Polariton condensation

J. M. J. Keeling
N. G. Berloff, M. O. Borgh, P. R. Eastham, P. B. Littlewood,
F. M. Marchetti, M. H. Szymanska.

Cambridge-ITAP school, 2009

Acknowledgements

People:

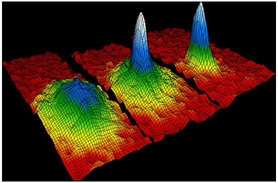
Funding:

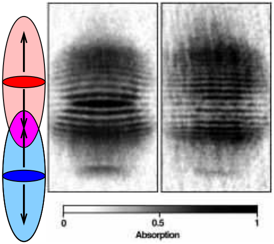
EPSRC Engineering and Physical Sciences Research Council

Pembroke College

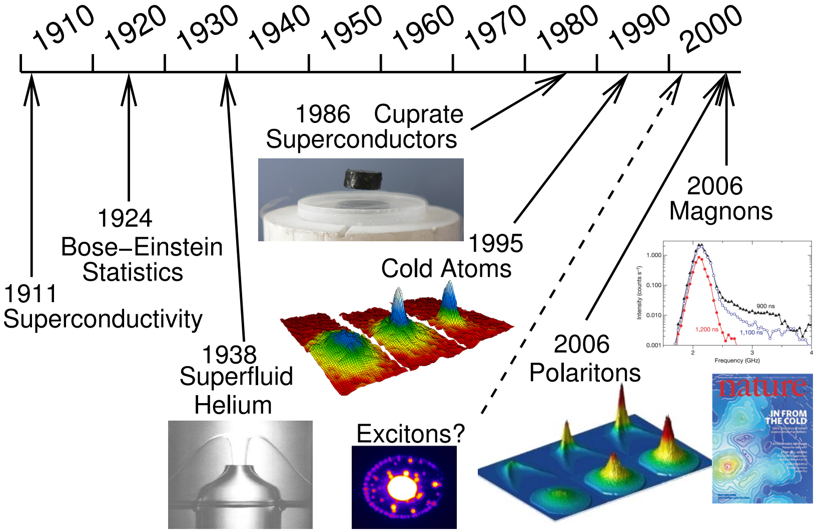
Bose-Einstein condensation

- Macroscopic occupation of ground state
  - Weakly interacting atoms, $T_c \propto n^{2/3}/m$.
- Macroscopic quantum coherence
  - No fragmentation
  - Macroscopic phase
- Superfluidity
  - Rigidity of wavefunction
  - New sound modes





Condensation timeline



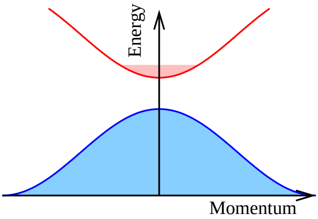
Outline

- Excitons in semiconductors — electron-hole quasiparticles
- Polariton quasiparticles — mixtures of light and matter
  - Light mass $10^{-5} m_e$ — high condensation T.
  - Some interaction (from exciton)
- Microcavity system; some recent experiments
- Coherent states of BEC, excitons, magnons, polaritons.
- Consequences of non-equilibrium BEC
  - Decoherence; lasing
  - Strong coupling laser without inversion.

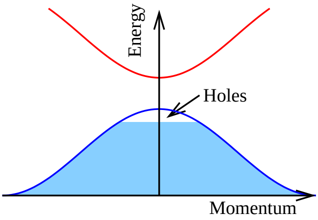
Overview

1. Introduction to BEC
2. Introduction to microcavity polaritons
   - Connection of broken symmetries
3. Model and review of equilibrium results
   - Disorder-localised exciton model
   - Equilibrium mean field theory
4. Non-equilibrium

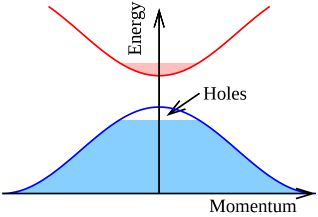
Excitons in semiconductors



Excitons in semiconductors



Excitons in semiconductors

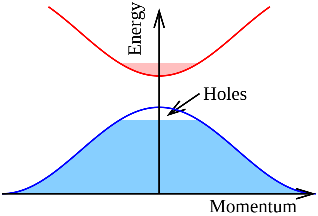


$$H = \sum_i T_i^e + T_i^h + \sum_{ij} V_{ij}^{ee} + V_{ij}^{hh} - V_{ij}^{eh}$$

$$T_i = \frac{p_i^2}{2m} \qquad V_{ij} = \frac{e^2}{\epsilon_r |r_i - r_j|}$$

- In GaAs $m^* = 0.1 m_e, \epsilon_r = 13$, so
  - $\mathcal{R}y = 5\text{meV}$ (13.6eV for H)
  - $a_B = 7\text{nm}$ (0.05nm for H).

