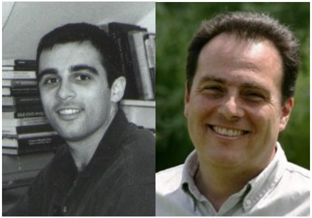
Collective dynamics of Bose–Einstein condensates in optical cavities

J. Keeling, M. J. Bhaseen, B. D. Simons

CEWQO St Andrews, June 2010

Acknowledgements

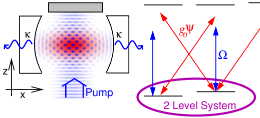
People:



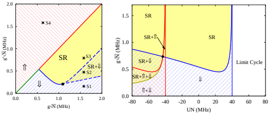
Funding:

EPSRC
Engineering and Physical Sciences Research Council

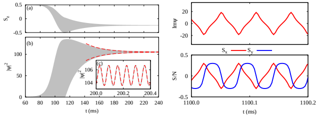
- Experimental realisation of superradiance transition



- "Feedback" induces extra phases

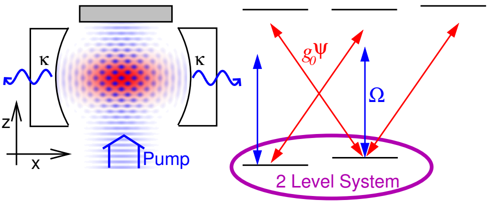


- Slowly decaying oscillations despite fast cavity loss — persistent oscillations possible.



Extended Dicke model

[Baumann *et al.* Nature 2010]



2 Level system, $|\Downarrow\rangle, |\Uparrow\rangle$:

$\Downarrow$: $\quad |k_x, k_z\rangle = |0, 0\rangle$,

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$\omega_0 = 2\omega_{\text{recoil}}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z$$

N atoms: $|\mathbf{S}| = N/2$

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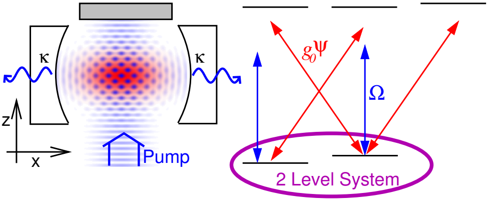

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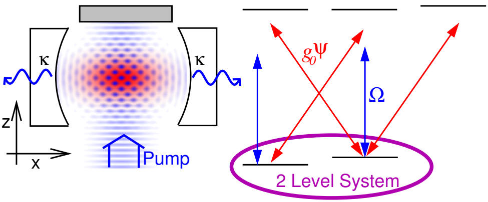

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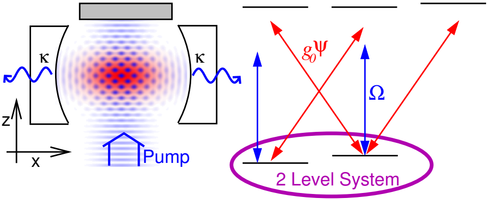

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Add decay:

$$\dot{S}^- = -i(\omega_0 + U\psi^\dagger\psi)S^- + 2i(g\psi + g'\psi^\dagger)S^z$$

$$\dot{S}^z = -ig(\psi S^+ - \psi^\dagger S^-) + ig'(\psi S^- - \psi^\dagger S^+)$$

$$\dot{\psi} = -[\kappa + i(\omega + US^z)]\psi - igS^- - ig'S^+$$

Fixed points at $U = 0$

Fixed points $\dot{\mathbf{S}}, \dot{\psi} = 0$.

- $\Uparrow, \Downarrow$: $S^z = \pm N/2, \psi = 0$ always
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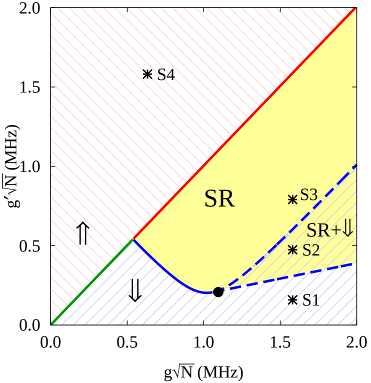

