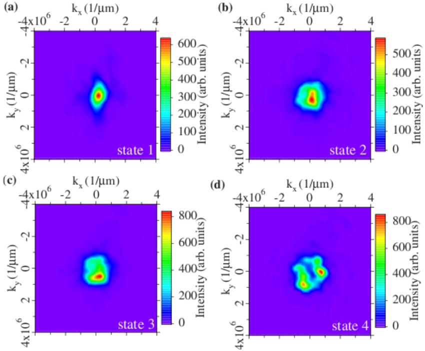
Non-equilibrium coherence in strongly coupled light-matter systems

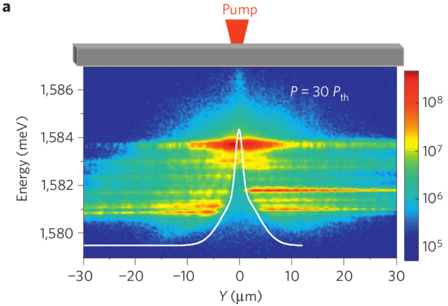
Jonathan Keeling

Telluride, July 2011

Funding: EPSRC
Engineering and Physical Sciences Research Council

Non-equilibrium features in experiment



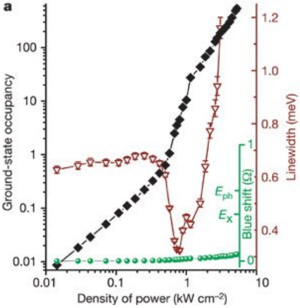


$|\psi(\mathbf{k})|^2 \neq |\psi(-\mathbf{k})|^2$:
Broken time-reversal symmetry.
[Krizhanovskii *et al.* PRB (2009)]

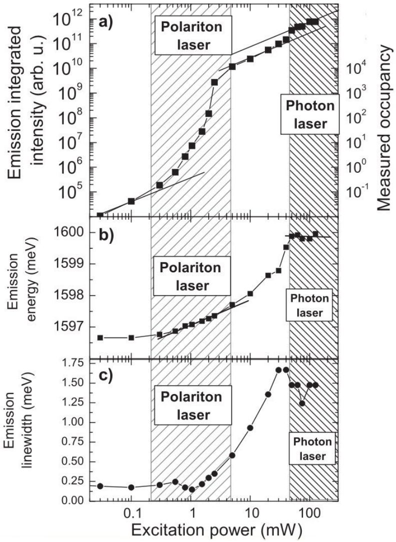
Flow from pumping spot
[Wertz *et al.* Nat. Phys. (2010)]

Strong coupling in experiment

[Bajoni *et al.* PRL 2008]



[Kasprzak, *et al.* Nature '06]



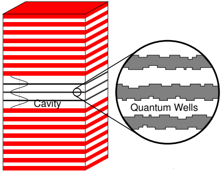
[Bajoni, *et al.* PRL '08]

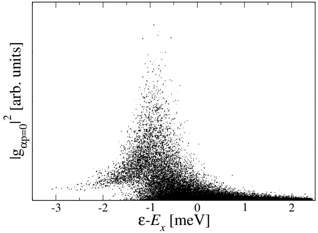
1. Microscopic model: lasing vs condensation
   - Model: localised excitons, propagating photons
   - Simple laser: Maxwell-Bloch
   - Non-equilibrium polaritons: coherence and strong coupling
2. Coherence of polariton condensate
   - Condensed spectrum
   - Power law decay of coherence
   - Finite size and Schawlow-Townes

Most results in review: `arXiv:1001.3338`

Polariton system model

- Disorder-localised excitons
- Treat disorder sites as 2-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling g.





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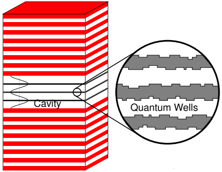

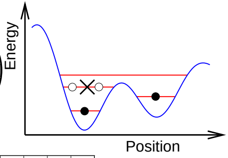

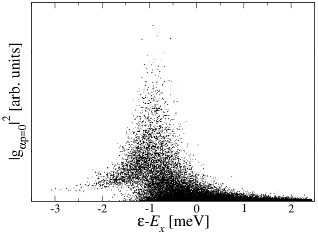

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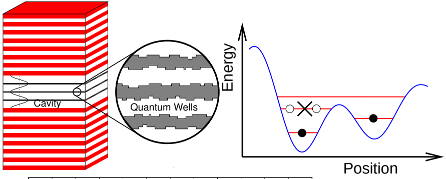

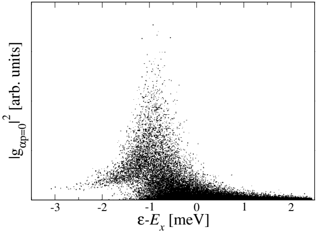

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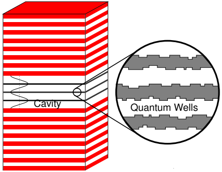
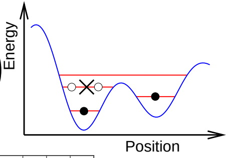
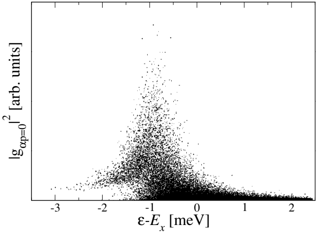
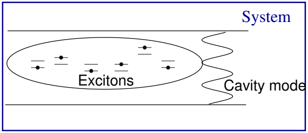






