Polariton condensation — A Green's function approach

J. M. J. Keeling
N. G. Berloff, M. O. Borgh, P. R. Eastham, P. B. Littlewood, F. M. Marchetti, M. H. Szymanska.

L'Aquila IP 2010

Acknowledgements

People:

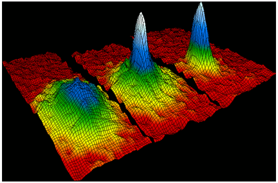
Funding:

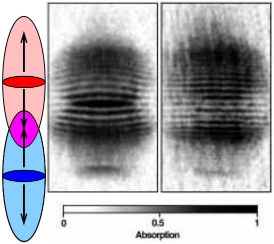
EPSRC Engineering and Physical Sciences Research Council

Pembroke College

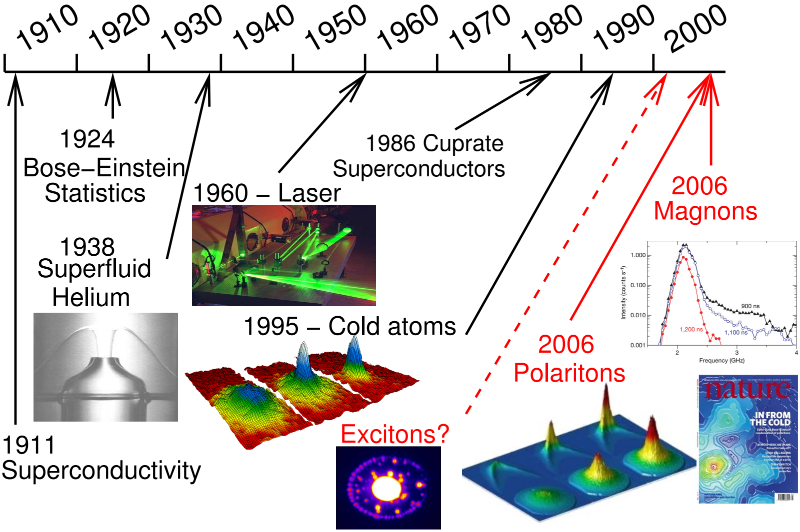
Bose-Einstein condensation

- Macroscopic occupation of ground state
  - Non-interacting bosons, $\mu \rightarrow 0$ at $T_c \propto n^{2/3}/m$.
- Macroscopic quantum coherence
  - No fragmentation
  - Macroscopic phase
- Superfluidity
  - Rigidity of wavefunction
  - New sound modes





Condensation timeline



Overview

Weakly Interacting Dilute Bose Gas model

Model for, e.g. dilute atomic systems

$$H - \mu N = \sum_k (\epsilon_k - \mu)\psi^\dagger_k \psi_k + \sum_{k,k',q} \frac{U}{2V} \psi^\dagger_{k+q} \psi^\dagger_{k'-q} \psi_{k'} \psi_k$$

- Mean-field theory: consider $|\Psi\rangle = (\psi^\dagger_0)^N |0\rangle$ *or* $|\Psi\rangle = e^{\lambda \psi^\dagger_0 - \lambda^2/2} |0\rangle$
- Demand state minimise $F = -k_B T \ln \mathrm{Tr}(e^{-\beta H})$
  Get: $N = \lambda^2 = \mu V / U$
- Describes macroscopically occupied state, neglects fluctuations
  - Want: spectrum, temperature dependence, modified ground state.

Weakly Interacting Dilute Bose Gas model

Model for, e.g. dilute atomic systems

$$H - \mu N = \sum_k (\epsilon_k - \mu)\psi_k^\dagger \psi_k + \sum_{k,k',q} \frac{U}{2V} \psi_{k+q}^\dagger \psi_{k'-q}^\dagger \psi_{k'} \psi_k$$

- Mean-field theory: consider $|\Psi\rangle = (\psi_0^\dagger)^N |0\rangle$ *or* $|\Psi\rangle = e^{\lambda\psi_0^\dagger - \lambda^2/2}|0\rangle$
- Demand state minimise $F = -k_B T \ln \mathrm{Tr}(e^{-\beta H})$
  Get: $N = \lambda^2 = \mu V / U$
- Describes macroscopically occupied state, neglects fluctuations
  - Want: spectrum, temperature dependence, modified ground state.

Weakly Interacting Dilute Bose Gas model

Model for, e.g. dilute atomic systems

$$H - \mu N = \sum_k (\epsilon_k - \mu)\psi_k^\dagger \psi_k + \sum_{k,k',q} \frac{U}{2V} \psi_{k+q}^\dagger \psi_{k'-q}^\dagger \psi_{k'} \psi_k$$

- Mean-field theory: consider $|\Psi\rangle = (\psi_0^\dagger)^N |0\rangle$ *or* $|\Psi\rangle = e^{\lambda\psi_0^\dagger - \lambda^2/2}|0\rangle$
- Demand state minimise $F = -k_B T \ln \mathrm{Tr}(e^{-\beta H})$
  Get: $N = \lambda^2 = \mu V / U$
- Describes macroscopically occupied state, neglects fluctuations
  - Want: spectrum, temperature dependence, modified ground state.
  - Consider $\psi_k \to \lambda\delta_k + \psi_k$

$$H_{\text{fluct}} = \sum_k (\epsilon_k - \mu)\psi_k^\dagger \psi_k + \frac{U}{2}\frac{\lambda^2}{V}\left(4\psi_k^\dagger \psi_k + \psi_k^\dagger \psi_{-k}^\dagger + \psi_k \psi_{-k}\right)$$

Weakly Interacting Dilute Bose Gas model

Model for, e.g. dilute atomic systems

$$H - \mu N = \sum_k (\epsilon_k - \mu)\psi^\dagger_k \psi_k + \sum_{k,k',q} \frac{U}{2V} \psi^\dagger_{k+q} \psi^\dagger_{k'-q} \psi_{k'} \psi_k$$

- Mean-field theory: consider $|\Psi\rangle = (\psi^\dagger_0)^N |0\rangle$ *or* $|\Psi\rangle = e^{\lambda \psi^\dagger_0 - \lambda^2/2}|0\rangle$
- Demand state minimise $F = -k_B T \ln \mathrm{Tr}(e^{-\beta H})$
  Get: $N = \lambda^2 = \mu V / U$
- Describes macroscopically occupied state, neglects fluctuations
  - Want: spectrum, temperature dependence, modified ground state.
  - Consider $\psi_k \to \lambda \delta_k + \psi_k$

$$H_{\text{fluct}} = \sum_k (\epsilon_k - \mu)\psi^\dagger_k \psi_k + \frac{U}{2} \frac{\lambda^2}{V} \left(4\psi^\dagger_k \psi_k + \psi^\dagger_k \psi^\dagger_{-k} + \psi_k \psi_{-k} \right),$$

WIDBG fluctuations

- Using $\lambda^2 = N = \mu V/U$,

$$H_{\text{fluct}} = \sum_k \frac{1}{2} \begin{pmatrix} \psi^\dagger_k & \psi_{-k} \end{pmatrix} \begin{pmatrix} \epsilon_k + \mu & \mu \\ \mu & \epsilon_k + \mu \end{pmatrix} \begin{pmatrix} \psi_k \\ \psi^\dagger_{-k} \end{pmatrix} - \frac{(\epsilon_k + \mu)}{2}.$$

- Diagonalise by $\phi_k = \cosh(\theta_k)\psi_k + \sinh(\theta_k)\psi^\dagger_k$
- Find $H_{\text{fluct}} = \sum_k \xi_k \phi^\dagger_k \phi_k + \frac{1}{2}(\xi_k - \epsilon_k - \mu)$

with:
$$\tanh(2\theta_k) = -\frac{\mu}{\epsilon_k + \mu}$$
$$\xi_k = \sqrt{\epsilon_k(\epsilon_k + 2\mu)}$$

- Features of fluctuation spectrum:
  - Spectrum is ξ_k, independent of temperature.

