Collective Dynamics of Generalized Dicke Models

J. Keeling, J. A. Mayoh, M. J. Bhaseen, B. D. Simons

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Coupling many atoms to light

Old question: *What happens to radiation when many atoms interact "collectively" with light.*

Superradiance — dynamical and steady state.

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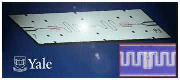
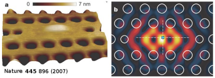
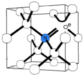
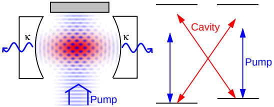
Superradiance — dynamical and steady state.

New relevance

- Superconducting qubits 

- Quantum dots 

- Nitrogen-Vacancies in diamond 
- Ultra-cold atoms 

- Rydberg atoms

Dicke effect: Enhanced emission

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

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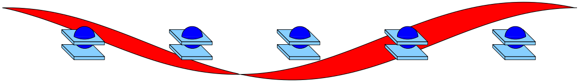

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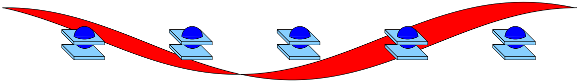

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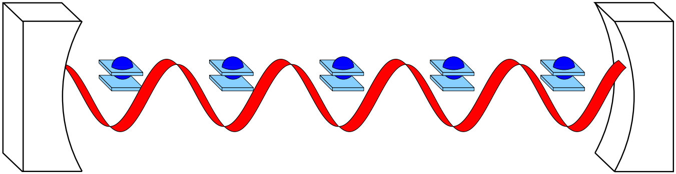
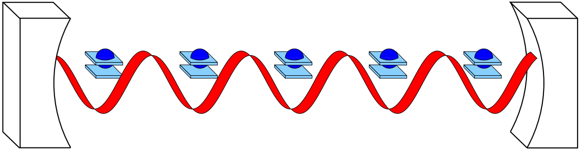
$$H = \omega\psi^\dagger\psi + \sum_i \omega_0 S_i^z + g\left(\psi^\dagger S_i^- + \psi S_i^+\right).$$



Dicke model: Equilibrium superradiance transition



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- Coherent state: $|\Psi\rangle \to e^{\lambda\psi^\dagger + \eta S^+}|\Omega\rangle$
- Energy:

$$E = \omega|\lambda|^2 + \frac{\omega_0 N}{2}\frac{|\eta|^2 - 1}{|\eta|^2 + 1} + gN\frac{\eta^*\lambda + \lambda^*\eta}{1 + |\eta|^2}$$

- Small g, min at $\lambda, \eta = 0$

[Hepp, Lieb, Ann. Phys. '73]

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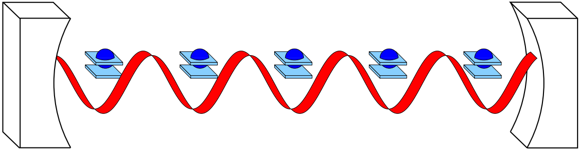

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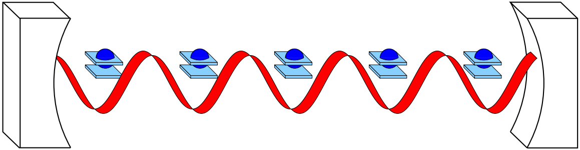

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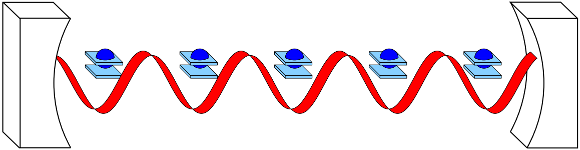

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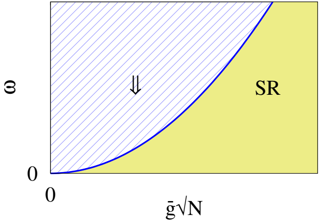
Spontaneous polarisation if: $Ng^2 > \omega\omega_0$



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No go theorem and transition

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Solutions:

- Fixed excitation density
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- Dissociate g, ω_0, e.g. Raman Scheme:
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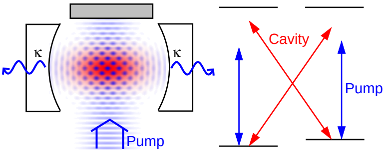


