Collective Dynamics of a Generalized Dicke Model

J. Keeling, J. A. Mayoh, M. J. Bhaseen, B. D. Simons

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SUPA

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Coupling many atoms to light

Old question: *What happens to radiation when many atoms interact "collectively" with light.*

Superradiance — dynamical and steady state.

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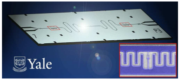
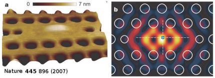
New relevance

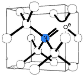
- Superconducting qubits



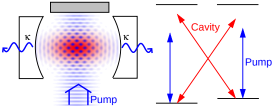
- Quantum dots



- Nitrogen-vacancies in diamond



- Ultra-cold atoms



- Rydberg atoms

Dicke effect: Enhanced emission

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

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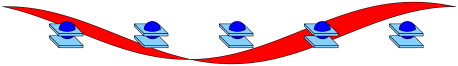

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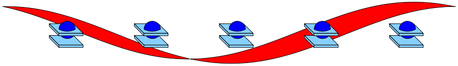

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Collective decay:

$$\frac{d\rho}{dt} = -\frac{\Gamma}{2}\left[S^+ S^- \rho - S^- \rho S^+ + \rho S^+ S^- \right]$$

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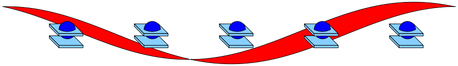

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If $S^z = |S| = N/2$ initially:

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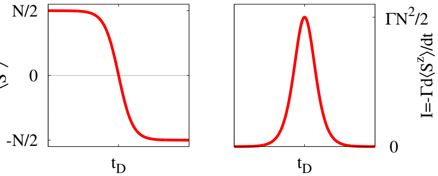

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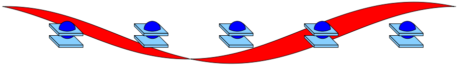

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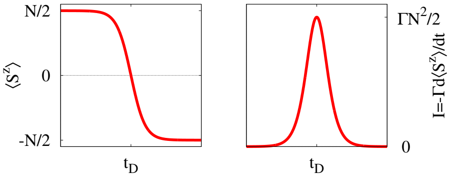
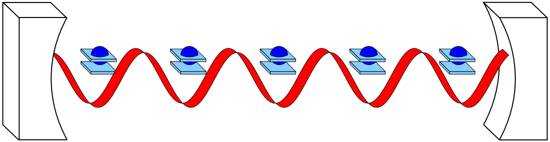


Problem: dipole interactions dephase. [Friedberg et al, Phys. Lett. 1972]

Collective radiation **with a cavity**: Dynamics



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Single cavity mode: oscillations

[Bonifacio and Preparata PRA 1970; **JK** PRA 2009]

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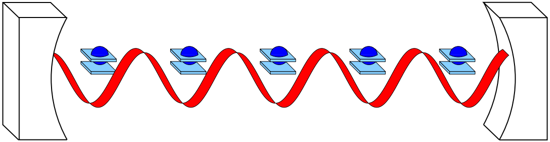

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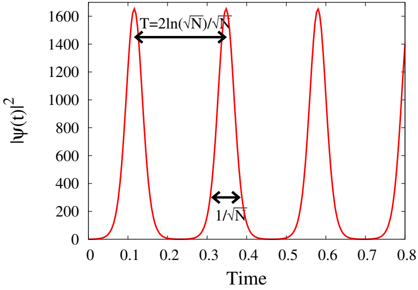

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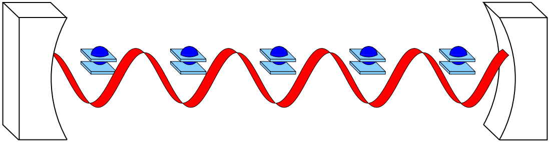

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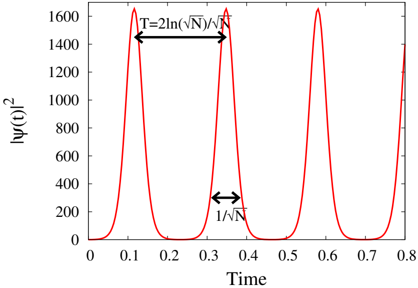

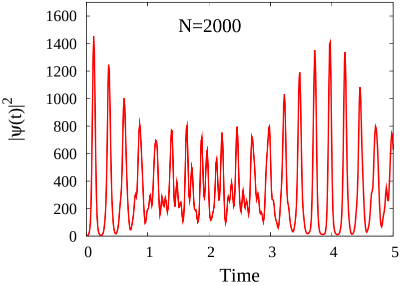

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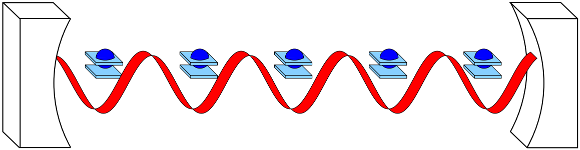
Dicke model: Equilibrium superradiance transition



$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g\left(\psi^\dagger S^- + \psi S^+\right).$$

- Coherent state: $|\Psi\rangle \to e^{\lambda\psi^\dagger + \eta S^+}|\Omega\rangle$
- Energy:

$$E = \omega|\lambda|^2 + \frac{\omega_0 N}{2}\frac{|\eta|^2-1}{|\eta|^2+1} + gN\frac{\eta^*\lambda+\lambda^*\eta}{1+|\eta|^2}$$

- Small g, min at $\lambda, \eta = 0$

[Hepp, Lieb, Ann. Phys. '73]

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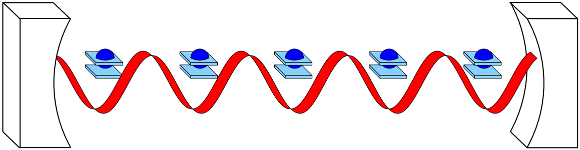

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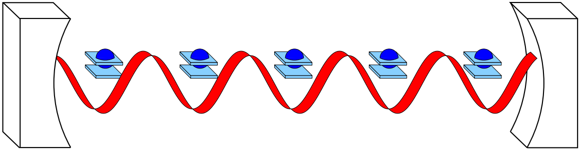

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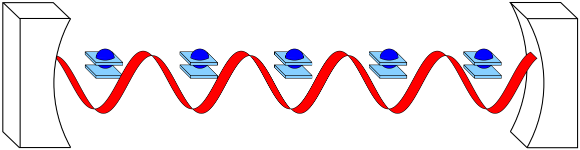