WIDBG fluctuations

- Using $\lambda^2 = N = \mu V/U$,

$$H_{\text{fluct}} = \sum_k \frac{1}{2} \begin{pmatrix} \psi_k^\dagger & \psi_{-k} \end{pmatrix} \begin{pmatrix} \epsilon_k + \mu & \mu \\ \mu & \epsilon_k + \mu \end{pmatrix} \begin{pmatrix} \psi_k \\ \psi_{-k}^\dagger \end{pmatrix} - \frac{(\epsilon_k + \mu)}{2}.$$

- Diagonalise by $\phi_k = \cosh(\theta_k)\psi_k + \sinh(\theta_k)\psi_k^\dagger$
- Find $H_{\text{fluct}} = \sum_k \xi_k \phi_k^\dagger \phi_k + \frac{1}{2}(\xi_k - \epsilon_k - \mu)$

with: $$\tanh(2\theta_k) = -\frac{\mu}{\epsilon_k + \mu}$$ $$\xi_k = \sqrt{\epsilon_k(\epsilon_k + 2\mu)}$$

- Features of fluctuation spectrum:
  - Spectrum is ξ_k, independent of temperature.

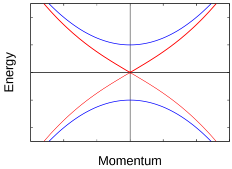
WIDBG fluctuations

- Using $\lambda^2 = N = \mu V/U$,

$$H_{\text{fluct}} = \sum_k \frac{1}{2} \begin{pmatrix} \psi_k^\dagger & \psi_{-k} \end{pmatrix} \begin{pmatrix} \epsilon_k + \mu & \mu \\ \mu & \epsilon_k + \mu \end{pmatrix} \begin{pmatrix} \psi_k \\ \psi_{-k}^\dagger \end{pmatrix} - \frac{(\epsilon_k + \mu)}{2}.$$

- Diagonalise by $\phi_k = \cosh(\theta_k)\psi_k + \sinh(\theta_k)\psi_k^\dagger$
- Find $H_{\text{fluct}} = \sum_k \xi_k \phi_k^\dagger \phi_k + \frac{1}{2}(\xi_k - \epsilon_k - \mu)$

with:
$$\tanh(2\theta_k) = -\frac{\mu}{\epsilon_k + \mu}$$
$$\xi_k = \sqrt{\epsilon_k(\epsilon_k + 2\mu)}$$



- Features of fluctuation spectrum:
  - Spectrum is ξ_k, independent of temperature.

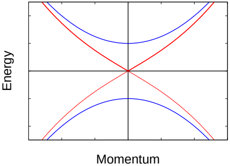
WIDBG fluctuations

- Using $\lambda^2 = N = \mu V/U$,

$$H_{\text{fluct}} = \sum_k \frac{1}{2} \begin{pmatrix} \psi_k^\dagger & \psi_{-k} \end{pmatrix} \begin{pmatrix} \epsilon_k + \mu & \mu \\ \mu & \epsilon_k + \mu \end{pmatrix} \begin{pmatrix} \psi_k \\ \psi_{-k}^\dagger \end{pmatrix} - \frac{(\epsilon_k + \mu)}{2}.$$

- Diagonalise by $\phi_k = \cosh(\theta_k)\psi_k + \sinh(\theta_k)\psi_k^\dagger$
- Find $H_{\text{fluct}} = \sum_k \xi_k \phi_k^\dagger \phi_k + \frac{1}{2}(\xi_k - \epsilon_k - \mu)$

with: $$\tanh(2\theta_k) = -\frac{\mu}{\epsilon_k + \mu}$$
$$\xi_k = \sqrt{\epsilon_k(\epsilon_k + 2\mu)}$$



- Features of fluctuation spectrum:
  - Spectrum is ξ_k, independent of temperature.
  - Ground state is not $\psi_k|\Omega\rangle = 0$ but $\phi_k|\Omega\rangle = 0$, find:

$$|\Omega\rangle = \prod \exp\left(-\tanh(\theta_k)\psi_k^\dagger\psi_{-k}^\dagger\right)|0\rangle.$$

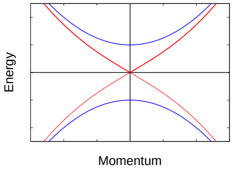
WIDBG fluctuations

- Using $\lambda^2 = N = \mu V/U$,

$$H_{\text{fluct}} = \sum_k \frac{1}{2} \begin{pmatrix} \psi_k^\dagger & \psi_{-k} \end{pmatrix} \begin{pmatrix} \epsilon_k + \mu & \mu \\ \mu & \epsilon_k + \mu \end{pmatrix} \begin{pmatrix} \psi_k \\ \psi_{-k}^\dagger \end{pmatrix} - \frac{(\epsilon_k + \mu)}{2}.$$

- Diagonalise by $\phi_k = \cosh(\theta_k)\psi_k + \sinh(\theta_k)\psi_k^\dagger$
- Find $H_{\text{fluct}} = \sum_k \xi_k \phi_k^\dagger \phi_k + \frac{1}{2}(\xi_k - \epsilon_k - \mu)$

with:
$$\tanh(2\theta_k) = -\frac{\mu}{\epsilon_k + \mu}$$
$$\xi_k = \sqrt{\epsilon_k(\epsilon_k + 2\mu)}$$

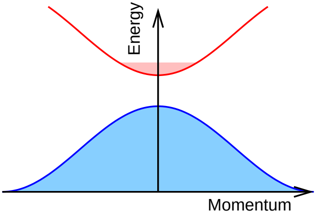


- Features of fluctuation spectrum:
  - Spectrum is ξ_k, independent of temperature.
  - Ground state is not $\psi_k|\Omega\rangle = 0$ but $\phi_k|\Omega\rangle = 0$, find:
    $$|\Omega\rangle = \prod \exp\left(-\tanh(\theta_k)\psi_k^\dagger\psi_{-k}^\dagger\right)|0\rangle.$$

Overview

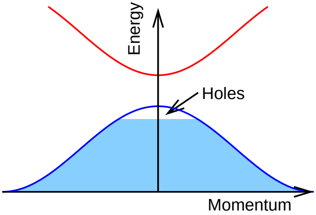
Excitons in semiconductors

Light quasiparticles with approximate Bose statistics.



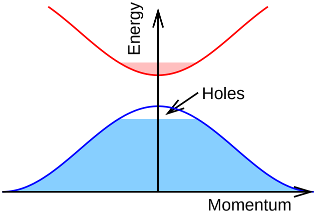
Excitons in semiconductors

Light quasiparticles with approximate Bose statistics.