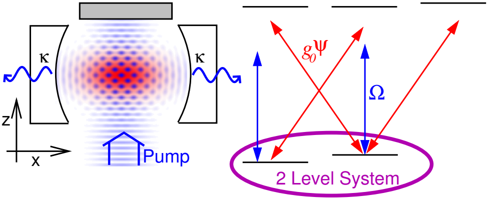
Outline

1. Introduction: Dicke model and superradiance
   - Rayleigh scheme: Generalised Dicke model
2. Attractors of dynamics (fixed points)
3. Approach to attractors: timescales
4. Attractors of dynamics (oscillations)

Outline

Extended Dicke model

[Baumann *et al.* Nature 2010]



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$\Downarrow$: $\Psi(x,z) = 1$

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$\omega_0 = 2\omega_{\text{recoil}}$

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$$\partial_t \rho = -i[H,\rho] - \kappa(\psi^\dagger\psi\rho - 2\psi\rho\psi^\dagger + \rho\psi^\dagger\psi)$$

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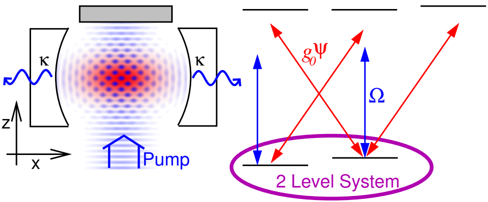

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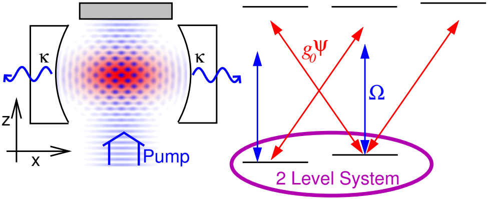

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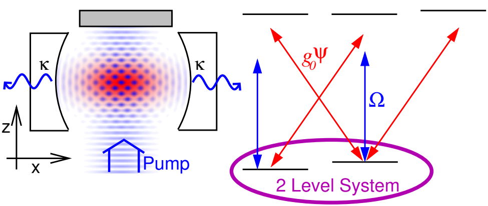

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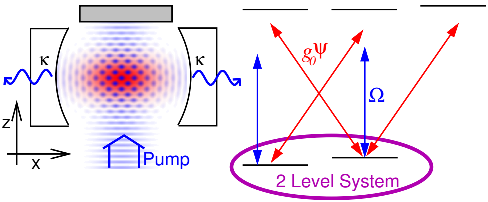

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$(|\mathbf{S}| = N/2 \gg 1)$

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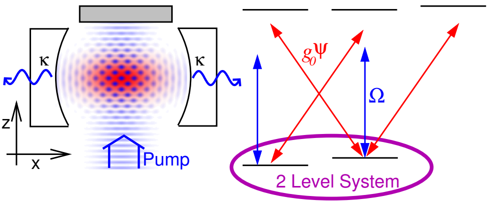

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$\omega_0 \sim \text{kHz} \ll \omega, \kappa, g\sqrt{N} \sim \text{MHz}.$

Fixed points (steady states)

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- $\psi = 0, \mathbf{S} = (0, 0, \pm N/2)$ always a solution.
- If $g > g_c$, $\psi \neq 0$ too
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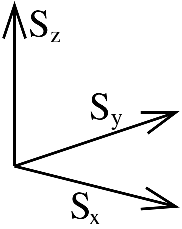

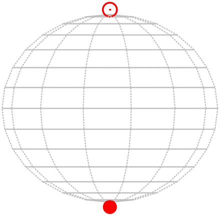

Small g: ⇑, ⇓ only.
($\omega = 30$MHz, $UN = -40$MHz)

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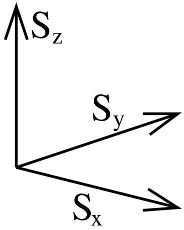

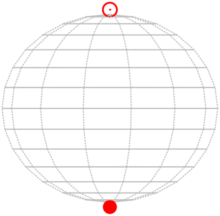

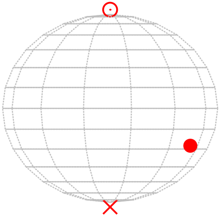

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Steady state phase diagram

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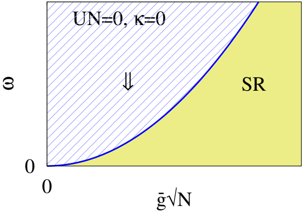