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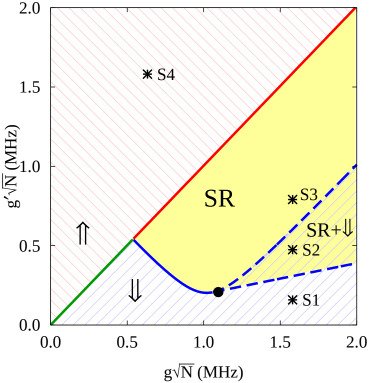

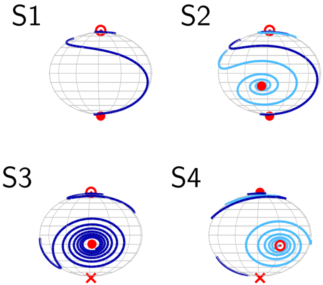

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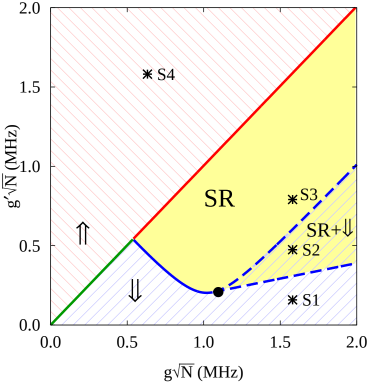
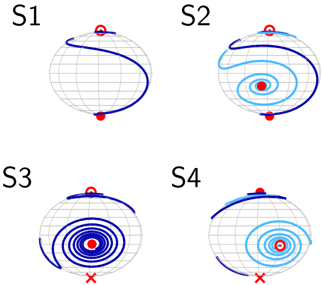
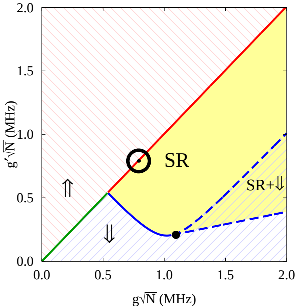
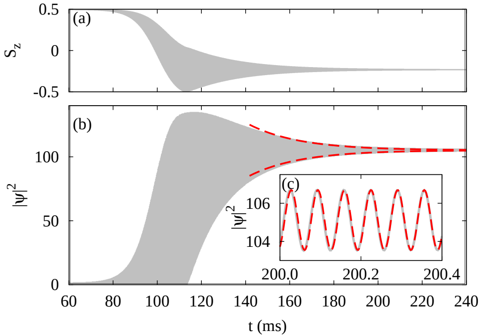




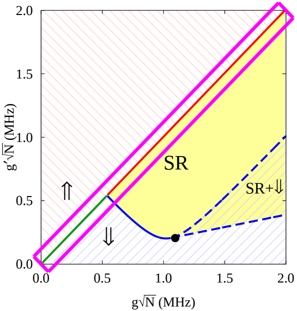
—, — $\dfrac{g'}{g} = \sqrt{\dfrac{(\omega + \omega_0)^2 + \kappa^2}{(\omega - \omega_0)^2 + \kappa^2}}$

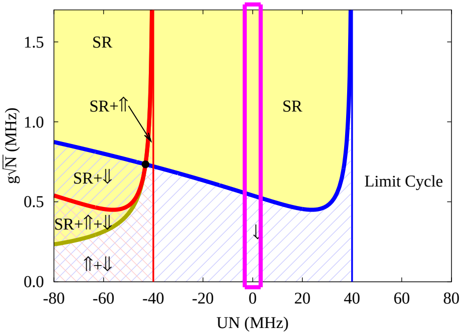
Slow dynamics near critical g'/g





$H \to H + US^z\psi^\dagger\psi$ phase diagram, $g = g'$





- $|UN| < \omega/2$, Regular SR, $S^+ = S^-$
- $UN < -\omega/2$, 2nd SR soln $\psi = -\psi^\dagger$
- $UN > \omega/2$ No SR Fixed point

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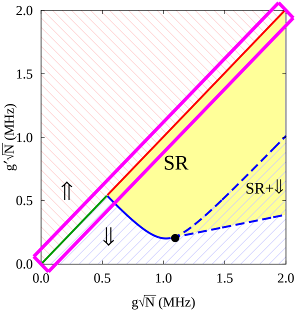

