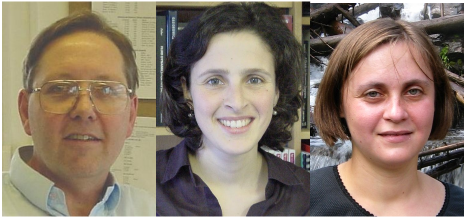
Non-equilibrium coherence in light-matter systems: condensation, lasing and the superradiance transition.

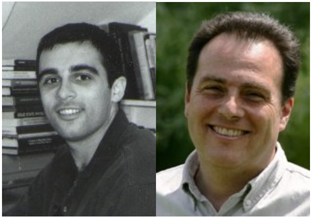
J. M. J. Keeling
P. B. Littlewood, F. M. Marchetti, M. H. Szymanska.
M. J. Bhaseen, B. D. Simons.

Herriot-Watt, November 2010

Acknowledgements

People:





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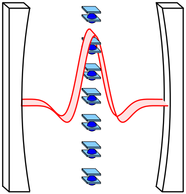
Dicke model & Superradiance phase transition



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[Hepp, Lieb, Ann. Phys. 1973]

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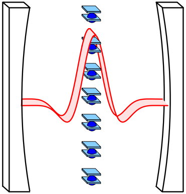

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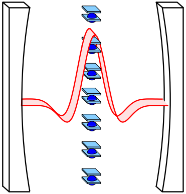

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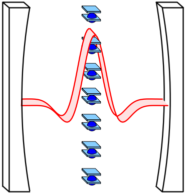

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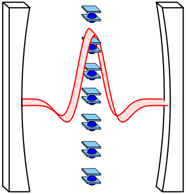

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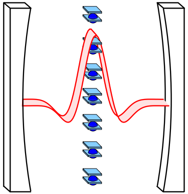

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For large N, $\omega \to \omega + 4N\zeta$. Need $Ng^2 > \omega_0(\omega + 4N\zeta)$.

But $g^2/\omega_0 < 4\zeta$. **No transition** [Rzazewski *et al* Phys. Rev. Lett 1975]

Dicke phase transition: ways out

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  - $H \to H - \mu(S^z + \psi^\dagger\psi)$, need: $g^2 N = (\omega_0 - \mu)(\omega - \mu)$
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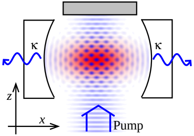


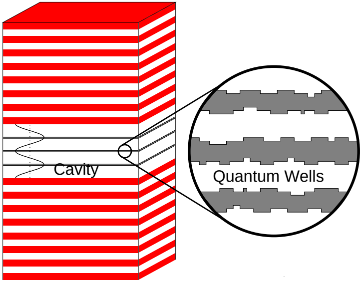
1. Dicke model and superradiance

2. Microcavity Polariton condensation
   - Polariton experiments
   - Non-equilibrium condensation and lasing
   - Non-equilibrium pattern formation

3. Superradiance in atom-cavity system
   - Superradiant steady states
   - Long-lived & persistent oscillations

4. Conclusions

Microcavity polaritons



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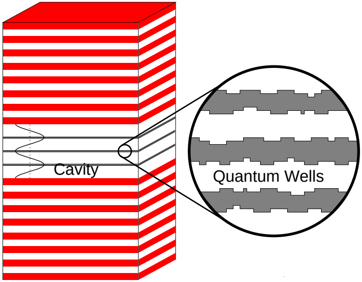

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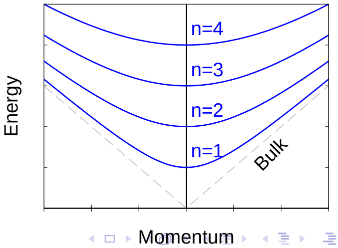
$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$



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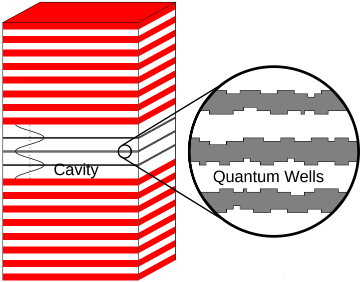

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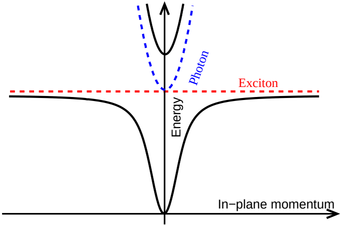

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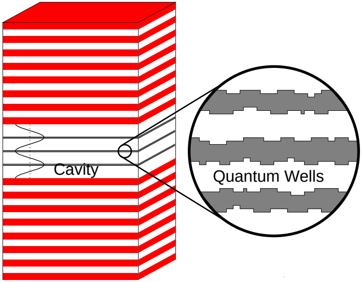

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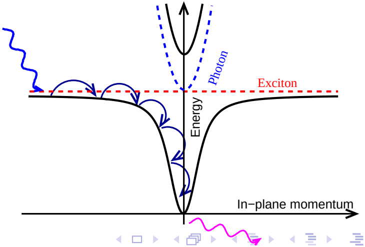

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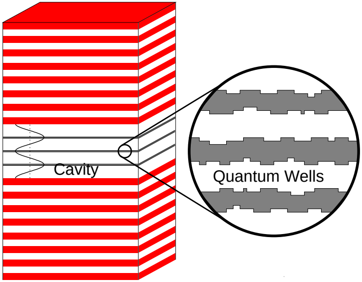

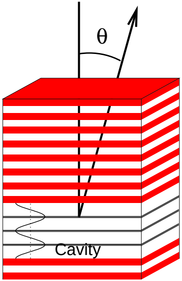

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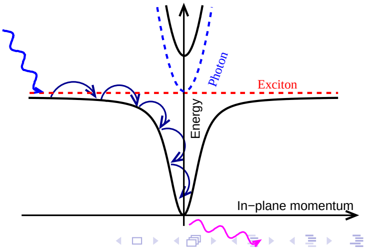
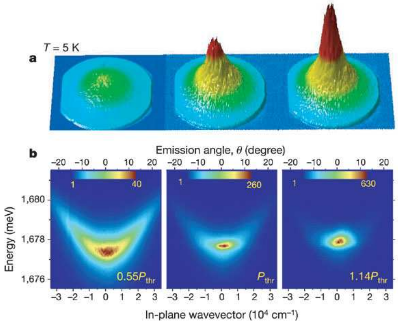
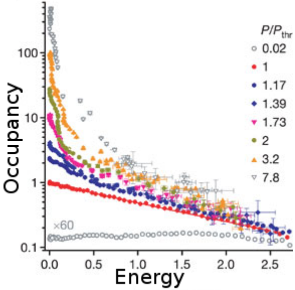


Polariton experiments: Momentum/Energy distribution





[Kasprzak, et al., Nature, 2006]

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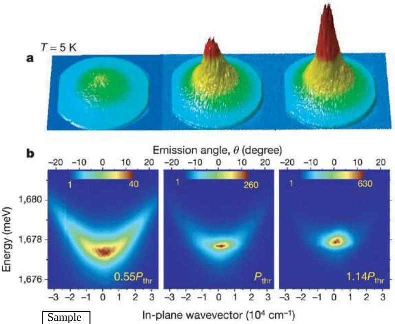

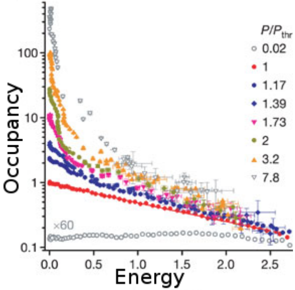

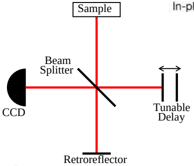

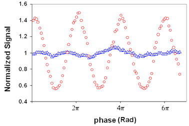

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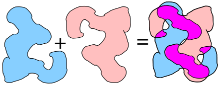
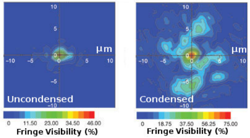
Coherence map:





Polariton system model

Polariton model

- Disorder-localised excitons
- Treat disorder sites as two-level (exciton/no-exciton)
- Propagating (2D) photons
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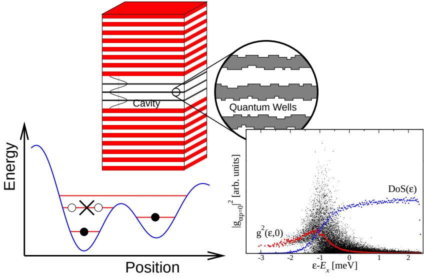

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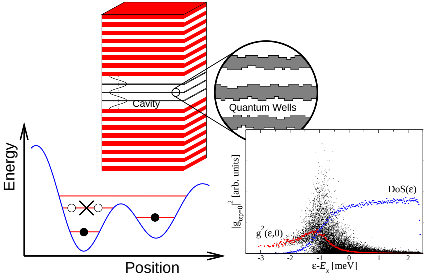

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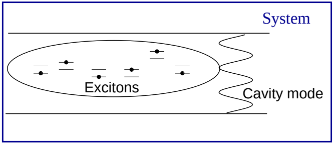

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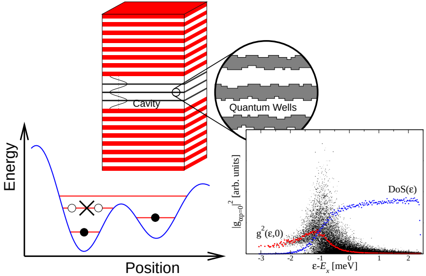

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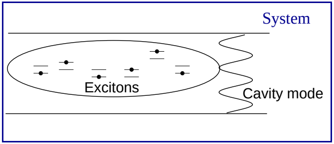

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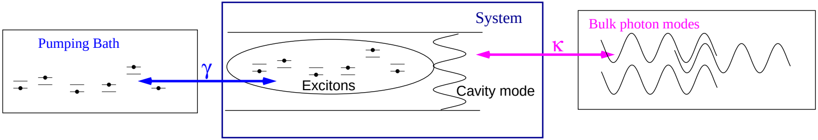

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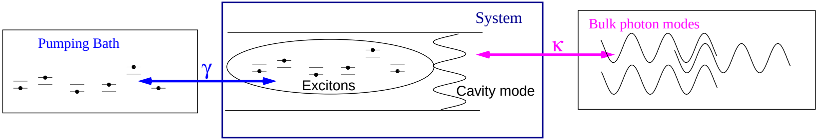

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Schematically: pump γ, decay κ

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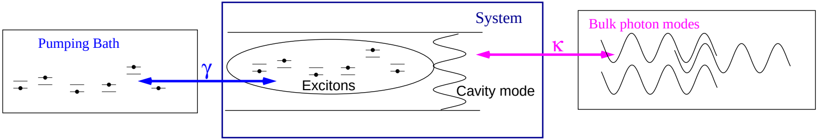

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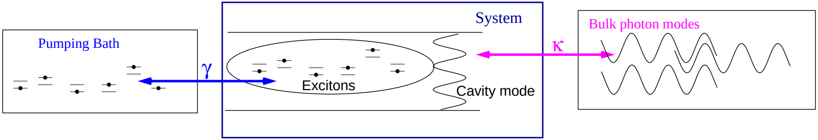

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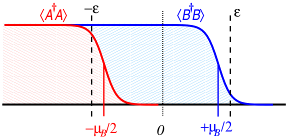
Ψ bath is empty. Pumping bath thermal, μ_B, T:



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Susceptibility:

$$\chi(\psi_0, \mu_s) = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

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- Laser limit: Gain vs Loss. If $F_b(\omega) - F_a(\omega) \to -2N_0$

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Limits of gap equation

$$\mu_s - \omega_0 + i\kappa = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

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$$\omega_0 - \mu_s = \frac{g^2}{2E} \tanh\left(\frac{\beta E}{2}\right)$$

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Nonlinear, complex susceptibility $\chi(\psi(r,t))$

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Gross-Pitaevskii equation: Harmonic trap

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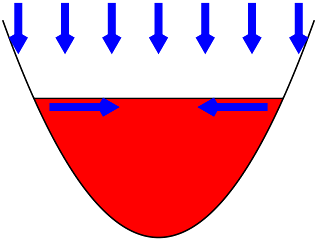

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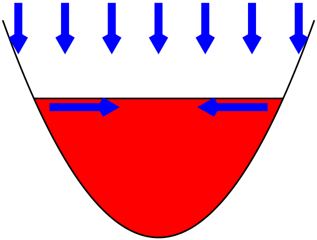

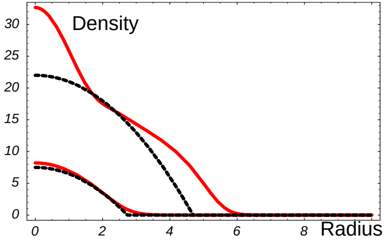

Stability of Thomas-Fermi solution

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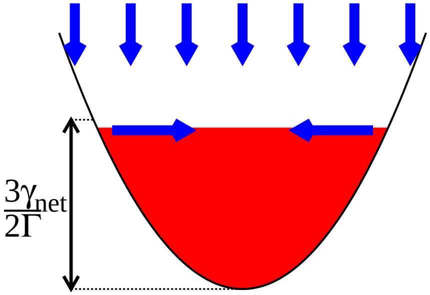

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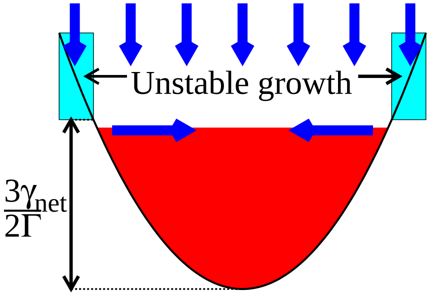

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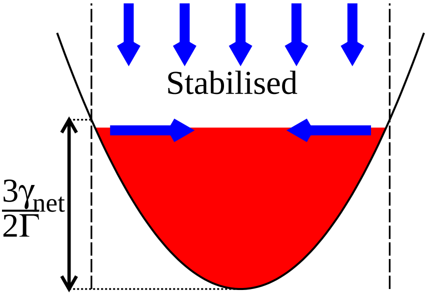

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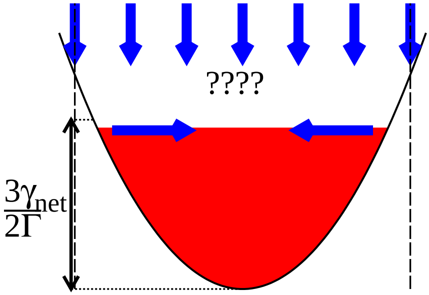
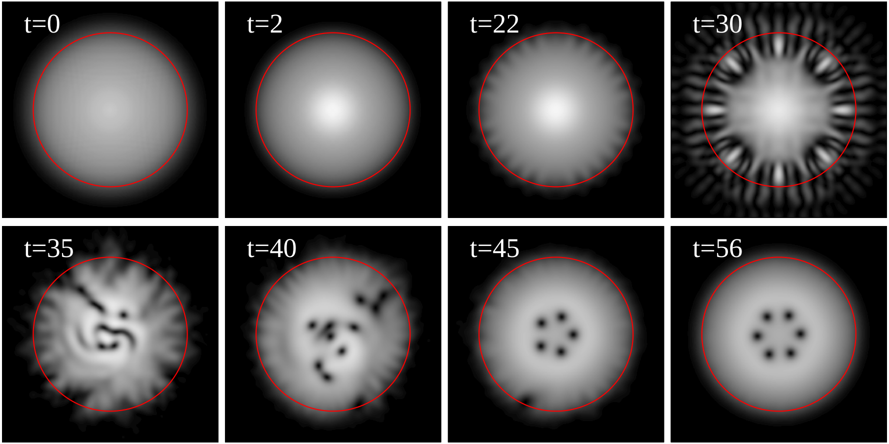
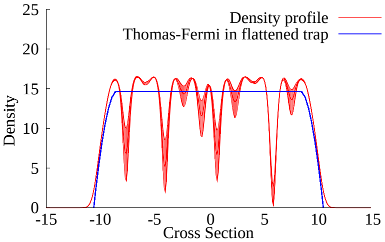
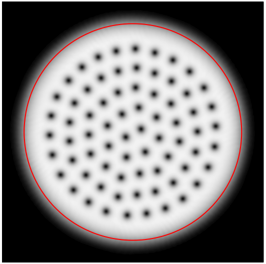


