Nonequilibrium quantum condensates: from microscopic theory to macroscopic phenomenology

J. M. J. Keeling
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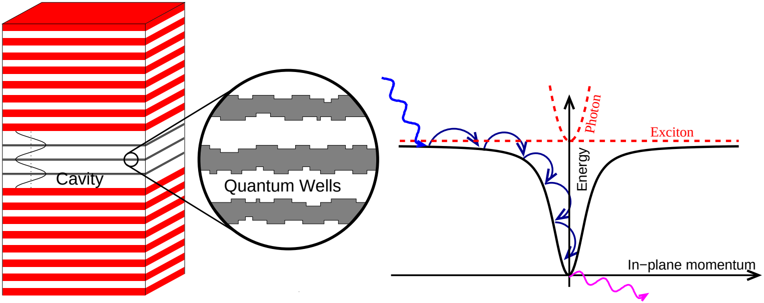
Non-equilibrium polariton condensate timescales



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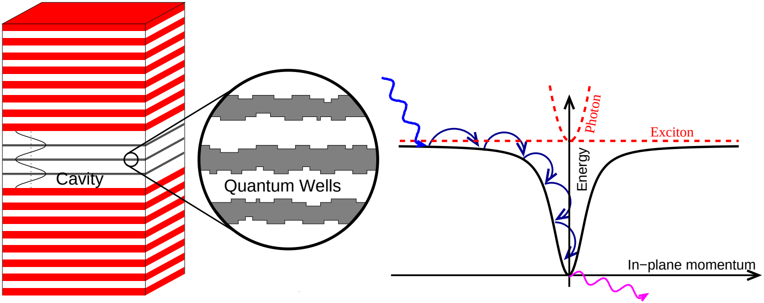

	Lifetime	Thermalisation
Atoms	10s	10ms
Excitons[a]	50ns	0.2ns
Polaritons	5ps	0.5ps
Magnons[b]	1μs(??)	100ns(?)

[a] Coupled quantum wells. [Hammack et al PRB 76 193308 (2007)]
[b] Yttrium Iron Garnett. [Demokritov et al, Nature 443 430 (2007)]

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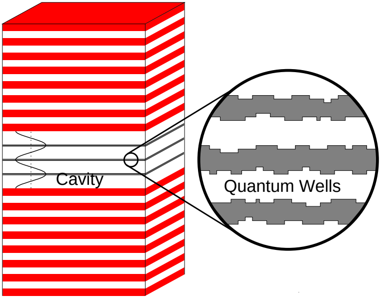

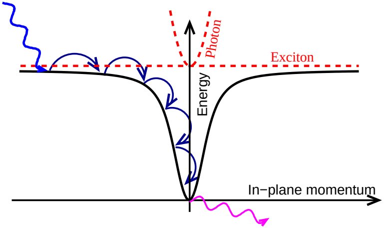

	Lifetime	Thermalisation	Linewidth	Temperature	
Atoms	10s	10ms	2.5×10^{-13}meV	10^{-8}K	10^{-9}meV
Excitons[a]	50ns	0.2ns	5×10^{-5}meV	1K	0.1meV
Polaritons	5ps	0.5ps	0.5meV	20K	2meV
Magnons[b]	1μs(??)	100ns(?)	2.5×10^{-6}meV	300K	30meV

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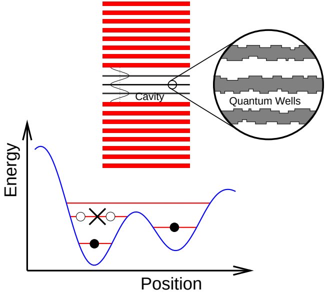
Overview

1. Microscopic non-equilibrium model
    - Fluctuations and stability of normal state
    - Fluctuations in a finite size non-equilibrium condensate
    - Superfluidity
2. Polariton spin degree of freedom
    - Equilibrium phase diagram
    - Non-equilibrium spinor condensate
        - Uniform system: stability and dispersion
        - Harmonic trapped system
        - Spectrum of vortex lattice

Polariton system model

Polariton model

- Disorder-localised excitons
- Treat disorder sites as two-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling g.



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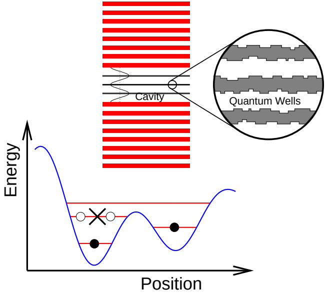
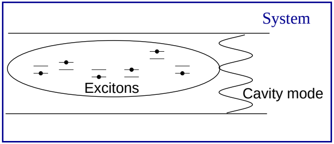
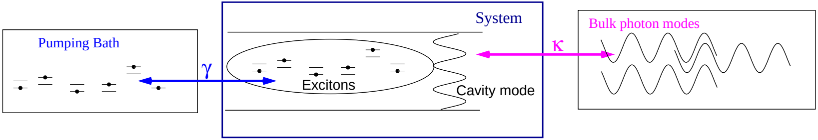


$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_{\alpha} \left(b^{\dagger}_{\alpha} b_{\alpha} - a^{\dagger}_{\alpha} a_{\alpha} \right) + \frac{1}{\sqrt{\text{Area}}} g_{\alpha,\mathbf{k}} \psi_{\mathbf{k}} b^{\dagger}_{\alpha} a_{\alpha} + \text{H.c.} \right]$$



Non-equilibrium model: baths



$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

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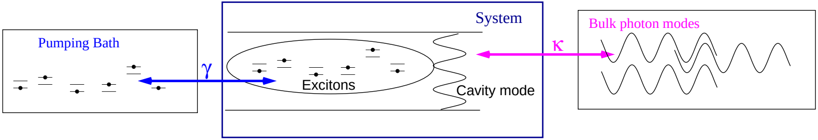

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Schematically: pump γ, decay κ

$$H_{\text{sys,bath}} \simeq \sum_{\mathbf{p},\vec{k}} \sqrt{\kappa}\psi_{\mathbf{k}}\Psi^\dagger_{\mathbf{p}} + \sum_{\alpha,\beta} \sqrt{\gamma}\left(a^\dagger_\alpha A_\beta + b^\dagger_\alpha B_\beta\right) + \text{H.c.}$$

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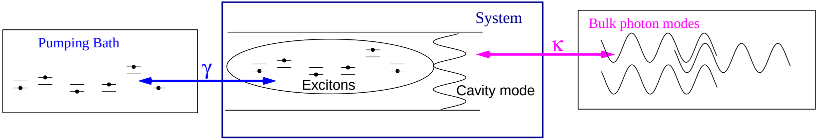

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Bath correlations, $\langle \Psi^\dagger \Psi \rangle$, $\langle A^\dagger A \rangle$, $\langle B^\dagger B \rangle$ fixed:

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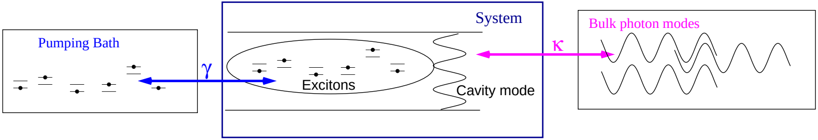

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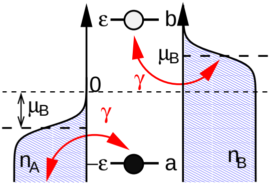
Ψ bath is empty. Pumping bath thermal, μ_B, T:



Fluctuations → Stability, Luminescence, Absorption

Green's functions:

$$D^R = i\theta[t - t'] \left\langle \left[\psi, \psi^\dagger\right]_- \right\rangle$$

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$$\left[D^R(\omega)\right]^{-1} = A(\omega) + iB(\omega),$$

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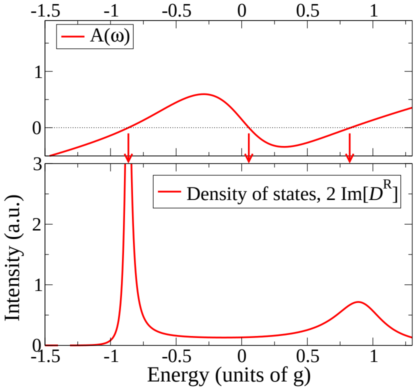

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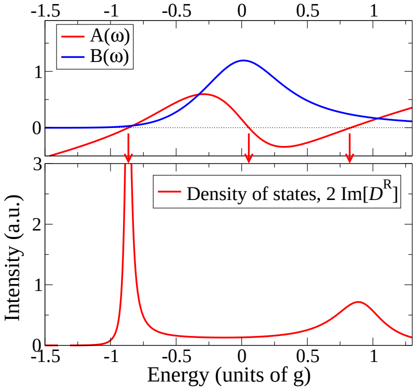