See also Domokos and Ritsch PRL '02, Domokos *et al.* PRL '10

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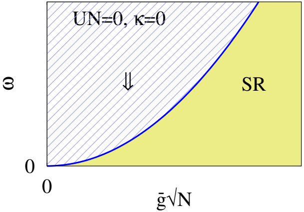

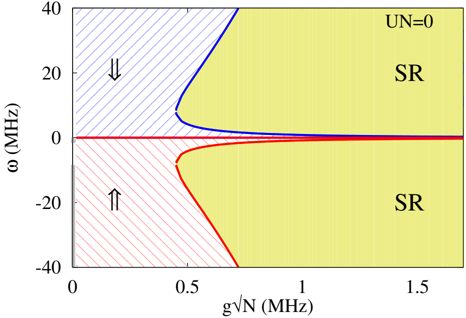

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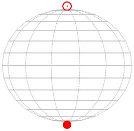
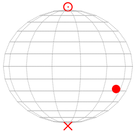


SR(A): $S_y = 0$



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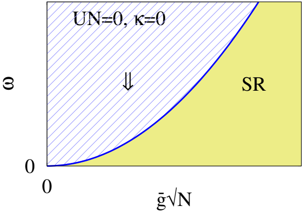

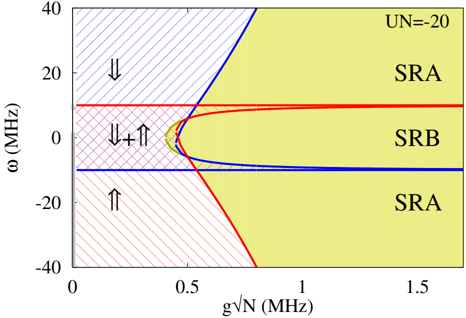

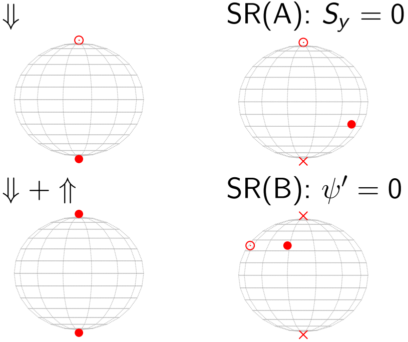

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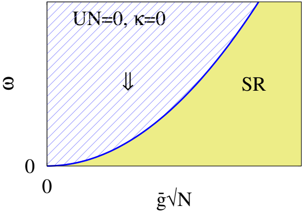

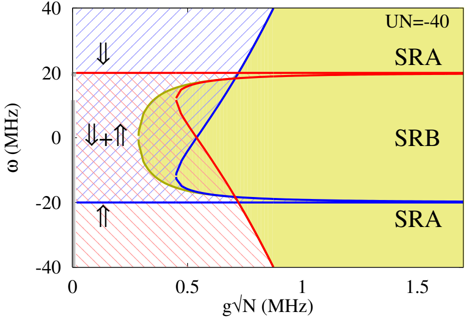

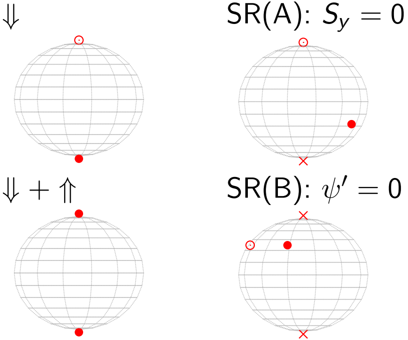

See also Domokos and Ritsch PRL '02, Domokos *et al.* PRL '10

Steady state phase diagram

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
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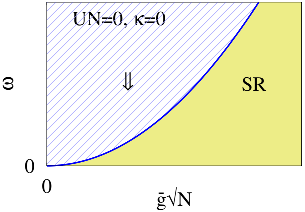
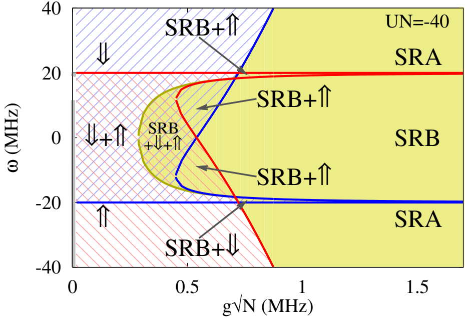
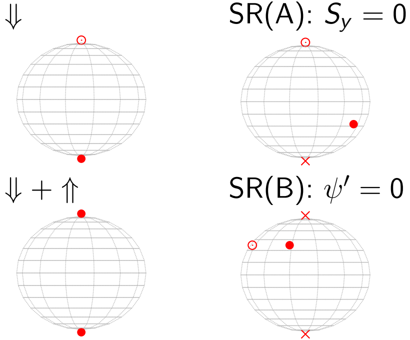




