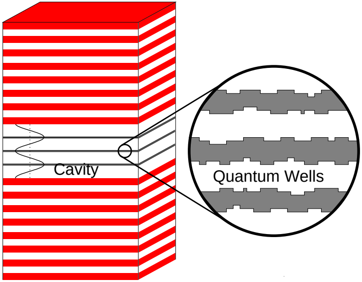
Superfluidity and pattern formation in non-equilibrium polariton condensates

Jonathan Keeling

The Burn, April 2011

Microcavity polaritons



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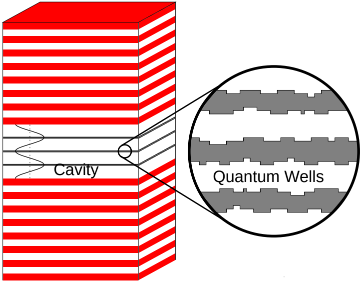

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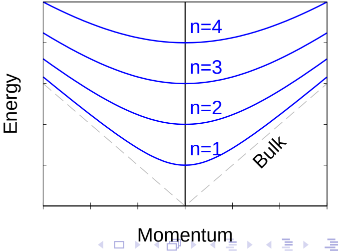
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$



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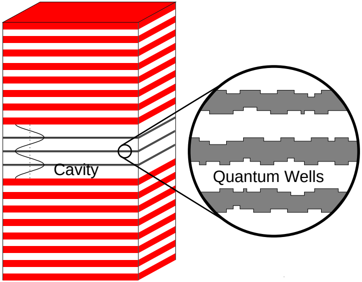

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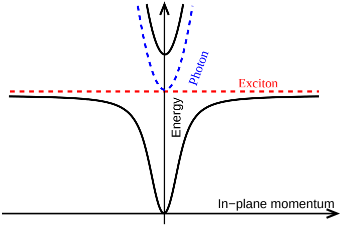

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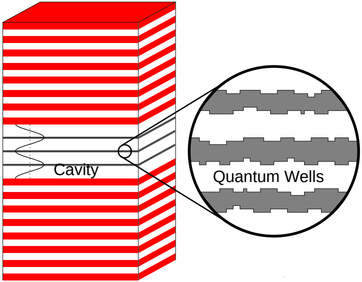

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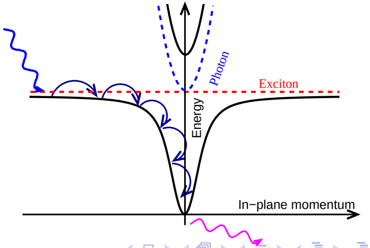

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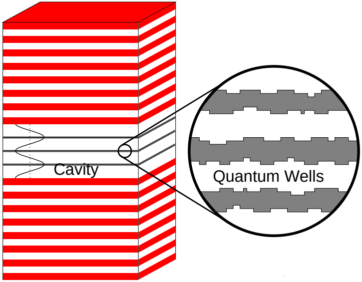

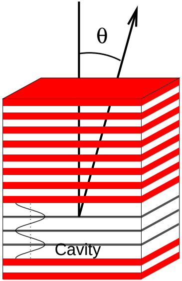

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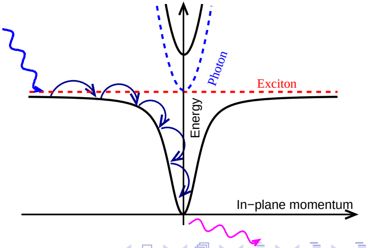
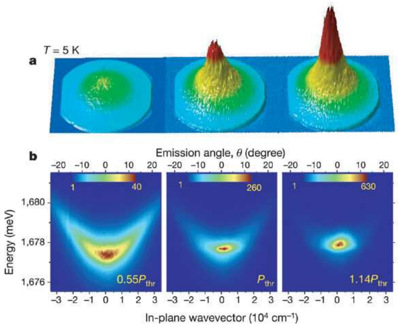
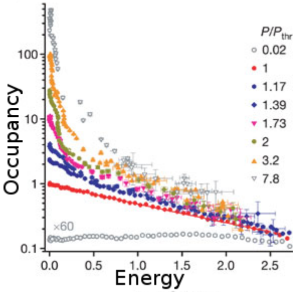


Polariton experiments: Momentum/Energy distribution





[Kasprzak, et al., Nature, 2006]

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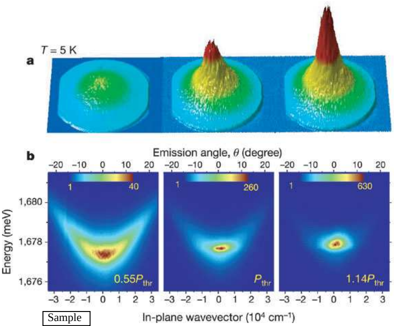

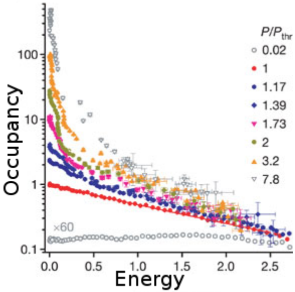

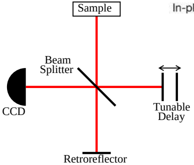

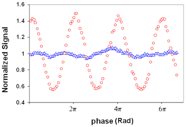

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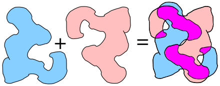
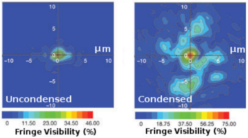
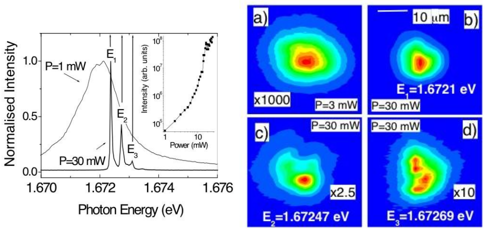
Coherence map:

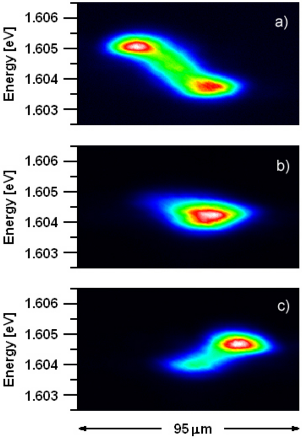




Other polariton condensation experiments

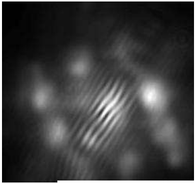
- Stress traps for polaritons
  [Balili *et al* Science 316 1007 (2007)]
- Temporal coherence and line narrowing
  [Love *et al* Phys. Rev. Lett. 101 067404 (2008)]