Time evolution:



Why vortices





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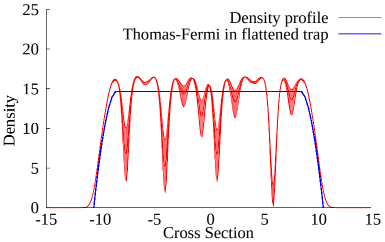

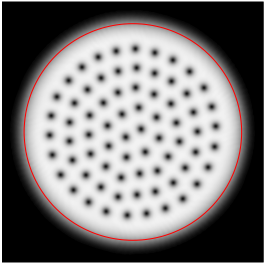

Rotating solution: $i\partial_t\psi = (\mu - 2\Omega L_z)\psi$

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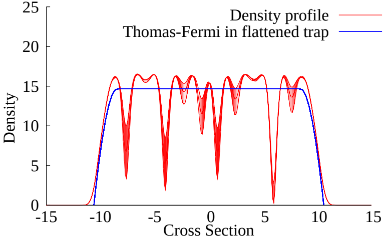

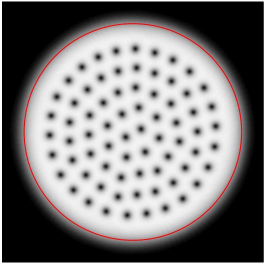

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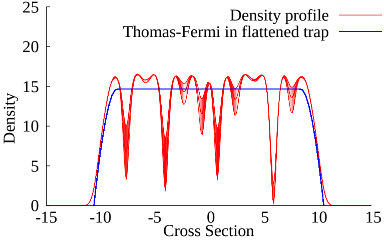

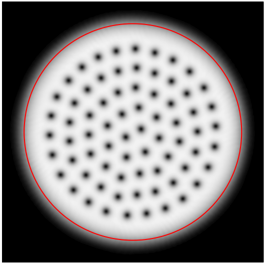

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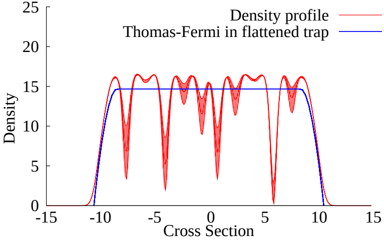

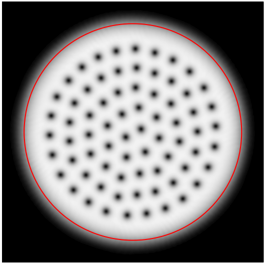

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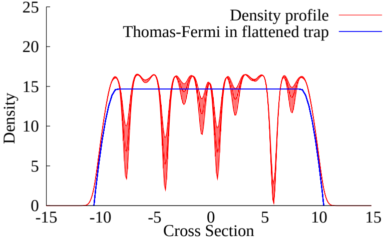

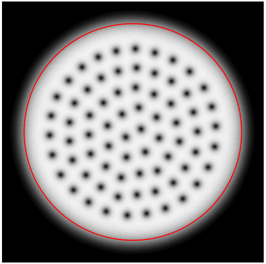

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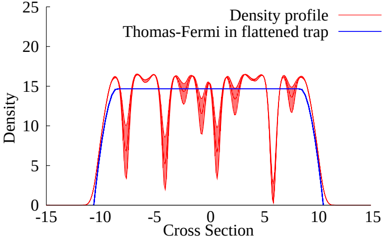

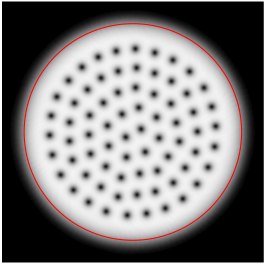

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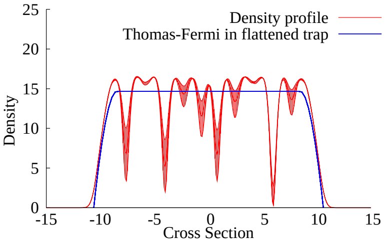

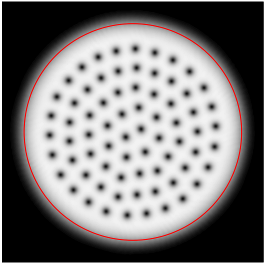

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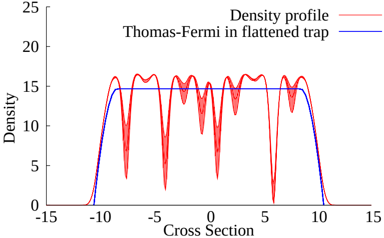

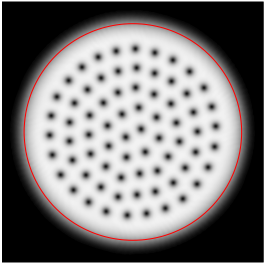

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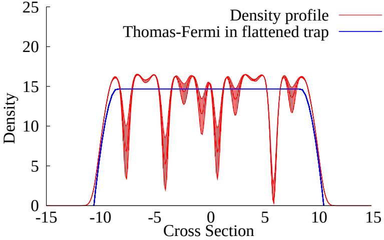

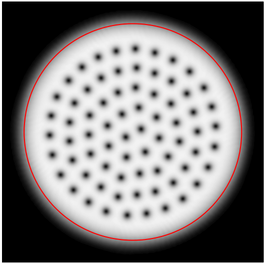

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1 Dicke model and superradiance

2 Microcavity Polariton condensation
- Polariton experiments
- Non-equilibrium condensation and lasing
- Non-equilibrium pattern formation

3 Superradiance in atom-cavity system
- Superradiant steady states
- Long-lived & persistent oscillations

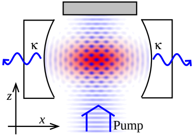
4 Conclusions

Dicke phase transition: ways out

Problem: $g^2/\omega_0 < 4\zeta$ for intrinsic parameters. **Solutions**:

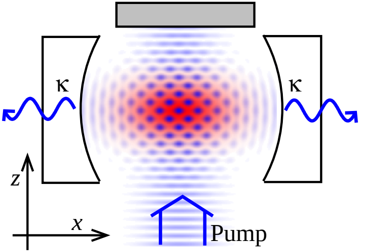
- Introduce chemical potential:
  - $H \to H - \mu(S^z + \psi^\dagger\psi)$, need: $g^2 N = (\omega_0 - \mu)(\omega - \mu)$
  - Pumped system — polariton condensation/lasing

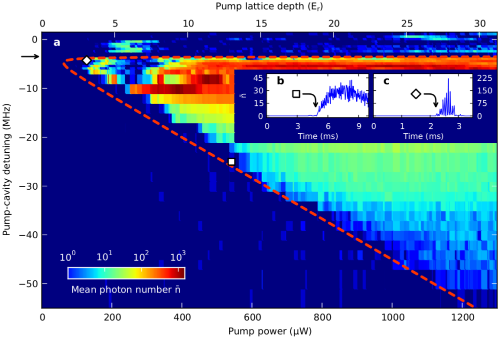
- Dissociate g, ω_0, e.g. Raman scheme: $\omega_0 \ll \omega$.
  [Baumann *et al* Nature 2010]



