Collective dynamics of Bose–Einstein condensate in optical cavities

J. Keeling, J. A. Mayoh, M. J. Bhaseen, B. D. Simons

Munich, April 2011

Acknowledgements

People:

Funding:

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Engineering and Physical Sciences
Research Council

Coupling many atoms to light

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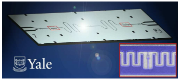
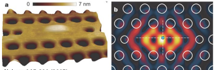
- Superconducting qubits 

- Quantum dots 

Nature 445 896 (2007)
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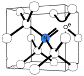
- Nitrogen-Vacancies in diamond 
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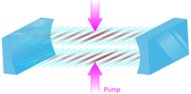
- Ultra-cold atoms 

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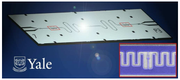
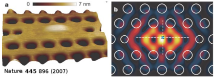
- Superconducting qubits 

- Quantum dots 

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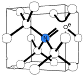
- Nitrogen-Vacancies in diamond 
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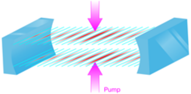
- Ultra-cold atoms 

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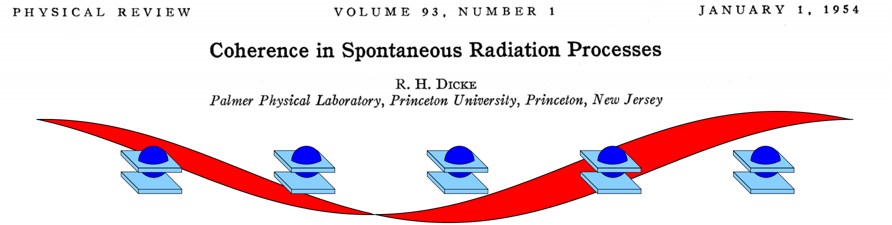
Superradiance — dynamical and steady state.

Dicke effect: Enhanced emission



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PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

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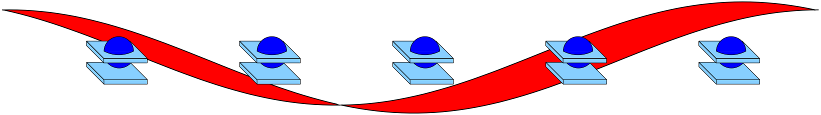

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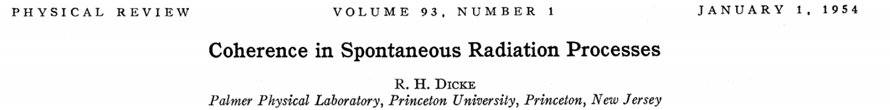
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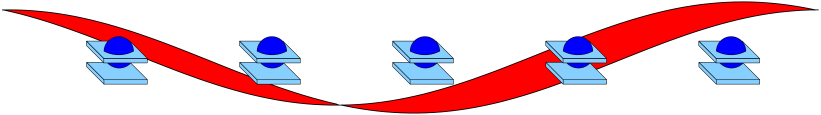

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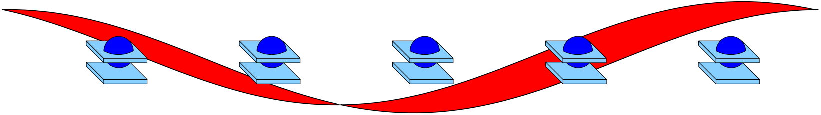

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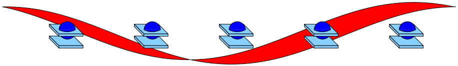

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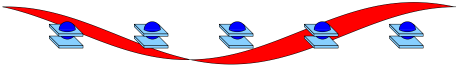

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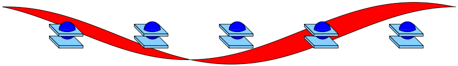

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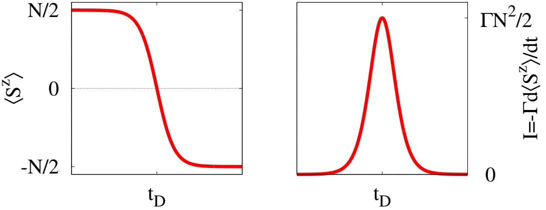

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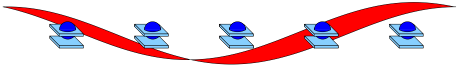

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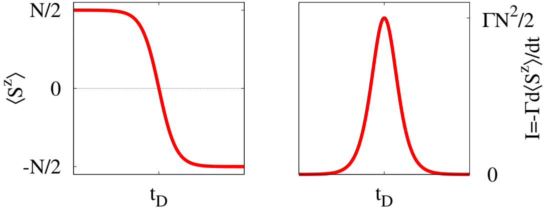
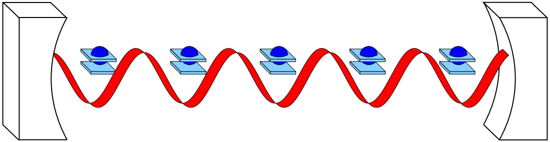


Problem: dipole-dipole interactions dephase.

