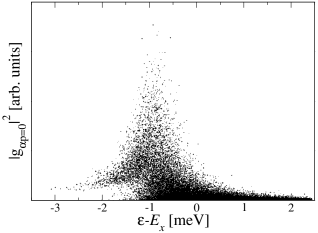




Polariton system model

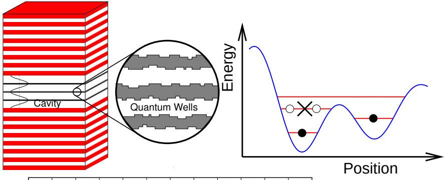
- Disorder-localised excitons
- Treat disorder sites as 2-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling g.

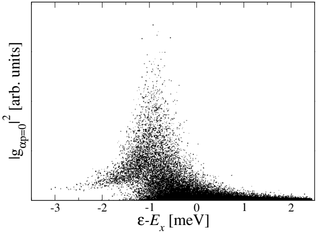




Polariton system model

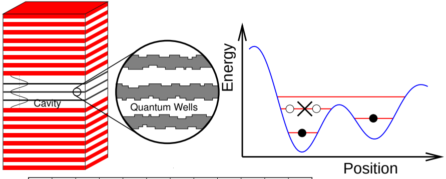
- Disorder-localised excitons
- Treat disorder sites as 2-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling *g*.

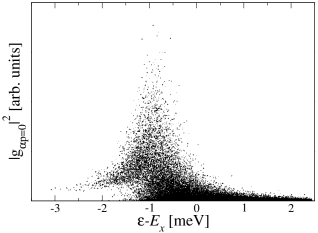




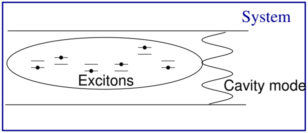
Polariton system model

- Disorder-localised excitons
- Treat disorder sites as 2-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling g.





$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \epsilon_\alpha S^z_\alpha + \sum_\alpha \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} S^+_\alpha \quad + \text{H.c.}$$



Equilibrium: Mean-field theory

Self-consistent polarisation and field

$$\left(-i\partial_t - \omega_0\right)\psi = -\sum_\alpha \frac{g_\alpha}{\sqrt{A}} S^-_\alpha$$

Equilibrium: Mean-field theory

Self-consistent polarisation and field

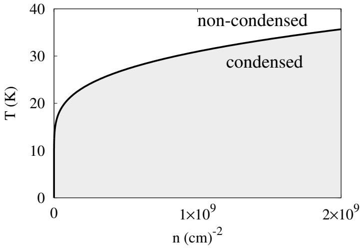
$$(\quad \mu - \omega_0)\psi = -\sum_{\alpha} \frac{g_\alpha}{\sqrt{A}} \frac{g_\alpha \psi}{2E_\alpha} \tanh(\beta E_\alpha), \qquad {E_\alpha}^2 = \left(\frac{\epsilon_\alpha - \mu}{2}\right)^2 + g_\alpha^2 |\psi|^2$$

Equilibrium: Mean-field theory

Self-consistent polarisation and field

$$(\mu - \omega_0)\psi = -\sum_\alpha \frac{g_\alpha}{\sqrt{A}} \frac{g_\alpha \psi}{2E_\alpha} \tanh(\beta E_\alpha), \quad {E_\alpha}^2 = \left(\frac{\epsilon_\alpha - \mu}{2}\right)^2 + g_\alpha^2 |\psi|^2$$

Phase diagram:

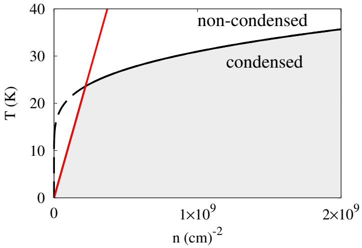


Equilibrium: Mean-field theory

Self-consistent polarisation and field

$$(\quad \mu - \omega_0)\psi = -\sum_\alpha \frac{g_\alpha}{\sqrt{A}} \frac{g_\alpha \psi}{2E_\alpha} \tanh(\beta E_\alpha), \qquad E_\alpha{}^2 = \left(\frac{\epsilon_\alpha - \mu}{2}\right)^2 + g_\alpha^2 |\psi|^2$$

Phase diagram:

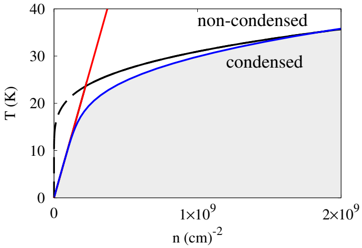


Equilibrium: Mean-field theory

Self-consistent polarisation and field

$$(\mu - \omega_0)\psi = -\sum_\alpha \frac{g_\alpha}{\sqrt{A}} \frac{g_\alpha \psi}{2E_\alpha} \tanh(\beta E_\alpha), \qquad E_\alpha{}^2 = \left(\frac{\epsilon_\alpha - \mu}{2}\right)^2 + g_\alpha^2 |\psi|^2$$

Phase diagram:

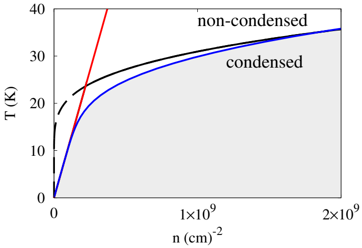


Equilibrium: Mean-field theory

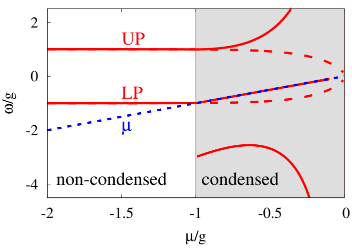
Self-consistent polarisation and field

$$(\quad \mu - \omega_0) \psi = -\sum_\alpha \frac{g_\alpha}{\sqrt{A}} \frac{g_\alpha \psi}{2E_\alpha} \tanh(\beta E_\alpha), \qquad {E_\alpha}^2 = \left(\frac{\epsilon_\alpha - \mu}{2}\right)^2 + g_\alpha^2 |\psi|^2$$

Phase diagram:

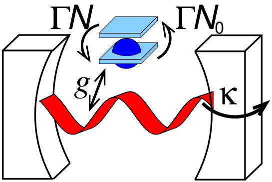


Modes (at $k = 0$)



Simple Laser: Maxwell Bloch equations

$$H = \omega_0 \psi^\dagger \psi + \sum_\alpha \epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi S^+_\alpha + \text{H.c.}$$



Maxwell-Bloch eqns: $P = -i\langle S^- \rangle, N = 2\langle S^z \rangle$

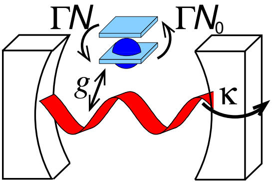
$$\partial_t \psi = -i\omega_0 \psi - \kappa \psi + \sum_\alpha g_\alpha P_\alpha$$

$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$

$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P^*_\alpha \psi)$$

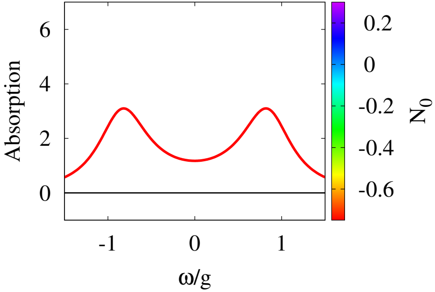
Simple Laser: Maxwell Bloch equations

$$H = \omega_0 \psi^\dagger \psi + \sum_\alpha \epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi S^+_\alpha + \text{H.c.}$$



Maxwell-Bloch eqns: $P = -i\langle S^- \rangle, N = 2\langle S^z \rangle$

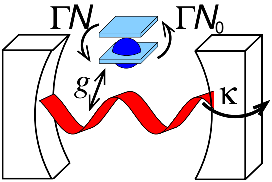
$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$
$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$
$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P^*_\alpha \psi)$$



- Strong coupling. $\kappa, \gamma < g\sqrt{n}$
- Inversion causes collapse before lasing

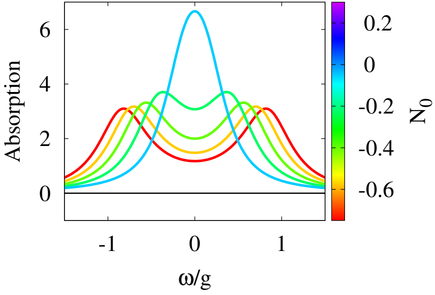
Simple Laser: Maxwell Bloch equations

$$H = \omega_0 \psi^\dagger \psi + \sum_\alpha \epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi S^+_\alpha + \text{H.c.}$$



Maxwell-Bloch eqns: $P = -i\langle S^- \rangle, N = 2\langle S^z \rangle$

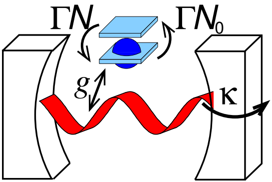
$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$
$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$
$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P^*_\alpha \psi)$$



- Strong coupling. $\kappa, \gamma < g\sqrt{n}$
- Inversion causes collapse before lasing

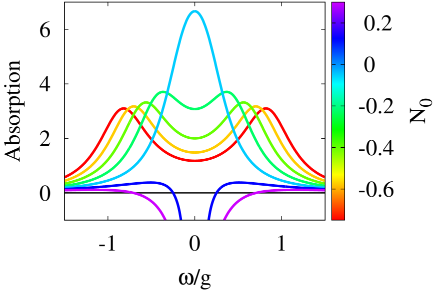
Simple Laser: Maxwell Bloch equations

$$H = \omega_0 \psi^\dagger \psi + \sum_\alpha \epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi S^+_\alpha + \text{H.c.}$$



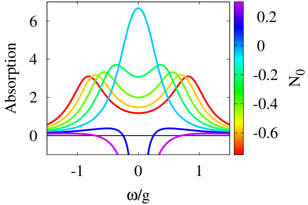
Maxwell-Bloch eqns: $P = -i\langle S^- \rangle, N = 2\langle S^z \rangle$

$$\partial_t \psi = -i\omega_0 \psi - \kappa \psi + \sum_\alpha g_\alpha P_\alpha$$
$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$
$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P^*_\alpha \psi)$$



- Strong coupling. $\kappa, \gamma < g\sqrt{n}$
- Inversion causes collapse before lasing

