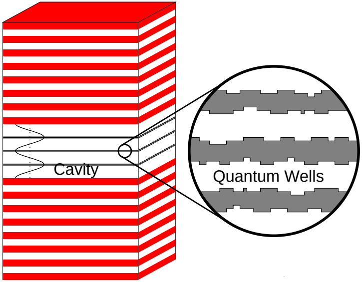
Condensation, superfluidity, and lasing of coupled light-matter systems.

Jonathan Keeling

IOP TCM Meeting, June 2011

Funding: EPSRC
Engineering and Physical Sciences
Research Council

Microcavity polaritons



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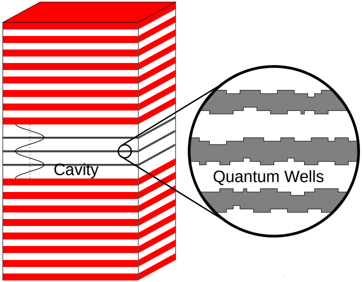

Cavity photons:

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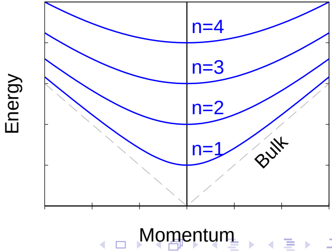
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$



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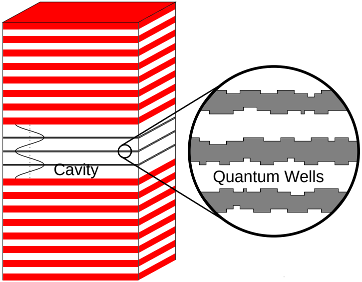

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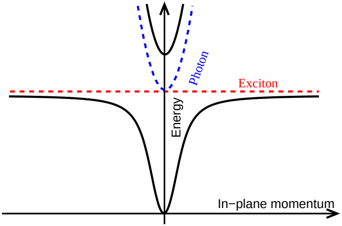

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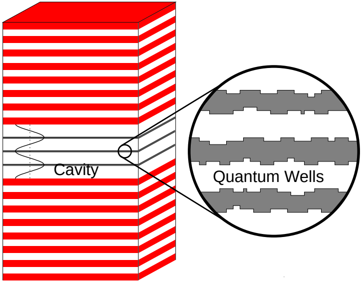

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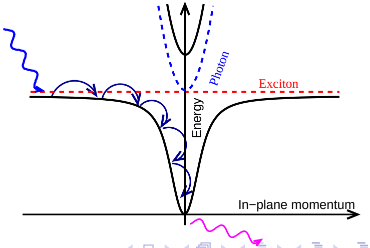

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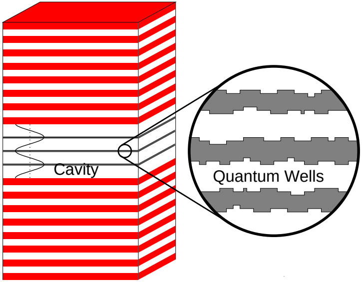

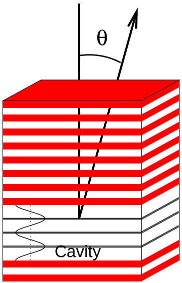

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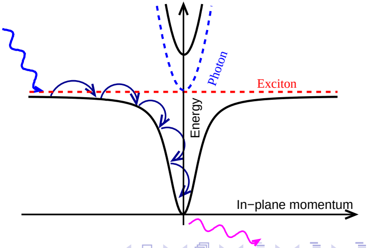
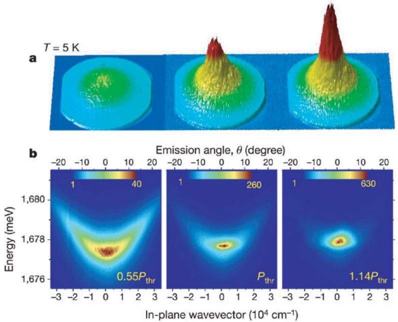
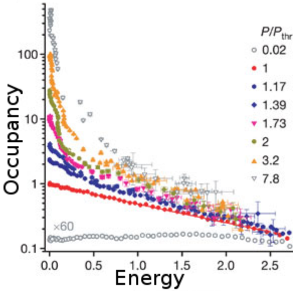