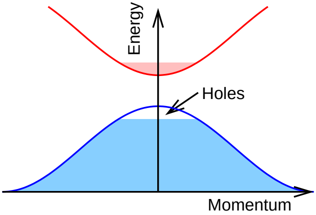
Excitons in semiconductors

Light quasiparticles with approximate Bose statistics.



Excitons in semiconductors

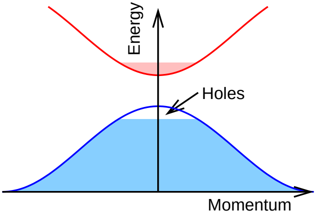
Light quasiparticles with approximate Bose statistics.



$$H = \sum_k \left[\epsilon_c(k) a^\dagger_{ck} a_{ck} + \epsilon_v(k) a^\dagger_{vk} a_{vk}\right]$$
$$+\frac{1}{2}\sum_q \left[V^{ee}_q \rho^e_q \rho^e_{-q} + V^{hh}_q \rho^h_q \rho^h_{-q} - 2V^{eh}_q \rho^e_q \rho^h_{-q}\right]$$

Excitons in semiconductors

Light quasiparticles with approximate Bose statistics.



$$H = \sum_k \left[\epsilon_c(k) a^\dagger_{ck} a_{ck} + \epsilon_v(k) a^\dagger_{vk} a_{vk}\right]$$
$$+\frac{1}{2}\sum_q \left[V^{ee}_q \rho^e_q \rho^e_{-q} + V^{hh}_q \rho^h_q \rho^h_{-q} - 2V^{eh}_q \rho^e_q \rho^h_{-q}\right]$$

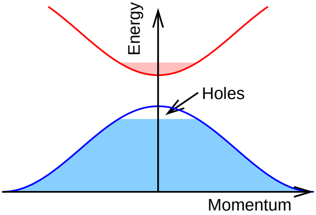
With parabolic dispersion:

$$\epsilon_c(k) = \frac{\hbar^2 k^2}{2m_c}, \quad \epsilon_v(k) = -E_g - \frac{\hbar^2 k^2}{2|m_v|}$$

Coulomb interaction: $V(q) = \dfrac{e^2 4\pi}{\epsilon q^2}$.

Excitons in semiconductors

Light quasiparticles with approximate Bose statistics.



$$H = \sum_k \left[\epsilon_c(k) a^\dagger_{ck} a_{ck} + \epsilon_v(k) a^\dagger_{vk} a_{vk}\right]$$
$$+\frac{1}{2}\sum_q \left[V^{ee}_q \rho^e_q \rho^e_{-q} + V^{hh}_q \rho^h_q \rho^h_{-q} - 2V^{eh}_q \rho^e_q \rho^h_{-q}\right]$$

With parabolic dispersion:

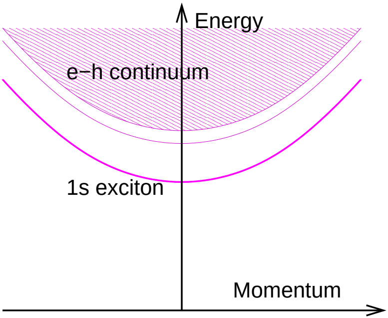
$$\epsilon_c(k) = \frac{\hbar^2 k^2}{2m_c}, \quad \epsilon_v(k) = -E_g - \frac{\hbar^2 k^2}{2|m_v|}$$

Exciton state:
$$\frac{1}{\sqrt{V}}\sum_k \phi_{1s}(k) a^\dagger_{ck} a_{vk} |\Omega\rangle$$

Coulomb interaction: $V(q) = \dfrac{e^2 4\pi}{\epsilon q^2}$.

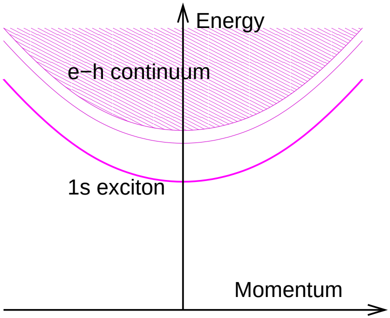
Quantum well excitons

- In GaAs $\mu = 0.1m^*, \epsilon_r = 13$, so
  - $\mathcal{R}y = 5\text{meV}$ (13.6eV for H)
  - $a_B = 7\text{nm}$ (0.05nm for H)
  - $M_{\text{total}} \sim m^*$



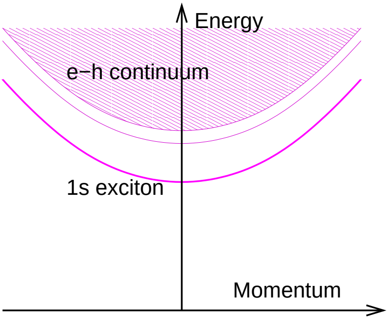
Quantum well excitons

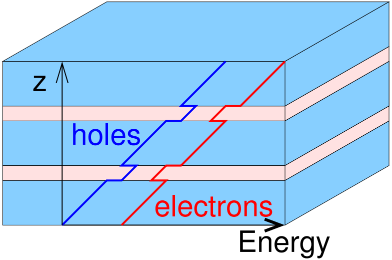
- In GaAs $\mu = 0.1m^*, \epsilon_r = 13$, so
  - $\mathcal{R}y = 5\text{meV}$ (13.6eV for H)
  - $a_B = 7\text{nm}$ (0.05nm for H)
  - $M_{\text{total}} \sim m^*$
- Finite lifetimes: enhance by
  - Optically forbidden transition
  - Spatial separation



Quantum well excitons

- In GaAs $\mu = 0.1m^*, \epsilon_r = 13$, so
  - $\mathcal{R}y = 5\text{meV}$ (13.6eV for H)
  - $a_B = 7\text{nm}$ (0.05nm for H)
  - $M_{\text{total}} \sim m^*$
- Finite lifetimes: enhance by
  - Optically forbidden transition
  - Spatial separation





- Strongly interacting dipoles — avoids bi-exciton formation.

