Nonequilibrium quantum condensates: from microscopic theory to macroscopic phenomenology

J. M. J. Keeling
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Sheffield LDSD Seminar. February 2009

Acknowledgements

People:

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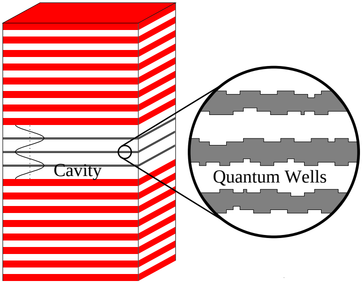
EPSRC Engineering and Physical Sciences Research Council

Pembroke College

Microcavity Polaritons



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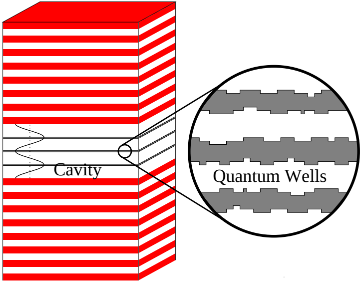

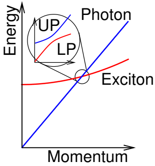

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[Hopfield, Phys. Rev.(1958)]

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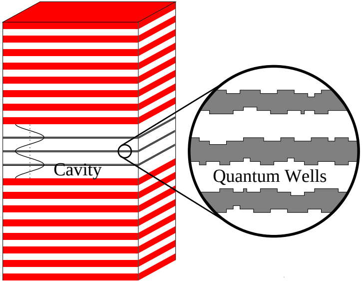

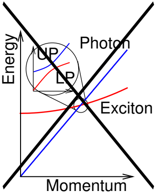

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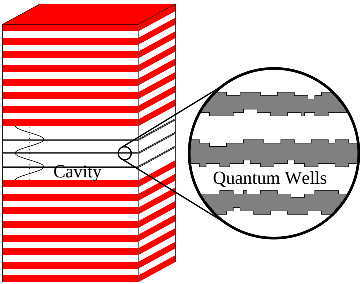

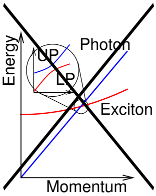

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Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$

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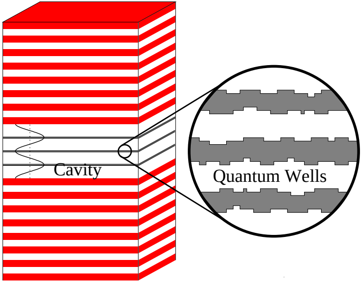

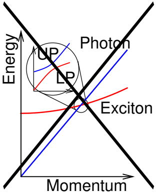

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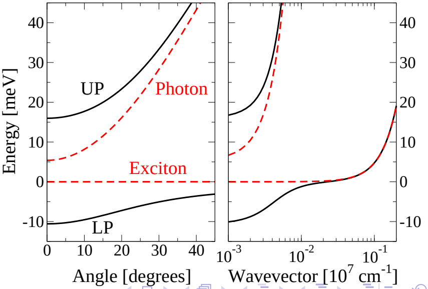
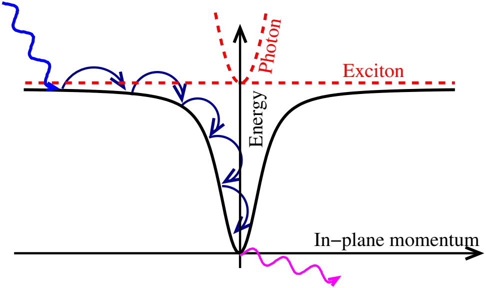
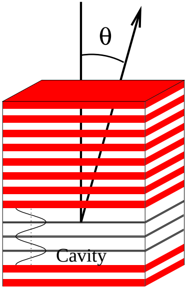
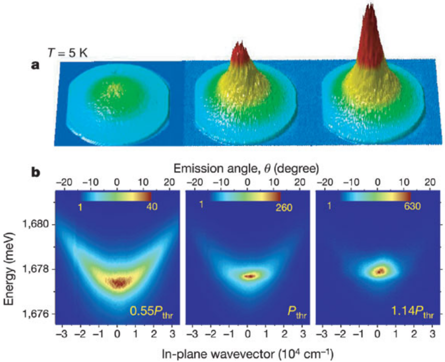


Nonequilibrium system

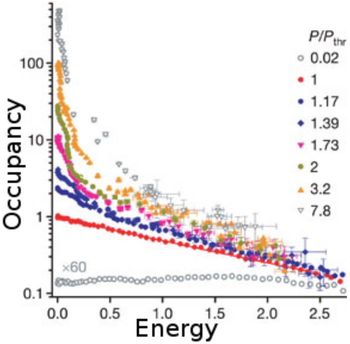




Polariton experiments: Momentum/Energy distribution

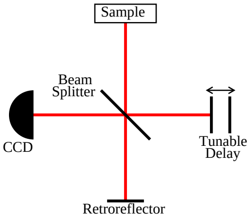


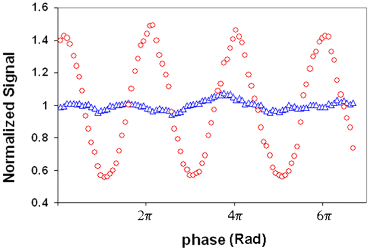
[Kasprzak, et al., Nature, 2006]



Polariton experiments: Coherence

Basic idea:





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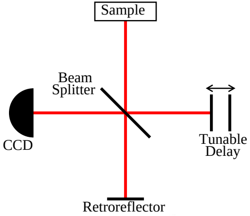
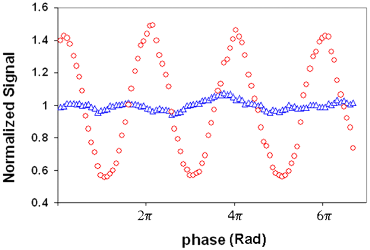
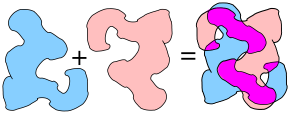
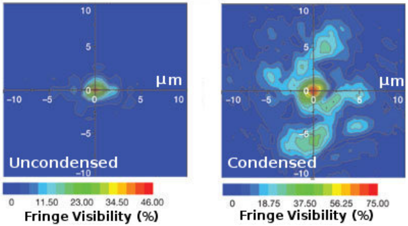




Coherence map:





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Other polariton condensation experiments

- Stress traps for polaritons
  [Balili *et al* Science 316 1007 (2007)]
- Temporal coherence and line narrowing
  [Love *et al* Phys. Rev. Lett. 101 067404 (2008)]
- Quantised vortices in disorder potential
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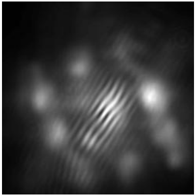

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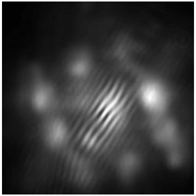
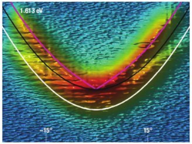
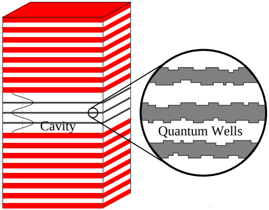




