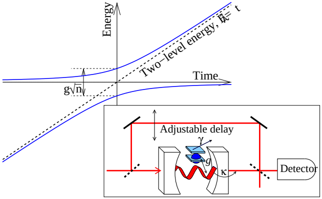
Collapse and revivals of the photon field in a Landau-Zener process

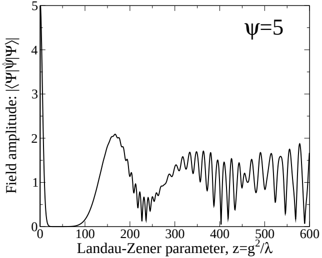
Jonathan Keeling

Central European Workshop on Quantum Optics

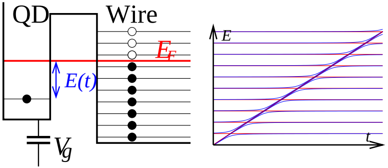
Generalisations of Landau-Zener problems

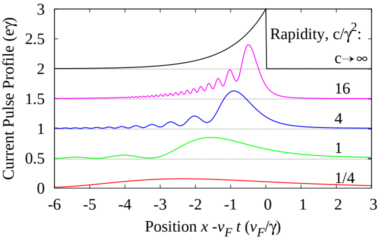
Two-level atom coupled to photon field



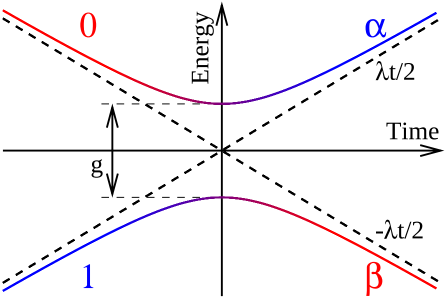


Quantum dot coupled to 1D wire

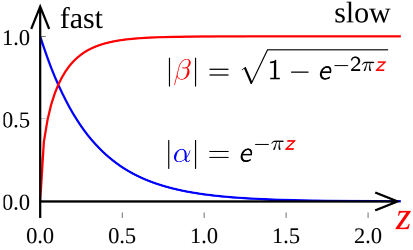




The Landau-Zener problem



$$\psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \qquad z = g^2/\lambda$$



$$i\partial_t \psi = \begin{pmatrix} \lambda t/2 & g \\ g & -\lambda t/2 \end{pmatrix} \psi$$

Coherent field Landau-Zener problem

- Hamiltonian: $H = \frac{\lambda t}{2} s^z + g\left(\psi^\dagger s^- + \psi\, s^+\right),$
- Initial coherent state: $|\Psi(-\infty)\rangle = e^{-|\psi|^2/2} \sum_n \frac{\psi^n}{\sqrt{n!}} |n, \uparrow\rangle$
- Each pair $|n, \uparrow\rangle \leftrightarrow |n+1, \downarrow\rangle$ undergoes LZ transition

$$H_{n,n+1} = \begin{pmatrix} \lambda t/2 & g\sqrt{n+1} \\ g\sqrt{n+1} & -\lambda t/2 \end{pmatrix}, \qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix}$$

- Final state:

$$|\Psi(+\infty)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1}|n, \uparrow\rangle + B_{n+1}|n+1, \downarrow\rangle \right]$$

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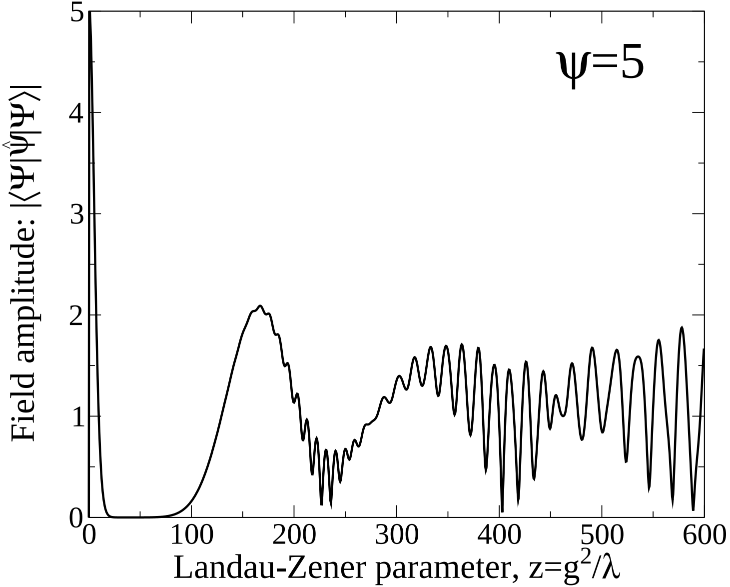
Collapse and revivals of field amplitude



Explaining results: expansion

Adiabatic limit: $z = g^2/\lambda \gg 1$

$$|\Psi(T)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1} |n, \uparrow\rangle + \qquad B_{n+1} |n+1, \downarrow\rangle \right]$$

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$$\Delta\phi_n = z \left[(n+1)\ln(n+1) - n \ln n \qquad \right]$$

Want $\Delta\phi_n$ for $n \simeq |\psi|^2$.

$$\Delta\phi_{n=|\psi|^2+m} \simeq \Delta\phi_{|\psi|^2} + \frac{zm}{|\psi|^2} - \frac{zm^2}{2|\psi|^4} + \cdots$$

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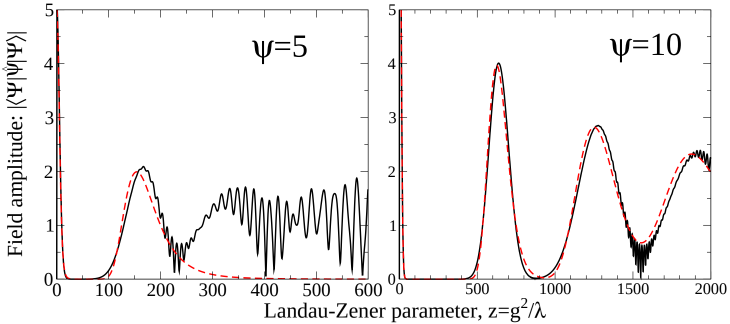
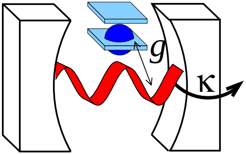
Explaining results: comparison

$$\langle\Psi|\psi|\Psi\rangle = \frac{|\psi|}{(1+z^2/|\psi|^4)^{1/4}} \sum_{N=0}^{N_{max}} \exp\left[-\frac{(z-2\pi N|\psi|^2)^2}{2|\psi|^2(1+z^2/|\psi|^4)}\right]$$



Effects of photon decay



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$$L_\kappa[\hat{\rho}] = -\frac{\kappa}{2}\left(\hat{\psi}^\dagger\hat{\psi}\hat{\rho} + \hat{\rho}\hat{\psi}^\dagger\hat{\psi} - 2\hat{\psi}\hat{\rho}\hat{\psi}^\dagger\right)$$

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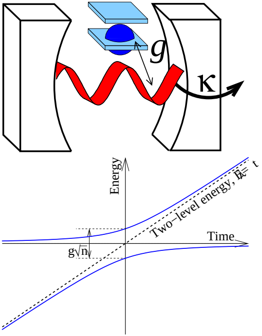

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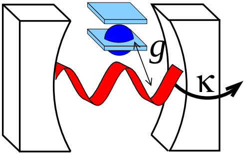
Photon loss does not switch branch

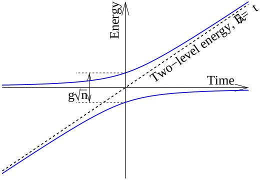
$$P_{\text{trans}} = \frac{\left| \langle n-1, - | \hat{\psi} | n, + \rangle \right|^2}{\langle n, + | \hat{\psi}^\dagger \hat{\psi} | n, + \rangle} \leq \frac{27}{256 n^2} \ll 1$$

Consider, $\Lambda_n = \rho_{n,n+1}$:

$$\frac{d\Lambda_n}{dt} = i \left[\frac{d\Delta\phi_{n-1}}{dt} \right] \Lambda_n - \kappa \left[\left(n - \frac{1}{2} \right) \Lambda_n - \sqrt{n(n+1)} \Lambda_{n+1} \right]$$

When $|t| \leq g\sqrt{n}/\lambda$, decay rate $\kappa n \simeq \kappa |\psi|^2 \gg \kappa$.