[Friedberg et al, Phys. Lett. 1972]

Collective radiation **with a cavity**: Dynamics



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Single cavity mode: oscillations

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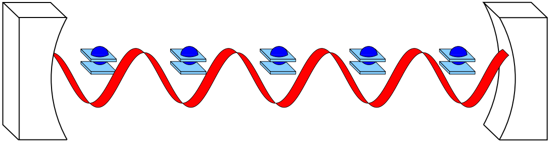

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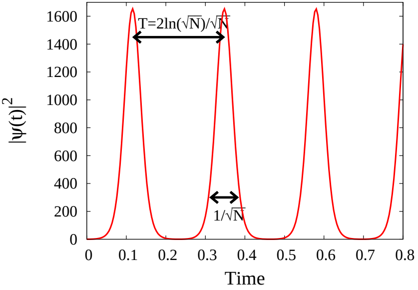

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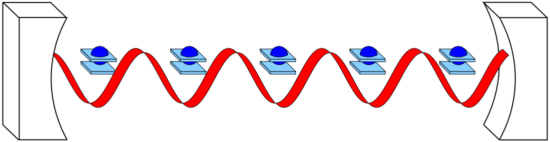

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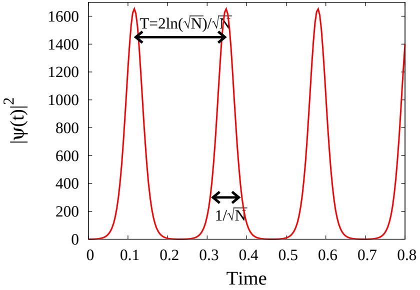

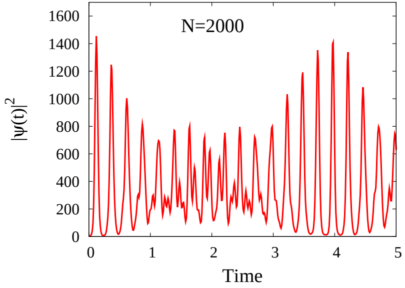

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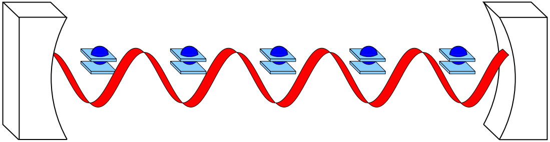
With a cavity: Superradiance phase transition



With detuning: $H = \omega_0 S^z + \omega\psi^\dagger\psi + g\left(\psi^\dagger S^- + \psi S^+\right)$

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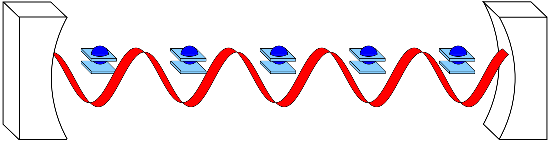

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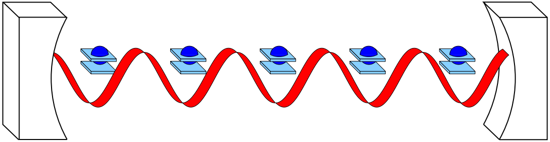

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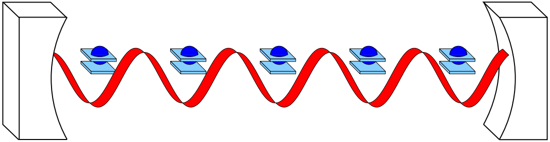

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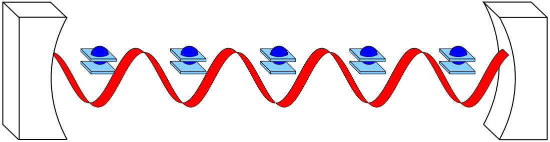

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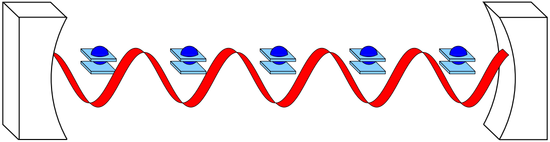

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For large N, $\omega \to \omega + 4N\zeta$. Need $Ng^2 > \omega_0(\omega + 4N\zeta)$.
But $g^2/\omega_0 < 4\zeta$. **No transition** [Rzazewski *et al* Phys. Rev. Lett 1975]

Dicke phase transition: ways out

Problem: $g^2/\omega_0 < 4\zeta$ for intrinsic parameters.
Solutions:

- **Non-solution** Ferroelectric transition in $\mathbf{D}\cdot\mathbf{r}$ gauge. [JK JPCM 2007]
- Grand canonical ensemble:
  - If $H \to H - \mu(S^z + \psi^\dagger\psi)$, need only: $g^2N > (\omega - \mu)(\omega_0 - \mu)$
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- Dissociate g, ω_0, e.g. Raman Scheme: $\omega_0 \ll \omega$. [Dimer *et al* PRA 2007; Baumann *et al* Nature 2010]
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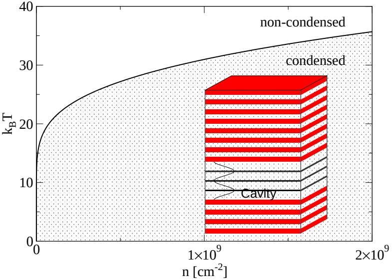

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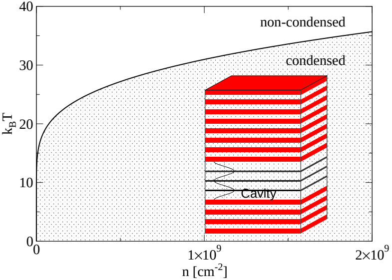

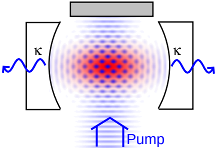

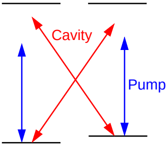

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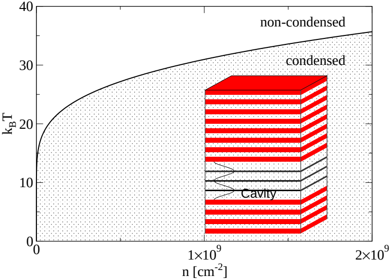

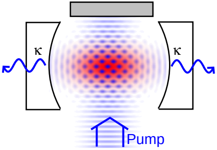

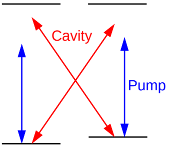

- See also [Nataf and Ciuti, Nat. Comm. 2010; Viehmann *et al* 1103.4639]

Overview

1. Dicke model and collective emission
   - Ferroelectric transition and gauges
2. Optical lattice realisation and dynamics
   - Fixed points and phase diagram
   - Dynamics and critical slowing down
   - Regions without fixed points
3. Hyperfine levels and extra phases
4. Conclusions

Ferroelectric transition

Atoms in Coulomb gauge

$$H = \sum \omega_k a_k^\dagger a_k + \sum_i [p_i - eA(r_i)]^2 + V_{\text{coul}}$$

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Two-level systems — dipole-dipole coupling

$$H = \omega_0 S^z + \omega\psi^\dagger\psi + g(S^+ + S^-)(\psi + \psi^\dagger) + N\zeta(\psi + \psi^\dagger)^2 - \eta(S^+ - S^-)^2$$

(nb $g^2, \zeta, \eta \propto 1/V$).

