Comparing polariton condensation and lasing

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Engineering and Physical Sciences
Research Council

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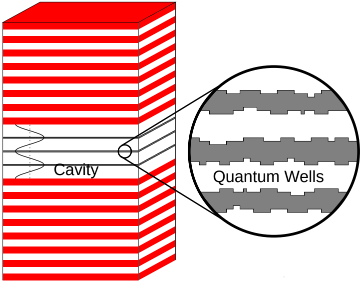
Microcavity Polaritons



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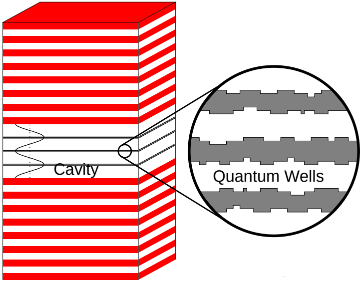

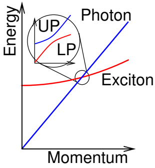

[Pekar, JETP(1958)]
[Hopfield, Phys. Rev.(1958)]

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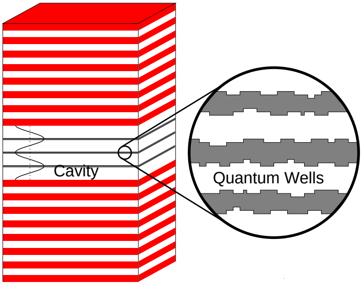

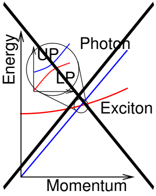

[Pekar, JETP(1958)]
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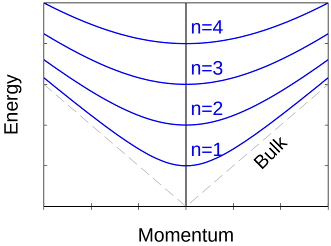
Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$



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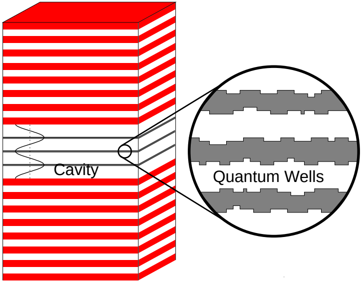

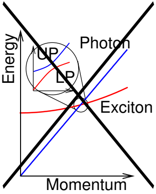

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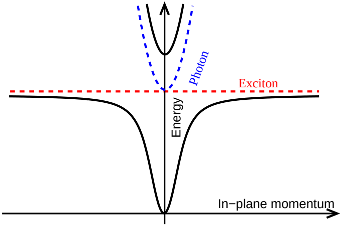
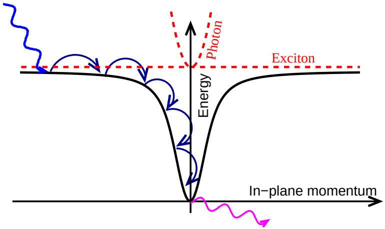


Non-equilibrium system



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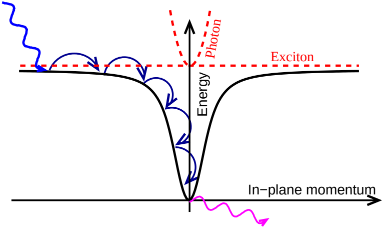

	Lifetime	Thermalisation
Atoms	10s	10ms
Excitons[a]	50ns	0.2ns
Polaritons	5ps	0.5ps
Magnons[b]	1μs(??)	100ns(?)

[a] Coupled quantum wells. [Hammack et al PRB 76 193308 (2007)]
[b] Yttrium Iron Garnett. [Demokritov et al, Nature 443 430 (2007)]

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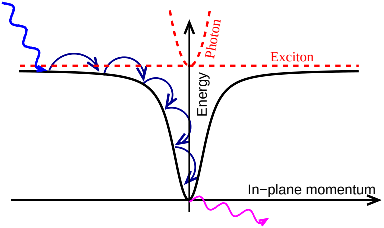

	Lifetime	Thermalisation	Linewidth	Temperature	
Atoms	10s	10ms	2.5×10^{-13}meV	10^{-8}K	10^{-9}meV
Excitons[a]	50ns	0.2ns	5×10^{-5}meV	1K	0.1meV
Polaritons	5ps	0.5ps	0.5meV	20K	2meV
Magnons[b]	1μs(??)	100ns(?)	2.5×10^{-6}meV	300K	30meV

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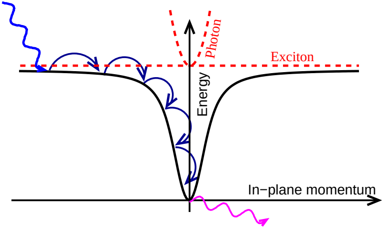

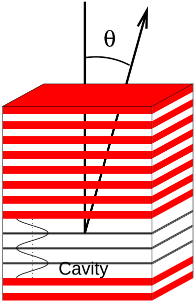

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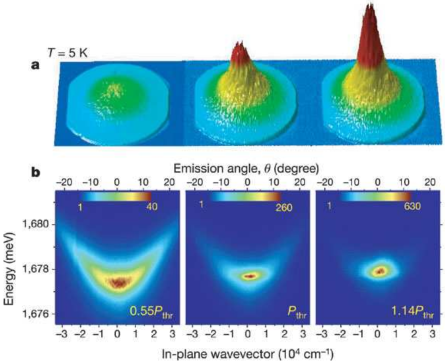
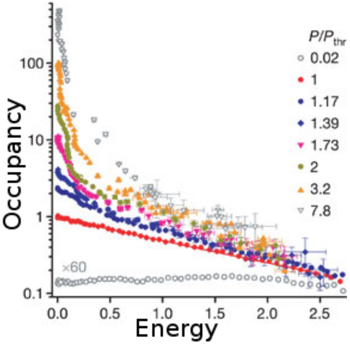
Polariton experiments: Momentum/Energy distribution