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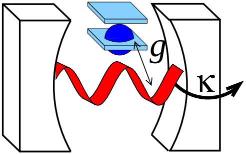

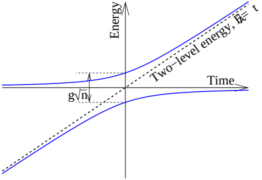

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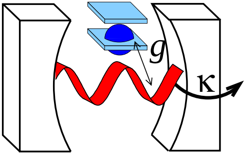

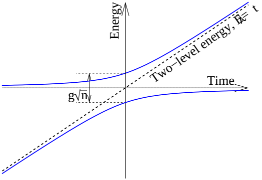

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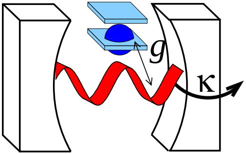

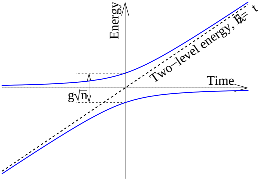

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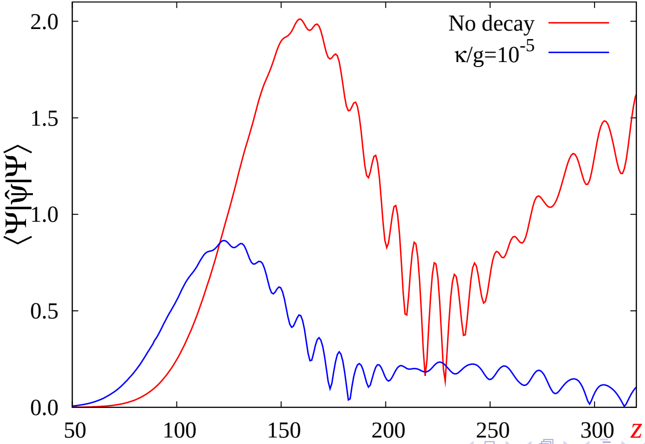
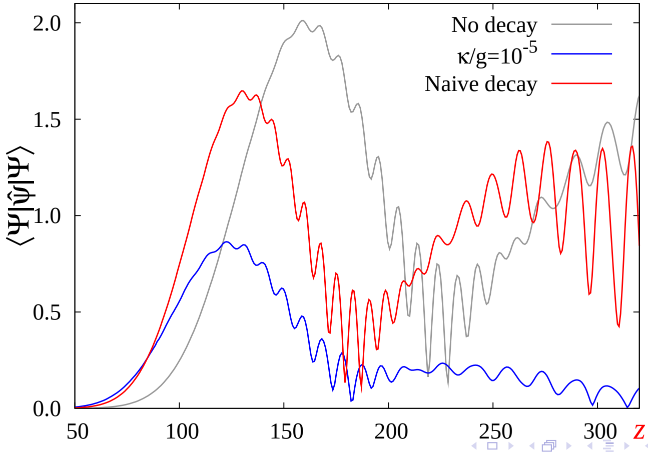
$\langle\Psi|\,\hat{\psi}(\psi_0,\kappa)\,|\Psi\rangle$



Comparison of photon decay

$$\langle\Psi|\,\hat{\psi}(\psi_0,\kappa)\,|\Psi\rangle \simeq \langle\Psi|\,\hat{\psi}(\psi_0 e^{-\kappa T/2},0)\,|\Psi\rangle \exp\left[-\frac{\kappa T}{2}\right]$$



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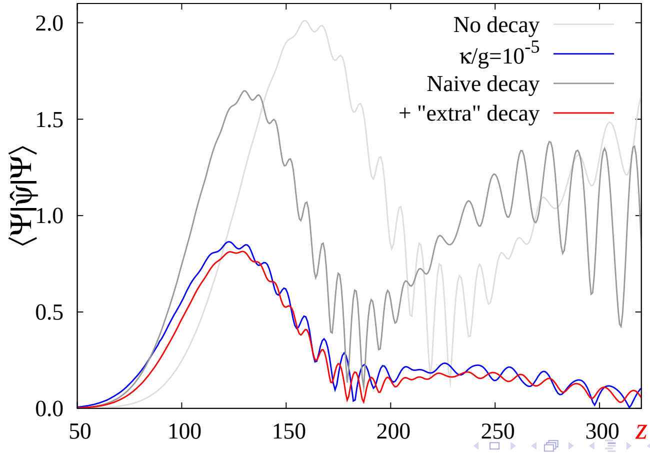

Many two-level systems

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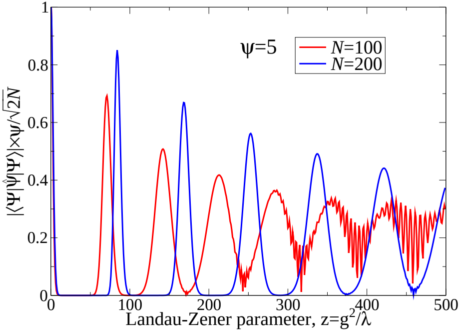
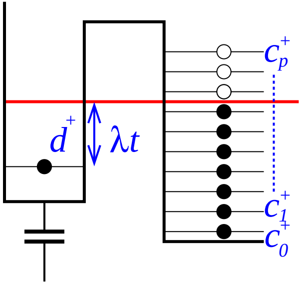


Quantum dot coupled to 1D wire



$$H = \lambda t d^\dagger d + \sum \varepsilon_p c_p^\dagger c_p + g(d^\dagger c_p + c_p^\dagger d)$$

- Initially: Filled fermi sea $\prod_{\varepsilon_p < \varepsilon_F} c_p^\dagger |\Omega\rangle$
- Find final state of wire

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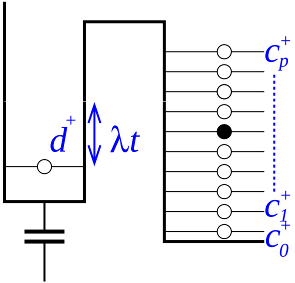

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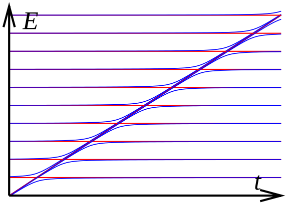
1 particle: Demkov-Osherov problem



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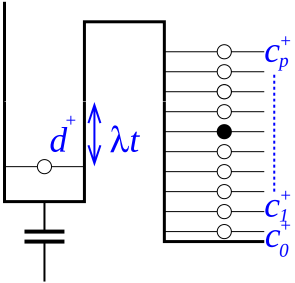

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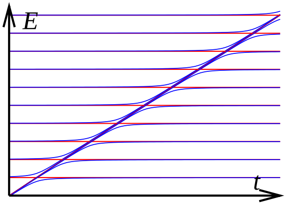