Excitons in semiconductors



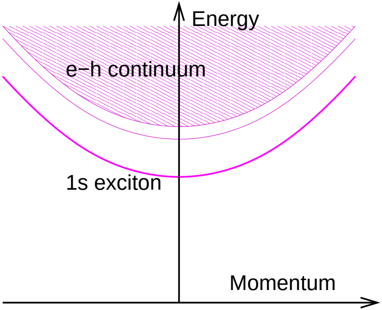
$$H = \sum_i T_i^e + T_i^h + \sum_{ij} V_{ij}^{ee} + V_{ij}^{hh} - V_{ij}^{eh}$$

$$T_i = \frac{p_i^2}{2m} \qquad V_{ij} = \frac{e^2}{\epsilon_r |r_i - r_j|}$$

- High density: e–h plasma
- Low density: excitons

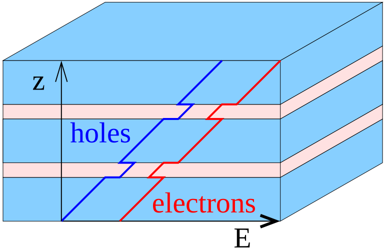
- In GaAs $m^* = 0.1 m_e, \epsilon_r = 13$, so
  - $\mathcal{R}y = 5\text{meV}$ (13.6eV for H)
  - $a_B = 7\text{nm}$ (0.05nm for H).

Quantum well excitons



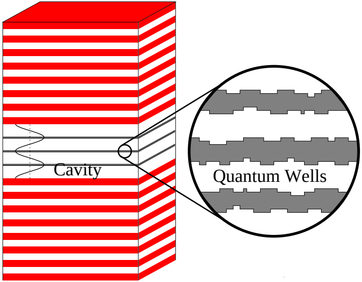
- Enhance lifetime
  - Spatial separation
  - Optically forbidden transition

- Mass $\sim m_e$.
- Strongly interacting dipoles.

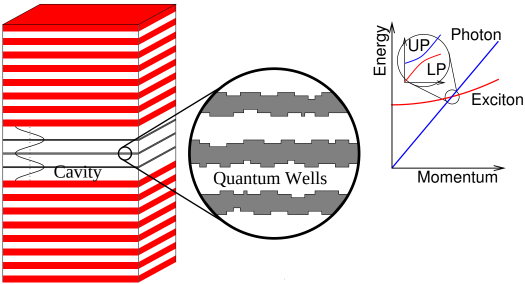


[See D. Snoke lectures]

Microcavity Polaritons

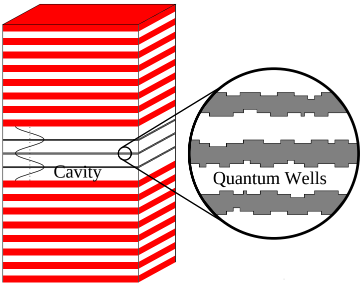


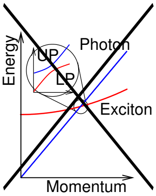
Microcavity Polaritons



[Pekar, JETP(1958)]
[Hopfield, Phys. Rev.(1958)]

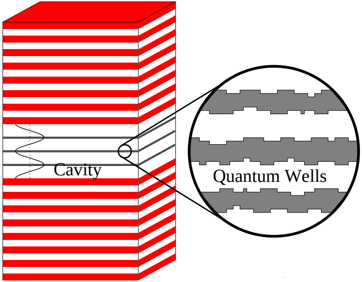
Microcavity Polaritons

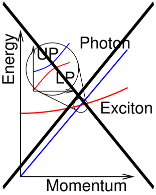




[Pekar, JETP(1958)]
[Hopfield, Phys. Rev.(1958)]

Microcavity Polaritons



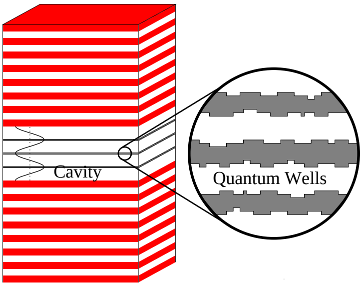


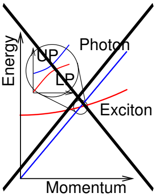
[Pekar, JETP(1958)]
[Hopfield, Phys. Rev.(1958)]

Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$

Microcavity Polaritons

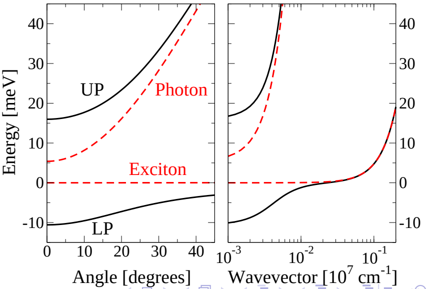




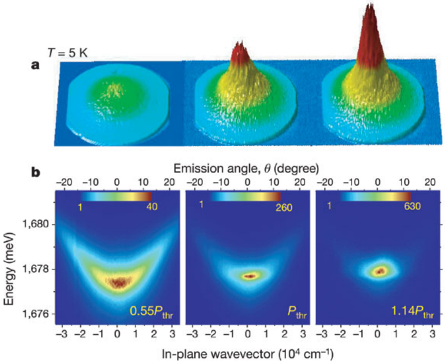
[Pekar, JETP(1958)]
[Hopfield, Phys. Rev.(1958)]