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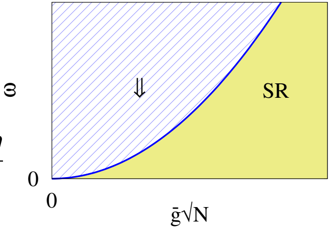
Spontaneous polarisation if: $Ng^2 > \omega\omega_0$



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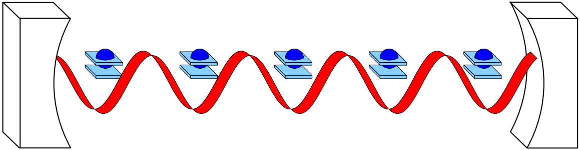
No go theorem and transition



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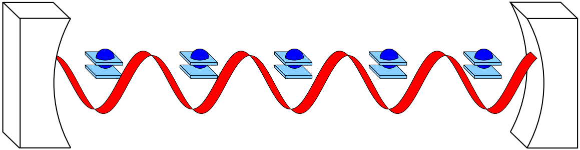

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$$-\sum_i \frac{e}{m} A \cdot p_i \Leftrightarrow g(\psi^\dagger S^- + \psi S^+), \qquad \sum_i \frac{A^2}{2m} \Leftrightarrow N\zeta(\psi + \psi^\dagger)^2$$

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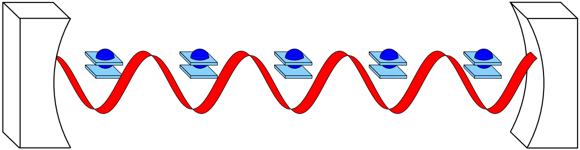

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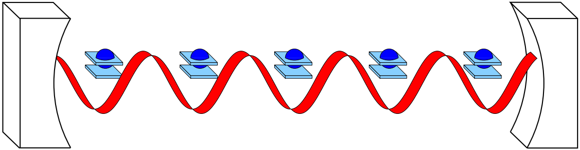

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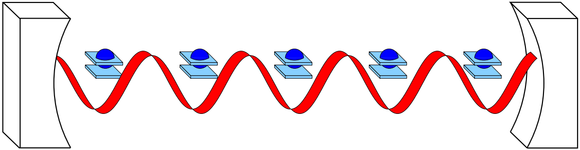

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But Thomas-Reiche-Kuhn sum rule states: $g^2/\omega_0 < 2\zeta$. **No transition**

[Rzazewski *et al* PRL '75]

Dicke phase transition: ways out

Problem: $g^2/\omega_0 < 2\zeta$ for intrinsic parameters.
Solutions:

- **Non-solution**
  Ferroelectric transition in $\mathbf{D}\cdot\mathbf{r}$ gauge.
  [JK JPCM '07]
- Grand canonical ensemble:
  - If $H \to H - \mu(S^z + \psi^\dagger\psi)$, need only:
    $g^2 N > (\omega - \mu)(\omega_0 - \mu)$
  - Incoherent pumping — polariton condensation.
- Dissociate g, ω_0.
  e.g. Raman scheme: $\omega_0 \ll \omega$.
  [Dimer *et al* PRA '07; Baumann *et al* Nature '10]
- See also [Nataf and Ciuti, Nat. Comm. '10; Viehmann *et al* PRL '11]

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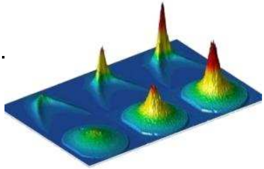

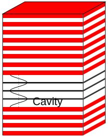

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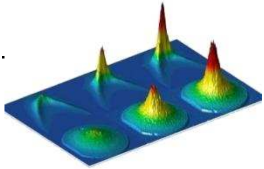

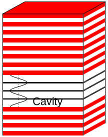

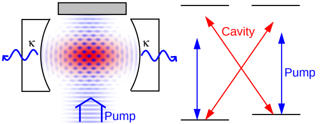

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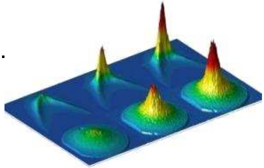

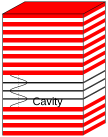

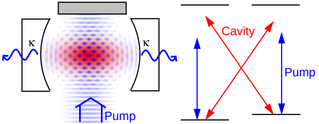

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   - Rayleigh scheme: Generalised Dicke model
2. Attractors of dynamics (fixed points)
   - Phase diagram: comparison to experiment
3. Approach to attractors: timescales
   - Why slow timescales emerge
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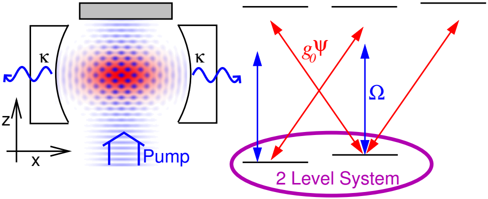
Extended Dicke model

[Baumann *et al.* Nature 2010]



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$\Downarrow$: $\Psi(x,z) = 1$

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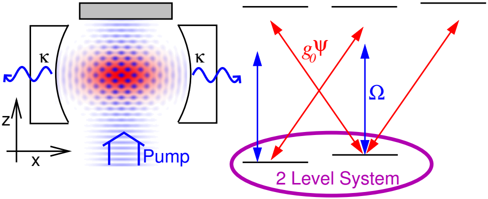

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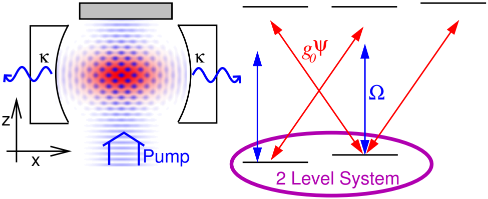
Extended Dicke model

[Baumann *et al.* Nature 2010]



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$\Downarrow$: $\Psi(x,z) = 1$

$\Uparrow$: $\Psi(x,z) = \sum_{\sigma,\sigma'=\pm} e^{ik(\sigma x + \sigma' z)}$

$\omega_0 = 2\omega_{\text{recoil}}$

Feedback: $U \propto \dfrac{g_0^2}{\omega_c - \omega_a}$

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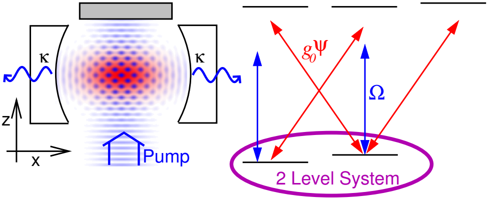

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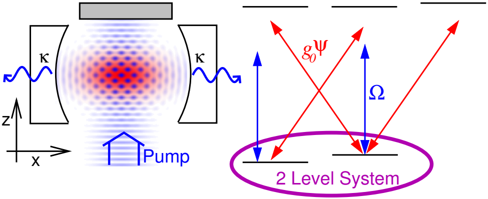

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Semiclassical EOM
$(|\mathbf{S}| = N/2 \gg 1)$

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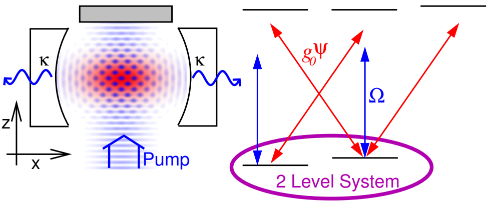

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$\omega_0 \sim$ kHz $\ll \omega, \kappa, g\sqrt{N} \sim$ MHz.