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Ferroelectric polarisation if $\omega_0 < 2\eta N$

Gauge transform to dipole gauge $\mathbf{D} \cdot \mathbf{r}$

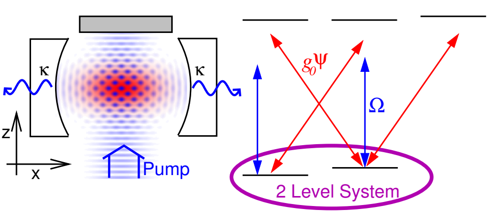
$$H = \omega_0 S^z + \omega \psi^\dagger \psi + \bar{g}(S^+ - S^-)(\psi - \psi^\dagger)$$

"Dicke" transition at $\omega_0 < N\bar{g}^2/\omega \equiv 2\eta N$

But, ψ describes **electric displacement**

Overview

Extended Dicke model [Baumann *et al.* Nature 2010]



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$\Downarrow$: $|k_x, k_z\rangle = |0, 0\rangle$,

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$\omega_0 = 2\omega_{\mathrm{recoil}}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi^\dagger S^- + \psi S^+) + g'(\psi^\dagger S^+ + \psi S^-) + U S_z \psi^\dagger\psi.$$

N atoms: $|\mathbf{S}| = N/2$

Extended Dicke model

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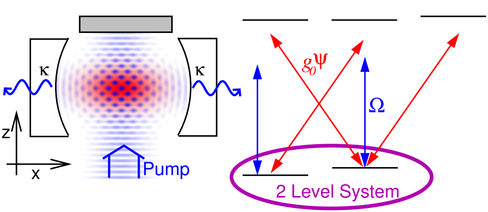

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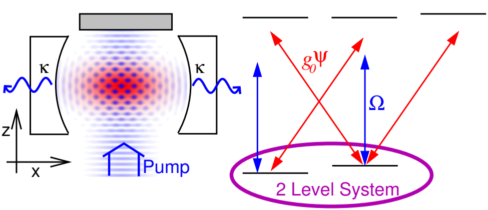

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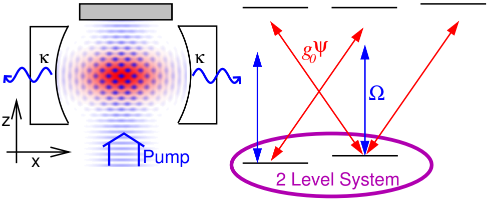

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Add decay:

$$\dot{S}^- = -i(\omega_0 + U\psi^\dagger\psi)S^- + 2i(g\psi + g'\psi^\dagger)S^z$$
$$\dot{S}^z = -ig(\psi S^+ - \psi^\dagger S^-) + ig'(\psi S^- - \psi^\dagger S^+)$$
$$\dot{\psi} = -\left[\kappa + i(\omega + US^z)\right]\psi - igS^- - ig'S^+$$

Fixed points at $U = 0$

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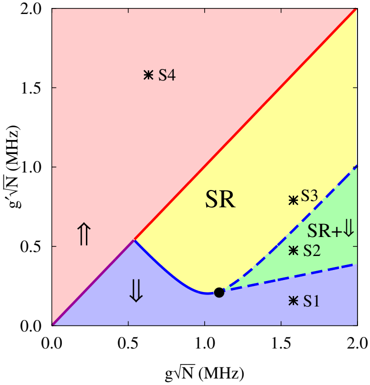

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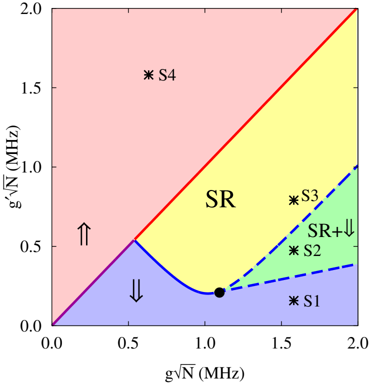
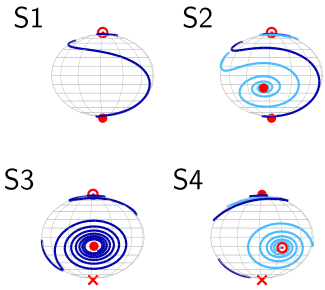
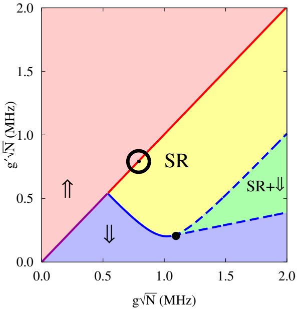
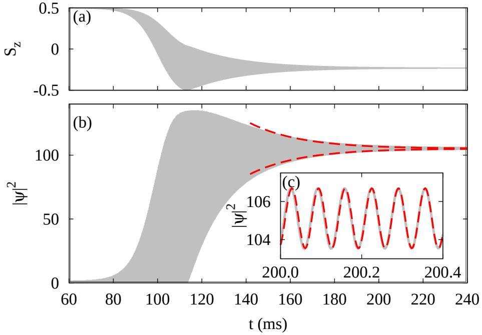




Slow dynamics near critical g'/g





$\omega, \kappa, g\sqrt{N} \sim$ MHz, $\omega_0 \sim$ kHz. Much slower decay.