Exciton condensation

- Single exciton state
$$\frac{1}{\sqrt{V}}\sum_k \phi_{1s}(k) a^\dagger_{ck} a_{vk} |\Omega\rangle$$
- **E**xciton **c**ondensate state:

$$|\Psi_{\text{Ec}}\rangle = \mathcal{N} \exp\left(\sum_k \lambda_k a^\dagger_{ck} a_{vk}\right)|\Omega\rangle$$

  - Low density: $v_k \simeq \lambda_k = \sqrt{N/V}\phi_{1s}(k)$
  - High density: saturation, $v_k < 1$

Exciton condensation

- Single exciton state
$$\frac{1}{\sqrt{V}} \sum_k \phi_{1s}(k) a^\dagger_{ck} a_{vk} |\Omega\rangle$$
- **E**xciton **c**ondensate state:

$$\begin{aligned} |\Psi_{\mathrm{Ec}}\rangle &= \mathcal{N} \exp\left(\sum_k \lambda_k a^\dagger_{ck} a_{vk} \right) |\Omega\rangle \\ &= \prod_k \left(u_k + v_k a^\dagger_{ck} a_{vk} \right) |\Omega\rangle \end{aligned}$$

- Low density: $v_k \simeq \lambda_k = \sqrt{N/V} \phi_{1s}(k)$
- High density: saturation, $v_k < 1$

Exciton condensation

- Single exciton state
  $$\frac{1}{\sqrt{V}}\sum_k \phi_{1s}(k)a^\dagger_{ck}a_{vk}|\Omega\rangle$$
- **E**xciton **c**ondensate state:

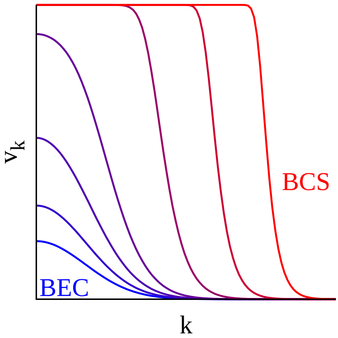
$$\begin{aligned}|\Psi_{\mathrm{Ec}}\rangle &= \mathcal{N}\exp\left(\sum_k \lambda_k a^\dagger_{ck}a_{vk}\right)|\Omega\rangle \\ &= \prod_k \left(u_k + v_k a^\dagger_{ck}a_{vk}\right)|\Omega\rangle\end{aligned}$$

  - Low density: $v_k \simeq \lambda_k = \sqrt{N/V}\phi_{1s}(k)$
  - High density: saturation, $v_k < 1$

Exciton condensation

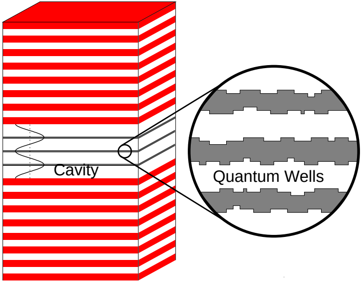
- Single exciton state
  $$\frac{1}{\sqrt{V}}\sum_k \phi_{1s}(k) a^\dagger_{ck} a_{vk} |\Omega\rangle$$
- **E**xciton **c**ondensate state:

$$|\Psi_{\mathrm{Ec}}\rangle = \mathcal{N} \exp\left(\sum_k \lambda_k a^\dagger_{ck} a_{vk}\right)|\Omega\rangle$$
$$= \prod_k \left(u_k + v_k a^\dagger_{ck} a_{vk}\right)|\Omega\rangle$$

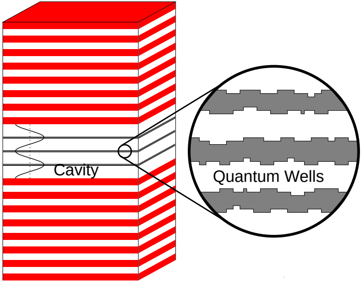


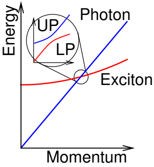
  - Low density: $v_k \simeq \lambda_k = \sqrt{N/V}\phi_{1s}(k)$
  - High density: saturation, $v_k < 1$

Microcavity Polaritons



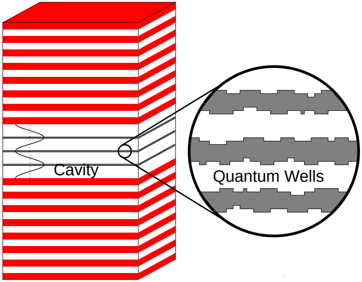
Microcavity Polaritons

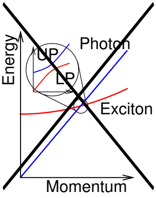




[Pekar, JETP(1958)]
[Hopfield, Phys. Rev.(1958)]

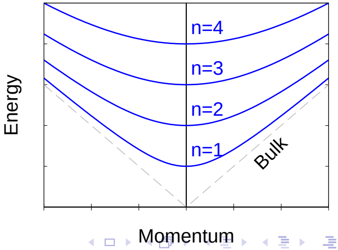
Microcavity Polaritons



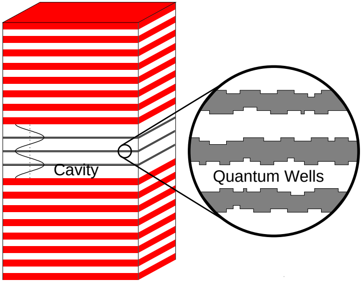


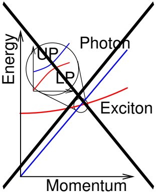
[Pekar, JETP(1958)]
[Hopfield, Phys. Rev.(1958)]
Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$



Microcavity Polaritons



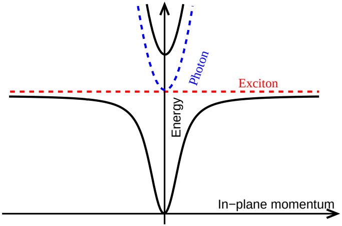


[Pekar, JETP(1958)]
[Hopfield, Phys. Rev.(1958)]
Cavity photons:

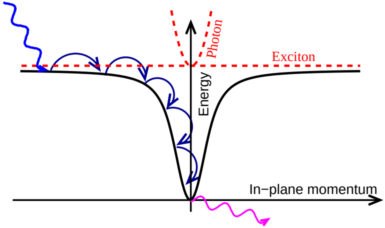
$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$

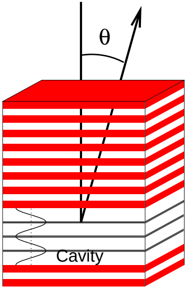
Bosonic excitons D_k

$$H = \begin{pmatrix} \psi_k^\dagger & D_k^\dagger \end{pmatrix} \begin{pmatrix} \omega_k & \Omega_R/2 \\ \Omega_R/2 & \epsilon_k \end{pmatrix} \begin{pmatrix} \psi_k \\ D_k \end{pmatrix}$$

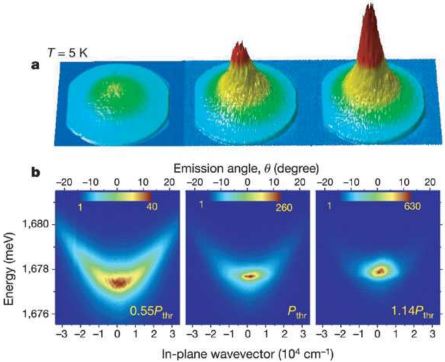


Non-equilibrium: flux and baths

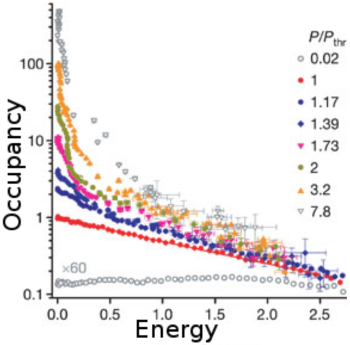




Polariton experiments: Momentum/Energy distribution

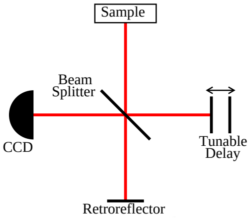


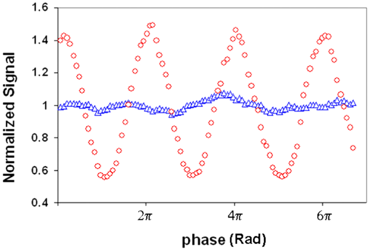
[Kasprzak, et al., Nature, 2006]