Outline

Fixed points (steady states)

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
$$0 = ig(\psi + \psi^*)(S^- - S^+)$$
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- $\psi = 0, \mathbf{S} = (0, 0, \pm N/2)$ always a solution.
- If $g > g_c$, $\psi \neq 0$ too
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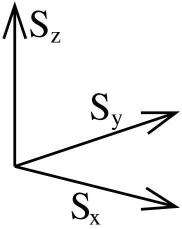

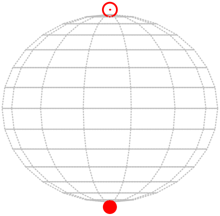

Small g: $\Uparrow, \Downarrow$ only.
($\omega = 30\text{MHz}$, $UN = -40\text{MHz}$)

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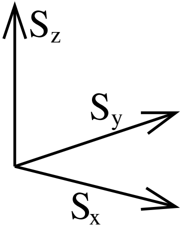
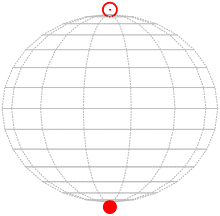
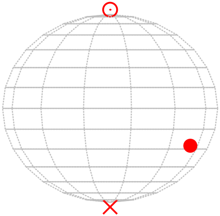




Small g: ⇑, ⇓ only.



Larger g: SR too.

($\omega = 30$MHz, $UN = -40$MHz)

Steady state phase diagram

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
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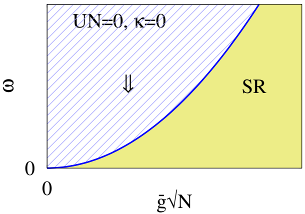

See also Domokos and Ritsch PRL '02, Domokos *et al.* PRL '10

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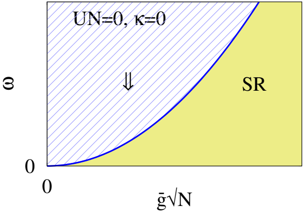

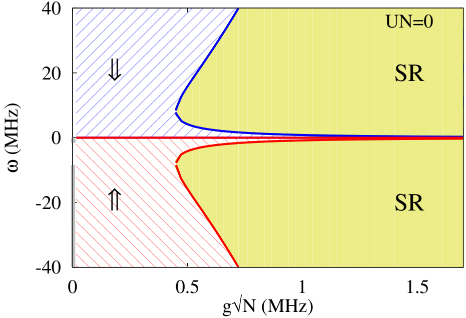

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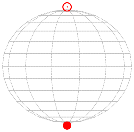
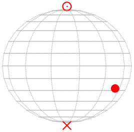


SR(A): $S_y = 0$



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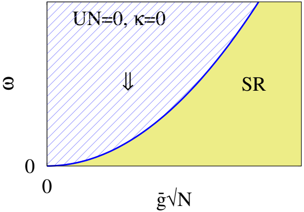

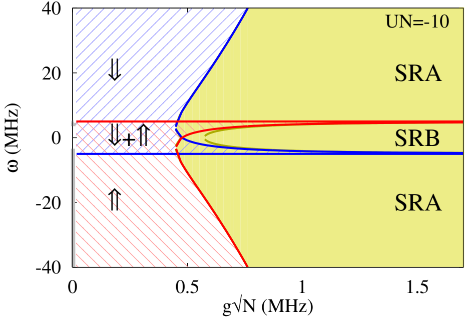

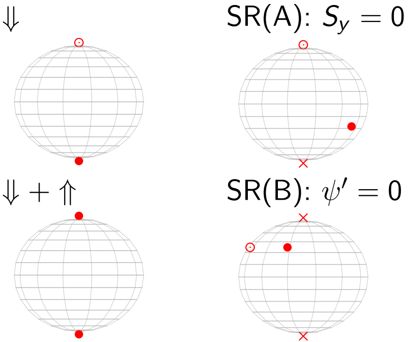

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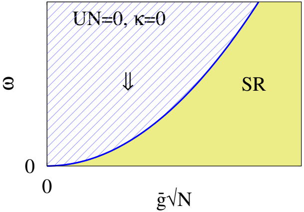

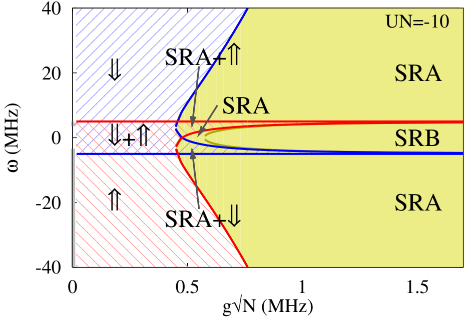

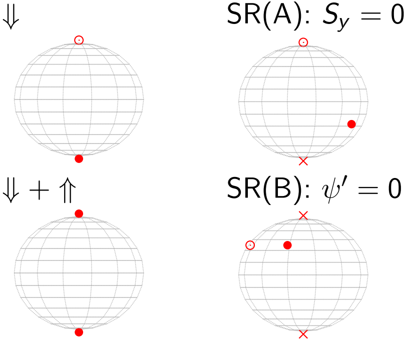

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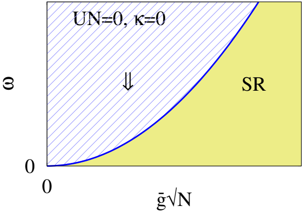