$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \epsilon_{\alpha} S^{z}_{\alpha} + \sum_{\alpha} \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} S^{+}_{\alpha} + \text{H.c.}$$



Equilibrium: Mean-field theory

Self-consistent polarisation and field

$$\left(-i\partial_t - \omega_0\right)\psi = -\sum_\alpha \frac{g_\alpha}{\sqrt{A}} S^-_\alpha$$

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$$(\quad \mu - \omega_0)\,\psi = -\sum_\alpha \frac{g_\alpha}{\sqrt{A}} \frac{g_\alpha \psi}{2E_\alpha} \tanh(\beta E_\alpha), \qquad {E_\alpha}^2 = \left(\frac{\epsilon_\alpha - \mu}{2}\right)^2 + g_\alpha^2 |\psi|^2$$

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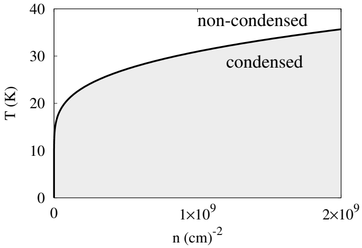

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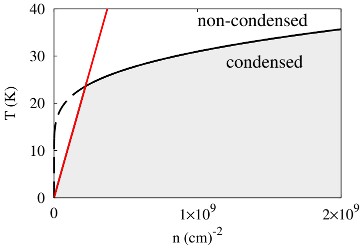

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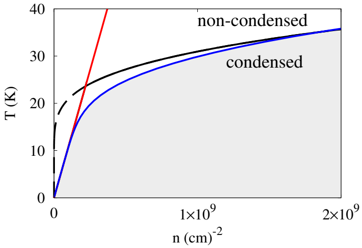

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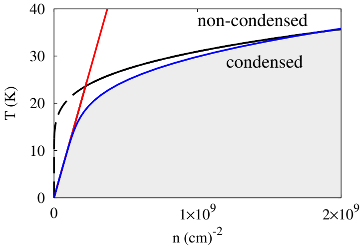
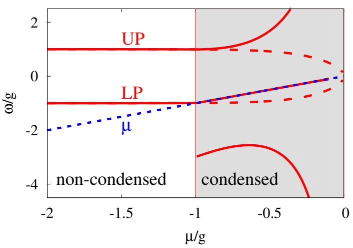
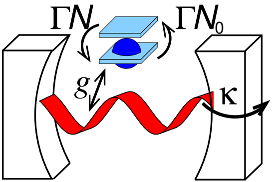


Modes (at $k = 0$)



Simple Laser: Maxwell Bloch equations

$$H = \omega_0 \psi^\dagger \psi + \sum_\alpha \epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{\mathrm{A}}} \psi S^+_\alpha + \text{H.c.}$$



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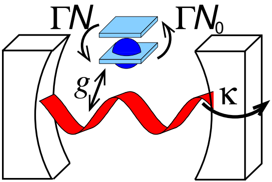

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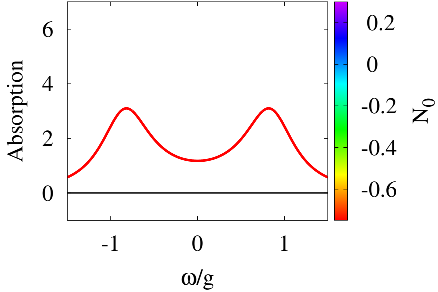

- Strong coupling. $\kappa, \gamma < g\sqrt{n}$
- Inversion causes collapse before lasing

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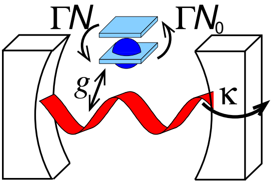

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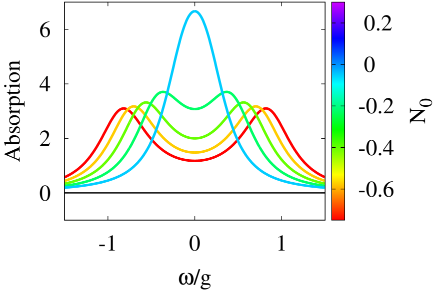

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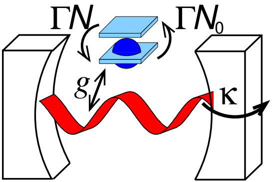

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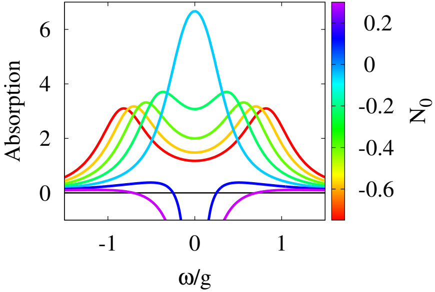

