

Spinor dynamics in a polariton condensate: Macroscopic effects of non-equilibrium condensation

J. Keeling

N. G. Berloff, M. O. Borgh, P. B. Littlewood, M. H. Szymanska.

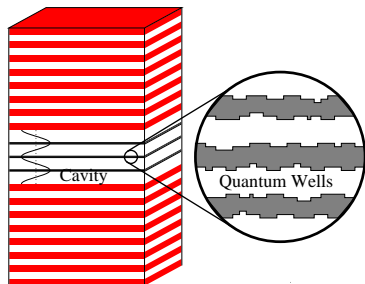


CMMP, December 2009

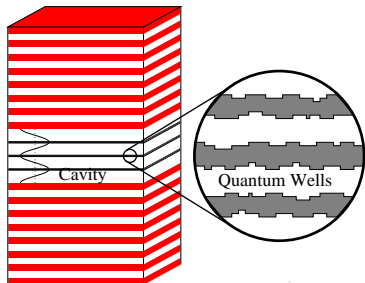


Funding: **EPSRC**
Engineering and Physical Sciences
Research Council

Microcavity Polaritons

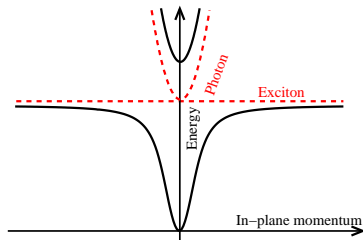


Microcavity Polaritons

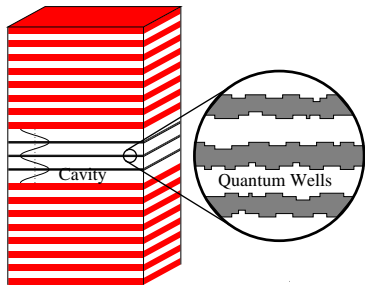


Cavity photons:

$$\begin{aligned}\omega_k &= \sqrt{\omega_0^2 + c^2 k^2} \\ &\simeq \omega_0 + k^2/2m^* \\ m^* &\sim 10^{-4} m_e\end{aligned}$$

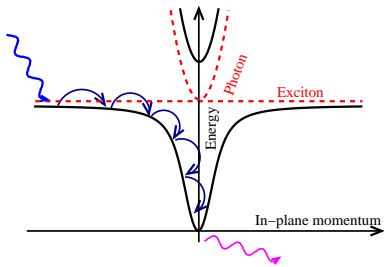
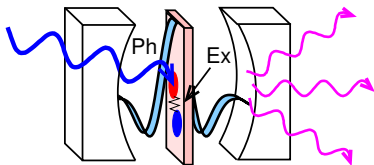


Microcavity Polaritons

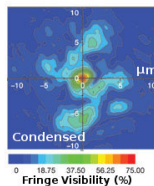
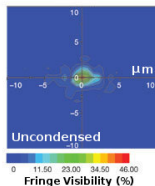
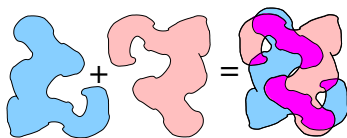
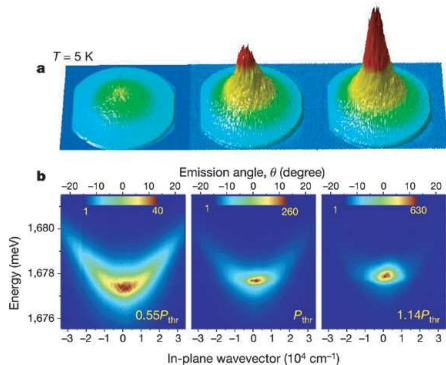


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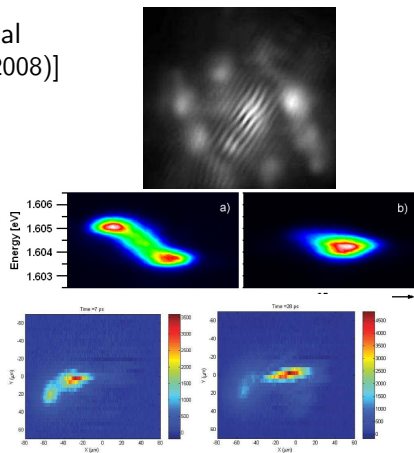
Polariton condensation: distribution and coherence