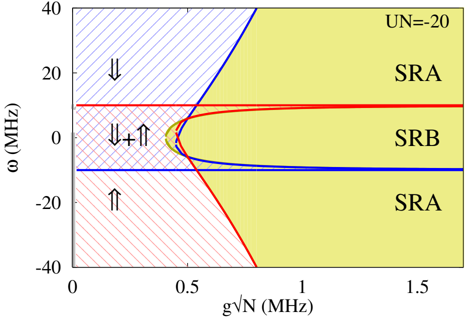

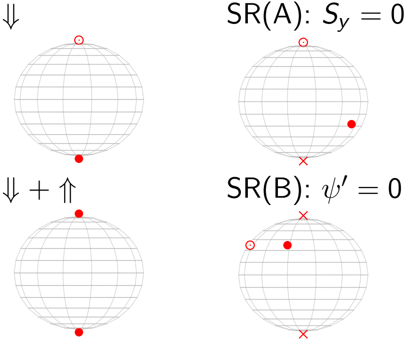

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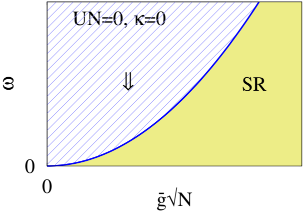

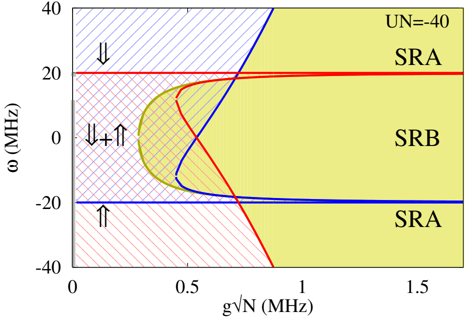

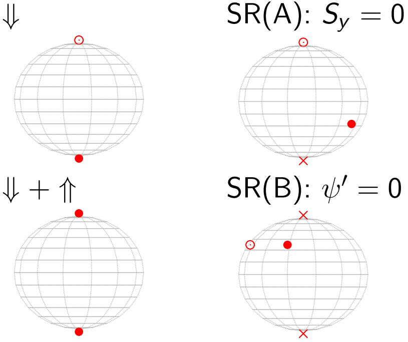

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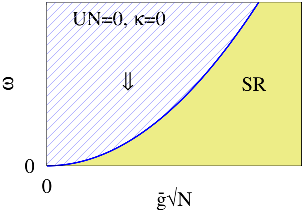

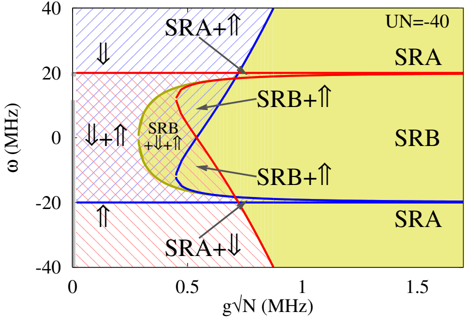

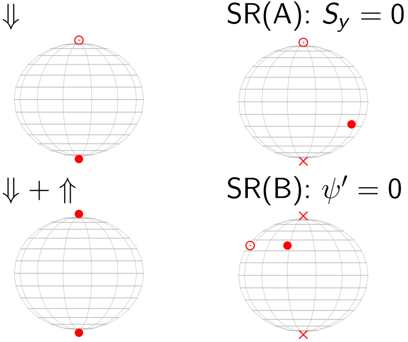

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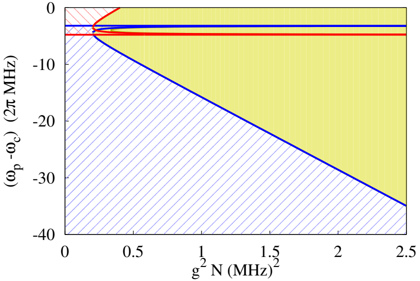
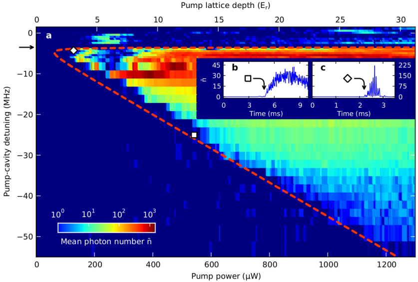
Comparison to experiment





$$UN = -10\text{MHz}$$

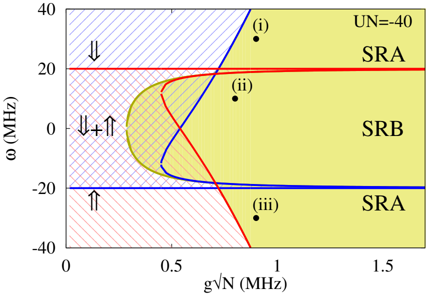
Adapted from: [Bhaseen *et al.* in prep.]

[Baumann *et al* Nature 2010]

$$\omega = \omega_c - \omega_p + \frac{5}{2} UN, \qquad UN = -\frac{g_0^2}{4(\omega_a - \omega_c)}$$

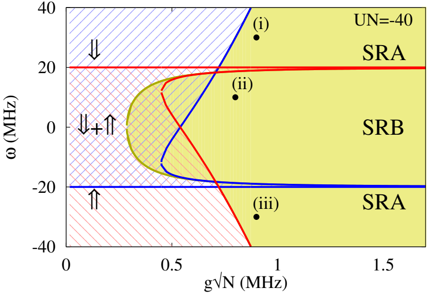
Outline

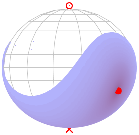
Dynamics: Evolution from normal state

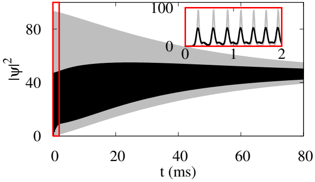


Dynamics: Evolution from normal state

Gray: $\mathbf{S} = (\sqrt{N}, \sqrt{N}, -N/2)$
Black: Wigner distribution of $\mathbf{S}, \psi$







Oscillations: $\sim$ 0.1ms
Decay: 20ms

Dynamics: Evolution from normal state

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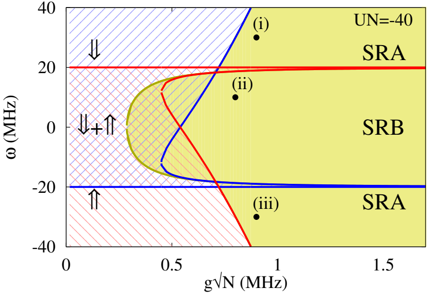