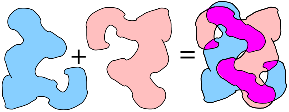
Polariton experiments: Coherence

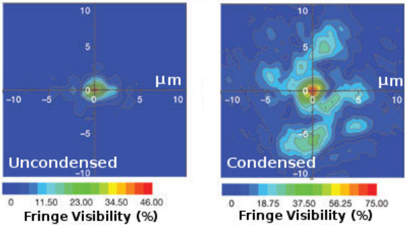
Basic idea:





Coherence map:

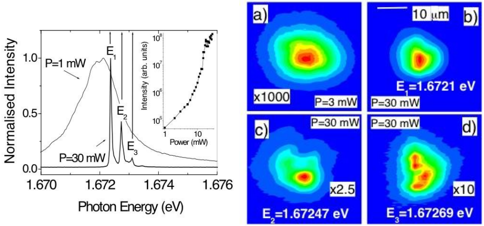


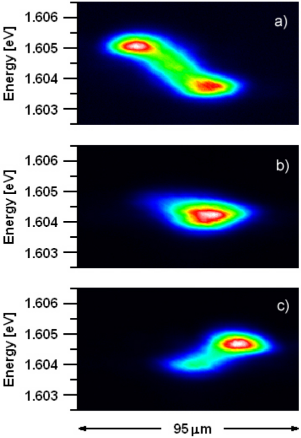


[Kasprzak, et al., Nature, 2006]

Other polariton condensation experiments

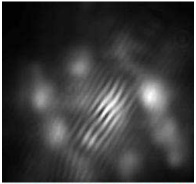
- Stress traps for polaritons
  [Balili *et al* Science 316 1007 (2007)]
- Temporal coherence and line narrowing
  [Love *et al* Phys. Rev. Lett. 101 067404 (2008)]



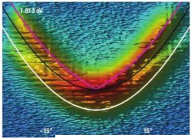


Other polariton condensation experiments

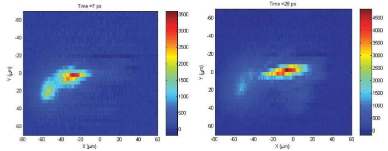
- Quantised vortices in disorder potential
  [Lagoudakis *et al* Nature Phys. 4, 706 (2008)]



- Changes to excitation spectrum
  [Utsunomiya *et al* Nature Phys. 4 700 (2008)]



- Soliton propagation
  [Amo *et al* Nature 457 291 (2009)]
- Driven superfluidity
  [Amo *et al* Nature Phys. (2009)]



Distinguishing features of polaritons

- Composite electron–hole–photon particle:
  - Similar energy scales
- Naturally two-dimensional, but finite
  - Berezhinskii-Kosterlitz-Thouless vs BEC
- Short polariton lifetime
  - Non-equilibrium distributions
  - Relationship to Laser.

Distinguishing features of polaritons

- Composite electron–hole–photon particle:
  - Similar energy scales
- Naturally two-dimensional, but finite
  - Berezhinskii-Kosterlitz-Thouless vs BEC
- Short polariton lifetime
  - Non-equilibrium distributions
  - Relationship to Laser.

Distinguishing features of polaritons

- Composite electron–hole–photon particle:
  - Similar energy scales
- Naturally two-dimensional, but finite
  - Berezhinskii-Kosterlitz-Thouless vs BEC
- Short polariton lifetime
  - Non-equilibrium distributions
  - Relationship to Laser.

Distinguishing features of polaritons

- Composite electron–hole–photon particle:
  - Similar energy scales
- Naturally two-dimensional, but finite
  - Berezhinskii-Kosterlitz-Thouless vs BEC
- Short polariton lifetime
  - Non-equilibrium distributions
  - Relationship to Laser.

	Lifetime	Thermalisation
Atoms	10s	10ms
Excitons[a]	50ns	0.2ns
Polaritons	5ps	0.5ps
Magnons[b]	1μs(??)	100ns(?)

[a] Coupled quantum wells. [Hammack et al PRB 76 193308 (2007)]
[b] Yttrium Iron Garnett. [Demokritov et al, Nature 443 430 (2007)]

Distinguishing features of polaritons

- Composite electron–hole–photon particle:
  - Similar energy scales
- Naturally two-dimensional, but finite
  - Berezhinskii-Kosterlitz-Thouless vs BEC
- Short polariton lifetime
  - Non-equilibrium distributions
  - Relationship to Laser.

	Lifetime	Thermalisation	Linewidth	Temperature	
Atoms	10s	10ms	2.5×10^{-13}meV	10^{-8}K	10^{-9}meV
Excitons[a]	50ns	0.2ns	5×10^{-5}meV	1K	0.1meV
Polaritons	5ps	0.5ps	0.5meV	20K	2meV
Magnons[b]	1μs(??)	100ns(?)	2.5×10^{-6}meV	300K	30meV

[a] Coupled quantum wells. [Hammack et al PRB 76 193308 (2007)]
[b] Yttrium Iron Garnett. [Demokritov et al, Nature 443 430 (2007)]

Overview

Models of polariton condensates

- Full model: $H = H_{\rm eh} + H_{\rm coul} + H_{\rm photon} + H_{\rm dipole} + H_{\rm disorder}$
  - Include photonic & excitonic disorder
  - Model allows unnecessary high energy degrees of freedom
- **Approximate model 1** — WIDBG

$$H = \sum_{\mathbf{k}} \begin{pmatrix} \psi_k^\dagger & D_k^\dagger \end{pmatrix} \begin{pmatrix} \omega_k & \frac{1}{2}\Omega_R \\ \frac{1}{2}\Omega_R & \epsilon_k \end{pmatrix} \begin{pmatrix} \psi_k \\ D_k \end{pmatrix} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \frac{U}{2} D^\dagger_{\mathbf{k}+\mathbf{q}} D^\dagger_{\mathbf{k}'-\mathbf{q}} D_{\mathbf{k}'} D_{\mathbf{k}}$$

$$- \frac{\Omega_R}{2\rho_{sat}} \left(D^\dagger_{\mathbf{k}'-\mathbf{q}} D^\dagger_{\mathbf{k}+\mathbf{q}} D_{\mathbf{k}} \psi_{\mathbf{k}'} + \psi^\dagger_{\mathbf{k}'-\mathbf{q}} D^\dagger_{\mathbf{k}+\mathbf{q}} D_{\mathbf{k}} D_{\mathbf{k}'} \right)$$