[Kasprzak, et al., Nature, 2006]

Other (relevant) experiments

- Quantised vortices in disorder potential [Lagoudakis *et al* Nature Phys. 4, 706 (2008)]
- Stress traps for polaritons [Balili *et al* Science 316 1007 (2007)]
- Soliton propagation [Amo *et al* Nature 457 291 (2009)]
- Driven superfluidity [Amo *et al* Nature Phys. (2009)]



Overview

- 1 Introduction to microcavity polaritons
- 2 Gross Pitaevskii equation and spatial profile
- 3 Spin degree of freedom
- 4 Conclusions

Gross-Pitaevskii equation: Harmonic trap

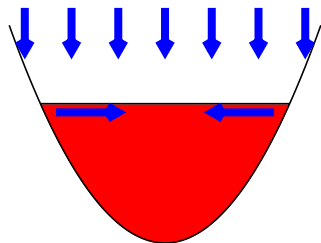
$$i\partial_t\psi = \left[-\frac{\nabla^2}{2m} + \frac{m\omega^2}{2}r^2 + U|\psi|^2 \right] \psi$$

Gross-Pitaevskii equation: Harmonic trap

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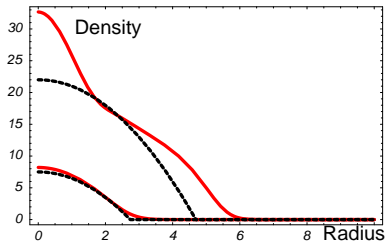
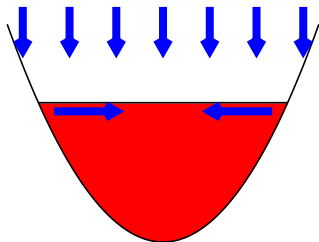
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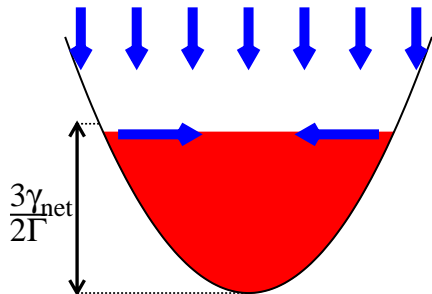
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Stability of Thomas-Fermi solution

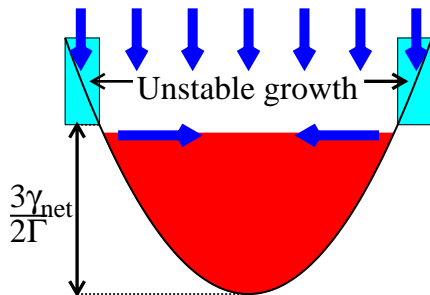
$$\frac{1}{2}\partial_t\rho + \nabla \cdot (\rho\mathbf{v}) = \frac{1}{\hbar}(\gamma_{\text{net}} - \Gamma\rho)\rho$$



Stability of Thomas-Fermi solution

High m modes: $\delta\rho_{n,m} \simeq e^{im\theta} r^m \dots$

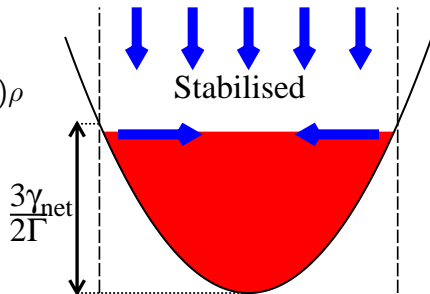
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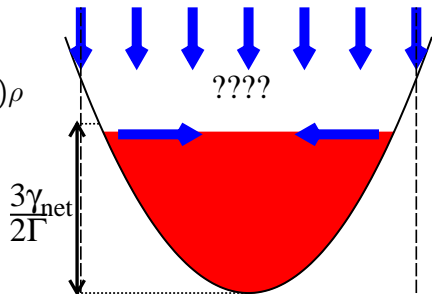
$$\frac{1}{2}\partial_t\rho + \nabla \cdot (\rho\mathbf{v}) = \frac{1}{\hbar}(\gamma_{\text{net}}\Theta(r_0-r) - \Gamma\rho)\rho$$