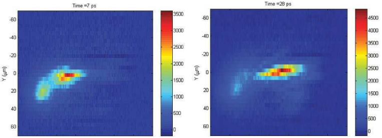


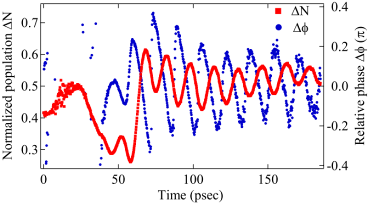
Other polariton condensation experiments

- Quantised vortices in disorder potential
  [Lagoudakis *et al* Nature Phys. 4, 706 (2008)]



- Soliton propagation
  [Amo *et al* Nature 457 291 (2009)]
- Driven superfluidity
  [Amo *et al* Nature Phys. (2009)
- Josephson Oscillations
  [Lagoudakis *et al* PRL (2010)]





1. Non-equilibrium model — coherence and strong coupling

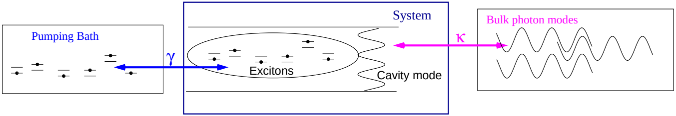
2. Pattern formation
   - Instability of Thomas-Fermi profile
   - Polarisation degree of freedom
   - Steady vortex lattices

3. Condensed spectrum and superfluidity
   - Condensed spectrum
   - Current-current response function
   - Power law decay of coherence

Non-equilibrium approach: Steady state, and fluctuations

$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}},$$

$$H_{\text{sys}} = \sum_k \omega_k \psi_k \psi_k^\dagger + \sum_\alpha g_\alpha (\phi_\alpha^\dagger \psi_k + \text{H.c.}) + H_{\text{ex}}[\phi_\alpha, \phi_\alpha^\dagger]$$



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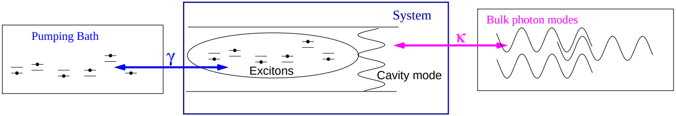

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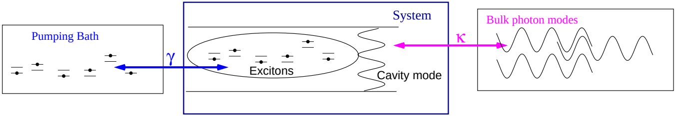

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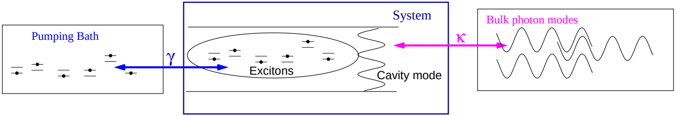

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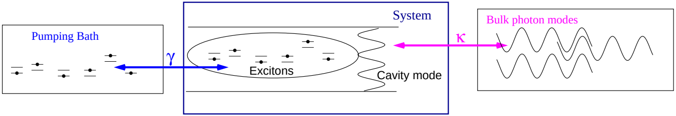

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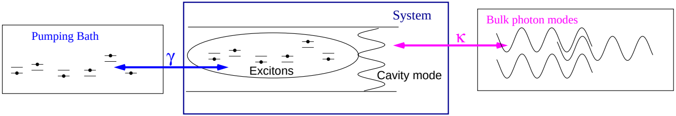

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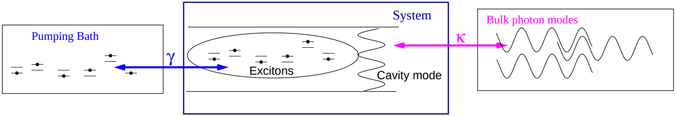

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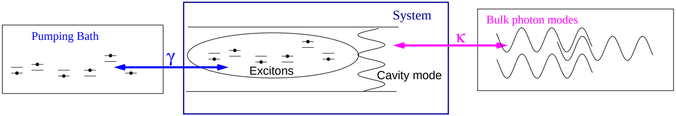

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   - Polarisation degree of freedom
   - Steady vortex lattices

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- Local density limit:

Complex Gross-Pitaevskii equation

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Nonlinear, complex susceptibility

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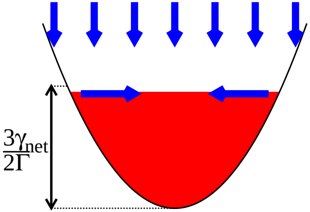

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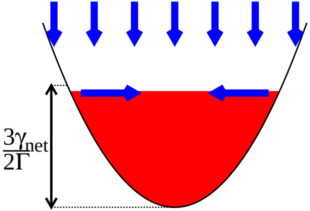

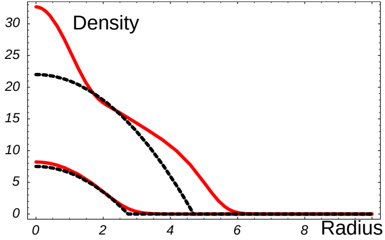

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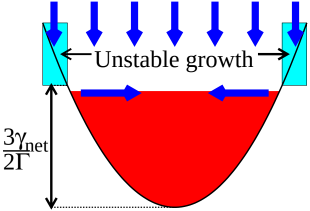

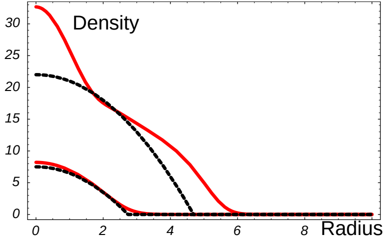

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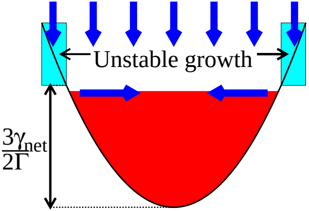
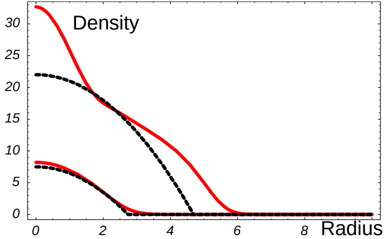
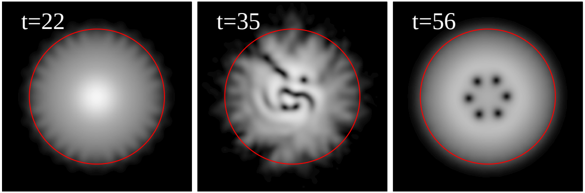




Pattern formation: Vortex lattices



Polariton spin degree of freedom

- Left- and Right-circular polarised states.
- For weakly-interacting dilute Bose gas model:
- Tendency to biexciton formation → U_1.
- Magnetic field: Δ
- $\Delta_{\parallel}$ — inequivalent axes.