Polariton experiments: occupation and coherence





[Kasprzak, *et al.* Nature, 2006]

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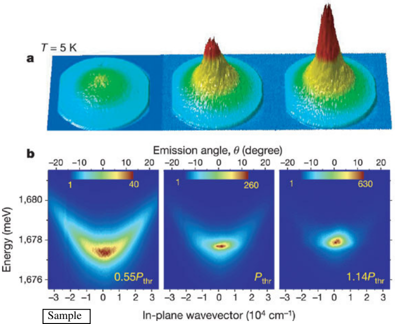
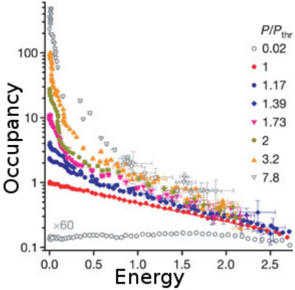
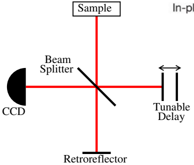
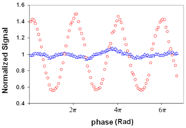
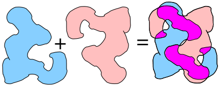


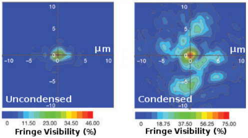






Coherence map:

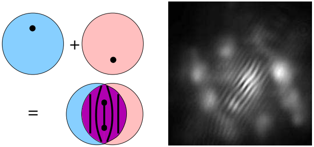




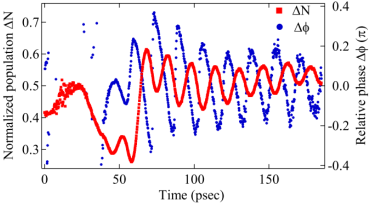
[Kasprzak, *et al.* Nature, 2006]

(Some) other polariton condensation experiments

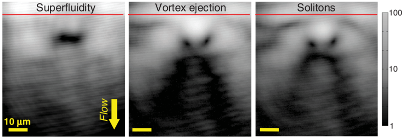
- Quantised vortices
  [**Lagoudakis *et al.* Nat. Phys. ’08**. Science ’09, PRL ’10; Sanvitto *et al.* Nat. Phys. ’10; Roumpos *et al.* Nat. Phys. ’10]



- Josephson oscillations
  [**Lagoudakis *et al.* PRL ’10**]



- Pattern formation/Hydrodynamics
  [**Amo *et al.* Science ’11**, Nature ’09; Wertz *et al.* Nat. Phys ’10]



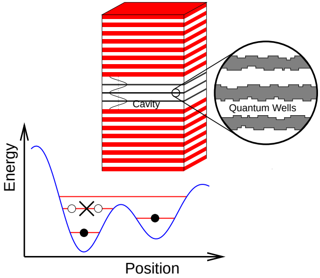
1. Microscopic model and lasing vs condensation
   - Introducing model
   - Stability of normal state
   - Comparison to standard laser

2. Coherence and superfluidity of condensate
   - Condensed spectrum
   - Aspects of superfluidity
   - Current-current response function
   - Power law decay of coherence

Microscopic model

Polariton model

- Disorder-localised excitons
- Treat sites as 2-level systems (exciton/no-exciton)
- Propagating (2D) photons



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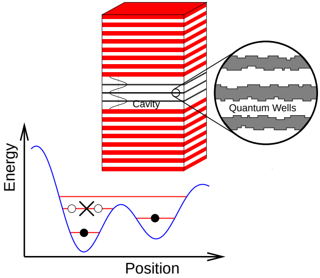
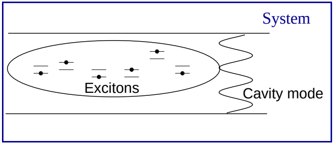