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- For large κ/ω_0, adiabatically eliminate ψ:
  $\partial_t \mathbf{S} = \{\mathbf{S}, H\} - \Gamma \mathbf{S} \times (\mathbf{S} \times \hat{z})$, $\quad H = \omega_0 S_z - \Lambda_+ S_x^2 - \Lambda_- S_y^2$
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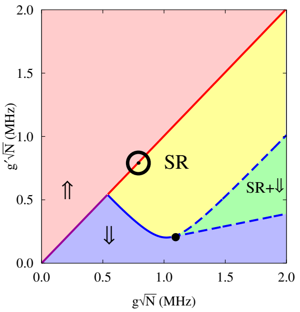

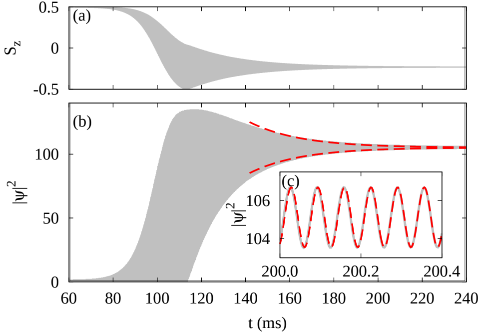

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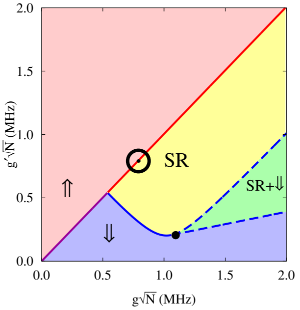

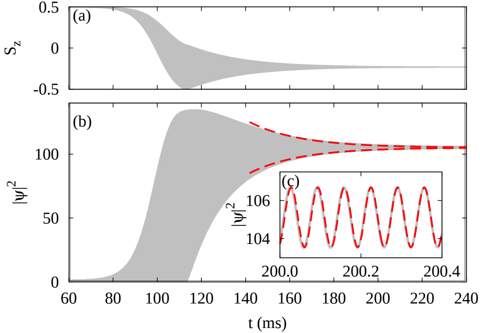

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Finite U phase diagram, $g = g'$

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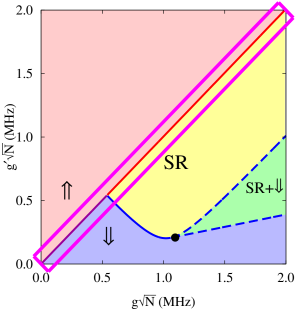
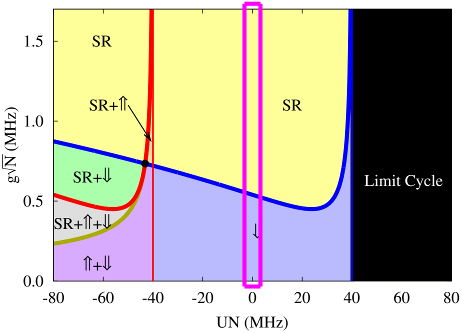
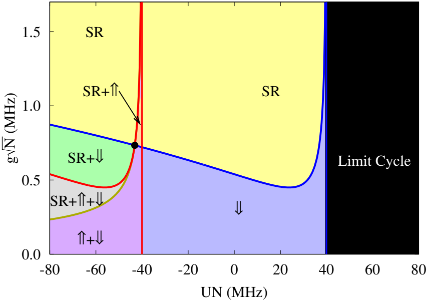




Explaining finite U phase diagram



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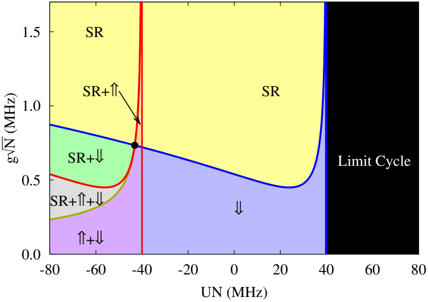

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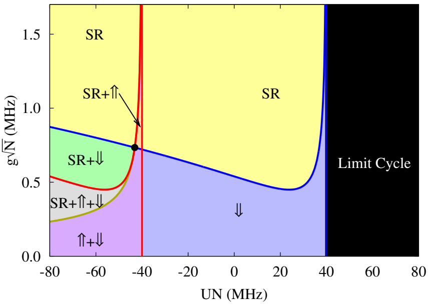

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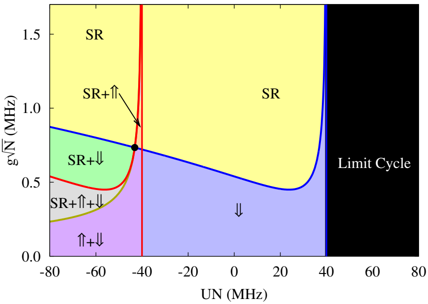

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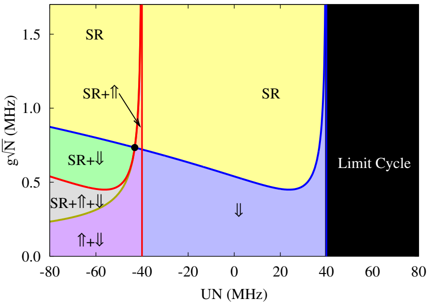

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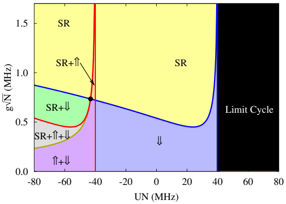
- If $UN > 2\omega$ **No stable fixed points**