Cavity photons:

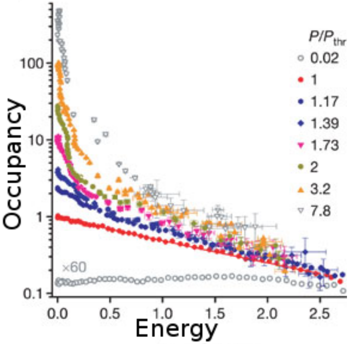
$$\omega_k = \sqrt{\omega_0^2 + c^2k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$



Polariton experiments: Momentum/Energy distribution

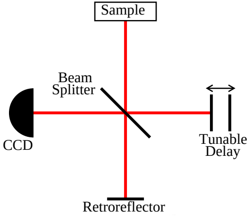


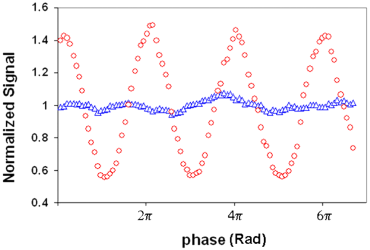
[Kasprzak, et al., Nature, 2006]



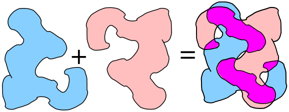
Polariton experiments: Coherence

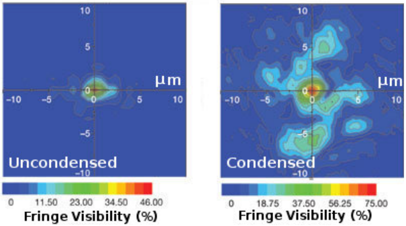
Basic idea:





Coherence map:

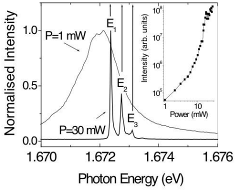


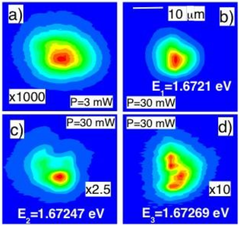


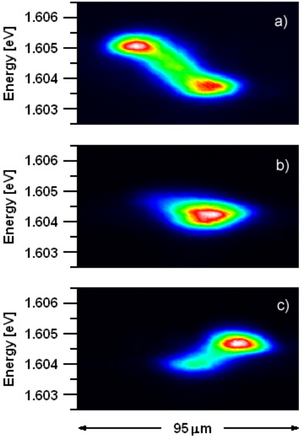
[Kasprzak, et al., Nature, 2006]

Other polariton condensation experiments

- Old measurements of $\langle N(t)N(t+\tau)\rangle$ [Deng *et al* PNAS 100 15318 (2003)]
- Stress traps for polaritons [Balili *et al* Science 316 1007 (2007)]
- Temporal coherence and line narrowing [Love *et al* Phys. Rev. Lett. 101 067404 (2008)]

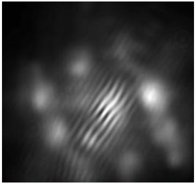




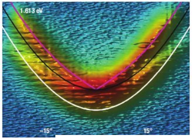


Other polariton condensation experiments

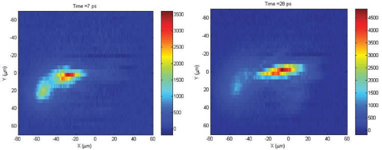
- Quantised vortices in disorder potential
[Lagoudakis *et al* Nature Phys. 4, 706 (2008)]



- Changes to excitation spectrum
[Utsunomiya *et al* Nature Phys. 4 700 (2008)]



- Soliton propagation
[Amo *et al* Nature 457 291 (2009)]
- Driven superfluidity
[Amo *et al* `arXiv:0812:2748`]



Distinguishing features of polaritons

- Composite electron–hole–photon particle:
  - Similar energy scales
- Light mass — strong overlap at low density.
- Naturally two-dimensional, but finite
  - Berezhinskii-Kosterlitz-Thouless vs BEC
- Short polariton lifetime
  - Non-equilibrium distributions
  - Decoherence
  - Relationship to Laser.

Distinguishing features of polaritons

- Composite electron–hole–photon particle:
  - Similar energy scales
- Light mass — strong overlap at low density.
- Naturally two-dimensional, but finite
  - Berezhinskii-Kosterlitz-Thouless vs BEC
- Short polariton lifetime
  - Non-equilibrium distributions
  - Decoherence
  - Relationship to Laser.

Distinguishing features of polaritons

- Composite electron–hole–photon particle:
  - Similar energy scales
- Light mass — strong overlap at low density.
- Naturally two-dimensional, but finite
  - Berezhinskii-Kosterlitz-Thouless vs BEC
- Short polariton lifetime
  - Non-equilibrium distributions
  - Decoherence
  - Relationship to Laser.