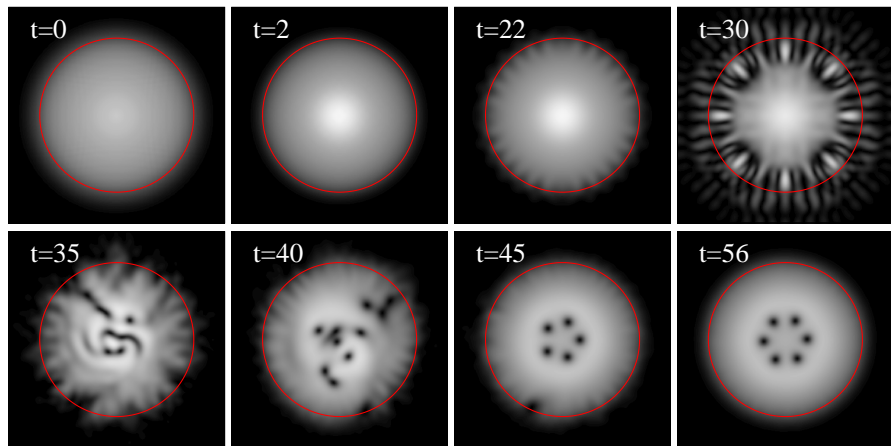
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Time evolution:



[Keeling & Berloff, PRL, '08]

Polariton spin degree of freedom

- Left- and Right-circular polarised states.

- Spinor Gross-Pitaevskii equation:

$$i\partial_t \psi_L = \left[-\frac{\nabla^2}{2m} + V(r) + U_0 |\psi_L|^2 + i(\gamma_{\text{eff}} - \kappa - \Gamma |\psi_L|^2) \right] \psi_L$$

- Two-mode case (neglect spatial variation): [Wouters PRB '08]

- Many modes — interaction of J_x and currents.

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- Tendency to biexciton formation: U_0
- Magnetic fields: Δ
- Broken rotation symmetry: \mathcal{A}
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- ▶ Tendency to biexciton formation: U_1

- ▶ Magnetic fields: Δ
- ▶ Broken rotation symmetry: Δ

- Two-mode case (neglect spatial variation): [Wouters PRB '08]
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Non-equilibrium spinor system: two-mode model

Write:

$$\psi_L = \sqrt{R+z} e^{i\phi+i\theta/2}, \quad \psi_R = \sqrt{R-z} e^{i\phi-i\theta/2}$$

Josephson regime: $J_1 \ll U_1 R$, $z \ll R$,

$$\dot{\theta} = -\Delta - 4U_1 z,$$

$$\dot{z} = -2\gamma_{\text{net}} z - 2J_1 \frac{\gamma_{\text{net}}}{\Gamma} \sin(\theta)$$

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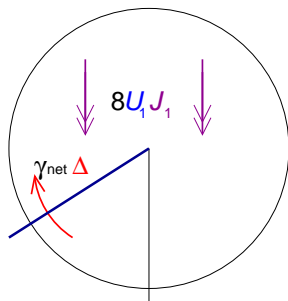
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Damped, driven pendulum

$$\ddot{\theta} + 2\gamma_{\text{net}} \dot{\theta} = 8U_1 J_1 \frac{\gamma_{\text{net}}}{\Gamma} \sin(\theta) - 2\gamma_{\text{net}} \Delta$$