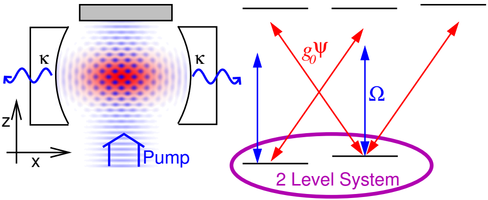
Raman scheme for Dicke model

[Baumann et al, Nature 2010]





Extended Dicke model



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$\Downarrow$: $|k_x, k_z\rangle = |0, 0\rangle$,

$\Uparrow$: $|k_x, k_z\rangle = |\pm k, \pm k\rangle$,

$\omega_0 = 2\omega_{\text{recoil}}$

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N atoms: $|\mathbf{S}| = N/2$

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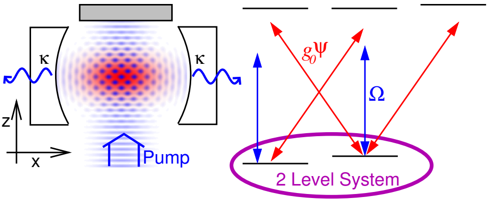

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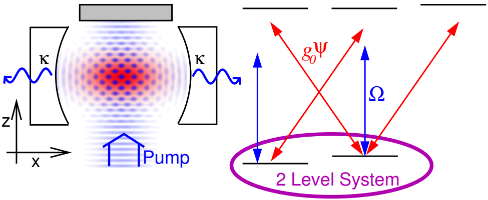

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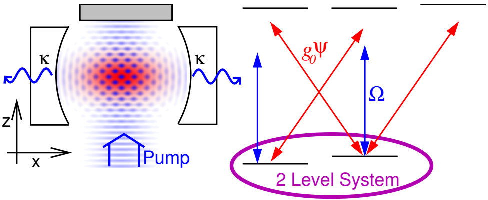

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Add decay:

$$\partial_t S^- = -i(\omega_0 + U\psi^\dagger\psi)S^- + 2ig(\psi + \psi^\dagger)S^z$$

$$\partial_t S^z = +ig(\psi + \psi^\dagger)(S^- - S^+)$$

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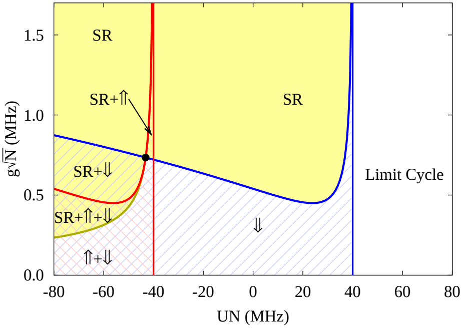
Phase diagram



- $|UN| < \omega/2$, Regular SR, $S^+ = S^-$
- $UN < -\omega/2$, 2nd SR soln $\psi = -\psi^\dagger$
- $UN > \omega/2$ **No SR Fixed point**

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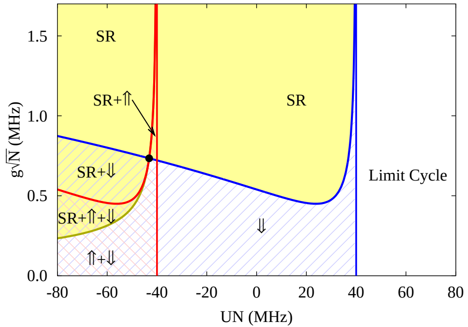

SR: Need $\partial_t S^z = ig(\psi + \psi^\dagger)(S^- - S^+) = 0$

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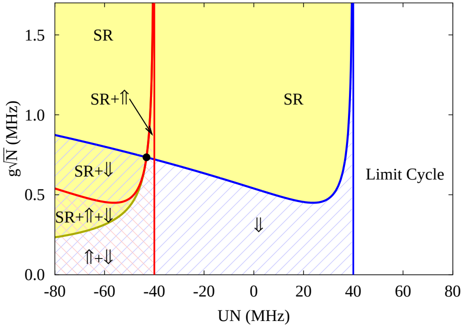

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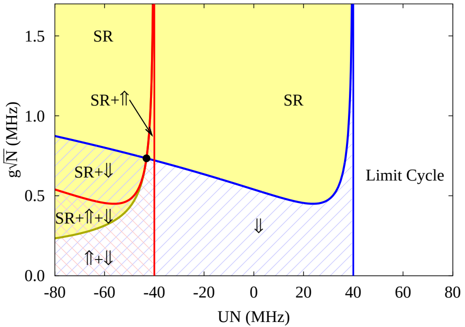

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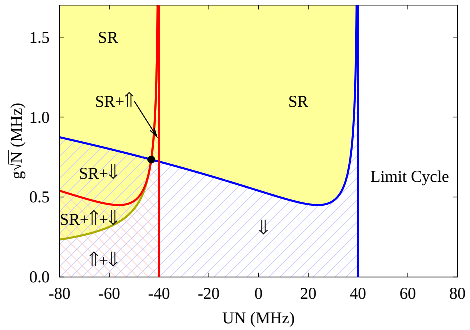

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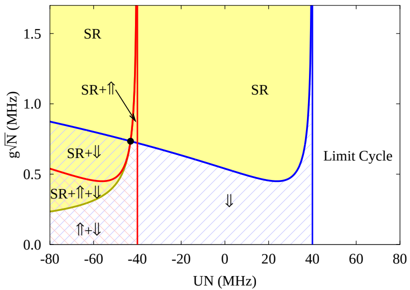
Large U and persistent oscillations



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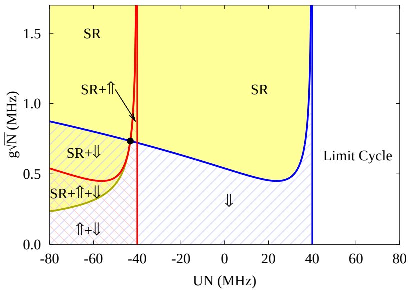

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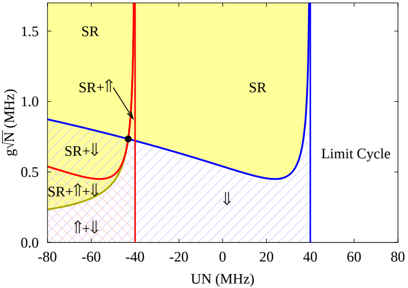

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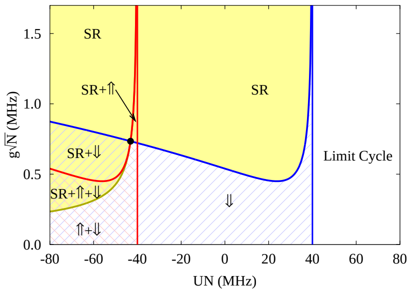

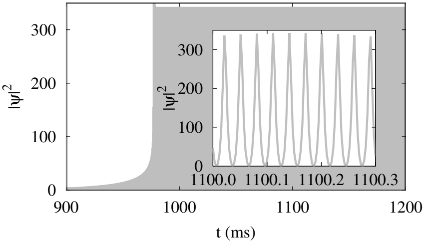

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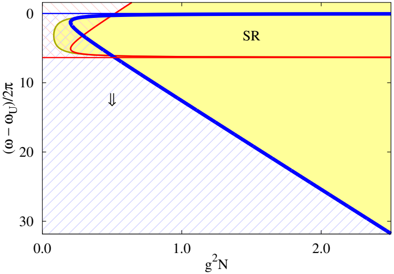
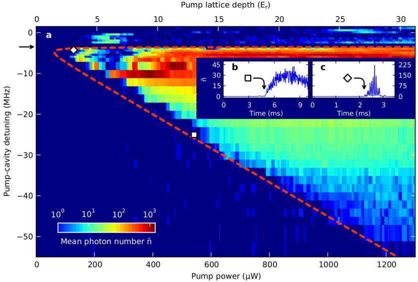
Comparison to experiment $UN = -40\text{MHz}$