Distinguishing features of polaritons

- Composite electron–hole–photon particle:
  - Similar energy scales
- Light mass — strong overlap at low density.
- Naturally two-dimensional, but finite
  - Berezhinskii-Kosterlitz-Thouless vs BEC
- Short polariton lifetime
  - Non-equilibrium distributions
  - Decoherence
  - Relationship to Laser.

Broken symmetries and condensate

- Bosonic coherent state $|\lambda\rangle = e^{\lambda\psi^\dagger}|0\rangle$.
- Excitonic Insulator ↔ BCS superconductor

$$|w_k\rangle = \exp\left[\sum_k w_k a^\dagger_{ck} a_{vk}\right]|0\rangle$$

- As a spin model $a^\dagger_{ci}a_{vi} \to s_i^+$, $a^\dagger_{ci}a_{ci} - a^\dagger_{vi}a_{vi} \to s_i^z$.
- XY Ferromagneti / Quantum Hall Bilayer / Magnon condensates

$$|w_i\rangle = \exp\left(\sum_i w_i s_i^+\right)|0\rangle$$

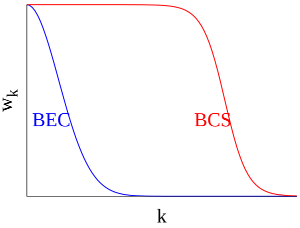
- Coupled to bosonic mode — polaritons; cold fermions → molecules.

$$|\lambda, w_i\rangle = \exp\left(\lambda\psi^\dagger + \sum_i w_i s_i^+\right)|0\rangle$$

Broken symmetries and condensate

- Bosonic coherent state $|\lambda\rangle = e^{\lambda\psi^\dagger}|0\rangle$.
- Excitonic Insulator $\leftrightarrow$ BCS superconductor

$$|w_k\rangle = \exp\left[\sum_k w_k a^\dagger_{ck} a_{vk}\right]|0\rangle$$



- As a spin model $a^\dagger_{ci} a_{vi} \to s_i^+$, $a^\dagger_{ci} a_{ci} - a^\dagger_{vi} a_{vi} \to s_i^z$.
- XY Ferromagneti / Quantum Hall Bilayer / Magnon condensates

$$|w_i\rangle = \exp\left(\sum_i w_i s_i^+\right)|0\rangle$$

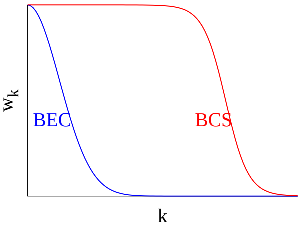
- Coupled to bosonic mode — polaritons; cold fermions → molecules.

$$|\lambda, w_i\rangle = \exp\left(\lambda\psi^\dagger + \sum_i w_i s_i^+\right)|0\rangle$$

Broken symmetries and condensate

- Bosonic coherent state $|\lambda\rangle = e^{\lambda\psi^\dagger}|0\rangle$.
- Excitonic Insulator $\leftrightarrow$ BCS superconductor

$$|w_k\rangle = \exp\left[\sum_k w_k a^\dagger_{ck} a_{vk}\right]|0\rangle$$



- As a spin model $a^\dagger_{ci} a_{vi} \to s_i^+$, $a^\dagger_{ci} a_{ci} - a^\dagger_{vi} a_{vi} \to s_i^z$,
- XY Ferromagneti / Quantum Hall Bilayer / Magnon condensates

$$|w_i\rangle = \exp\left(\sum_i w_i s_i^+\right)|0\rangle$$

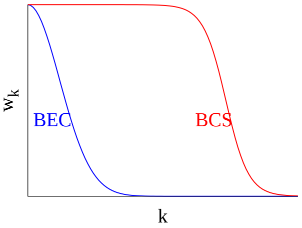
- Coupled to bosonic mode — polaritons; cold fermions → molecules.

$$|\lambda, w_i\rangle = \exp\left(\lambda\psi^\dagger + \sum_i w_i s_i^+\right)|0\rangle$$

Broken symmetries and condensate

- Bosonic coherent state $|\lambda\rangle = e^{\lambda\psi^\dagger}|0\rangle$.
- Excitonic Insulator $\leftrightarrow$ BCS superconductor

$$|w_k\rangle = \exp\left[\sum_k w_k a^\dagger_{ck} a_{vk}\right]|0\rangle$$



- As a spin model $a^\dagger_{ci}a_{vi} \rightarrow s^+_i$, $a^\dagger_{ci}a_{ci} - a^\dagger_{vi}a_{vi} \rightarrow s^z_i$,
- XY Ferromagneti / Quantum Hall Bilayer / Magnon condensates

$$|w_i\rangle = \exp\left(\sum_i w_i s^+_i\right)|0\rangle$$

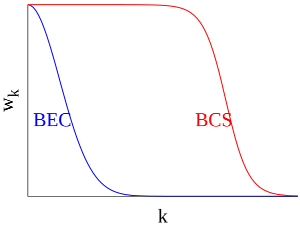
- Coupled to bosonic mode — polaritons; cold fermions → molecules.