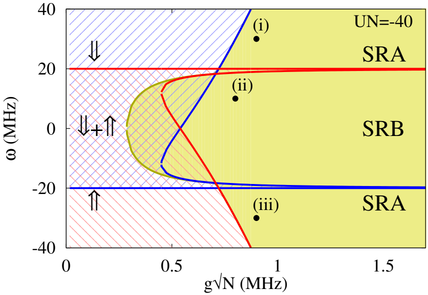


See also Domokos and Ritsch PRL '02, Domokos *et al.* PRL '10

Outline

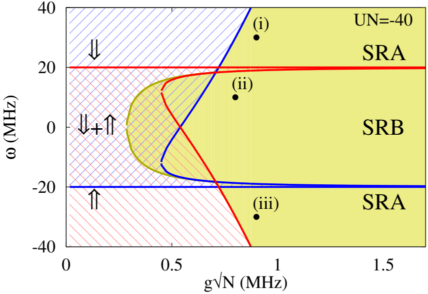
1. Introduction: Dicke model and superradiance
   - Rayleigh scheme: Generalised Dicke model
2. Attractors of dynamics (fixed points)
3. Approach to attractors: timescales
4. Attractors of dynamics (oscillations)

Dynamics: Evolution from normal state

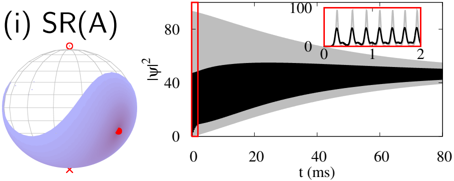


Dynamics: Evolution from normal state

Gray: $\mathbf{S} = (\sqrt{N}, \sqrt{N}, -N/2)$
Black: Wigner distribution of $\mathbf{S}, \psi$



(i) SR(A)



Oscillations: ~ 0.1ms
Decay: 20ms

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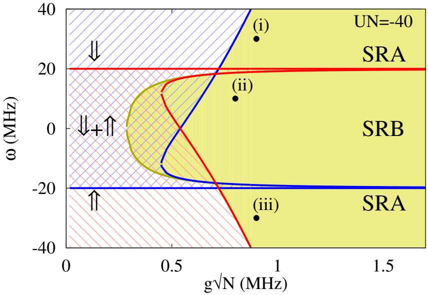

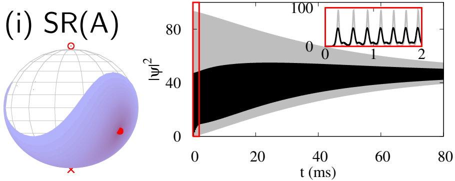

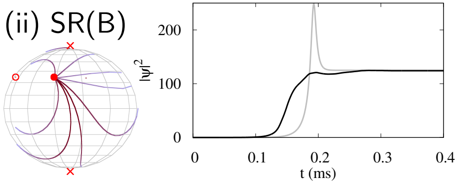

Oscillations: $\sim$ 0.1ms

Decay: 20ms, 0.1ms

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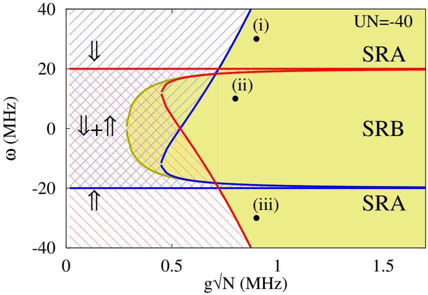

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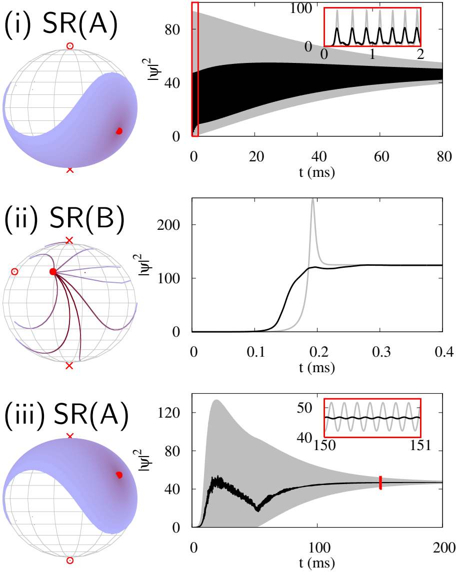