[**JK** *et al* `arXiv:1002.3108`]

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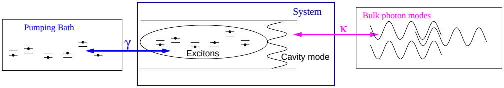
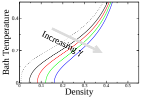
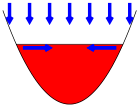
Summary

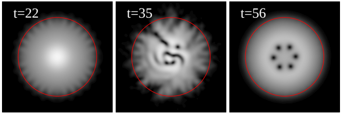
- Non-equilibrium mean field theory of polaritons



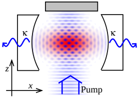


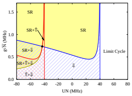
- Instability of Thomas-Fermi and spontaneous rotation

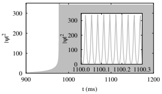




- Atomic realisation: many phases & non-trivial dynamics







Extra slides

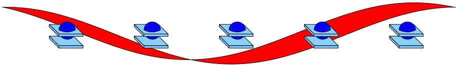
5 Introduction
- Other types of superradiance
- Ferroelectric transition

6 Polaritons
- Other polariton experiments
- Equilibrium results
- Non-equilibrium polariton timescales
- Condensation vs lasing
- Spinor problem
- T=0 Keldysh results

7 Cold atom Dicke
- Zero U boundaries
- Fixed points vs U.

Dicke effect and superradiance without a cavity

$$H_{\text{int}} = \sum_{k,i} g_k \left(\psi_k^\dagger S_i^- e^{-i\mathbf{k}\cdot\mathbf{r}_i} + \text{H.c.} \right)$$



[Dicke, Phys. Rev. 1954]

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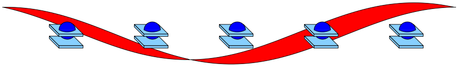

If $|\mathbf{r}_i - \mathbf{r}_j| \ll \lambda$, use $\sum_i \mathbf{S}_i \to \mathbf{S}$. Many modes ψ_k — integrate out:

$$\frac{d\rho}{dt} = -\frac{\Gamma}{2} \left[S^+ S^- \rho - S^- \rho S^+ + \rho S^+ S^- \right]$$

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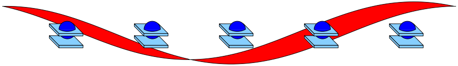

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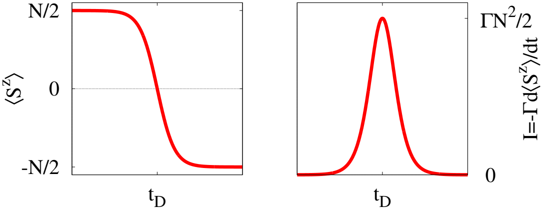

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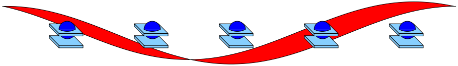

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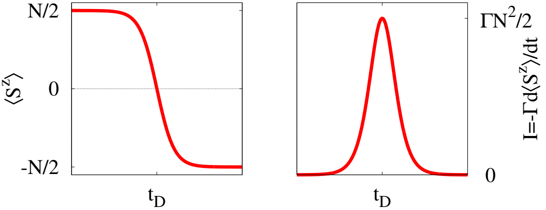
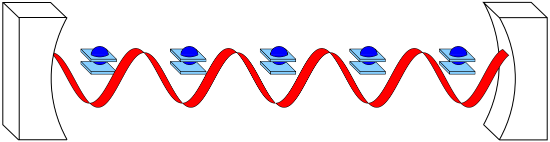


[Dicke, Phys. Rev. 1954]

Problem: dipole-dipole interactions dephase.

Collective radiation with a cavity: Dynamics



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Single cavity mode: oscillations

[Bonifacio and Preparata PRA 1970; **JK** PRA 2009]

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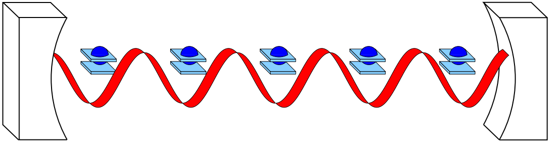

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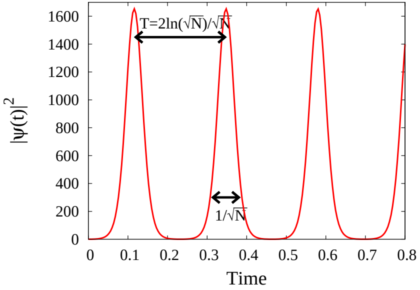

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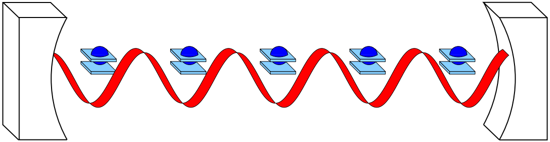

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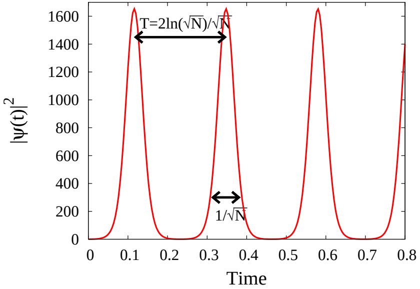

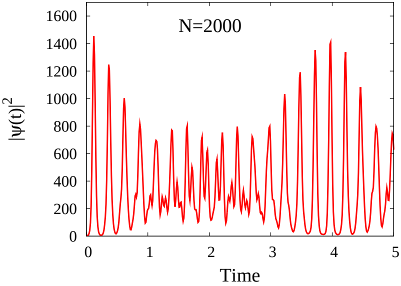

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Ferroelectric transition

Atoms in Coulomb gauge

$$H = \sum \omega_k a_k^\dagger a_k + \sum_i [p_i - eA(r_i)]^2 + V_{\text{coul}}$$

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(nb $g^2, \zeta, \eta \propto 1/V$).

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Ferroelectric polarisation if $\omega_0 < 2\eta N$

Gauge transform to dipole gauge $\mathbf{D} \cdot \mathbf{r}$

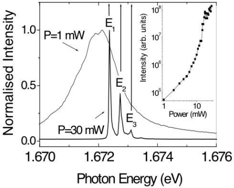
$$H = \omega_0 S^z + \omega\psi^\dagger\psi + \bar{g}(S^+ - S^-)(\psi - \psi^\dagger)$$

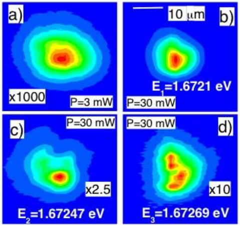
"Dicke" transition at $\omega_0 < N\bar{g}^2/\omega \equiv 2\eta N$

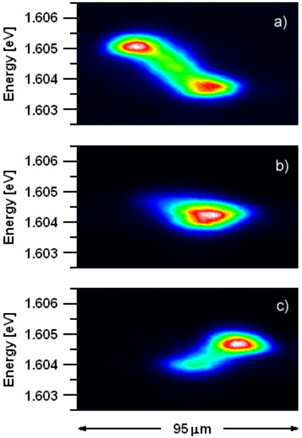
But, ψ describes **electric displacement**

Other polariton condensation experiments

- Stress traps for polaritons
  [Balili *et al* Science 316 1007 (2007)]
- Temporal coherence and line narrowing
  [Love *et al* Phys. Rev. Lett. 101 067404 (2008)]