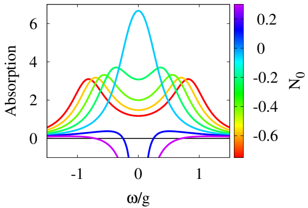
Maxwell-Bloch Equations: Retarded Green's function



- Introduce $D^R(\omega)$:
  Response to perturbation

- Absorption $= -2\Im[D^R(\omega)]$

Maxwell-Bloch Equations: Retarded Green's function



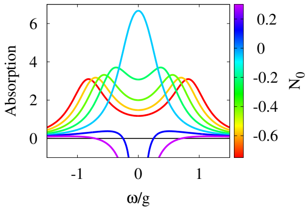
- Introduce $D^R(\omega)$:
  Response to perturbation

$$\partial_t\psi = -i\omega_0\psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$
$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$
$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P_\alpha^* \psi)$$

- Absorption $= -2\Im[D^R(\omega)]$

$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma}$$

Maxwell-Bloch Equations: Retarded Green's function



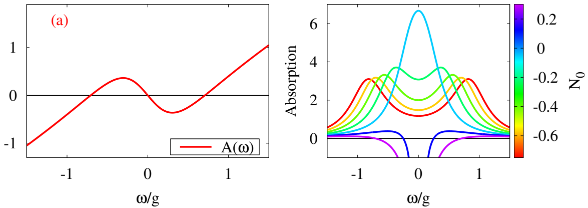
- Introduce $D^R(\omega)$: Response to perturbation

$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$
$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$
$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P^*_\alpha \psi)$$

- Absorption $= -2\Im[D^R(\omega)] = \dfrac{2B(\omega)}{A(\omega)^2 + B(\omega)^2}$

$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma} = A(\omega) + iB(\omega)$$

Maxwell-Bloch Equations: Retarded Green's function



$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$

- Introduce $D^R(\omega)$: Response to perturbation

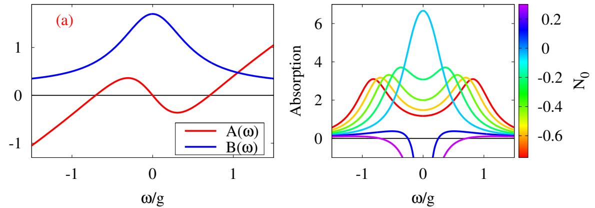
$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$

$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P_\alpha^* \psi)$$

- Absorption $= -2\Im[D^R(\omega)] = \dfrac{2B(\omega)}{A(\omega)^2 + B(\omega)^2}$

$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma} = A(\omega) + iB(\omega)$$

Maxwell-Bloch Equations: Retarded Green's function



$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$

- Introduce $D^R(\omega)$: Response to perturbation

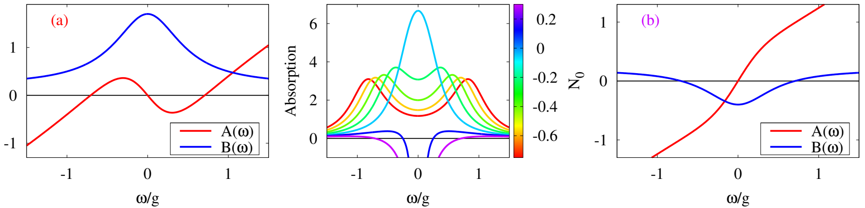
$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$

$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P_\alpha^* \psi)$$

- Absorption $= -2\Im[D^R(\omega)] = \dfrac{2B(\omega)}{A(\omega)^2 + B(\omega)^2}$

$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma} = A(\omega) + iB(\omega)$$

Maxwell-Bloch Equations: Retarded Green's function



$$\partial_t \psi = -i\omega_0 \psi - \kappa\psi + \sum_\alpha g_\alpha P_\alpha$$

- Introduce $D^R(\omega)$:
  Response to perturbation

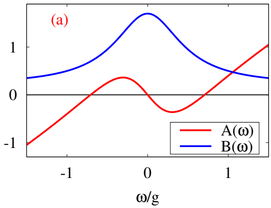
$$\partial_t P_\alpha = -2i\epsilon_\alpha P_\alpha - 2\gamma P + g_\alpha \psi N_\alpha$$

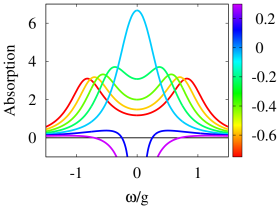
$$\partial_t N_\alpha = 2\gamma(N_0 - N_\alpha) - 2g_\alpha(\psi^* P_\alpha + P_\alpha^* \psi)$$

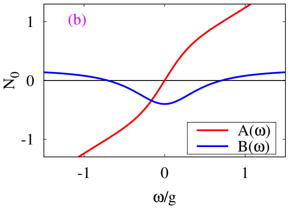
- Absorption $= -2\Im[D^R(\omega)] = \dfrac{2B(\omega)}{A(\omega)^2 + B(\omega)^2}$

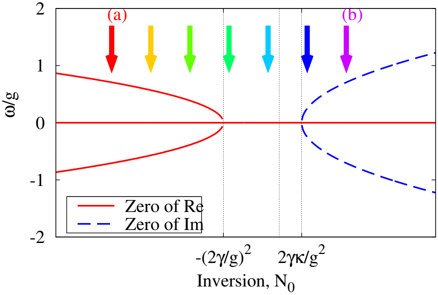
$$\left[D^R(\omega)\right]^{-1} = \omega - \omega_k + i\kappa + \frac{g^2 N_0}{\omega - 2\epsilon + i2\gamma} = A(\omega) + iB(\omega)$$