Overview

Excitons in a disorderd Quantum well



Exciton states in disorder:

$$\left[-\frac{\nabla_{\mathbf{R}}^2}{2m_X} + V(\mathbf{R})\right]\Phi_\alpha(\mathbf{R}) = \varepsilon_\alpha \Phi_\alpha(\mathbf{R})$$

$V(\vec{R})$ smoothed by exciton Bohr radius

[PRL 96 066405 (2006); PRB 76 115326 (2007)]

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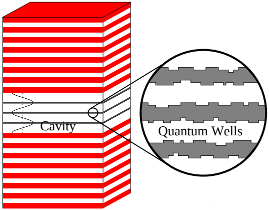

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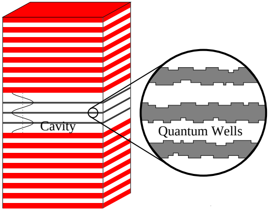

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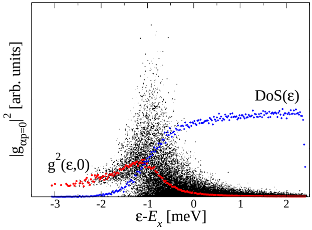

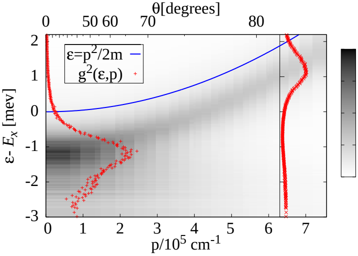

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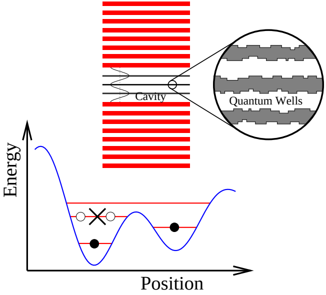
Polariton system model

Polariton model

- Disorder-localised excitons
- Treat disorder sites as two-level (exciton/no-exciton)
- Propagating (2D) photons
- Exciton–photon coupling g.



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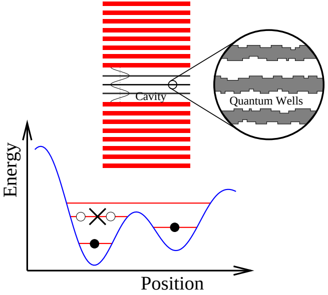
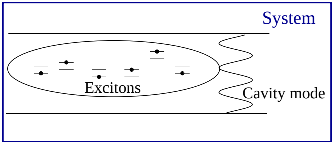


$$H_{\text{sys}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\alpha} \left[\epsilon_\alpha \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \frac{1}{\sqrt{\text{Area}}} g_{\alpha,\mathbf{k}} \psi_{\mathbf{k}} b^\dagger_\alpha a_\alpha + \text{H.c.} \right]$$



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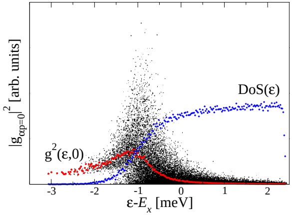

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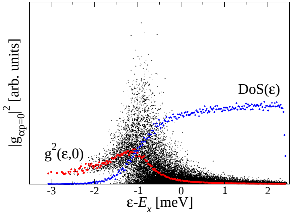
Mean-field theory:
Self-consistent polarisation and field

$$\left[i\partial_t + \omega_0 - \frac{\nabla^2}{2m} \right] \psi = \frac{1}{\sqrt{A}} \sum_\alpha g_\alpha P_\alpha$$



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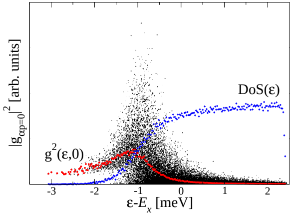

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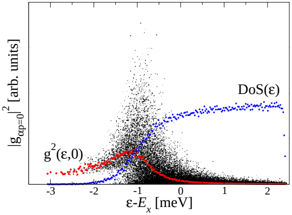
$$E_\alpha{}^2 = \left(\frac{\tilde{\epsilon}_\alpha}{2}\right)^2 + g_\alpha^2 \psi^2, \quad \tilde{\epsilon}_\alpha = \epsilon_\alpha - \mu$$



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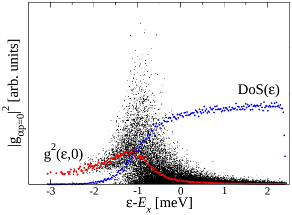

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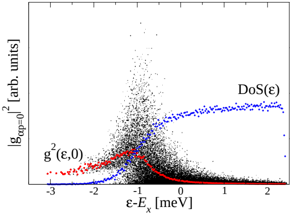

Density

$$\rho = |\psi|^2 + \frac{1}{A} \sum_\alpha \left[\frac{1}{2} - \frac{\tilde{\epsilon}_\alpha}{4 E_\alpha} \tanh(\beta E_\alpha) \right]$$

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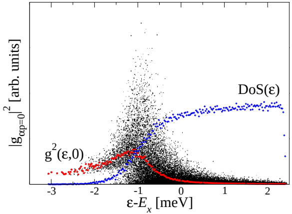

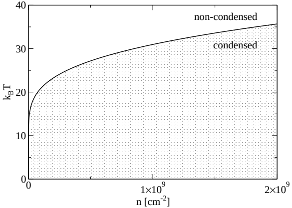

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Equilibrium: Fluctuations about mean-field

Fluctuations $\psi \rightarrow \psi + \delta\psi$; correction to action

$$S = S_0 + \sum_{\nu,\mathbf{p},\mathbf{q}} \begin{pmatrix} \delta\psi^*_{\mathbf{p}} \\ \delta\psi_{-\mathbf{p}} \end{pmatrix}^T \mathcal{G}^{-1}_{\mathbf{pq}}(\nu) \begin{pmatrix} \delta\psi_{\mathbf{q}} \\ \delta\psi^*_{-\mathbf{q}} \end{pmatrix}$$

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- Optical response:
  Treat $\mathbf{p} \neq \mathbf{q}$ perturbatively [D. M. Whittaker PRL 80 4791]
  - Spectral weight $W(\nu, \mathbf{p}) = 2\Im\left[\mathcal{G}^{11}_{\mathbf{pp}}(i\nu)\right]$

Equilibrium: Fluctuations about mean-field

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$$S = S_0 + \sum_{\nu,\mathbf{p},\mathbf{q}} \begin{pmatrix} \delta\psi^*_{\mathbf{p}} \\ \delta\psi_{-\mathbf{p}} \end{pmatrix}^T \mathcal{G}^{-1}_{\mathbf{pq}}(\nu) \begin{pmatrix} \delta\psi_{\mathbf{q}} \\ \delta\psi^*_{-\mathbf{q}} \end{pmatrix}$$

$$\mathcal{G}^{-1}_{\mathbf{pq}}(\nu) = (\omega_{\mathbf{p}} + i\nu)\mathbb{1}\delta_{\mathbf{pq}} + \sum_\alpha g^*_{\alpha\mathbf{p}} g_{\alpha\mathbf{q}} \begin{pmatrix} \chi^{(1)}_\alpha(\nu) & \chi^{(2)}_\alpha(\nu) \\ \chi^{(2)}_\alpha(\nu) & \chi^{(1)*}_\alpha(\nu) \end{pmatrix}$$