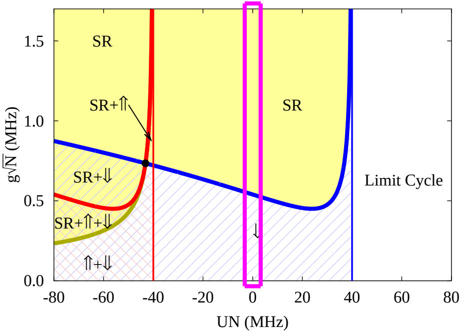

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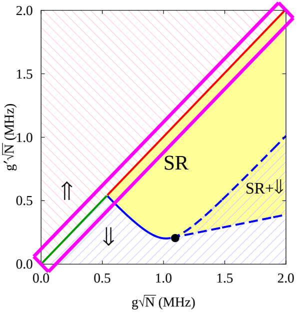

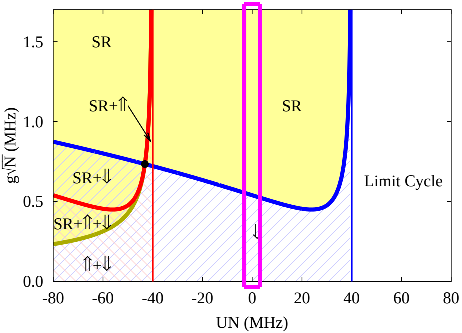

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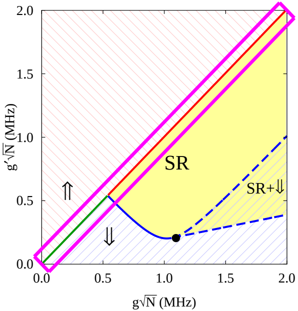

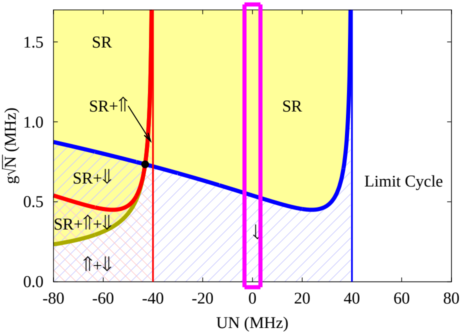

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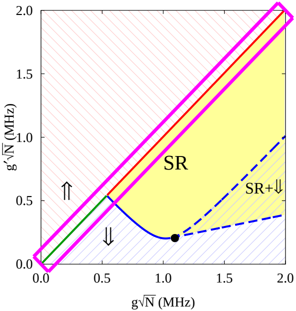

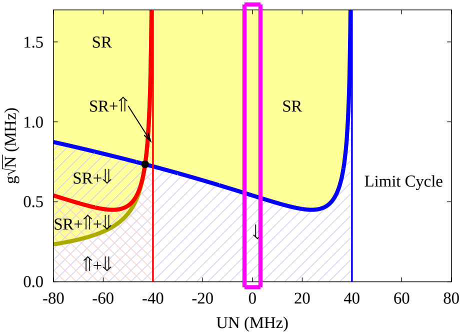

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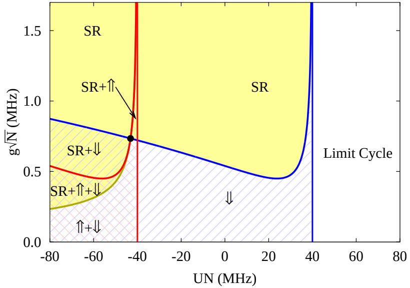
Large U and persistent oscillations



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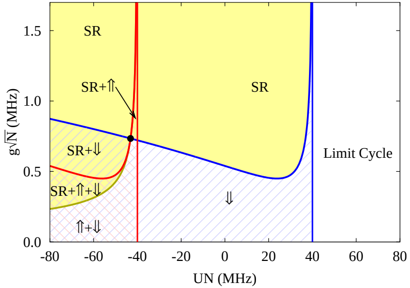

