Nonequilibrium quantum condensates: from microscopic theory to macroscopic phenomenology

J. M. J. Keeling
N. G. Berloff, P. B. Littlewood, F. M. Marchetti, M. H. Szymanska.

EasyMeeting, Leiden, May 2009

Acknowledgements

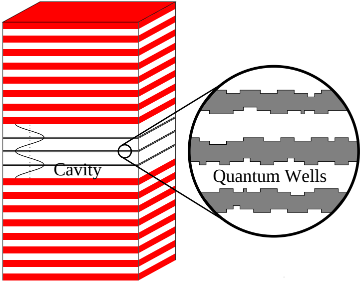
People:

Funding:

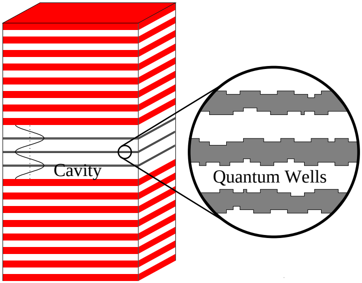
EPSRC Engineering and Physical Sciences Research Council

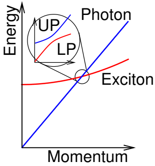
Pembroke College

Microcavity Polaritons



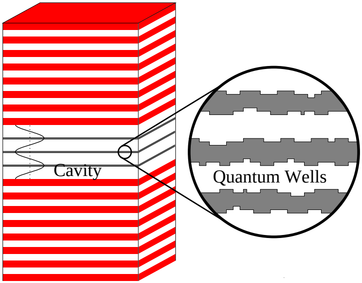
Microcavity Polaritons

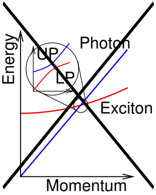




[Pekar, JETP(1958)]
[Hopfield, Phys. Rev.(1958)]

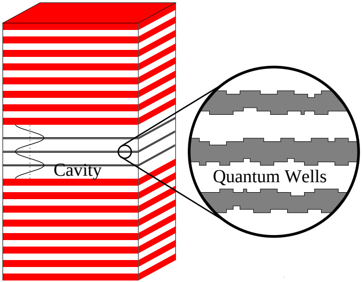
Microcavity Polaritons

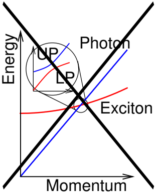




[Pekar, JETP(1958)]
[Hopfield, Phys. Rev.(1958)]

Microcavity Polaritons



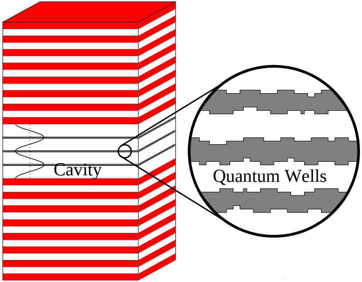


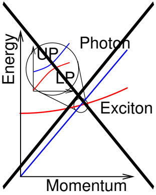
[Pekar, JETP(1958)]
[Hopfield, Phys. Rev.(1958)]

Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$

Microcavity Polaritons

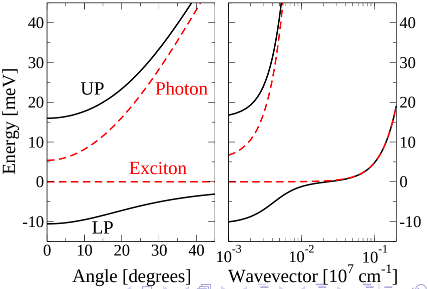




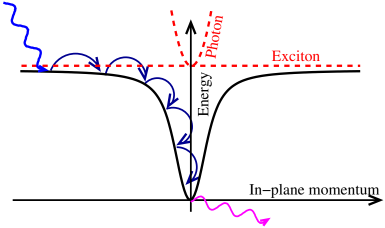
[Pekar, JETP(1958)]
[Hopfield, Phys. Rev.(1958)]

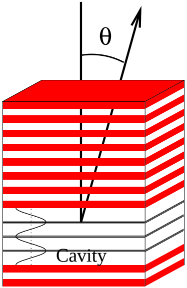
Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$

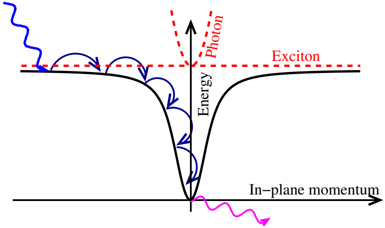


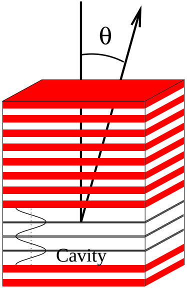
Non-equilibrium system





Non-equilibrium system

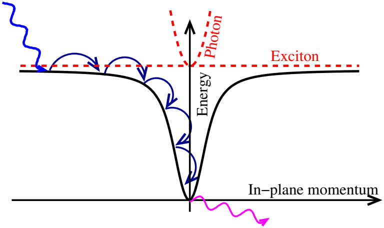


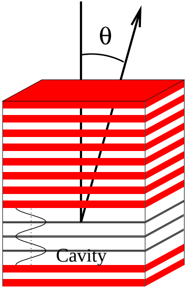


	Lifetime	Thermalisation
Atoms	10s	10ms
Excitons[a]	50ns	0.2ns
Polaritons	5ps	0.5ps
Magnons[b]	1μs(??)	100ns(?)

[a] Coupled quantum wells. [Hammack et al PRB 76 193308 (2007)]
[b] Yttrium Iron Garnett. [Demokritov et al, Nature 443 430 (2007)]

Non-equilibrium system

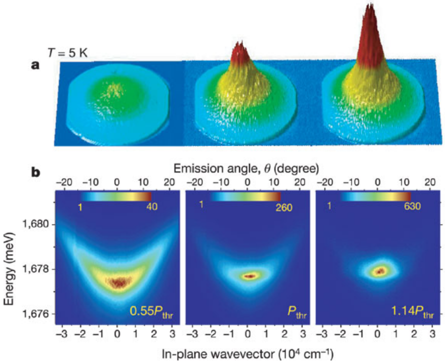




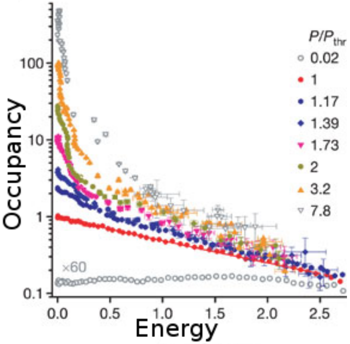
	Lifetime	Thermalisation	Linewidth	Temperature	
Atoms	10s	10ms	2.5×10^{-13}meV	10^{-8}K	10^{-9}meV
Excitons[a]	50ns	0.2ns	5×10^{-5}meV	1K	0.1meV
Polaritons	5ps	0.5ps	0.5meV	20K	2meV
Magnons[b]	1μs(??)	100ns(?)	2.5×10^{-6}meV	300K	30meV

[a] Coupled quantum wells. [Hammack et al PRB 76 193308 (2007)]
[b] Yttrium Iron Garnett. [Demokritov et al, Nature 443 430 (2007)]

Polariton experiments: Momentum/Energy distribution

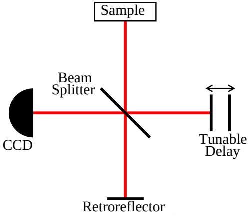


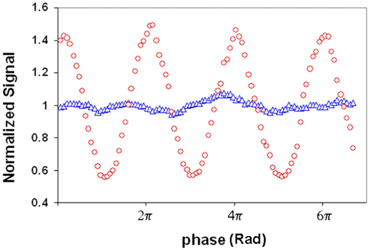
[Kasprzak, et al., Nature, 2006]



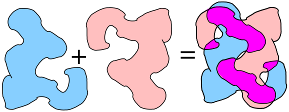
Polariton experiments: Coherence

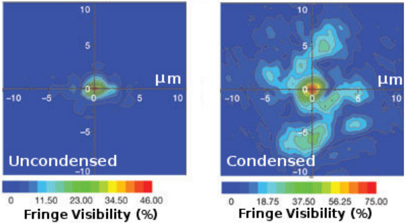
Basic idea:





Coherence map:

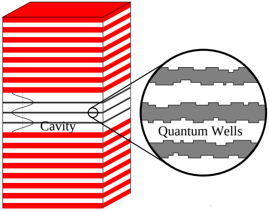




[Kasprzak, et al., Nature, 2006]

Overview

Excitons in a disorderd Quantum well



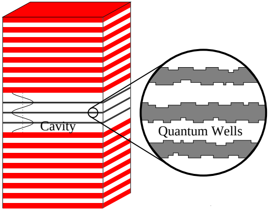
Exciton states in disorder:

$$\left[-\frac{\nabla^2_{\mathbf{R}}}{2m_X} + V(\mathbf{R})\right]\Phi_\alpha(\mathbf{R}) = \varepsilon_\alpha \Phi_\alpha(\mathbf{R})$$

$V(\vec{R})$ smoothed by exciton Bohr radius

[PRL 96 066405 (2006); PRB 76 115326 (2007)]

Excitons in a disorderd Quantum well



Exciton states in disorder:

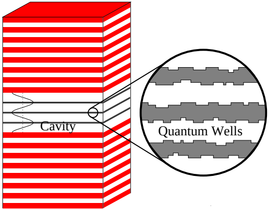
$$\left[-\frac{\nabla^2_{\mathbf{R}}}{2m_X} + V(\mathbf{R})\right]\Phi_\alpha(\mathbf{R}) = \varepsilon_\alpha \Phi_\alpha(\mathbf{R})$$

$V(\vec{R})$ smoothed by exciton Bohr radius

Want: Energies ε_α Oscillator strengths: $g_{\alpha,\mathbf{p}} \propto \psi_{1s}(0)\Phi_{\alpha,\mathbf{p}}$

[PRL 96 066405 (2006); PRB 76 115326 (2007)]

Excitons in a disorderd Quantum well

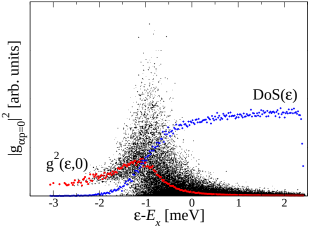


Exciton states in disorder:

$$\left[-\frac{\nabla^2_{\mathbf{R}}}{2m_X} + V(\mathbf{R})\right]\Phi_\alpha(\mathbf{R}) = \varepsilon_\alpha \Phi_\alpha(\mathbf{R})$$

$V(\vec{R})$ smoothed by exciton Bohr radius

Want: Energies ε_α Oscillator strengths: $g_{\alpha,\mathbf{p}} \propto \psi_{1s}(0)\Phi_{\alpha,\mathbf{p}}$

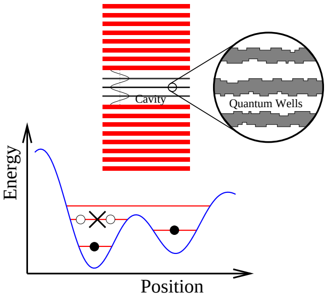


[PRL 96 066405 (2006); PRB 76 115326 (2007)]

Polariton system model

Polariton model

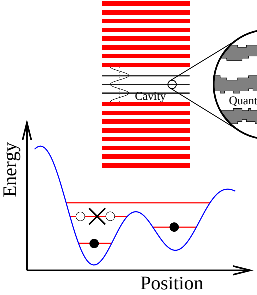
- Disorder-localised excitons
- Treat disorder sites as two-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling g.



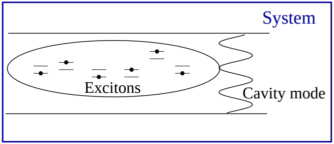
Polariton system model

Polariton model

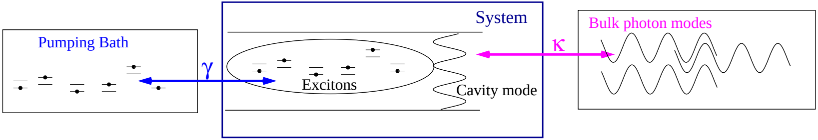
- Disorder-localised excitons
- Treat disorder sites as two-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling g.



$$H_{sys} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{1}{\sqrt{\mathrm{Area}}} g_{\alpha,\mathbf{k}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \mathrm{H.c.} \right]$$

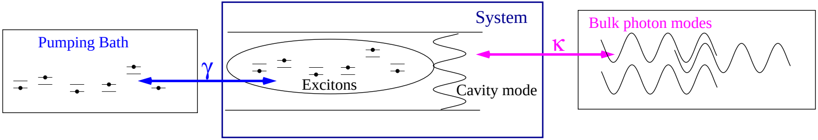


Non-equilibrium model: baths



$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

Non-equilibrium model: baths

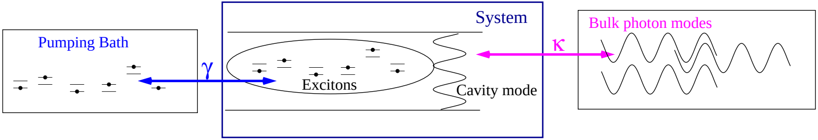


$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

Schematically: pump γ, decay κ

$$H_{\text{sys,bath}} \simeq \sum_{\mathbf{p},\vec{k}} \sqrt{\kappa}\psi_{\mathbf{k}}\Psi^{\dagger}_{\mathbf{p}} + \sum_{\alpha,\beta} \sqrt{\gamma}\left(a^{\dagger}_{\alpha}A_{\beta} + b^{\dagger}_{\alpha}B_{\beta}\right) + \text{H.c.}$$

Non-equilibrium model: baths



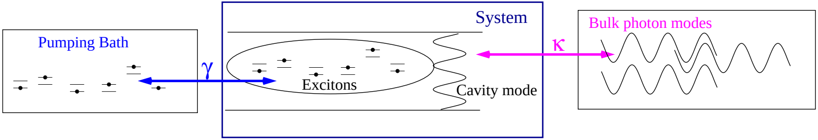
$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

Schematically: pump γ, decay κ

$$H_{\text{sys,bath}} \simeq \sum_{\mathbf{p},\vec{k}} \sqrt{\kappa}\psi_{\mathbf{k}}\Psi^\dagger_{\mathbf{p}} + \sum_{\alpha,\beta} \sqrt{\gamma}\left(a^\dagger_\alpha A_\beta + b^\dagger_\alpha B_\beta\right) + \text{H.c.}$$