- Optical response:
  Treat $\mathbf{p} \neq \mathbf{q}$ perturbatively [D. M. Whittaker PRL 80 4791]
  - Spectral weight $\quad W(\nu, \mathbf{p}) = 2\Im\left[\mathcal{G}^{11}_{\mathbf{pp}}(i\nu)\right]$
  - Emission $\quad P_{\text{emit}}(\nu, \mathbf{p}) = n_B(\nu) W(\nu, \mathbf{p})$
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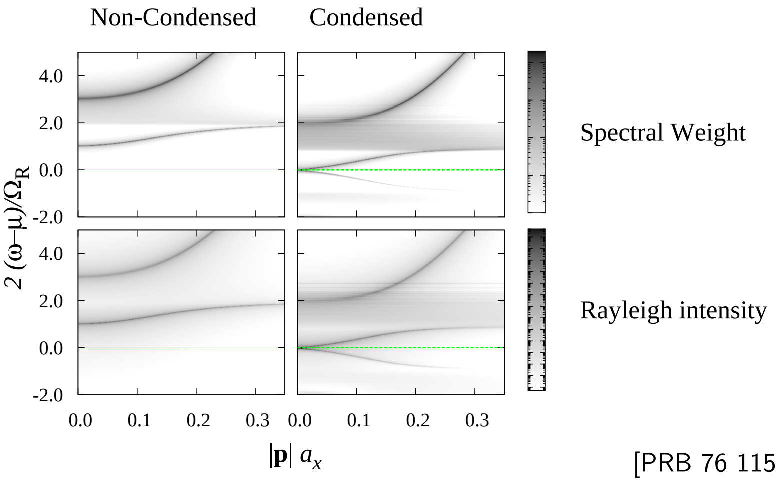
  - Rayleigh scattering $I_{\mathbf{p}\neq\mathbf{q}}(\nu) = |\mathcal{G}^{11}_{\mathbf{pq}}(i\nu)|^2$

Fluctuations and optical spectra



[PRB 76 115326 (2007)]

- Phase sensitive Rayleigh → "negative energy" Bogoliubov modes.

Fluctuation corrections to phase boundary

Fluctuation corrections to density

$$\rho \to \rho_0 + \sum_q \langle \delta\psi^\dagger \delta\psi \rangle$$

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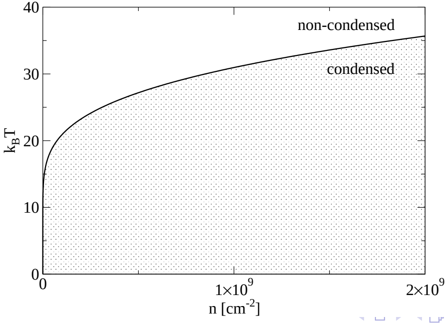

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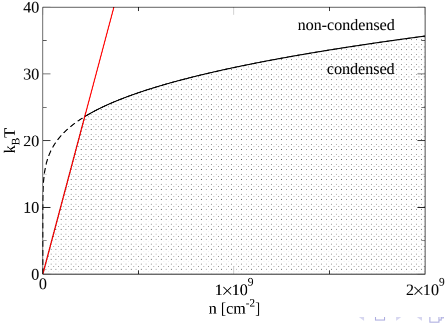

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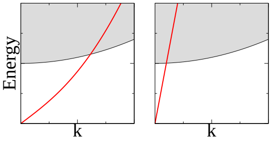

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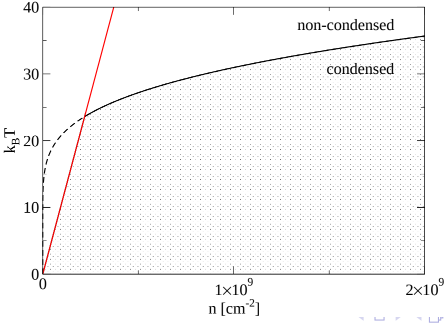

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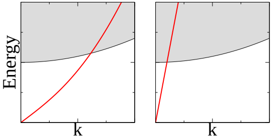

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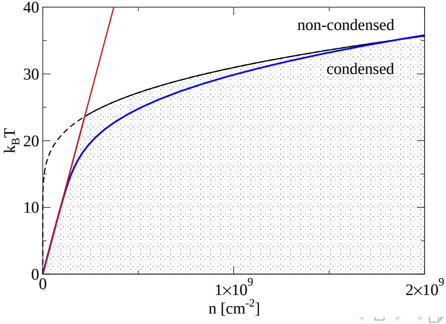

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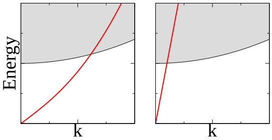

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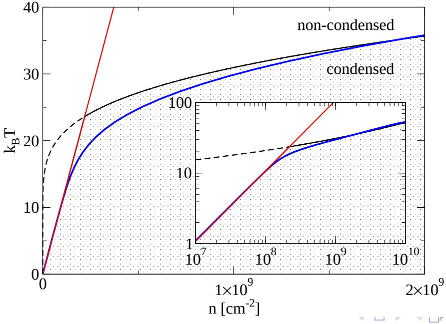
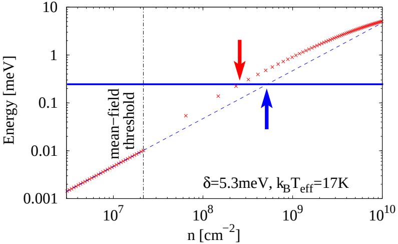


Blueshift:



Clean limit:

$$\delta E_{LP} \simeq \mathcal{R}y_X a_X^2 n + \Omega_R a_X^2 n$$

Here: $\Omega_R a_X^2 \to \Omega_R \xi^2$
[PRB 77 235313]

Blueshfit and experimental phase boundary

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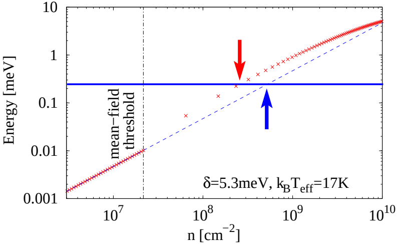

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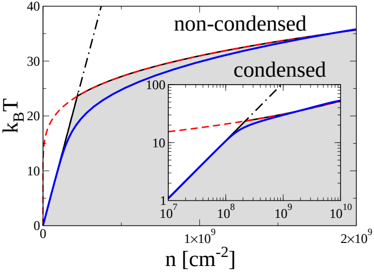
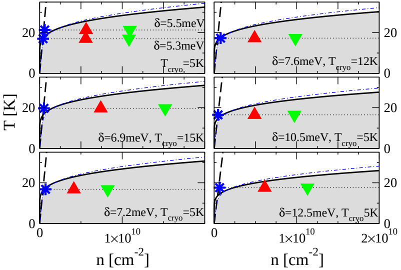
Phase diagram:

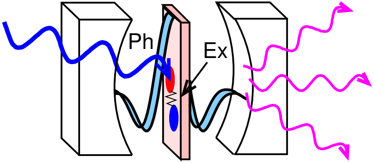


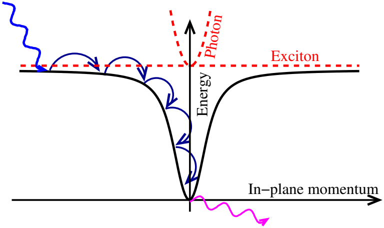
CdTe:

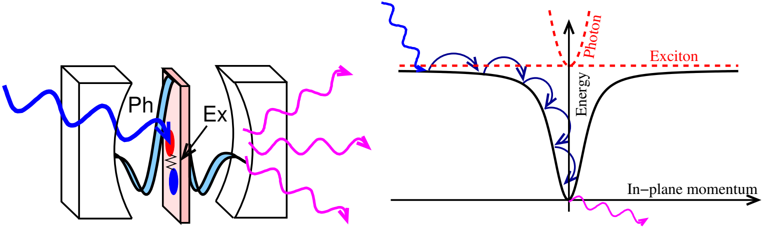


Overview

Non-equilibrium: Timescales







	Lifetime	Thermalisation
Atoms	10s	10ms
Excitons[a]	50ns	0.2ns
Polaritons	5ps	0.5ps
Magnons[b]	1μs(??)	100ns(?)

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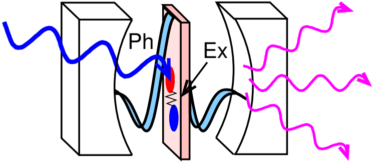

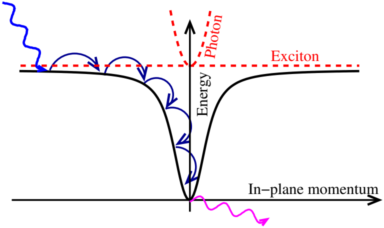

	Lifetime	Thermalisation	Linewidth	Temperature	
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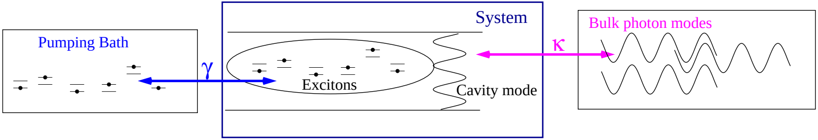
Non-equilibrium model: baths



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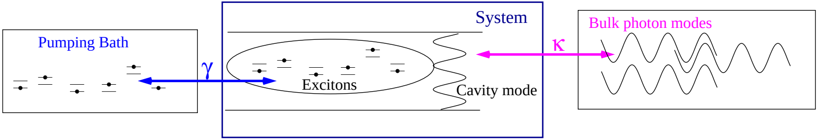

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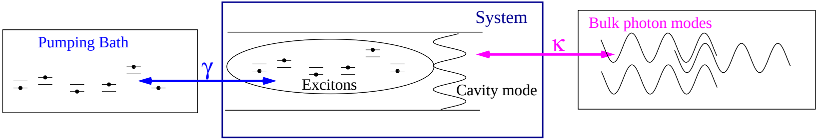

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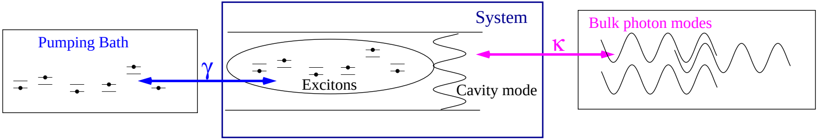

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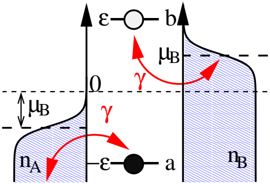

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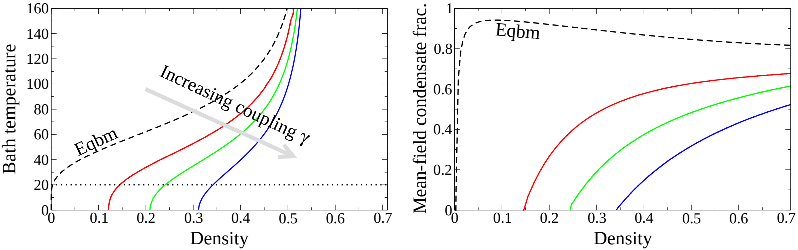

Limits of gap equation

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$$\omega_0 - \mu_s = \frac{g^2}{2E} \tanh\left(\frac{\beta E}{2}\right)$$

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Nonlinear, complex susceptibility $\chi(\psi(r,t))$

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$$i\hbar\partial_t\psi|_{\text{nlin}} = U|\psi|^2\psi$$

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$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit: Gross-Pitaevskii equation

$$\left(i\hbar\partial_t + i\kappa - \left[V(r) - \frac{\hbar^2\nabla^2}{2m}\right]\right)\psi(r) = \chi(\psi(r,t))\psi(r,t)$$

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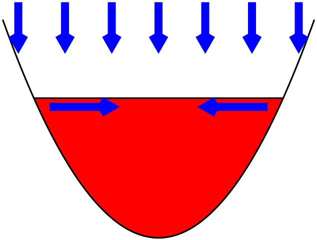
Gross-Pitaevskii equation: Harmonic trap

$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2\nabla^2}{2m} + \frac{m\omega^2}{2}r^2 + U|\psi|^2 + i\left(\gamma_{\text{eff}} - \kappa - \Gamma|\psi|^2\right)\right]\psi$$



[Keeling & Berloff, PRL, '08]

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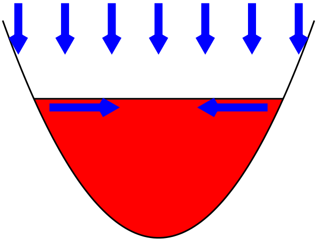

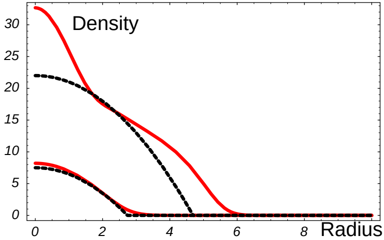

[Keeling & Berloff, PRL, '08]

Instability of Thomas-Fermi: details

$\psi = \sqrt{\rho}e^{i\phi} \quad \mathbf{v} = \frac{\hbar}{m}\nabla\phi$

$$\frac{1}{2}\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$

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$$\delta\rho_{n,m}(r,\theta,t) = e^{im\theta} h_{n,m}(r) e^{i\omega_{n,m}t}$$

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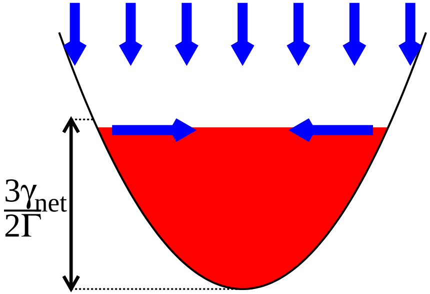
Add weak pumping/decay:

$$\omega_{n,n} \to \omega_{n,m} + i\gamma_{\text{net}} \left[\frac{m(1+2n) + 2n(n+1) - m^2}{2m(1+2n) + 4n(n+1) + m^2} \right]$$