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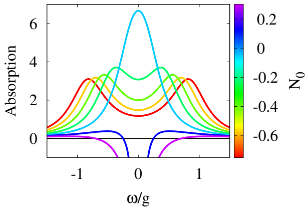
Maxwell-Bloch Equations: Retarded Green's function



- Introduce $D^R(\omega)$:
  Response to perturbation

- Absorption $= -2\Im[D^R(\omega)]$

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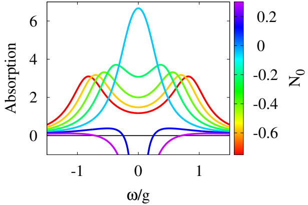

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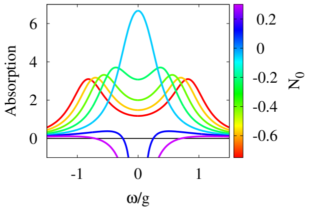

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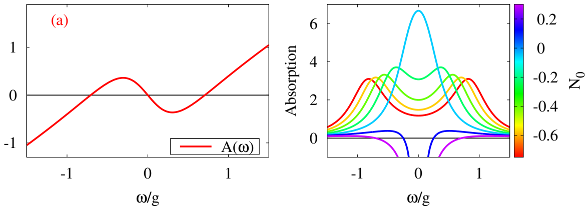

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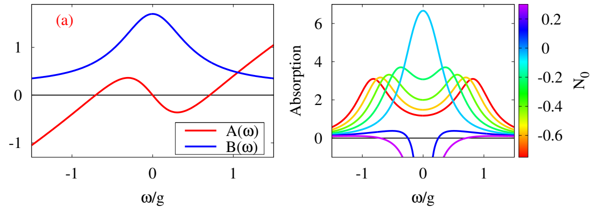

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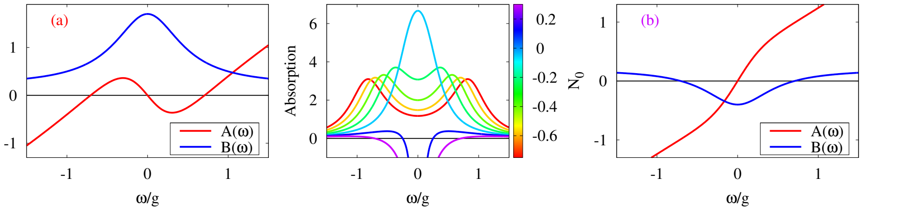

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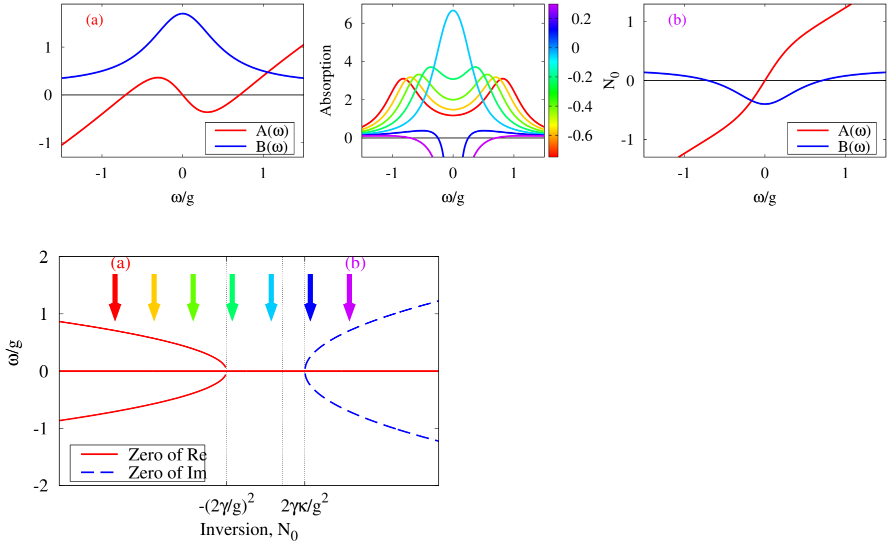
Evolution of poles with Inversion



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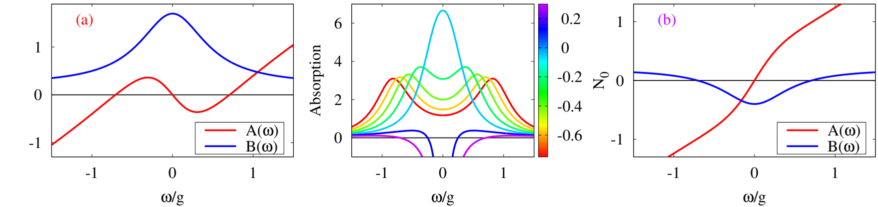
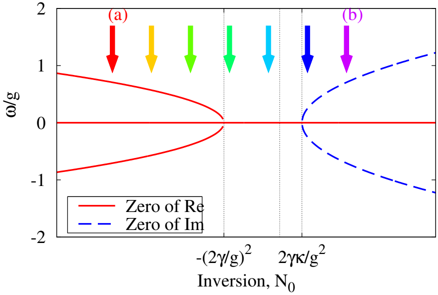
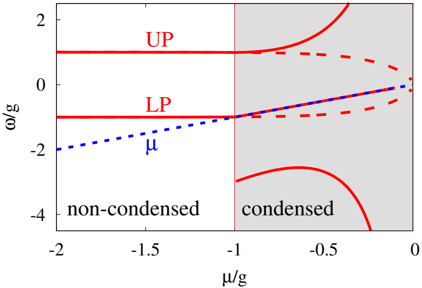