$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_{\alpha} \left(b^{\dagger}_{\alpha} b_{\alpha} - a^{\dagger}_{\alpha} a_{\alpha} \right) + \frac{1}{\sqrt{A}} g_{\alpha,\mathbf{k}} \psi_{\mathbf{k}} b^{\dagger}_{\alpha} a_{\alpha} + \text{H.c.} \right]$$



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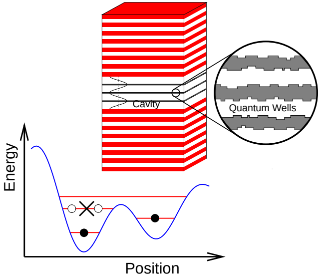

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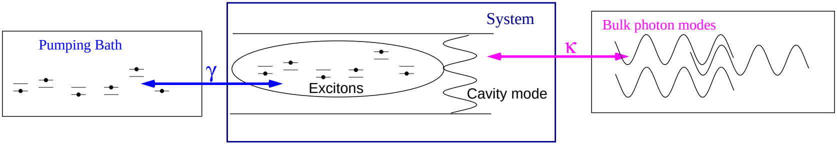

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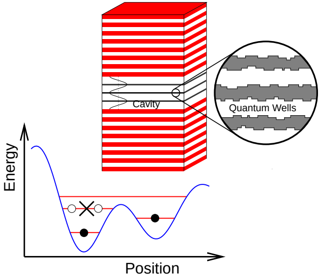

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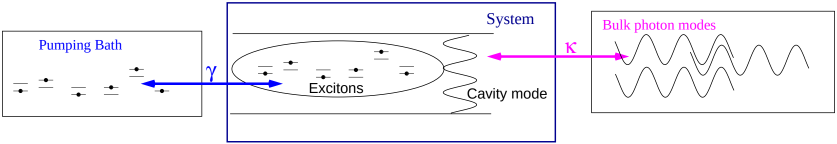

Pumping bath thermal, μ_B, T_B, Photon bath empty

Luminescence spectrum and Green's functions

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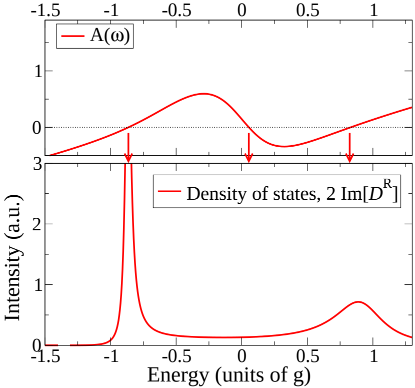

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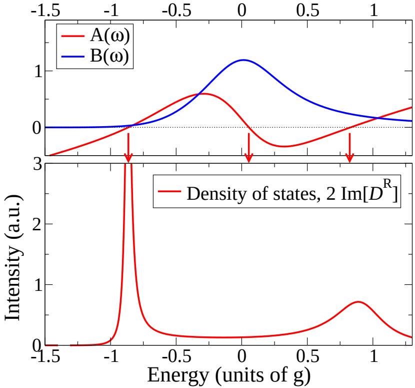

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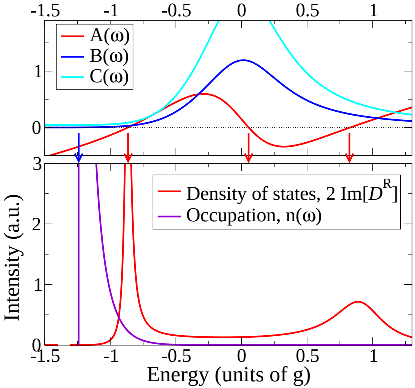
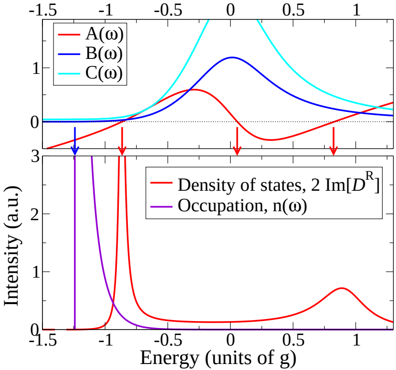