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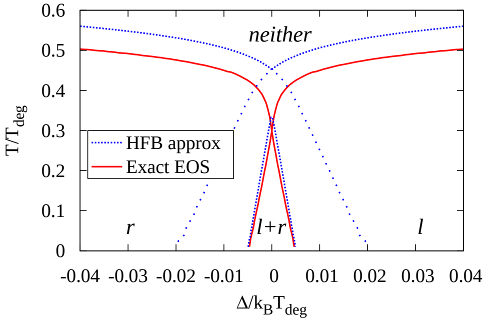
$\Delta_{\parallel} = 0$.
For $U_1 = 0.5$, $\Psi_{L,R}$ decouple.



Eliptical $\rightarrow$ Circular transitions.

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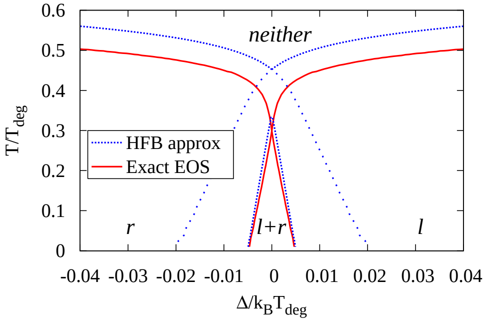

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$\Delta_{\parallel} \neq 0$: Eqbm state locked.

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Non-equilibrium spinor condensate

$$H = H_0[\psi_L] + H_0[\psi_R] + (U_0 - 2U_1)|\psi_L|^2|\psi_R|^2 + \frac{\Delta_\perp}{2}\left(|\psi_L|^2 - |\psi_R|^2\right) + \Delta_\parallel(\psi_L^\dagger\psi_R + \text{H.c.})$$

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$$i\partial_t\psi_L = \left[-\frac{\nabla^2}{2m} + V(r) + U_0|\psi_L|^2 + i\left(\gamma_{\text{eff}} - \kappa - \Gamma_0|\psi_L|^2 \qquad\qquad \right)\right]\psi_L$$

- Left-right coupling: U_1
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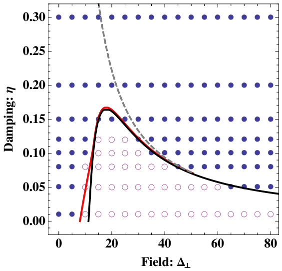

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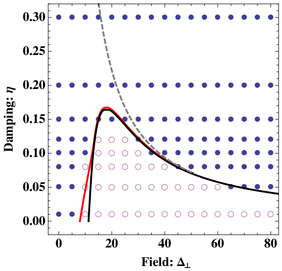
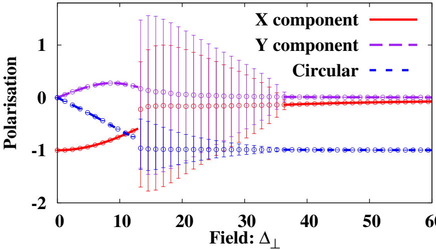
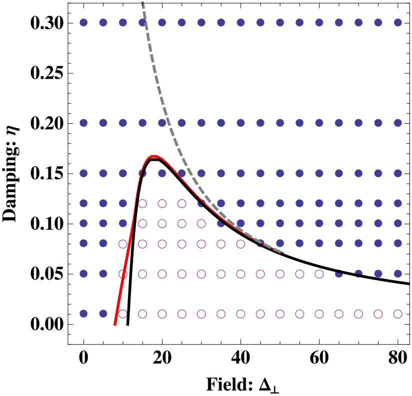




Desynchronisation and pattern formation

Trapping:

Synchronisation unaffected



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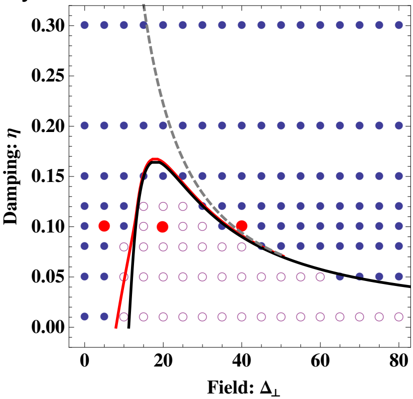

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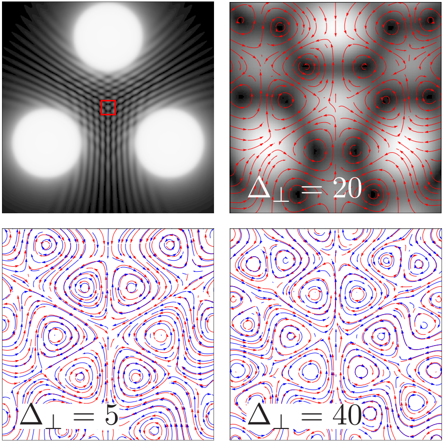

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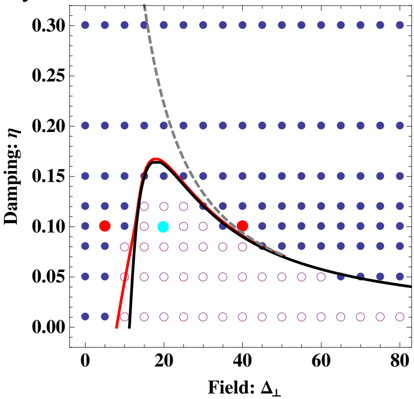

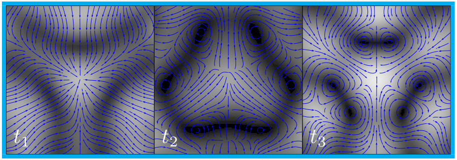

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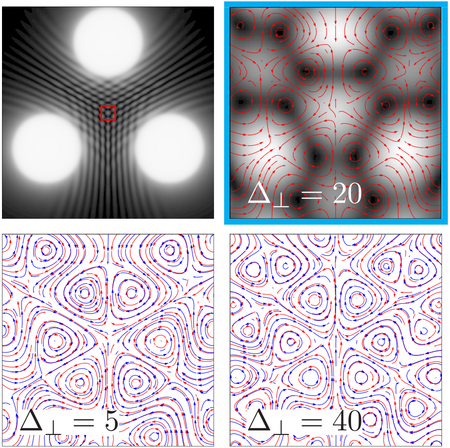