Laser:



Equilibrium:



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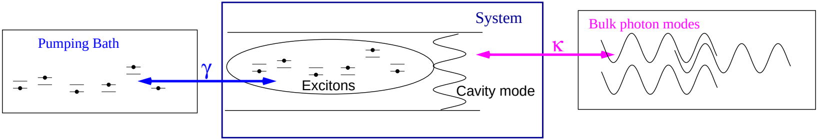
Non-equilibrium description: baths



$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

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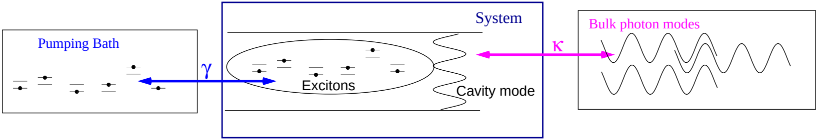

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Schematically: pump γ, decay κ

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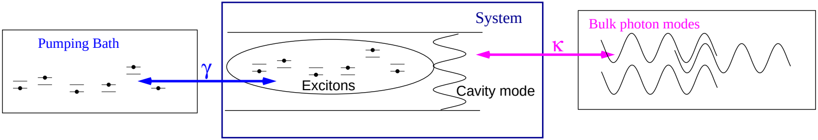

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Bath correlations, $\langle \Psi^{\dagger}\Psi \rangle$, $\langle A^{\dagger}A \rangle$, $\langle B^{\dagger}B \rangle$ fixed:

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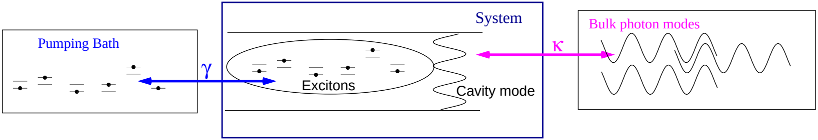

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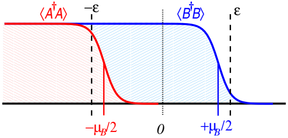
Schematically: pump γ, decay κ

$$H_{\text{sys,bath}} \simeq \sum_{\mathbf{p},\mathbf{k}} \sqrt{\kappa}\psi_{\mathbf{k}}\Psi^\dagger_{\mathbf{p}} + \sum_{\alpha,\beta} \sqrt{\gamma}\left(a^\dagger_\alpha A_\beta + b^\dagger_\alpha B_\beta\right) + \text{H.c.}$$

Bath correlations, $\langle \Psi^\dagger \Psi \rangle$, $\langle A^\dagger A \rangle$, $\langle B^\dagger B \rangle$ fixed:
Ψ bath is empty. Pumping bath thermal, μ_B, T_B:



Non-equilibrium mean-field theory

Look for mean-field solution, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$. Gap equation:

$$(\mu_S - \omega_0 + i\kappa)\,\psi_0 = \chi(\psi_0, \mu_S)\psi_0$$

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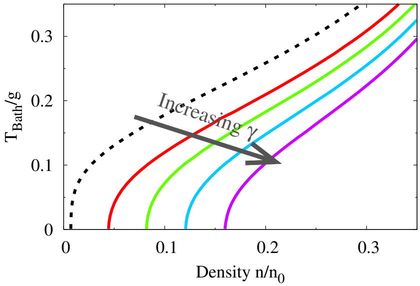

Luminescence spectrum and Green's functions

$$-2\Im[D^R(\omega)] = \mathrm{DoS}(\omega)$$

$$\left[D^R(\omega)\right]^{-1} = A(\omega) + iB(\omega)$$

$$\mathrm{DoS}(\omega) = \frac{2B(\omega)}{B(\omega)^2 + A(\omega)^2}$$

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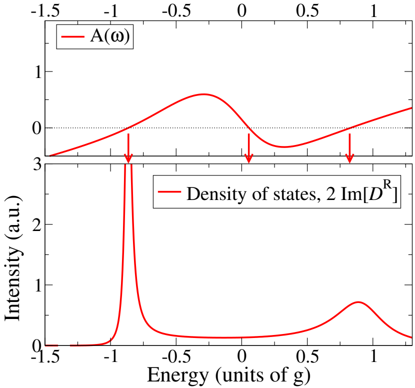