Stability and evolution with pumping



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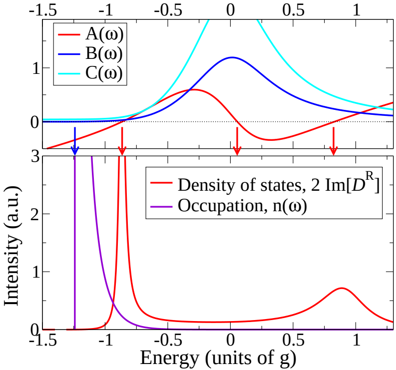

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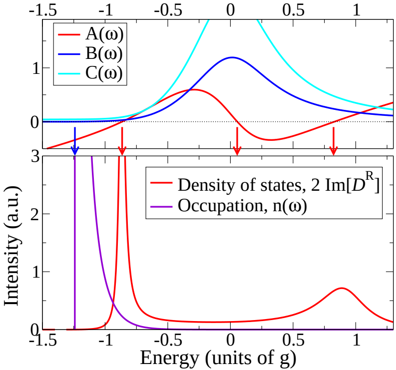

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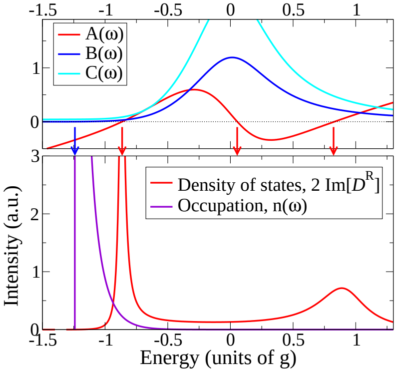

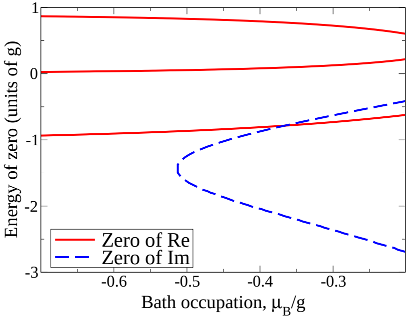

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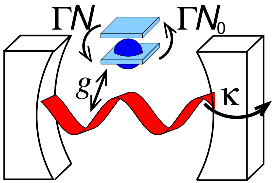
Green's function and stability for a laser



Maxwell-Bloch equations:

$$\partial_t \psi = -i\omega_k \psi - \kappa\psi + gP$$
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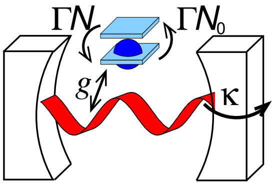

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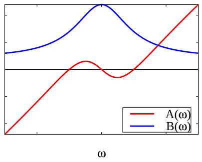

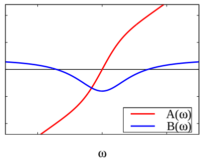

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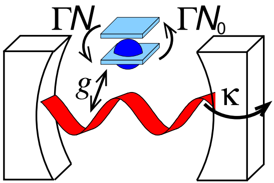

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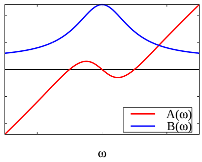

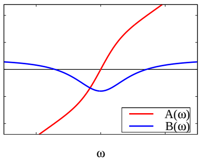

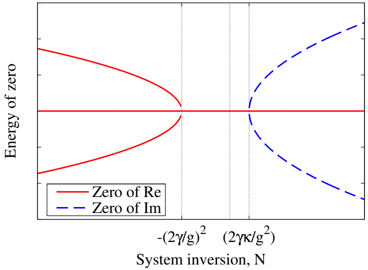

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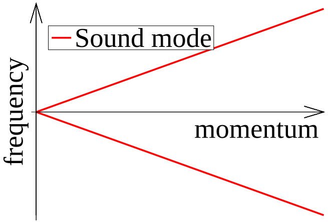
Spectrum above transition