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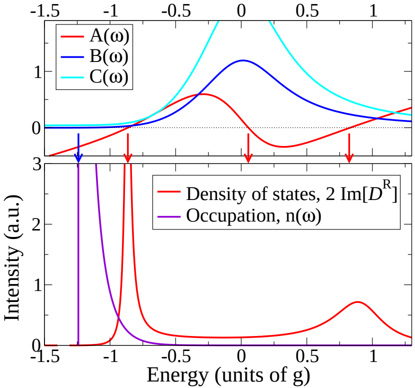

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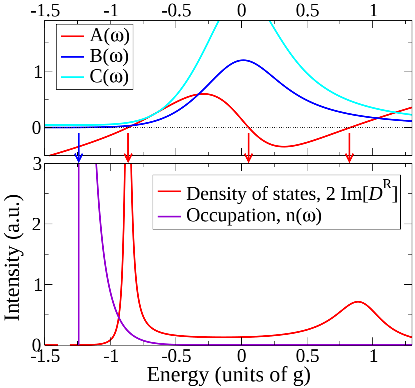
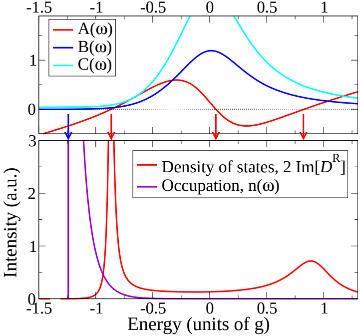
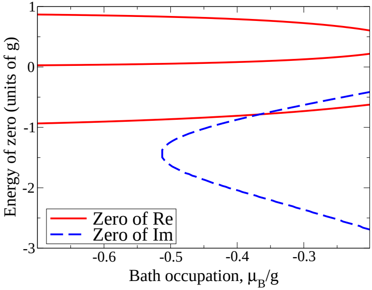
$$\left[D^R(\omega)\right]^{-1} = (\omega - \omega_k^*) + i\alpha(\omega - \mu_{\text{eff}})$$

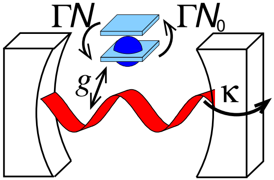


Linewidth, inverse Green's function and gap equation





$[D^R]^{-1}$ for a laser



Maxwell-Bloch equations:

$$\partial_t \psi = -i\omega_k \psi - \kappa\psi + gP$$
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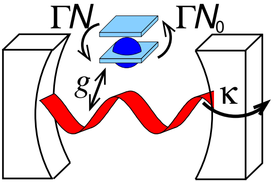

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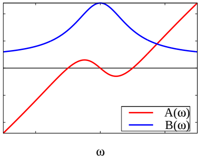

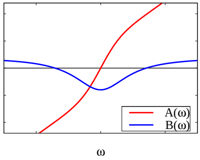

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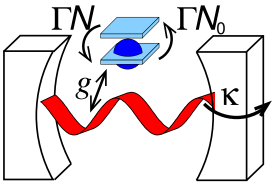

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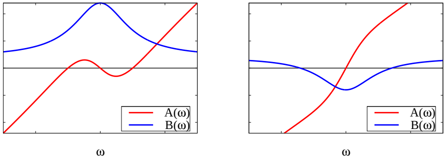
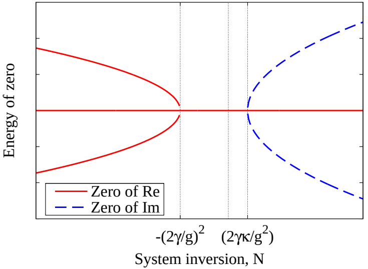




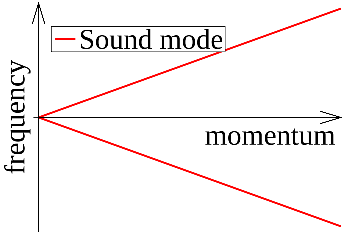
Fluctuations above transition

When condensed

$$\text{Det}\left[D^R(\omega, \mathbf{k})\right]^{-1} = \omega^2 - c^2\mathbf{k}^2$$

Poles:

$$\omega^* = c|\mathbf{k}|$$



[Szymańska, PRL '06; Wouters, PRB '06]

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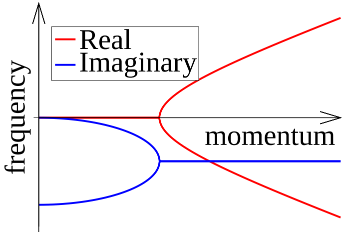
$$\omega^* = -i\eta \pm \sqrt{c^2\mathbf{k}^2-\eta^2}$$



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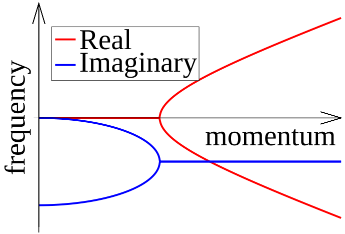

Correlations (in 2D): $\left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0)\right\rangle \simeq |\psi_0|^2 \exp\left[-\mathcal{D}_{\phi\phi}(t, \mathbf{r}, \mathbf{r}')\right]$

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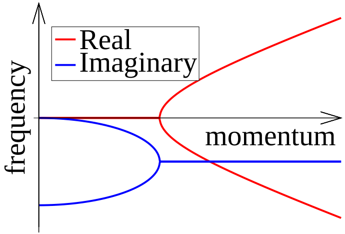

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$$\left\langle\psi^\dagger(\mathbf{r},t)\psi(0,0)\right\rangle \simeq |\psi_0|^2\exp\left[-\eta\begin{cases}\ln(r/\xi) & r\to\infty, t\simeq 0\\ \frac{1}{2}\ln(c^2t/\eta\xi^2) & r\simeq 0, t\to\infty\end{cases}\right]$$

[Szymańska, PRL '06; Wouters, PRB '06]

Finite size effects: Single mode vs many mode

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$$\mathcal{D}_{\phi\phi}(t,\mathbf{r},\mathbf{r}) \propto \sum_n^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(\mathbf{r})|^2(1-e^{i\omega t})}{|(\omega+i\eta)^2+\eta^2-\zeta_n^2|^2}$$

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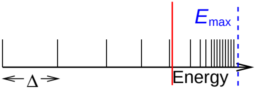

$\mathcal{D}_{\phi\phi} \sim 1 + \ln(E_{\max}\sqrt{t/\eta})$

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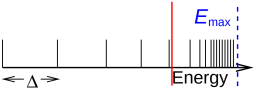

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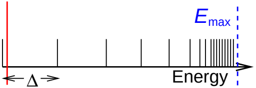

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Asking about non-equilibrium superfluidity

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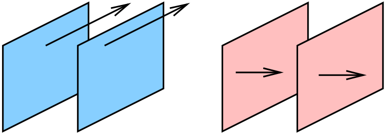

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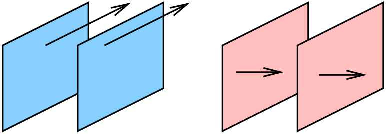

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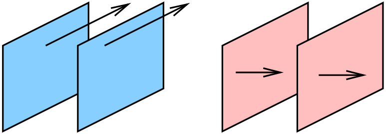

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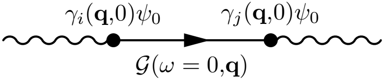
$\Delta\chi_{ij}(q) =$  $+ \ldots$

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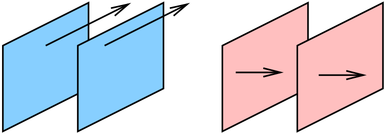

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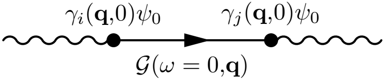
$\Delta\chi_{ij}(q) =$ 

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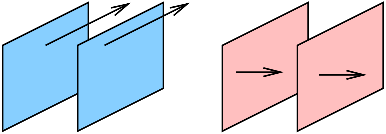

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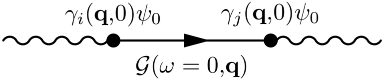
$\Delta\chi_{ij}(q) =$ 

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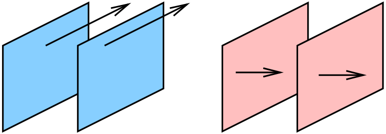

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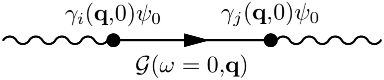
$\Delta\chi_{ij}(q) =$ 