Bath correlations, $\langle\Psi^\dagger\Psi\rangle$, $\langle A^\dagger A\rangle$, $\langle B^\dagger B\rangle$ fixed:

Non-equilibrium model: baths

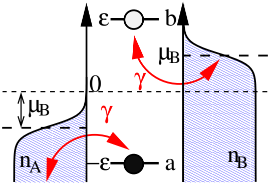


$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

Schematically: pump γ, decay κ

$$H_{\text{sys,bath}} \simeq \sum_{\mathbf{p},\vec{k}} \sqrt{\kappa}\psi_{\mathbf{k}}\Psi^\dagger_{\mathbf{p}} + \sum_{\alpha,\beta} \sqrt{\gamma}\left(a^\dagger_\alpha A_\beta + b^\dagger_\alpha B_\beta\right) + \text{H.c.}$$

Bath correlations, $\langle \Psi^\dagger \Psi \rangle$, $\langle A^\dagger A \rangle$, $\langle B^\dagger B \rangle$ fixed:
Ψ bath is empty. Pumping bath thermal, μ_B, T:



Non-equilibrium theory; mean-field

Look for mean-field solution, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$. Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi_0 = P = \chi(\psi_0, \mu_s)\psi_0$$

Non-equilibrium theory; mean-field

Look for mean-field solution, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$. Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi_0 = P = \chi(\psi_0, \mu_s)\psi_0$$

Susceptibility:

$$\chi(\psi_0, \mu_s) = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

Non-equilibrium theory; mean-field

Look for mean-field solution, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$. Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi_0 = P = \chi(\psi_0, \mu_s)\psi_0$$

Susceptibility: $E_\alpha^2 = \epsilon_\alpha^2 + g^2|\psi_0|^2$

$$\chi(\psi_0, \mu_s) = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

Non-equilibrium theory; mean-field

Look for mean-field solution, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$. Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi_0 = P = \chi(\psi_0, \mu_s)\psi_0$$

Susceptibility: $E_\alpha^2 = \epsilon_\alpha^2 + g^2|\psi_0|^2$, $F_{a,b}(\nu) = F[\nu \mp (\frac{1}{2}\mu_s - \mu_B)]$

$$\chi(\psi_0, \mu_s) = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

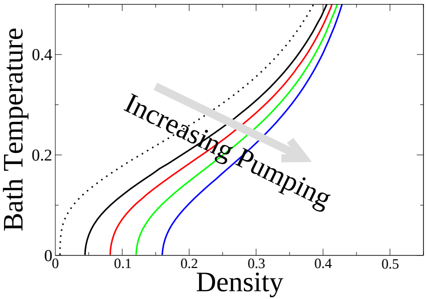
Non-equilibrium theory; mean-field

Look for mean-field solution, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$. Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi_0 = P = \chi(\psi_0, \mu_s)\psi_0$$

Susceptibility: $E_\alpha^2 = \epsilon_\alpha^2 + g^2|\psi_0|^2$, $F_{a,b}(\nu) = F[\nu \mp (\frac{1}{2}\mu_s - \mu_B)]$

$$\chi(\psi_0, \mu_s) = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$



Limits of gap equation

Gap equation:

$$\mu_s - \omega_0 + i\kappa = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

Limits of gap equation

Gap equation:

$$\mu_s - \omega_0 + i\kappa = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

- Laser Limit Imaginary part: Gain vs Loss.

Limits of gap equation

Gap equation:

$$\mu_s - \omega_0 + i\kappa = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

- Laser Limit Imaginary part: Gain vs Loss. If $T \gg \gamma$

$$\kappa = -g^2\gamma \sum_{\text{excitons}} \frac{[F_b(E_\alpha) - F_a(E_\alpha)]}{4E_\alpha^2 + 4\gamma^2}$$

Limits of gap equation

Gap equation:

$$\mu_s - \omega_0 + i\kappa = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

- Laser Limit Imaginary part: Gain vs Loss. If $T \gg \gamma$

$$\kappa = -g^2\gamma \sum_{\text{excitons}} \frac{[F_b(E_\alpha) - F_a(E_\alpha)]}{4E_\alpha^2 + 4\gamma^2} = \frac{g^2}{2\gamma} \times \text{Inversion}$$

Limits of gap equation

Gap equation:

$$\mu_s - \omega_0 + i\kappa = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

- Laser Limit Imaginary part: Gain vs Loss. If $T \gg \gamma$

$$\kappa = -g^2\gamma \sum_{\text{excitons}} \frac{[F_b(E_\alpha) - F_a(E_\alpha)]}{4E_\alpha^2 + 4\gamma^2} = \frac{g^2}{2\gamma} \times \text{Inversion}$$

- Equilibrium limit: finite T set by pumping, need $\kappa \ll \gamma$.

Limits of gap equation

Gap equation:

$$\mu_s - \omega_0 + i\kappa = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

- Laser Limit Imaginary part: Gain vs Loss. If $T \gg \gamma$

$$\kappa = -g^2\gamma \sum_{\text{excitons}} \frac{[F_b(E_\alpha) - F_a(E_\alpha)]}{4E_\alpha^2 + 4\gamma^2} = \frac{g^2}{2\gamma} \times \text{Inversion}$$

- Equilibrium limit: finite T set by pumping, need $\kappa \ll \gamma$.
  Require: $F_a = F_b$ so $\mu_S = 2\mu_B$

Limits of gap equation

Gap equation:

$$\mu_s - \omega_0 + i\kappa = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

- Laser Limit Imaginary part: Gain vs Loss. If $T \gg \gamma$

$$\kappa = -g^2\gamma \sum_{\text{excitons}} \frac{[F_b(E_\alpha) - F_a(E_\alpha)]}{4E_\alpha^2 + 4\gamma^2} = \frac{g^2}{2\gamma} \times \text{Inversion}$$

- Equilibrium limit: finite T set by pumping, need $\kappa \ll \gamma$.
  Require: $F_a = F_b$ so $\mu_S = 2\mu_B$

Limits of gap equation

Gap equation:

$$\mu_s - \omega_0 + i\kappa = -g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(i\gamma + \epsilon_\alpha - \frac{1}{2}\mu_S)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}$$

- Laser Limit Imaginary part: Gain vs Loss. If $T \gg \gamma$

$$\kappa = -g^2\gamma \sum_{\text{excitons}} \frac{[F_b(E_\alpha) - F_a(E_\alpha)]}{4E_\alpha^2 + 4\gamma^2} = \frac{g^2}{2\gamma} \times \text{Inversion}$$

- Equilibrium limit: finite T set by pumping, need $\kappa \ll \gamma$.
  Require: $F_a = F_b$ so $\mu_S = 2\mu_B$

$$\omega_0 - \mu_s = \frac{g^2}{2E} \tanh\left(\frac{\beta E}{2}\right)$$

Limits of gap equation

Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit:

Limits of gap equation

Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit: Gross-Pitaevskii equation

$$\left(i\hbar\partial_t + i\kappa - \left[V(r) - \frac{\hbar^2\nabla^2}{2m}\right]\right)\psi(r) = \chi(\psi(r,t))\psi(r,t)$$

Nonlinear, complex susceptibility $\chi(\psi(r,t))$

Limits of gap equation

Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit: Gross-Pitaevskii equation

$$\left(i\hbar\partial_t + i\kappa - \left[V(r) - \frac{\hbar^2\nabla^2}{2m}\right]\right)\psi(r) = \chi(\psi(r,t))\psi(r,t)$$

Nonlinear, complex susceptibility $\chi[E(\psi(r,t))]$, $E_\alpha^2 = \epsilon_\alpha^2 + g^2|\psi_0|^2$

$$i\hbar\partial_t\psi|_{\text{nlin}} = U|\psi|^2\psi$$

Limits of gap equation

Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit: Gross-Pitaevskii equation

$$\left(i\hbar\partial_t + i\kappa - \left[V(r) - \frac{\hbar^2\nabla^2}{2m}\right]\right)\psi(r) = \chi(\psi(r,t))\psi(r,t)$$

Nonlinear, complex susceptibility $\chi[E(\psi(r,t))]$, $E_\alpha^2 = \epsilon_\alpha^2 + g^2|\psi_0|^2$

$$i\hbar\partial_t\psi|_{\text{nlin}} = U|\psi|^2\psi$$

$$i\hbar\partial_t\psi|_{\text{loss}} = -i\kappa\psi \qquad i\hbar\partial_t\psi|_{\text{gain}} = i\gamma_{\text{eff}}(\mu_B)\psi$$

Limits of gap equation

Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit: Gross-Pitaevskii equation

$$\left(i\hbar\partial_t + i\kappa - \left[V(r) - \frac{\hbar^2\nabla^2}{2m}\right]\right)\psi(r) = \chi(\psi(r,t))\psi(r,t)$$

Nonlinear, complex susceptibility $\chi[E(\psi(r,t))]$, $E_\alpha^2 = \epsilon_\alpha^2 + g^2|\psi_0|^2$

$$i\hbar\partial_t\psi|_{\text{nlin}} = U|\psi|^2\psi$$

$$i\hbar\partial_t\psi|_{\text{loss}} = -i\kappa\psi \qquad i\hbar\partial_t\psi|_{\text{gain}} = i\gamma_{\text{eff}}(\mu_B)\psi - i\Gamma|\psi|^2\psi$$

Limits of gap equation

Gap equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit: Gross-Pitaevskii equation

$$\left(i\hbar\partial_t + i\kappa - \left[V(r) - \frac{\hbar^2\nabla^2}{2m}\right]\right)\psi(r) = \chi(\psi(r,t))\psi(r,t)$$

Nonlinear, complex susceptibility $\chi[E(\psi(r,t))]$, $E_\alpha^2 = \epsilon_\alpha^2 + g^2|\psi_0|^2$

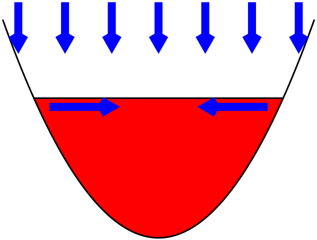
$$i\hbar\partial_t\psi|_{\text{nlin}} = U|\psi|^2\psi$$

$$i\hbar\partial_t\psi|_{\text{loss}} = -i\kappa\psi \qquad i\hbar\partial_t\psi|_{\text{gain}} = i\gamma_{\text{eff}}(\mu_B)\psi - i\Gamma|\psi|^2\psi$$

$$i\partial_t\psi = \left[-\frac{\hbar^2\nabla^2}{2m} + V(r) + U|\psi|^2 + i\left(\gamma_{\text{eff}}(\mu_B) - \kappa - \Gamma|\psi|^2\right)\right]\psi$$

Gross-Pitaevskii equation: Harmonic trap

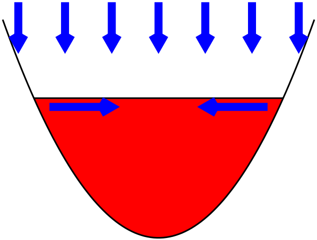
$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2\nabla^2}{2m} + \frac{m\omega^2}{2}r^2 + U|\psi|^2 + i\left(\gamma_{\mathrm{eff}} - \kappa - \Gamma|\psi|^2\right)\right]\psi$$

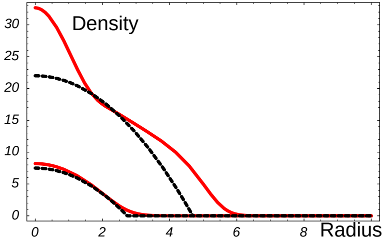


[Keeling & Berloff, PRL, '08]

Gross-Pitaevskii equation: Harmonic trap

$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2\nabla^2}{2m} + \frac{m\omega^2}{2}r^2 + U|\psi|^2 + i\left(\gamma_{\text{eff}} - \kappa - \Gamma|\psi|^2\right)\right]\psi$$

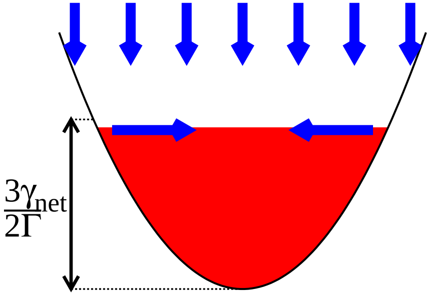




[Keeling & Berloff, PRL, '08]

Stability of Thomas-Fermi solution

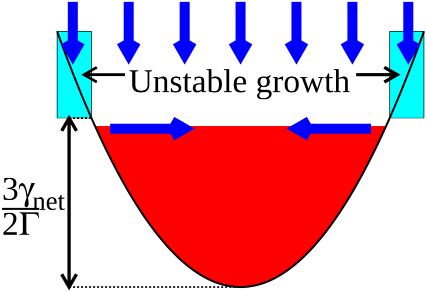
$$\frac{1}{2}\partial_t \rho + \nabla\cdot(\rho \mathbf{v}) = \frac{1}{\hbar}(\gamma_{\text{net}} - \Gamma\rho)\rho$$



Stability of Thomas-Fermi solution

High m modes: $\delta\rho_{n,m} \simeq e^{im\theta} r^m \dots$

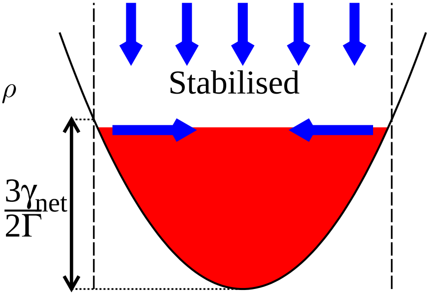
$$\frac{1}{2}\partial_t\rho + \nabla\cdot(\rho\mathbf{v}) = \frac{1}{\hbar}(\gamma_{\text{net}} - \Gamma\rho)\rho$$