Instability

Stability of Thomas-Fermi solution

$$\frac{1}{2}\partial_t \rho + \nabla\cdot(\rho \mathbf{v}) = \frac{1}{\hbar}(\gamma_{\text{net}} - \Gamma\rho)\rho$$



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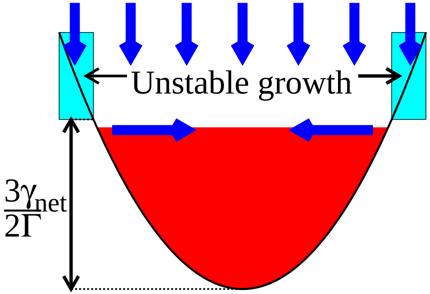

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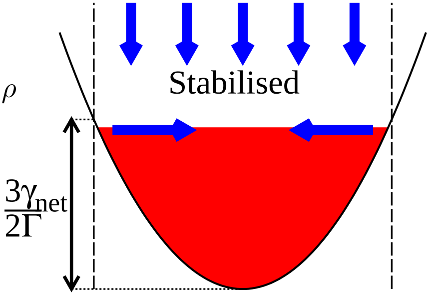

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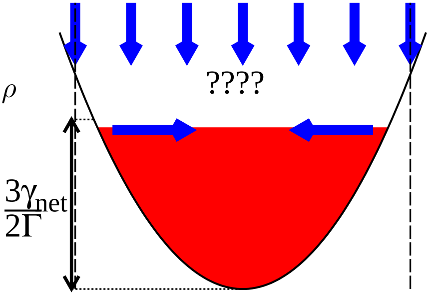
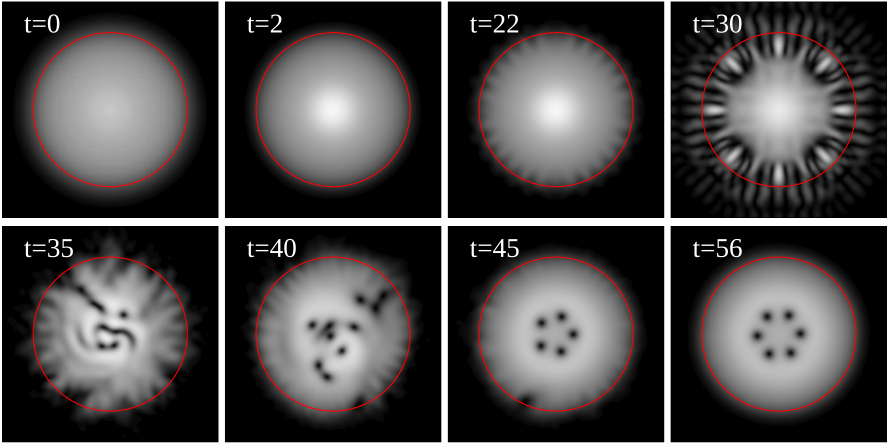
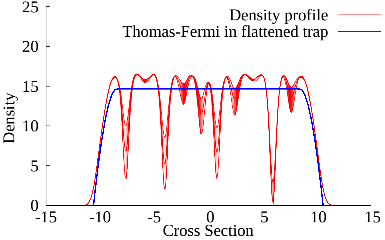


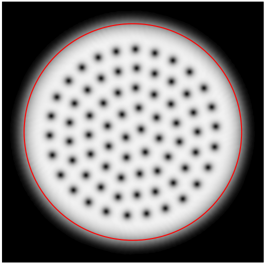
Time evolution:



[Keeling & Berloff, PRL, '08]

Why vortices





$$\nabla \cdot [\rho(\mathbf{v} - \Omega \times \mathbf{r})] = (\gamma_{\text{net}} \Theta(R - r) - \Gamma\rho)\,\rho,$$

$$\mu = \frac{m}{2}|\mathbf{v} - \Omega \times \mathbf{r}|^2 + \frac{m}{2} r^2(\omega^2 - \Omega^2) + U\rho - \frac{\hbar^2 \nabla^2 \sqrt{\rho}}{2m\sqrt{\rho}}$$

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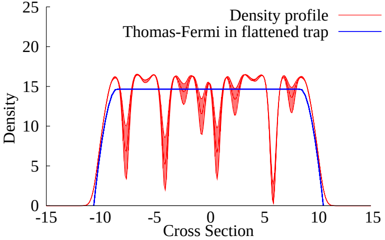

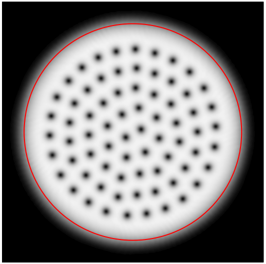

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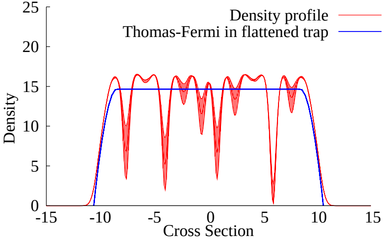

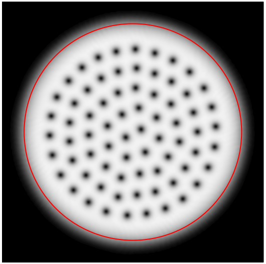

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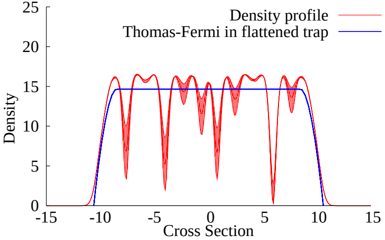

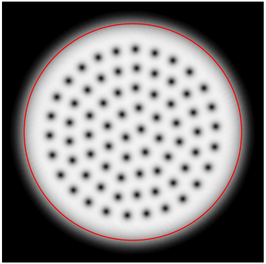

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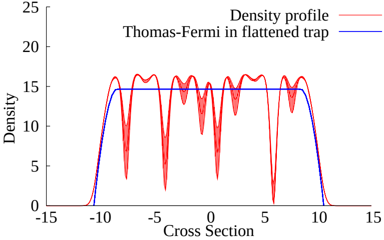

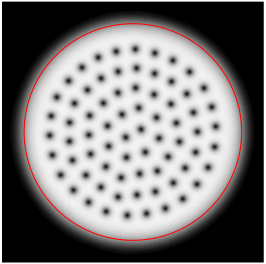

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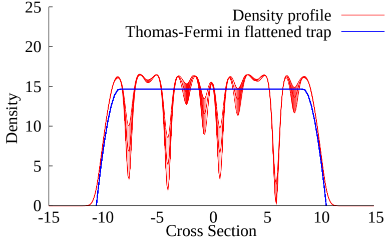

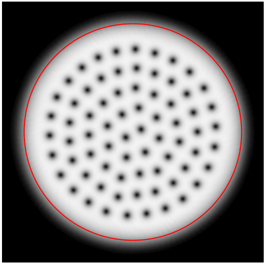

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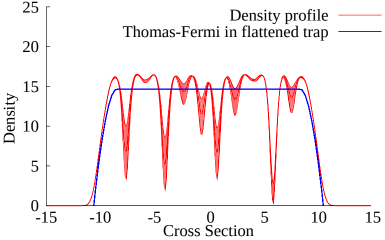