Evolution of poles with Inversion

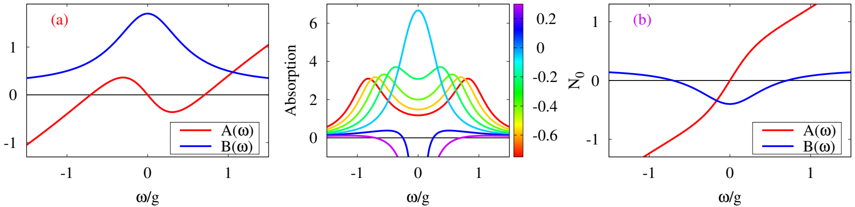




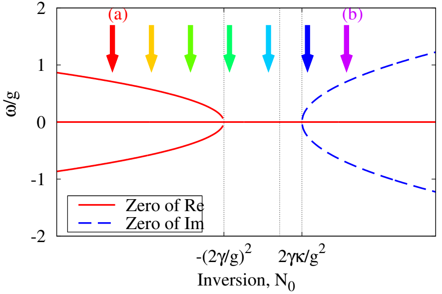




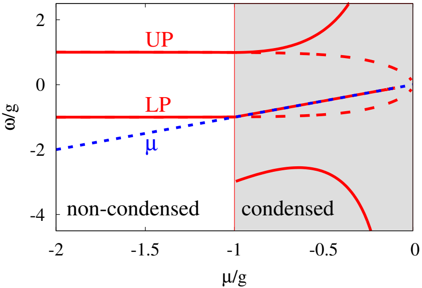
Evolution of poles with Inversion



Laser:



Equilibrium:



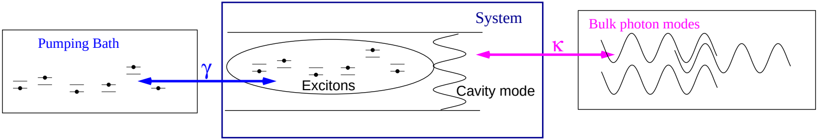
5 Microscopic model: lasing vs condensation

- Model: localised excitons, propagating photons
- Simple laser: Maxwell-Bloch
- Non-equilibrium polaritons: coherence and strong coupling

6 Measuring superfluid density

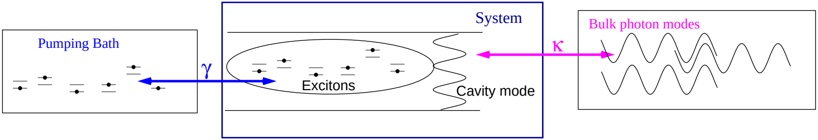
7 Coherence Finite size and Schawlow-Townes

Non-equilibrium description: baths



$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

Non-equilibrium description: baths

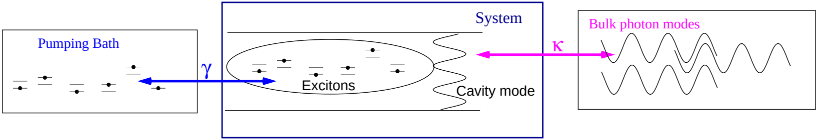


$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

Schematically: pump γ, decay κ

$$H_{\text{sys,bath}} \simeq \sum_{\mathbf{p},\mathbf{k}} \sqrt{\kappa} \psi_{\mathbf{k}} \Psi^{\dagger}_{\mathbf{p}} + \sum_{\alpha,\beta} \sqrt{\gamma} \left(a^{\dagger}_{\alpha} A_{\beta} + b^{\dagger}_{\alpha} B_{\beta} \right) + \text{H.c.}$$

Non-equilibrium description: baths



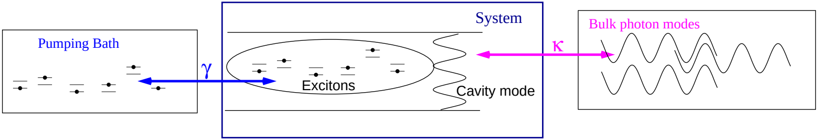
$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

Schematically: pump γ, decay κ

$$H_{\text{sys,bath}} \simeq \sum_{\mathbf{p},\mathbf{k}} \sqrt{\kappa}\psi_{\mathbf{k}}\Psi^{\dagger}_{\mathbf{p}} + \sum_{\alpha,\beta} \sqrt{\gamma}\left(a^{\dagger}_{\alpha}A_{\beta} + b^{\dagger}_{\alpha}B_{\beta}\right) + \text{H.c.}$$

Bath correlations, $\langle \Psi^{\dagger}\Psi \rangle$, $\langle A^{\dagger}A \rangle$, $\langle B^{\dagger}B \rangle$ fixed:

Non-equilibrium description: baths

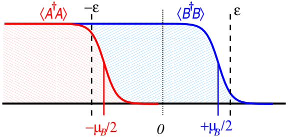


$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

Schematically: pump γ, decay κ

$$H_{\text{sys,bath}} \simeq \sum_{\mathbf{p},\mathbf{k}} \sqrt{\kappa}\psi_{\mathbf{k}}\Psi^\dagger_{\mathbf{p}} + \sum_{\alpha,\beta} \sqrt{\gamma}\left(a^\dagger_\alpha A_\beta + b^\dagger_\alpha B_\beta\right) + \text{H.c.}$$

Bath correlations, $\langle\Psi^\dagger\Psi\rangle$, $\langle A^\dagger A\rangle$, $\langle B^\dagger B\rangle$ fixed:
Ψ bath is empty. Pumping bath thermal, μ_B, T_B:



Non-equilibrium mean-field theory

Look for mean-field solution, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$. Gap equation:

$$(\mu_S - \omega_0 + i\kappa)\,\psi_0 = \chi(\psi_0, \mu_S)\psi_0$$

Non-equilibrium mean-field theory

Look for mean-field solution, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$. Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi_0 = \chi(\psi_0, \mu_s)\psi_0$$

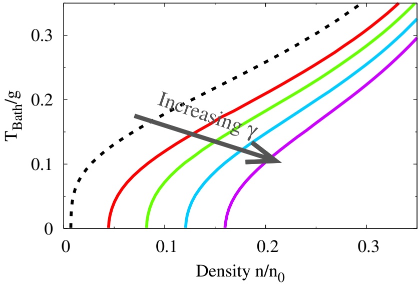
Susceptibility $\chi = \chi(\psi_0, \mu_s, \mu_B, T_b, \gamma)$

Non-equilibrium mean-field theory

Look for mean-field solution, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_s t}$. Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi_0 = \chi(\psi_0, \mu_s)\psi_0$$

Susceptibility $\chi = \chi(\psi_0, \mu_s, \mu_B, T_b, \gamma)$



Luminescence spectrum and Green's functions

$$-2\Im[D^R(\omega)] = \mathrm{DoS}(\omega)$$

$$\left[D^R(\omega)\right]^{-1} = A(\omega) + iB(\omega)$$

$$\mathrm{DoS}(\omega) = \frac{2B(\omega)}{B(\omega)^2 + A(\omega)^2}$$

Luminescence spectrum and Green's functions

$$-2\Im[D^R(\omega)] = \mathrm{DoS}(\omega) \qquad iD^K(\omega) = (2n(\omega)+1)\mathrm{DoS}(\omega)$$

$$\left[D^R(\omega)\right]^{-1} = A(\omega) + iB(\omega)$$

$$\mathrm{DoS}(\omega) = \frac{2B(\omega)}{B(\omega)^2 + A(\omega)^2}$$

Luminescence spectrum and Green's functions

$$-2\Im[D^R(\omega)] = \text{DoS}(\omega) \qquad iD^K(\omega) = (2n(\omega)+1)\text{DoS}(\omega)$$

$$\left[D^R(\omega)\right]^{-1} = A(\omega) + iB(\omega)$$

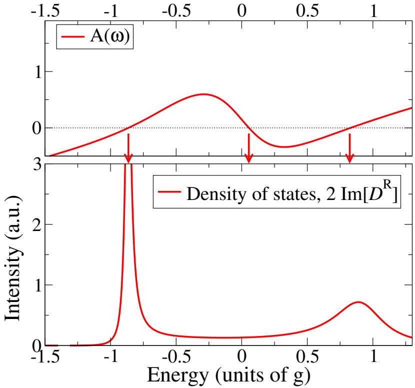
$$\text{DoS}(\omega) = \frac{2B(\omega)}{B(\omega)^2 + A(\omega)^2}$$

Luminescence spectrum and Green's functions

$$-2\Im[D^R(\omega)] = \text{DoS}(\omega) \qquad iD^K(\omega) = (2n(\omega)+1)\text{DoS}(\omega)$$

$$\left[D^R(\omega)\right]^{-1} = A(\omega) + iB(\omega)$$

$$\text{DoS}(\omega) = \frac{2B(\omega)}{B(\omega)^2 + A(\omega)^2}$$

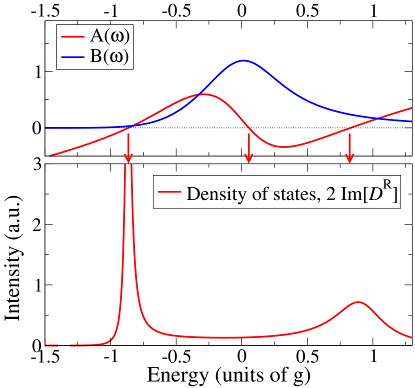


Luminescence spectrum and Green's functions

$$-2\Im[D^R(\omega)] = \text{DoS}(\omega) \qquad iD^K(\omega) = (2n(\omega)+1)\text{DoS}(\omega)$$

$$\left[D^R(\omega)\right]^{-1} = A(\omega) + iB(\omega)$$

$$\text{DoS}(\omega) = \frac{2B(\omega)}{B(\omega)^2 + A(\omega)^2}$$



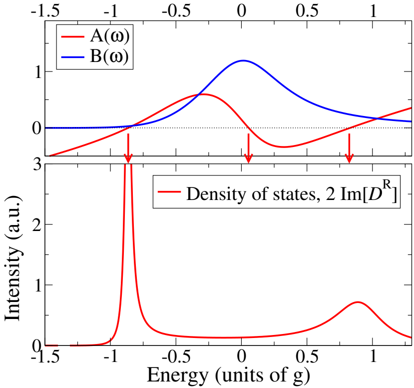
Luminescence spectrum and Green's functions