When condensed

$$\text{Det}\left[D^R(\omega,k)\right]^{-1} = \omega^2 - \xi_k^2$$

With $\xi_k \simeq ck$
Poles:

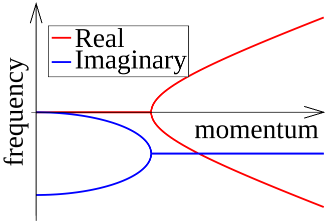
$$\omega^* = \xi_k$$



Spectrum above transition

When condensed

$$\text{Det}\left[D^R(\omega,k)\right]^{-1} = (\omega + i\gamma_{\text{net}})^2 + \gamma_{\text{net}}^2 - \xi_k^2$$

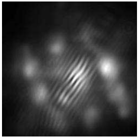


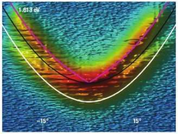
With $\xi_k \simeq ck$
Poles:

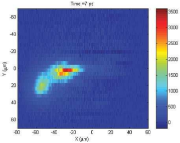
$$\omega^* = -i\gamma_{\text{net}} \pm \sqrt{\xi_k^2 - \gamma_{\text{net}}^2}$$

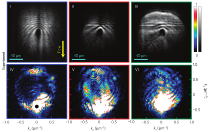
Aspects of superfluidity

	Quantised vortices	Landau critical velocity	Metastable persistent flow	Two-fluid hydrodynamics	Local thermal equilibrium	Solitary waves
Superfluid ^{4}He/cold atom Bose-Einstein condensate	✔	✔	✔	✔	✔	✔
Non-interacting Bose-Einstein condensate	✔	✗	✗	✔	✔	✗
Classical irrotational fluid	✗	✔	✗	✔	✔	✔
Incoherently pumped polariton condensates	✔	✗	?	?	✗	?









Lagoudakis *et al.* Nat. Phys. '08. Utsunomiya *et al.* Nat. Phys. '08. Amo *et al.* Nature '09; Nat. Phys. '09

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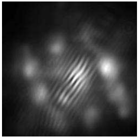

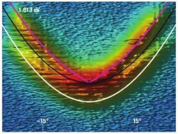

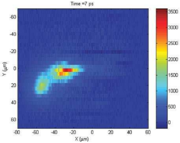

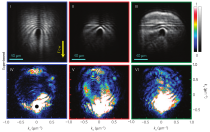

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Superfluid density

- Currrent:

$$\mathbf{J} = \rho \mathbf{v} = \psi^\dagger i \nabla \psi = |\psi|^2 \nabla \phi$$

$$J_i(\mathbf{q}) = \psi^\dagger_{\mathbf{k}+\mathbf{q}} \frac{2k_i + q_i}{2m} \psi_{\mathbf{k}}$$

- Response function:

$$H \to H - \sum_q \mathbf{f}(\mathbf{q}) \cdot \mathbf{J}_i(\mathbf{q}) \qquad J_i(\mathbf{q}) = \chi_{ij}(\mathbf{q}) f_j(\mathbf{q})$$

- Vertex corrections essential for superfluid part.

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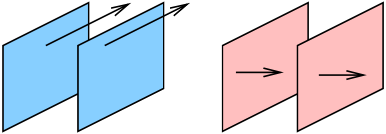

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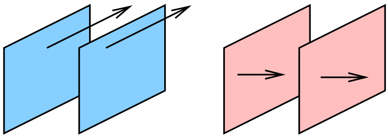

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Non-equilibrium current response functions

- Superfluid response exists because:

$$= \left(\frac{i\psi_0 q_i}{2m}\right) D^R(q, \omega = 0) \left(\frac{i\psi_0 q_j}{2m}\right)$$

- $D^R(\omega = 0) \propto 1/q^2$ despite pumping/decay — superfluid response exists.
- Normal density:

$$\rho_N = \int d^d k \epsilon_k \int \frac{d\omega}{2\pi} \mathrm{Tr}\left[\sigma_z D^K \sigma_z (D^R + D^A)\right]$$

- Is affected by pump/decay:
  Does not vanish at $T \to 0$.