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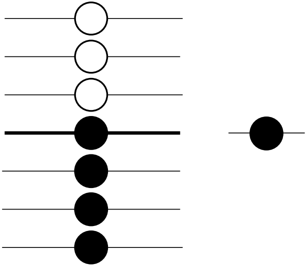

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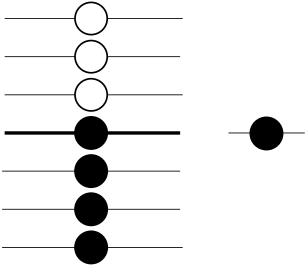

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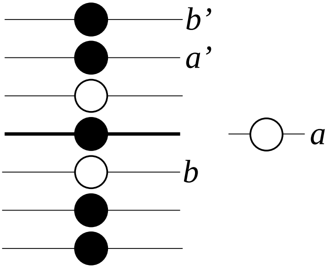

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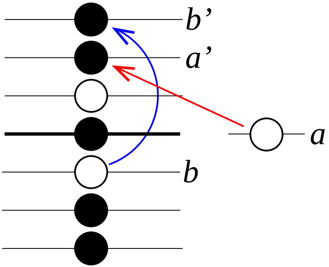

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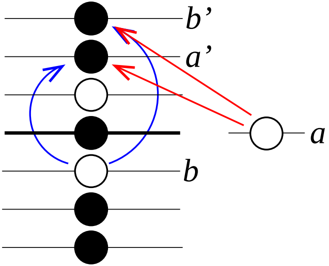

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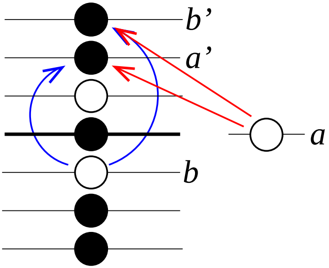

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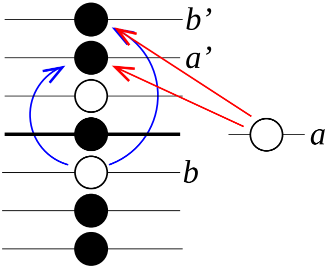

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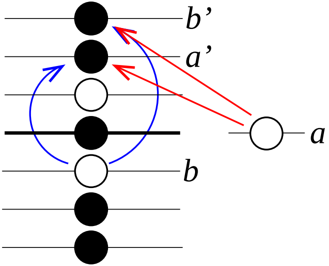

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Current pulse profile

Driving need not be adiabatic

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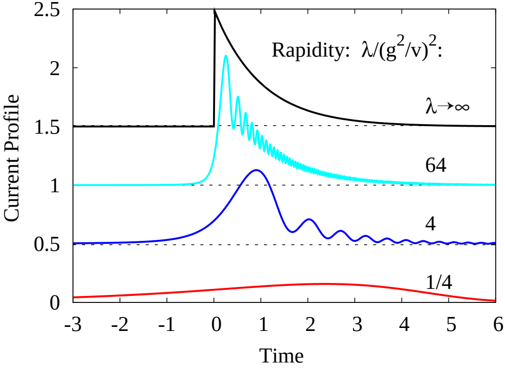


Acknowledgements

Two-level system and photon field

Victor Gurarie.
Chris Pointon.

Quantum dot and 1D wire

Leonid Levitov.
Andriy Shytov.

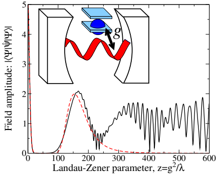
Funding:

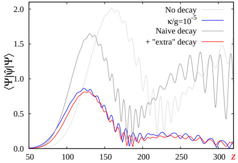
EPSRC Engineering and Physical Sciences Research Council

Pembroke College

Conclusions

- Dynamically driven single two-level system:





[Keeling and Gurarie, Phys. Rev. Lett. 101 033001 (2008)]

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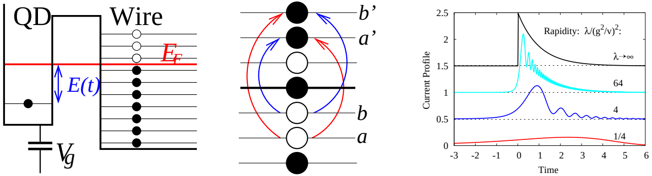


[Keeling, Shytov and Levitov, Phys. Rev. Lett. 101 196404 (2008)]

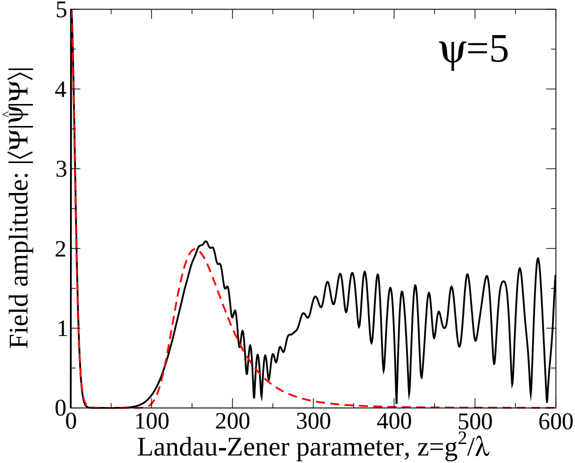
Extra material

3. Coherent Landau-Zener process
   - Wigner function
   - Graphical description of revivals
   - Decay effects
   - Systems for LZ problem
   - Many spins LZ problem

4. Localised fermion coupled to continuum
   - Finding $T(\varepsilon, \varepsilon')$
   - Periodic driving
   - Measuring Noise
   - Noisy driving

Revivals are not coherent states

$W(x,p) = \frac{1}{\pi}\int dy\, \Psi^*(x+y)\Psi(x-y)e^{2ipy}$.



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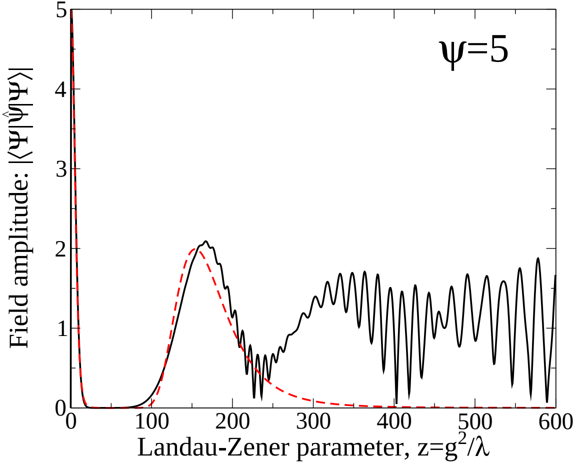

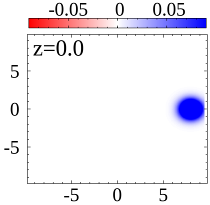

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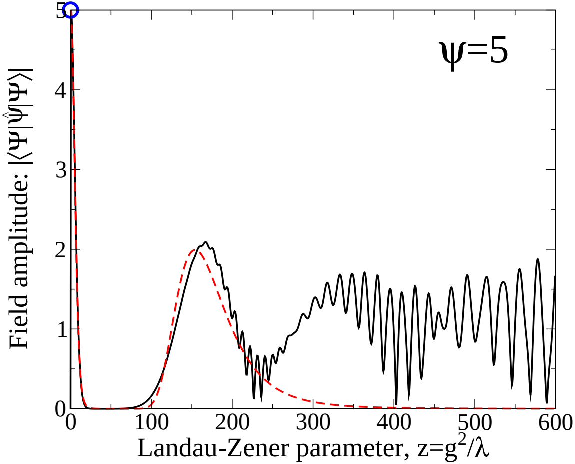