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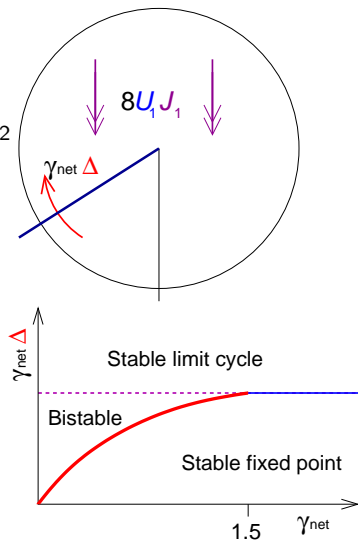
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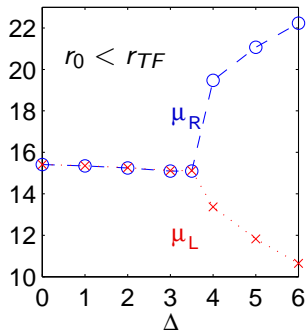
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Trapped spinor system

$$V(r) = m\omega^2 \frac{r^2}{2}, \quad \gamma_{\text{net}}(r) = J_1 \Theta(r_0 - r).$$

Plot $\mu_{L,R} = \partial_t \phi \pm \partial_t \theta / 2$ vs Δ .

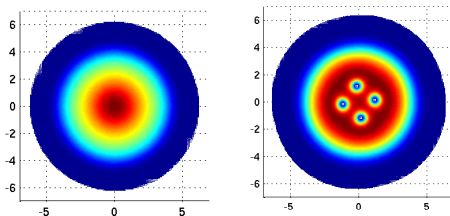
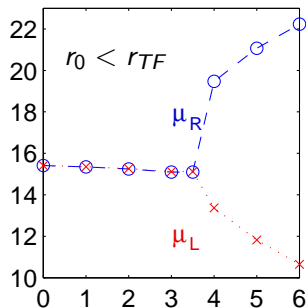
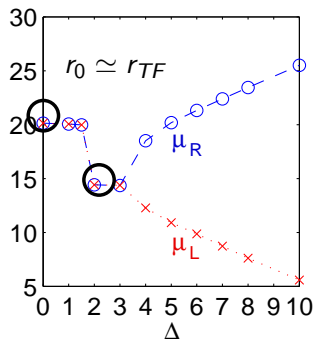


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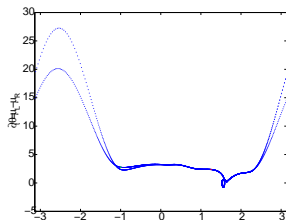
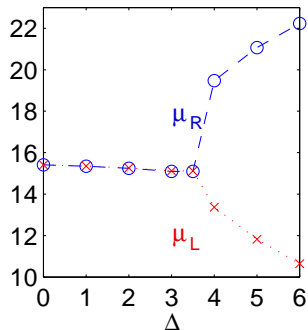
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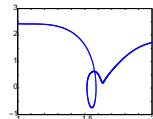
$$\dot{\theta} = -\Delta - 4U_1 z = 0$$



Trapped spinor system — phase portraits



$$\Delta = 3.20$$



Examine phase portrait $\partial_t \theta$ vs θ

Conclusions

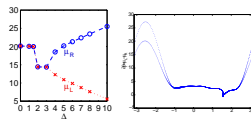
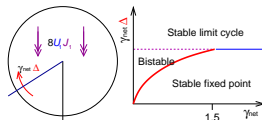
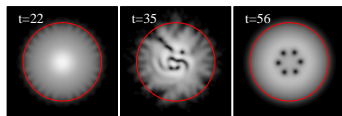
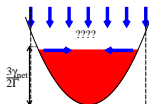
- Modification to Thomas-Fermi profile
Spontaneous rotating vortex lattice

[JK, NGB. PRL **100** 250401 (2008)]

- Spinor model: Steady states

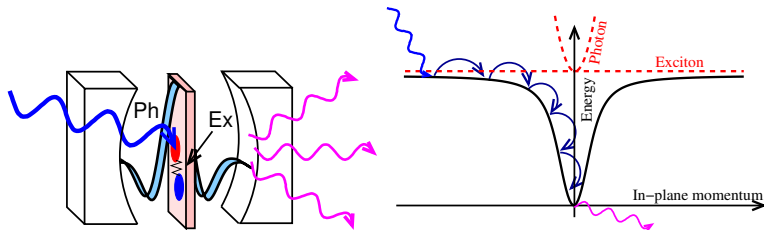
- Coupled spin and spatial dynamics

[MOB, JK, NGB. arXiv:0911:4486]