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Polariton spin degree of freedom

- Left- and Right-circular polarised states.
- For weakly-interacting dilute Bose gas model:
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- J_2 Circular Symmetry → D_{2d} — two equivalent axes.
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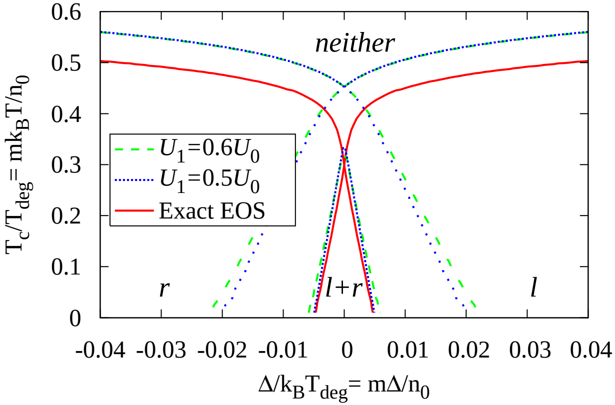

[Rubo *et al* Phys. Lett. A '06; Keeling, Phys. Rev. B '08]

Mathematical outline

Two approaches:

- For $(U_0 - 2U_1) = 0$ use one-component equation of state:

$$n_0 = T\left[f\left(\frac{\mu+\Delta}{T}\right) + f\left(\frac{\mu-\Delta}{T}\right)\right]$$

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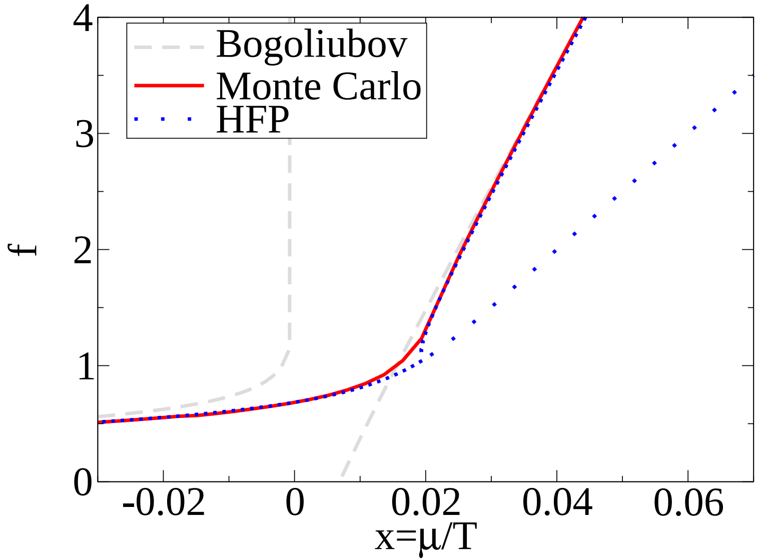
Graphical implementation of $T = n_0 / \left[f_c + f\left(x_c + \frac{2\Delta}{T} \right) \right]$



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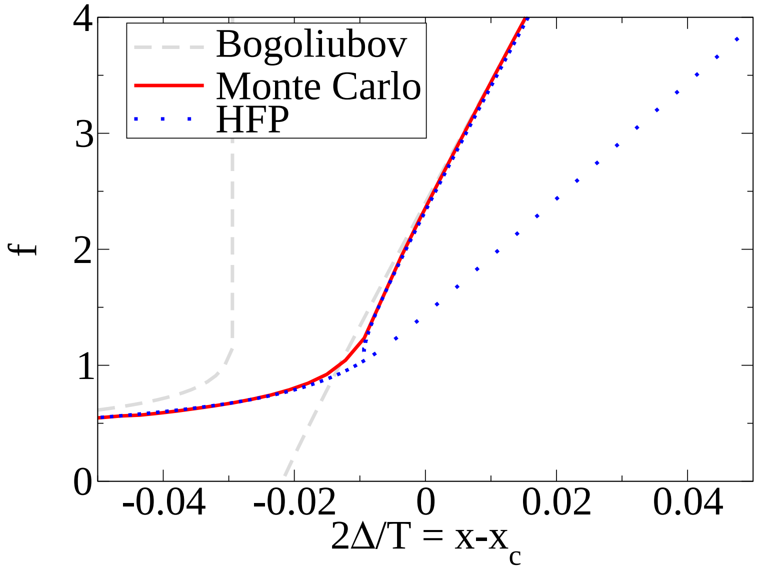

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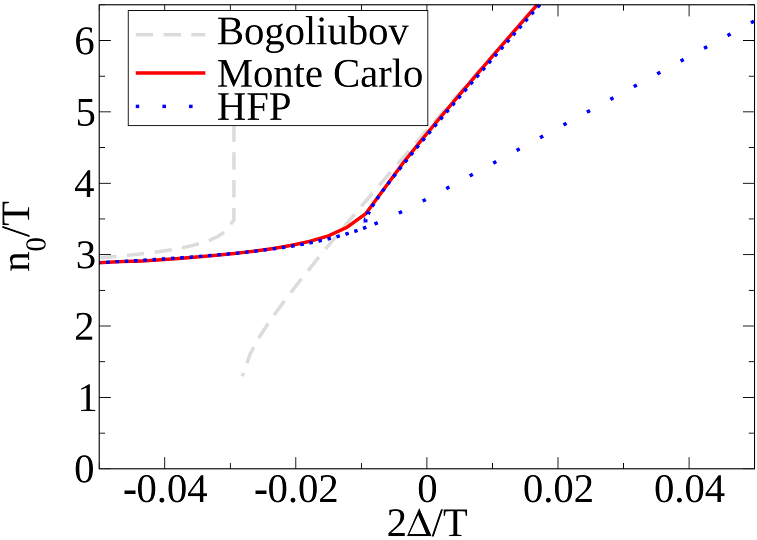

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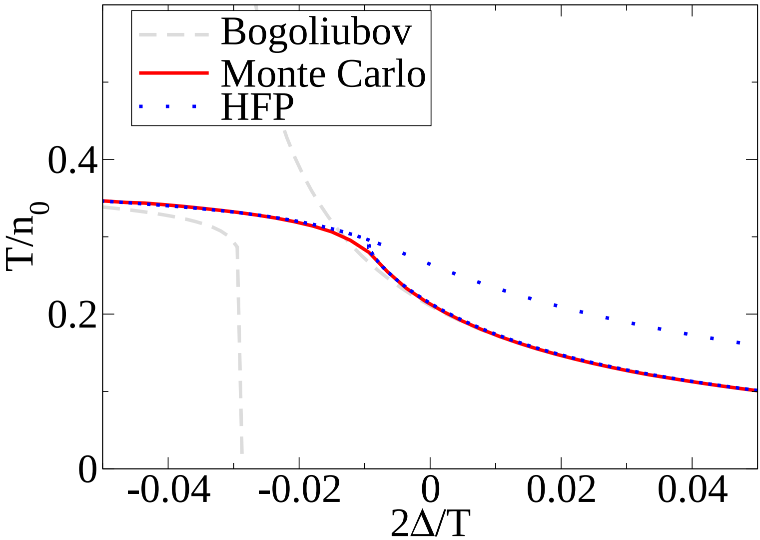

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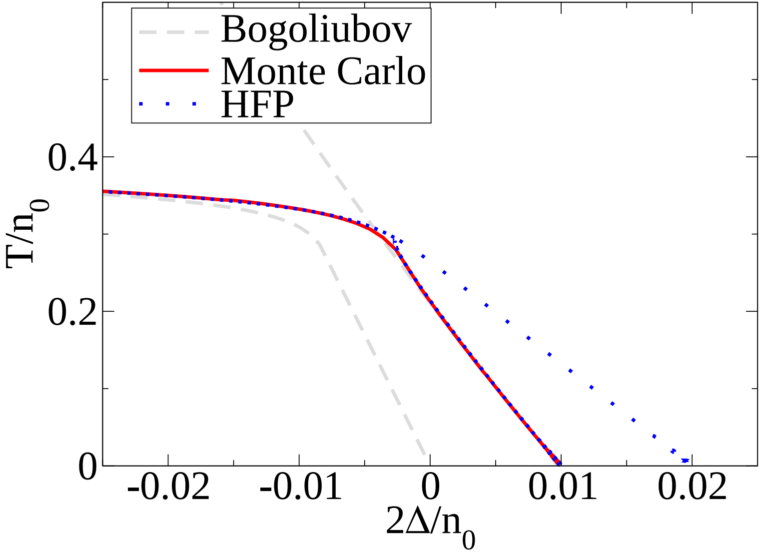