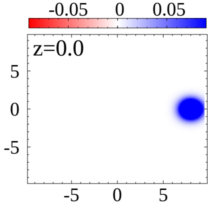

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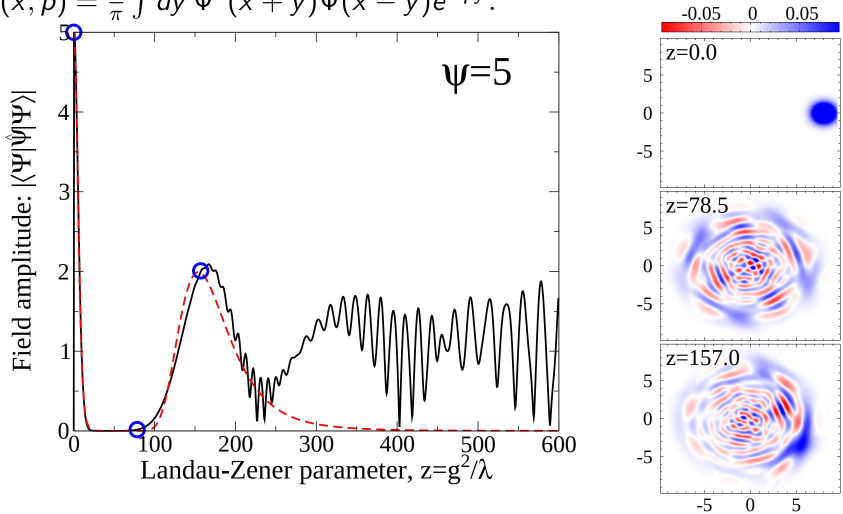

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$$\langle\Psi|\psi|\Psi\rangle = \psi e^{-|\psi|^2} \sum_n \frac{|\psi|^{2n}}{n!} \sqrt{\frac{n+2}{n+1}} e^{i(\phi_{n+2}-\phi_{n+1})}$$

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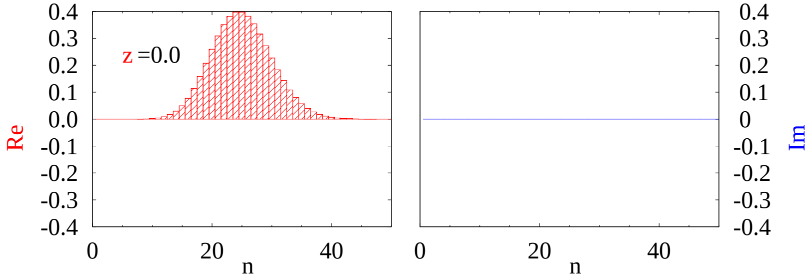

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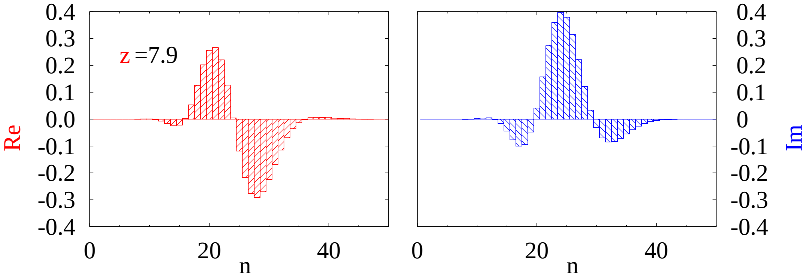

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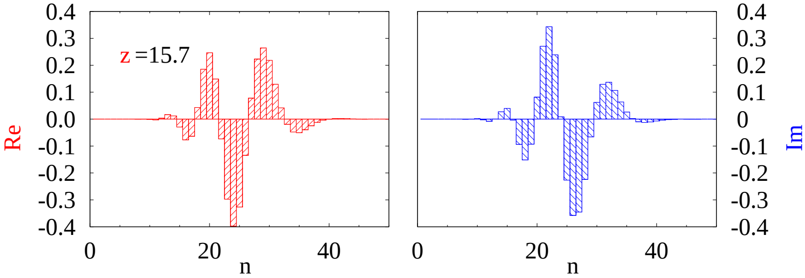

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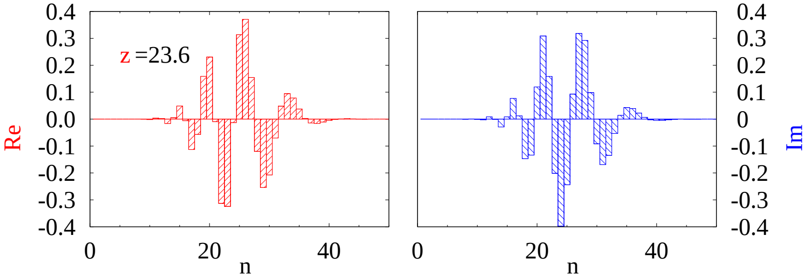

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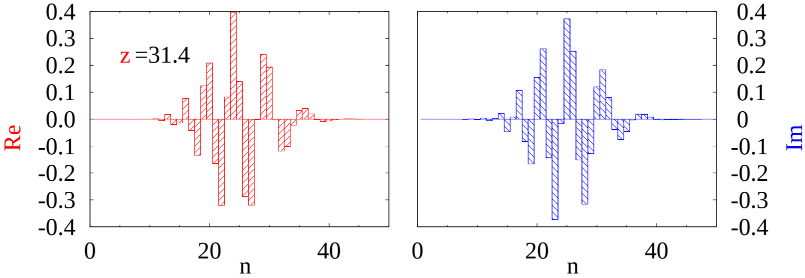

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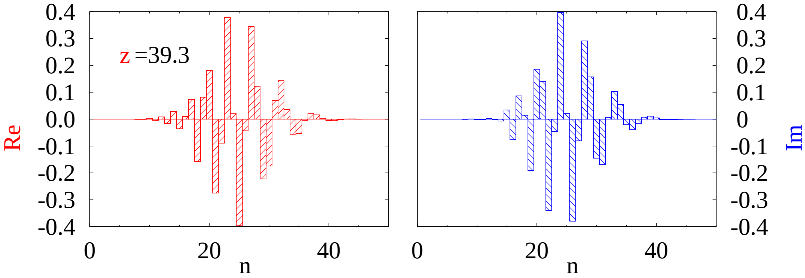

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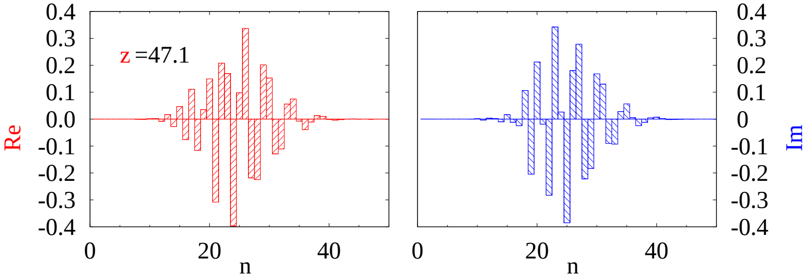

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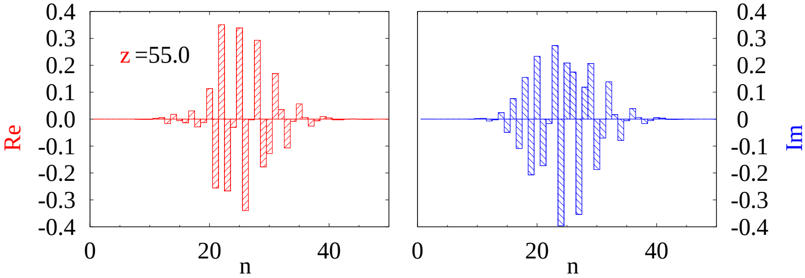