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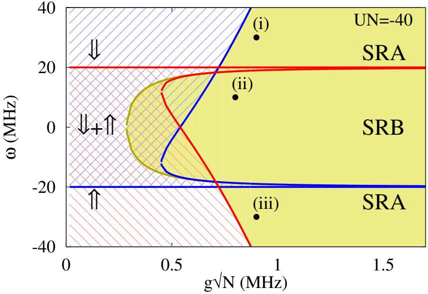
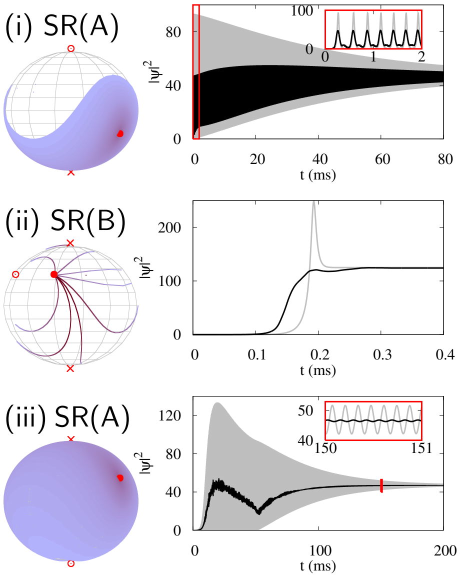


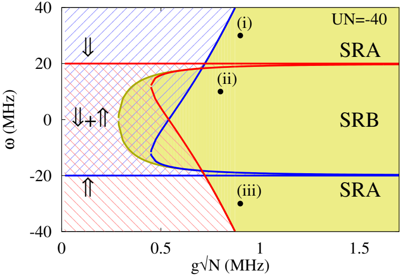


Oscillations: ~ 0.1ms

Decay: 20ms, 0.1ms, 20ms

Asymptotic state: Evolution from normal state

All stable attractors:



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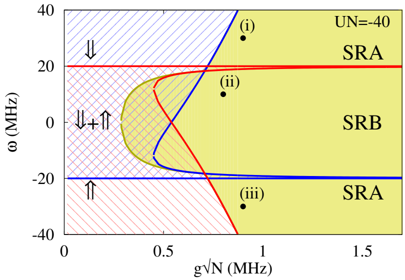
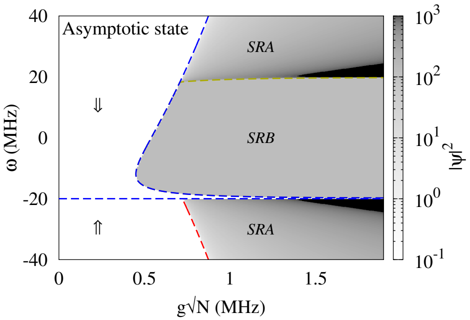
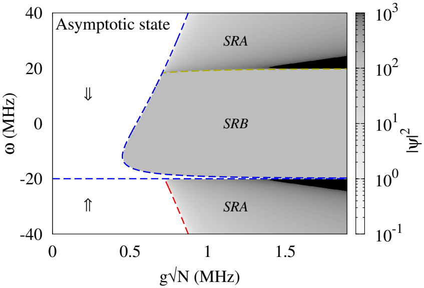
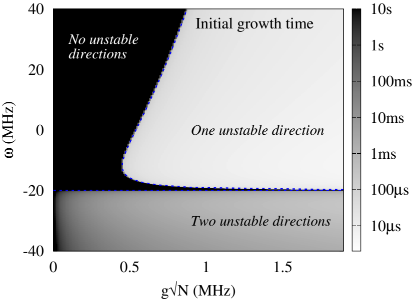


Starting from ⇓



Timescales for dynamics: What are they?