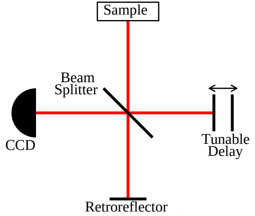


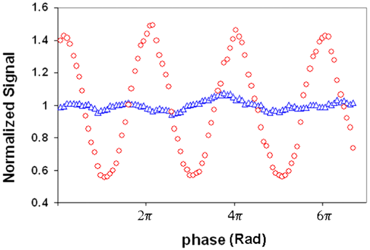
[Kasprzak, et al., Nature, 2006]



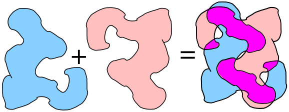
Polariton experiments: Coherence

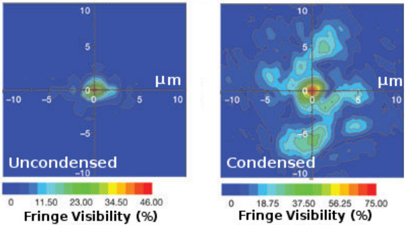
Basic idea:





Coherence map:

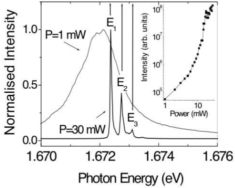


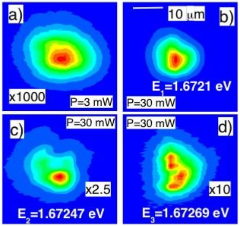


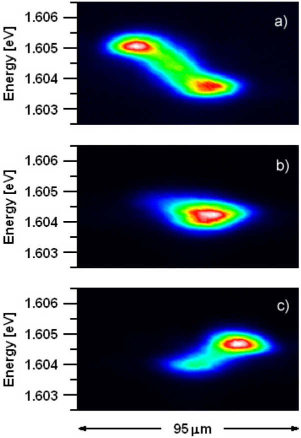
[Kasprzak, et al., Nature, 2006]

Other polariton condensation experiments

- Stress traps for polaritons
  [Balili *et al* Science 316 1007 (2007)]
- Temporal coherence and line narrowing
  [Love *et al* Phys. Rev. Lett. 101 067404 (2008)]

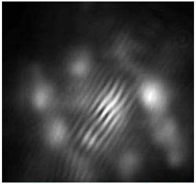




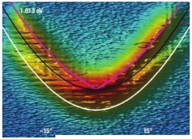


Other polariton condensation experiments

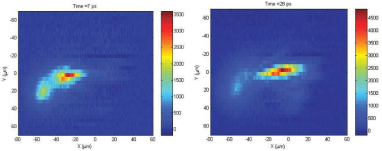
- Quantised vortices in disorder potential
  [Lagoudakis *et al* Nature Phys. 4, 706 (2008)]



- Changes to excitation spectrum
  [Utsunomiya *et al* Nature Phys. 4 700 (2008)]



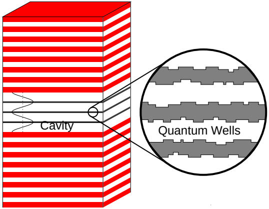
- Soliton propagation
  [Amo *et al* Nature 457 291 (2009)]
- Driven superfluidity
  [Amo *et al* Nature Phys. (2009)]



Overview

1. Introduction to microcavity polaritons
2. Microscopic Hamiltonian & equilibrium results
   - Disorder-localised exciton model
   - Equilibrium mean-field theory
3. Including pumping and decay and mean-field theory
   - Limits of mean-field theory
4. Fluctuations
   - Stability of normal state — lasing vs condensation
   - Condensed spectrum
5. Conclusions

Excitons in a disorderd Quantum well



Exciton states in disorder:

$$\left[-\frac{\nabla_{\mathbf{R}}^2}{2m_X} + V(\mathbf{R})\right]\Phi_\alpha(\mathbf{R}) = \varepsilon_\alpha \Phi_\alpha(\mathbf{R})$$

$V(\mathbf{R})$ smoothed by exciton Bohr radius

[Marchetti *et al.* PRL 96 066405 (2006); PRB 76 115326 (2007)]

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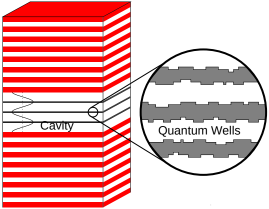

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Want: Energies ε_α Oscillator strengths: $g_{\alpha,\mathbf{p}} \propto \psi_{1s}(0)\Phi_{\alpha,\mathbf{p}}$

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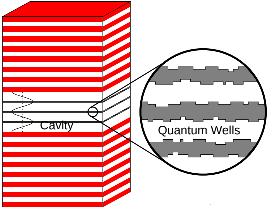

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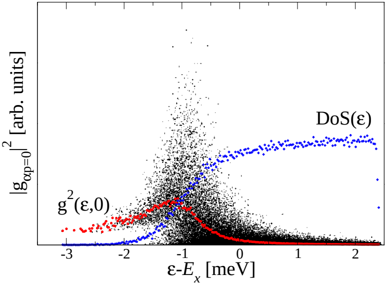

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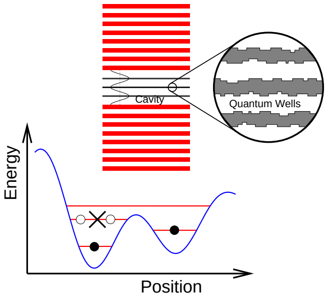
Polariton system model

Polariton model

- Disorder-localised excitons
- Treat disorder sites as two-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling g.