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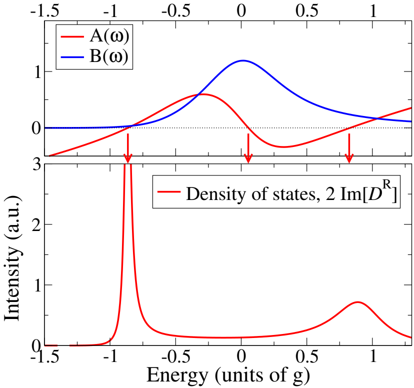

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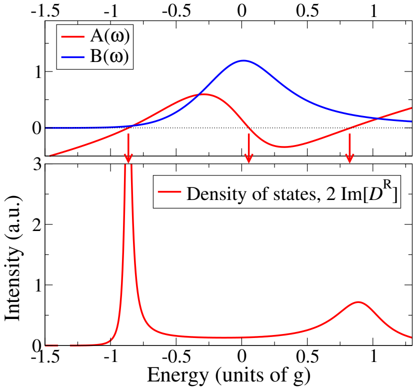

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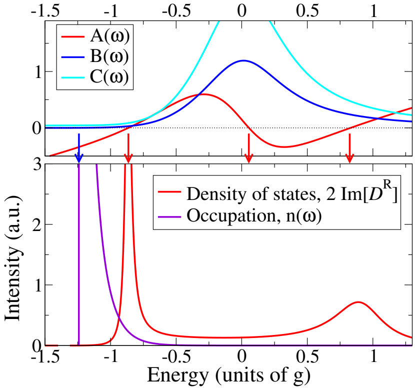
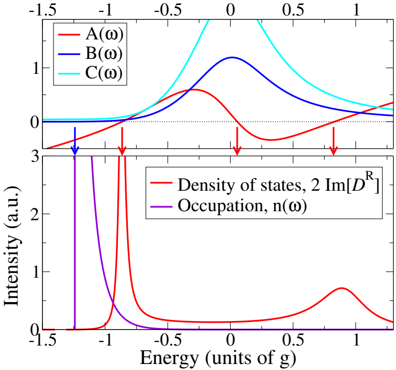


Stability and evolution with pumping



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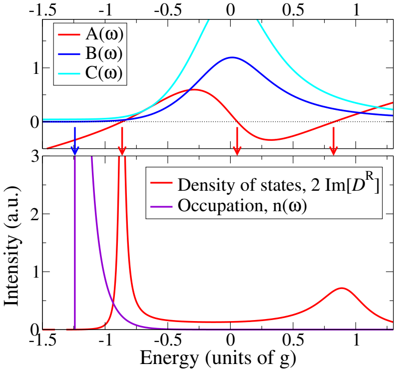

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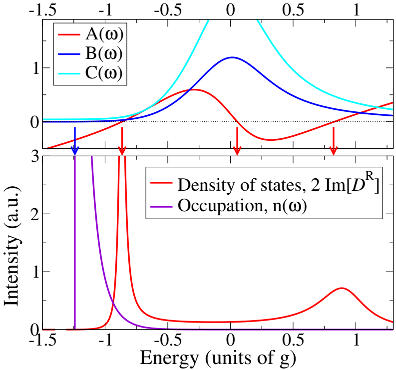

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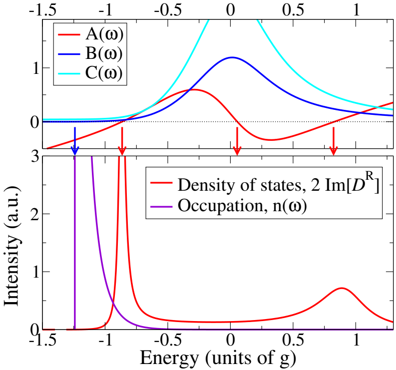

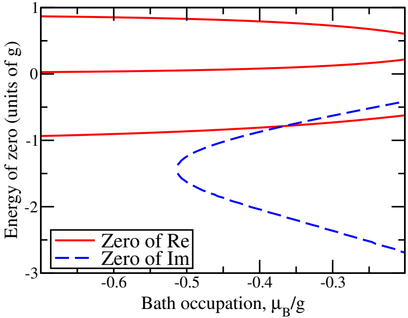

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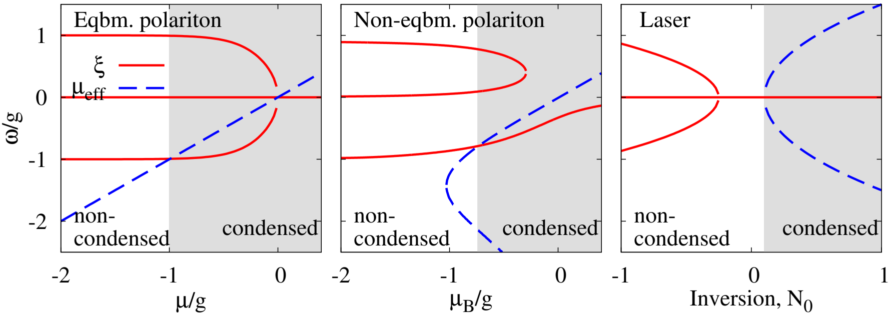
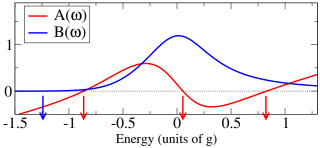
Strong coupling and lasing — low temperature phenomenon