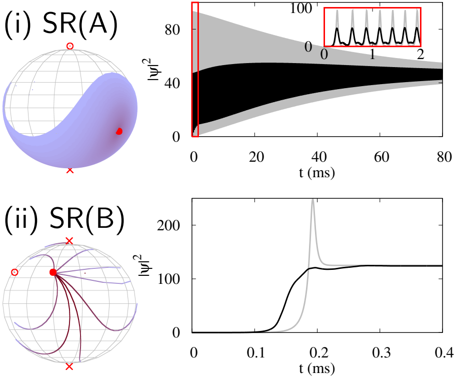

Oscillations: $\sim$ 0.1ms
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Dynamics: Evolution from normal state

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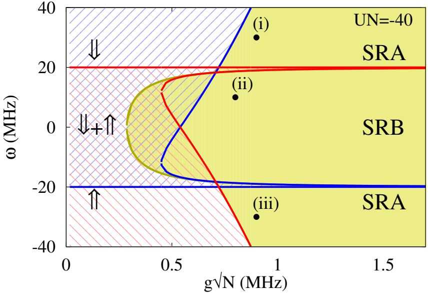
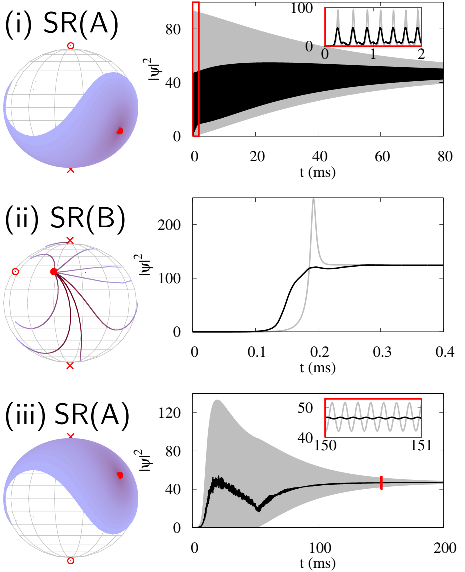


Oscillations: ~ 0.1ms
Decay: 20ms, 0.1ms, 20ms



Dynamics: Evolution from normal state

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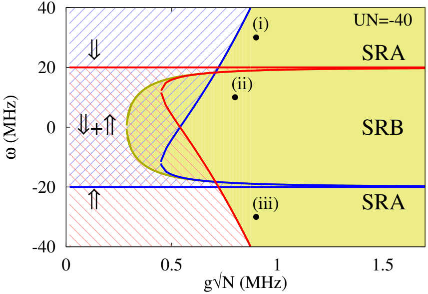
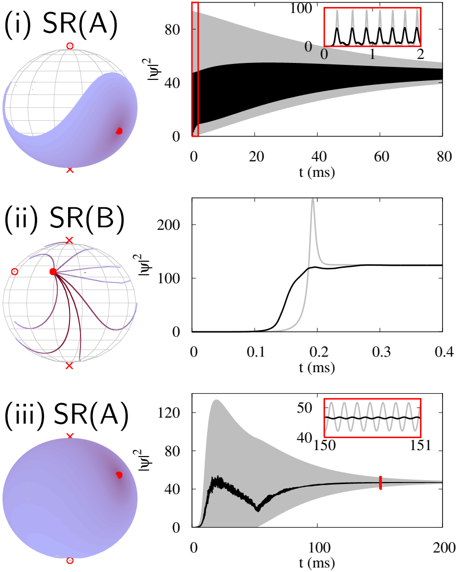


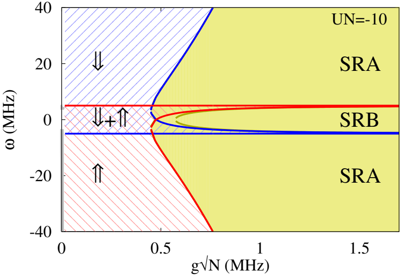


Oscillations: ~ 0.1ms
Decay: 20ms, 0.1ms, 20ms

Asymptotic state: Evolution from normal state

(Near to experimental $UN = -13$MHz).

All stable attractors:



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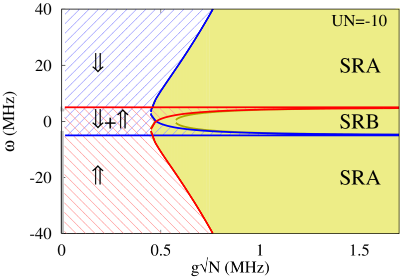
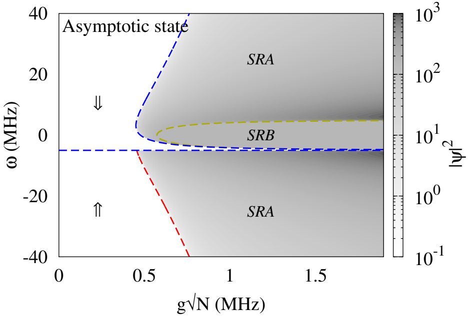
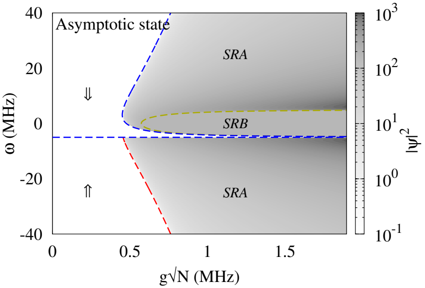
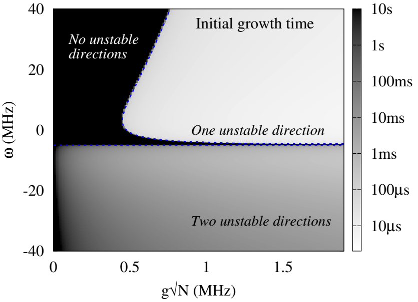


Starting from ⇓



Timescales for dynamics: What are they?

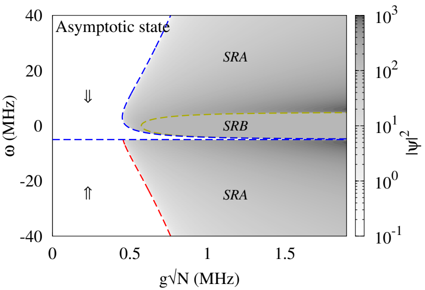


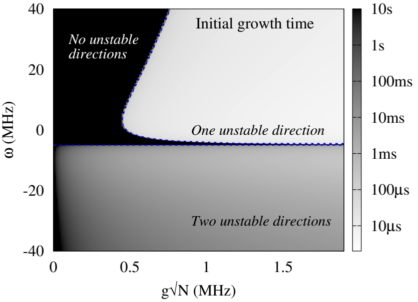


Growth Most unstable eigenvalues near $\mathbf{S} = (0, 0, -N/2)$

Decay Slowest stable eigenvalues near final state

Timescales for dynamics: What are they?