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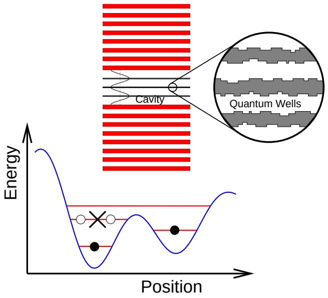
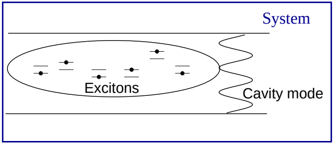


$$H_{\mathrm{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{1}{\sqrt{\mathrm{Area}}} g_{\alpha,\mathbf{k}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \mathrm{H.c.} \right]$$



Equilibrium: Mean-field theory

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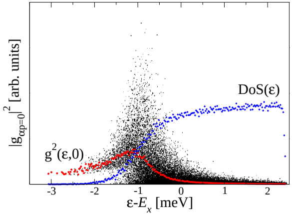

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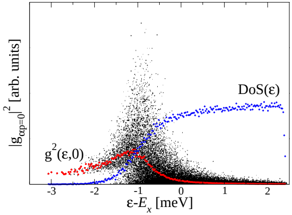
Mean-field theory:
Self-consistent polarisation and field

$$\left[i\partial_t + \omega_0 - \frac{\nabla^2}{2m} \right] \psi = \frac{1}{\sqrt{A}} \sum_\alpha g_\alpha P_\alpha$$



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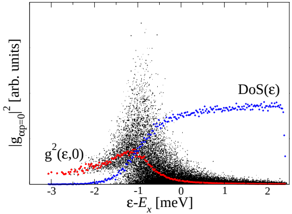
$$\left[-\mu + \omega_0 - \frac{\nabla^2}{2m} \right] \psi = \frac{1}{\sqrt{A}} \sum_{\alpha} g_{\alpha} \langle a^{\dagger}_{\alpha} b_{\alpha} \rangle$$



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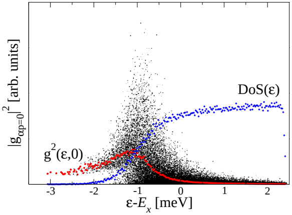
$$\left[-\mu + \omega_0 - \frac{\nabla^2}{2m} \right] \psi = \frac{1}{\sqrt{A}} \sum_\alpha g_\alpha \frac{g_\alpha \psi}{2E_\alpha} \tanh\left(\beta E_\alpha\right)$$

$$E^2_\alpha = \left(\frac{\epsilon_\alpha - \mu}{2} \right)^2 + g^2_\alpha \psi^2$$



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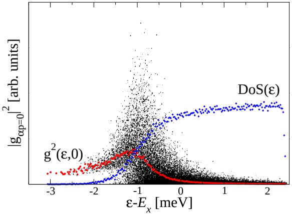

Density

$$\rho = |\psi|^2 + \frac{1}{A} \sum_{\alpha} \left[\frac{1}{2} - \frac{\epsilon_{\alpha} - \mu}{4E_{\alpha}} \tanh(\beta E_{\alpha}) \right]$$

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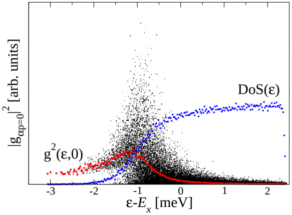

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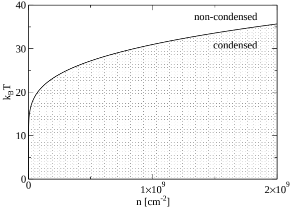

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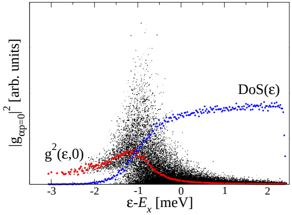

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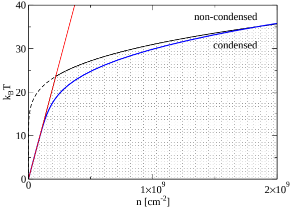
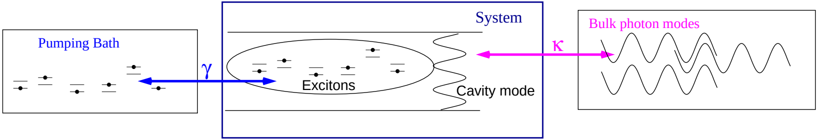


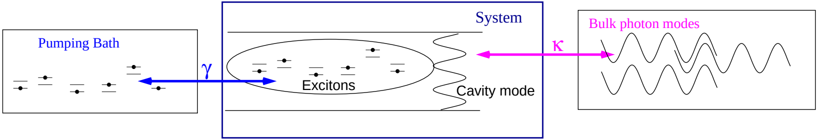
Overview

Non-equilibrium model: baths



$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

Non-equilibrium model: baths



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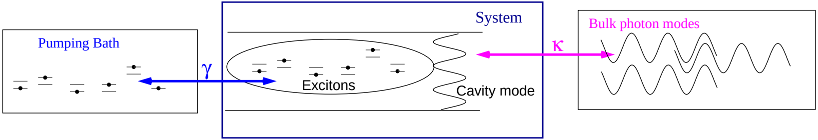

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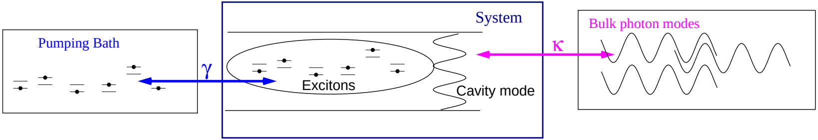

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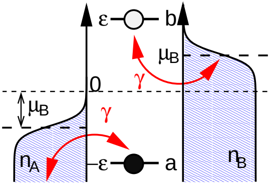
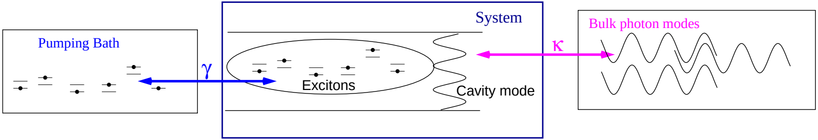
Ψ bath is empty. Pumping bath thermal, μ_B, T:



Non-equilibrium mean-field theory



- Look for mean-field solution, $\langle \psi(\mathbf{r}, t) \rangle = \psi_0 e^{-i\mu_S t}$.
- Must satisfy EOM: $\langle i\partial_t \psi \rangle = \langle [\psi, H] \rangle$

$$\mu_S \psi_0(t) = \omega_0 \psi_0(t) + \sum_i g_i \langle a_i^\dagger(t) b_i(t) \rangle + \sum_p \zeta_{p,0} \langle \Psi_p(t) \rangle$$

- Coupled to Markovian decay bath.

$$\sum_p \zeta_{p,0} \langle \Psi_p(t) \rangle = -i \int^t dt' \sum_p \zeta_{p,0}^2 e^{-i\omega_p^S (t-t')} \psi_0(t') = -i\kappa \psi_0(t)$$

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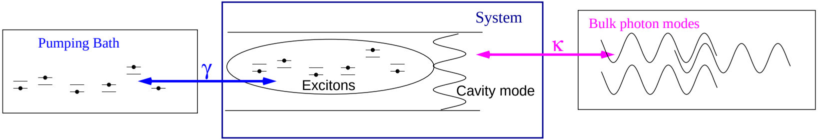

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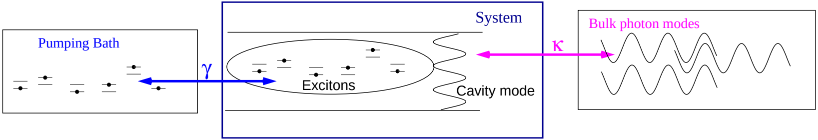

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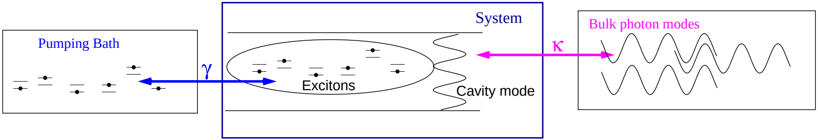

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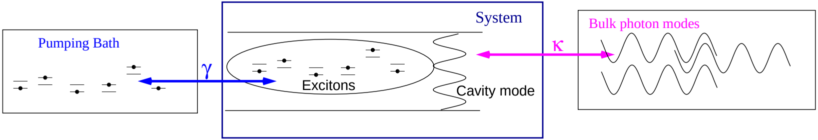

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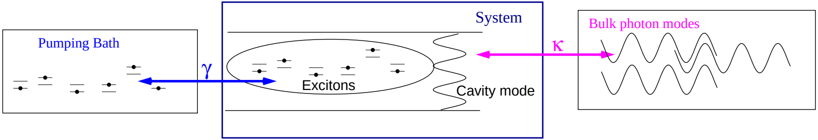

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Non-equilibrium mean-field theory

Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi_0 = \chi(\psi_0, \mu_s)\psi_0$$

Susceptibility:

$$\chi(\psi_0, \mu_s) = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

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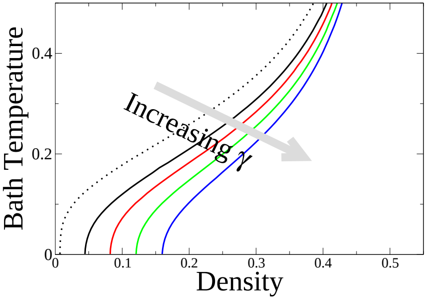

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$$\omega_0 - \mu_s = \frac{g^2}{2E} \tanh\left(\frac{\beta E}{2}\right)$$

Overview

Fluctuations: Photon Green's function

Keldysh approach:

$$D^{R,A} = \mp i\theta[\pm(t-t')] \left\langle \left[\psi, \psi^\dagger\right]_- \right\rangle$$

$$D^{K} = -i \left\langle \left[\psi, \psi^\dagger\right]_+ \right\rangle$$

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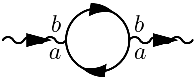

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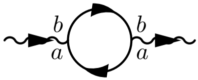
To find D, use $D^{-1} = D_0^{-1} - \Sigma_\kappa - \Sigma_{\psi^\dagger\psi}$, $\Sigma_{\psi^\dagger\psi} =$ 

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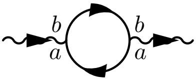
To find D, use $D^{-1} = D_0^{-1} - \Sigma_\kappa - \Sigma_{\psi^\dagger\psi}$, $\Sigma_{\psi^\dagger\psi} =$ 

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Fluctuations → Stability, Luminescence, Absorption

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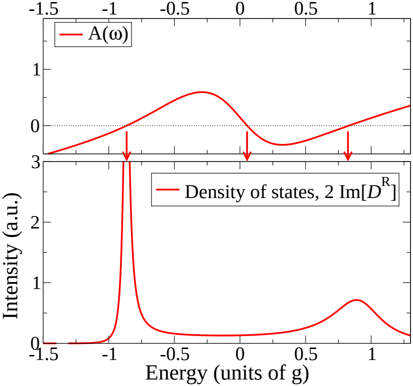

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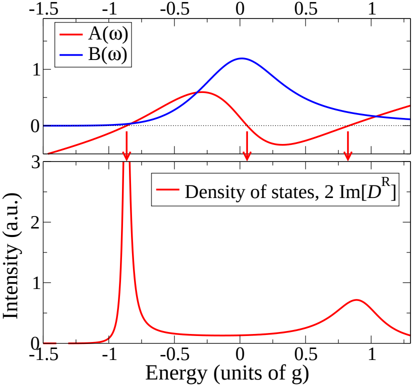

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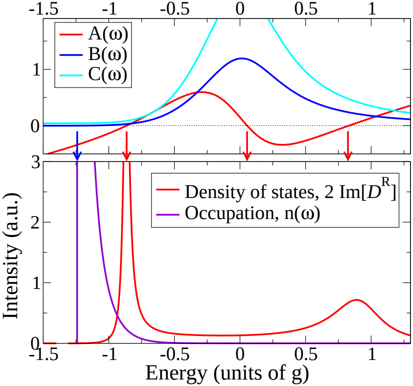