- Laser: Uniformly invert TLS
- Non-equilibrium polaritons: Cold bath
- If $T_B \gg \gamma \rightarrow$ Laser limit



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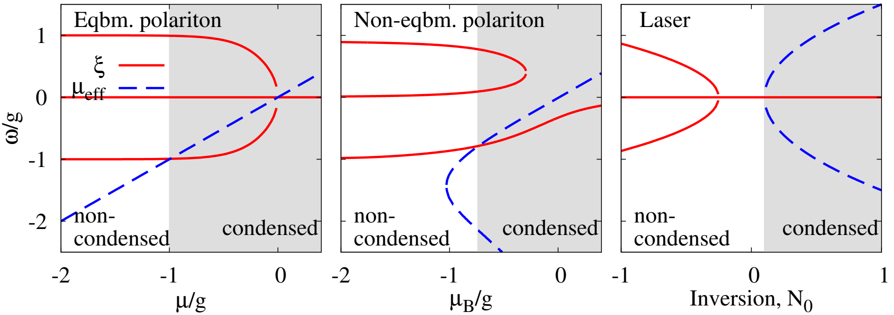

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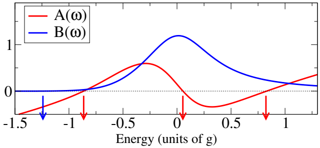

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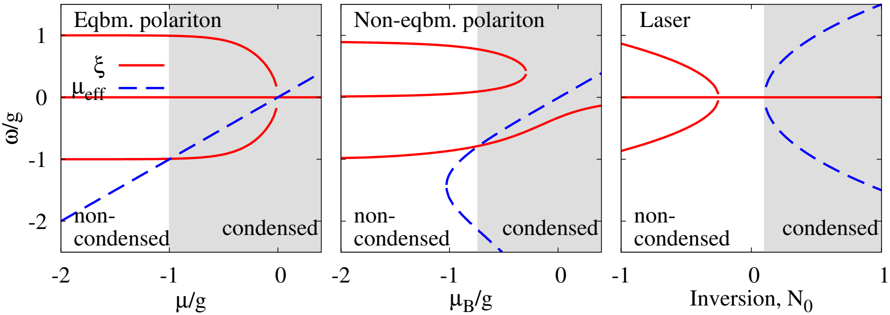

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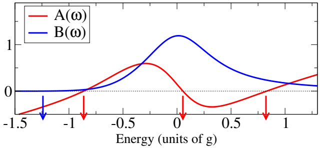

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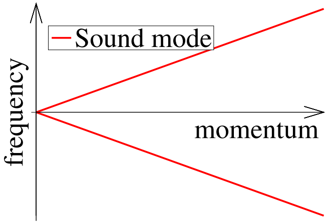
Spectrum above transition

When condensed

$$\text{Det}\left[D^R(\omega, k)\right]^{-1} = \omega^2 - \xi_k^2$$

With $\xi_k \simeq ck$
Poles:

$$\omega^* = \xi_k$$



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$$\mathrm{Det}\left[D^R(\omega, k)\right]^{-1} = (\omega + i\gamma_{\mathrm{net}})^2 + \gamma_{\mathrm{net}}^2 - \xi_k^2$$

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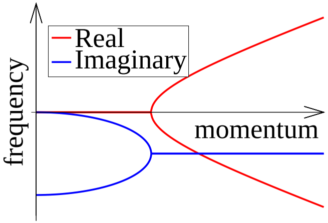
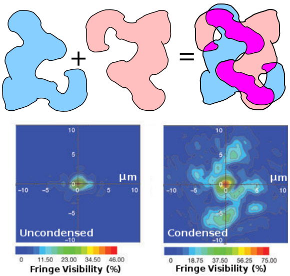


Correlations in a 2D Gas

Correlations:

$$g_1(\mathbf{r}, \mathbf{r}', t) = \left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0) \right\rangle$$



[Szymańska *et al.* PRL '06; PRB '07]

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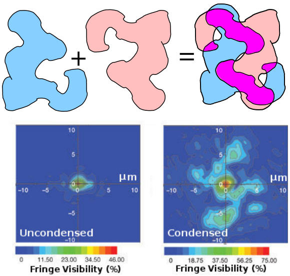
Correlations: (in 2D)

$$g_1(\mathbf{r}, \mathbf{r}', t) = \left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0) \right\rangle \simeq |\psi_0|^2 \exp\left[-D^<_{\phi\phi}(\mathbf{r}, \mathbf{r}', t) \right]$$