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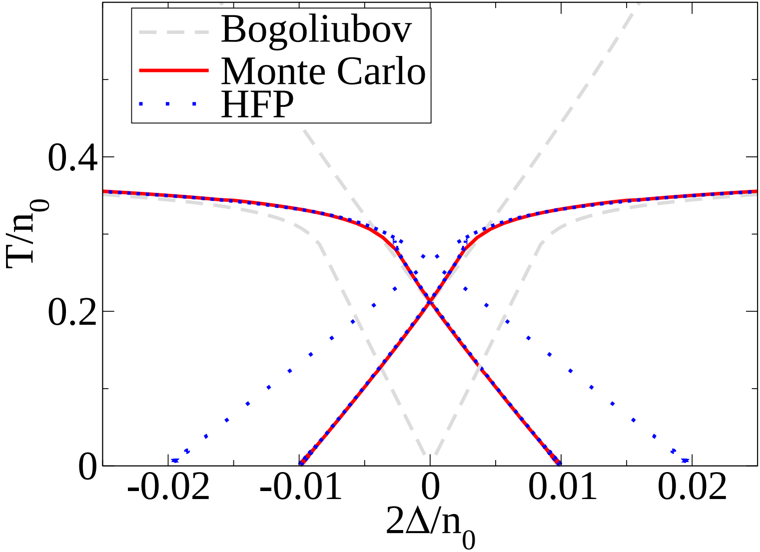
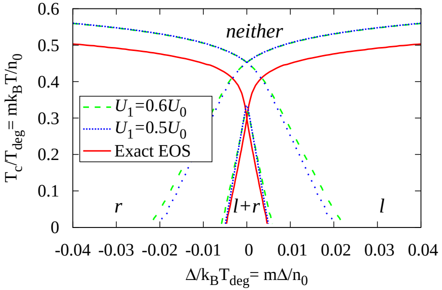


Equilibrium phase diagrams

$J_1 = J_2 = 0$.



Circular $\rightarrow$ Eliptical transitions.

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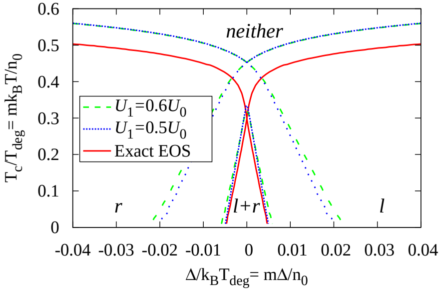

Circular → Eliptical transitions.

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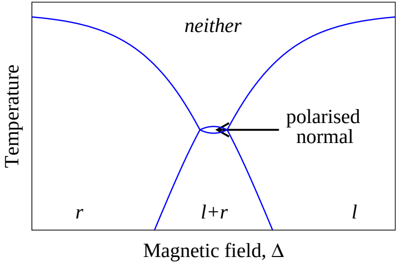

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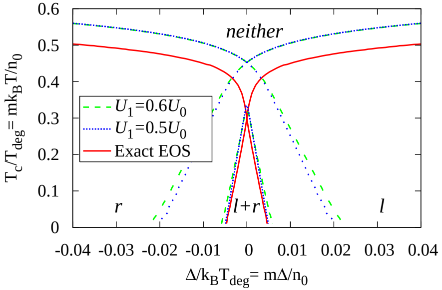

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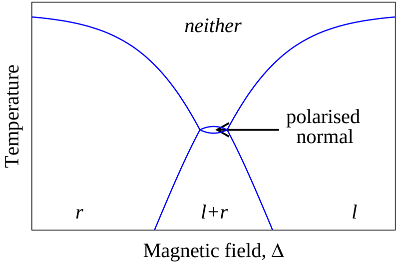

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Non-equilibrium spinor system

Spinor Gross-Pitaevskii equation:

$$i\partial_t \psi_L = \left[-\frac{\nabla^2}{2m} + V(r) + \frac{\Delta}{2} + U_0|\psi_L|^2 + (U_0 - 2U_1)|\psi_R|^2 \right. \\ \left. + i\left(\gamma_{\text{eff}}(\mu_B) - \kappa - \Gamma|\psi_L|^2\right)\right]\psi_L + J_1\psi_R$$

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- Two-mode case (neglect spatial variation): [Wouters PRB '08]
- To recap results write $\psi_{L,R} = \sqrt{\rho_{L,R}}e^{i(\phi\pm\theta/2)}$, $\rho_{L,R} = R \pm z$.

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Non-equilibrium spinor system

Spinor Gross-Pitaevskii equation:

$$i\partial_t\psi_L = \left[-\frac{\nabla^2}{2m} + V(r) + \frac{\Delta}{2} + U_0|\psi_L|^2 + (U_0 - 2U_1)|\psi_R|^2 + i\left(\gamma_{\text{eff}}(\mu_B) - \kappa - \Gamma|\psi_L|^2\right)\right]\psi_L + J_1\psi_R$$

- $J_1 \rightarrow$ interconversion. **How does this interact with currents.**
- Two-mode case (neglect spatial variation): [Wouters PRB '08]
- To recap results write $\psi_{L,R} = \sqrt{\rho_{L,R}}e^{i(\phi\pm\theta/2)}$, $\quad \rho_{L,R} = R \pm z$.

$$\dot{R} = 2U_0\left(R\frac{\gamma_{\text{net}}}{\Gamma} - R^2 - z^2\right) \qquad \dot{\theta} = -\Delta - 4U_1 z + 2\frac{J_1 z\cos(\theta)}{\sqrt{R^2 - z^2}}$$

$$\dot{z} = 2(\gamma_{\text{net}} - 2\Gamma R)z - 2J_1\sqrt{R^2 - z^2}\sin(\theta)$$

Two-mode system summary

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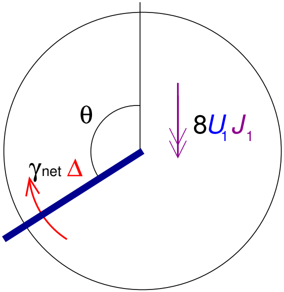

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Damped, driven pendulum

$$\ddot{\theta} + 2\gamma_{\text{net}}\dot{\theta} = 8U_1 J_1 R_0 \sin(\theta) - 2\gamma_{\text{net}}\Delta$$

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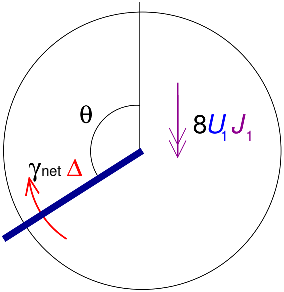
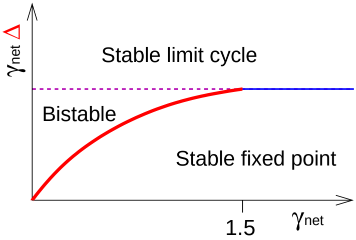
[e.g. Strogatz, Nonlinear dynamics and chaos]



(Cartoon)



Two-mode model bistability

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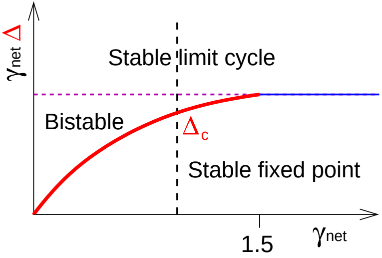

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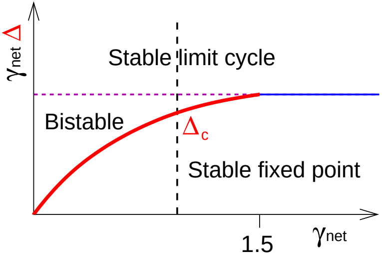
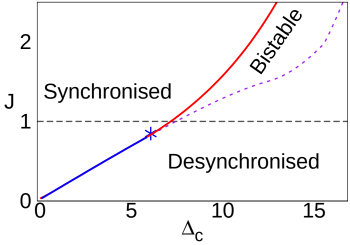


(Actual:)



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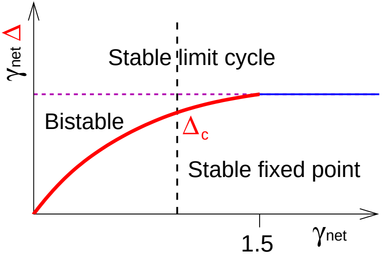