1. Non-equilibrium model — coherence and strong coupling

2. Pattern formation
   - Instability of Thomas-Fermi profile
   - Polarisation degree of freedom
   - Steady vortex lattices

3. Condensed spectrum and superfluidity
   - Condensed spectrum
   - Current-current response function
   - Power law decay of coherence

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Why superfluidity:

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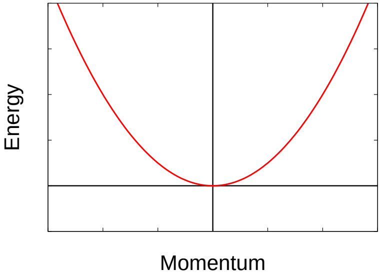

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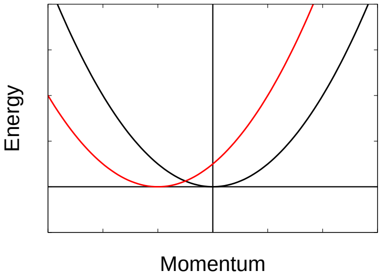

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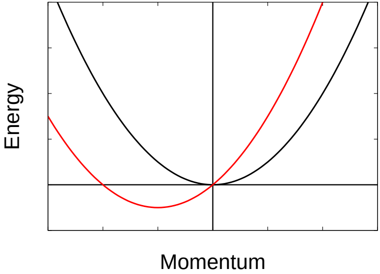

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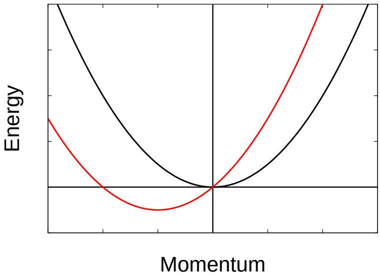

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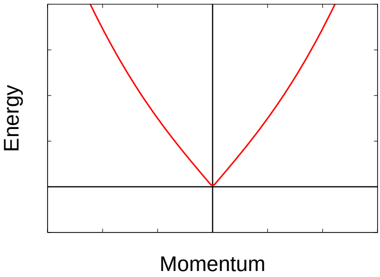

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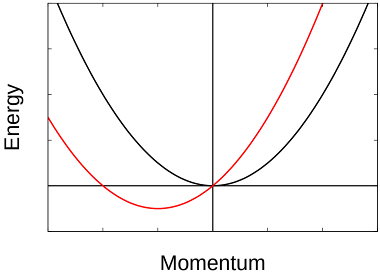

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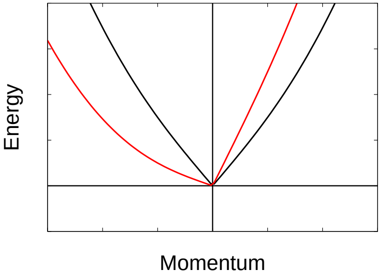


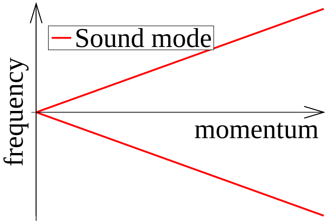
Fluctuations above transition

When condensed

$$\text{Det}\left[D^R(\omega,k)\right]^{-1} = \omega^2 - \xi_k^2$$

With $\xi_k \simeq ck$
Poles:

$$\omega^* = \xi_k$$



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$$\text{Det}\left[D^R(\omega, k)\right]^{-1} = (\omega + i\gamma_{\text{net}})^2 + \gamma_{\text{net}}^2 - \xi_k^2$$

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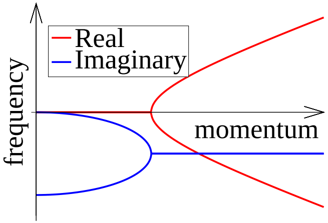
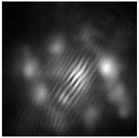
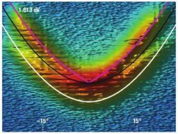
$$\omega^* = -i\gamma_{\text{net}} \pm \sqrt{\xi_k^2 - \gamma_{\text{net}}^2}$$

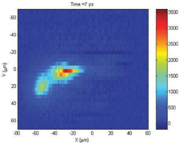


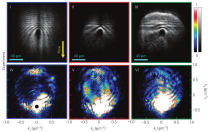
Non-equilibrium superfluidity checklist

	Quantised vortices	Landau critical velocity	Metastable persistent flow	Two-fluid hydrodynamics	Local thermal equilibrium	Solitary waves
Superfluid ^{4}He/cold atom Bose-Einstein condensate	✔	✔	✔	✔	✔	✔
Non-interacting Bose-Einstein condensate	✔	✗	✗	✗	✔	✗
Classical irrotational fluid	✗	✔	✗	✔	✔	✔
Incoherently pumped polariton condensates	✔	✗	?	?	✗	?









Lagoudakis *et al* Nature Phys. 4, 706 (2008). Utsunomiya *et al* Nature Phys. 4 700 (2008). Amo *et al* Nature 457 291 (2009); Nature Phys (2009)

Asking about non-equilibrium superfluidity

Currrent:

$$\mathbf{J} = \rho\mathbf{v} = \psi^\dagger i\nabla\psi = |\psi|^2\nabla\phi$$

- Response function:

$$\chi_{ij}(\omega = 0, \mathbf{q} \to 0) = \langle [J_i(\mathbf{q}), J_j(-\mathbf{q})] \rangle = \frac{\rho_S}{m}\frac{q_i q_j}{q^2} + \frac{\rho_N}{m}\delta_{ij}$$

- Given D and $J_i = \psi^\dagger_{k+q}\frac{2k_i + q_i}{2m}\psi_k$
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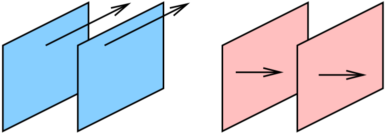

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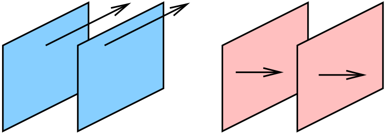

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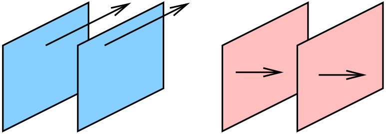