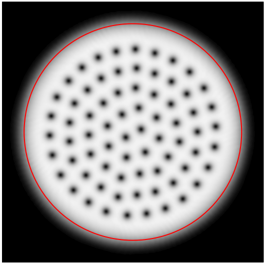

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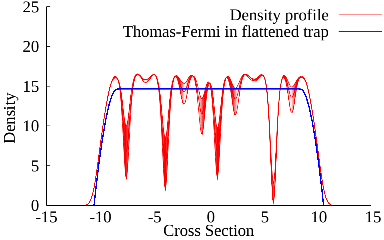

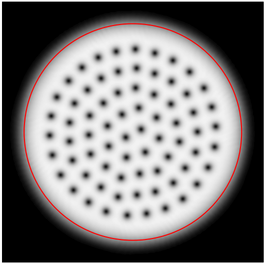

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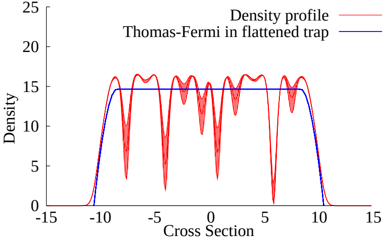

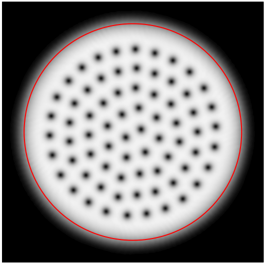

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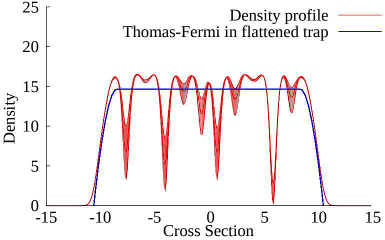

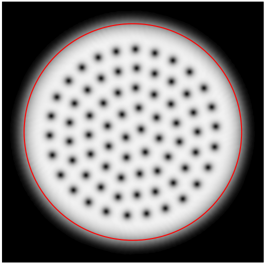

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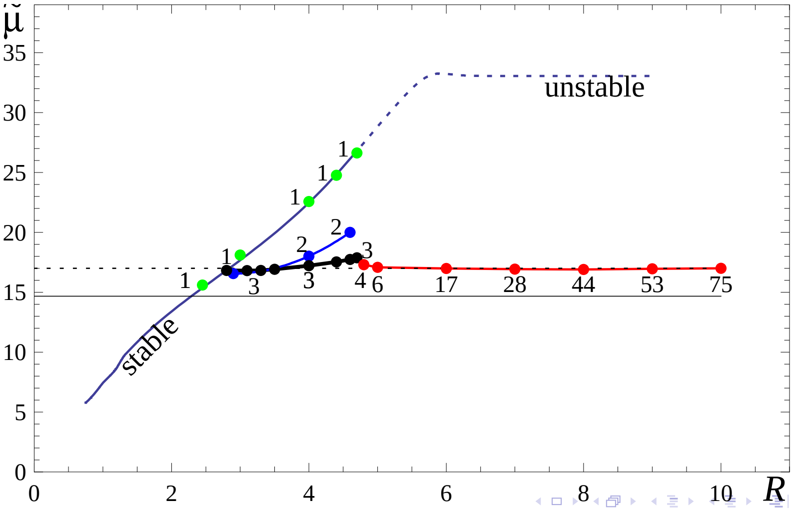
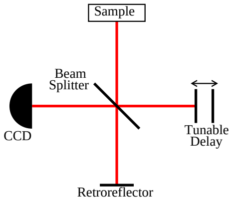
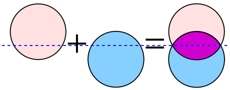
Why vortices: chemical potential vs size

$$\text{Thomas-Fermi} : \mu \propto R^2 \quad \text{Vortex} : \mu = \frac{U\gamma_{\text{net}}}{\Gamma}$$



Observing vortices: fringe pattern





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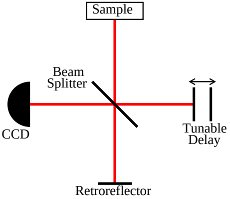

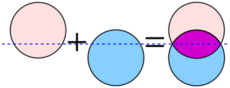

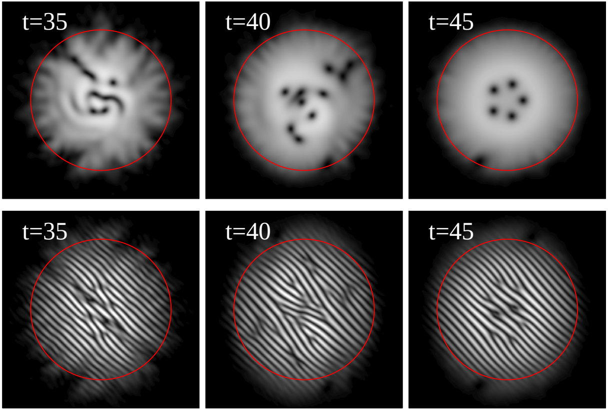

Overview

Fluctuations → Stability, Luminescence, Absorption

Keldysh approach:

$$\mathcal{G}_{R,A} = \mp i\theta[\pm(t-t')] \left\langle \left[\psi^\dagger, \psi\right]_- \right\rangle$$

$$\mathcal{G}_K = -i \left\langle \left[\psi^\dagger, \psi\right]_+ \right\rangle$$

Fluctuations → Stability, Luminescence, Absorption

Keldysh approach:

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Fluctuations → Stability, Luminescence, Absorption

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Fluctuations → Stability, Luminescence, Absorption

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$$\mathcal{G}_R - \mathcal{G}_A = \frac{1}{{\mathcal{G}_\mathcal{R}}^{-1}} - \frac{1}{[{\mathcal{G}_\mathcal{R}}^{-1}]^\dagger}$$

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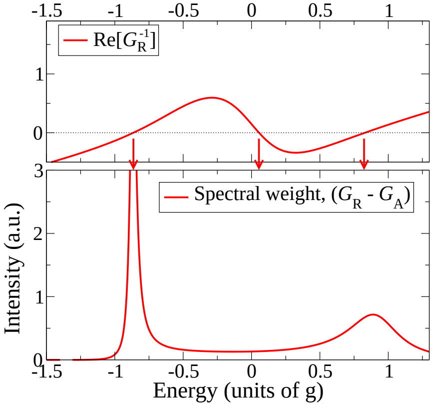

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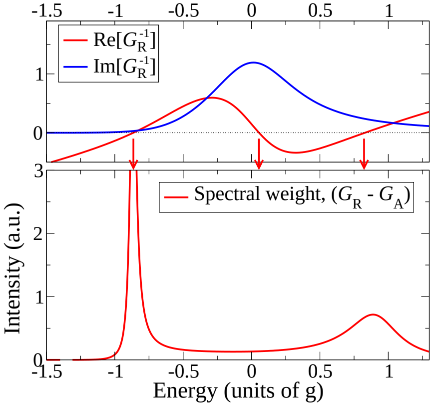