Fluctuations → Stability, Luminescence, Absorption

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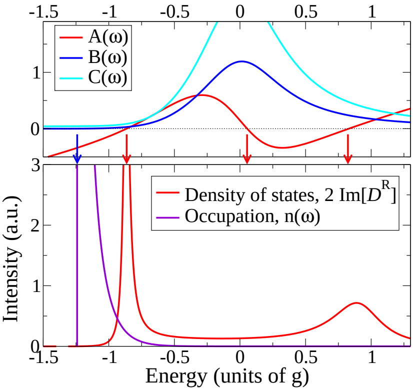

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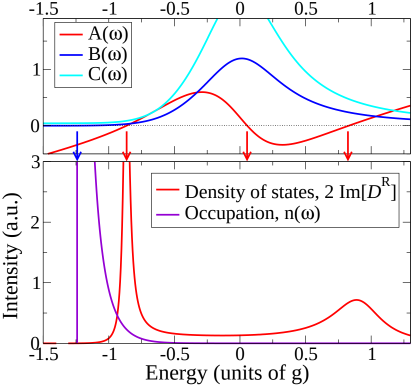
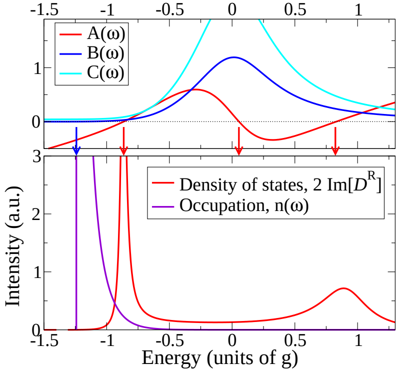
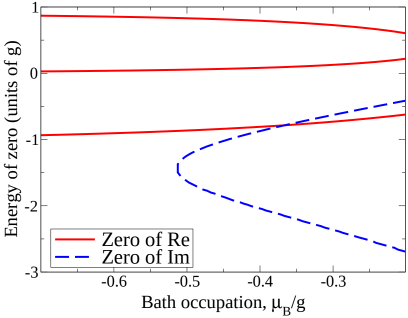
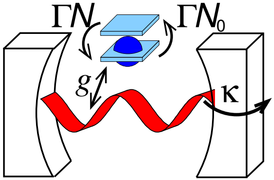


Linewidth, inverse Green's function and gap equation





$[D^R]^{-1}$ for a laser



Maxwell-Bloch equations:

$$\partial_t \psi = -i\omega_k \psi - \kappa\psi + gP$$
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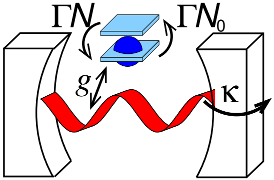

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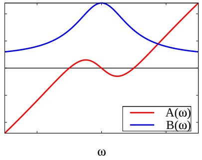

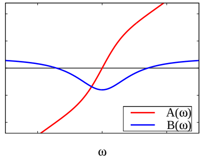

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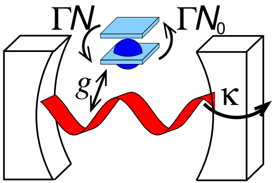

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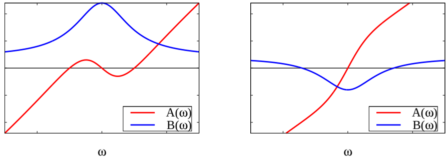
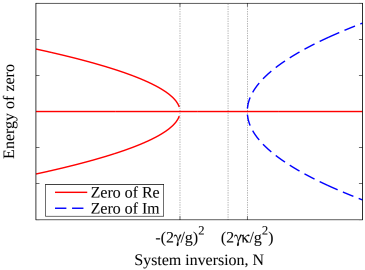
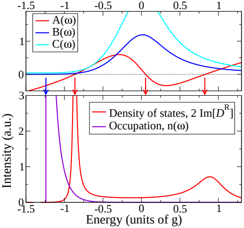
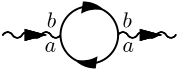




Gain/loss rate $B(\omega)$ for non-equilibrium polaritons



$$A(\omega) + iB(\omega) = \omega - \omega_k + i\kappa -$$ 

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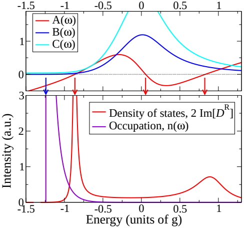
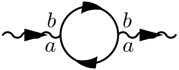


$$A(\omega) + iB(\omega) = \omega - \omega_k + i\kappa -$$ 

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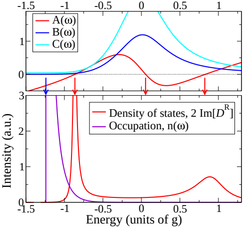
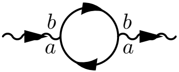


$$A(\omega) + iB(\omega) = \omega - \omega_k + i\kappa -$$ 

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For $T \gg g$, replace $n[F_B(\epsilon) - F_A(\epsilon - \omega)] \to -N_0$. **Maxwell-Bloch**

Overview

Condensed spectrum: Hugenholtz-Pines

Anomolous Green's function $\rightarrow 4 \times 4$ Photon Greens' function.

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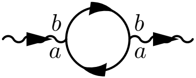
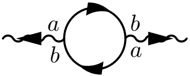
with $\Sigma_{\psi^\dagger\psi} =$ 

, $\Sigma_{\psi^\dagger\psi^\dagger} =$ 

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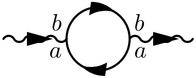
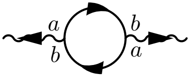
with $\Sigma_{\psi^\dagger\psi} =$ 

, $\Sigma_{\psi^\dagger\psi^\dagger} =$ 

Gapless spectrum as mean-field condition implies $\mathrm{Det}\left[D^R(0,0)\right]^{-1} = 0$.