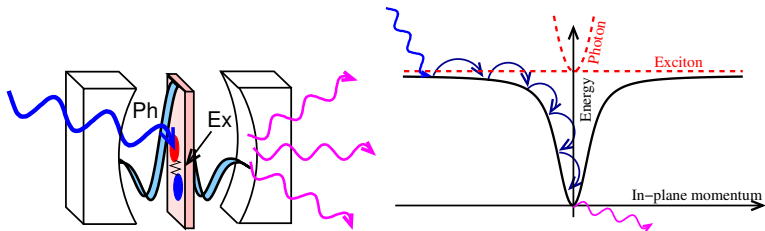


- 5 Non-equilibrium timescales
- 6 Polariton experiments
- 7 Spinor bistability
- 8 Spinor spectrum

Non-equilibrium: Timescales



Non-equilibrium: Timescales

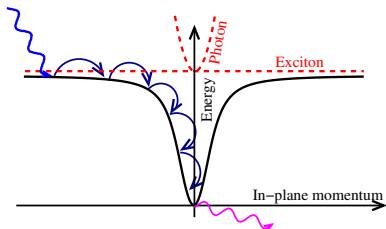
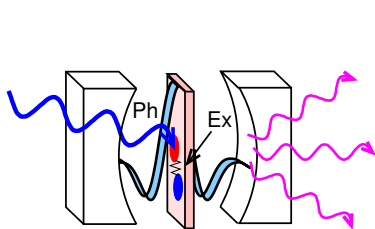


	Lifetime	Thermalisation
Atoms	10s	10ms
Excitons ^a	50ns	0.2ns
Polaritons	5ps	0.5ps
Magnons ^b	1 μ s(??)	100ns(?)

^aCoupled quantum wells. [Hammack et al PRB 76 193308 (2007)]

^bYttrium Iron Garnett. [Demokritov et al, Nature 443 430 (2007)]

Non-equilibrium: Timescales

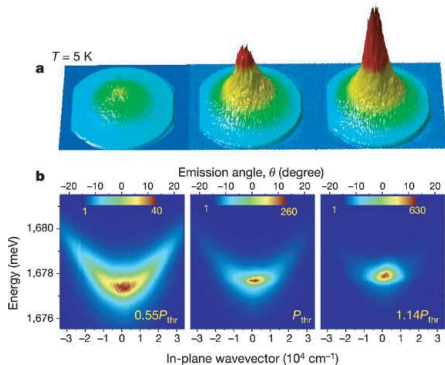


	Lifetime	Thermalisation	Linewidth	Temperature	
Atoms	10s	10ms	2.5×10^{-13} meV	10^{-8} K	10^{-9} meV
Excitons ^a	50ns	0.2ns	5×10^{-5} meV	1K	0.1meV
Polaritons	5ps	0.5ps	0.5meV	20K	2meV
Magnons ^b	$1\mu\text{s}(??)$	100ns(?)	2.5×10^{-6} meV	300K	30meV

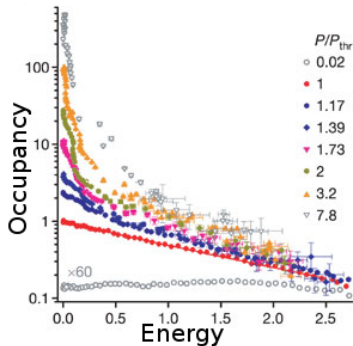
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Polariton experiments: Momentum/Energy distribution