$$|\lambda, w_i\rangle = \exp\left(\lambda\psi^\dagger + \sum_i w_i s^+_i\right)|0\rangle$$

Broken symmetries and condensate

- Bosonic coherent state $|\lambda\rangle = e^{\lambda\psi^\dagger}|0\rangle$.
- Excitonic Insulator $\leftrightarrow$ BCS superconductor

$$|w_k\rangle = \exp\left[\sum_k w_k a^\dagger_{ck} a_{vk}\right]|0\rangle$$



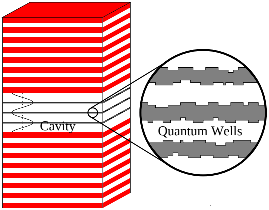
- As a spin model $a^\dagger_{ci}a_{vi} \to s^+_i$, $a^\dagger_{ci}a_{ci} - a^\dagger_{vi}a_{vi} \to s^z_i$,
- XY Ferromagneti / Quantum Hall Bilayer / Magnon condensates

$$|w_i\rangle = \exp\left(\sum_i w_i s^+_i\right)|0\rangle$$

- Coupled to bosonic mode — polaritons; cold fermions → molecules.

$$|\lambda, w_i\rangle = \exp\left(\lambda\psi^\dagger + \sum_i w_i s^+_i\right)|0\rangle$$

Excitons in a disorderd Quantum well



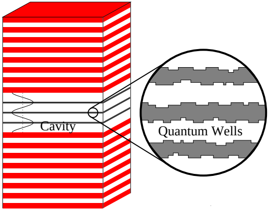
Exciton states in disorder:

$$\left[-\frac{\nabla^2_{\mathbf{R}}}{2m_X} + V(\mathbf{R})\right]\Phi_\alpha(\mathbf{R}) = \varepsilon_\alpha \Phi_\alpha(\mathbf{R})$$

$V(\vec{R})$ smoothed by exciton Bohr radius

[PRL 96 066405 (2006); PRB 76 115326 (2007)]

Excitons in a disorderd Quantum well



Exciton states in disorder:

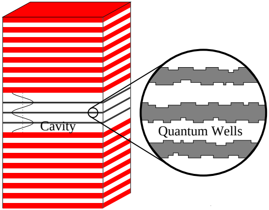
$$\left[-\frac{\nabla^2_{\mathbf{R}}}{2m_X} + V(\mathbf{R})\right]\Phi_\alpha(\mathbf{R}) = \varepsilon_\alpha \Phi_\alpha(\mathbf{R})$$

$V(\vec{R})$ smoothed by exciton Bohr radius

Want: Energies ε_α Oscillator strengths: $g_{\alpha,\mathbf{p}} \propto \psi_{1s}(0)\Phi_{\alpha,\mathbf{p}}$

[PRL 96 066405 (2006); PRB 76 115326 (2007)]

Excitons in a disorderd Quantum well

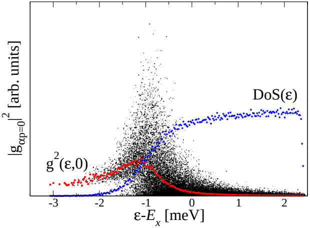


Exciton states in disorder:

$$\left[-\frac{\nabla^2_{\mathbf{R}}}{2m_X} + V(\mathbf{R})\right]\Phi_\alpha(\mathbf{R}) = \varepsilon_\alpha \Phi_\alpha(\mathbf{R})$$

$V(\vec{R})$ smoothed by exciton Bohr radius

Want: Energies ε_α Oscillator strengths: $g_{\alpha,\mathbf{p}} \propto \psi_{1s}(0)\Phi_{\alpha,\mathbf{p}}$

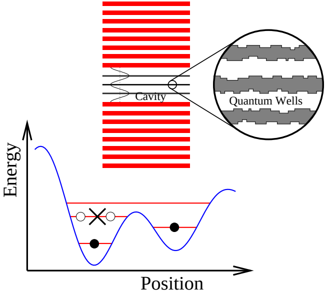


[PRL 96 066405 (2006); PRB 76 115326 (2007)]

Polariton system model

Polariton model

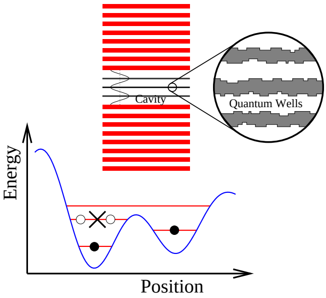
- Disorder-localised excitons
- Treat disorder sites as two-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling g.



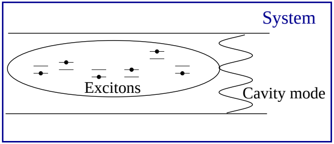
Polariton system model

Polariton model

- Disorder-localised excitons
- Treat disorder sites as two-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling g.