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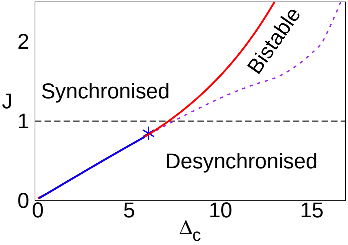
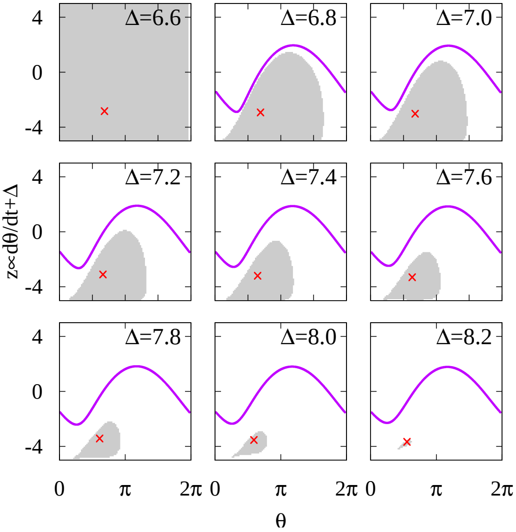
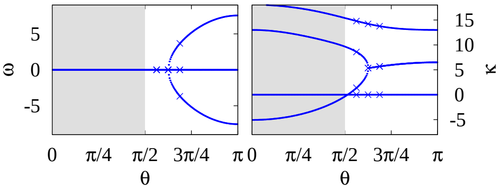




Stability & dispersion at $\Delta < \Delta_c$

Consider: $\psi_\alpha \rightarrow e^{-i\mu t}\left(\psi_\alpha^0 + u_\alpha e^{-i\mathbf{k}\cdot\mathbf{r}+(-i\omega-\kappa)t} + v_\alpha^* e^{i\mathbf{k}\cdot\mathbf{r}+(i\omega-\kappa)t}\right)$

First $k=0$ fluctuations. Four normal modes:



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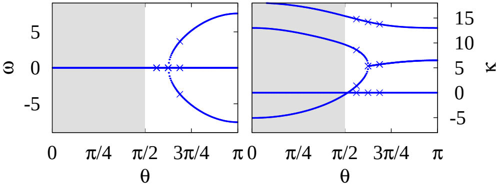

- Global phase:
  $\omega - i\kappa = 0$
- Global density:
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- Relative phase/density

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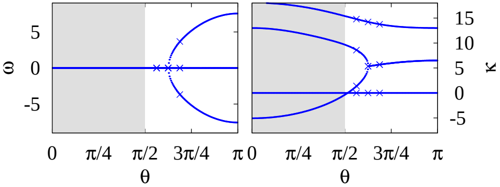

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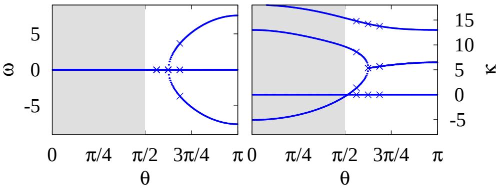

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Near steady state

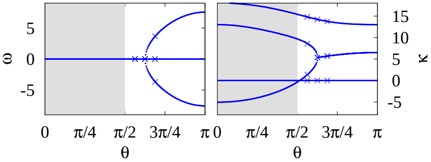
$$\ddot{\theta} + 2\gamma_{\text{net}}\theta + \Omega_p^2\theta = 0$$

$$\omega - i\kappa \simeq -i\gamma_{\text{net}} \pm i\sqrt{\gamma_{\text{net}} - \Omega_p^2}$$

Stable if $\Omega_p^2 = -8 U_\alpha J_\alpha R_0 \cos(\theta) > 0$

Stability & dispersion: finite k

Consider: $\psi_\alpha \rightarrow e^{-i\mu t}\left(\psi_\alpha^0 + u_\alpha e^{-i\mathbf{k}\cdot\mathbf{r}+(-i\omega-\kappa)t} + v_\alpha^* e^{i\mathbf{k}\cdot\mathbf{r}+(i\omega-\kappa)t}\right)$



If $\Omega_p^2 = \gamma_{\text{net}}$, finite $k \rightarrow$
degenerate perturbation theory

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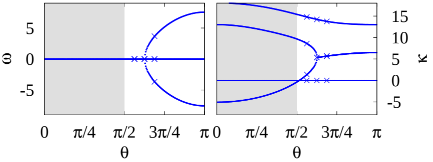

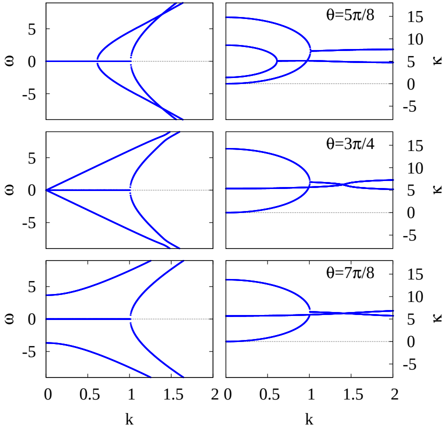

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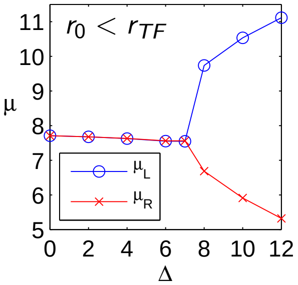
Overview

1. Microscopic non-equilibrium model
   - Fluctuations and stability of normal state
   - Fluctuations in a finite size non-equilibrium condensate
   - Superfluidity
2. Polariton spin degree of freedom
   - Equilibrium phase diagram
   - Non-equilibrium spinor condensate
     - Uniform system: stability and dispersion
     - Harmonic trapped system
     - Spectrum of vortex lattice

Trapped spinor system

$$V(r) = m\omega^2 \frac{r^2}{2}, \quad \gamma_{\rm net}(r) = J_1 \Theta(r_0 - r).$$

Plot $\mu_{L,R} = \dot{\phi} \pm \frac{1}{2}\dot{\theta}$ vs Δ.



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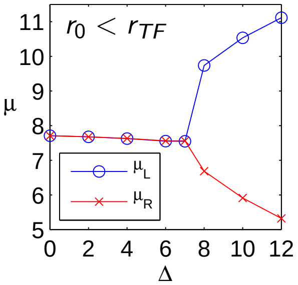
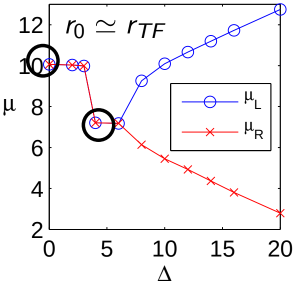
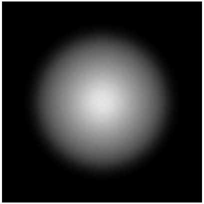
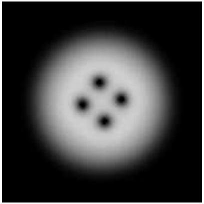
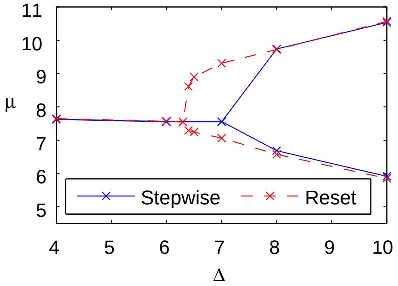








Extended model bistability



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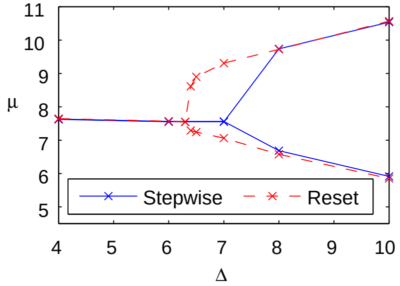
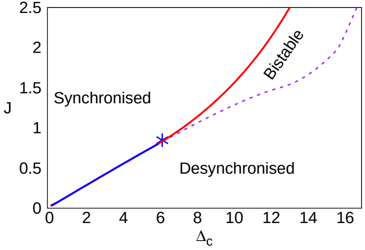
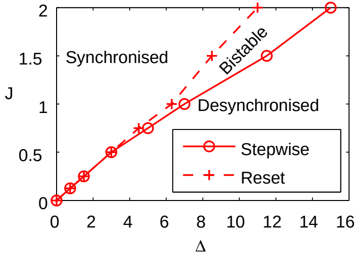


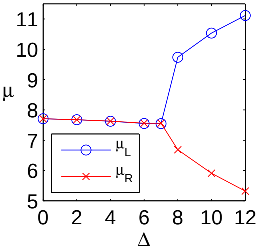
Uniform:



Harmonic trap:

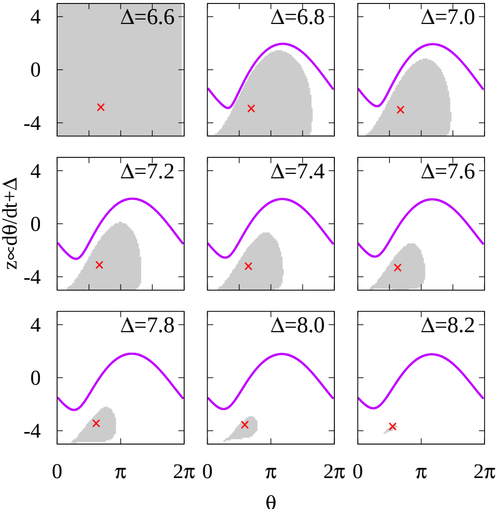


Trapped spinor system — phase portraits



Examine phase portrait $\partial_t \theta$ vs θ

- Retrograde motion
- Limit cycle winding $\# = 0, 1, 2$
- Chaos (large J_1, Δ)



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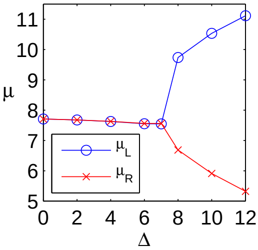
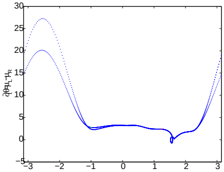
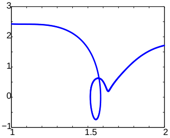




$J_1 = 1$,
$\Delta = 6.4$



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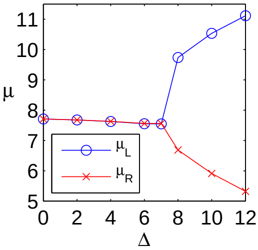

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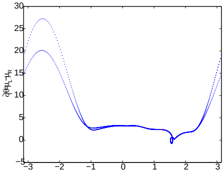

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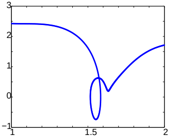

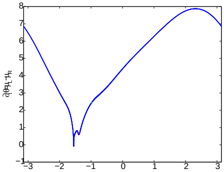

$J_1 = 1,$
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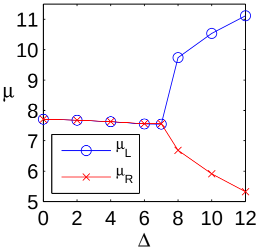
Trapped spinor system — phase portraits



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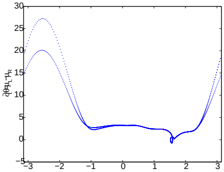

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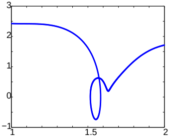

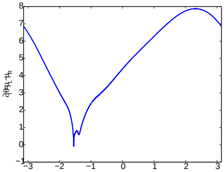

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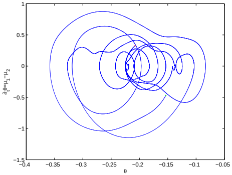
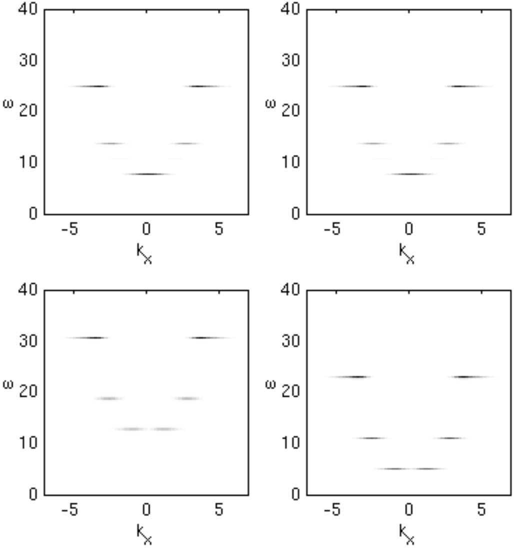


$J_1 = 2,$
$\Delta = 10.0$

Vortex lattices, desynchronisation and spectrum

- Non-steady condensate — non-trivial spectrum
- Presence/absence of central vortex clear.
- NB $\omega \sim 2\frac{\hbar^2 k^2}{2m}$ due to virial theorem in harmonic trap.



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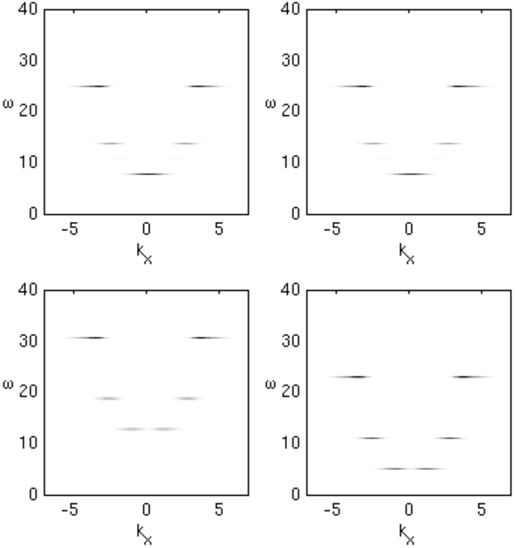

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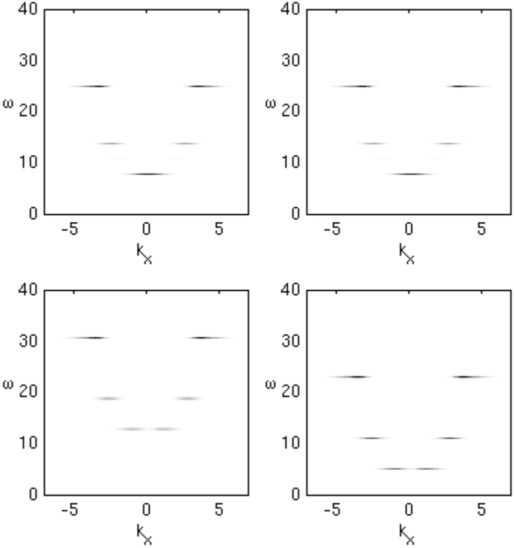

Acknowledgements

People:

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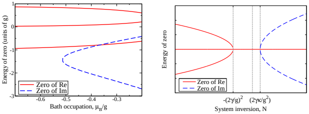
EPSRC
Engineering and Physical Sciences Research Council

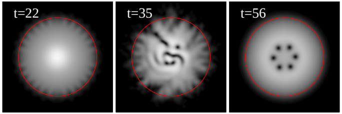
Pembroke College

Conclusions

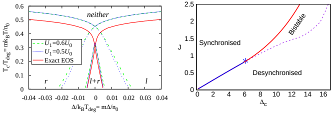
- Instability of normal state
  Translating: condensation ↔ lasing



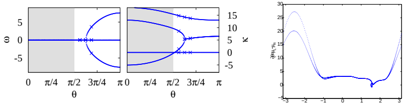
- Spontaneous rotating vortex lattice



- Spinor phase transitions



- Density profile and spectrum



Extra slides

3. Mean-field Keldysh theory
4. Spinor TLS
5. Fluctuations of non-equilibrium condensate
6. Vortex lattice in an harmonic trap
7. μ vs pump spot size
8. Observation vortex lattices

Limits of gap equation

Gap equation:

$$\mu_s - \omega_0 + i\kappa = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

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Spin in terms of ~~two~~four-level systems

To include spin, replace 2 level system with 4 levels: $|0\rangle, |L\rangle, |R|\rangle, |LR\rangle$

- Bi-exciton binding $E_{XX} \leftrightarrow U_1$
- Mean-field: find polarisation given ψ_L, ψ_R.
- E_{XX} has weak effect on T_c

[Marchetti *et al* PRB, '08]

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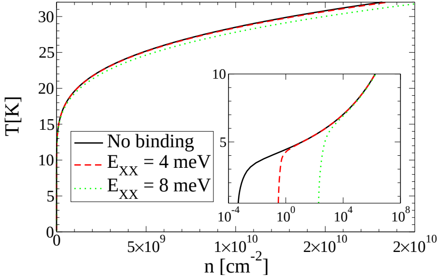

[Marchetti *et al* PRB, '08]

Relating finite-size spectrum to self phase modulation

Single mode spectrum:

$$\mathcal{D}_{\phi\phi}(t, r, r) \propto \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2 (1 - e^{i\omega t})}{|(\omega + ix)^2 + x^2 - (n\Delta)^2|^2}$$