Stability of Thomas-Fermi solution

High m modes: $\delta\rho_{n,m} \simeq e^{im\theta} r^m \dots$

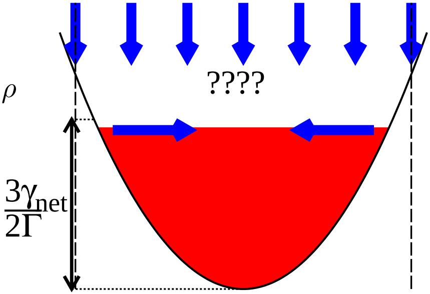
$$\frac{1}{2}\partial_t \rho + \nabla\cdot(\rho \mathbf{v}) = \frac{1}{\hbar}(\gamma_{\text{net}}\Theta(R-r) - \Gamma\rho)\rho$$



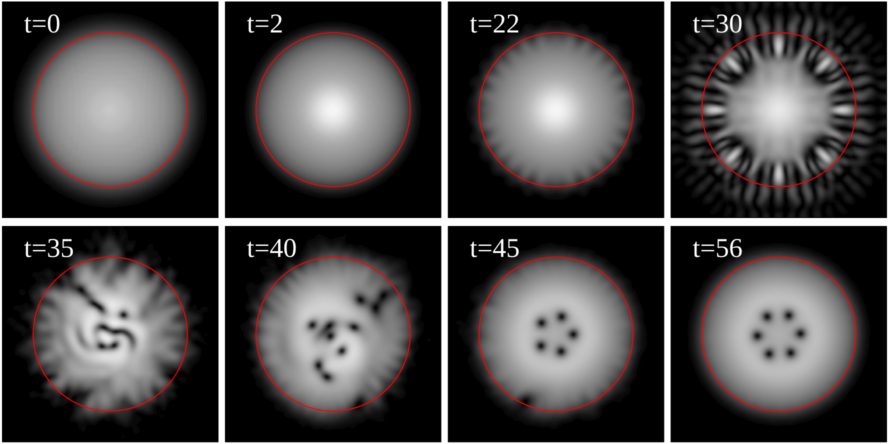
Stability of Thomas-Fermi solution

High m modes: $\delta\rho_{n,m} \simeq e^{im\theta} r^m \ldots$

$$\frac{1}{2}\partial_t \rho + \nabla\cdot(\rho \mathbf{v}) = \frac{1}{\hbar}(\gamma_{\text{net}}\Theta(R-r) - \Gamma\rho)\rho$$



Time evolution:



[Keeling & Berloff, PRL, '08]

Overview

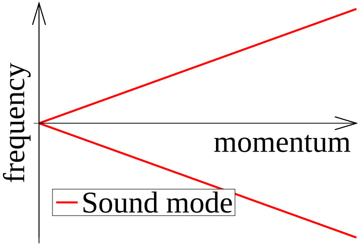
Fluctuations above transition

When condensed

$\text{Det}\left[\mathcal{G}_R^{-1}(\omega, \mathbf{k})\right] = \omega^2 - c^2\mathbf{k}^2$

Poles:

$$\omega^* = c|\mathbf{k}|$$



[Szymańska et al., PRL '06; PRB '07]

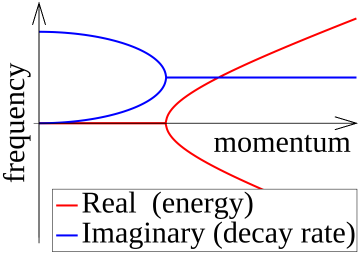
Fluctuations above transition

When condensed

$$\text{Det}\left[\mathcal{G}_R^{-1}(\omega,\mathbf{k})\right] = (\omega+ix)^2+x^2-c^2\mathbf{k}^2$$

Poles:

$$\omega^* = -ix \pm \sqrt{c^2\mathbf{k}^2 - x^2}$$



[Szymańska et al., PRL '06; PRB '07]

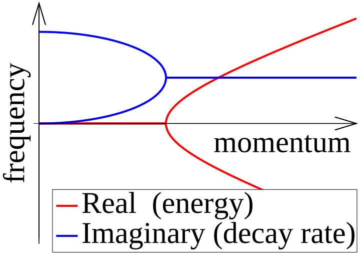
Fluctuations above transition

When condensed

Det $\left[\mathcal{G}_R^{-1}(\omega, \mathbf{k})\right] = (\omega + ix)^2 + x^2 - c^2\mathbf{k}^2$

Poles:

$$\omega^* = -ix \pm \sqrt{c^2\mathbf{k}^2 - x^2}$$



Correlations (in 2D): $\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, r')\right\rangle \simeq |\psi_0|^2 \exp\left[-\mathcal{D}_{\phi\phi}(t, r, r')\right]$

[Szymańska et al., PRL '06; PRB '07]

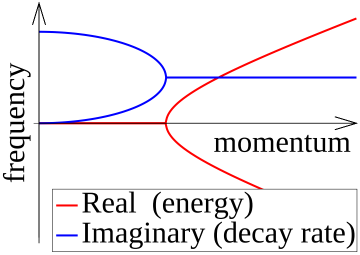
Fluctuations above transition

When condensed

$$\text{Det}\left[\mathcal{G}_R^{-1}(\omega, \mathbf{k})\right] = (\omega + ix)^2 + x^2 - c^2\mathbf{k}^2$$

Poles:

$$\omega^* = -ix \pm \sqrt{c^2\mathbf{k}^2 - x^2}$$



Correlations (in 2D): $\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, r')\right\rangle \simeq |\psi_0|^2 \exp\left[-\mathcal{D}_{\phi\phi}(t, r, r')\right]$

$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, 0)\right\rangle \simeq |\psi_0|^2 \exp\left[-\eta \begin{cases} \ln(r/\xi) & r \to \infty, t \simeq 0 \\ \frac{1}{2}\ln(c^2 t/x\xi^2) & r \simeq, t \to \infty \end{cases}\right]$$

[Szymańska et al., PRL '06; PRB '07]

Finite size effects: Single mode vs many mode

$$\left\langle \psi^\dagger(r,t)\psi(0,r')\right\rangle \simeq |\psi_0|^2 \exp\left[-\mathcal{D}_{\phi\phi}(t,r,r')\right]$$

Finite size effects: Single mode vs many mode

$$\left\langle \psi^\dagger(r,t)\psi(0,r')\right\rangle \simeq |\psi_0|^2 \exp\left[-\mathcal{D}_{\phi\phi}(t,r,r')\right]$$

$\mathcal{D}_{\phi\phi}(t,r,r')$ from sum of phase modes. Study $ct \gg r$ limit:

$$\mathcal{D}_{\phi\phi}(t,r,r) \propto \sum_n^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2(1-e^{i\omega t})}{|(\omega+ix)^2+x^2-(n\Delta)^2|^2}$$

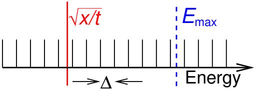
Finite size effects: Single mode vs many mode

$$\left\langle \psi^\dagger(r,t)\psi(0,r')\right\rangle \simeq |\psi_0|^2 \exp\left[-\mathcal{D}_{\phi\phi}(t,r,r')\right]$$

$\mathcal{D}_{\phi\phi}(t,r,r')$ from sum of phase modes. Study $ct \gg r$ limit:

$$\mathcal{D}_{\phi\phi}(t,r,r) \propto \sum_{n}^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2(1-e^{i\omega t})}{|(\omega+ix)^2+x^2-(n\Delta)^2|^2}$$

$\Delta \ll \sqrt{x/t} \ll E_{\max}$



$\mathcal{D}_{\phi\phi} \sim 1 + \ln(E_{\max}\sqrt{t/x})$

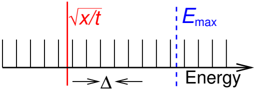
Finite size effects: Single mode vs many mode

$$\left\langle \psi^\dagger(r,t)\psi(0,r')\right\rangle \simeq |\psi_0|^2 \exp\left[-\mathcal{D}_{\phi\phi}(t,r,r')\right]$$

$\mathcal{D}_{\phi\phi}(t,r,r')$ from sum of phase modes. Study $ct \gg r$ limit:

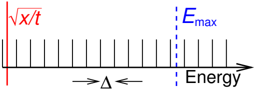
$$\mathcal{D}_{\phi\phi}(t,r,r) \propto \sum_n^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2(1-e^{i\omega t})}{|(\omega+ix)^2 + x^2 - (n\Delta)^2|^2}$$

$\Delta \ll \sqrt{x/t} \ll E_{\max}$



$\mathcal{D}_{\phi\phi} \sim 1 + \ln(E_{\max}\sqrt{t/x})$

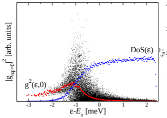
$\sqrt{x/t} \ll \Delta \ll E_{\max}$

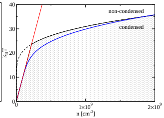


$\mathcal{D}_{\phi\phi} \sim \left(\frac{\pi C}{2x}\right)\left(\frac{t}{2x}\right)$

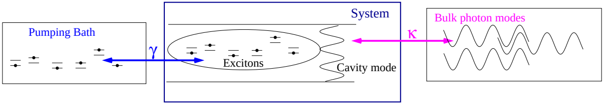
Conclusions

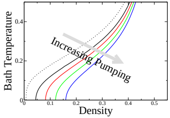
- Localised two-level system model
  Mean-field and fluctuations



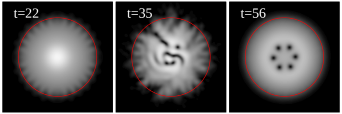


- Effects of pumping on mean-field theory

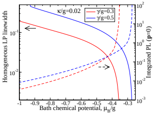


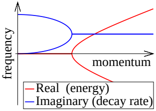


- Modification to Thomas-Fermi profile
  Spontaneous rotating vortex lattice



- Change to spectrum and correlations
  Phase modes and finite size

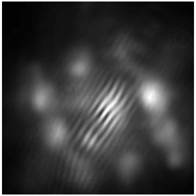


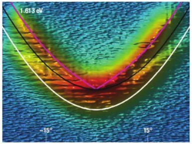


Extra slides

Other polariton condensation experiments

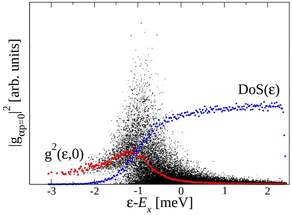
- Old measurements of $\langle N(t)N(t+\tau)\rangle$
  [Deng *et al* PNAS 100 15318 (2003)]
- Stress traps for polaritons
  [Balili *et al* Science 316 1007 (2007)]
- Temporal coherence and line narrowing
  [Love *et al* Phys. Rev. Lett. 101 067404 (2008)]
- Quantised vortices in disorder potential
  [Lagoudakis *et al* Nature Phys. 4, 706 (2008)]
- Changes to excitation spectrum
  [Utsunomiya *et al* Nature Phys. 4 700 (2008)]
- Soliton propagation
  [Amo *et al* Nature 457 291 (2009)]





Equilibrium: Mean-field theory

$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \text{H.c.} \right]$$



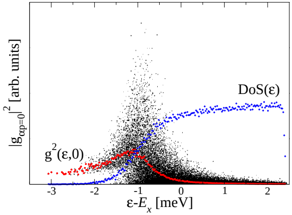
Equilibrium: Mean-field theory

$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \text{H.c.} \right]$$

Mean-field theory:

Self-consistent polarisation and field

$$\left[i\partial_t + \omega_0 - \frac{\nabla^2}{2m} \right] \psi = \frac{1}{\sqrt{A}} \sum_\alpha g_\alpha P_\alpha$$

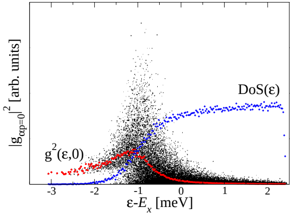


Equilibrium: Mean-field theory

$$H_{sys} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \text{H.c.} \right]$$

Mean-field theory:
Self-consistent polarisation and field

$$\left[i\partial_t + \omega_0 - \frac{\nabla^2}{2m} \right] \psi = \frac{1}{\sqrt{A}} \sum_{\alpha} g_\alpha \langle a^\dagger_\alpha b_\alpha \rangle$$



Equilibrium: Mean-field theory

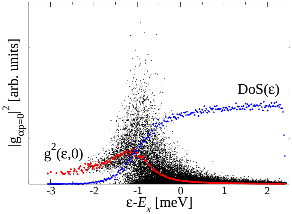
$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{\text{A}}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \text{H.c.} \right]$$

Mean-field theory:

Self-consistent polarisation and field

$$\left[i\partial_t + \omega_0 - \frac{\nabla^2}{2m} \right] \psi = \frac{1}{\sqrt{A}} \sum_\alpha g_\alpha \frac{g_\alpha \psi}{2E_\alpha} \tanh\left(\beta E_\alpha\right)$$

$$E_\alpha{}^2 = \left(\frac{\tilde{\epsilon}_\alpha}{2} \right)^2 + g_\alpha^2 \psi^2, \quad \tilde{\epsilon}_\alpha = \epsilon_\alpha - \mu$$



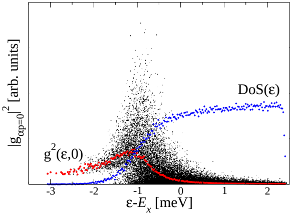
Equilibrium: Mean-field theory

$$H_{\mathrm{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{\mathrm{A}}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \mathrm{H.c.} \right]$$

Mean-field theory:
Self-consistent polarisation and field

$$\left[-\mu + \omega_0 - \frac{\nabla^2}{2m} \right] \psi = \frac{1}{\sqrt{A}} \sum_\alpha g_\alpha \frac{g_\alpha \psi}{2 E_\alpha} \tanh\left(\beta E_\alpha\right)$$

$$E_\alpha{}^2 = \left(\frac{\tilde{\epsilon}_\alpha}{2} \right)^2 + g_\alpha^2 \psi^2, \quad \tilde{\epsilon}_\alpha = \epsilon_\alpha - \mu$$



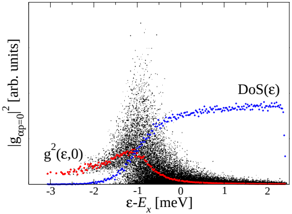
Equilibrium: Mean-field theory

$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \text{H.c.} \right]$$

Mean-field theory:
Self-consistent polarisation and field

$$\left[-\mu + \omega_0 - \frac{\nabla^2}{2m} \right] \psi = \frac{1}{\sqrt{A}} \sum_\alpha g_\alpha \frac{g_\alpha \psi}{2 E_\alpha} \tanh\left(\beta E_\alpha \right)$$

$$E_\alpha{}^2 = \left(\frac{\tilde{\epsilon}_\alpha}{2} \right)^2 + g_\alpha^2 \psi^2, \quad \tilde{\epsilon}_\alpha = \epsilon_\alpha - \mu$$