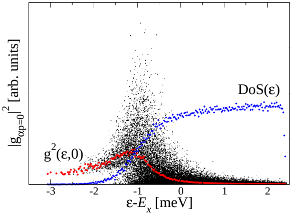


$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{1}{\sqrt{\text{Area}}} g_{\alpha,\mathbf{k}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \text{H.c.} \right]$$



Equilibrium: Mean-field theory

$$H_{\mathrm{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{\mathrm{A}}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \mathrm{H.c.} \right]$$

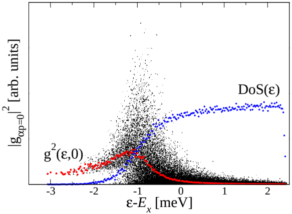


Equilibrium: Mean-field theory

$$H_{\mathrm{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_{\alpha} \left(b^{\dagger}_{\alpha} b_{\alpha} - a^{\dagger}_{\alpha} a_{\alpha} \right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{\mathrm{A}}} \psi_{\mathbf{k}} b^{\dagger}_{\alpha} a_{\alpha} + \mathrm{H.c.} \right]$$

Mean-field theory:
Self-consistent polarisation and field

$$\left[i\partial_t + \omega_0 - \frac{\nabla^2}{2m} \right] \psi = \frac{1}{\sqrt{A}} \sum_{\alpha} g_{\alpha} \langle a^{\dagger}_{\alpha} b_{\alpha} \rangle$$



Equilibrium: Mean-field theory

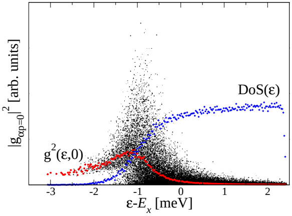
$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \text{H.c.} \right]$$

Mean-field theory:
Self-consistent polarisation and field

$$\left[i\partial_t + \omega_0 - \frac{\nabla^2}{2m} \right] \psi = \frac{1}{\sqrt{A}} \sum_\alpha g_\alpha \langle a^\dagger_\alpha b_\alpha \rangle$$

Density

$$\rho = |\psi|^2 + \frac{1}{A} \sum_\alpha \langle b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \rangle$$



Fluctuation corrections to phase boundary

Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$

In 2D system: modified critical condition:

$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$

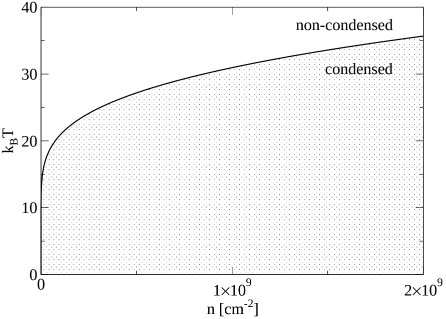
Fluctuation corrections to phase boundary

Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$

In 2D system: modified critical condition:

$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



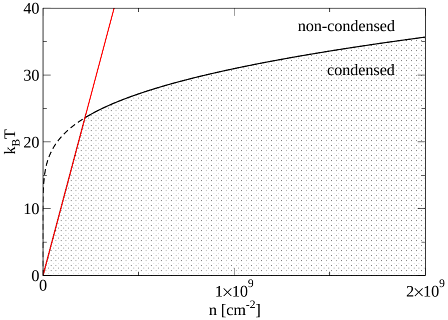
Fluctuation corrections to phase boundary

Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$

In 2D system: modified critical condition:

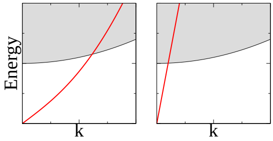
$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



Fluctuation corrections to phase boundary

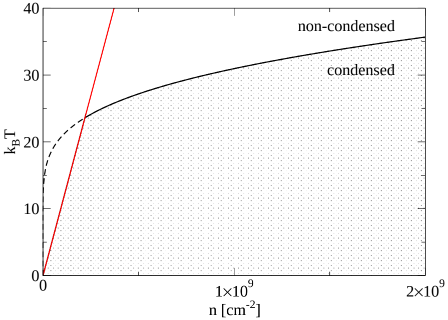
Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$



In 2D system: modified critical condition:

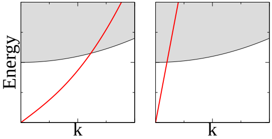
$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



Fluctuation corrections to phase boundary

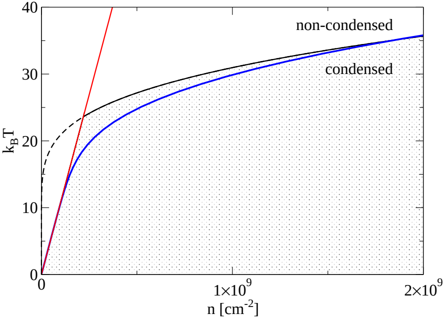
Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$



In 2D system: modified critical condition:

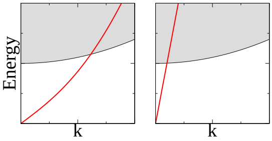
$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