Connection to Kubo Formula [Eastham and Whittaker, arXiv:0811.4333]

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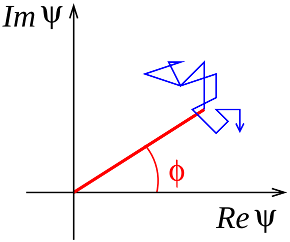

$$\partial_t \phi = U \delta N$$

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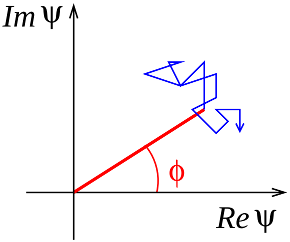

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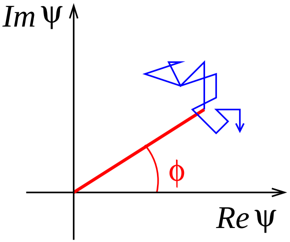

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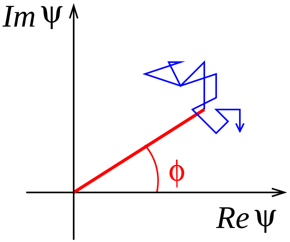

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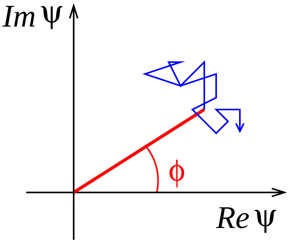

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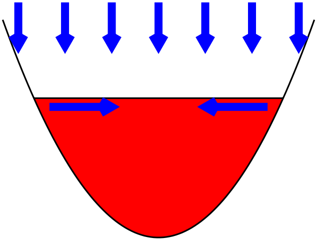
&= \frac{CU^2}{2\Gamma^3}\left[\Gamma t - 1 + e^{-\Gamma t}\right]
\end{aligned}$$

Overview

Gross-Pitaevskii equation: Harmonic trap

$$i\partial_t \psi = \left[-\frac{\nabla^2}{2m} + \frac{m\omega^2}{2} r^2 + U|\psi|^2 + i\left(\gamma_{\text{eff}} - \kappa - \Gamma|\psi|^2\right)\right]\psi$$



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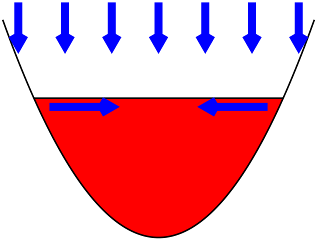
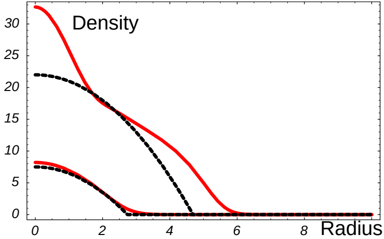
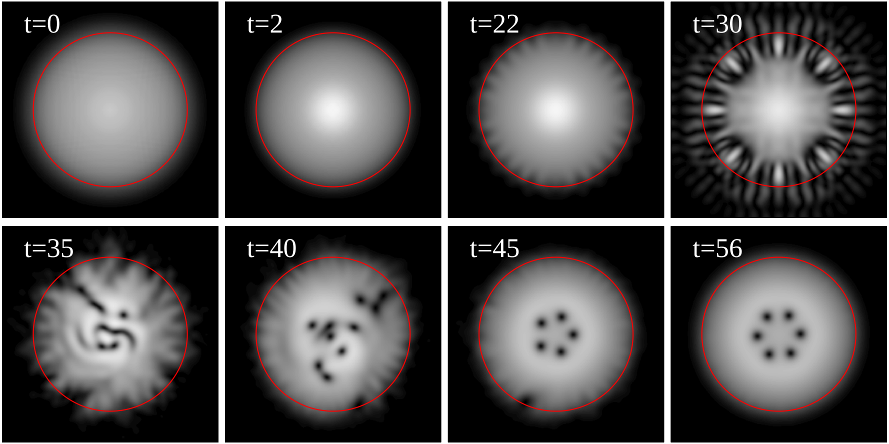




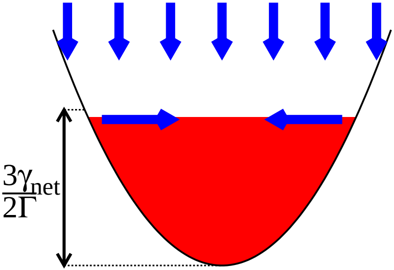
Time evolution:



Instability of Thomas-Fermi: details

$$\frac{1}{2}\partial_t\rho + \nabla\cdot(\rho\mathbf{v}) = (\gamma_{\mathrm{net}} - \Gamma\rho)\rho$$

$$\partial_t\mathbf{v} + \nabla\left(U\rho + \frac{m\omega^2}{2}r^2 + \frac{m}{2}|\mathbf{v}|^2\right) = 0$$



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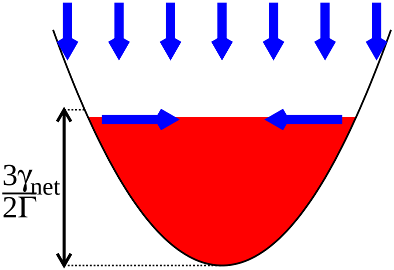

Normal modes for $\gamma_{\text{net}}, \Gamma \to 0$:

$$\delta\rho_{n,m}(r, \theta, t) = e^{im\theta} h_{n,m}(r) e^{i\omega_{n,m} t}$$

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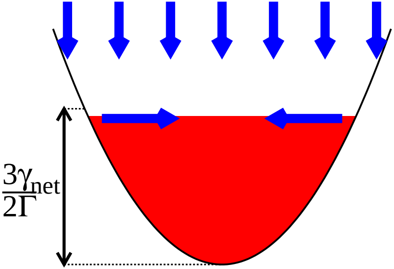

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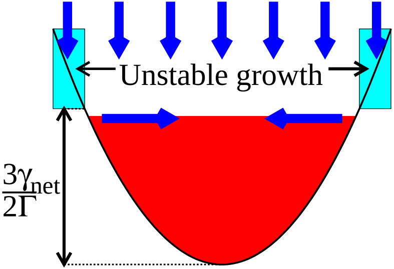

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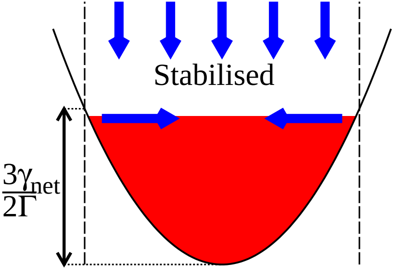

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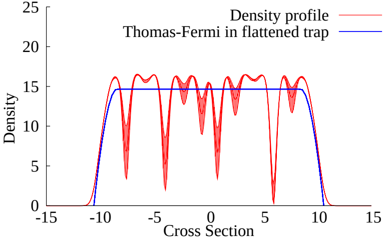
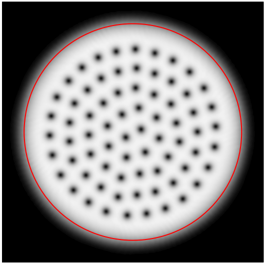
Why vortices





$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\text{net}}\Theta(r_0 - r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}|\mathbf{v} - \Omega \times \mathbf{r}|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{\text{net}}}{\Gamma}\Theta(r_0 - r) = \frac{\mu}{U}$$

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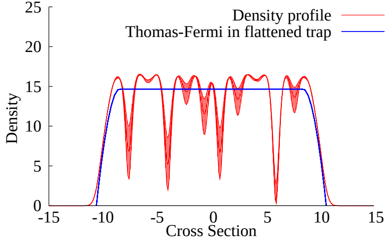

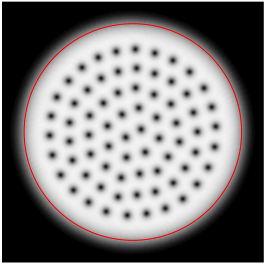

Rotating solution: $i\partial_t \psi = (\mu - 2\Omega L_z)\psi$

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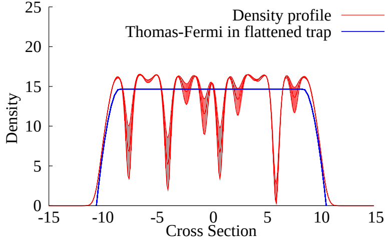