$$-2\Im[D^R(\omega)] = \mathrm{DoS}(\omega) \qquad iD^K(\omega) = (2n(\omega)+1)\mathrm{DoS}(\omega)$$

$$\left[D^R(\omega)\right]^{-1} = A(\omega) + iB(\omega)$$

$$\mathrm{DoS}(\omega) = \frac{2B(\omega)}{B(\omega)^2 + A(\omega)^2}$$

$$iD^K(\omega) = \frac{C(\omega)}{B(\omega)^2 + A(\omega)^2}$$



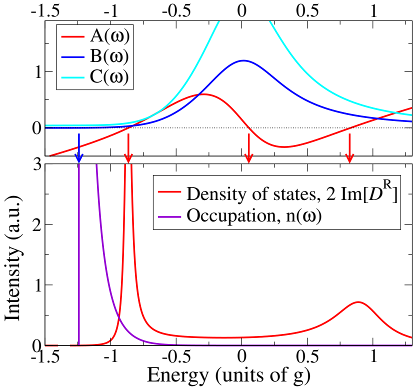
Luminescence spectrum and Green's functions

$$-2\Im[D^R(\omega)] = \mathrm{DoS}(\omega) \qquad iD^K(\omega) = (2n(\omega)+1)\mathrm{DoS}(\omega)$$

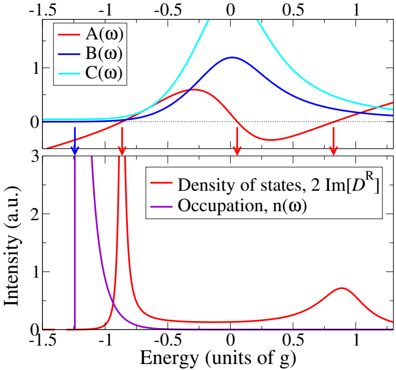
$$\left[D^R(\omega)\right]^{-1} = A(\omega) + iB(\omega)$$

$$\mathrm{DoS}(\omega) = \frac{2B(\omega)}{B(\omega)^2 + A(\omega)^2}$$

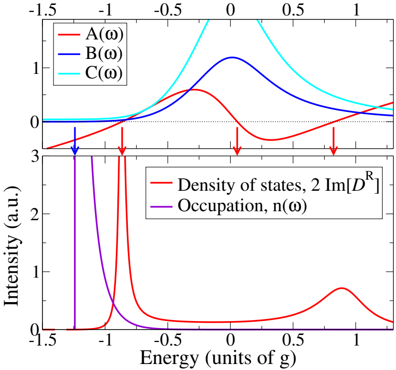
$$iD^K(\omega) = \frac{C(\omega)}{B(\omega)^2 + A(\omega)^2}$$



Stability and evolution with pumping

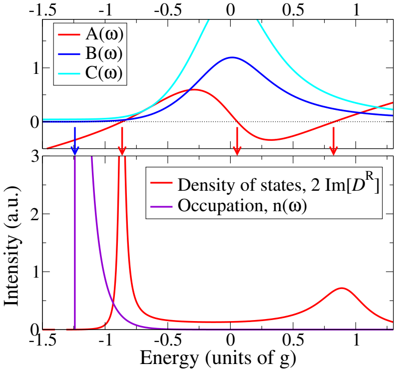


Stability and evolution with pumping



$$\left[D^R(\omega)\right]^{-1} = (\omega - \xi_k) + i\alpha(\omega - \mu_{\text{eff}})$$

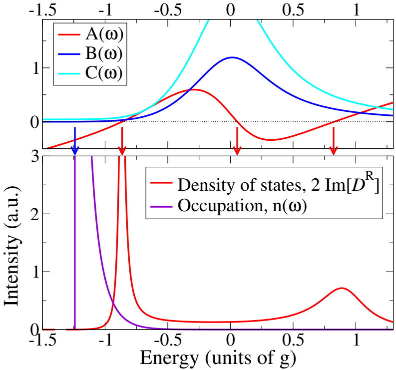
Stability and evolution with pumping

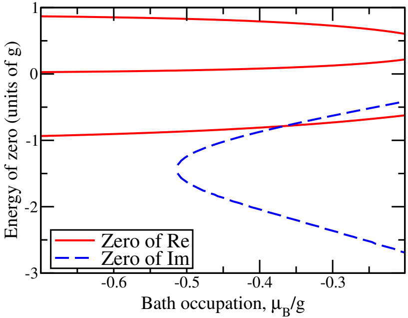


$$\left[D^R(\omega)\right]^{-1} = (\omega - \xi_k) + i\alpha(\omega - \mu_{\text{eff}})$$

$$\left[D^R(\omega_k^*)\right]^{-1} = 0 \rightarrow \Im(\omega^*) \propto \mu_{\text{eff}} - \xi_k$$