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 $$= \frac{i\psi_0 q_i}{2m}\,(1,-1)\,D^R(q,\omega=0)\begin{pmatrix}1\\-1\end{pmatrix}\frac{i\psi_0 q_j}{2m}$$

- $D^R(\omega=0) \propto 1/\epsilon_q$ despite pumping/decay — superfluid response exists.
- Normal density:

$$\rho_N = \int d^d k \epsilon_k \int \frac{d\omega}{2\pi} \mathrm{Tr}\left[\sigma_z D^K \sigma_z (D^R + D^A)\right]$$

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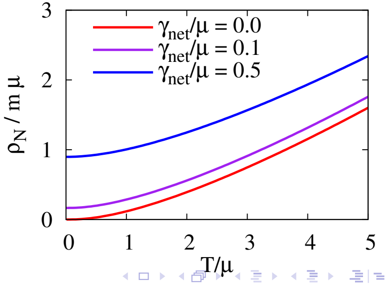

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Correlations in a 2D Gas

Correlations (in 2D):

$$g_1(\mathbf{r},\mathbf{r}') = \left\langle \psi^\dagger(\mathbf{r},t)\psi(0,r') \right\rangle \simeq |\psi_0|^2 \exp\left[-D^<_{\phi\phi}(t,r,r')\right]$$

- $D^< = D^K - D^R + D^A$
- Generally, get: $\left\langle \psi^\dagger(\mathbf{r},t)\psi(0,0) \right\rangle \simeq$
  $$|\psi_0|^2 \exp\left[-a_p \begin{cases} \ln(r/r_0) & r\to\infty, t\simeq 0 \\ \frac{1}{2}\ln(c^2 t/\gamma_{net} r_0^2) & r\simeq, t\to\infty \end{cases}\right]$$

[Szymańska et al., PRL '06; PRB '07]

Correlations in a 2D Gas

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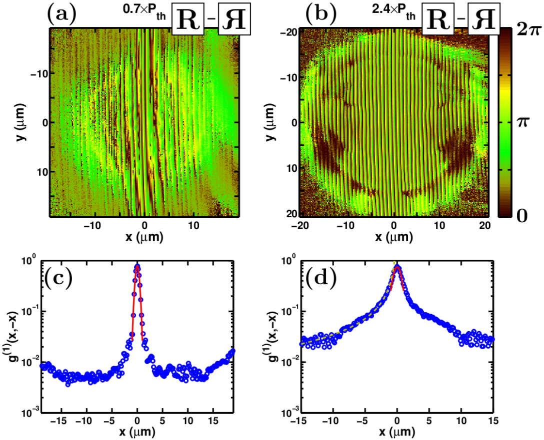
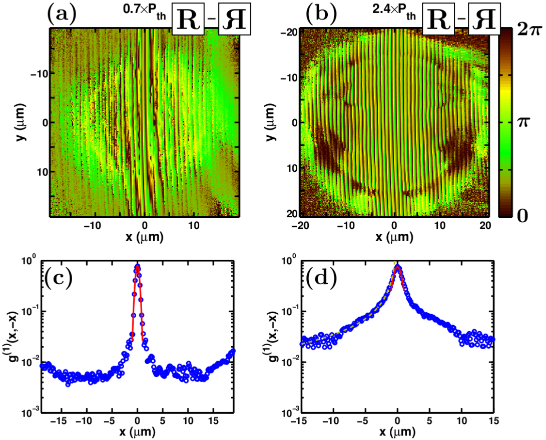
[Szymańska et al., PRL ’06; PRB ’07]

Power law experiment

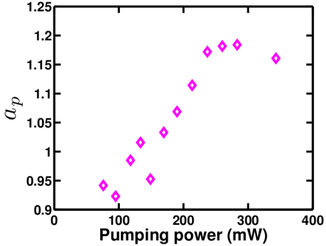


G. Rompos, Y. Yamamoto et al., submitted

Power law experiment



$$g_1(r,-r) \propto \left(\frac{r}{r_0}\right)^{-a_p}$$



G. Rompos, Y. Yamamoto et al., submitted

Power law experiment — non-equilibrium theory

$$\lim_{r\to\infty}\left\langle \psi^\dagger(\mathbf{r},0)\psi(-\mathbf{r},0)\right\rangle = |\psi_0|^2 \exp\left[-D^{<}_{\phi\phi}(r,-r)\right] \propto \exp\left[-a_p \ln\left(\frac{2r}{r_0}\right)\right]$$

- Experimentally, $a_P \simeq 1.1$
- In equilibrium $a_p = mk_B T/2\pi\hbar^2 n_s < 1/4$ (BKT transition)
- Non-equilibrium theory depends on thermalisation.

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Acknowledgements

People:

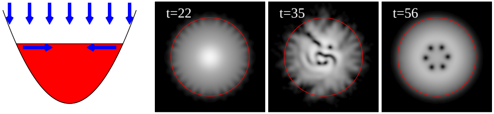
Funding:

EPSRC

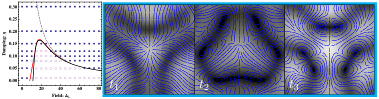
Engineering and Physical Sciences
Research Council

Summary

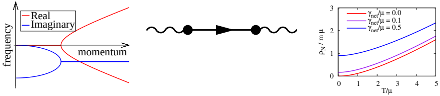
- Instability of Thomas-Fermi and spontaneous rotation



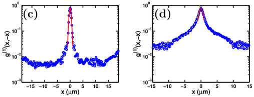
- Desynchronisation and vortex lattices



- Survival of superfluid response vs change to spectrum



- Power law decay of correlations



Extra slides

4 Non-equilibrium pattern formation

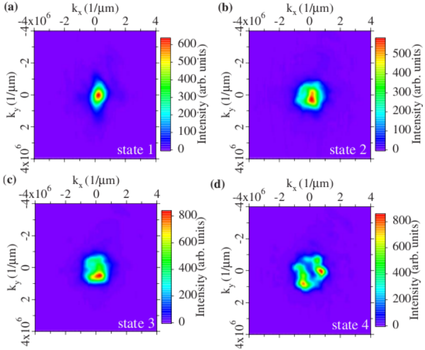
5 Other polariton experiments

6 Green's functions and stability

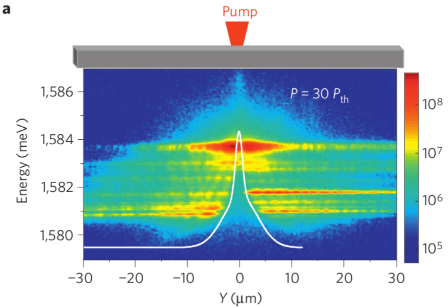
7 Spinor problem

8 T=0 Keldysh results

Other polariton condensation experiments: Non-equilibrium features



$|\psi(\mathbf{k})|^2 \neq |\psi(-\mathbf{k})|^2$:
Broken time-reversal symmetry.
[Krizhanovskii *et al,* PRB (2009)]