Density

$$\rho = |\psi|^2 + \frac{1}{A} \sum_\alpha \left[\frac{1}{2} - \frac{\tilde{\epsilon}_\alpha}{4 E_\alpha} \tanh(\beta E_\alpha) \right]$$

Equilibrium: Mean-field theory

$$H_{sys} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_\alpha \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \text{H.c.} \right]$$

Mean-field theory:

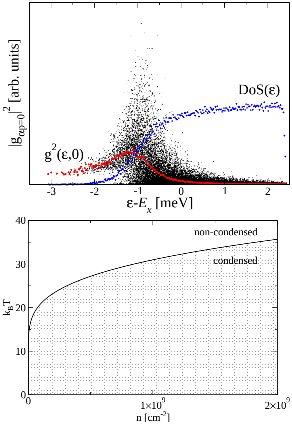
Self-consistent polarisation and field

$$\left[-\mu + \omega_0 - \frac{\nabla^2}{2m} \right] \psi = \frac{1}{\sqrt{A}} \sum_\alpha g_\alpha \frac{g_\alpha \psi}{2 E_\alpha} \tanh\left(\beta E_\alpha\right)$$

$$E_\alpha{}^2 = \left(\frac{\tilde{\epsilon}_\alpha}{2} \right)^2 + g_\alpha^2 \psi^2, \quad \tilde{\epsilon}_\alpha = \epsilon_\alpha - \mu$$

Density

$$\rho = |\psi|^2 + \frac{1}{A} \sum_\alpha \left[\frac{1}{2} - \frac{\tilde{\epsilon}_\alpha}{4 E_\alpha} \tanh(\beta E_\alpha) \right]$$



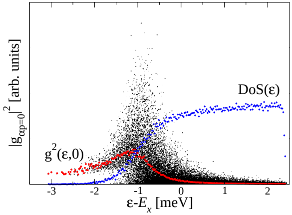
Equilibrium: Mean-field theory

$$H_{\mathrm{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_\alpha \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{A}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \mathrm{H.c.} \right]$$

Mean-field theory:
Self-consistent polarisation and field

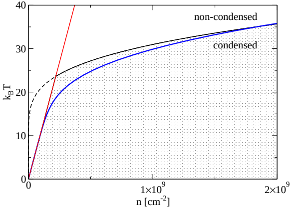
$$\left[-\mu + \omega_0 - \frac{\nabla^2}{2m} \right] \psi = \frac{1}{\sqrt{A}} \sum_\alpha g_\alpha \frac{g_\alpha \psi}{2E_\alpha} \tanh\left(\beta E_\alpha \right)$$

$$E_\alpha{}^2 = \left(\frac{\tilde{\epsilon}_\alpha}{2} \right)^2 + g_\alpha^2 \psi^2, \quad \tilde{\epsilon}_\alpha = \epsilon_\alpha - \mu$$



Density

$$\rho = |\psi|^2 + \frac{1}{A} \sum_\alpha \left[\frac{1}{2} - \frac{\tilde{\epsilon}_\alpha}{4E_\alpha} \tanh(\beta E_\alpha) \right]$$



Equilibrium: Fluctuations about mean-field

Fluctuations $\psi \to \psi + \delta\psi$; correction to action

$$S = S_0 + \sum_{\nu,\mathbf{p},\mathbf{q}} \begin{pmatrix} \delta\psi^*_{\mathbf{p}} \\ \delta\psi_{-\mathbf{p}} \end{pmatrix}^T \mathcal{G}^{-1}_{\mathbf{pq}}(\nu) \begin{pmatrix} \delta\psi_{\mathbf{q}} \\ \delta\psi^*_{-\mathbf{q}} \end{pmatrix}$$

Equilibrium: Fluctuations about mean-field

Fluctuations $\psi \to \psi + \delta\psi$; correction to action

$$S = S_0 + \sum_{\nu,\mathbf{p},\mathbf{q}} \begin{pmatrix} \delta\psi^*_{\mathbf{p}} \\ \delta\psi_{-\mathbf{p}} \end{pmatrix}^T \mathcal{G}^{-1}_{\mathbf{pq}}(\nu) \begin{pmatrix} \delta\psi_{\mathbf{q}} \\ \delta\psi^*_{-\mathbf{q}} \end{pmatrix}$$

$$\mathcal{G}^{-1}_{\mathbf{pq}}(\nu) = (\omega_{\mathbf{p}} + i\nu)\mathbb{1}\delta_{\mathbf{pq}} + \sum_{\alpha} g^*_{\alpha\mathbf{p}} g_{\alpha\mathbf{q}} \begin{pmatrix} \chi^{(1)}_{\alpha}(\nu) & \chi^{(2)}_{\alpha}(\nu) \\ \chi^{(2)}_{\alpha}(\nu) & \chi^{(1)*}_{\alpha}(\nu) \end{pmatrix}$$

Equilibrium: Fluctuations about mean-field

Fluctuations $\psi \to \psi + \delta\psi$; correction to action

$$S = S_0 + \sum_{\nu,\mathbf{p},\mathbf{q}} \begin{pmatrix} \delta\psi^*_{\mathbf{p}} \\ \delta\psi_{-\mathbf{p}} \end{pmatrix}^T \mathcal{G}^{-1}_{\mathbf{pq}}(\nu) \begin{pmatrix} \delta\psi_{\mathbf{q}} \\ \delta\psi^*_{-\mathbf{q}} \end{pmatrix}$$

$$\mathcal{G}^{-1}_{\mathbf{pq}}(\nu) = (\omega_{\mathbf{p}} + i\nu)\mathbb{1}\delta_{\mathbf{pq}} + \sum_{\alpha} g^*_{\alpha\mathbf{p}} g_{\alpha\mathbf{q}} \begin{pmatrix} \chi^{(1)}_{\alpha}(\nu) & \chi^{(2)}_{\alpha}(\nu) \\ \chi^{(2)}_{\alpha}(\nu) & \chi^{(1)*}_{\alpha}(\nu) \end{pmatrix}$$

- Optical response:
  Treat $\mathbf{p} \neq \mathbf{q}$ perturbatively [D. M. Whittaker PRL 80 4791]
  - Spectral weight $\quad W(\nu, \mathbf{p}) \quad = \quad 2\Im\left[\mathcal{G}^{11}_{\mathbf{pp}}(i\nu)\right]$

Equilibrium: Fluctuations about mean-field

Fluctuations $\psi \to \psi + \delta\psi$; correction to action

$$S = S_0 + \sum_{\nu,\mathbf{p},\mathbf{q}} \begin{pmatrix} \delta\psi^*_{\mathbf{p}} \\ \delta\psi_{-\mathbf{p}} \end{pmatrix}^T \mathcal{G}^{-1}_{\mathbf{pq}}(\nu) \begin{pmatrix} \delta\psi_{\mathbf{q}} \\ \delta\psi^*_{-\mathbf{q}} \end{pmatrix}$$

$$\mathcal{G}^{-1}_{\mathbf{pq}}(\nu) = (\omega_{\mathbf{p}} + i\nu)\mathbb{1}\delta_{\mathbf{pq}} + \sum_{\alpha} g^*_{\alpha\mathbf{p}} g_{\alpha\mathbf{q}} \begin{pmatrix} \chi^{(1)}_{\alpha}(\nu) & \chi^{(2)}_{\alpha}(\nu) \\ \chi^{(2)}_{\alpha}(\nu) & \chi^{(1)*}_{\alpha}(\nu) \end{pmatrix}$$

- Optical response:
  Treat $\mathbf{p} \neq \mathbf{q}$ perturbatively [D. M. Whittaker PRL 80 4791]
  - Spectral weight $W(\nu, \mathbf{p}) = 2\Im\left[\mathcal{G}^{11}_{\mathbf{pp}}(i\nu)\right]$
  - Emission $P_{\text{emit}}(\nu, \mathbf{p}) = n_B(\nu) W(\nu, \mathbf{p})$
  - Absorption $P_{\text{absorb}}(\nu, \mathbf{p}) = (1 + n_B(\nu)) W(\nu, \mathbf{p})$

Equilibrium: Fluctuations about mean-field

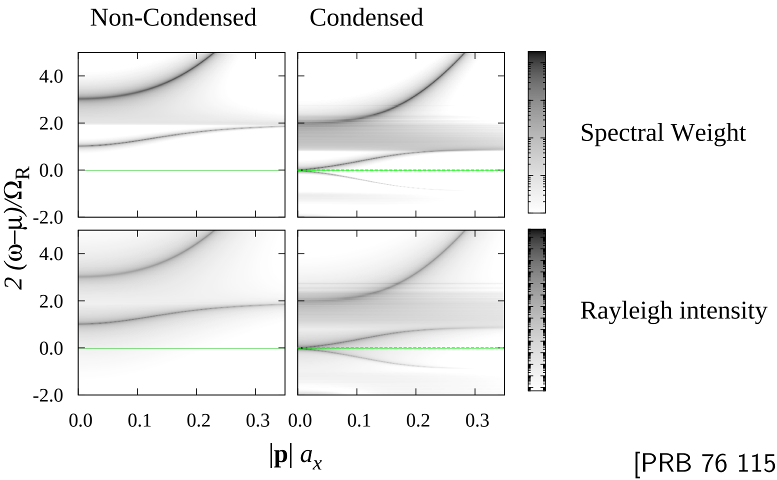
Fluctuations $\psi \to \psi + \delta\psi$; correction to action

$$S = S_0 + \sum_{\nu,\mathbf{p},\mathbf{q}} \begin{pmatrix} \delta\psi^*_{\mathbf{p}} \\ \delta\psi_{-\mathbf{p}} \end{pmatrix}^T \mathcal{G}^{-1}_{\mathbf{pq}}(\nu) \begin{pmatrix} \delta\psi_{\mathbf{q}} \\ \delta\psi^*_{-\mathbf{q}} \end{pmatrix}$$

$$\mathcal{G}^{-1}_{\mathbf{pq}}(\nu) = (\omega_{\mathbf{p}} + i\nu)\mathbb{1}\delta_{\mathbf{pq}} + \sum_{\alpha} g^*_{\alpha\mathbf{p}} g_{\alpha\mathbf{q}} \begin{pmatrix} \chi^{(1)}_{\alpha}(\nu) & \chi^{(2)}_{\alpha}(\nu) \\ \chi^{(2)}_{\alpha}(\nu) & \chi^{(1)*}_{\alpha}(\nu) \end{pmatrix}$$

- Optical response:
  Treat $\mathbf{p} \neq \mathbf{q}$ perturbatively [D. M. Whittaker PRL 80 4791]
  - Spectral weight $W(\nu, \mathbf{p}) = 2\Im\left[\mathcal{G}^{11}_{\mathbf{pp}}(i\nu)\right]$
  - Emission $P_{\text{emit}}(\nu, \mathbf{p}) = n_B(\nu) W(\nu, \mathbf{p})$
  - Absorption $P_{\text{absorb}}(\nu, \mathbf{p}) = (1 + n_B(\nu)) W(\nu, \mathbf{p})$
  - Rayleigh scattering $I_{\mathbf{p}\neq\mathbf{q}}(\nu) = |\mathcal{G}^{11}_{\mathbf{pq}}(i\nu)|^2$

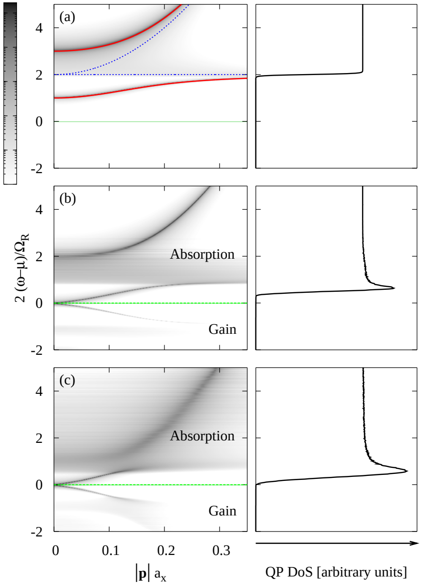
Fluctuations and optical spectra



[PRB 76 115326 (2007)]

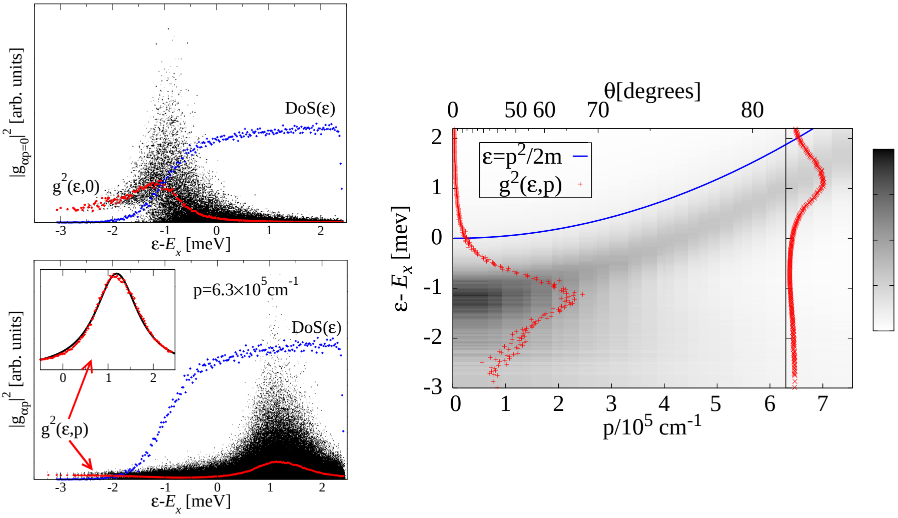
- Phase sensitive Rayleigh → "negative energy" Bogoliubov modes.

Exciton disorder and polariton density of states



- "Dark" exciton states left at exciton energy.
- Dark states not truly dark, but weak coupling.
- No gap in condensate due to weak coupling tail.