Stability and evolution with pumping

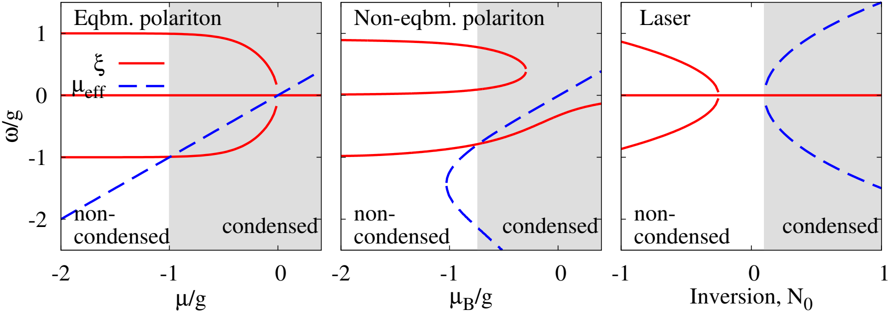




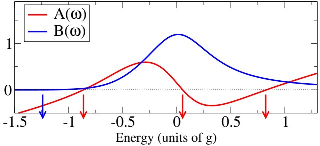
$$\left[D^R(\omega)\right]^{-1} = (\omega - \xi_k) + i\alpha(\omega - \mu_{\text{eff}})$$

$$\left[D^R(\omega_k^*)\right]^{-1} = 0 \rightarrow \Im(\omega^*) \propto \mu_{\text{eff}} - \xi_k$$

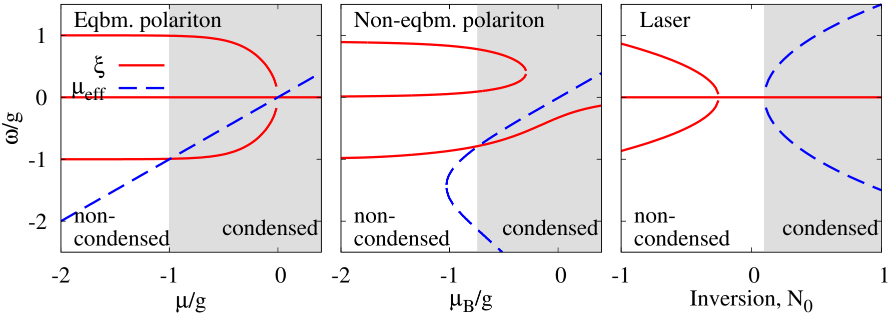
Strong coupling and lasing — low temperature phenomenon



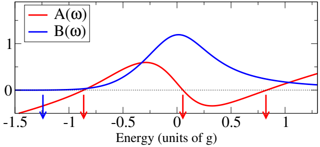
- Laser: Uniformly invert TLS
- Non-equilibrium polaritons: Cold bath
- If $T_B \gg \gamma \rightarrow$ Laser limit



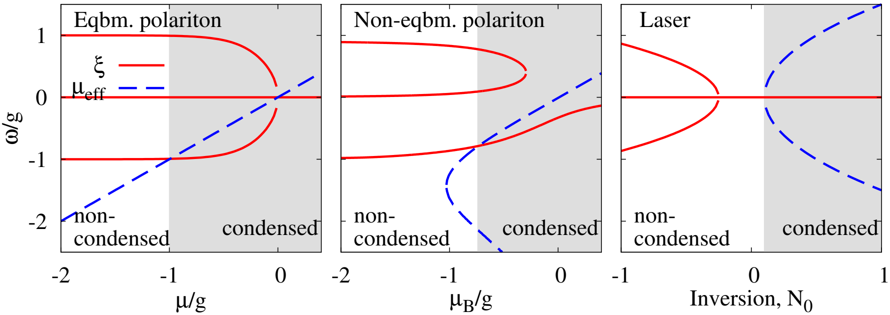
Strong coupling and lasing — low temperature phenomenon



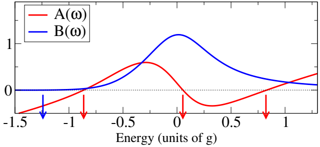
- Laser: Uniformly invert TLS
- Non-equilibrium polaritons: Cold bath



Strong coupling and lasing — low temperature phenomenon



- Laser: Uniformly invert TLS
- Non-equilibrium polaritons: Cold bath
- If $T_B \gg \gamma \rightarrow$ Laser limit

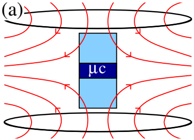


Measuring superfluid density

1. Effect rotating frame

Polariton polarization: $(\psi_{\circlearrowleft}, \psi_{\circlearrowright})$

$$H = \lambda \begin{pmatrix} \ell^2 & r^2 e^{2i\phi} \\ r^2 e^{-2i\phi} & -\ell^2 \end{pmatrix}$$



Measuring superfluid density

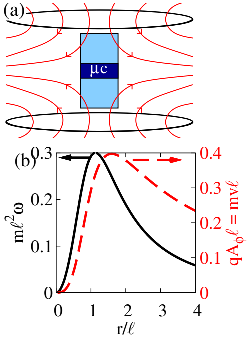
1. Effect rotating frame

Polariton polarization: $(\psi_{\circlearrowleft}, \psi_{\circlearrowright})$

$$H = \lambda \begin{pmatrix} \ell^2 & r^2 e^{2i\phi} \\ r^2 e^{-2i\phi} & -\ell^2 \end{pmatrix}$$