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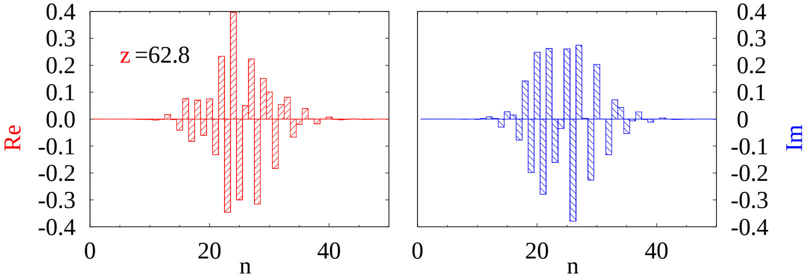

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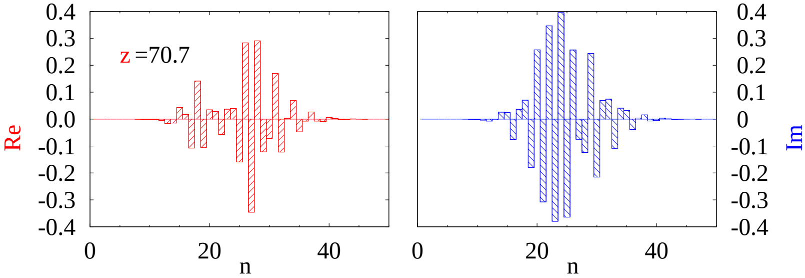

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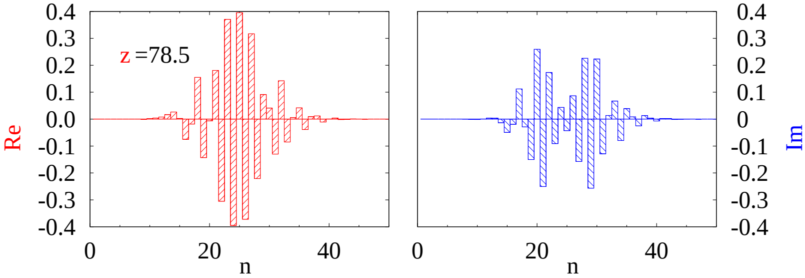

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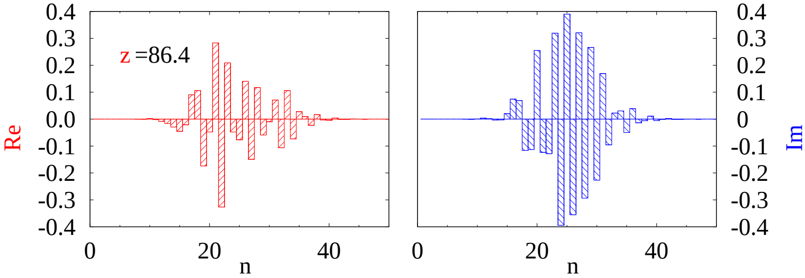

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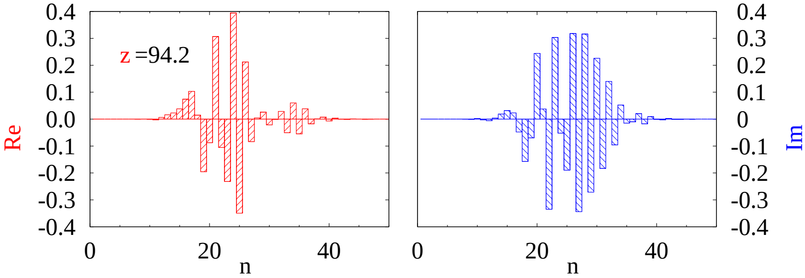

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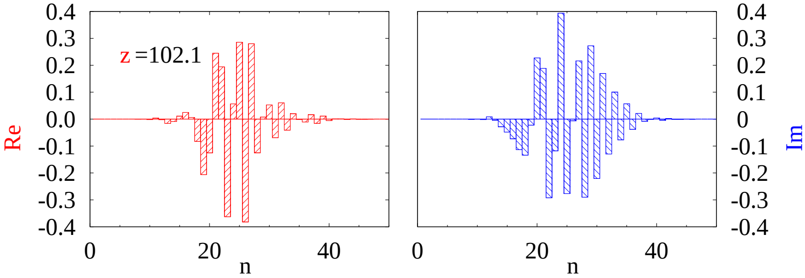

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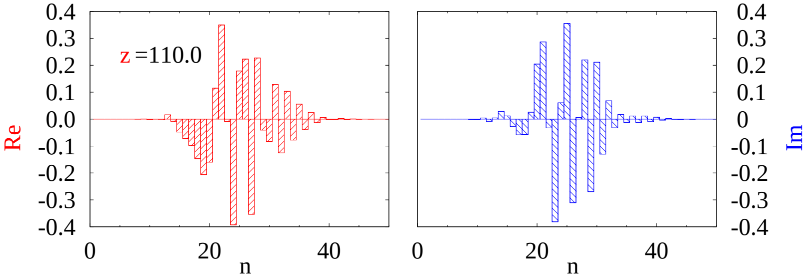

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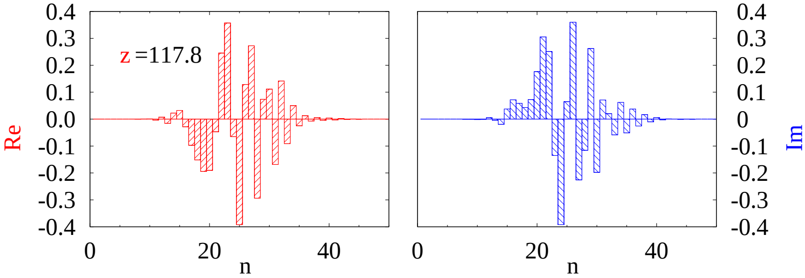

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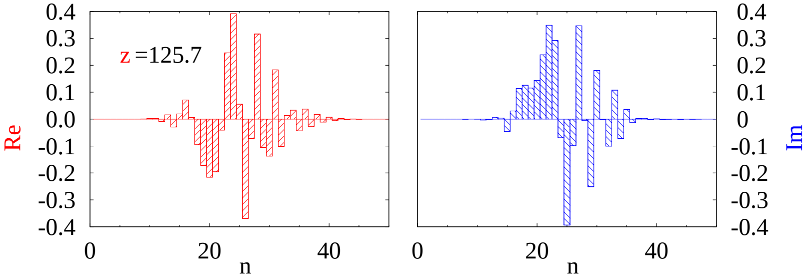

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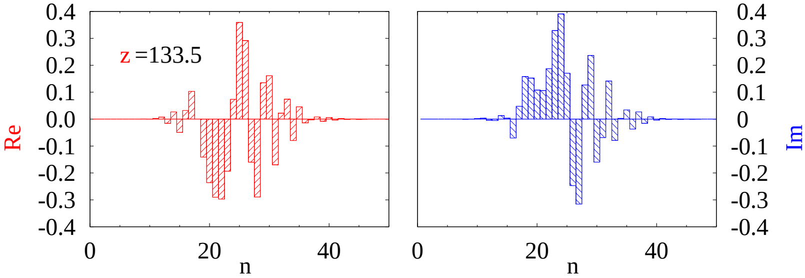