- $D^< = D^K - D^R + D^A$
- Generally, get: $\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0,0) \right\rangle \simeq$

$$|\psi_0|^2 \exp\left[-a_p \begin{cases} \ln(r/r_0) & r \to \infty, t \simeq 0 \\ \frac{1}{2}\ln(c^2 t/\gamma_{net} r_0^2) & r \simeq, t \to \infty \end{cases} \right]$$

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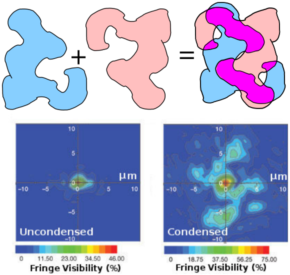

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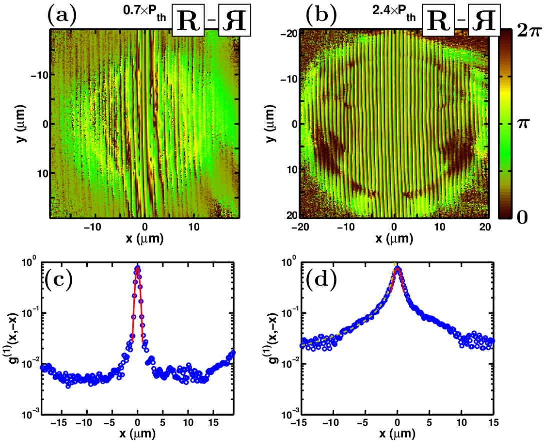

G. Rompos, Y. Yamamoto *et al.* submitted

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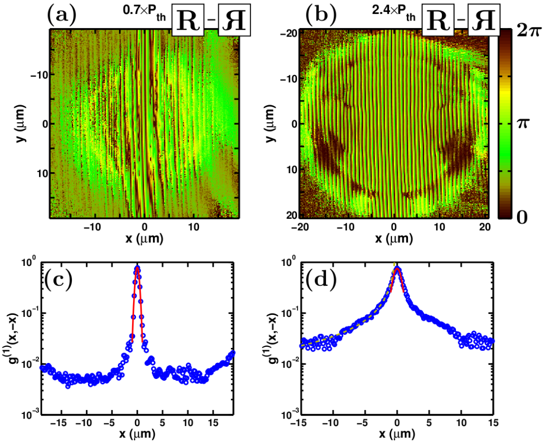
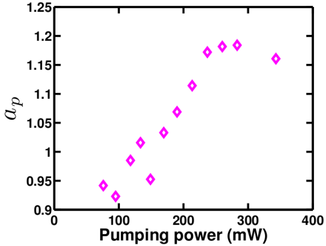


$$g_1(\mathbf{r}, -\mathbf{r}) \propto \left(\frac{r}{r_0}\right)^{-a_p}$$



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Exponent in a non-equilibrium 2D gas

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- In equilibrium $a_p = \frac{mk_B T}{2\pi\hbar^2 n_s} < \frac{1}{4}$ (BKT transition)
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Finite size effects: Single mode vs many mode

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$$D^<_{\phi\phi}(\mathbf{r},\mathbf{r},t) \propto \sum_n^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(\mathbf{r})|^2(1-e^{i\omega t})}{\left|(\omega+i\gamma_{\text{net}})^2+\gamma_{\text{net}}^2-\xi_n^2\right|^2}$$

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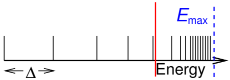

$$D^<_{\phi\phi} \sim 1 + \ln\left(E_{\text{max}}\sqrt{\frac{t}{\gamma_{\text{net}}}}\right)$$

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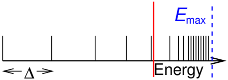

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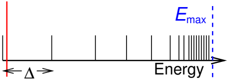
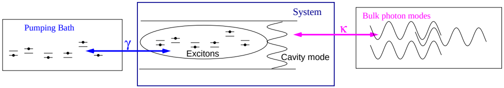


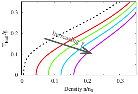
$$D^{<}_{\phi\phi} \sim \left(\frac{\pi C}{2\gamma_{\text{net}}}\right)\left(\frac{t}{2\gamma_{\text{net}}}\right)$$

(Recovers Schawlow-Townes laser linewidth)

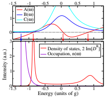
Summary

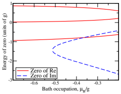
- Non-equilibrium mean field theory of polaritons

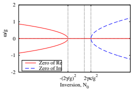




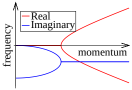
- Strong-coupling & condensation vs lasing.

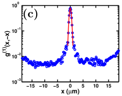


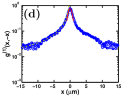




- Change to condensate spectrum and consequences





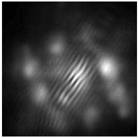


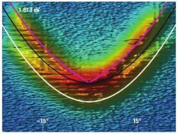
Extra slides

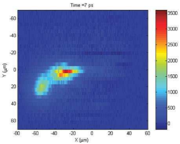
3. Superfluidity
   - Aspects of superfluidity
   - Current-current response function
   - Measuring superfluid density

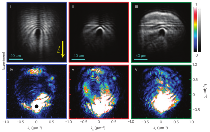
Aspects of superfluidity

	Quantised vortices	Landau critical velocity	Metastable persistent flow	Two-fluid hydrody-namics	Local thermal equilib-rium	Solitary waves
Superfluid ^{4}He/cold atom Bose-Einstein condensate	✔	✔	✔	✔	✔	✔
Non-interacting Bose-Einstein condensate	✔	✗	✗	✔	✔	✗
Classical irrotational fluid	✗	✔	✗	✔	✔	✔
Incoherently pumped polariton condensates	✔	✗	?	?	✗	?