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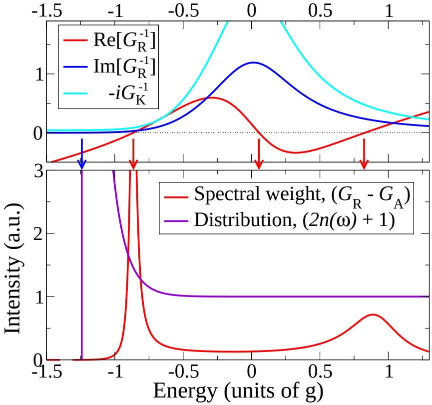

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$$\mathcal{L} = \left\langle \psi^\dagger \psi \right\rangle = \frac{i}{2}\left[\mathcal{G}_K + (\mathcal{G}_R - \mathcal{G}_A)\right]$$

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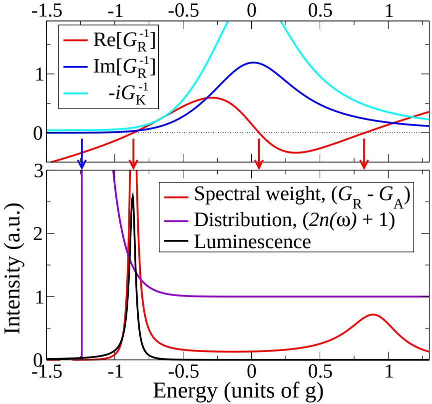
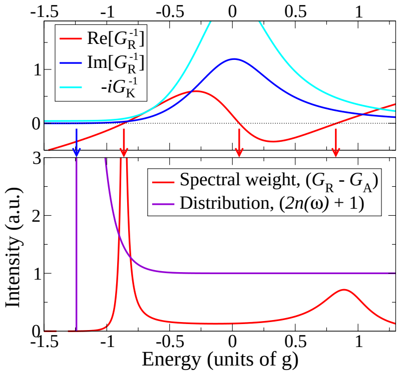


Linewidth, inverse Green's function and gap equation



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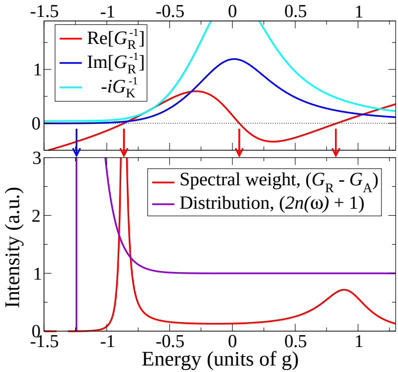

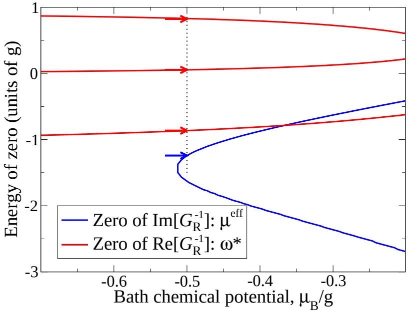

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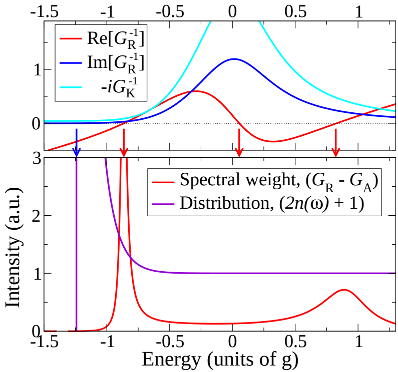
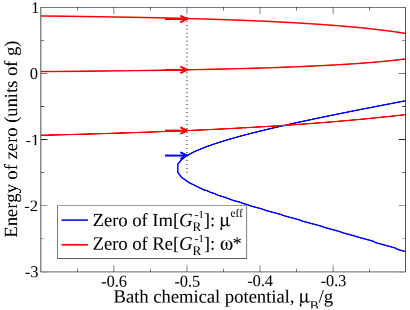
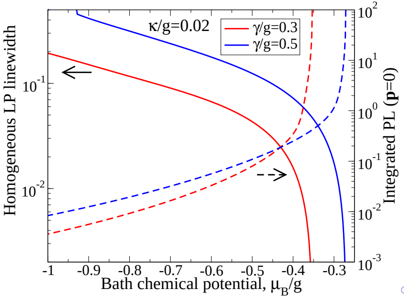




At transition, Gap Equation implies:

$$\mathcal{G}_R^{-1}(\omega = \mu_S, k = 0) = 0$$



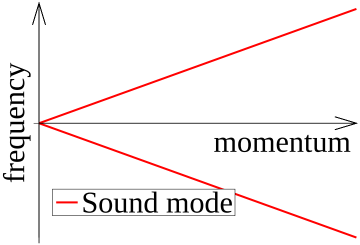
Fluctuations above transition

When condensed

$$\text{Det}\left[\mathcal{G}_R^{-1}(\omega, \mathbf{k})\right] = \omega^2 - c^2\mathbf{k}^2$$

Poles:

$$\omega^* = c|\mathbf{k}|$$



[Szymańska et al., PRL '06; PRB '07]

Fluctuations above transition

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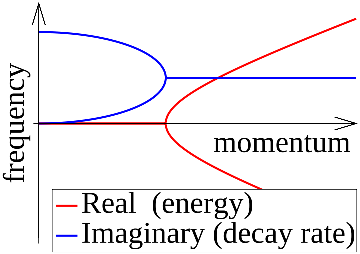
$$\text{Det}\left[\mathcal{G}_R^{-1}(\omega,\mathbf{k})\right] = (\omega + ix)^2 + x^2 - c^2\mathbf{k}^2$$

Poles:

$$\omega^* = -ix \pm \sqrt{c^2\mathbf{k}^2 - x^2}$$



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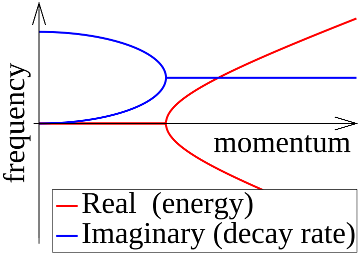

Correlations (in 2D): $\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, r')\right\rangle \simeq |\psi_0|^2 \exp\left[-\mathcal{D}_{\phi\phi}(t, r, r')\right]$

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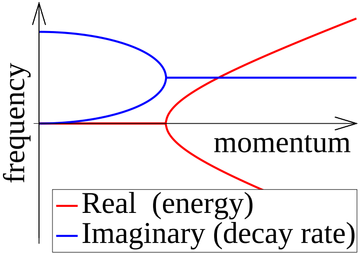

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$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0, 0)\right\rangle \simeq |\psi_0|^2 \exp\left[-\eta \begin{cases} \ln(r/\xi) & r \to \infty, t \simeq 0 \\ \frac{1}{2}\ln(c^2 t/x\xi^2) & r \simeq, t \to \infty \end{cases}\right]$$

[Szymańska et al., PRL '06; PRB '07]