Flow from pumping spot
[Wertz *et al.*, Nat. Phys. (2010)]

Complex Gross-Pitaevskii equation

Steady state equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit:

Complex Gross-Pitaevskii equation

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$$\left(i\partial_t + i\kappa - \left[V(r) - \frac{\nabla^2}{2m}\right]\right)\psi(r) = \chi(\psi(r,t))\psi(r,t)$$

Nonlinear, complex susceptibility

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Gross-Pitaevskii equation: Harmonic trap

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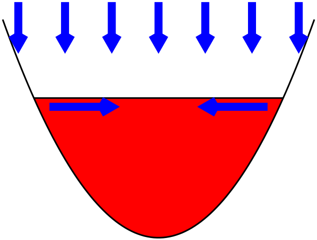

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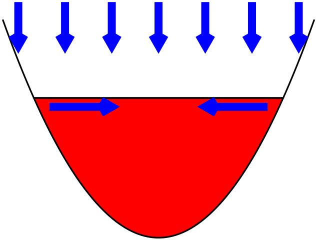
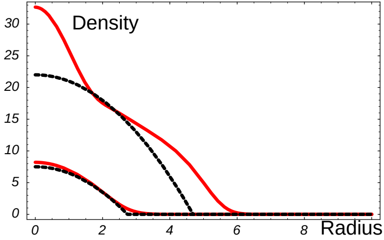
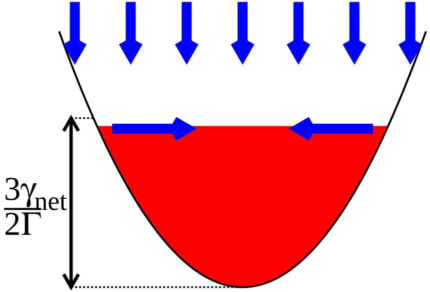




Stability of Thomas-Fermi solution

$$\frac{1}{2}\partial_t\rho+\nabla\cdot(\rho\mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$



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High m modes: $\delta\rho_{n,m} \simeq e^{im\theta} r^m \ldots$

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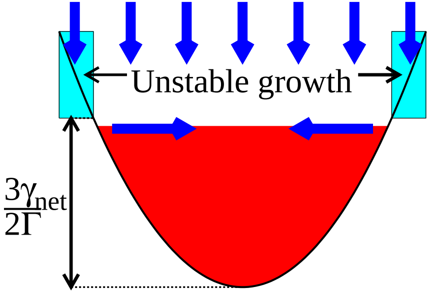

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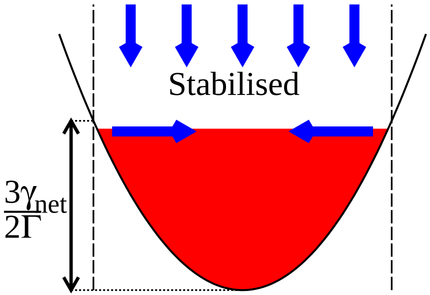

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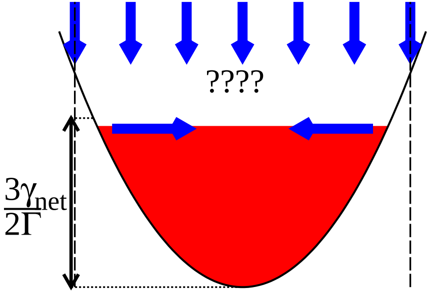
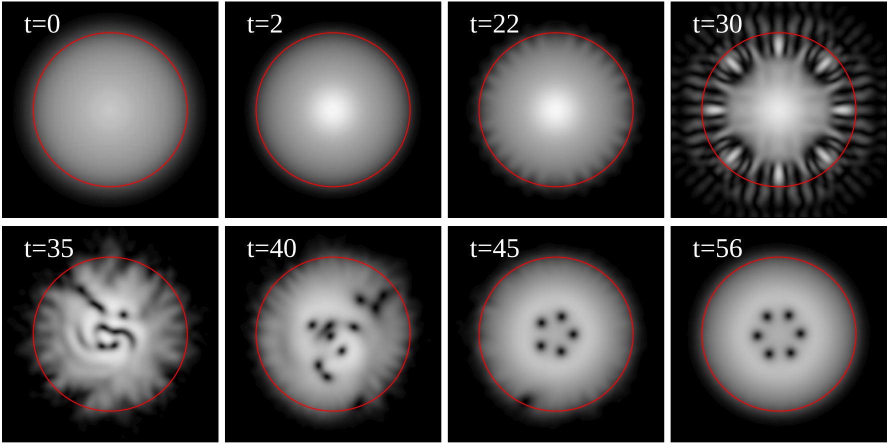
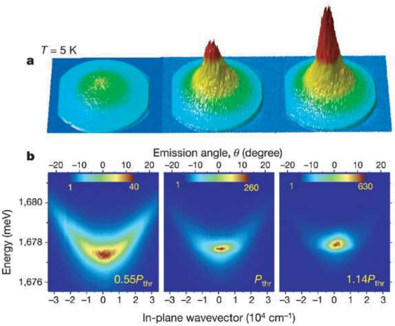
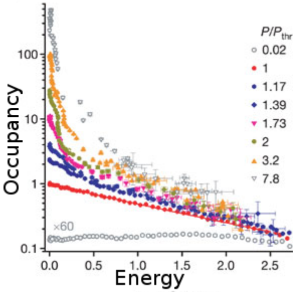


Time evolution:



Polariton experiments: Momentum/Energy distribution





[Kasprzak, et al., Nature, 2006]

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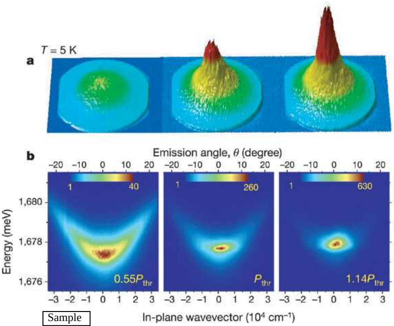
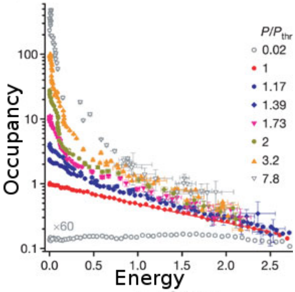
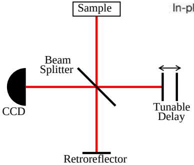
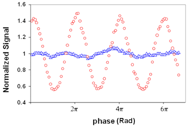
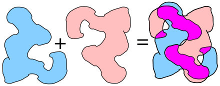