- **Approximate model 2** — Disorder localised excitons

Models of polariton condensates

- Full model: $H = H_{\mathrm{eh}} + H_{\mathrm{coul}} + H_{\mathrm{photon}} + H_{\mathrm{dipole}} + H_{\mathrm{disorder}}$
  - Include photonic & excitonic disorder
  - Model allows unnecessary high energy degrees of freedom
- **Approximate model 1 — WIDBG**

$$H = \sum_{\mathbf{k}} \begin{pmatrix} \psi_k^\dagger & D_k^\dagger \end{pmatrix} \begin{pmatrix} \omega_k & \frac{1}{2}\Omega_R \\ \frac{1}{2}\Omega_R & \epsilon_k \end{pmatrix} \begin{pmatrix} \psi_k \\ D_k \end{pmatrix} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \frac{U}{2} D^\dagger_{\mathbf{k}+\mathbf{q}} D^\dagger_{\mathbf{k}'-\mathbf{q}} D_{\mathbf{k}'} D_{\mathbf{k}}$$
$$- \frac{\Omega_R}{2\rho_{sat}} \left(D^\dagger_{\mathbf{k}'-\mathbf{q}} D^\dagger_{\mathbf{k}+\mathbf{q}} D_{\mathbf{k}} \psi_{\mathbf{k}'} + \psi^\dagger_{\mathbf{k}'-\mathbf{q}} D^\dagger_{\mathbf{k}+\mathbf{q}} D_{\mathbf{k}} D_{\mathbf{k}'} \right)$$

- **Approximate model 2 — Disorder localised excitons**

Models of polariton condensates

- Full model: $H = H_{\mathrm{eh}} + H_{\mathrm{coul}} + H_{\mathrm{photon}} + H_{\mathrm{dipole}} + H_{\mathrm{disorder}}$
  - Include photonic & excitonic disorder
  - Model allows unnecessary high energy degrees of freedom
- **Approximate model 1** — WIDBG

$$H = \sum_{\mathbf{k}} \begin{pmatrix} \psi_k^\dagger & D_k^\dagger \end{pmatrix} \begin{pmatrix} \omega_k & \frac{1}{2}\Omega_R \\ \frac{1}{2}\Omega_R & \epsilon_k \end{pmatrix} \begin{pmatrix} \psi_k \\ D_k \end{pmatrix} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \frac{U}{2} D^\dagger_{\mathbf{k}+\mathbf{q}} D^\dagger_{\mathbf{k}'-\mathbf{q}} D_{\mathbf{k}'} D_{\mathbf{k}}$$

$$- \frac{\Omega_R}{2\rho_{sat}} \left(D^\dagger_{\mathbf{k}'-\mathbf{q}} D^\dagger_{\mathbf{k}+\mathbf{q}} D_{\mathbf{k}} \psi_{\mathbf{k}'} + \psi^\dagger_{\mathbf{k}'-\mathbf{q}} D^\dagger_{\mathbf{k}+\mathbf{q}} D_{\mathbf{k}} D_{\mathbf{k}'} \right)$$

  - Includes exciton interaction and saturation
  - At low densities, project to lower polariton

$$H_{\mathrm{LP}} = \sum_{\mathbf{k}} E^{\mathrm{LP}}_{\mathbf{k}} L^\dagger_{\mathbf{k}} L_{\mathbf{k}} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V^{\mathrm{eff}}_{\mathbf{k},\mathbf{k}',\mathbf{q}} L^\dagger_{\mathbf{k}+\mathbf{q}} L^\dagger_{\mathbf{k}'-\mathbf{q}} L_{\mathbf{k}'} L_{\mathbf{k}}$$

- **Approximate model 2** — Disorder localised excitons

Models of polariton condensates

- Full model: $H = H_{\rm eh} + H_{\rm coul} + H_{\rm photon} + H_{\rm dipole} + H_{\rm disorder}$
  - Include photonic & excitonic disorder
  - Model allows unnecessary high energy degrees of freedom
- **Approximate model 1** — WIDBG

$$H = \sum_{\mathbf{k}} \begin{pmatrix} \psi_k^\dagger & D_k^\dagger \end{pmatrix} \begin{pmatrix} \omega_k & \frac{1}{2}\Omega_R \\ \frac{1}{2}\Omega_R & \epsilon_k \end{pmatrix} \begin{pmatrix} \psi_k \\ D_k \end{pmatrix} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \frac{U}{2} D^\dagger_{\mathbf{k}+\mathbf{q}} D^\dagger_{\mathbf{k}'-\mathbf{q}} D_{\mathbf{k}'} D_{\mathbf{k}} - \frac{\Omega_R}{2\rho_{sat}} \left(D^\dagger_{\mathbf{k}'-\mathbf{q}} D^\dagger_{\mathbf{k}+\mathbf{q}} D_{\mathbf{k}} \psi_{\mathbf{k}'} + \psi^\dagger_{\mathbf{k}'-\mathbf{q}} D^\dagger_{\mathbf{k}+\mathbf{q}} D_{\mathbf{k}} D_{\mathbf{k}'} \right)$$

  - Includes exciton interaction and saturation
  - At low densities, project to lower polariton

$$H_{\rm LP} = \sum_{\mathbf{k}} E^{\rm LP}_{\mathbf{k}} L^\dagger_{\mathbf{k}} L_{\mathbf{k}} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V^{\rm eff}_{\mathbf{k},\mathbf{k}',\mathbf{q}} L^\dagger_{\mathbf{k}+\mathbf{q}} L^\dagger_{\mathbf{k}'-\mathbf{q}} L_{\mathbf{k}'} L_{\mathbf{k}}$$

- **Approximate model 2** — Disorder localised excitons

Models of polariton condensates

- Full model: $H = H_{\rm eh} + H_{\rm coul} + H_{\rm photon} + H_{\rm dipole} + H_{\rm disorder}$
  - Include photonic & excitonic disorder
  - Model allows unnecessary high energy degrees of freedom
- **Approximate model 1** — WIDBG

$$H = \sum_{\mathbf{k}} \begin{pmatrix} \psi_k^\dagger & D_k^\dagger \end{pmatrix} \begin{pmatrix} \omega_k & \frac{1}{2}\Omega_R \\ \frac{1}{2}\Omega_R & \epsilon_k \end{pmatrix} \begin{pmatrix} \psi_k \\ D_k \end{pmatrix} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \frac{U}{2} D^\dagger_{\mathbf{k}+\mathbf{q}} D^\dagger_{\mathbf{k}'-\mathbf{q}} D_{\mathbf{k}'} D_{\mathbf{k}}$$
$$- \frac{\Omega_R}{2\rho_{sat}} \left(D^\dagger_{\mathbf{k}'-\mathbf{q}} D^\dagger_{\mathbf{k}+\mathbf{q}} D_{\mathbf{k}} \psi_{\mathbf{k}'} + \psi^\dagger_{\mathbf{k}'-\mathbf{q}} D^\dagger_{\mathbf{k}+\mathbf{q}} D_{\mathbf{k}} D_{\mathbf{k}'} \right)$$

  - Includes exciton interaction and saturation
  - At low densities, project to lower polariton

$$H_{\rm LP} = \sum_{\mathbf{k}} E^{\rm LP}_{\mathbf{k}} L^\dagger_{\mathbf{k}} L_{\mathbf{k}} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V^{\rm eff}_{\mathbf{k},\mathbf{k}',\mathbf{q}} L^\dagger_{\mathbf{k}+\mathbf{q}} L^\dagger_{\mathbf{k}'-\mathbf{q}} L_{\mathbf{k}'} L_{\mathbf{k}}$$