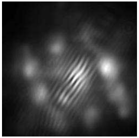


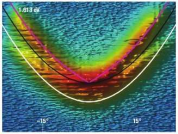


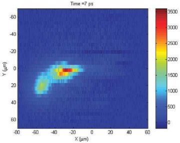
Lagoudakis *et al.* Nat. Phys. '08. Utsunomiya *et al.* Nat. Phys. '08. Amo *et al.* Nature '09; Nat. Phys. '09

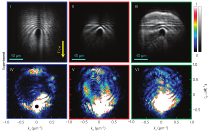
Aspects of superfluidity

	Quantised vortices	Landau critical velocity	Metastable persistent flow	Two-fluid hydrodynamics	Local thermal equilibrium	Solitary waves
Superfluid ^{4}He/cold atom Bose-Einstein condensate	✓	✓	✓	✓	✓	✓
Non-interacting Bose-Einstein condensate	✓	✗	✗	✓	✓	✗
Classical irrotational fluid	✗	✓	✗	✓	✓	✓
Incoherently pumped polariton condensates	✓	✗	?	?	✗	?









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Superfluid density

- Currrent:

$$\mathbf{J} = \rho \mathbf{v} = \psi^\dagger i \nabla \psi = |\psi|^2 \nabla \phi$$

$$J_i(\mathbf{q}) = \psi^\dagger_{\mathbf{k}+\mathbf{q}} \frac{2k_i + q_i}{2m} \psi_{\mathbf{k}}$$

- Response function:

$$H \to H - \sum_q \mathbf{f}(\mathbf{q}) \cdot \mathbf{J}_i(\mathbf{q}) \qquad J_i(\mathbf{q}) = \chi_{ij}(\mathbf{q}) f_j(\mathbf{q})$$

- Vertex corrections essential for superfluid part.

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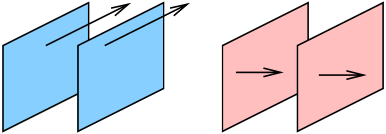

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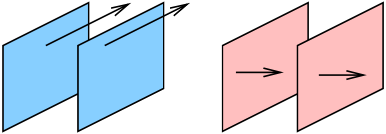

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Non-equilibrium current response functions

- Superfluid response exists because:

 $$= \left(\frac{i\psi_0 q_i}{2m}\right) D^R(q,\omega=0) \left(\frac{i\psi_0 q_j}{2m}\right)$$

- $D^R(\omega=0) \propto 1/q^2$ despite pumping/decay — superfluid response exists.
- Normal density:

$$\rho_N = \int d^d k\, \epsilon_k \int \frac{d\omega}{2\pi} \mathrm{Tr}\left[\sigma_z D^K \sigma_z (D^R + D^A)\right]$$

- Is affected by pump/decay.
  Does not vanish at $T \to 0$.

[JK, arXiv:1106.0682]

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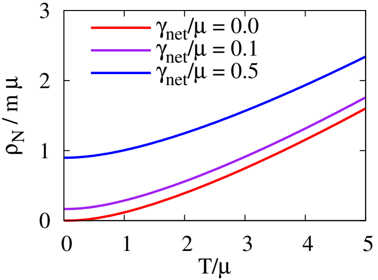

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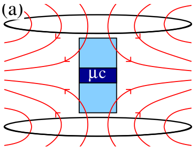
Measuring superfluid density

1. Effect rotating frame

Polariton polarization: $(\psi_{\circlearrowleft}, \psi_{\circlearrowright})$

$$H = \lambda \begin{pmatrix} \ell^2 & r^2 e^{2i\phi} \\ r^2 e^{-2i\phi} & -\ell^2 \end{pmatrix}$$



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Ground state Berry phase:

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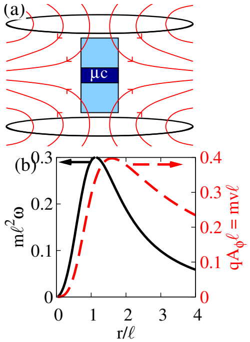

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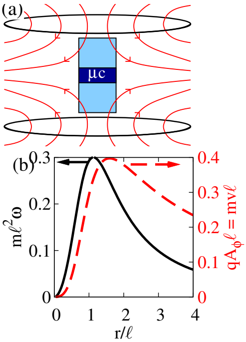

2. Measure resulting current

Energy shift of normal state:
$\Delta E = (1/2)mv^2 = 0.08/m\ell^2 \simeq 0.1\text{meV}$