Fluctuation corrections to phase boundary

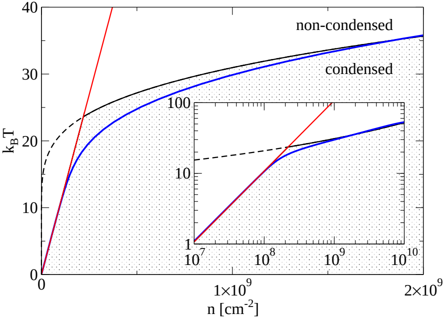
Fluctuation corrections to density

$$\rho \rightarrow \rho_0 + \sum_{q} \langle \delta\psi^\dagger \delta\psi \rangle$$



In 2D system: modified critical condition:

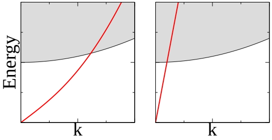
$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



Fluctuation corrections to phase boundary

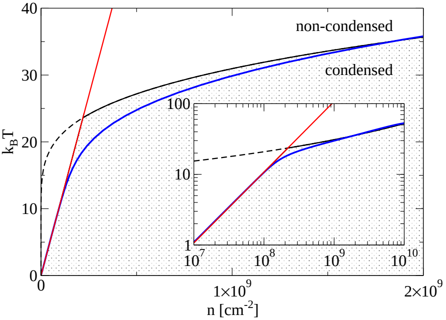
Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$



In 2D system: modified critical condition:

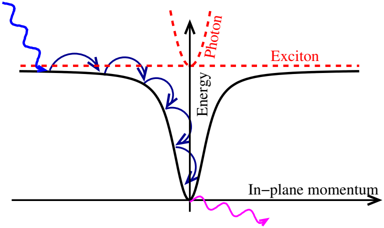
$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$

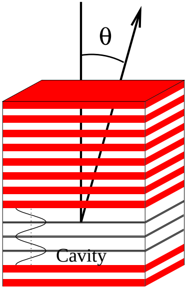


Second BCS crossover at $na_B^2 \simeq 1$

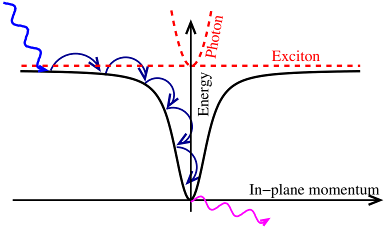
Overview

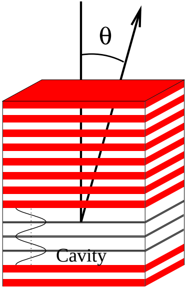
Non-equilibrium system





Non-equilibrium system

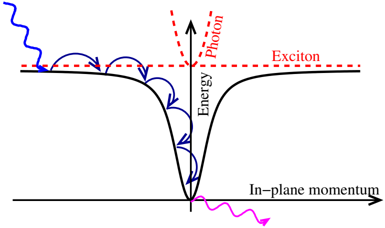


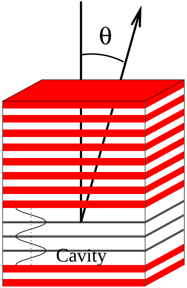


	Lifetime	Thermalisation
Atoms	10s	10ms
Excitons[a]	50ns	0.2ns
Polaritons	5ps	0.5ps
Magnons[b]	1μs(??)	100ns(?)

[a] Coupled quantum wells. [Hammack et al PRB 76 193308 (2007)]
[b] Yttrium Iron Garnett. [Demokritov et al, Nature 443 430 (2007)]

Non-equilibrium system

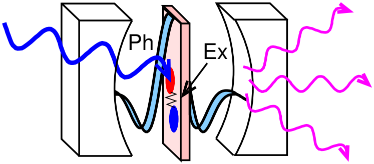


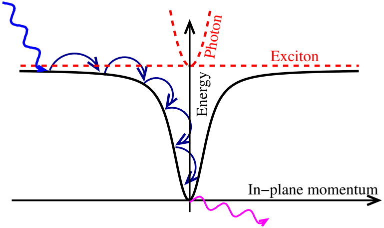


	Lifetime	Thermalisation	Linewidth	Temperature	
Atoms	10s	10ms	2.5×10^{-13}meV	10^{-8}K	10^{-9}meV
Excitons[a]	50ns	0.2ns	5×10^{-5}meV	1K	0.1meV
Polaritons	5ps	0.5ps	0.5meV	20K	2meV
Magnons[b]	1μs(??)	100ns(?)	2.5×10^{-6}meV	300K	30meV

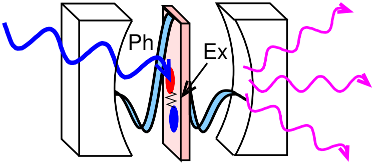
[a] Coupled quantum wells. [Hammack et al PRB 76 193308 (2007)]
[b] Yttrium Iron Garnett. [Demokritov et al, Nature 443 430 (2007)]