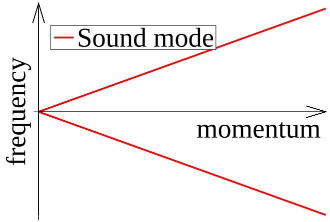
Fluctuations above transition

When condensed

$$\text{Det}\left[D^R(\omega, \mathbf{k})\right]^{-1} = \omega^2 - c^2\mathbf{k}^2$$

Poles:

$$\omega_k^* = c|\mathbf{k}|$$



Fluctuations above transition

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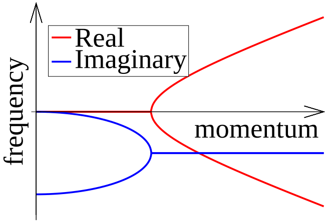

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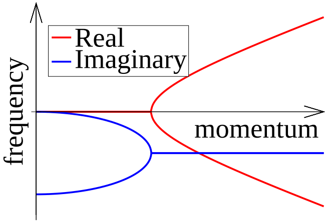

Correlations (in 2D): $\left\langle \psi^\dagger(\mathbf{r},t)\psi(0,r')\right\rangle \simeq |\psi_0|^2 \exp\left[-\mathcal{D}_{\phi\phi}(t,r,r')\right]$

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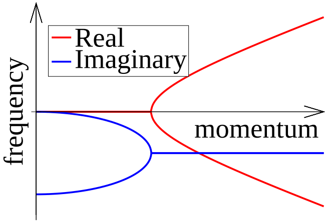

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$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, 0)\right\rangle \simeq |\psi_0|^2 \exp\left[-\eta \begin{cases} \ln(r/\xi) & r \to \infty, t \simeq 0 \\ \frac{1}{2}\ln(c^2 t / x\xi^2) & r \simeq, t \to \infty \end{cases}\right]$$

Finite size effects: Single mode vs many mode

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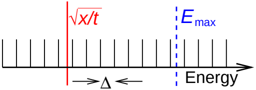

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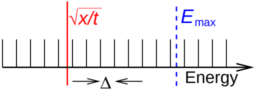

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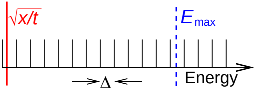
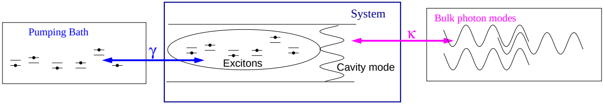
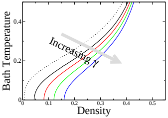


$\mathcal{D}_{\phi\phi} \sim \left(\frac{\pi C}{2x}\right)\left(\frac{t}{2x}\right)$

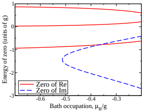
Conclusions

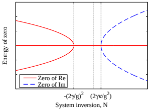
- Effects of pumping on mean-field theory



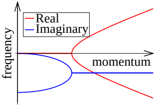


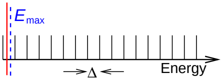
- Instability of normal state
  Translating: condensation ↔ lasing





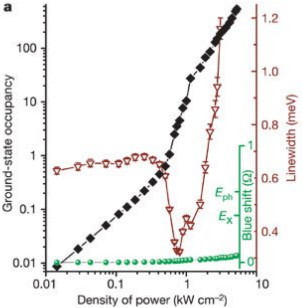
- Change to spectrum and correlations
  Phase modes and finite size



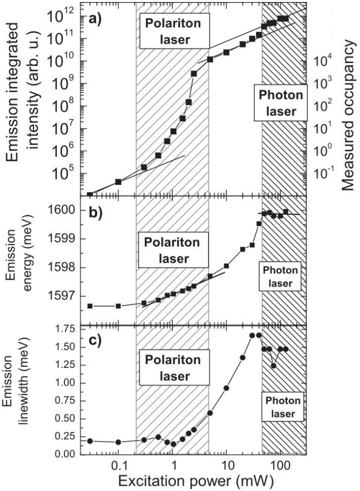


Extra slides

Polariton experiments: Strong coupling



[Kasprzak, et al., Nature, 2006]



[Bajoni *et al* PRL 2008]

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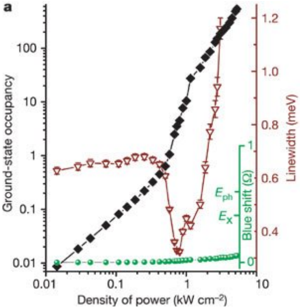
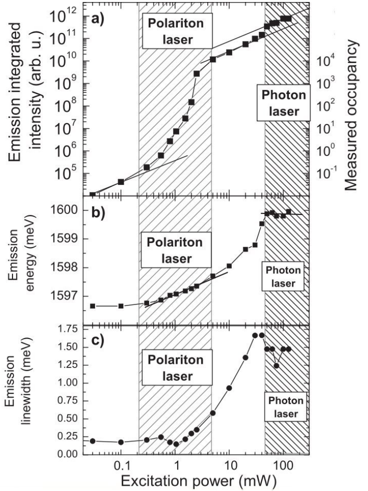


[Kasprzak, et al., Nature, 2006]

Strong coupling via:

- Small blueshift compared to Ω_R
- Polaritonic dispersion, $m > m_{\text{phot}}$
- Separate photon threshold



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Fluctuation corrections to phase boundary

Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$

In 2D system: modified critical condition:

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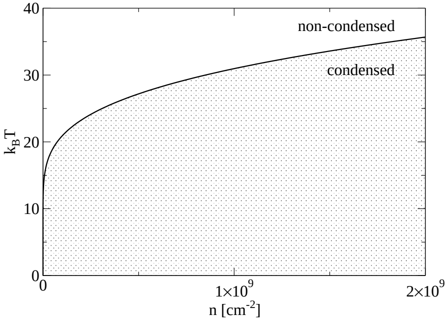