Disorder localised states



Fluctuation corrections to phase boundary

Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$

In 2D system: modified critical condition:

$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$

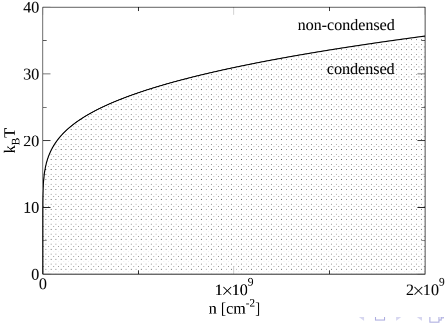
Fluctuation corrections to phase boundary

Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$

In 2D system: modified critical condition:

$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



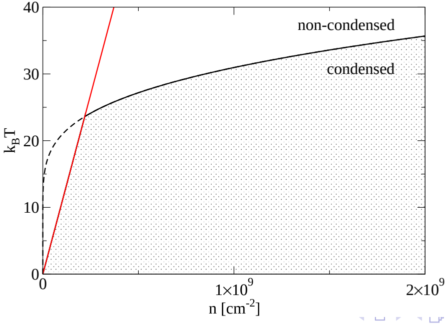
Fluctuation corrections to phase boundary

Fluctuation corrections to density

$$\rho \rightarrow \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$

In 2D system: modified critical condition:

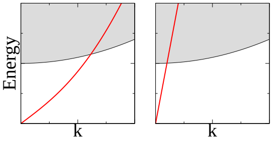
$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



Fluctuation corrections to phase boundary

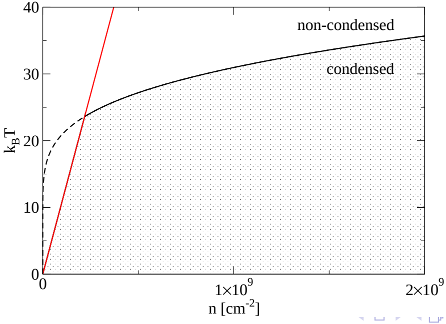
Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$



In 2D system: modified critical condition:

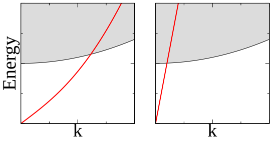
$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



Fluctuation corrections to phase boundary

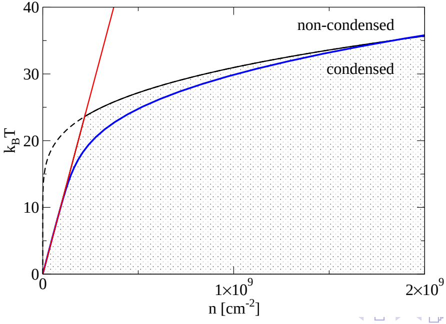
Fluctuation corrections to density

$$\rho \rightarrow \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$



In 2D system: modified critical condition:

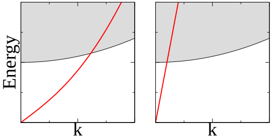
$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



Fluctuation corrections to phase boundary

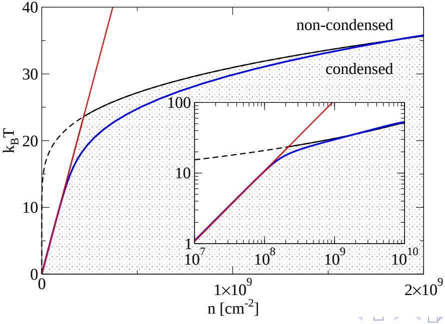
Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_{q} \langle \delta\psi^\dagger \delta\psi \rangle$$



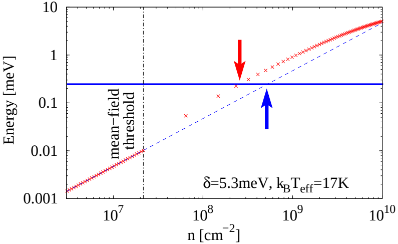
In 2D system: modified critical condition:

$$\rho_s = \rho - \rho_{\text{normal}} = \# \frac{2m^2}{\hbar} k_B T$$



Blueshfit and experimental phase boundary

Blueshift:



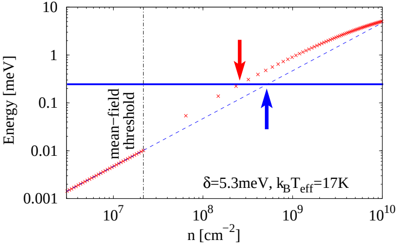
Clean limit:

$$\delta E_{\text{LP}} \simeq \mathcal{R}y_{\text{X}} a_{\text{X}}^2 n + \Omega_{\text{R}} a_{\text{X}}^2 n$$

Here: $\Omega_{\text{R}} a_{\text{X}}^2 \rightarrow \Omega_{\text{R}} \xi^2$
[PRB 77 235313]

Blueshfit and experimental phase boundary

Blueshift:

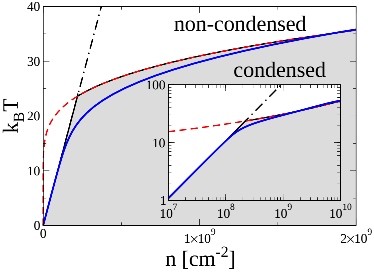


Clean limit:

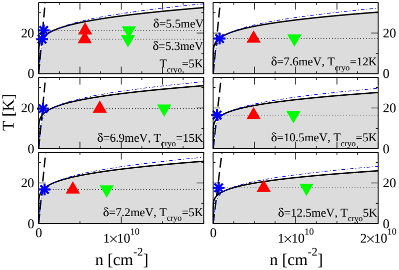
$$\delta E_{LP} \simeq \mathcal{R}y_X a_X^2 n + \Omega_R a_X^2 n$$

Here: $\Omega_R a_X^2 \to \Omega_R \xi^2$
[PRB 77 235313]

Phase diagram:



CdTe:



Instability of Thomas-Fermi: details

$\psi = \sqrt{\rho}e^{i\phi} \quad \mathbf{v} = \frac{\hbar}{m}\nabla\phi$

$$\frac{1}{2}\partial_t\rho + \nabla\cdot(\rho\mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$

$$\partial_t\mathbf{v} + \nabla(U\rho + \frac{m\omega^2}{2}r^2 + \frac{m}{2}|\mathbf{v}|^2) = 0$$

Instability of Thomas-Fermi: details

$\psi = \sqrt{\rho} e^{i\phi} \quad \mathbf{v} = \frac{\hbar}{m} \nabla \phi$

$$\frac{1}{2} \partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = (\gamma_{\text{net}} - \Gamma \rho) \rho$$

$$\partial_t \mathbf{v} + \nabla \left(U\rho + \frac{m\omega^2}{2} r^2 + \frac{m}{2} |\mathbf{v}|^2 \right) = 0$$

Consider $\rho \rightarrow \rho + \delta\rho, \mathbf{v} \rightarrow \mathbf{v} + \delta\mathbf{v}$.

Instability of Thomas-Fermi: details

$\psi = \sqrt{\rho}e^{i\phi} \quad \mathbf{v} = \frac{\hbar}{m}\nabla\phi$

$$\frac{1}{2}\partial_t\rho + \nabla\cdot(\rho\mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$

$$\partial_t\mathbf{v} + \nabla(U\rho + \frac{m\omega^2}{2}r^2 + \frac{m}{2}|\mathbf{v}|^2) = 0$$

Consider $\rho \to \rho + \delta\rho, \mathbf{v} \to \mathbf{v} + \delta\mathbf{v}$.

If $\gamma_{\text{net}}, \Gamma \to 0$, can find normal modes in 2D trap:

$$\delta\rho_{n,m}(r,\theta,t) = e^{im\theta}h_{n,m}(r)e^{i\omega_{n,m}t}$$

$$\omega_{n,m} = \omega 2\sqrt{m(1+2n) + 2n(n+1)}$$

Instability of Thomas-Fermi: details

$\psi = \sqrt{\rho} e^{i\phi} \quad \mathbf{v} = \frac{\hbar}{m}\nabla\phi$

$$\frac{1}{2}\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$

$$\partial_t \mathbf{v} + \nabla(U\rho + \frac{m\omega^2}{2} r^2 + \frac{m}{2}|\mathbf{v}|^2) = 0$$

If $\gamma_{\text{net}}, \Gamma \to 0$, can find normal modes in 2D trap:

$$\delta\rho_{n,m}(r,\theta,t) = e^{im\theta} h_{n,m}(r) e^{i\omega_{n,m} t}$$

$$\omega_{n,m} = \omega 2\sqrt{m(1+2n) + 2n(n+1)}$$

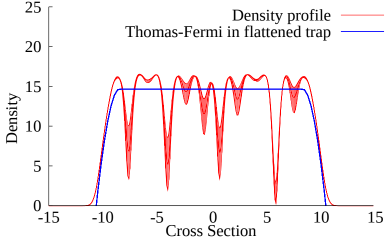
Consider $\rho \to \rho + \delta\rho, \mathbf{v} \to \mathbf{v} + \delta\mathbf{v}$.

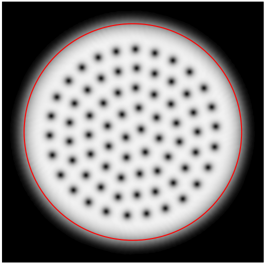
Add weak pumping/decay:

$$\omega_{n,n} \to \omega_{n,m} + i\gamma_{\text{net}} \left[\frac{m(1+2n) + 2n(n+1) - m^2}{2m(1+2n) + 4n(n+1) + m^2} \right]$$

Instability

Why vortices



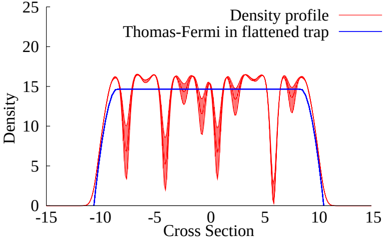


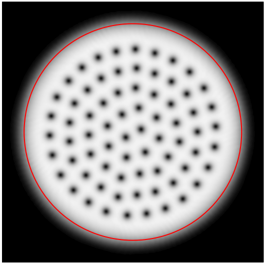
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\mathrm{net}}\Theta(R - r) - \Gamma\rho)\rho,$$

$$\mu = \frac{m}{2}|\mathbf{v} - \Omega \times \mathbf{r}|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{\mathrm{net}}}{\Gamma}\Theta(R - r) = \frac{\mu}{U}$$

Why vortices





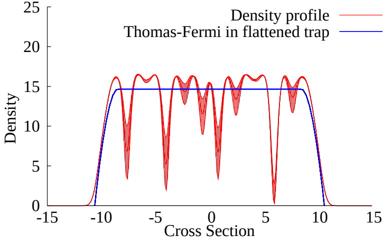
Rotating solution: $i\partial_t\psi = (\mu - 2\Omega L_z)\psi$

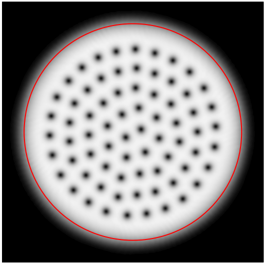
$$\nabla\cdot[\rho(\mathbf{v}-\Omega\times\mathbf{r})] = (\gamma_{\text{net}}\Theta(R-r)-\Gamma\rho)\rho,$$

$$\mu = \frac{m}{2}|\mathbf{v}-\Omega\times\mathbf{r}|^2 + \frac{m}{2}r^2(\omega^2-\Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega\times\mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{\text{net}}}{\Gamma}\Theta(R-r) = \frac{\mu}{U}$$

Why vortices



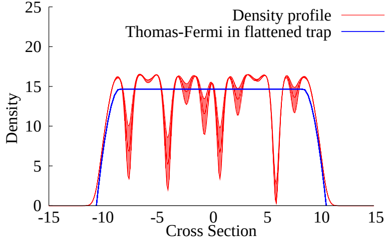


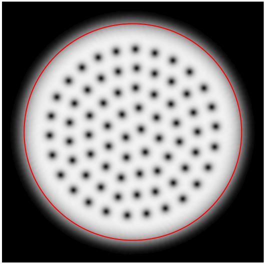
Rotating solution: $i\partial_t\psi = (\mu - 2\Omega L_z)\psi$

$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\text{net}}\Theta(R-r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}\left|\mathbf{v} - \Omega \times \mathbf{r}\right|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

Why vortices



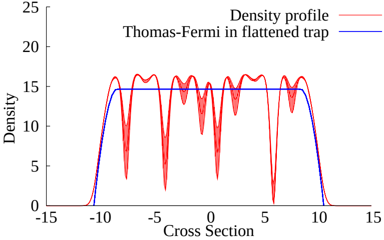


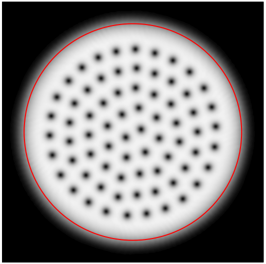
Rotating solution: $i\partial_t \psi = (\mu - 2\Omega L_z)\psi$

$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\mathrm{net}}\Theta(R - r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}\left|\mathbf{v} - \Omega \times \mathbf{r}\right|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r},$$





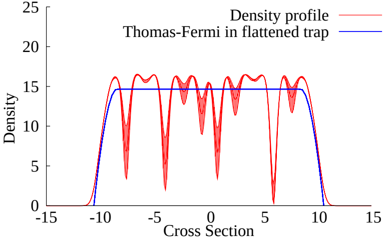
Rotating solution: $i\partial_t\psi = (\mu - 2\Omega L_z)\psi$

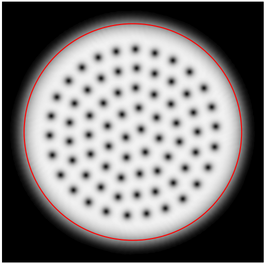
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\rm net}\Theta(R-r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}\left|\mathbf{v} - \Omega \times \mathbf{r}\right|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r}, \qquad \Omega = \omega, \qquad \rho = \frac{\gamma_{\rm net}}{\Gamma}\Theta(R-r) = \frac{\mu}{U}$$