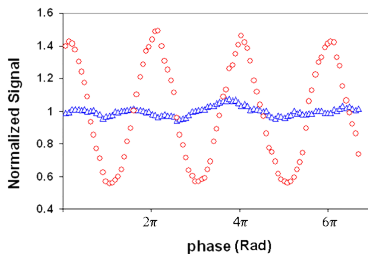
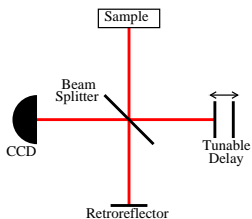


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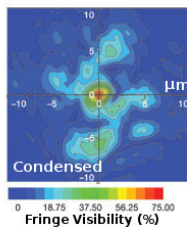
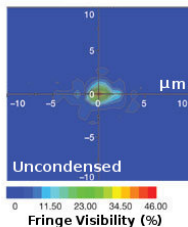
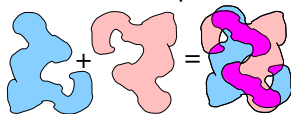


Polariton experiments: Coherence

Basic idea:



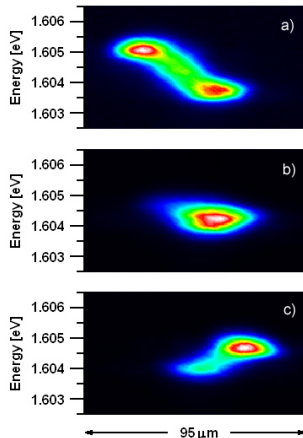
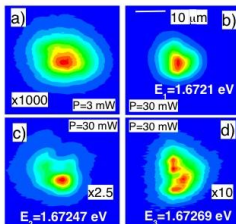
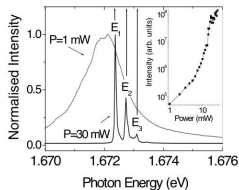
Coherence map:



[Kasprzak, et al., Nature, 2006]

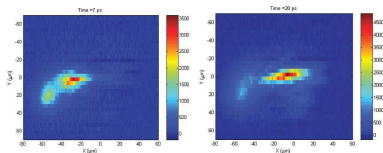
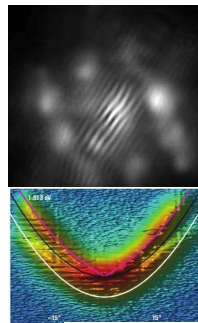
Other polariton condensation experiments

- Stress traps for polaritons [Balili *et al* Science 316 1007 (2007)]
- Temporal coherence and line narrowing [Love *et al* Phys. Rev. Lett. 101 067404 (2008)]

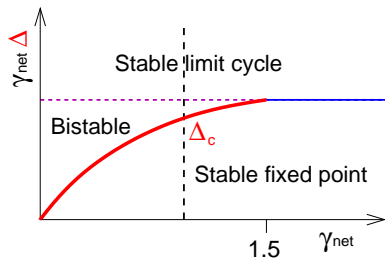


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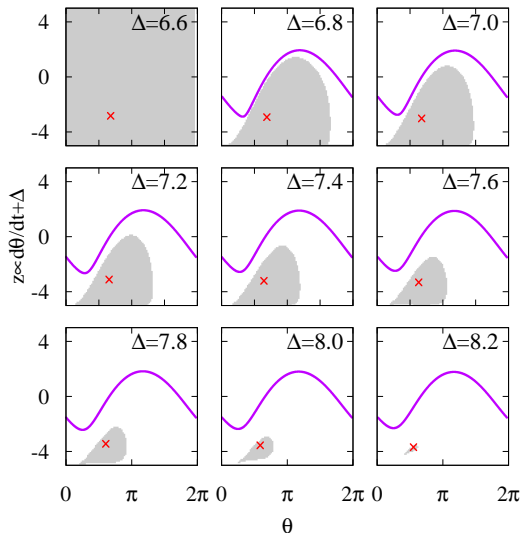
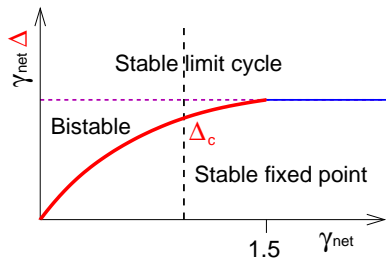
- Quantised vortices in disorder potential [Lagoudakis *et al* Nature Phys. 4, 706 (2008)]
- Changes to excitation spectrum [Utsunomiya *et al* Nature Phys. 4 700 (2008)]
- Soliton propagation [Amo *et al* Nature 457 291 (2009)]
- Driven superfluidity [Amo *et al* Nature Phys. (2009)]