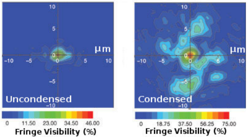






Coherence map:

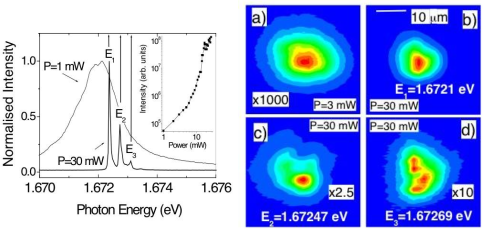


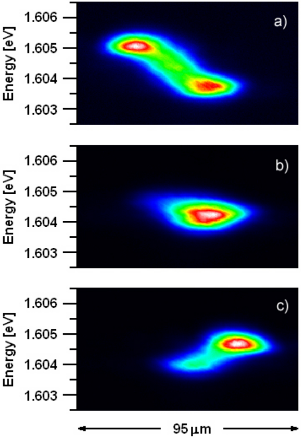


[Kasprzak, et al., Nature, 2006]

Other polariton condensation experiments

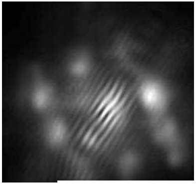
- Stress traps for polaritons
  [Balili *et al* Science 316 1007 (2007)]
- Temporal coherence and line narrowing
  [Love *et al* Phys. Rev. Lett. 101 067404 (2008)]



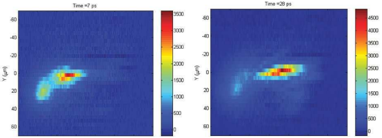


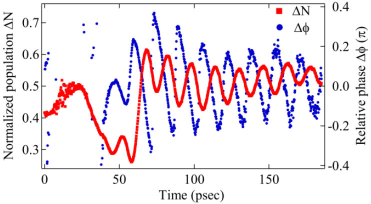
Other polariton condensation experiments

- Quantised vortices in disorder potential
  [Lagoudakis *et al* Nature Phys. 4, 706 (2008)]



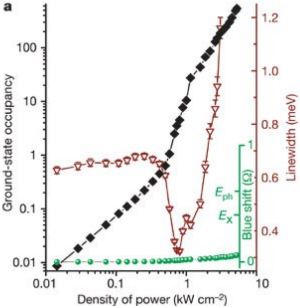
- Soliton propagation
  [Amo *et al* Nature 457 291 (2009)]
- Driven superfluidity
  [Amo *et al* Nature Phys. (2009)
- Josephson Oscillations
  [Lagoudakis *et al* PRL (2010)]





Polariton experiments: Strong coupling

[Bajoni *et al* PRL 2008]



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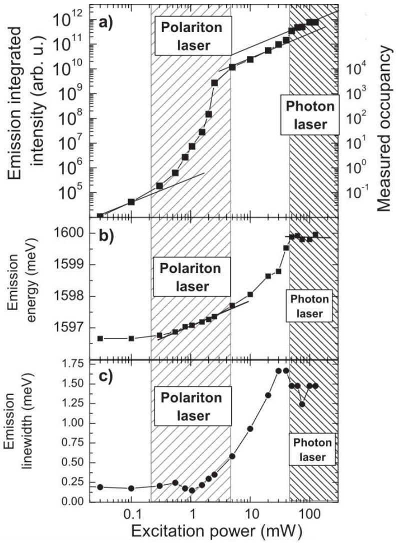

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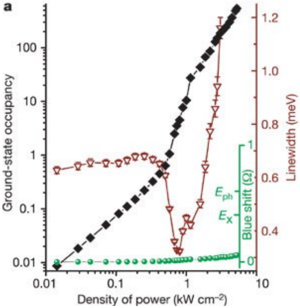

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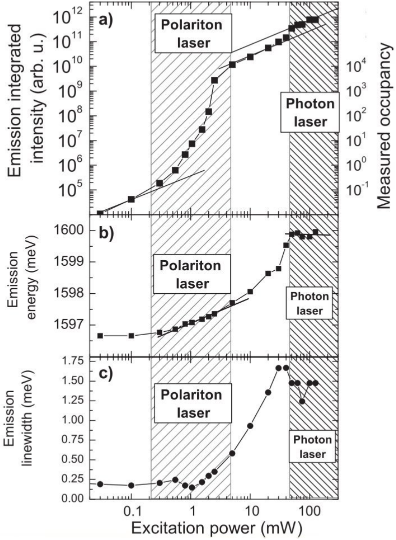
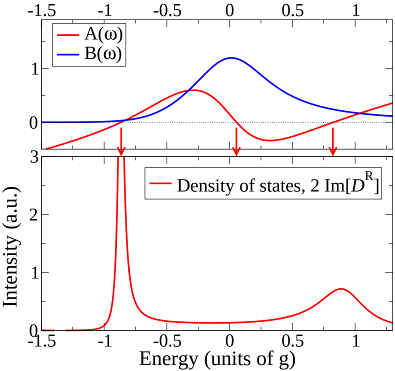


Strong coupling via:

- Small blueshift compared to Ω_R
- Polaritonic dispersion, $m > m_{\text{phot}}$
- Separate photon threshold

Poles of Green's function and stability

$$\left[D^R(\omega)\right]^{-1} = (\omega - \omega_k^*) + i\alpha(\omega - \mu_{\text{eff}})$$



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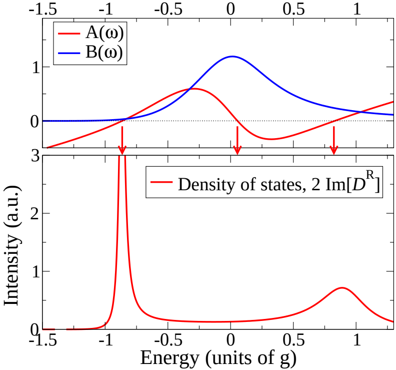