Growth: Most unstable eigenvalues near $\mathbf{S} = (0, 0, -N/2)$

Decay: Slowest stable eigenvalues near final state

Expand in ω_0/κ:
Oscillations: $\sim \omega_0$,
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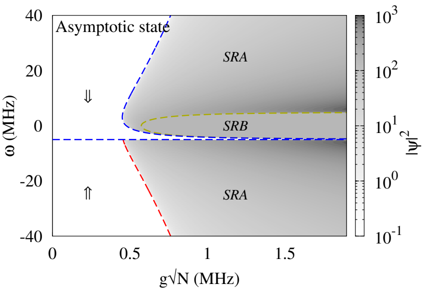
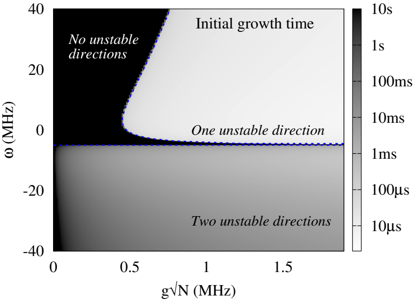
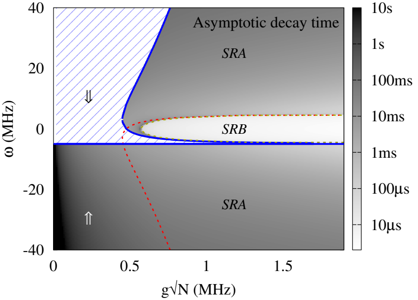
Timescales for dynamics: What are they?







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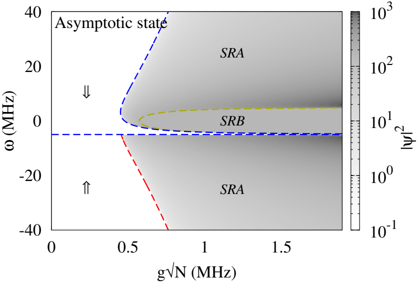

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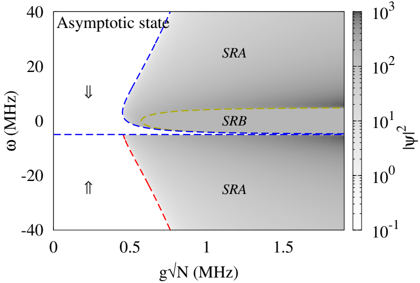

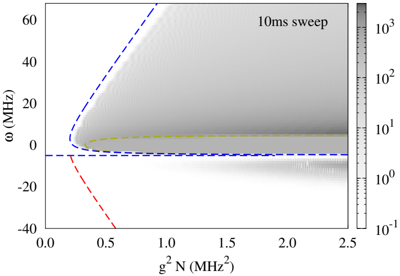

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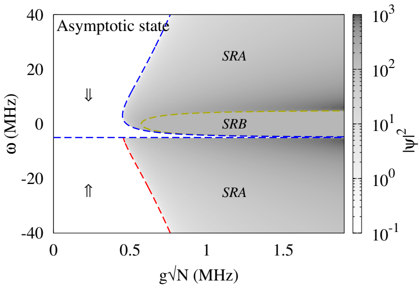
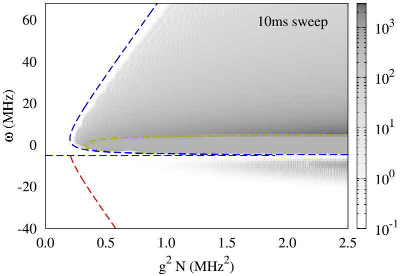
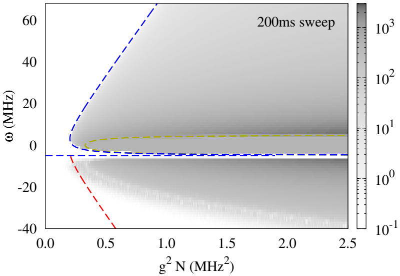
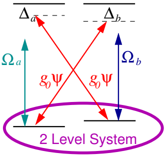






Timescales for dynamics: Why so slow and varied?

Suppose co- and counter-rotating terms differ



$$H = \ldots + g(\psi^\dagger S^- + \psi S^+) + g'(\psi^\dagger S^+ + \psi S^-) + \ldots$$

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- SR(A), SR(B) continuously connect

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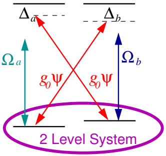

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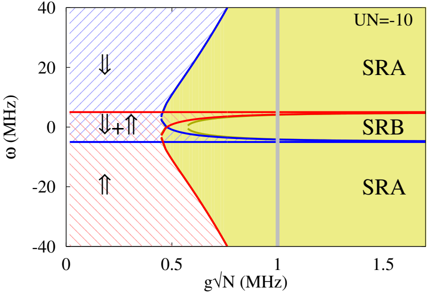

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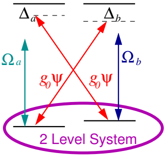

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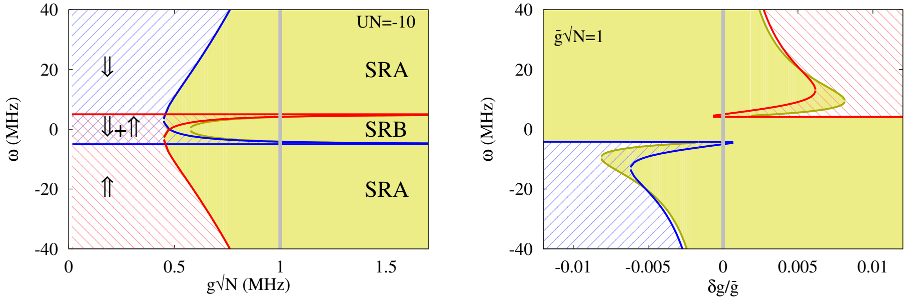

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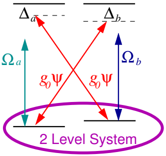

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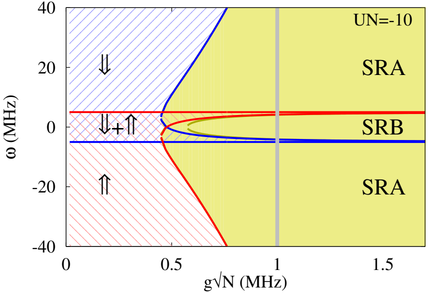