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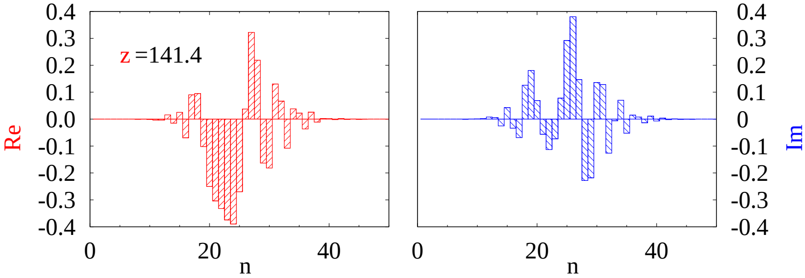

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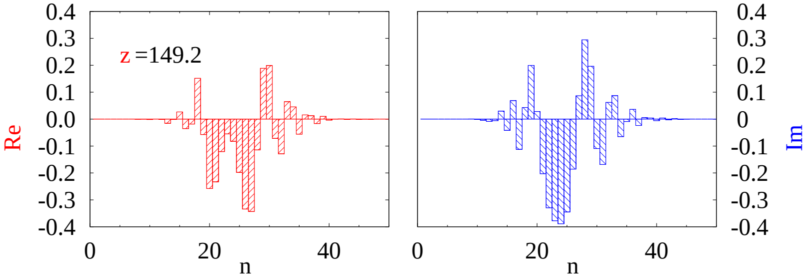

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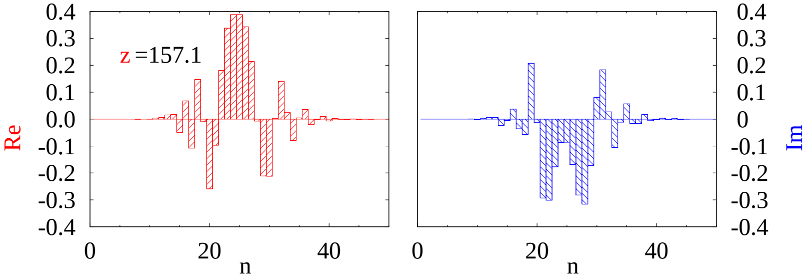
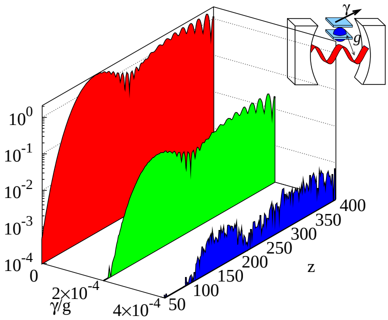
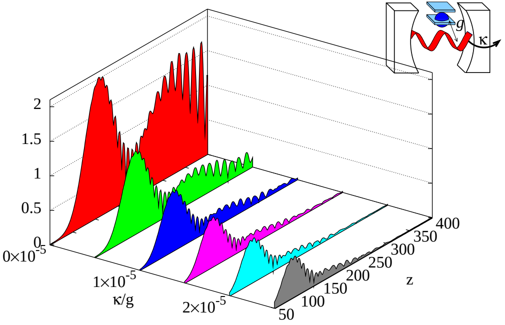


Results including decay





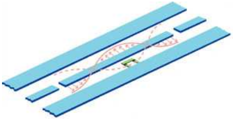
$\gamma_{\text{max}}/g \simeq 2 \times 10^{-4}$
$\gamma_{\text{naive}}/g \ll 10 \times 10^{-4}$

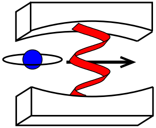
$\kappa_{\text{max}}/g \simeq 1 \times 10^{-5}$
$\kappa_{\text{naive}}/g \ll 100 \times 10^{-5}$

Possible systems

Requirement: $\kappa/g \lesssim 1 \times 10^{-5}$, $\gamma/g \lesssim 2 \times 10^{-4}$

System	**Source**	κ/g	γ/g
Quantum dots/Microdisk	CNRS 2005	2×10^{-1}	3×10^{-1}
Atom/Optical cavity	ETH 2007	1×10^{-1}	3×10^{-1}
Josephson junction/stripline	Yale, 2004	2×10^{-2}	2×10^{-3}
Atom/Microwave cavity	ENS, 2004	7×10^{-3}	2×10^{-4}
	ENS, 2007	2×10^{-4}	2×10^{-4}

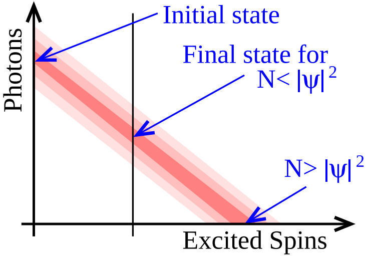


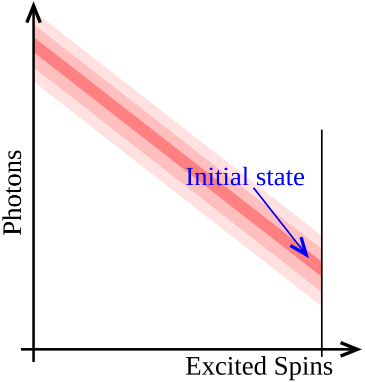


Many two-level systems

If many spins:

$$H = \sum_{i}^{N} \left(\lambda t s_i^z + g(s_i^+ \psi + s_i^- \psi^\dagger) \right)$$





Many two-level systems: Converting spins to photons

$$\langle\Psi|\psi|\Psi\rangle = \psi e^{-|\psi|^2} \sum_n \frac{|\psi|^{2n}}{n!} \sqrt{\frac{n+1+N}{n+1}} e^{i(\phi_{n+2}-\phi_{n+1})}$$

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If adiabatic ϕ dominated by logarithm, $t \to \pm\infty$

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For $t \to -\infty$:

$$E_- \simeq \frac{N}{2}\lambda t + \frac{g^2 N(n+1)}{\lambda t} - \frac{g^4 N(n+1)}{(\lambda t)^3}[1+(n+1)-N]$$

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Similar expression from $t \to +\infty$. For both:
$\phi = \phi_+ + \phi_- = A(N) + B(N)n + Czn^2 + \mathcal{O}(1/N)$

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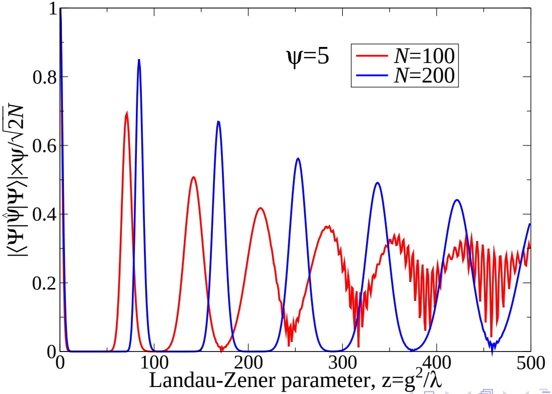
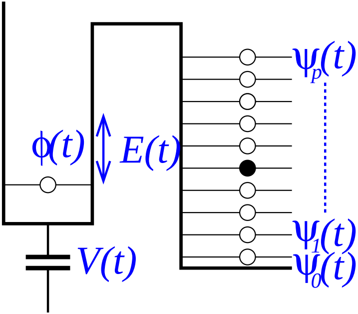


Single particle problem



Continuum states: $\psi(x,t) = \sum \psi_p(t) e^{ipx}$
Thus, continuum equations:

$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x,t)\delta(x)$$

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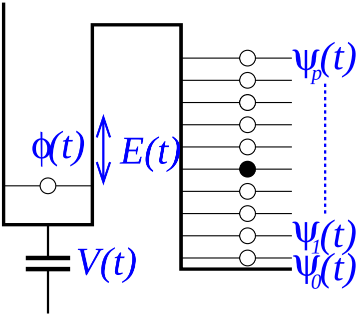