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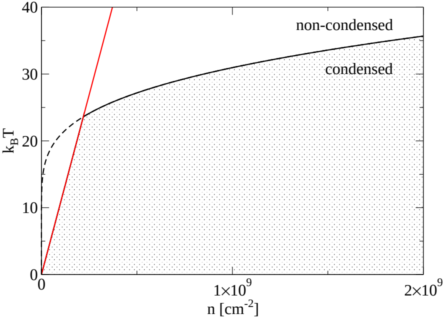

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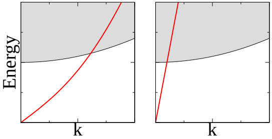

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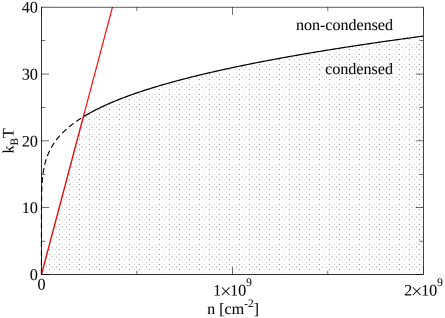

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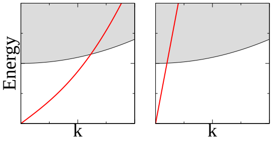

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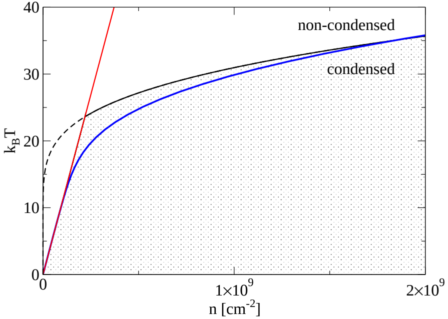

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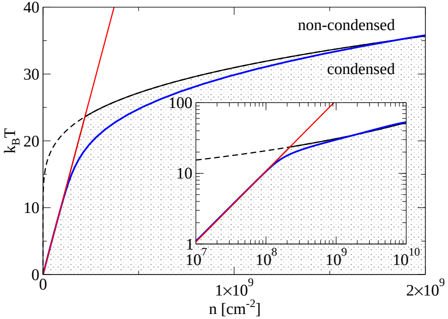
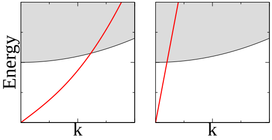
$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$

In 2D system: modified critical condition:

$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$





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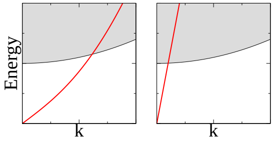

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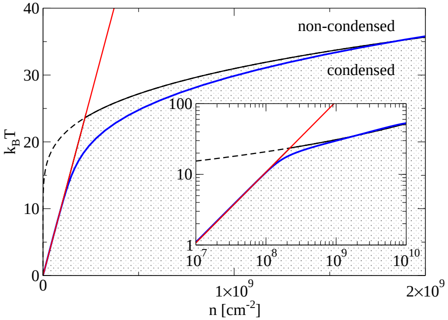
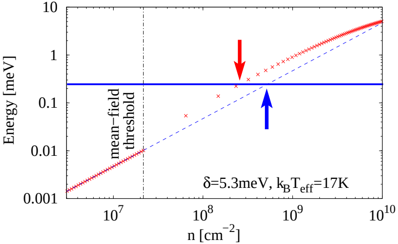


Second BCS crossover at $na_B^2 \simeq 1$

Blueshfit and experimental phase boundary

Blueshift:



Clean limit:

$$\delta E_{\mathrm{LP}} \simeq \mathcal{R}y_{\mathrm{X}} a_{\mathrm{X}}^2 n + \Omega_{\mathrm{R}} a_{\mathrm{X}}^2 n$$

Here: $\Omega_{\mathrm{R}} a_{\mathrm{X}}^2 \rightarrow \Omega_{\mathrm{R}} \xi^2$
[PRB 77 235313]

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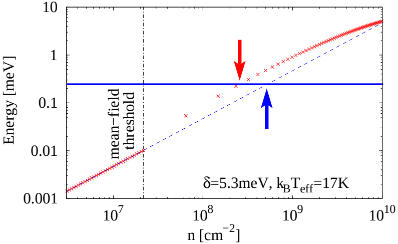

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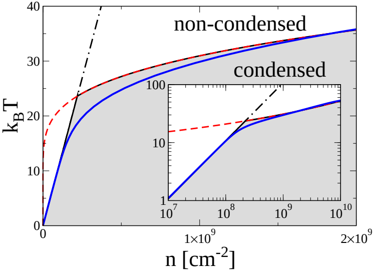

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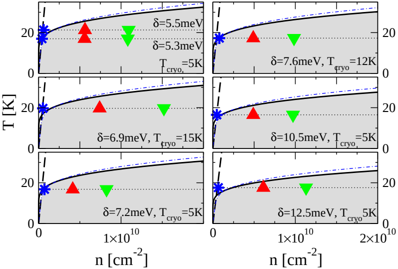
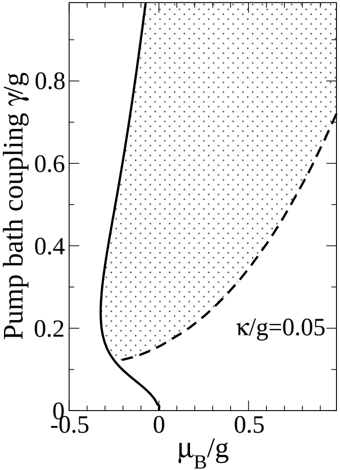


Zero temperature phase diagram

$$\tilde{\omega}_0 - i\kappa = g^2\gamma \sum_\alpha \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(\tilde{\epsilon}_\alpha + i\gamma)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}.$$



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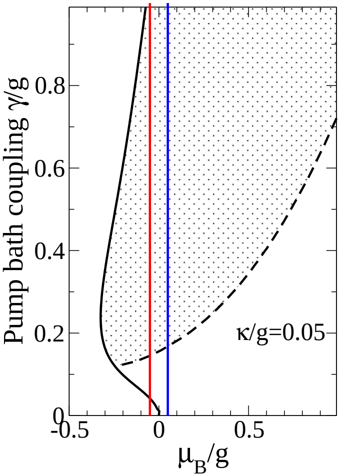