Persistent optomechanical oscillations



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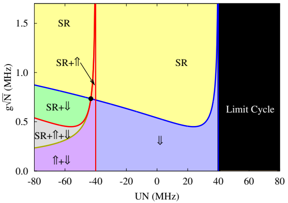

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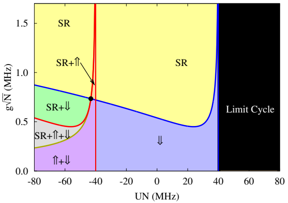

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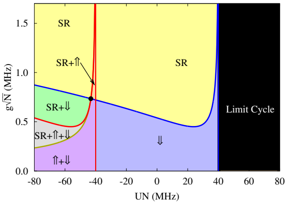

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$$S^- = re^{-i\theta} \quad r = \sqrt{\frac{N^2}{4} - (S^z)^2}$$

Get:

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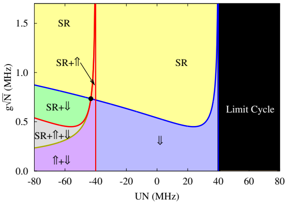

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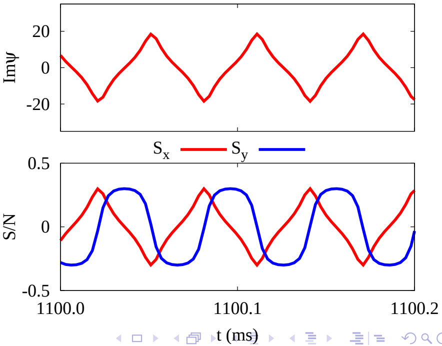
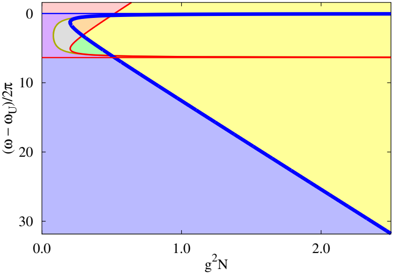
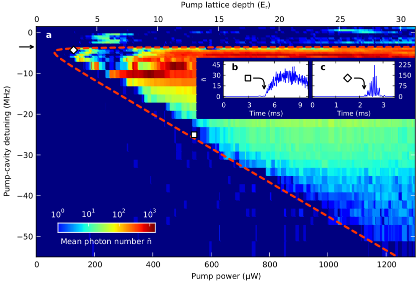


Comparison to experiment $UN = -40\text{MHz}$

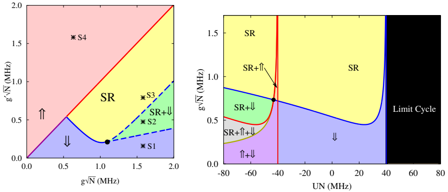


[**JK** *et al* PRL 2010]



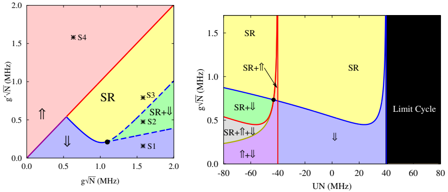
[Baumann *et al* Nature 2010]

Parameters and phases



Phase	Seen?	
$\Downarrow$	✓	
$\Uparrow$	$\times$	(not true TLS)
SR ($\psi \neq 0$)	✓	
SR + $\Downarrow$	?	
Limit cycle	$\times$	Need positive U

Parameters and phases



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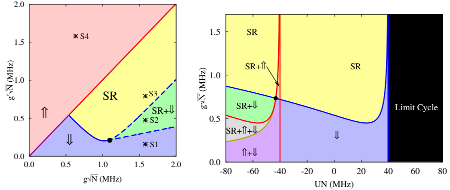
- **Tunable parameters**
  - g, g' $\checkmark (g = g')$
  - U ?
  - ω $\checkmark$
  - ϵ ?
  - κ $\times$

Parameters and phases



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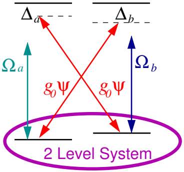
- **Tunable parameters**
  - g, g' ✓ $(g = g')$
  - U ?
  - ω ✓
  - ϵ ?
  - κ ×
- Can we tune $g \neq g'$?
- What other phases occur?

Tuning g, g', U

[Dimer *et al.* Phys. Rev. A. (2007)]



- Separate pump strength/detuning
- $g \sim \frac{g_0 \Omega_b}{\Delta_b}, g' \sim \frac{g_0 \Omega_a}{\Delta_a}, U \sim \frac{{g_0}^2}{\Delta_a} - \frac{{g_0}^2}{\Delta_b}$

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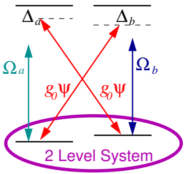
[Dimer *et al.* Phys. Rev. A. (2007)]



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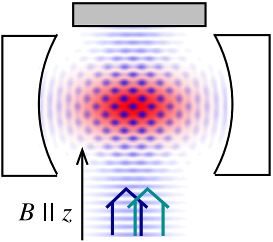
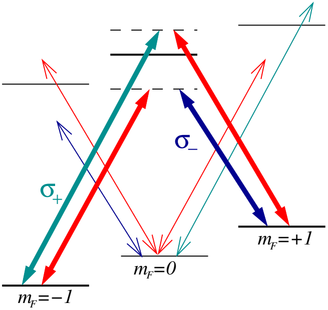
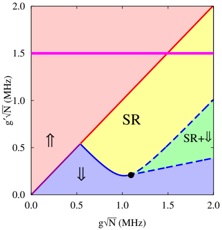
- $g \sim \frac{g_0\Omega_b}{\Delta_b}, g' \sim \frac{g_0\Omega_a}{\Delta_a}, U \sim \frac{{g_0}^2}{\Delta_a} - \frac{{g_0}^2}{\Delta_b}$