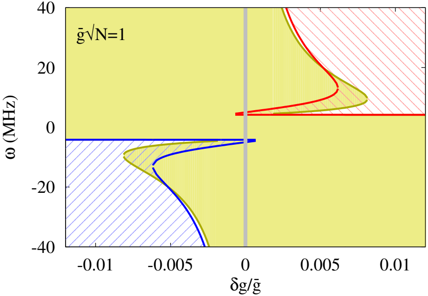

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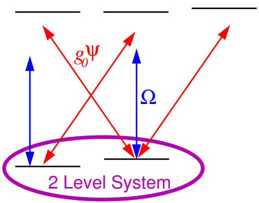
Outline

Regions without fixed points

Changing U:



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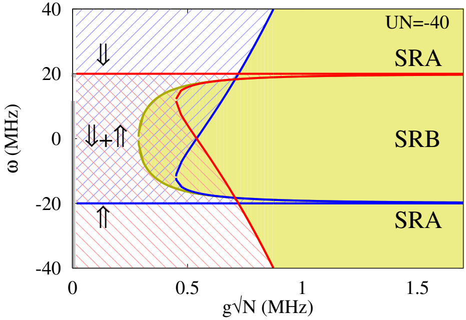

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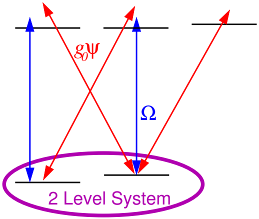

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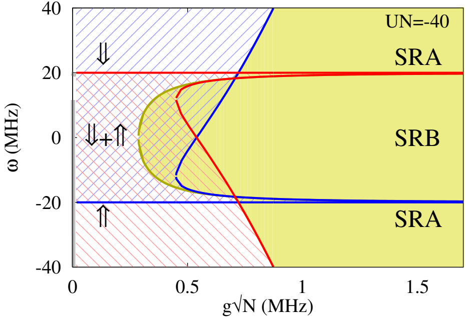

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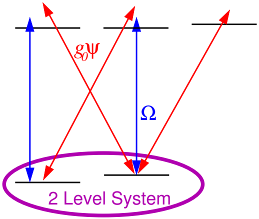

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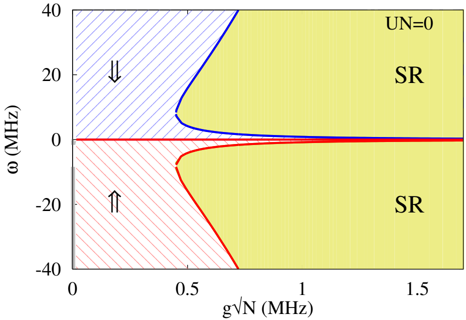

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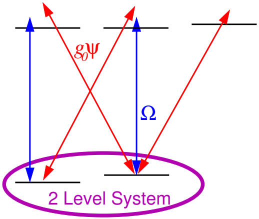

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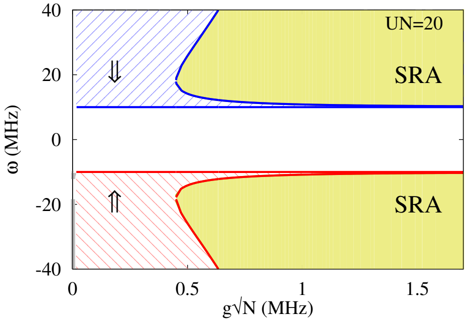

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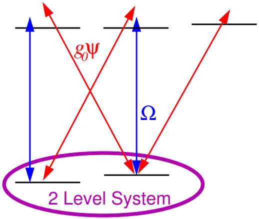

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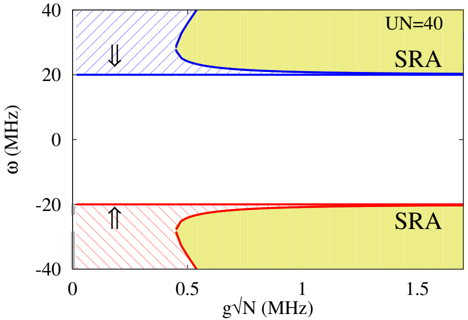

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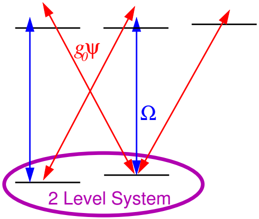

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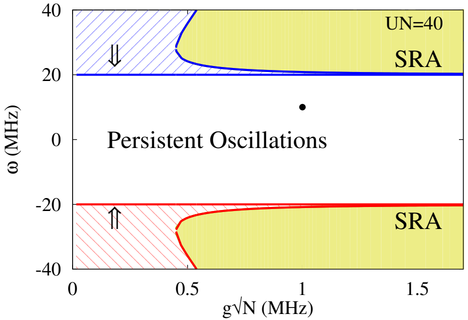
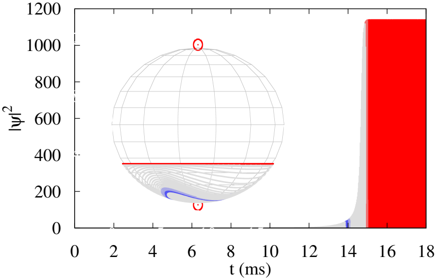
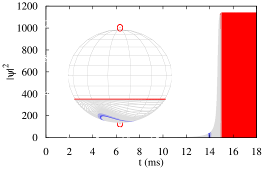




Persistent (optomechanical) oscillations



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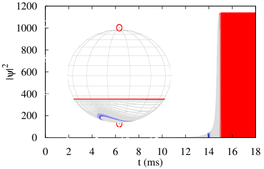

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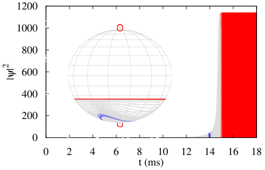

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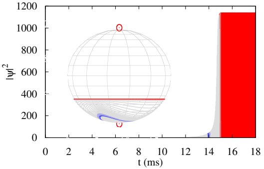

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Fix $\omega + US^z = 0$ **if** $\psi' = 0$.

$$S^- = re^{-i\theta} \quad r = \sqrt{\frac{N^2}{4} - (S^z)^2}$$

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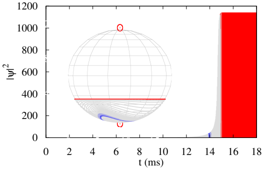