[JK, arXiv:1106.0682]

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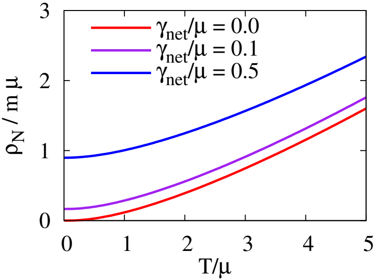
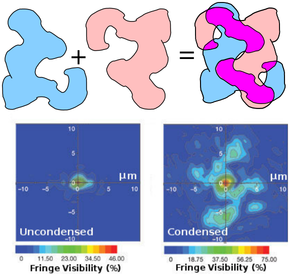


[JK, arXiv:1106.0682]

Correlations in a 2D Gas

Correlations:

$$g_1(\mathbf{r}, \mathbf{r}', t) = \left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, \mathbf{r}') \right\rangle$$

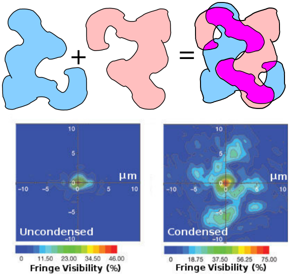


[Szymańska *et al.* PRL '06; PRB '07]

Correlations in a 2D Gas

Correlations: (in 2D)

$$g_1(\mathbf{r}, \mathbf{r}', t) = \left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, \mathbf{r}') \right\rangle$$
$$\simeq |\psi_0|^2 \exp\left[-D^{<}_{\phi\phi}(t, \mathbf{r}, \mathbf{r}')\right]$$



- $D^{<} = D^K - D^R + D^A$
- Generally, get: $\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, 0) \right\rangle \simeq$

$$|\psi_0|^2 \exp\left[-a_p \begin{cases} \ln(r/r_0) & r \to \infty, t \simeq 0 \\ \frac{1}{2}\ln(c^2 t/\gamma_{net} r_0^2) & r \simeq, t \to \infty \end{cases}\right]$$

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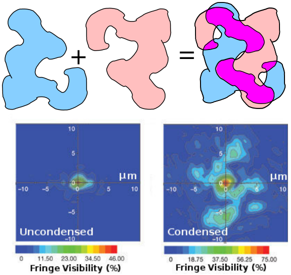

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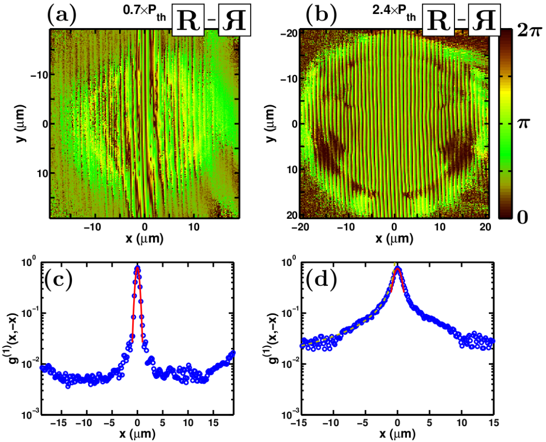
Experimental observation of power-law decay