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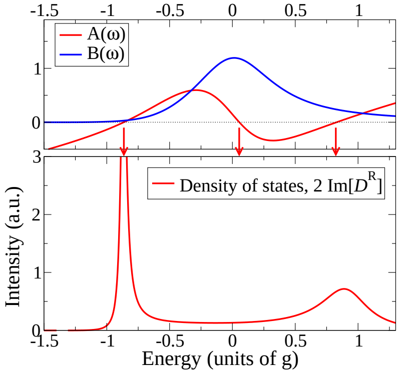

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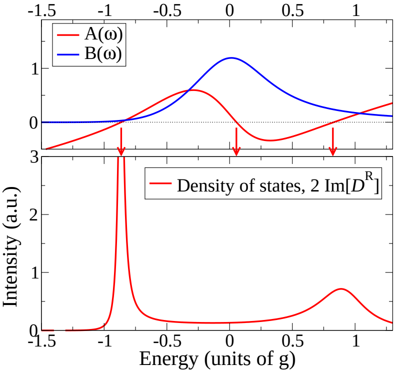
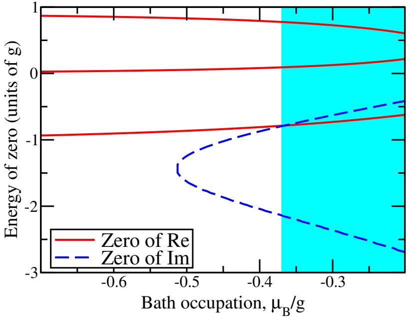
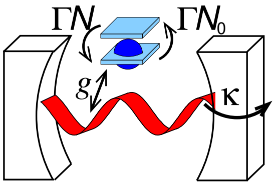




Poles and stability for a laser



Maxwell-Bloch equations:

$$\partial_t \psi = -i\omega_k \psi - \kappa\psi + gP$$
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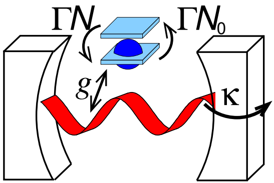

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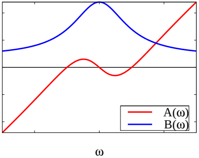

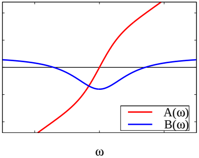

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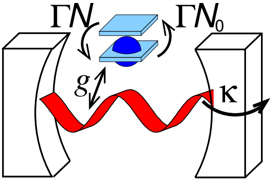

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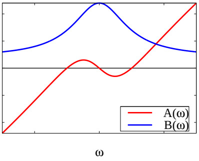

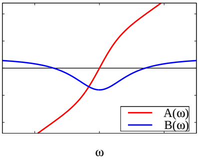

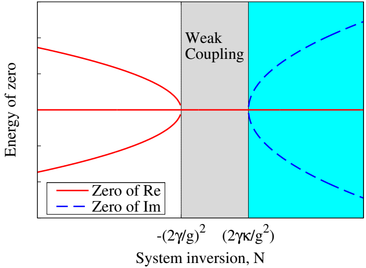

Spin in terms of ~~two~~four-level systems

To include spin, replace 2 level system with 4 levels: $|0\rangle, |L\rangle, |R|\rangle, |LR\rangle$

- Bi-exciton binding $E_{XX} \leftrightarrow U_1$
- Mean-field: find polarisation given ψ_L, ψ_R.
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[Marchetti *et al* PRB, '08]

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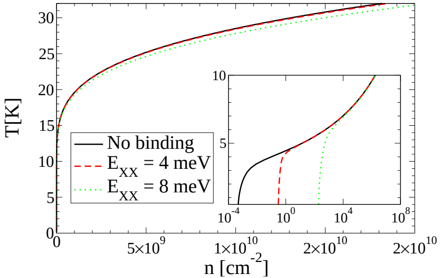

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Non-equilibrium spinor condensate

$$H = H_0[\psi_L] + H_0[\psi_R] + (U_0 - 2U_1)|\psi_L|^2|\psi_R|^2 + \frac{\Delta_\perp}{2}\left(|\psi_L|^2 - |\psi_R|^2\right) + \Delta_\parallel(\psi_L^\dagger\psi_R + \text{H.c.})$$

Spinor Gross-Pitaekvsii equation

$$i\partial_t\psi_L = \left[-\frac{\nabla^2}{2m} + V(r) + U_0|\psi_L|^2 + i\left(\gamma_{\text{eff}} - \kappa - \Gamma_0|\psi_L|^2 \qquad \right)\right]\psi_L$$

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Non-equilibrium spinor system: two-mode model

Two-mode case (neglect spatial variation) [Wouters PRB '08]

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Write:

$$\psi_L = \sqrt{R+z}e^{i\phi + i\theta/2},$$
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Simple case $\Gamma_1 = \eta = 0$

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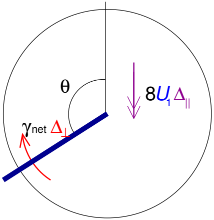

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Damped, driven pendulum

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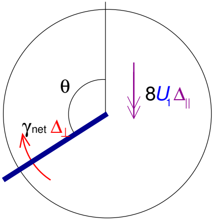

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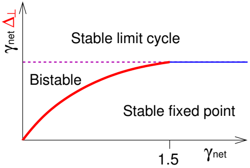
Cartoon:



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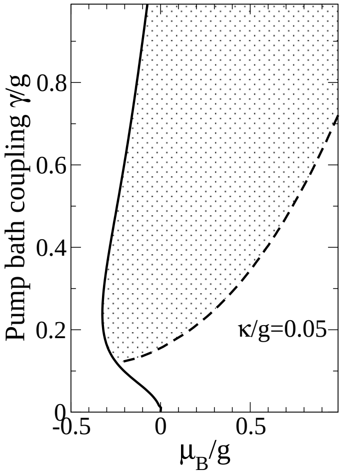
Zero temperature phase diagram

$$\tilde{\omega}_0 - i\kappa = g^2\gamma \sum_\alpha \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(\tilde{\epsilon}_\alpha + i\gamma)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}.$$



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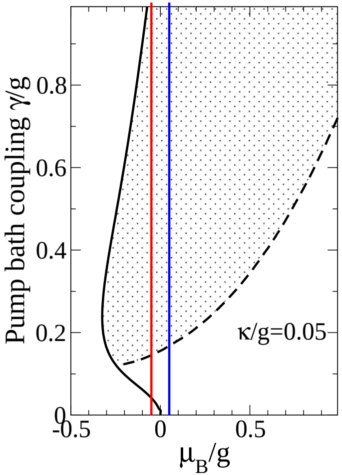

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