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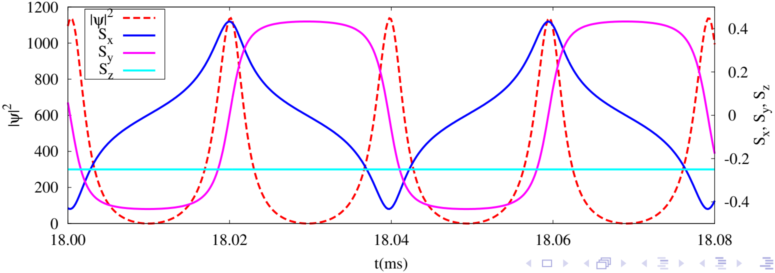
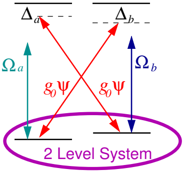


Tuning g, g', U

[Dimer *et al.* Phys. Rev. A. (2007)]



- Separate pump strength/detuning
- $g \sim \frac{g_0 \Omega_b}{\Delta_b}, g' \sim \frac{g_0 \Omega_a}{\Delta_a}, U \sim \frac{g_0{}^2}{\Delta_a} - \frac{g_0{}^2}{\Delta_b}$

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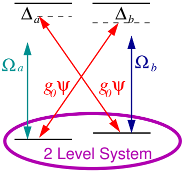

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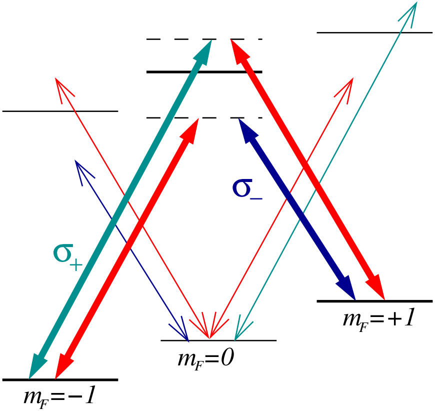
Possible realization: Hyperfine levels



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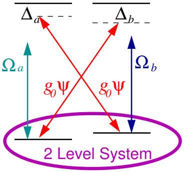

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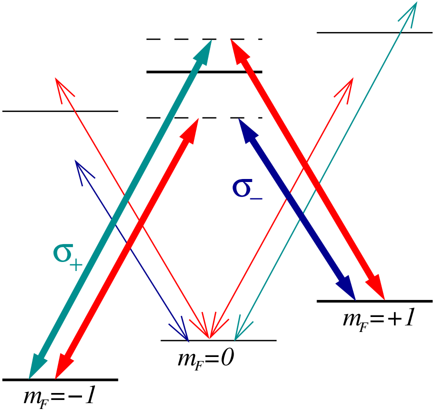
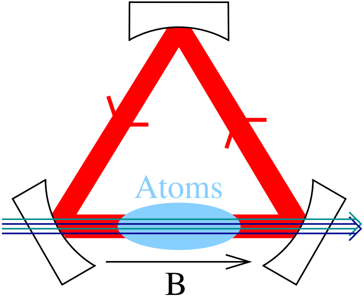
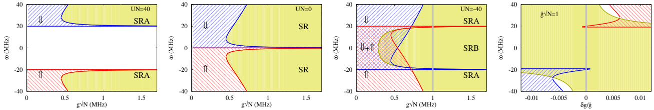


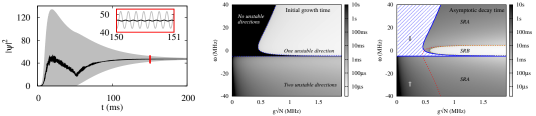


Summary

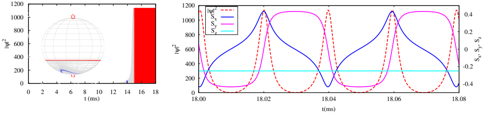
- Wide variety of dynamical phases



- Slow dynamics



- Persistent oscillations if $U > 0$



[Postdoc position available in St Andrews]

JK *et al.* PRL '10, Bhaseen *et al.* in preparation

Extra slides

5. Ferroelectric phase transition

6. Lipkin-Meshkov-Glick

7. Tricritical point

Ferroelectric transition

Atoms in Coulomb gauge

$$H = \sum \omega_k a_k^\dagger a_k + \sum_i [p_i - eA(r_i)]^2 + V_{\text{coul}}$$

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$$H = \omega_0 S^z + \omega \psi^\dagger \psi + g(S^+ + S^-)(\psi + \psi^\dagger) + N\zeta(\psi + \psi^\dagger)^2 - \eta(S^+ - S^-)^2$$

(nb $g^2, \zeta, \eta \propto 1/V$).

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Ferroelectric polarisation if $\omega_0 < 2\eta N$

Gauge transform to dipole gauge $\mathbf{D} \cdot \mathbf{r}$

$$H = \omega_0 S^z + \omega\psi^\dagger\psi + \bar{g}(S^+ - S^-)(\psi - \psi^\dagger)$$

"Dicke" transition at $\omega_0 < N\bar{g}^2/\omega \equiv 2\eta N$

But, ψ describes **electric displacement**

Timescales for dynamics: $U = 0$ and Lipkin-Meshkov-Glick

- Since $\kappa \gg \omega_0$, can consider eliminating ψ

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$$\partial_t \mathbf{S} = \{\mathbf{S}, H\} - \Gamma \mathbf{S} \times (\mathbf{S} \times \hat{z}), \qquad H = \omega_0 S_z - \Lambda_+ S_x^2 - \Lambda_- S_y^2$$

with: $\Lambda_\pm = \frac{\omega}{\kappa^2+\omega^2}(g \pm g')^2$, $\Gamma = \frac{2\kappa}{\kappa^2+\omega^2}(g'^2 - g^2)$

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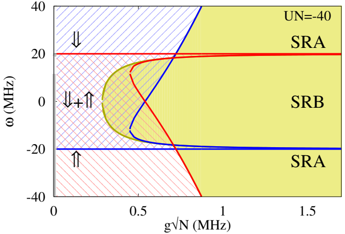
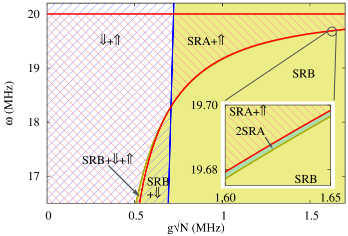
- NB, $g' = g$, no dissipation. Dissipation restored by finite κ.

Tricritical point





If $-\omega_u = -UN/2 > \kappa$, crossing of boundaries at:

$$\omega^* = \sqrt{\omega_u^2 - \kappa^2}$$

$$g^* \sqrt{N} = \sqrt{\frac{-\omega_0 UN}{4}}$$

For $g > g^*$, 2SRA region exists at edge of SRB.