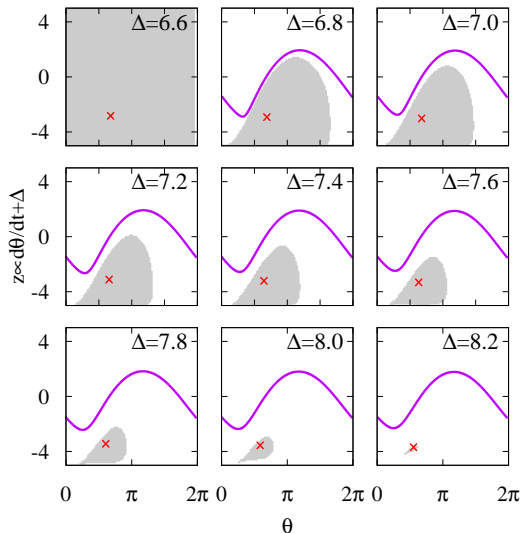
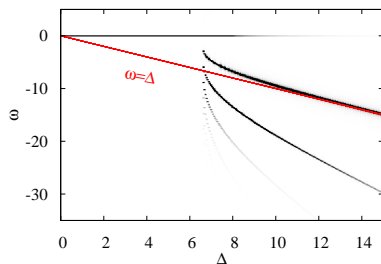
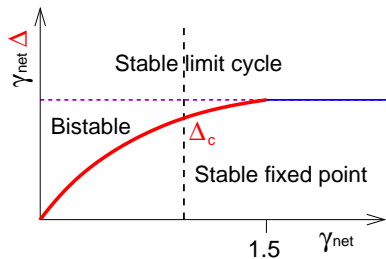
Two-mode model bistability



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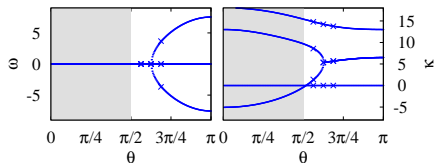
$$\Delta \gtrsim \Delta_c: \omega \simeq [\ln(\Delta - \Delta_c)]^{-1}$$

Homogenous case: stability at $\Delta < \Delta_c$

Damped oscillations

$$\Omega_p^2 = -8U_1 J_1 R_0 \cos(\theta)$$

$$\omega - i\kappa \simeq \begin{cases} 0 \\ -i\gamma_{\text{net}} \pm i\sqrt{\gamma_{\text{net}} - \Omega_p^2} \\ -2i\gamma_{\text{net}} \end{cases}$$



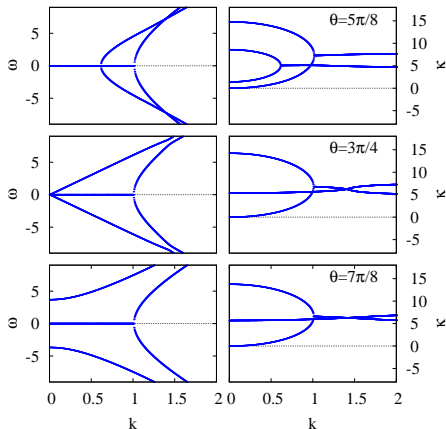
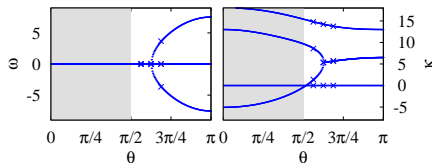
If $\Omega_p^2 = \gamma_{\text{net}}$ degenerate modes:

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If $\Omega_p^2 = \gamma_{\text{net}}$ degenerate modes:
 $\omega \propto k$ for spin wave.