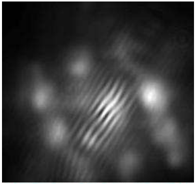




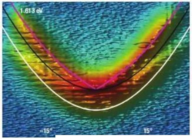


Other polariton condensation experiments

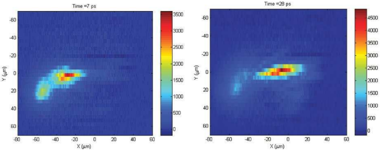
- Quantised vortices in disorder potential
  [Lagoudakis *et al* Nature Phys. 4, 706 (2008)]



- Changes to excitation spectrum
  [Utsunomiya *et al* Nature Phys. 4 700 (2008)]

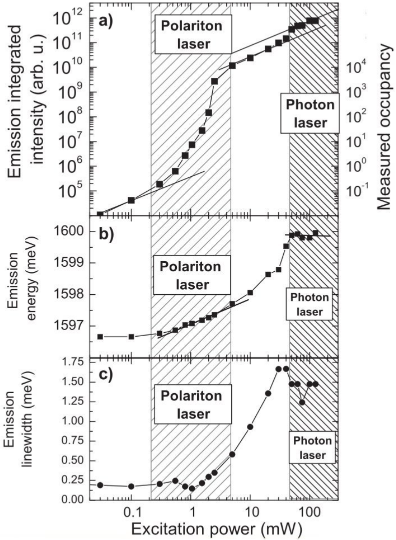


- Soliton propagation
  [Amo *et al* Nature 457 291 (2009)]
- Driven superfluidity
  [Amo *et al* Nature Phys. (2009)



Polariton experiments: Strong coupling

[Bajoni *et al* PRL 2008]



Equilibrium: Mean-field theory

$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_{\alpha} \left(b^{\dagger}_{\alpha} b_{\alpha} - a^{\dagger}_{\alpha} a_{\alpha} \right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{\mathrm{A}}} \psi_{\mathbf{k}} b^{\dagger}_{\alpha} a_{\alpha} + \mathrm{H.c.} \right]$$

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Self-consistent polarisation and field

$$\left[- i\partial_t - \omega_0 + \frac{\nabla^2}{2m} \right] \psi = -\frac{1}{\sqrt{A}} \sum_\alpha g_\alpha P_\alpha$$

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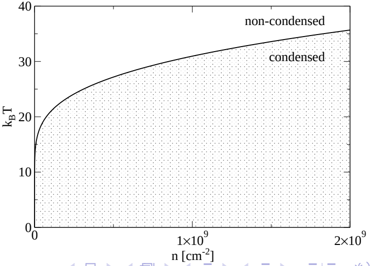
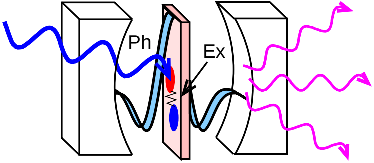
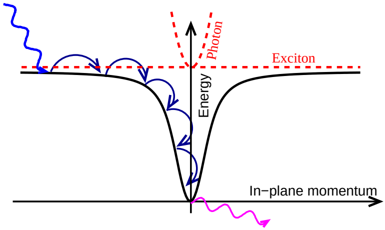


Non-equilibrium: Timescales





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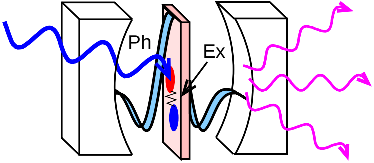

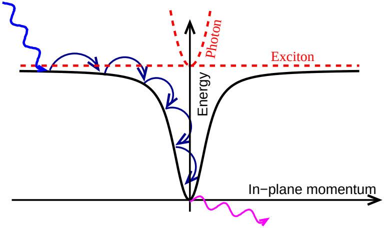

	Lifetime	Thermalisation
Atoms	10s	10ms
Excitons[a]	50ns	0.2ns
Polaritons	5ps	0.5ps
Magnons[b]	1μs(??)	100ns(?)

[a] Coupled quantum wells. [Hammack et al PRB 76 193308 (2007)]
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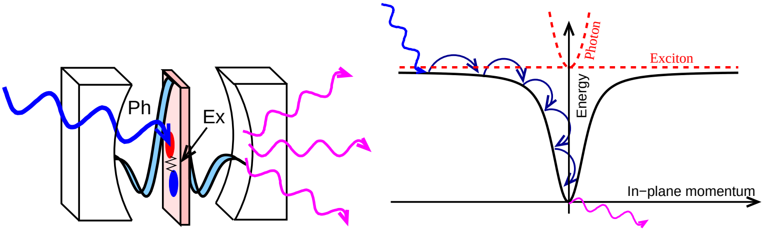

	Lifetime	Thermalisation	Linewidth	Temperature	
Atoms	10s	10ms	2.5×10^{-13}meV	10^{-8}K	10^{-9}meV
Excitons[a]	50ns	0.2ns	5×10^{-5}meV	1K	0.1meV
Polaritons	5ps	0.5ps	0.5meV	20K	2meV
Magnons[b]	1μs(??)	100ns(?)	2.5×10^{-6}meV	300K	30meV

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Fluctuations spectrum and stability

$$D^R - D^A = -i\left\langle \left[\psi, \psi^\dagger\right]_- \right\rangle$$

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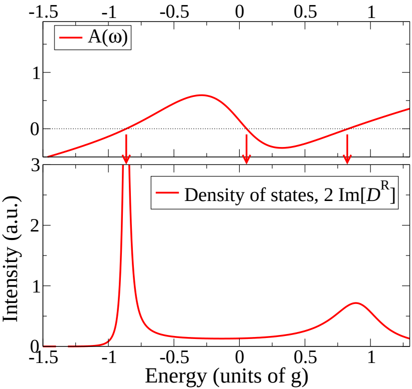

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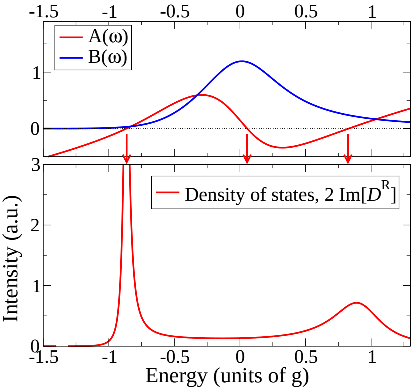

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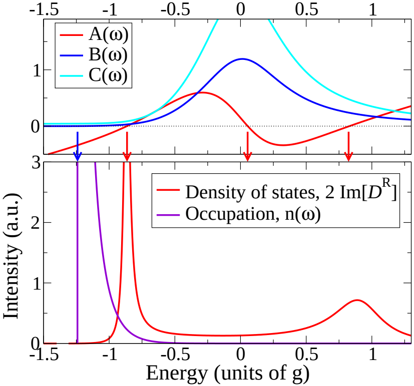

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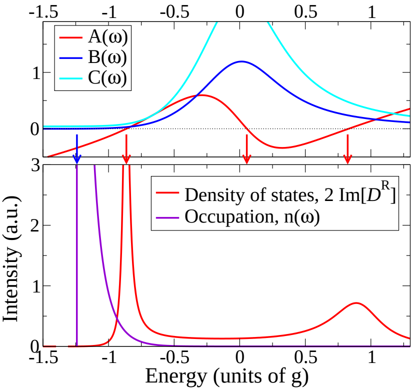
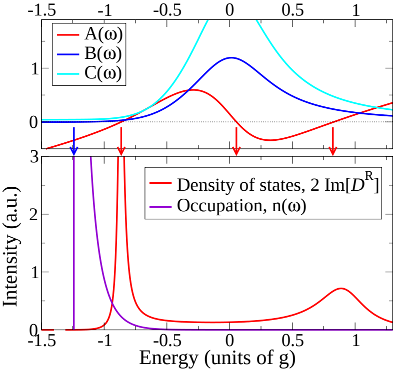
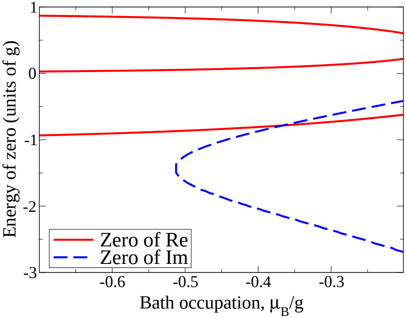
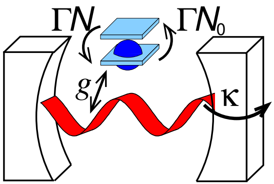