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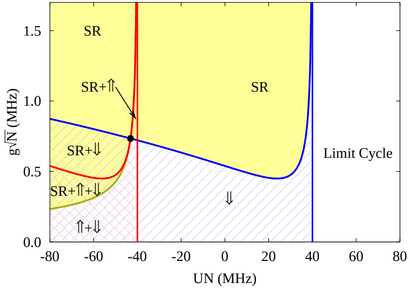

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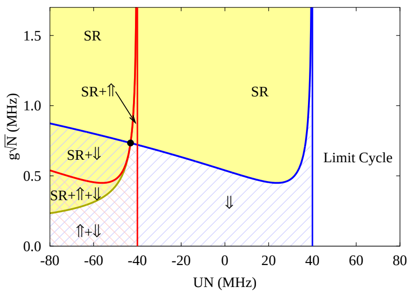

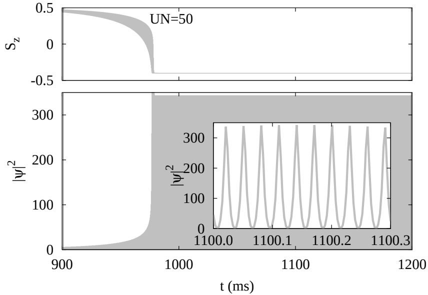

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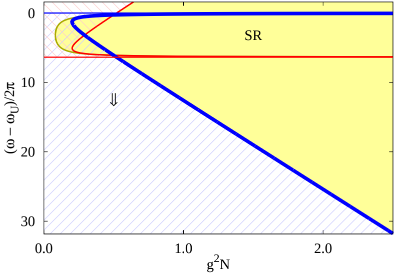
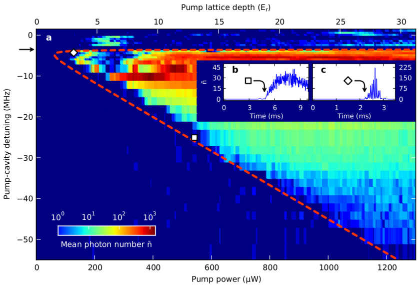
Comparison to experiment $UN = -40\text{MHz}$





[**JK** *et al* `arXiv:1002.3108`]

[Baumann *et al* Nature 2010]

Overview. arXiv:1002.3108

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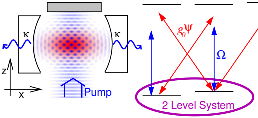

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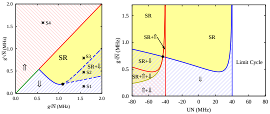

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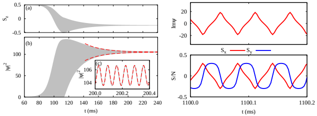


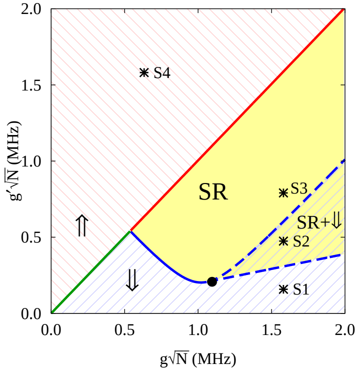
Extra slides

2. Zero U boundaries

3. Fixed points vs U.

Boundaries $U = 0$

$\kappa \neq 0$



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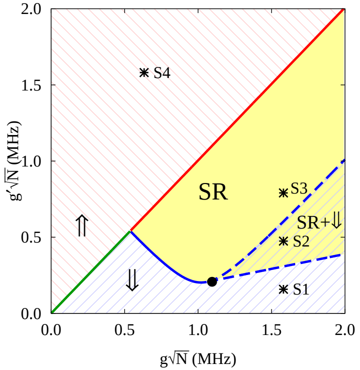