Why vortices





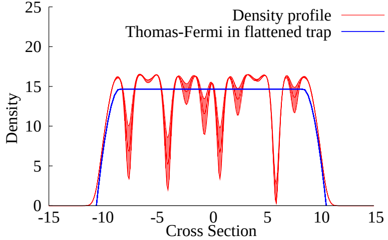
Rotating solution: $i\partial_t\psi = (\mu - 2\Omega L_z)\psi$

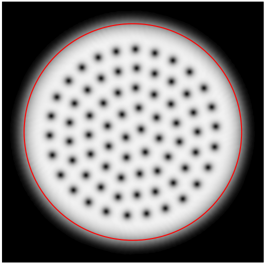
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\text{net}}\Theta(R-r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}\left|\mathbf{v} - \Omega \times \mathbf{r}\right|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{\text{net}}}{\Gamma}\Theta(R-r) = \frac{\mu}{U}$$

Why vortices





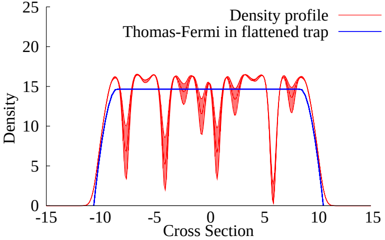
Rotating solution: $i\partial_t\psi = (\mu - 2\Omega L_z)\psi$

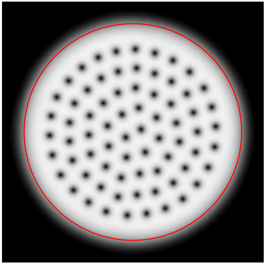
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\text{net}}\Theta(R-r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}\left|\mathbf{v} - \Omega \times \mathbf{r}\right|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{\text{net}}}{\Gamma}\Theta(R-r) = \frac{\mu}{U}$$

Why vortices





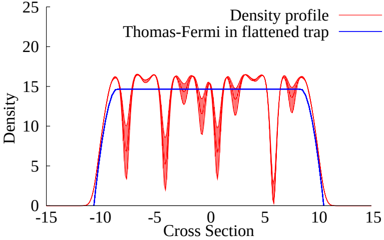
Rotating solution: $i\partial_t \psi = (\mu - 2\Omega L_z)\psi$

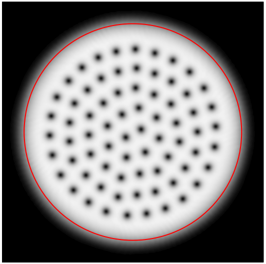
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\text{net}}\Theta(R-r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}|\mathbf{v} - \Omega \times \mathbf{r}|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{\text{net}}}{\Gamma}\Theta(R-r)$$

Why vortices





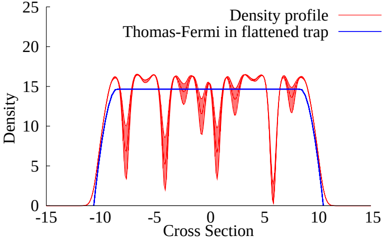
Rotating solution: $i\partial_t \psi = (\mu - 2\Omega L_z)\psi$

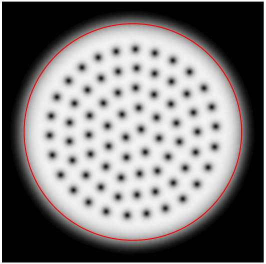
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\text{net}} \Theta(R - r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}\left|\mathbf{v} - \Omega \times \mathbf{r}\right|^2 + \frac{m}{2} r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2 \nabla^2 \sqrt{\rho}}{2m\sqrt{\rho}}$$

$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{\text{net}}}{\Gamma}\Theta(R - r)$$

Why vortices





Rotating solution: $i\partial_t\psi = (\mu - 2\Omega L_z)\psi$

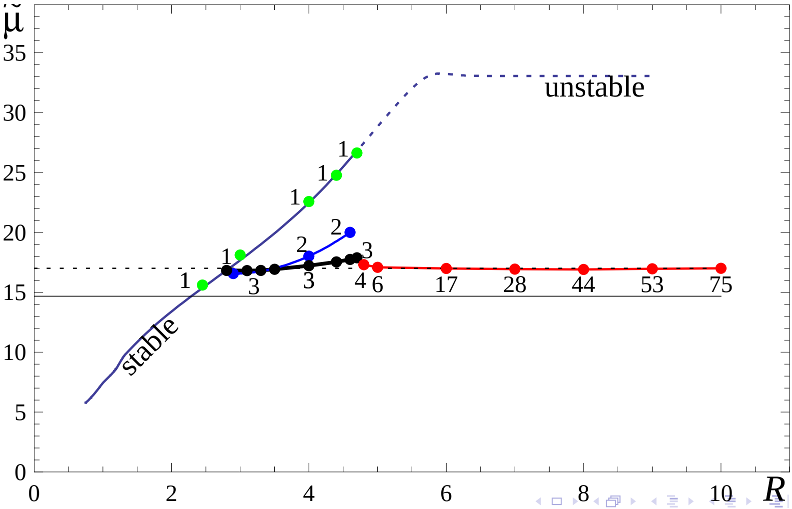
$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\text{net}}\Theta(R-r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}|\mathbf{v} - \Omega \times \mathbf{r}|^2 + \frac{m}{2}r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}$$

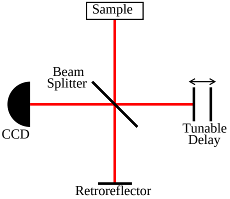
$$\mathbf{v} = \Omega \times \mathbf{r}, \quad \Omega = \omega, \quad \rho = \frac{\gamma_{\text{net}}}{\Gamma}\Theta(R-r) = \frac{\mu}{U}$$

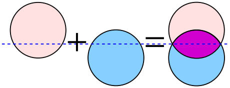
Why vortices: chemical potential vs size

Thomas-Fermi : $\mu \propto R^2$ Vortex : $\mu = \frac{U\gamma_{\text{net}}}{\Gamma}$

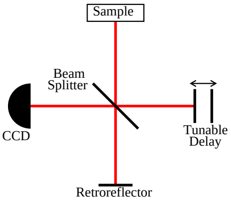


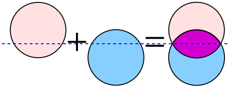
Observing vortices: fringe pattern

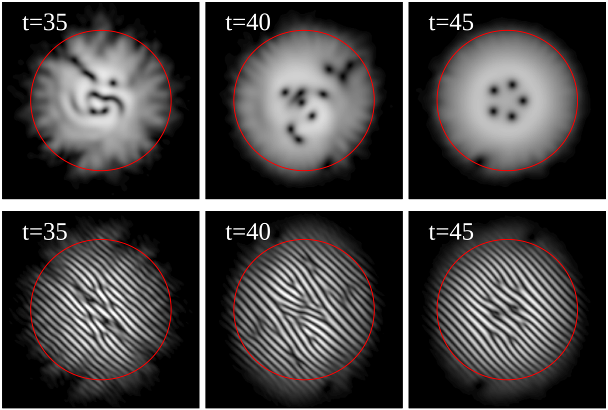




Observing vortices: fringe pattern







Superfluidity

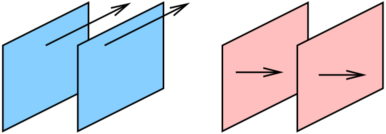
Superfluidity:

$$\mathbf{J} = \rho \mathbf{v} = \Psi^\dagger i\hbar\nabla\Psi = |\Psi|^2\nabla\phi$$

Superfluidity

Superfluidity:

$$\mathbf{J} = \rho \mathbf{v} = \Psi^\dagger i\hbar\nabla\Psi = |\Psi|^2\nabla\phi$$

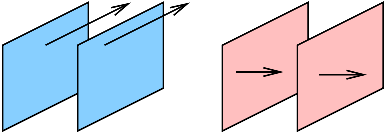


$$\chi_{ij}(\omega = 0, \mathbf{q} \to 0) = \langle J_i(\mathbf{q}) J_j(\mathbf{q}) \rangle$$
$$= \chi_T(q)\left(\delta_{ij} - \frac{q_i q_j}{q^2}\right) + \chi_L(q)\frac{q_i q_j}{q^2}$$

Superfluidity

Superfluidity:

$$\mathbf{J} = \rho \mathbf{v} = \Psi^\dagger i\hbar \nabla \Psi = |\Psi|^2 \nabla \phi$$



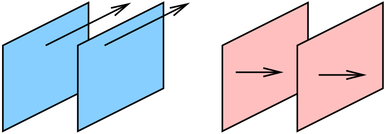
$$\chi_{ij}(\omega = 0, \mathbf{q} \to 0) = \langle J_i(\mathbf{q}) J_j(\mathbf{q}) \rangle$$
$$= \chi_T(q) \left(\delta_{ij} - \frac{q_i q_j}{q^2} \right) + \chi_L(q) \frac{q_i q_j}{q^2}$$

Superfluid part,
$\rho_s \propto \lim_{q \to 0} (\chi_L - \chi_T)$.

Superfluidity

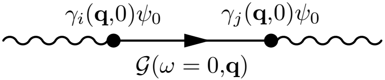
Superfluidity:

$$\mathbf{J} = \rho \mathbf{v} = \Psi^\dagger i\hbar\nabla\Psi = |\Psi|^2\nabla\phi$$



$$\chi_{ij}(\omega = 0, \mathbf{q} \to 0) = \langle J_i(\mathbf{q}) J_j(\mathbf{q})\rangle$$
$$= \chi_T(q)\left(\delta_{ij} - \frac{q_i q_j}{q^2}\right) + \chi_L(q)\frac{q_i q_j}{q^2}$$

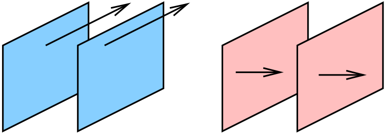
Superfluid part,
$\rho_s \propto \lim_{q\to 0}(\chi_L - \chi_T)$.
$J_i(q) = \langle \psi^\dagger \gamma_i(q)\psi\rangle$

$\Delta\chi_{ij}(q) =$  $+ \dots$

Superfluidity

Superfluidity:

$$\mathbf{J} = \rho \mathbf{v} = \Psi^\dagger i\hbar \nabla \Psi = |\Psi|^2 \nabla \phi$$

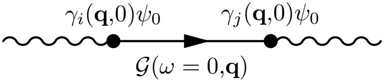


$$\chi_{ij}(\omega = 0, \mathbf{q} \to 0) = \langle J_i(\mathbf{q}) J_j(\mathbf{q}) \rangle$$
$$= \chi_T(q) \left(\delta_{ij} - \frac{q_i q_j}{q^2} \right) + \chi_L(q) \frac{q_i q_j}{q^2}$$

Superfluid part,

$\rho_s \propto \lim_{q \to 0} (\chi_L - \chi_T)$.

$J_i(q) = \langle \psi^\dagger \gamma_i(q) \psi \rangle$

$\Delta\chi_{ij}(q) =$ 

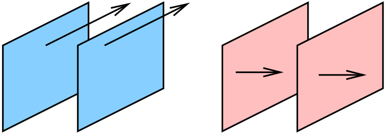
$+ \dots$

$$= \gamma_i(q) \mathcal{G}_R(\omega = 0, \vec{q}) \gamma_j(q) + \dots$$

Superfluidity

Superfluidity:

$$\mathbf{J} = \rho \mathbf{v} = \Psi^\dagger i\hbar\nabla\Psi = |\Psi|^2 \nabla\phi$$



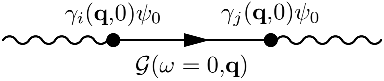
$$\chi_{ij}(\omega = 0, \mathbf{q} \to 0) = \langle J_i(\mathbf{q}) J_j(\mathbf{q}) \rangle$$
$$= \chi_T(q) \left(\delta_{ij} - \frac{q_i q_j}{q^2} \right) + \chi_L(q) \frac{q_i q_j}{q^2}$$

Superfluid part,

$\rho_s \propto \lim_{q \to 0} (\chi_L - \chi_T)$.

$J_i(q) = \langle \psi^\dagger \gamma_i(q) \psi \rangle$

$\Delta\chi_{ij}(q) =$



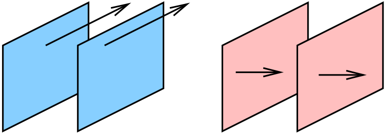
$+ \dots$

$$= \gamma_i(q) \mathcal{G}_R(\omega = 0, \vec{q}) \gamma_j(q) + \dots$$
$$\propto \frac{q_i \, (\dots) \, q_j}{(\omega + ix)^2 + x^2 - c^2 \mathbf{q}^2} + \dots$$

Superfluidity

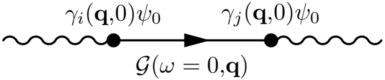
Superfluidity:

$$\mathbf{J} = \rho \mathbf{v} = \Psi^\dagger i\hbar\nabla\Psi = |\Psi|^2\nabla\phi$$



$$\chi_{ij}(\omega = 0, \mathbf{q} \to 0) = \langle J_i(\mathbf{q}) J_j(\mathbf{q})\rangle$$
$$= \chi_T(q)\left(\delta_{ij} - \frac{q_i q_j}{q^2}\right) + \chi_L(q)\frac{q_i q_j}{q^2}$$

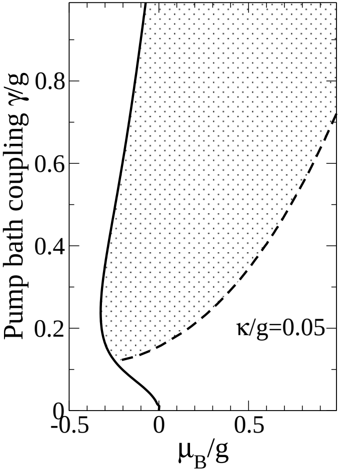
Superfluid part,
$\rho_s \propto \lim_{q\to 0}(\chi_L - \chi_T)$.
$J_i(q) = \langle\psi^\dagger\gamma_i(q)\psi\rangle$
Static ρ_S survies

$\Delta\chi_{ij}(q) =$  $+ \ldots$

$$= \gamma_i(q)\mathcal{G}_R(\omega = 0, \vec{q})\gamma_j(q) + \ldots$$
$$\propto \frac{q_i\,(\ldots)\,q_j}{(\omega + ix)^2 + x^2 - c^2\mathbf{q}^2} + \ldots$$