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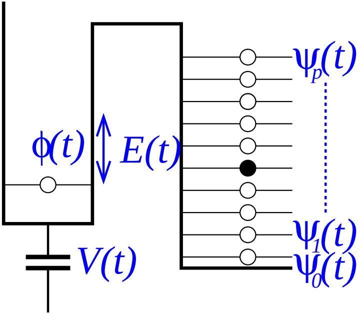

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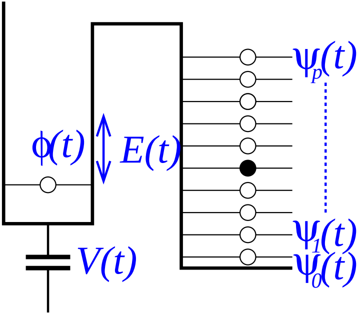

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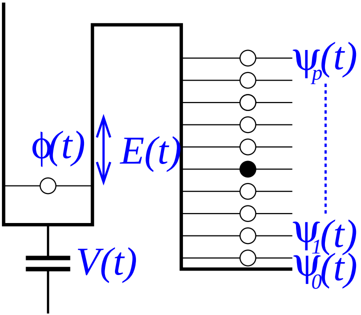

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Periodic driving: example results

Suppose $E(t) = E + A\sin(\Omega t)$: Find $N^{ex}(t)$ vs E.

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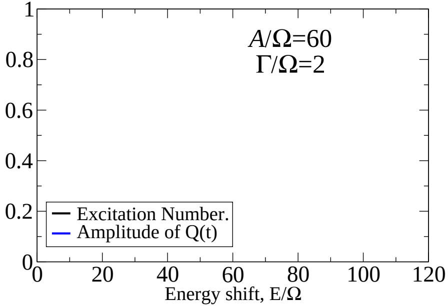

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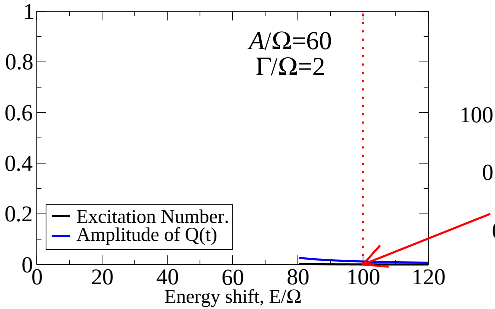

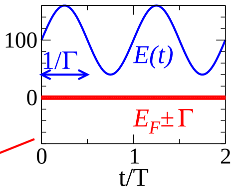

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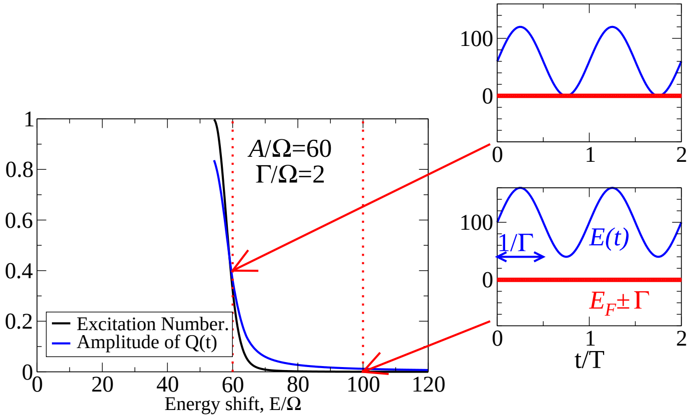

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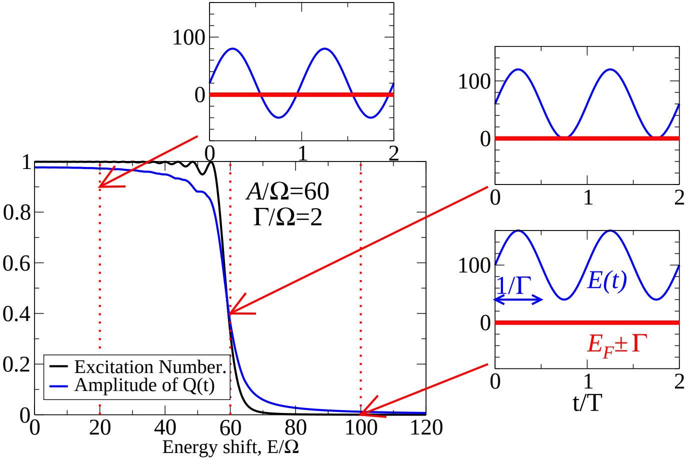

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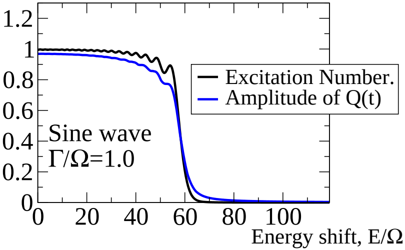

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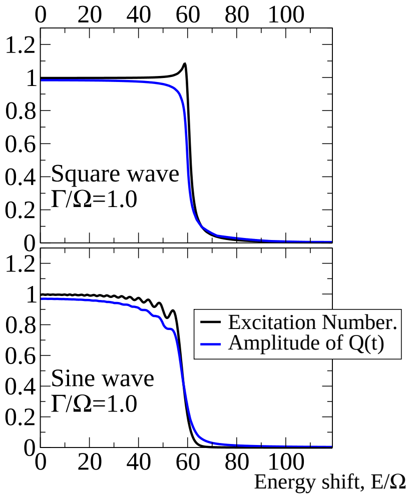

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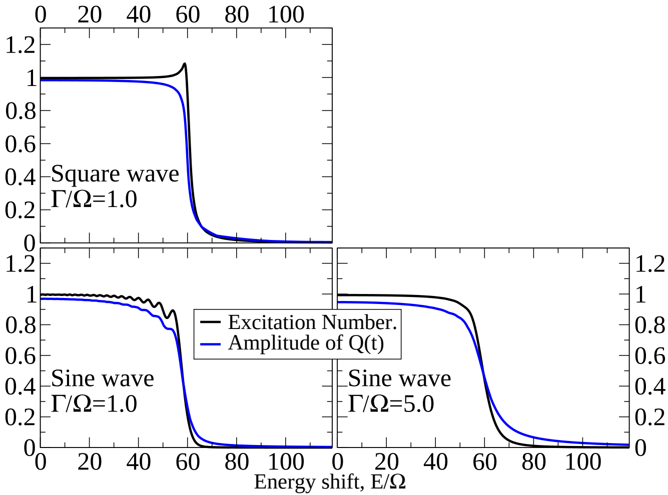

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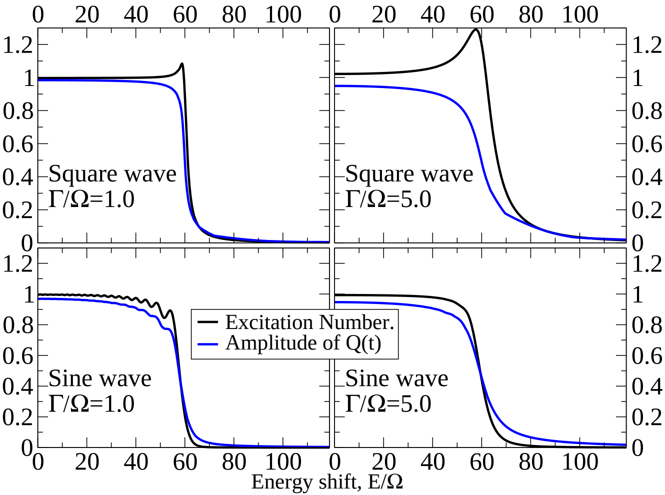