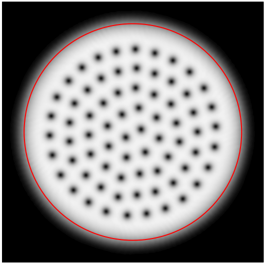

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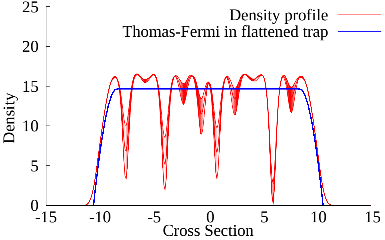

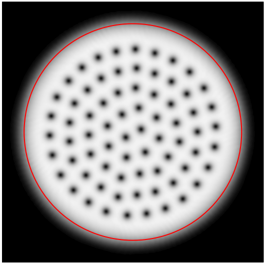

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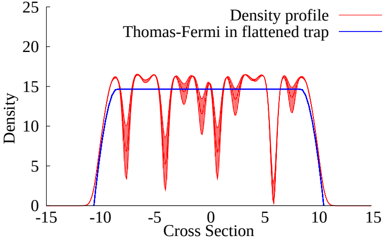

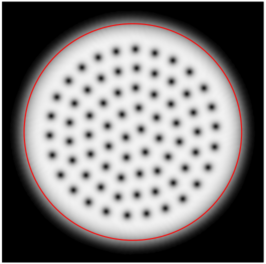

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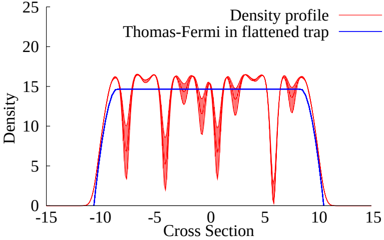

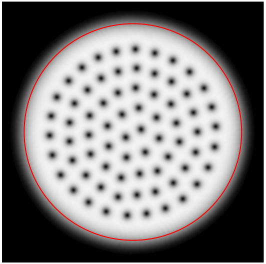

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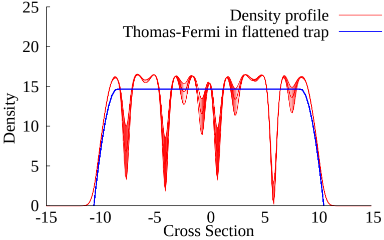

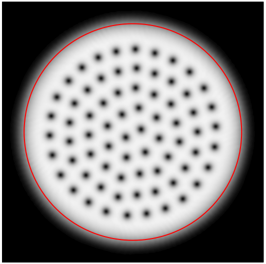

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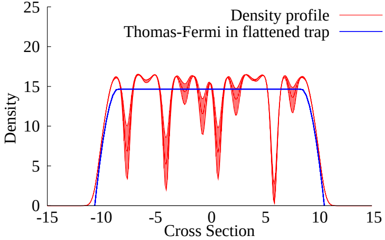

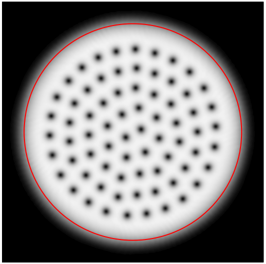

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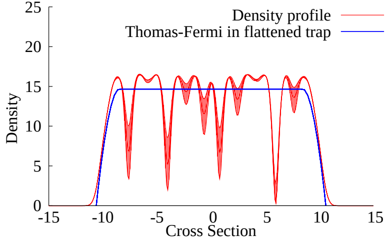

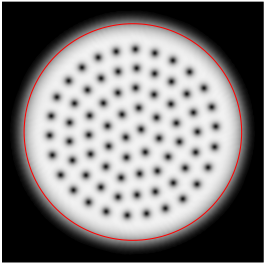

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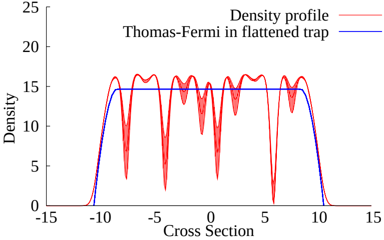

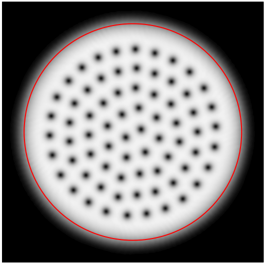

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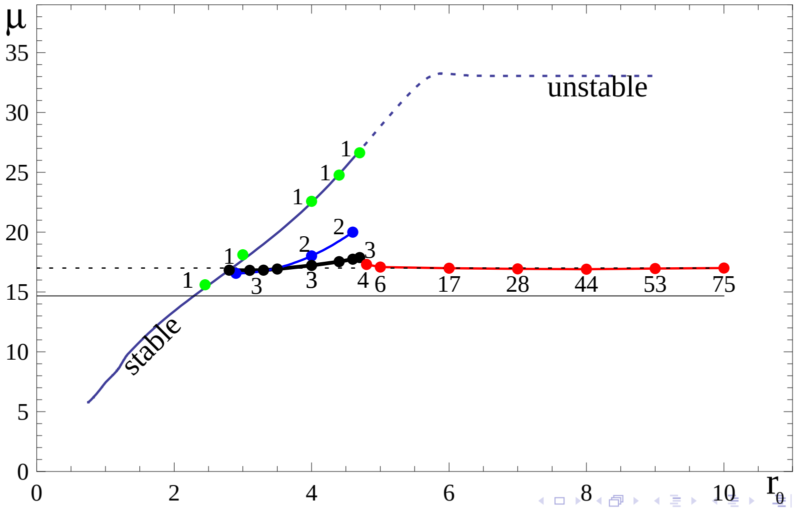
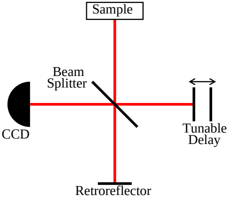
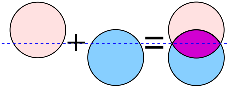
Why vortices: chemical potential vs size

$$\text{Thomas-Fermi}: \mu = f(r_0) \quad \text{Vortex}: \mu = \frac{U\gamma_{\text{net}}}{\Gamma}$$



Observing vortices: fringe pattern





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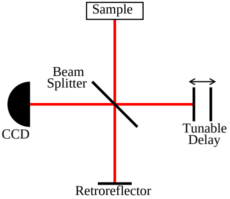
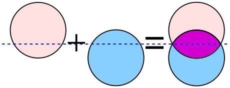
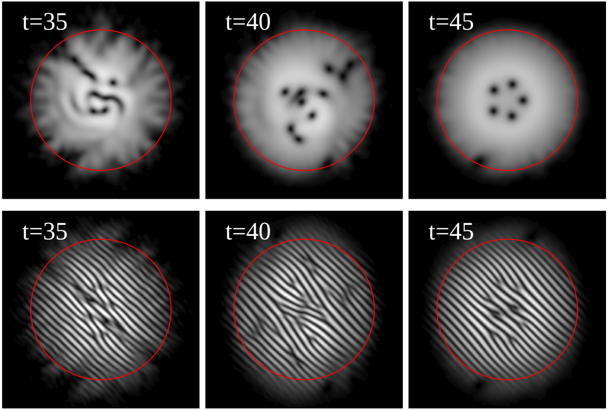
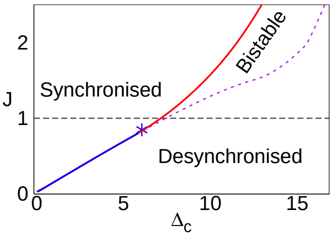




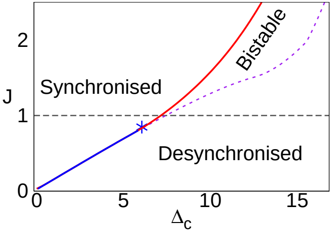


Singularity at critical Δ



Choose "most unstable" conditions

Singularity at critical Δ



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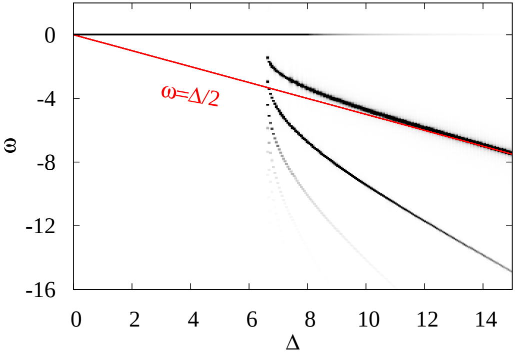

$$\Delta \gtrsim \Delta_c: \ \omega \equiv \langle \dot{\theta} \rangle \simeq \frac{1}{\ln(\Delta - \Delta_c)}$$

(Chemical potential mismatch)