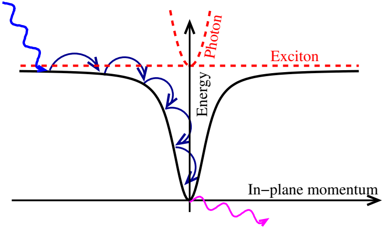
Non-equilibrium: flux and baths

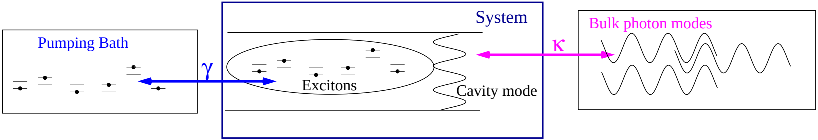




Non-equilibrium: flux and baths







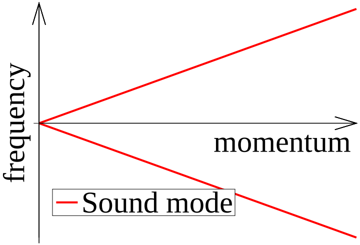
Fluctuations above transition

When condensed

$$\text{Det}\left[\mathcal{G}_R^{-1}(\omega, \mathbf{k})\right] = \omega^2 - c^2\mathbf{k}^2$$

Poles:

$$\omega^* = c|\mathbf{k}|$$



[Szymańska et al., PRL '06; PRB '07. Wouters and Carusotto PRL '07]

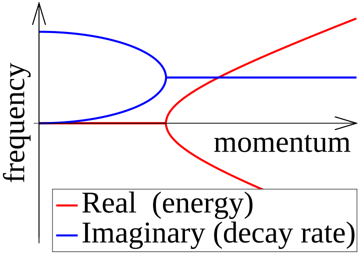
Fluctuations above transition

When condensed

$$\mathrm{Det}\left[\mathcal{G}_R^{-1}(\omega, \mathbf{k})\right] = (\omega + ix)^2 + x^2 - c^2\mathbf{k}^2$$

Poles:

$$\omega^* = -ix \pm \sqrt{c^2\mathbf{k}^2 - x^2}$$



[Szymańska et al., PRL '06; PRB '07. Wouters and Carusotto PRL '07]

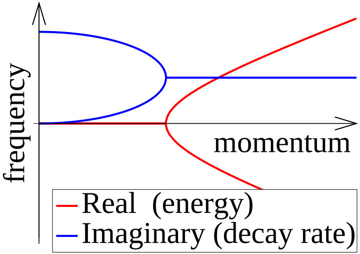
Fluctuations above transition

When condensed

$$\text{Det}\left[\mathcal{G}_R^{-1}(\omega, \mathbf{k})\right] = (\omega + ix)^2 + x^2 - c^2\mathbf{k}^2$$

Poles:

$$\omega^* = -ix \pm \sqrt{c^2\mathbf{k}^2 - x^2}$$



Correlations (in 2D): $\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, r')\right\rangle \simeq |\psi_0|^2 \exp\left[-\mathcal{D}_{\phi\phi}(t, r, r')\right]$

[Szymańska et al., PRL '06; PRB '07. Wouters and Carusotto PRL '07]

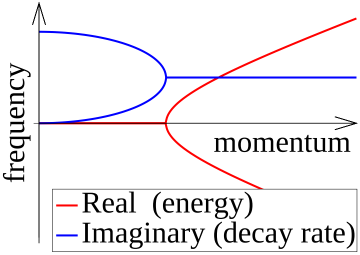
Fluctuations above transition

When condensed

$$\text{Det}\left[\mathcal{G}_R^{-1}(\omega,\mathbf{k})\right] = (\omega+ix)^2+x^2-c^2\mathbf{k}^2$$

Poles:

$$\omega^* = -ix \pm \sqrt{c^2\mathbf{k}^2 - x^2}$$



Correlations (in 2D): $\left\langle \psi^\dagger(\mathbf{r},t)\psi(0,r')\right\rangle \simeq |\psi_0|^2 \exp\left[-\mathcal{D}_{\phi\phi}(t,r,r')\right]$

$$\left\langle \psi^\dagger(\mathbf{r},t)\psi(0,0)\right\rangle \simeq |\psi_0|^2 \exp\left[-\eta\begin{cases}\ln(r/\xi) & r\rightarrow\infty, t\simeq 0\\ \frac{1}{2}\ln(c^2t/x\xi^2) & r\simeq, t\rightarrow\infty\end{cases}\right]$$

[Szymańska et al., PRL '06; PRB '07. Wouters and Carusotto PRL '07]

Conclusions: Theoretical aims of lectures

- Internal polariton structure
  - BEC/BCS crossover
- Two dimensional physics; lineshape and correlations
- Decoerence
  - Dephasing, changing T_c
  - Effect on spatial correlations
  - Strong coupling and lasing
- Experimental signatures
  - Superfluidity
  - Phase coherence, vortices
  - Excitation spectrum

Extra slides