Growth Most unstable eigenvalues near $\mathbf{S} = (0, 0, -N/2)$

Decay Slowest stable eigenvalues near final state

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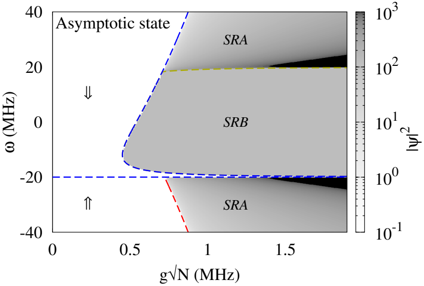

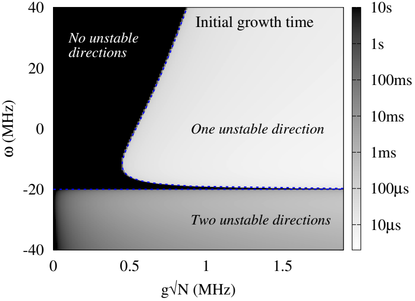

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Expand in ω_0/κ:
Oscillations: $\sim \omega_0$,
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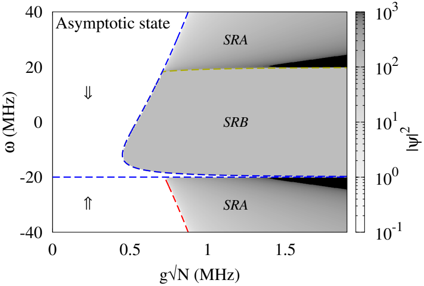

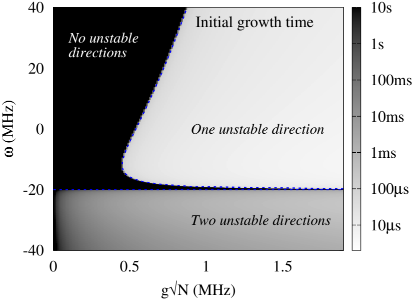

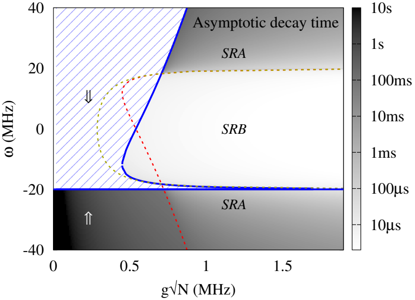

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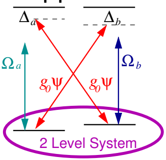
Timescales for dynamics: Why so slow and varied?

Suppose co- and counter-rotating terms differ



$$H = \ldots + g(\psi^\dagger S^- + \psi S^+) + g'(\psi^\dagger S^+ + \psi S^-) + \ldots$$

- SR(A) near phase boundary at small δg $\rightarrow$ Critical slowing down
- SR(A), SR(B) continuously connect

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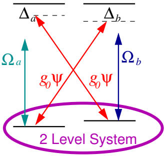

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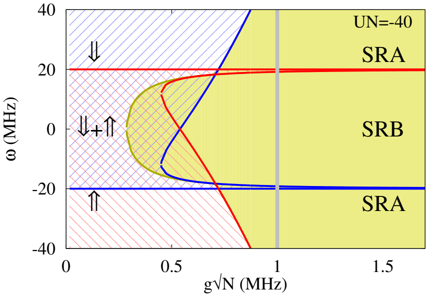

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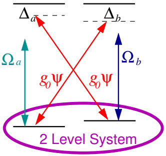

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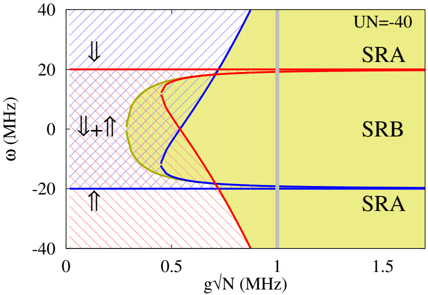

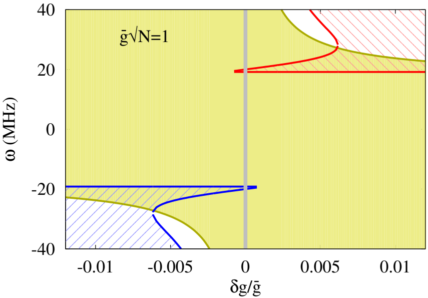

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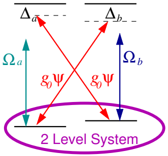