- **Approximate model 2** — Disorder localised excitons

Models of polariton condensates

- Full model: $H = H_{\text{eh}} + H_{\text{coul}} + H_{\text{photon}} + H_{\text{dipole}} + H_{\text{disorder}}$
  - Include photonic & excitonic disorder
  - Model allows unnecessary high energy degrees of freedom
- **Approximate model 1** — WIDBG

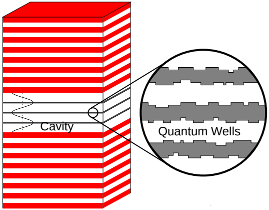
$$H = \sum_{\mathbf{k}} \begin{pmatrix} \psi_k^\dagger & D_k^\dagger \end{pmatrix} \begin{pmatrix} \omega_k & \frac{1}{2}\Omega_R \\ \frac{1}{2}\Omega_R & \epsilon_k \end{pmatrix} \begin{pmatrix} \psi_k \\ D_k \end{pmatrix} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \frac{U}{2} D^\dagger_{\mathbf{k}+\mathbf{q}} D^\dagger_{\mathbf{k}'-\mathbf{q}} D_{\mathbf{k}'} D_{\mathbf{k}}$$
$$- \frac{\Omega_R}{2\rho_{sat}} \left(D^\dagger_{\mathbf{k}'-\mathbf{q}} D^\dagger_{\mathbf{k}+\mathbf{q}} D_{\mathbf{k}} \psi_{\mathbf{k}'} + \psi^\dagger_{\mathbf{k}'-\mathbf{q}} D^\dagger_{\mathbf{k}+\mathbf{q}} D_{\mathbf{k}} D_{\mathbf{k}'} \right)$$

  - Includes exciton interaction and saturation
  - At low densities, project to lower polariton

$$H_{\text{LP}} = \sum_{\mathbf{k}} E^{\text{LP}}_{\mathbf{k}} L^\dagger_{\mathbf{k}} L_{\mathbf{k}} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V^{\text{eff}}_{\mathbf{k},\mathbf{k}',\mathbf{q}} L^\dagger_{\mathbf{k}+\mathbf{q}} L^\dagger_{\mathbf{k}'-\mathbf{q}} L_{\mathbf{k}'} L_{\mathbf{k}}$$

- **Approximate model 2** — Disorder localised excitons

Excitons in a disordered quantum well



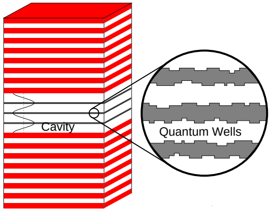
Exciton states in disorder:

$$\left[-\frac{\nabla^2_{\mathbf{R}}}{2m_X} + V(\mathbf{R})\right]\Phi_\alpha(\mathbf{R}) = 2\epsilon_\alpha\Phi_\alpha(\mathbf{R})$$

$V(\mathbf{R})$ smoothed by exciton Bohr radius

[PRL 96 066405 (2006); PRB 76 115326 (2007)]

Excitons in a disordered quantum well



Exciton states in disorder:

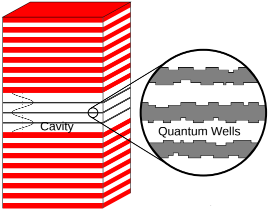
$$\left[-\frac{\nabla_{\mathbf{R}}^2}{2m_X} + V(\mathbf{R})\right]\Phi_\alpha(\mathbf{R}) = 2\epsilon_\alpha\Phi_\alpha(\mathbf{R})$$

$V(\mathbf{R})$ smoothed by exciton Bohr radius

Want: Energies ϵ_α Oscillator strengths: $g_{\alpha,\mathbf{p}} \propto \psi_{1s}(0)\Phi_{\alpha,\mathbf{p}}$

[PRL 96 066405 (2006); PRB 76 115326 (2007)]

Excitons in a disordered quantum well

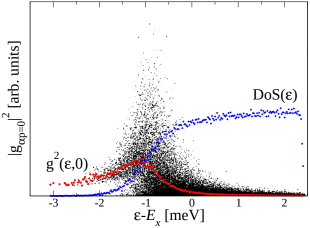


Exciton states in disorder:

$$\left[-\frac{\nabla_{\mathbf{R}}^2}{2m_X} + V(\mathbf{R})\right] \Phi_\alpha(\mathbf{R}) = 2\epsilon_\alpha \Phi_\alpha(\mathbf{R})$$

$V(\mathbf{R})$ smoothed by exciton Bohr radius

Want: Energies ϵ_α Oscillator strengths: $g_{\alpha,\mathbf{p}} \propto \psi_{1s}(0)\Phi_{\alpha,\mathbf{p}}$

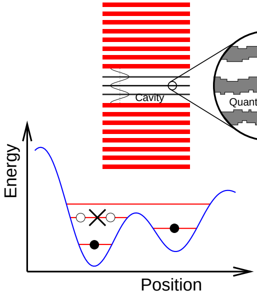


[PRL 96 066405 (2006); PRB 76 115326 (2007)]

Polariton system model

Polariton model

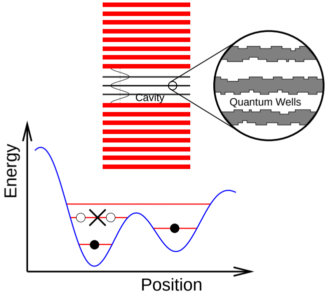
- Disorder-localised excitons
- Treat disorder sites as two-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling g.



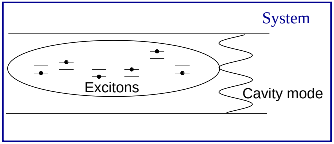
Polariton system model

Polariton model

- Disorder-localised excitons
- Treat disorder sites as two-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling g.



$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[2\epsilon_{\alpha} S^{z}_{\alpha} + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} S^{+}_{\alpha} + \text{H.c.} \right]$$



Equilibrium: Mean-field theory

Fermionic representation: $S^z \to \frac{1}{2}\left(b^\dagger b - a^\dagger a\right), \quad S^+ \to b^\dagger a.$

$$H_{\mathrm{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{\mathrm{A}}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \mathrm{H.c.} \right]$$

Equilibrium: Mean-field theory

Fermionic representation: $S^z \rightarrow \frac{1}{2}\left(b^\dagger b - a^\dagger a\right), \quad S^+ \rightarrow b^\dagger a.$

$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{\mathrm{A}}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \mathrm{H.c.} \right]$$

Mean-field theory:
Minimise F for coherent state

$$(\omega_0 - \mu)\lambda = \frac{1}{\sqrt{A}} \sum_{\alpha} g_\alpha \langle a^\dagger_\alpha b_\alpha \rangle$$

Equilibrium: Mean-field theory

Fermionic representation: $S^z \rightarrow \frac{1}{2}\left(b^\dagger b - a^\dagger a\right), \quad S^+ \rightarrow b^\dagger a.$

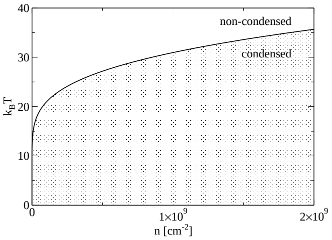
$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_\alpha \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha\right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{\mathrm{A}}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \mathrm{H.c.}\right]$$