Possible realization: Hyperfine levels





Phase diagrams vs g, g', U, ω



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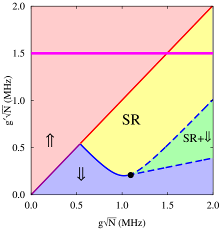

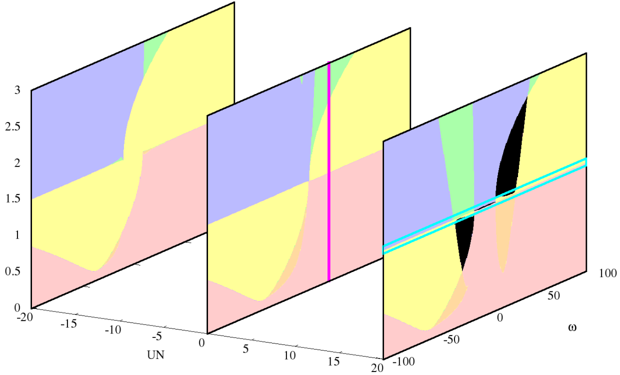

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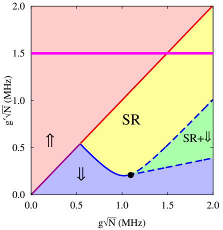
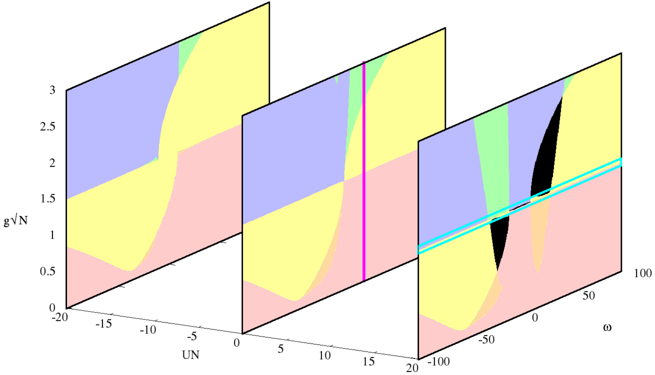
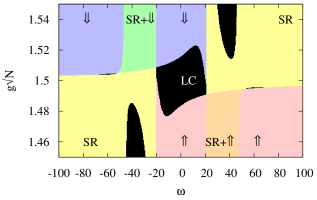
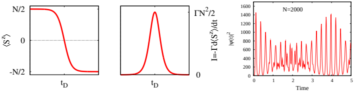




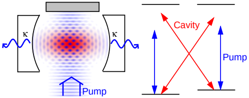


Summary

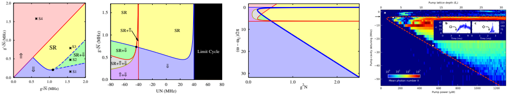
- Dynamical vs steady state superradiance



- Circumventing no-go theorem: (Ferroelectric?), Open systems, Raman scheme
- Realisation of (modified) superradiance transition



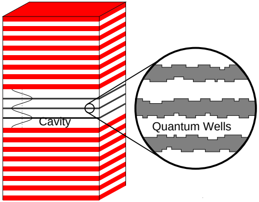
- For $g \neq g'$, $U \neq 0$, wide variety of dynamical phases



- Slow dynamics and persistent oscillations

Extra slides

Chemical potential and Dicke model



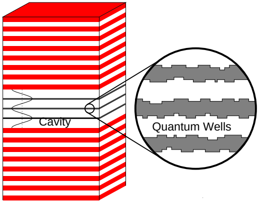
$$H = \omega\psi^\dagger\psi + \sum_i \omega_0 S_i^z + g(\psi S_i^+ + \text{H.c.})$$

Transition occurs at:

$$\omega - \mu = \frac{Ng^2}{\omega_0 - \mu} \tanh\left[\beta \frac{1}{2}(\omega_0 - \mu)\right]$$

- Analogy to dynamics:
  Pendulum equation **in frame rotating at μ**
- Open system; incoherent pumping:
  Polariton condensate

Chemical potential and Dicke model



How to introduce μ

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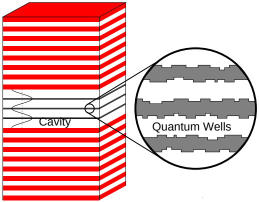

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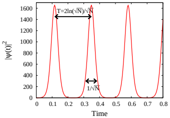

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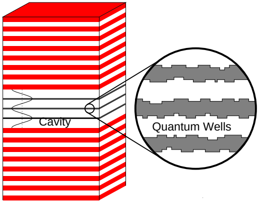

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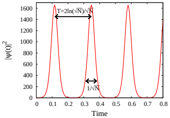

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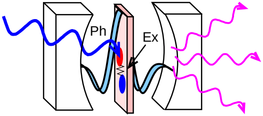
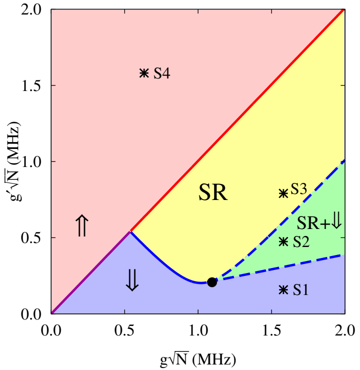


Boundaries $U = 0$

$\kappa \neq 0$



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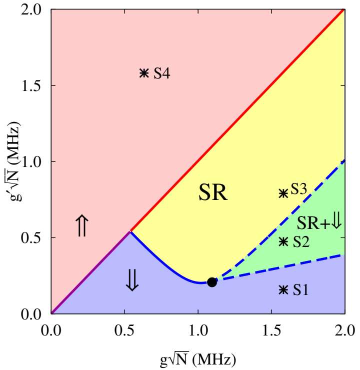