Linewidth, inverse Green's function and gap equation





$[D^R]^{-1}$ for a laser



Maxwell-Bloch equations:

$$\partial_t \psi = -i\omega_k \psi - \kappa\psi + gP$$
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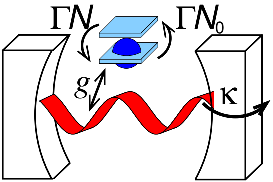

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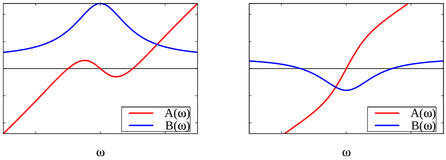

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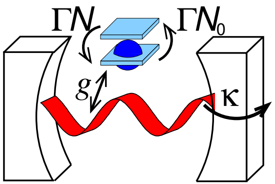

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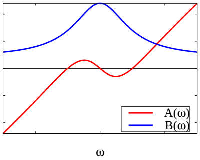

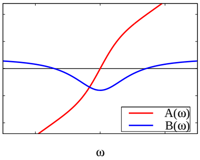

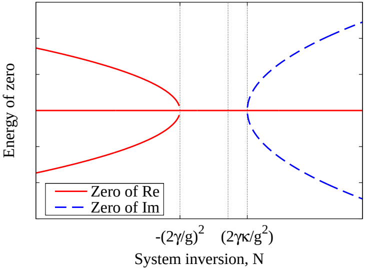

Spin in terms of ~~two~~four-level systems

To include spin, replace 2 level system with 4 levels: $|0\rangle, |L\rangle, |R|\rangle, |LR\rangle$

- Bi-exciton binding $E_{XX} \leftrightarrow U_1$
- Mean-field: find polarisation given ψ_L, ψ_R.
- E_{XX} has weak effect on T_c

[Marchetti *et al* PRB, '08]

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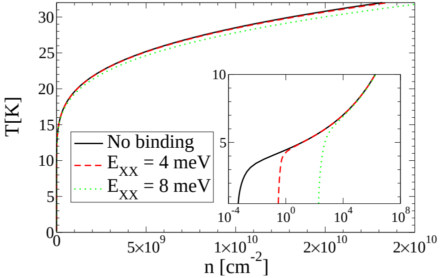

[Marchetti *et al* PRB, '08]

Polariton spin degree of freedom

- Left- and Right-circular polarised states.
- Spinor Gross-Pitaevskii equation:

$$i\partial_t \psi_L = \left[-\frac{\nabla^2}{2m} + V(r) + U_0 |\psi_L|^2 \right.$$
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Non-equilibrium spinor system: two-mode model

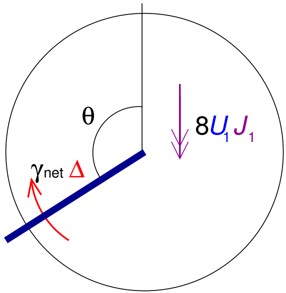
Write:

$$\psi_L = \sqrt{R+z}e^{i\phi+i\theta/2}, \quad \psi_R = \sqrt{R-z}e^{i\phi-i\theta/2}$$

Josephson regime: $J_1 \ll U_1 R$, $z \ll R$,

$$\dot{\theta} = -\Delta - 4U_1 z,$$
$$\dot{z} = -2\gamma_{\text{net}} z - 2J_1 \frac{\gamma_{\text{net}}}{\Gamma} \sin(\theta)$$

Non-equilibrium spinor system: two-mode model



Write:

$$\psi_L = \sqrt{R+z}e^{i\phi+i\theta/2}, \quad \psi_R = \sqrt{R-z}e^{i\phi-i\theta/2}$$

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Damped, driven pendulum

$$\ddot{\theta} + 2\gamma_{net}\dot{\theta} = 8U_1 J_1 \frac{\gamma_{net}}{\Gamma}\sin(\theta) - 2\gamma_{net}\Delta$$

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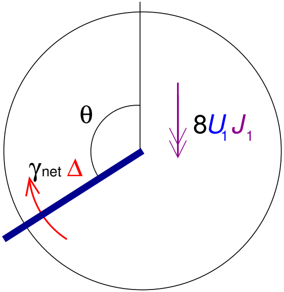
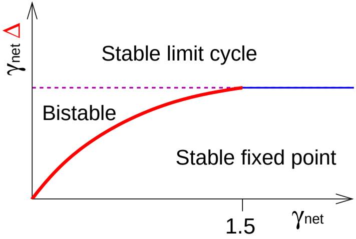
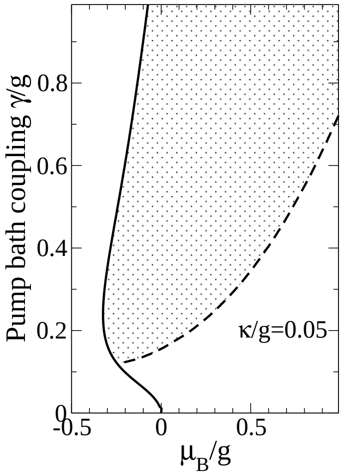




Zero temperature phase diagram

$$\tilde{\omega}_0 - i\kappa = g^2\gamma \sum_\alpha \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(\tilde{\epsilon}_\alpha + i\gamma)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}.$$



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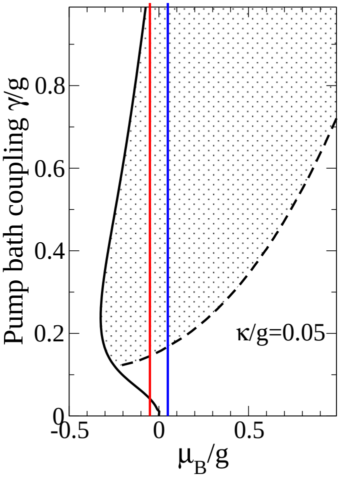

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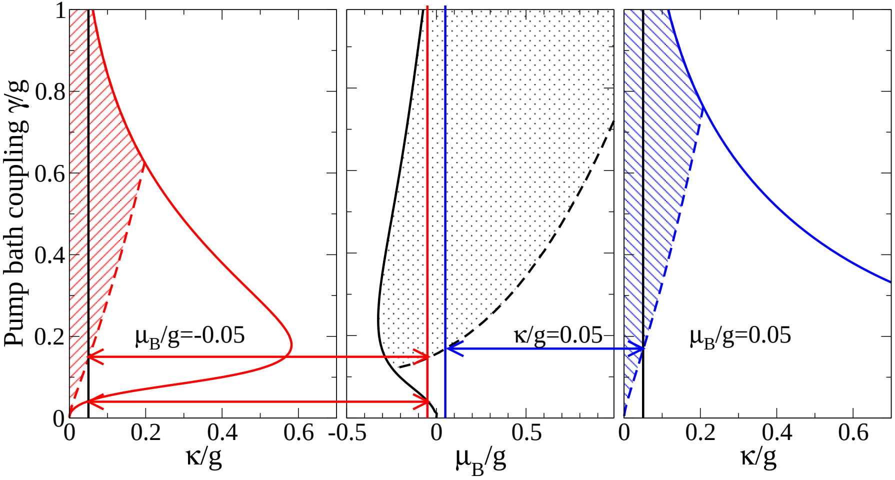
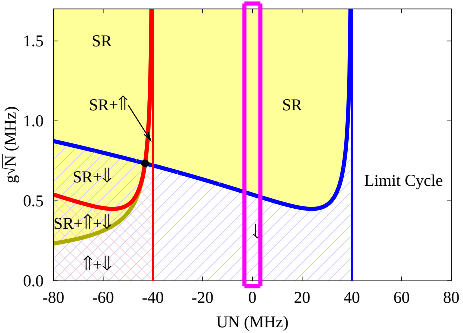
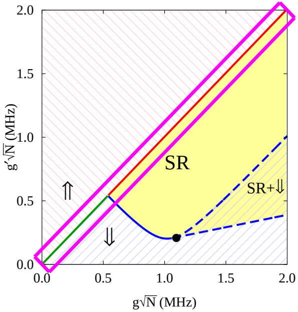


$U = 0$, different g, g'





$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi^\dagger S^- + \psi S^+) + g'(\psi^\dagger S^+ + \psi S^-) + US_z\psi^\dagger\psi.$$

Fixed points at $U = 0$

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Fixed points $\dot{\mathbf{S}}, \dot{\psi} = 0$.