Ground state Berry phase:

$$q\mathbf{A}_{\text{eff}} = m\omega \times \mathbf{r} = \frac{\hat{\phi}}{r}\left[1 - \frac{\ell^2}{\sqrt{r^4 + \ell^4}}\right]$$



Measuring superfluid density

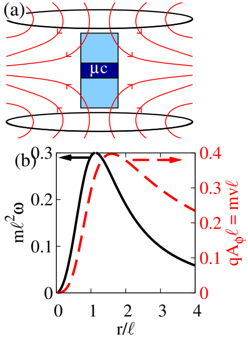
1. Effect rotating frame

Polariton polarization: $(\psi_{\circlearrowleft}, \psi_{\circlearrowright})$

$$H = \lambda \begin{pmatrix} \ell^2 & r^2 e^{2i\phi} \\ r^2 e^{-2i\phi} & -\ell^2 \end{pmatrix}$$

Ground state Berry phase:

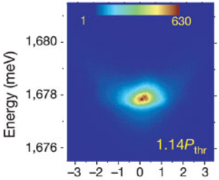
$$q\mathbf{A}_{\text{eff}} = m\omega \times \mathbf{r} = \frac{\hat{\phi}}{r}\left[1 - \frac{\ell^2}{\sqrt{r^4 + \ell^4}}\right]$$



2. Measure resulting current

Energy shift of normal state:

$\Delta E = (1/2)mv^2 = 0.08/m\ell^2 \simeq 0.1\text{meV}$



Finite size effects: Single mode vs many mode

$$\left\langle \psi^{\dagger}(\mathbf{r}, t)\psi(\mathbf{r}', 0)\right\rangle \simeq |\psi_0|^2 \exp\left[-D^{<}_{\phi\phi}(\mathbf{r}, \mathbf{r}', t)\right]$$

Finite size effects: Single mode vs many mode

$$\left\langle \psi^\dagger(\mathbf{r},t)\psi(\mathbf{r}',0)\right\rangle \simeq |\psi_0|^2 \exp\left[-D^<_{\phi\phi}(\mathbf{r},\mathbf{r}',t)\right]$$

$D^<_{\phi\phi}(\mathbf{r},\mathbf{r}',t)$ from sum of phase modes. Study $ct \gg r$ limit:

$$D^<_{\phi\phi}(\mathbf{r},\mathbf{r},t) \propto \sum_n^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(\mathbf{r})|^2(1-e^{i\omega t})}{\left|(\omega+i\gamma_{\rm net})^2+\gamma_{\rm net}^2-\xi_n^2\right|^2}$$

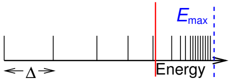
Finite size effects: Single mode vs many mode

$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0)\right\rangle \simeq |\psi_0|^2 \exp\left[-D^<_{\phi\phi}(\mathbf{r}, \mathbf{r}', t)\right]$$

$D^<_{\phi\phi}(\mathbf{r}, \mathbf{r}', t)$ from sum of phase modes. Study $ct \gg r$ limit:

$$D^<_{\phi\phi}(\mathbf{r}, \mathbf{r}, t) \propto \sum_n^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(\mathbf{r})|^2(1 - e^{i\omega t})}{\left|(\omega + i\gamma_{\text{net}})^2 + \gamma_{\text{net}}^2 - \xi_n^2\right|^2}$$

$$\Delta\xi \ll \sqrt{\frac{\gamma_{\text{net}}}{t}} \ll E_{\text{max}}$$



$$D^<_{\phi\phi} \sim 1 + \ln\left(E_{\text{max}}\sqrt{\frac{t}{\gamma_{\text{net}}}}\right)$$

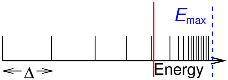
Finite size effects: Single mode vs many mode

$$\left\langle \psi^\dagger(\mathbf{r},t)\psi(\mathbf{r}',0)\right\rangle \simeq |\psi_0|^2 \exp\left[-D^{<}_{\phi\phi}(\mathbf{r},\mathbf{r}',t)\right]$$

$D^{<}_{\phi\phi}(\mathbf{r},\mathbf{r}',t)$ from sum of phase modes. Study $ct \gg r$ limit:

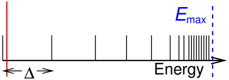
$$D^{<}_{\phi\phi}(\mathbf{r},\mathbf{r},t) \propto \sum_{n}^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(\mathbf{r})|^2(1-e^{i\omega t})}{\left|(\omega + i\gamma_{\text{net}})^2 + \gamma_{\text{net}}^2 - \xi_n^2\right|^2}$$

$$\Delta\xi \ll \sqrt{\frac{\gamma_{\text{net}}}{t}} \ll E_{\text{max}}$$



$$D^{<}_{\phi\phi} \sim 1 + \ln\left(E_{\text{max}}\sqrt{\frac{t}{\gamma_{\text{net}}}}\right)$$

$$\sqrt{\frac{\gamma_{\text{net}}}{t}} \ll \Delta\xi \ll E_{\text{max}}$$



$$D^{<}_{\phi\phi} \sim \left(\frac{\pi C}{2\gamma_{\text{net}}}\right)\left(\frac{t}{2\gamma_{\text{net}}}\right)$$

(Recovers Schawlow-Townes laser linewidth)