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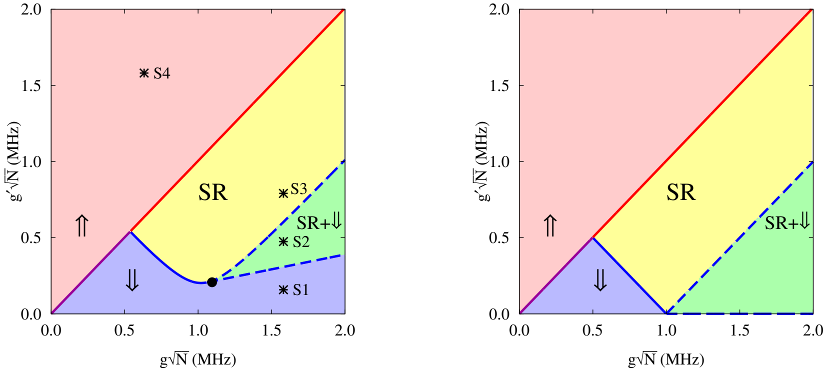
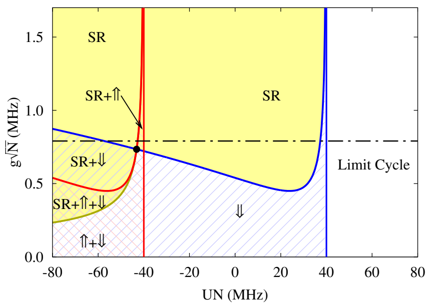
—$N(g+g')^2 = \omega\omega_0$



Numerical confirmation of fixed points



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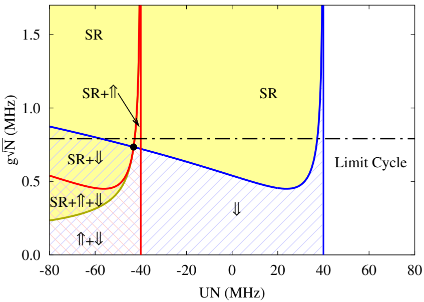

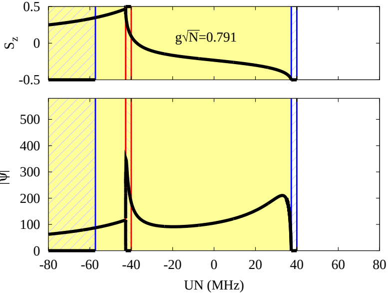

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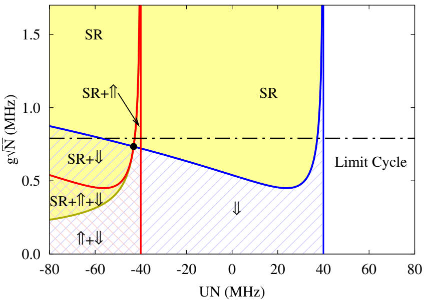

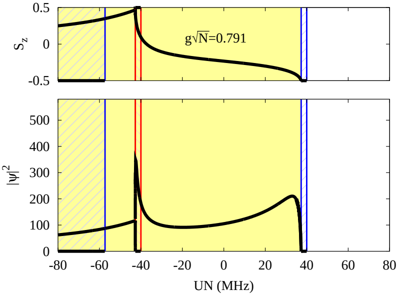

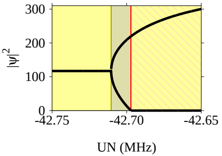

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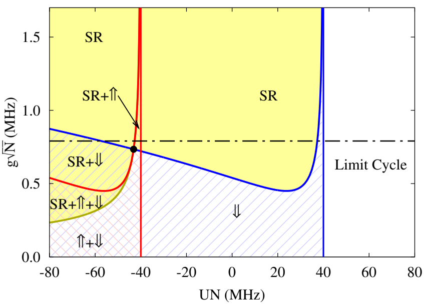
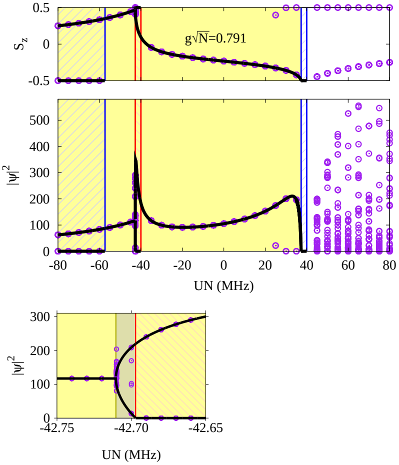
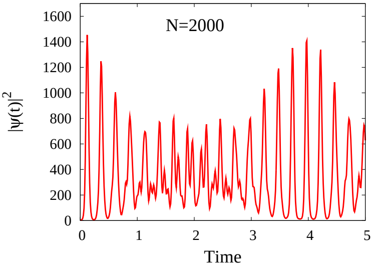


$T = 360\text{ms}$



How good is semiclassics?

From eigenstates $H|\Psi_q\rangle = E_q|\Psi_q\rangle$:



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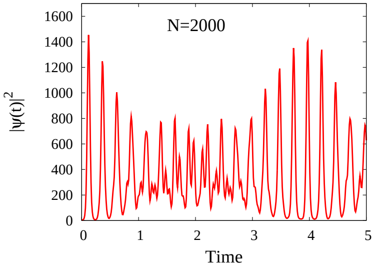