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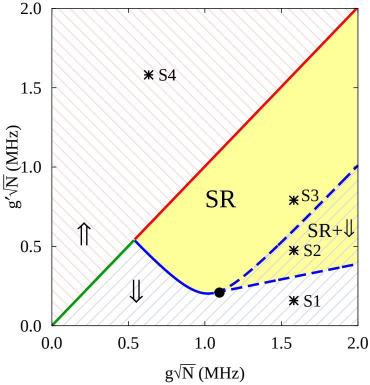

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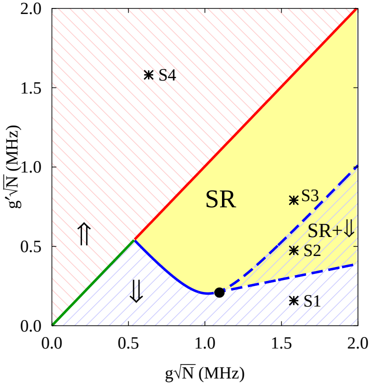
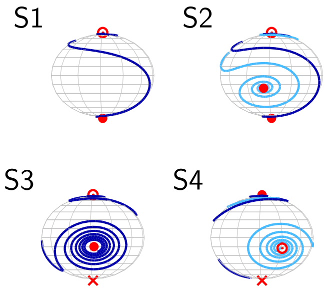
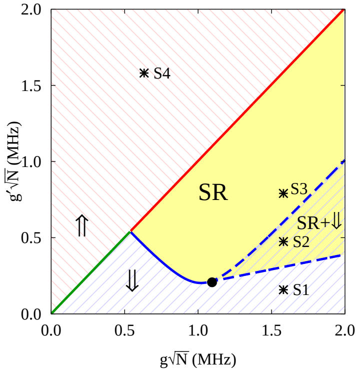




Boundaries $U = 0$

$\kappa \neq 0$



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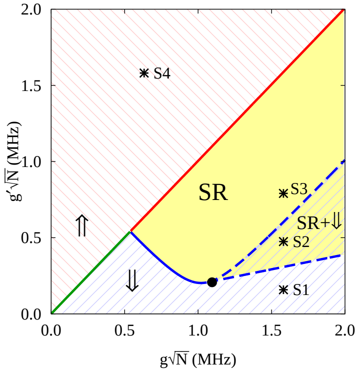

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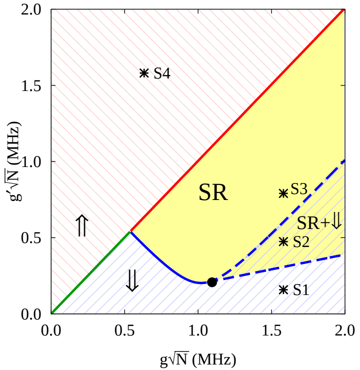
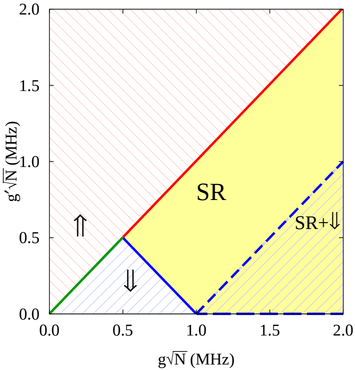
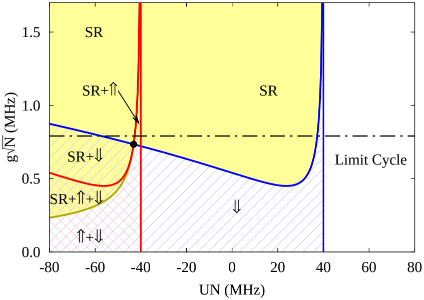


$\kappa = 0$:

—$N(g+g')^2 = \omega\omega_0$



Numerical confirmation of fixed points



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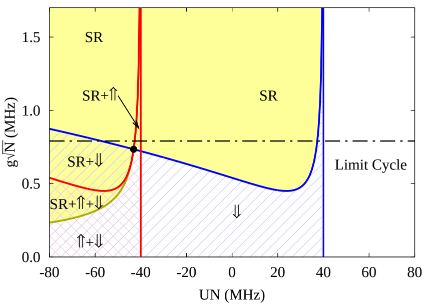

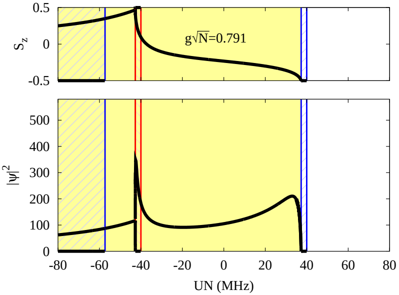

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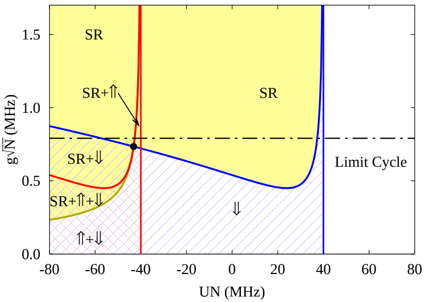

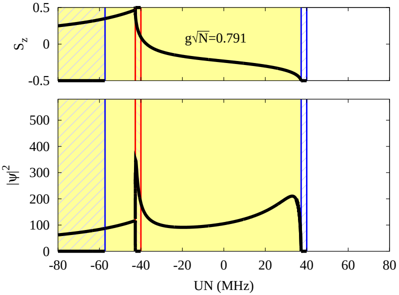

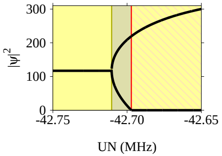

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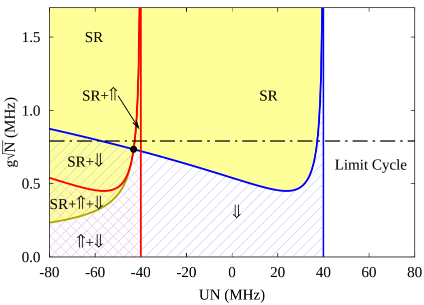

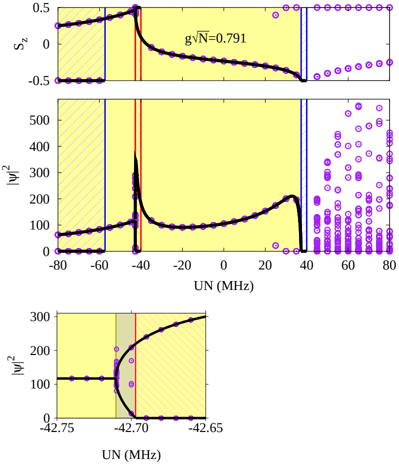

$T = 360\text{ms}$