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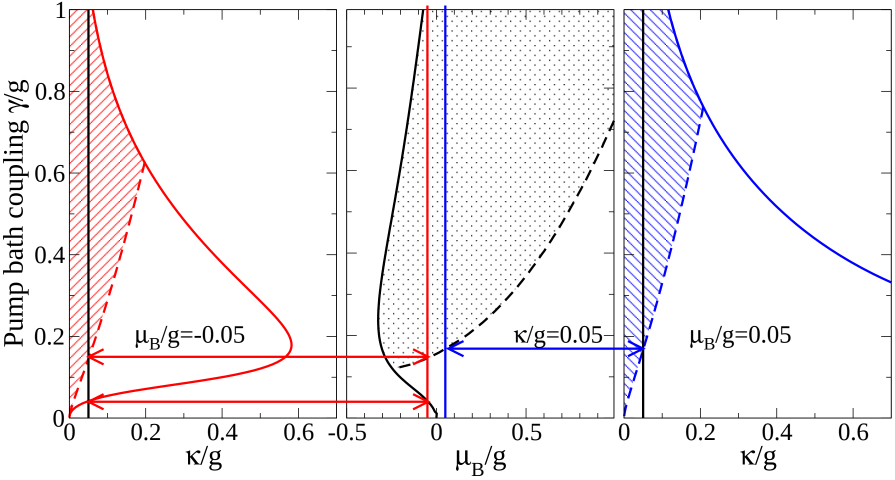

Occupation function at $T \gg \gamma$

Same approximation as used for $B(\omega)$ yields:

$$2n_\psi(\omega) + 1 = \frac{\kappa(2n_\Psi(\omega)+1) + \dfrac{ng^2\gamma}{(\omega-2\epsilon)^2+4\gamma^2}(1 - F_B(\epsilon)F_A(\epsilon-\omega))}{\kappa \qquad\qquad + \dfrac{ng^2\gamma}{(\omega-2\epsilon)^2+4\gamma^2}(F_B(\epsilon) - F_A(\epsilon-\omega))}.$$

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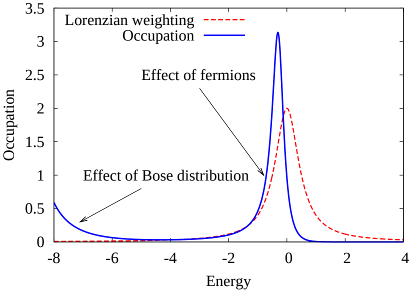

Relating finite-size spectrum to self phase modulation

Single mode spectrum:

$$\mathcal{D}_{\phi\phi}(t,r,r) \propto \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2(1-e^{i\omega t})}{|(\omega+ix)^2+x^2-(n\Delta)^2|^2}$$

Connection to Kubo Formula [Eastham and Whittaker, EPL 2009]

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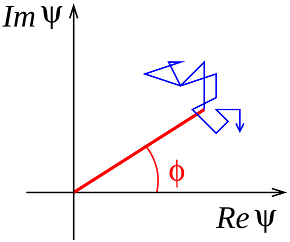

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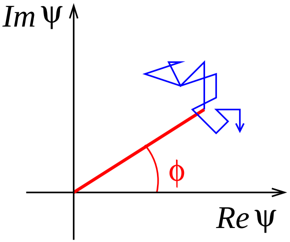

$$\begin{aligned} \partial_t \phi &= U \delta N \\ \partial_t \delta N &= -\Gamma \delta N + F(t), \quad \langle F(t) F(t') \rangle = C \delta(t - t') \end{aligned}$$

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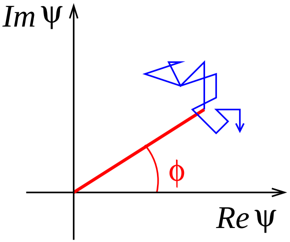

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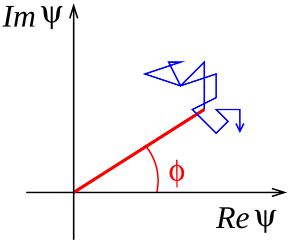

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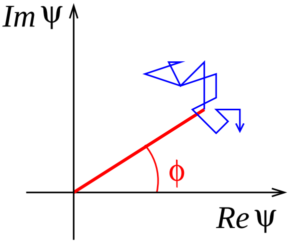

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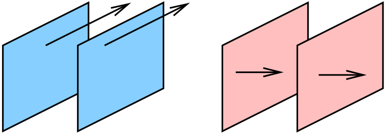

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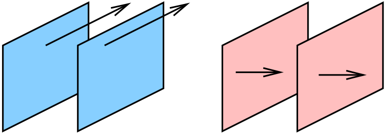

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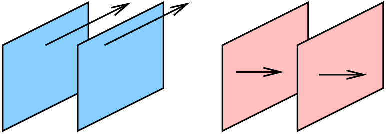

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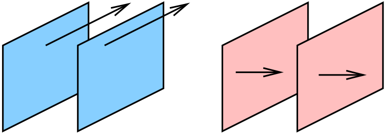

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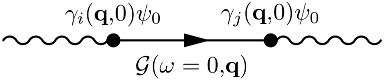
$\Delta\chi_{ij}(q) =$  $+ \ldots$

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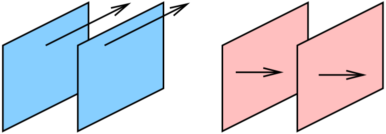

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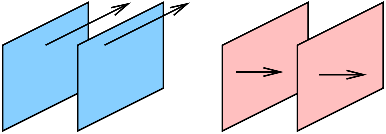

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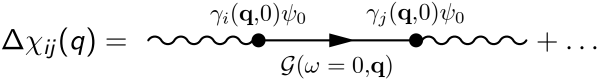
Static ρ_S survies



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