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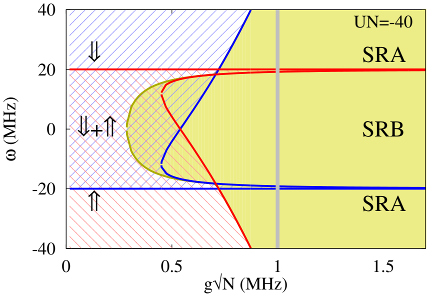

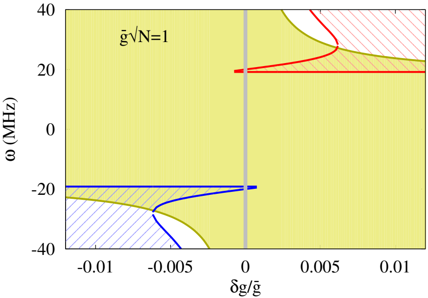

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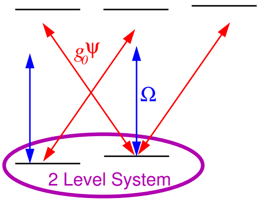
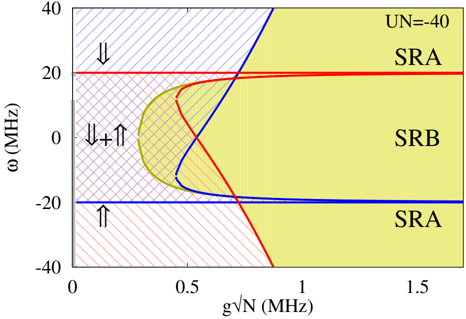
Outline

Regions without fixed points

Changing U:



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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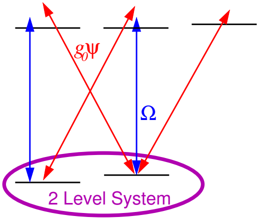

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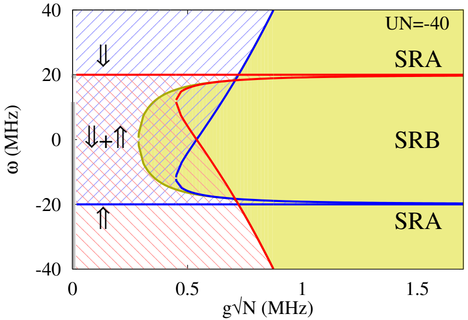

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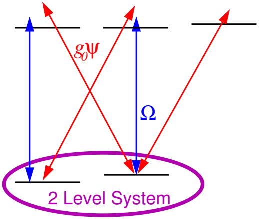

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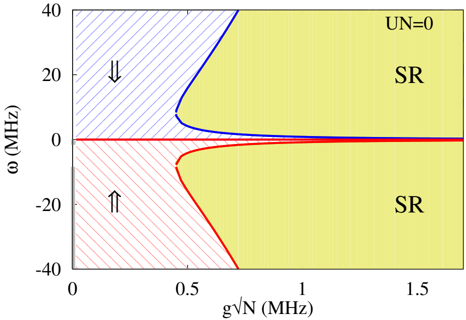

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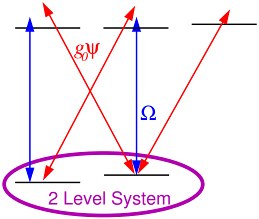

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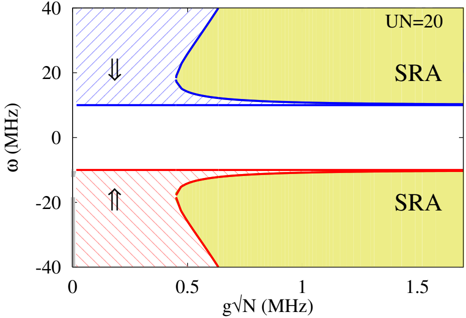

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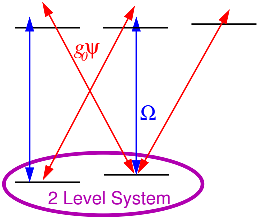

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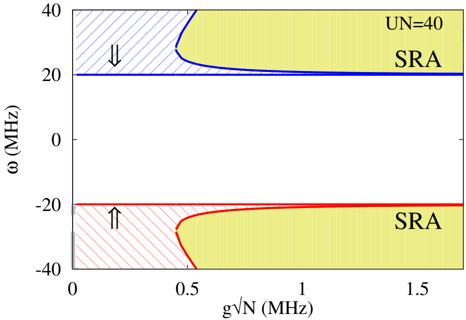