General time dependence: measuring noise

Number of excitations:

$$N^{\mathrm{ex}} = \sum_{\epsilon > \epsilon_{\mathrm{F}} > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_{\mathrm{F}} < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$

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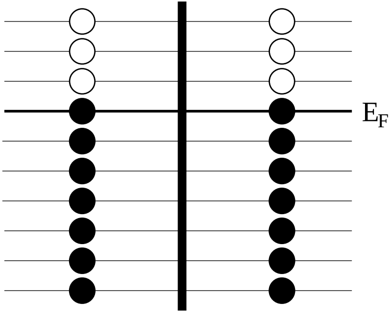

Define: $N^{\mathrm{ex}}_{r,l} = N^{e}_{r,l} + N^{h}_{r,l}$

$q_{r,l} = e(N^{e}_{r,l} - N^{h}_{r,l})$

Then

$$\langle q_r \rangle = T \langle q_l \rangle$$

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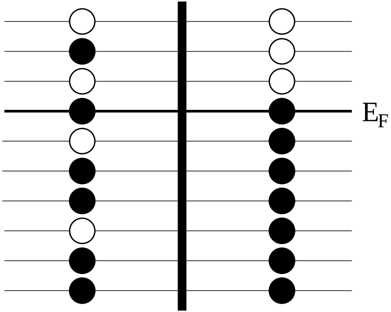

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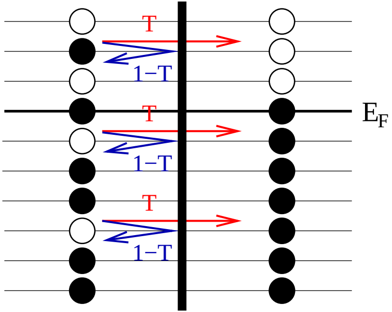

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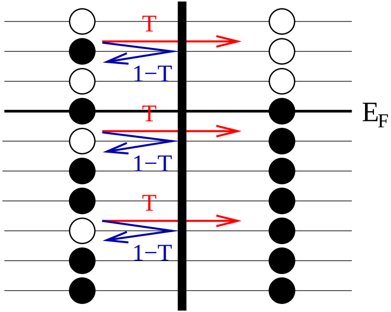

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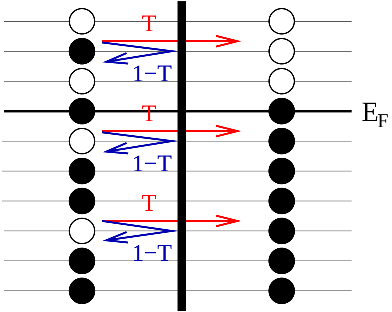

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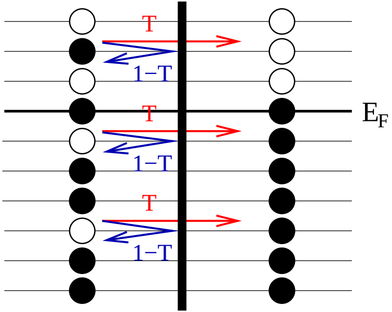

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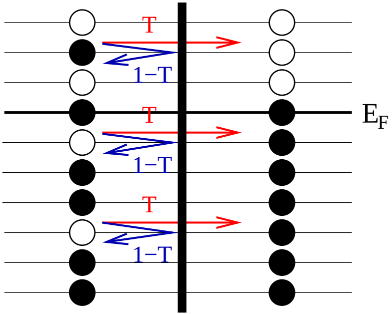

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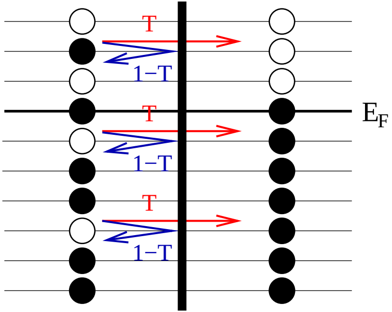

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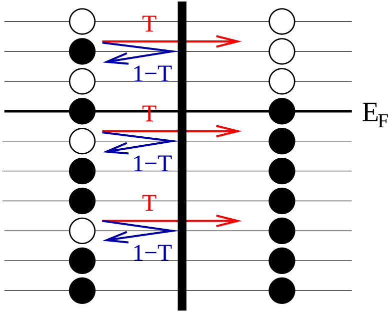

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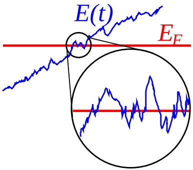

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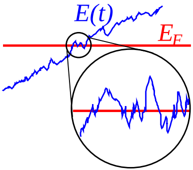

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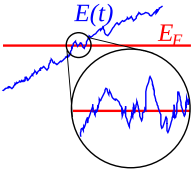

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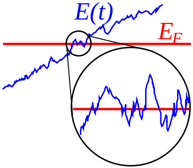

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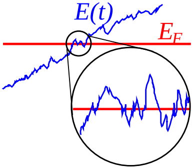

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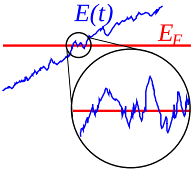

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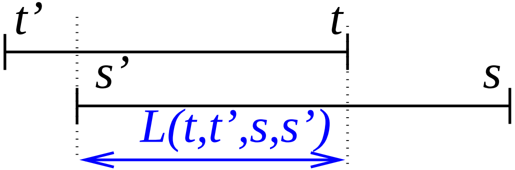

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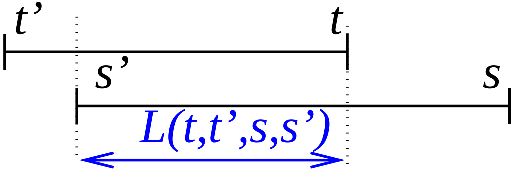

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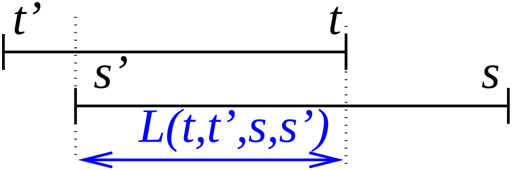

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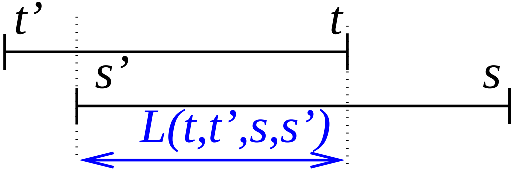

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- Integral log divergent: white noise limit
- Infinite no. crossings of Fermi surface
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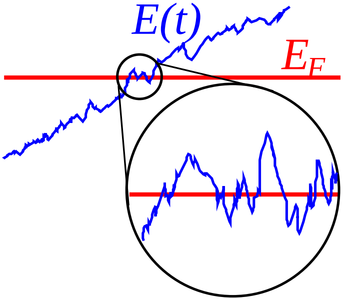

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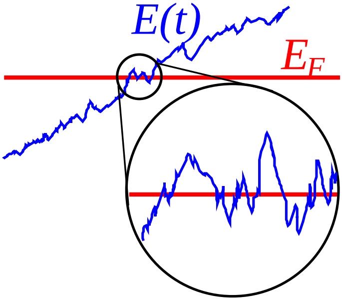

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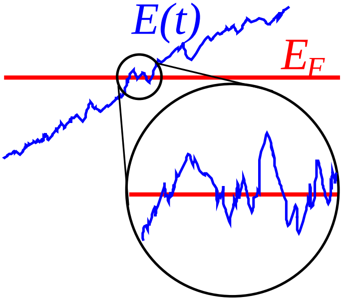

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