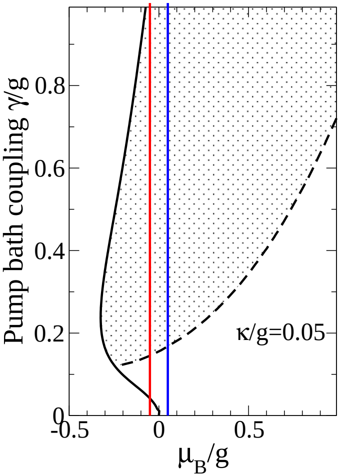
Zero temperature phase diagram

$$\tilde{\omega}_0 - i\kappa = g^2\gamma \sum_\alpha \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(\tilde{\epsilon}_\alpha + i\gamma)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}.$$



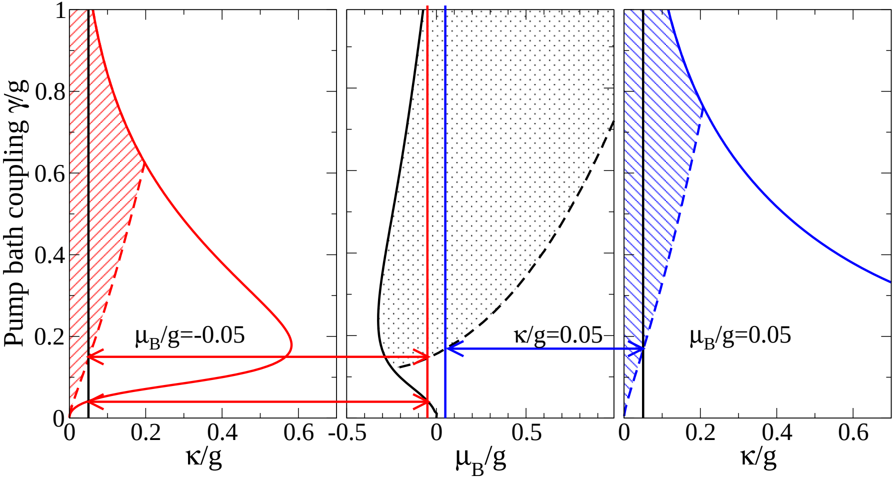
Zero temperature phase diagram

$$\tilde{\omega}_0 - i\kappa = g^2\gamma \sum_\alpha \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(\tilde{\epsilon}_\alpha + i\gamma)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}.$$



Zero temperature phase diagram

$$\tilde{\omega}_0 - i\kappa = g^2\gamma \sum_{\alpha} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(\tilde{\epsilon}_\alpha + i\gamma)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}.$$



Fluctuations → Stability, Luminescence, Absorption

Keldysh approach:

$$\mathcal{G}_{R,A} = \mp i\theta[\pm(t-t')]\left\langle\left[\psi^\dagger,\psi\right]_-\right\rangle$$

$$\mathcal{G}_K = -i\left\langle\left[\psi^\dagger,\psi\right]_+\right\rangle$$

Fluctuations → Stability, Luminescence, Absorption

Keldysh approach:

$$\mathcal{G}_R - \mathcal{G}_A = -i\left\langle \left[\psi^\dagger, \psi\right]_- \right\rangle$$

$$\mathcal{G}_K = -i\left\langle \left[\psi^\dagger, \psi\right]_+ \right\rangle$$

Fluctuations → Stability, Luminescence, Absorption

Keldysh approach:

$$\mathcal{G}_R - \mathcal{G}_A = -i\left\langle \left[\psi^\dagger, \psi\right]_- \right\rangle$$

$$\mathcal{G}_K = -i\left\langle \left[\psi^\dagger, \psi\right]_+ \right\rangle = (2n(\omega)+1)(\mathcal{G}_R - \mathcal{G}_A)$$

Fluctuations → Stability, Luminescence, Absorption

Keldysh approach:

$$\mathcal{G}_R - \mathcal{G}_A = -i\left\langle \left[\psi^\dagger, \psi\right]_- \right\rangle$$

$$\mathcal{G}_K = -i\left\langle \left[\psi^\dagger, \psi\right]_+ \right\rangle = (2n(\omega)+1)(\mathcal{G}_R - \mathcal{G}_A)$$

Fluctuations → Stability, Luminescence, Absorption

Keldysh approach:

$$\mathcal{G}_R - \mathcal{G}_A = -i\left\langle\left[\psi^\dagger,\psi\right]_-\right\rangle$$

$$\mathcal{G}_K = -i\left\langle\left[\psi^\dagger,\psi\right]_+\right\rangle = (2n(\omega)+1)(\mathcal{G}_R - \mathcal{G}_A)$$

$$\begin{aligned}\mathcal{G}_K &= -\frac{1}{{\mathcal{G}_\mathcal{R}}^{-1}}\mathcal{G}_K^{-1}\frac{1}{[{\mathcal{G}_\mathcal{R}}^{-1}]^\dagger}\\ \mathcal{G}_R - \mathcal{G}_A &= \frac{1}{{\mathcal{G}_\mathcal{R}}^{-1}} - \frac{1}{[{\mathcal{G}_\mathcal{R}}^{-1}]^\dagger}\end{aligned}$$

Fluctuations → Stability, Luminescence, Absorption

Keeldysh approach:

$$\mathcal{G}_R - \mathcal{G}_A = -i\left\langle \left[\psi^\dagger, \psi\right]_- \right\rangle$$

$$\mathcal{G}_K = -i\left\langle \left[\psi^\dagger, \psi\right]_+ \right\rangle = (2n(\omega)+1)(\mathcal{G}_R - \mathcal{G}_A)$$

$$\mathcal{G}_K = \frac{-{\mathcal{G}_{\mathcal{K}}}^{-1}}{\mathrm{Im}[{\mathcal{G}_{\mathcal{R}}}^{-1}]^2 + \mathrm{Re}[{\mathcal{G}_{\mathcal{R}}}^{-1}]^2}$$

$$\mathcal{G}_R - \mathcal{G}_A = \frac{2\mathrm{Im}[{\mathcal{G}_{\mathcal{R}}}^{-1}]}{\mathrm{Im}[{\mathcal{G}_{\mathcal{R}}}^{-1}]^2 + \mathrm{Re}[{\mathcal{G}_{\mathcal{R}}}^{-1}]^2}$$

Fluctuations → Stability, Luminescence, Absorption

Keldysh approach:

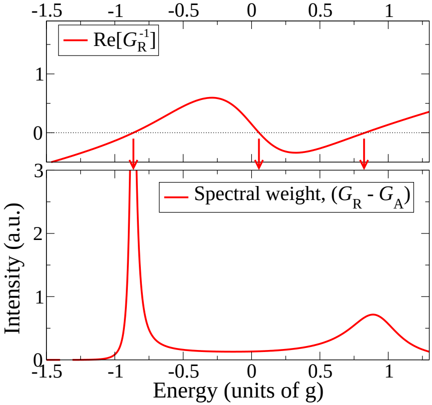
$$\mathcal{G}_R - \mathcal{G}_A = -i \left\langle \left[\psi^\dagger, \psi\right]_- \right\rangle$$

$$\mathcal{G}_K = -i \left\langle \left[\psi^\dagger, \psi\right]_+ \right\rangle = (2n(\omega)+1)(\mathcal{G}_R - \mathcal{G}_A)$$

$$\mathcal{G}_K = \frac{-\mathcal{G_K}^{-1}}{\mathrm{Im}[\mathcal{G_R}^{-1}]^2 + \mathrm{Re}[\mathcal{G_R}^{-1}]^2}$$

$$\mathcal{G}_R - \mathcal{G}_A = \frac{2\mathrm{Im}[\mathcal{G_R}^{-1}]}{\mathrm{Im}[\mathcal{G_R}^{-1}]^2 + \mathrm{Re}[\mathcal{G_R}^{-1}]^2}$$

$$\mathcal{G}_R^{-1}(\omega, k) = (\omega - \omega_k^*)$$



Fluctuations → Stability, Luminescence, Absorption

Keldysh approach:

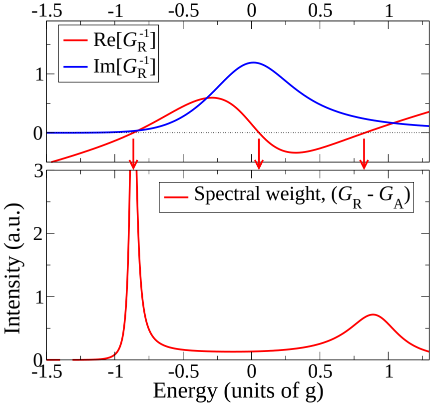
$$\mathcal{G}_R - \mathcal{G}_A = -i\left\langle \left[\psi^\dagger, \psi\right]_- \right\rangle$$

$$\mathcal{G}_K = -i\left\langle \left[\psi^\dagger, \psi\right]_+ \right\rangle = (2n(\omega)+1)(\mathcal{G}_R - \mathcal{G}_A)$$

$$\mathcal{G}_K = \frac{-\mathcal{G}_\mathcal{K}{}^{-1}}{\mathrm{Im}[\mathcal{G}_\mathcal{R}{}^{-1}]^2 + \mathrm{Re}[\mathcal{G}_\mathcal{R}{}^{-1}]^2}$$

$$\mathcal{G}_R - \mathcal{G}_A = \frac{2\mathrm{Im}[\mathcal{G}_\mathcal{R}{}^{-1}]}{\mathrm{Im}[\mathcal{G}_\mathcal{R}{}^{-1}]^2 + \mathrm{Re}[\mathcal{G}_\mathcal{R}{}^{-1}]^2}$$

$$\mathcal{G}_R^{-1}(\omega, k) = (\omega - \omega_k^*) + i\alpha$$



Fluctuations → Stability, Luminescence, Absorption

Keldysh approach:

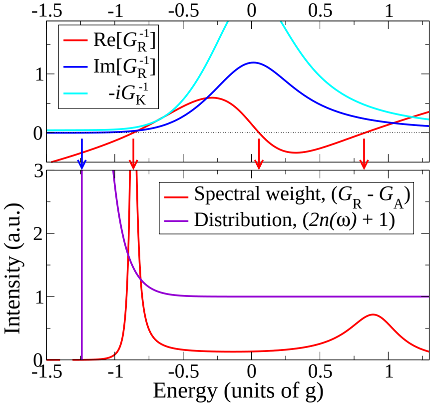
$$\mathcal{G}_R - \mathcal{G}_A = -i\left\langle \left[\psi^\dagger, \psi\right]_- \right\rangle$$

$$\mathcal{G}_K = -i\left\langle \left[\psi^\dagger, \psi\right]_+ \right\rangle = (2n(\omega)+1)(\mathcal{G}_R - \mathcal{G}_A)$$

$$\mathcal{G}_K = \frac{-{\mathcal{G}_\mathcal{K}}^{-1}}{\mathrm{Im}[{\mathcal{G}_\mathcal{R}}^{-1}]^2 + \mathrm{Re}[{\mathcal{G}_\mathcal{R}}^{-1}]^2}$$

$$\mathcal{G}_R - \mathcal{G}_A = \frac{2\mathrm{Im}[{\mathcal{G}_\mathcal{R}}^{-1}]}{\mathrm{Im}[{\mathcal{G}_\mathcal{R}}^{-1}]^2 + \mathrm{Re}[{\mathcal{G}_\mathcal{R}}^{-1}]^2}$$

$$\mathcal{G}_R^{-1}(\omega, k) = (\omega - \omega_k^*) + i\alpha(\omega - \mu_{\mathrm{eff}})$$



Fluctuations → Stability, Luminescence, Absorption

Keldysh approach:

$$\mathcal{G}_R - \mathcal{G}_A = -i\left\langle \left[\psi^\dagger, \psi\right]_- \right\rangle$$

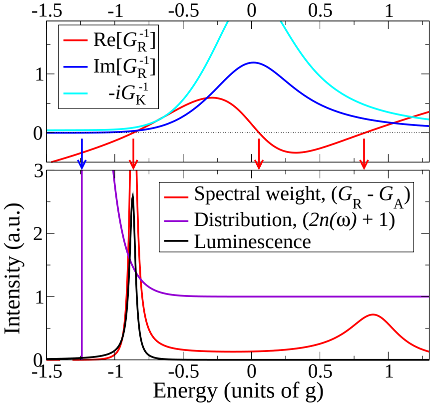
$$\mathcal{G}_K = -i\left\langle \left[\psi^\dagger, \psi\right]_+ \right\rangle = (2n(\omega)+1)(\mathcal{G}_R - \mathcal{G}_A)$$

$$\mathcal{L} = \left\langle \psi^\dagger \psi \right\rangle = \frac{i}{2}\left[\mathcal{G}_K + (\mathcal{G}_R - \mathcal{G}_A)\right]$$

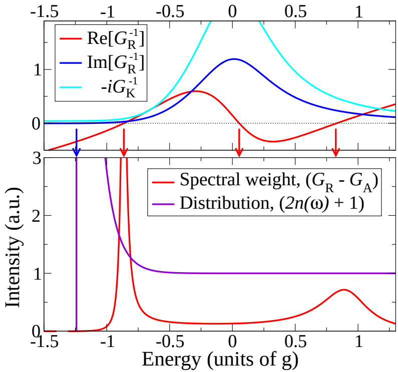
$$\mathcal{G}_K = \frac{-{\mathcal{G}_\mathcal{K}}^{-1}}{\mathrm{Im}[{\mathcal{G}_\mathcal{R}}^{-1}]^2 + \mathrm{Re}[{\mathcal{G}_\mathcal{R}}^{-1}]^2}$$

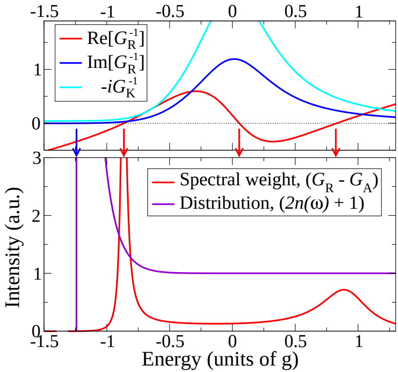
$$\mathcal{G}_R - \mathcal{G}_A = \frac{2\mathrm{Im}[{\mathcal{G}_\mathcal{R}}^{-1}]}{\mathrm{Im}[{\mathcal{G}_\mathcal{R}}^{-1}]^2 + \mathrm{Re}[{\mathcal{G}_\mathcal{R}}^{-1}]^2}$$

$$\mathcal{G}_R^{-1}(\omega, k) = (\omega - \omega^*_k) + i\alpha(\omega - \mu_{\mathrm{eff}})$$