G. Rompos, Y. Yamamoto *et al.* submitted

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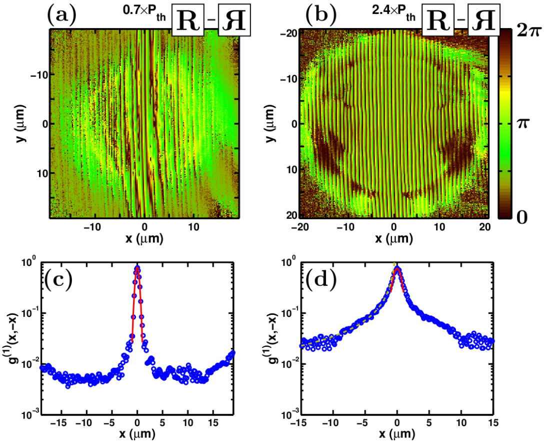
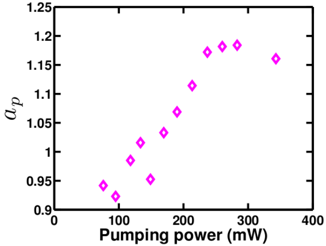


$$g_1(r,-r) \propto \left(\frac{r}{r_0}\right)^{-a_p}$$



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Exponent in a non-equilibrium 2D gas

$$\lim_{r\to\infty}\left\langle \psi^\dagger(\mathbf{r},0)\psi(-\mathbf{r},0)\right\rangle = |\psi_0|^2 \exp\left[-D^{<}_{\phi\phi}(\mathbf{r},-\mathbf{r})\right] \propto \exp\left[-a_p \ln\left(\frac{2r}{r_0}\right)\right]$$

- Experimentally, $a_P \simeq 1.2$
- In equilibrium $a_p = mk_BT/2\pi\hbar^2 n_s < 1/4$ (BKT transition)
- Non-equilibrium theory depends on thermalisation.

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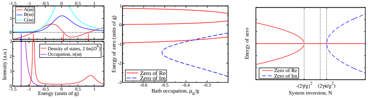
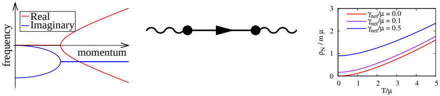
Conclusion

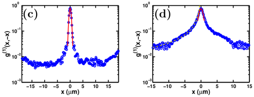
- Strong-coupling & condensation vs lasing.



- Survival of superfluid response



- Power law decay of correlations



Extra slides

3. Mean field theory

4. Green's functions and stability

5. Measuring superfluid density

Non-equilibrium theory; mean-field

Look for mean-field solution, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$.

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Look for mean-field solution, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$. Gap equation:

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Susceptibility:

$$\chi(\psi_0, \mu_s) = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon - \frac{1}{2}\mu_S)}{[(\nu - E)^2 + \gamma^2][(\nu + E)^2 + \gamma^2]}$$

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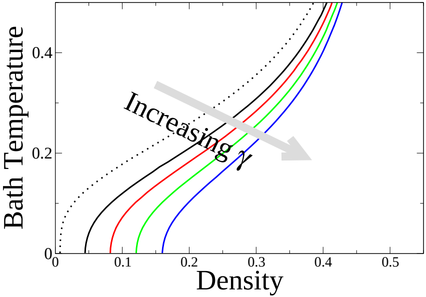
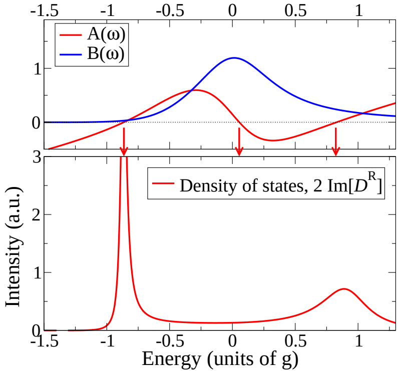


Poles of Green's function and stability

$$\left[D^R(\omega)\right]^{-1} = (\omega - \omega_k^*) + i\alpha(\omega - \mu_{\text{eff}})$$



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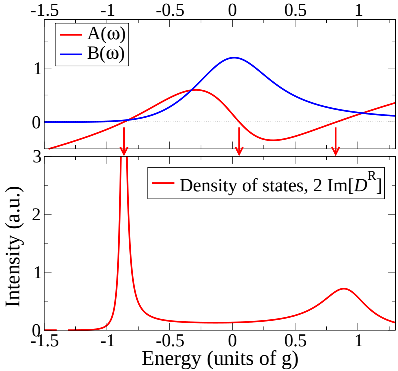