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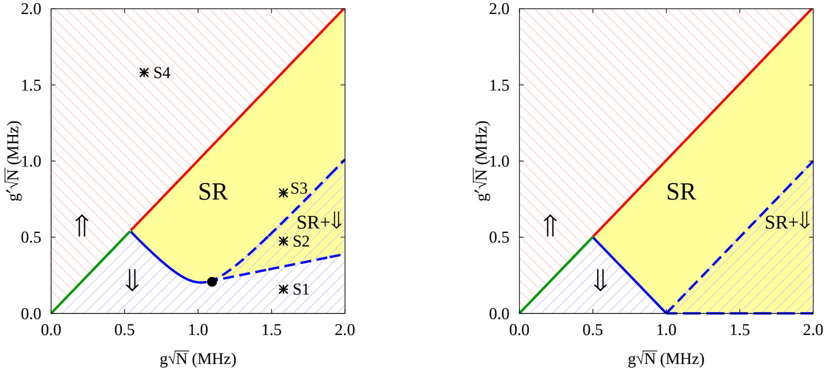
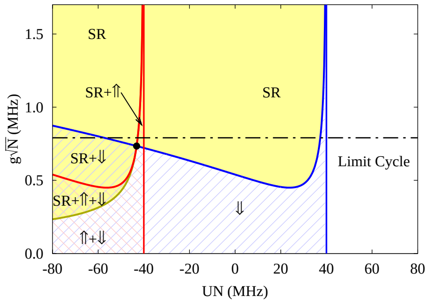
$\kappa = 0$:

— $N(g+g')^2 = \omega\omega_0$



Numerical confirmation of fixed points



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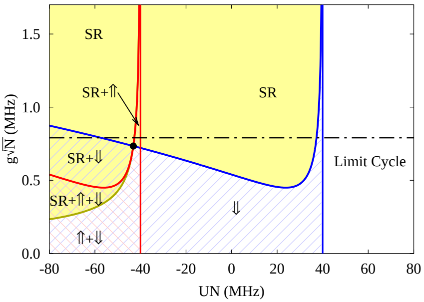

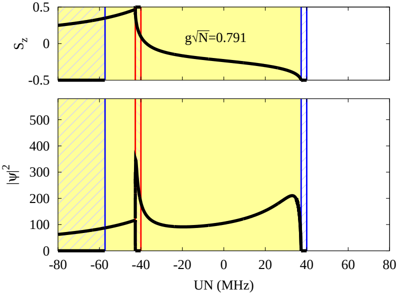

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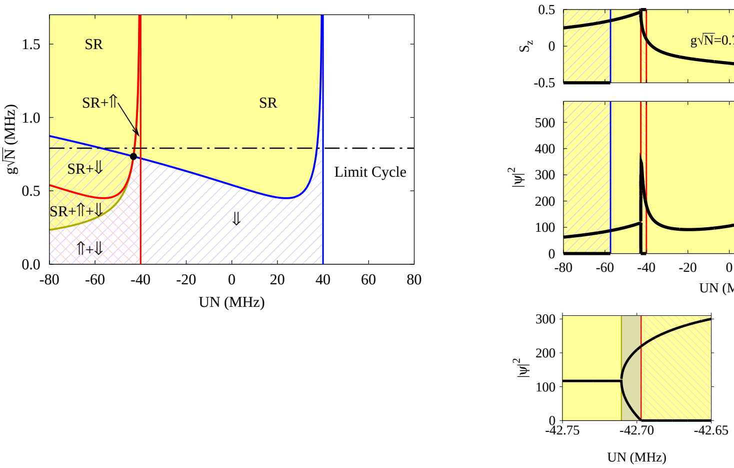

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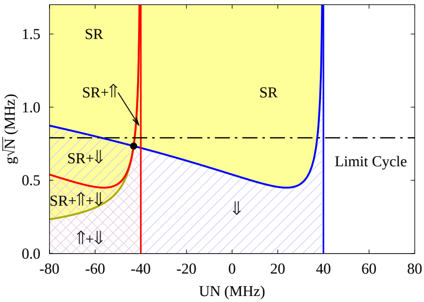

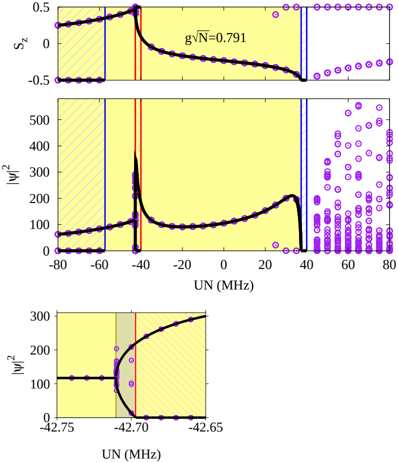

$T = 360\text{ms}$