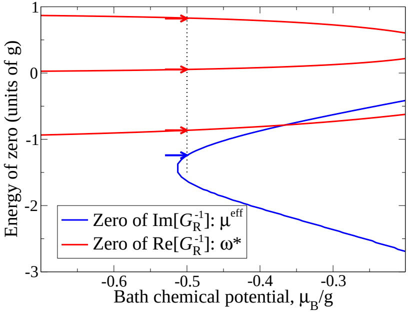


Linewidth, inverse Green's function and gap equation



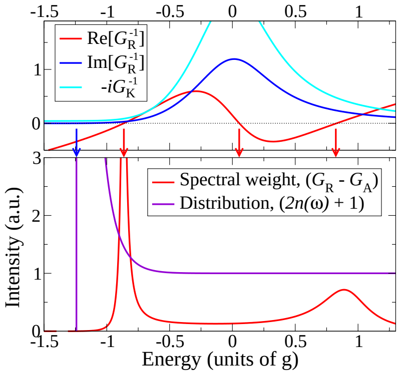


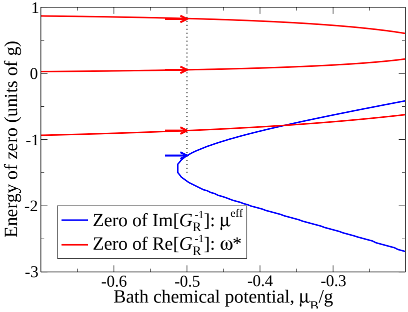
-1.5
-1
-0.5
0
0.5
1
Re[G_R^{-1}]
Im[G_R^{-1}]
-iG_K^{-1}
Spectral weight, (G_R - G_A)
Distribution, ($2n(\omega)$ + 1)
Intensity (a.u.)
Energy (units of g)

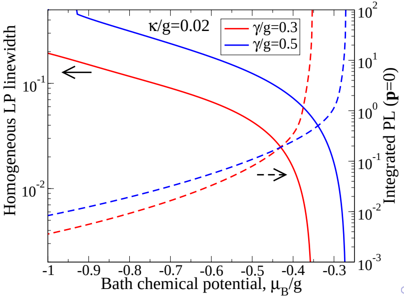


Energy of zero (units of g)
Zero of Im[G_R^{-1}]: μ^{eff}
Zero of Re[G_R^{-1}]: ω^*
-0.6
-0.5
-0.4
-0.3
Bath chemical potential, μ_B/g

Linewidth, inverse Green's function and gap equation







At transition, Gap Equation implies:

$$\mathcal{G}_R^{-1}(\omega = \mu_S, k = 0) = 0$$

Instability compared to a laser

Weak coupling: Maxwell-Bloch equations

$$\partial_t \psi = -\kappa\psi + gP$$
$$\partial_t P = -\gamma P + g\psi N$$
$$\partial_t N = \Gamma(N_0 - N) - g(\psi^* P + \psi P^*).$$

Instability compared to a laser

Weak coupling: Maxwell-Bloch equations

$$\partial_t \psi = -\kappa\psi + gP$$
$$\partial_t P = -\gamma P + g\psi N$$
$$\partial_t N = \Gamma(N_0 - N) - g(\psi^* P + \psi P^*).$$

Correspond to energy independent $F_{a,b}$,

$$\mathcal{G}_R^{-1} = \omega + i\kappa + \frac{g^2 N}{\omega + i\gamma}$$

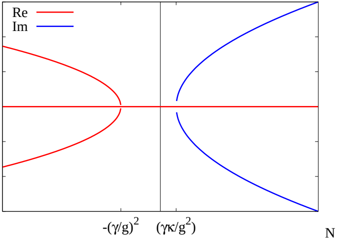
Instability compared to a laser

Weak coupling: Maxwell-Bloch equations

$$\partial_t \psi = -\kappa\psi + gP$$
$$\partial_t P = -\gamma P + g\psi N$$
$$\partial_t N = \Gamma(N_0 - N) - g(\psi^* P + \psi P^*).$$

Correspond to energy independent $F_{a,b}$,

$$\mathcal{G}_R^{-1} = \omega + i\kappa + \frac{g^2 N}{\omega + i\gamma}$$



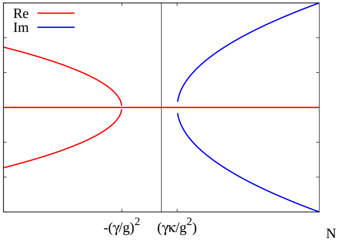
Instability compared to a laser

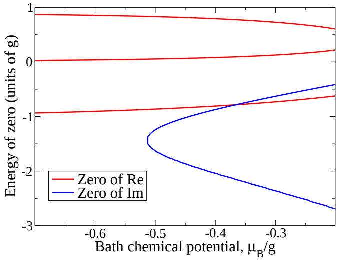
Weak coupling: Maxwell-Bloch equations

$$\partial_t \psi = -\kappa\psi + gP$$
$$\partial_t P = -\gamma P + g\psi N$$
$$\partial_t N = \Gamma(N_0 - N) - g(\psi^* P + \psi P^*).$$

Correspond to energy independent $F_{a,b}$,

$$\mathcal{G}_R^{-1} = \omega + i\kappa + \frac{g^2 N}{\omega + i\gamma}$$





Relating finite-size spectrum to self phase modulation

Finite-size spectrum:

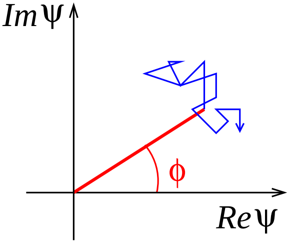
$$\mathcal{D}_{\phi\phi}(t,r,r) \propto \sum_{n}^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2 (1 - e^{i\omega t})}{\left|(\omega + ix)^2 + x^2 - (n\Delta)^2\right|^2}$$

Connection to Kubo Formula [Eastham and Whittaker, arXiv:0811.4333]

Finite-size spectrum:

$$\mathcal{D}_{\phi\phi}(t,r,r) \propto \sum_{n}^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2(1-e^{i\omega t})}{\left|(\omega+ix)^2+x^2-(n\Delta)^2\right|^2}$$

Connection to Kubo Formula [Eastham and Whittaker, arXiv:0811.4333]



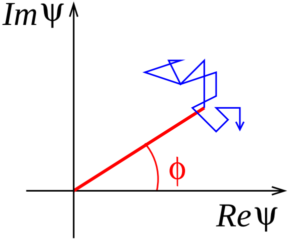
$$\partial_t \phi = U \delta N$$

Relating finite-size spectrum to self phase modulation

Finite-size spectrum:

$$\mathcal{D}_{\phi\phi}(t, r, r) \propto \sum_{n}^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2(1 - e^{i\omega t})}{|(\omega + ix)^2 + x^2 - (n\Delta)^2|^2}$$

Connection to Kubo Formula [Eastham and Whittaker, arXiv:0811.4333]



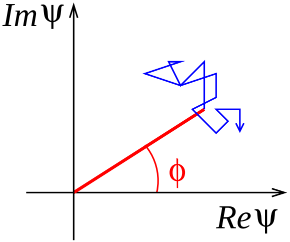
$$\partial_t \phi = U\delta N$$
$$\partial_t \delta N = -\Gamma\delta N + F(t), \quad \langle F(t)F(t')\rangle = C\delta(t - t')$$

Relating finite-size spectrum to self phase modulation

Finite-size spectrum:

$$\mathcal{D}_{\phi\phi}(t, r, r) \propto \sum_{n}^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2 (1 - e^{i\omega t})}{|(\omega + ix)^2 + x^2 - (n\Delta)^2|^2}$$

Connection to Kubo Formula [Eastham and Whittaker, arXiv:0811.4333]



$$\partial_t \phi = U\delta N$$

$$\partial_t \delta N = -\Gamma \delta N + F(t), \quad \langle F(t) F(t') \rangle = C\delta(t - t')$$

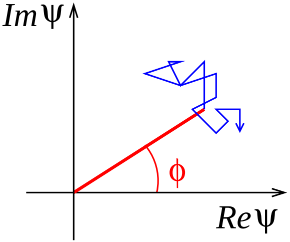
$$\mathcal{D}_{\phi\phi}(t) = \int \frac{d\omega}{2\pi} (1 - e^{i\omega t}) \langle \phi_\omega \phi_{-\omega} \rangle$$

Relating finite-size spectrum to self phase modulation

Finite-size spectrum:

$$\mathcal{D}_{\phi\phi}(t,r,r) \propto \sum_{n}^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2(1-e^{i\omega t})}{|(\omega+ix)^2+x^2-(n\Delta)^2|^2}$$

Connection to Kubo Formula [Eastham and Whittaker, arXiv:0811.4333]



$$\partial_t \phi = U\delta N$$

$$\partial_t \delta N = -\Gamma\delta N + F(t), \quad \langle F(t)F(t')\rangle = C\delta(t-t')$$

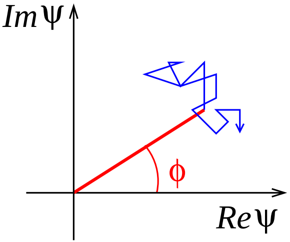
$$\mathcal{D}_{\phi\phi}(t) = \int \frac{d\omega}{2\pi}(1-e^{i\omega t})\frac{CU^2}{|\omega(\omega+i\Gamma)|^2}$$

Relating finite-size spectrum to self phase modulation

Finite-size spectrum:

$$\mathcal{D}_{\phi\phi}(t,r,r) \propto \sum_{n}^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2 (1 - e^{i\omega t})}{|(\omega + ix)^2 + x^2 - (n\Delta)^2|^2}$$

Connection to Kubo Formula [Eastham and Whittaker, arXiv:0811.4333]



$$\partial_t \phi = U \delta N$$

$$\partial_t \delta N = -\Gamma \delta N + F(t), \quad \langle F(t) F(t') \rangle = C\delta(t - t')$$

$$\mathcal{D}_{\phi\phi}(t) = \int \frac{d\omega}{2\pi} (1 - e^{i\omega t}) \frac{CU^2}{|\omega(\omega + i\Gamma)|^2}$$

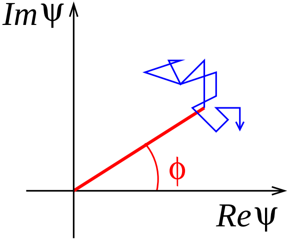
$$= \frac{CU^2}{2\Gamma^3} \left[\Gamma t - 1 + e^{-\Gamma t} \right]$$

Relating finite-size spectrum to self phase modulation

Finite-size spectrum:

$$\mathcal{D}_{\phi\phi}(t, r, r) \propto \sum_{n}^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2 (1 - e^{i\omega t})}{|(\omega + ix)^2 + x^2 - (n\Delta)^2|^2}$$

Connection to Kubo Formula [Eastham and Whittaker, arXiv:0811.4333]



$$\partial_t \phi = U \delta N$$

$$\partial_t \delta N = -\Gamma \delta N + F(t), \quad \langle F(t) F(t') \rangle = C\delta(t - t')$$

$$\mathcal{D}_{\phi\phi}(t) = \int \frac{d\omega}{2\pi} (1 - e^{i\omega t}) \frac{CU^2}{|\omega(\omega + i\Gamma)|^2}$$

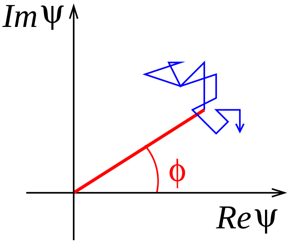
$$= \frac{CU^2}{2\Gamma^3} \begin{cases} \Gamma^2 t^2 / 2 & t \ll 1/\Gamma \\ \Gamma t & t \gg 1/\Gamma \end{cases}$$

Relating finite-size spectrum to self phase modulation

Finite-size spectrum:

$$\mathcal{D}_{\phi\phi}(t,r,r) \propto \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2(1-e^{i\omega t})}{|(\omega+ix)^2+x^2-(n\Delta)^2|^2}$$

Connection to Kubo Formula [Eastham and Whittaker, arXiv:0811.4333]



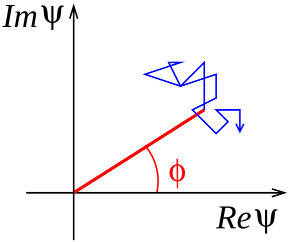
$$\partial_t \phi = U\delta N$$
$$\partial_t \delta N = -\Gamma\delta N + F(t), \quad \langle F(t)F(t')\rangle = C\delta(t-t')$$
$$\mathcal{D}_{\phi\phi}(t) = \int \frac{d\omega}{2\pi}(1-e^{i\omega t})\frac{CU^2}{|\omega(\omega+i\Gamma)|^2}$$
$$= \frac{CU^2}{2\Gamma^3}\begin{cases}\Gamma^2 t^2/2 & t \ll 1/\Gamma \\ \Gamma t & t \gg 1/\Gamma\end{cases}$$

Relating finite-size spectrum to self phase modulation

Finite-size spectrum:

$$\mathcal{D}_{\phi\phi}(t, r, r) \propto \qquad \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2 (1 - e^{i\omega t})}{|(\omega + ix)^2 + x^2|^2}$$

Connection to Kubo Formula [Eastham and Whittaker, arXiv:0811.4333]



$$\partial_t \phi = U \delta N$$

$$\partial_t \delta N = -\Gamma \delta N + F(t), \quad \langle F(t) F(t') \rangle = C \delta(t - t')$$

$$\mathcal{D}_{\phi\phi}(t) = \int \frac{d\omega}{2\pi} (1 - e^{i\omega t}) \frac{CU^2}{|\omega(\omega + i\Gamma)|^2}$$

$$= \frac{CU^2}{2\Gamma^3} \begin{cases} \Gamma^2 t^2/2 & t \ll 1/\Gamma \\ \Gamma t & t \gg 1/\Gamma \end{cases}$$