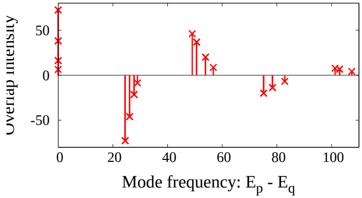

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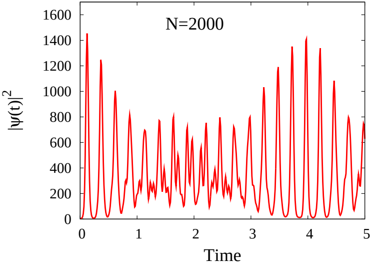
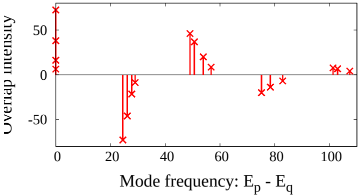
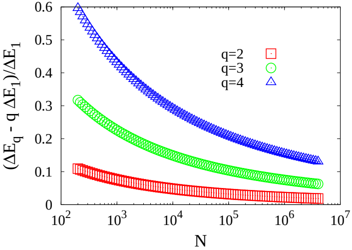




Anharmonicity: $\Delta E_q - q\Delta E_1$



Semiclassical approximation: WKB quantisation

Problem is **one dimensional**; $n_{phot} + S_z \equiv N/2$, find $\Psi(n_{phot})$:

$$(E + \Delta n)\Psi_n = gn\sqrt{N+1-n}\Psi_{n-1} + g(n+1)\sqrt{N-n}\Psi_{n+1}$$

[Keeling PRA **79** 053825; see also Babelon et al. J. Stat. Mech p.07011]

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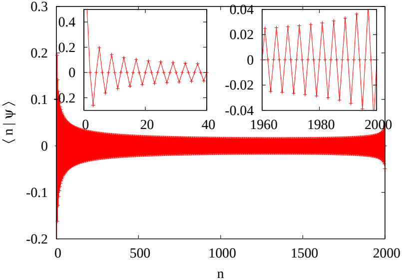

WKB wavefunction:

$$\Psi_n \simeq \frac{\cos(E\Phi_n + \phi + n\pi/2)}{\sqrt{(n+1/2)\sqrt{N-n+1/2}}}$$

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Find E, ϕ by matching assymptotics at $n \simeq 0, n \simeq N$.

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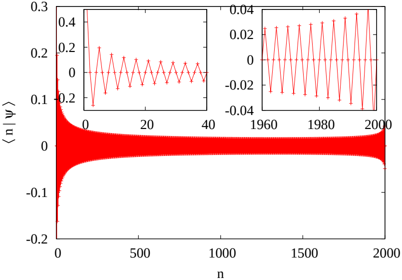

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Hard boundary at $n = 0$: breakdown of Bohr-Sommerfeld quantisation.

[Keeling PRA **79** 053825; see also Babelon et al. J. Stat. Mech p.07011]

Scaling with system size

Simple approximation: match WKB soln to eqn for Ψ_0, Ψ_1:

$$1 = -\frac{2}{\pi}\sqrt{N}\frac{E}{g}\tan\left[\frac{E}{g}\frac{\ln(\sqrt{N})}{\sqrt{N}}\right],$$

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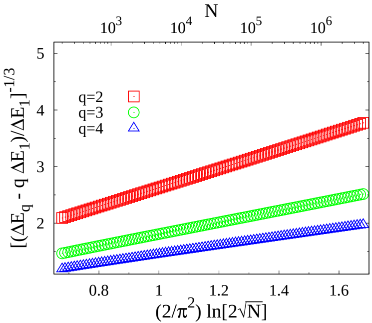
Semiclassics controlled by $1/\ln(N)$.



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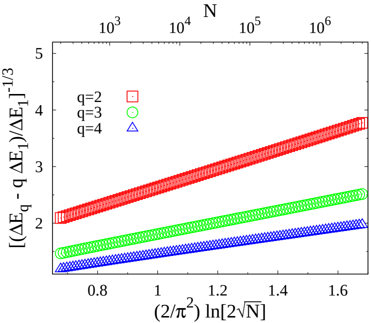
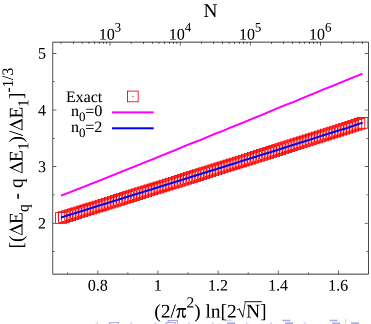




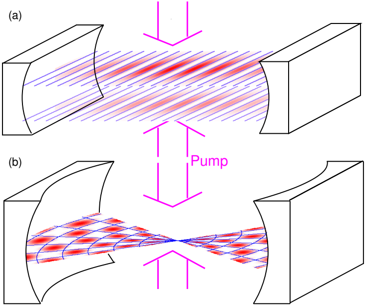
Overview

Many photon modes

nature physics
ARTICLES
PUBLISHED ONLINE: 4 OCTOBER 2009 | DOI: 10.1038/NPHYS1403

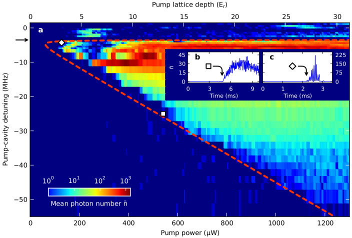
Emergent crystallinity and frustration with Bose–Einstein condensates in multimode cavities

Sarang Gopalakrishnan[1,2]*, Benjamin L. Lev[1] and Paul M. Goldbart[1,2,3]



- Transition breaks $Z_2 \otimes Z_n$ — crystallisation
- No cubic mode-mode coupling — Brazovskii transition
- "Supersmectic" phase

Dynamics during/following sweep



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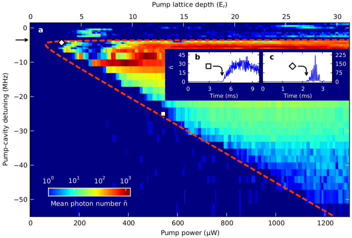

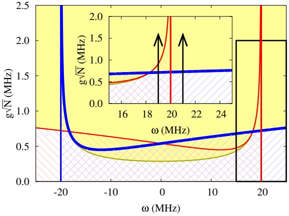

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