Mean-field theory:
Minimise F for coherent state

$$(\omega_0 - \mu)\lambda = \frac{1}{\sqrt{A}} \sum_\alpha g_\alpha \langle a^\dagger_\alpha b_\alpha \rangle$$

Density

$$\rho = |\lambda|^2 + \frac{1}{2A} \sum_\alpha \langle b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \rangle$$



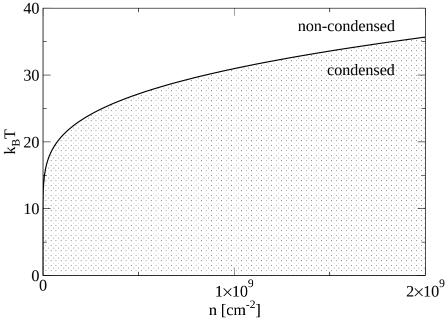
Fluctuation corrections to phase boundary

Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$

In 2D system: modified critical condition:

$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



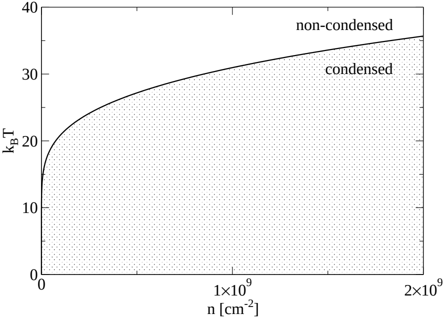
Fluctuation corrections to phase boundary

Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$

In 2D system: modified critical condition:

$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



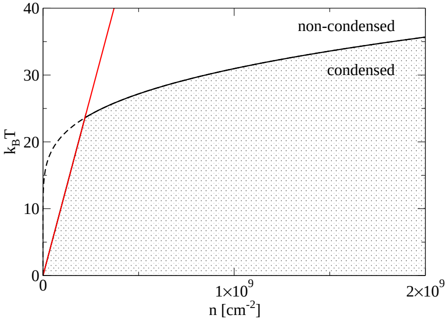
Fluctuation corrections to phase boundary

Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$

In 2D system: modified critical condition:

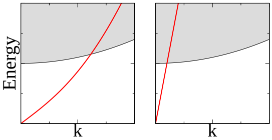
$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



Fluctuation corrections to phase boundary

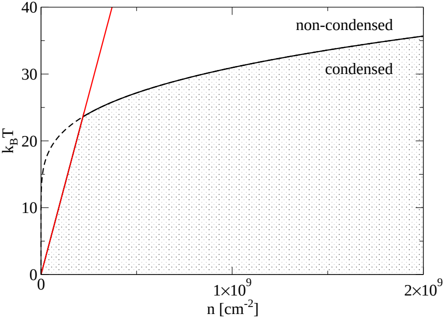
Fluctuation corrections to density

$$\rho \rightarrow \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$



In 2D system: modified critical condition:

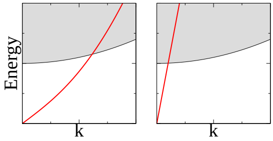
$$\rho_s = \rho - \rho_{\mathrm{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



Fluctuation corrections to phase boundary

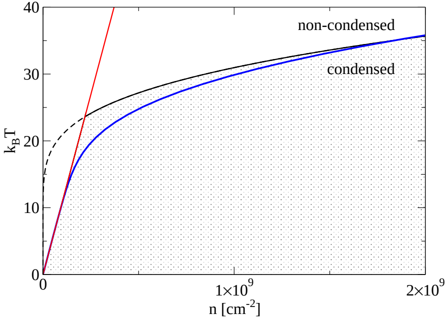
Fluctuation corrections to density

$$\rho \rightarrow \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$



In 2D system: modified critical condition:

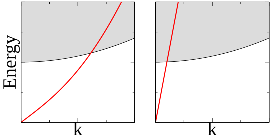
$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



Fluctuation corrections to phase boundary

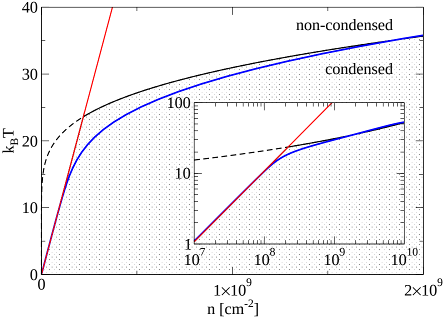
Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$



In 2D system: modified critical condition:

$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



Equilibrium: Recovering WIDBG at low density

Localised exciton model recovers WIDBG at low density.

$$H = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[2\epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{\mathsf{A}}} \psi_{\mathbf{k}} S^+_\alpha + \text{H.c.} \right]$$

Writing:

$S^z_\alpha = D^\dagger_\alpha D_\alpha - \frac{1}{2}, \quad S^+_\alpha = D^\dagger_\alpha \sqrt{1 - D^\dagger_\alpha D_\alpha}$

Equilibrium: Recovering WIDBG at low density

Localised exciton model recovers WIDBG at low density.

$$H = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[2\epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{\mathsf{A}}} \psi_{\mathbf{k}} S^+_\alpha + \text{H.c.} \right]$$

Writing:

$S^z_\alpha = D^\dagger_\alpha D_\alpha - \frac{1}{2}, \quad S^+_\alpha = D^\dagger_\alpha \sqrt{1 - D^\dagger_\alpha D_\alpha}$

Get:

$$H = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} 2\epsilon_\alpha D^\dagger_\alpha D_\alpha$$

$$+ \sum_{\alpha,\mathbf{k}} \left[\frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} D^\dagger_\alpha \left(1 - \frac{1}{2} D^\dagger_\alpha D_\alpha \right) + \text{H.c.} \right].$$

Equilibrium: Recovering WIDBG at low density

Localised exciton model recovers WIDBG at low density.

$$H = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[2\epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{\mathsf{A}}} \psi_{\mathbf{k}} S^+_\alpha + \text{H.c.} \right]$$

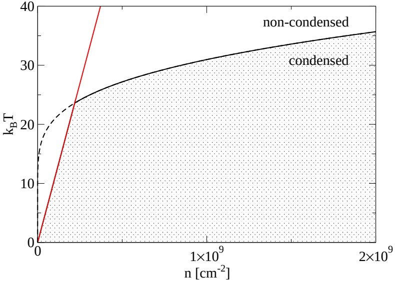
Writing:

$S^z_\alpha = D^\dagger_\alpha D_\alpha - \frac{1}{2}, \quad S^+_\alpha = D^\dagger_\alpha \sqrt{1 - D^\dagger_\alpha D_\alpha}$

Get:

$$H = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} 2\epsilon_\alpha D^\dagger_\alpha D_\alpha$$

$$+ \sum_{\alpha,\mathbf{k}} \left[\frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} D^\dagger_\alpha \left(1 - \frac{1}{2} D^\dagger_\alpha D_\alpha \right) + \text{H.c.} \right].$$



Conclusions: Theoretical aims of lectures

- Bose condensation via mean-field/fluctuations
- Localised exciton model of polaritons
- Green's function approach to condensation
  - Mean-field theory as basis
  - Equilibrium — thermal Green's function and spectrum.
- Consequences of non-equilibrium BEC
  - Decoherence; lasing
  - Strong coupling laser without inversion.