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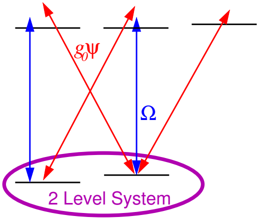

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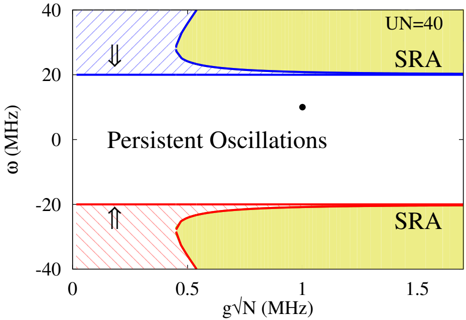
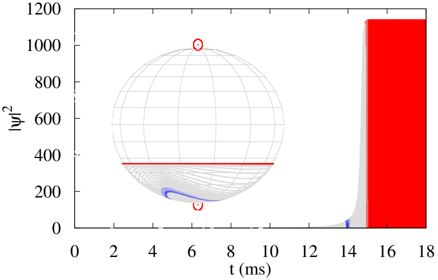
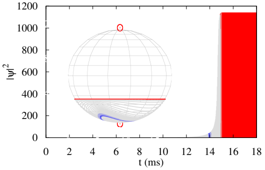




Persistent (optomechanical) oscillations



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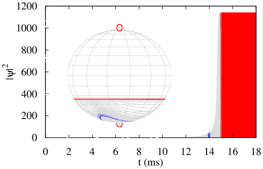

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Fix $\omega + US^z = 0$ **if** $\psi' = 0$.

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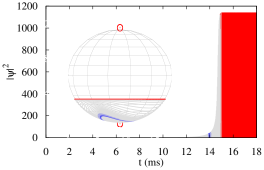

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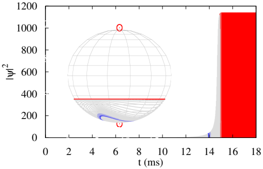

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Get:

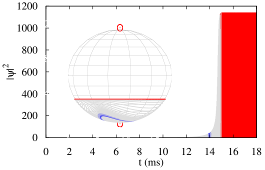
Fix $\omega + US^z = 0$ **if** $\psi' = 0$.

$$S^- = re^{-i\theta} \quad r = \sqrt{\frac{N^2}{4} - (S^z)^2}$$

$$\dot{\theta} = \omega_0 + U|\psi|^2$$

$$\dot{\psi} + \kappa\psi = -2igr\cos(\theta)$$

Persistent (optomechanical) oscillations



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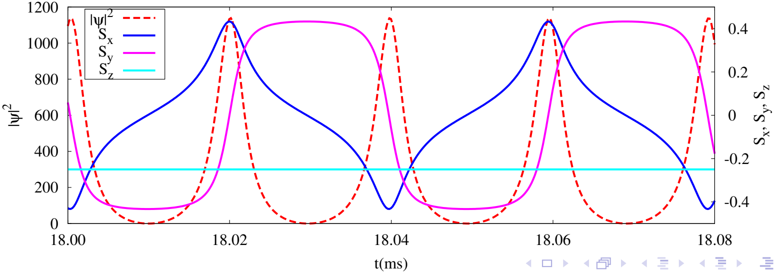
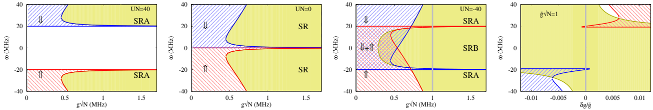
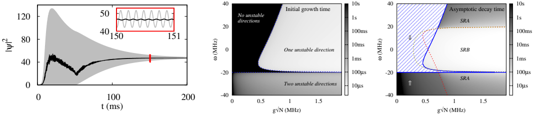


Summary

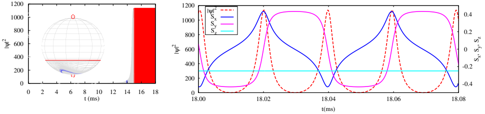
- Wide variety of dynamical phases



- Slow dynamics



- Persistent oscillations if $U > 0$



[Postdoc position available in St Andrews]

JK *et al.* PRL '10, M. J. Bhaseen *et al.* in preparation

Extra slides