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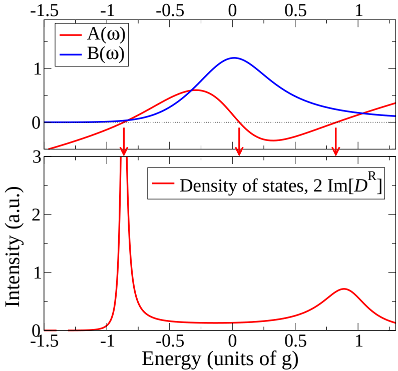

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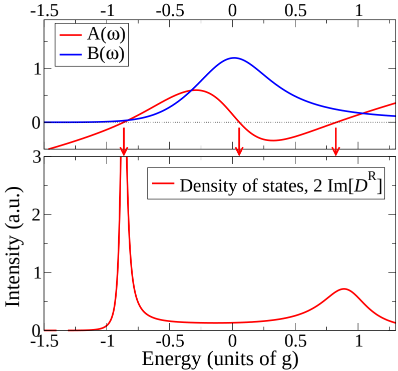

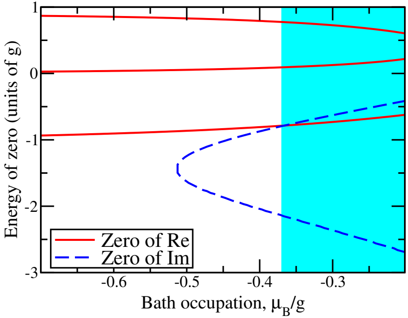

Strong coupling and lasing — low temperature phenomenon

- Laser result: $B(\omega) = \mathrm{Im}\left\{\left[D^R(\omega)\right]^{-1}\right\} = \kappa - 2\gamma\frac{g^2 N_0}{(\omega - 2\epsilon)^2 + 4\gamma^2}$

  Uniformly invert TLS

- Non-equilibrium model

$$B(\omega) = \kappa + \int \frac{d\nu}{2\pi}\sum_\alpha \frac{\gamma^2 g^2 (F_b(\nu+\omega) - F_a(\omega))}{\left[(\nu+\omega-\epsilon)^2+\gamma^2\right]\left[(\nu+\epsilon)^2+\gamma^2\right]}$$

  Inverts low energy part of homogeneous spectrum

- Non-equilibrium model, $T \gg \gamma$

$$B(\omega) \simeq \kappa + \sum_\alpha g_\alpha^2 \gamma \frac{[F_b(\epsilon_\alpha) - F_a(\epsilon_\alpha - \omega)]}{(\omega - 2\epsilon_\alpha)^2 + 4\gamma^2}$$

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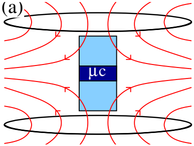
Measuring superfluid density

1. Effect rotating frame

Polariton polarization: $(\psi_{\circlearrowleft}, \psi_{\circlearrowright})$

$$H = \lambda \begin{pmatrix} \ell^2 & r^2 e^{2i\phi} \\ r^2 e^{-2i\phi} & -\ell^2 \end{pmatrix}$$



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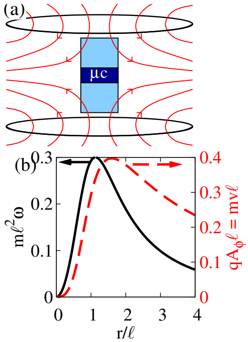
$$q\mathbf{A}_{\text{eff}} = m\omega \times \mathbf{r} = \frac{\hat{\phi}}{r}\left[1 - \frac{\ell^2}{\sqrt{r^4 + \ell^4}}\right]$$



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2. Measure resulting current

Energy shift of normal state:

$\Delta E = (1/2)mv^2 = 0.08/m\ell^2 \simeq 0.1\text{meV}$