Finite size effects: Single mode vs many mode

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$\mathcal{D}_{\phi\phi}(t,r,r')$ from sum of phase modes. Study $ct \gg r$ limit:

$$\mathcal{D}_{\phi\phi}(t,r,r) \propto \sum_n^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2(1-e^{i\omega t})}{|(\omega+ix)^2 + x^2 - (n\Delta)^2|^2}$$

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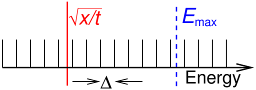
$\Delta \ll \sqrt{x/t} \ll E_{\text{max}}$



$\mathcal{D}_{\phi\phi} \sim 1 + \ln(E_{\text{max}}\sqrt{t/x})$

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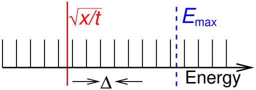

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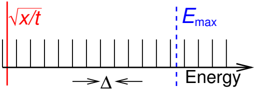

$\mathcal{D}_{\phi\phi} \sim \left(\frac{\pi C}{2x}\right)\left(\frac{t}{2x}\right)$

Relating finite-size spectrum to self phase modulation

Finite-size spectrum:

$$\mathcal{D}_{\phi\phi}(t, r, r) \propto \sum_{n}^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(r)|^2(1 - e^{i\omega t})}{|(\omega + ix)^2 + x^2 - (n\Delta)^2|^2}$$

Connection to Kubo Formula [Eastham and Whittaker, arXiv:0811.4333]

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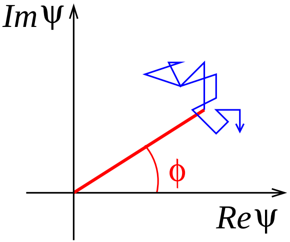

$$\partial_t \phi = U\delta N$$

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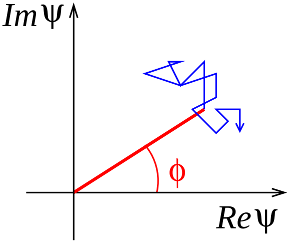

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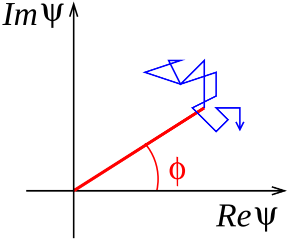

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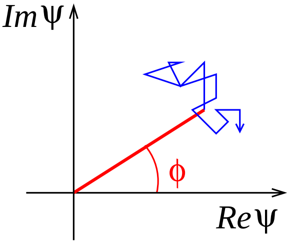

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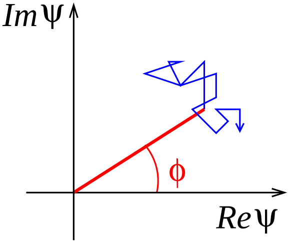

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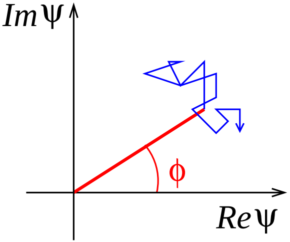

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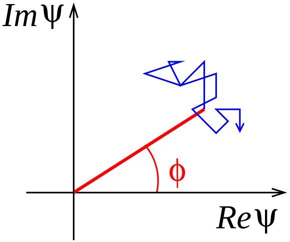

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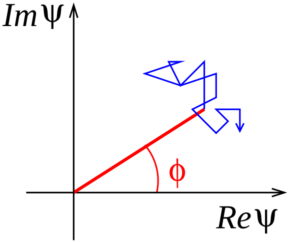

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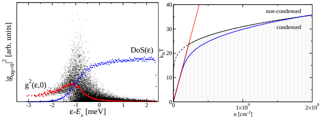
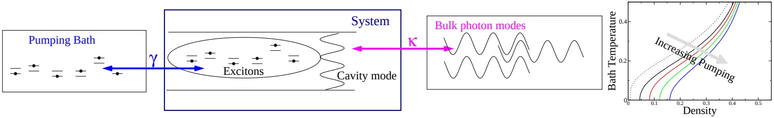
Conclusions

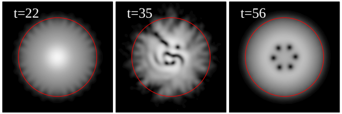
- Localised two-level system model
  Mean-field and fluctuations



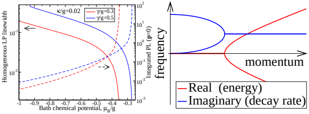
- Effects of pumping on mean-field theory



- Modification to Thomas-Fermi profile
  Spontaneous rotating vortex lattice



- Change to spectrum and correlations
  Phase modes and finite size



Acknowledgements

People:

Funding:

EPSRC Engineering and Physical Sciences Research Council

Pembroke College

Extra slides

Superfluidity

Superfluidity:

$$\mathbf{J} = \rho \mathbf{v} = \Psi^{\dagger} i\hbar \nabla \Psi = |\Psi|^2 \nabla \phi$$

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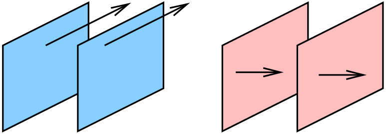

$$\begin{aligned}\chi_{ij}(\omega = 0, \mathbf{q} \to 0) &= \langle J_i(\mathbf{q}) J_j(\mathbf{q}) \rangle \\ &= \chi_T(q) \left(\delta_{ij} - \frac{q_i q_j}{q^2} \right) + \chi_L(q) \frac{q_i q_j}{q^2}\end{aligned}$$

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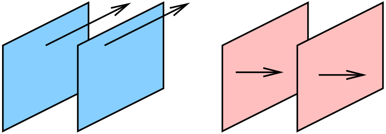

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Superfluid part,
$\rho_s \propto \lim_{q\to 0}(\chi_L - \chi_T)$.

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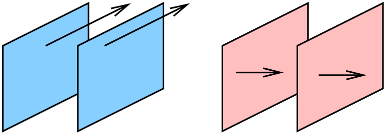

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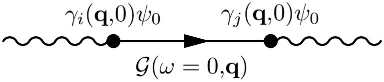
$\Delta\chi_{ij}(q) =$  $+ \ldots$

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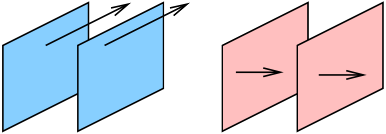

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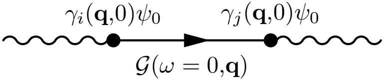
$\Delta\chi_{ij}(q) =$  $+ \ldots$

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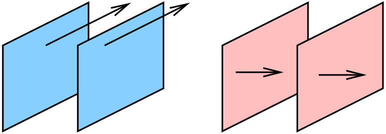

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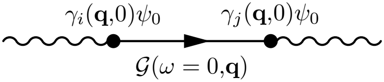
$\Delta\chi_{ij}(q) =$  $+ \dots$

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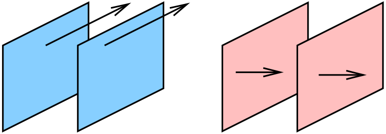

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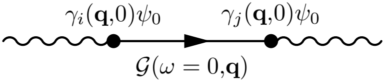
$\Delta\chi_{ij}(q) =$ 

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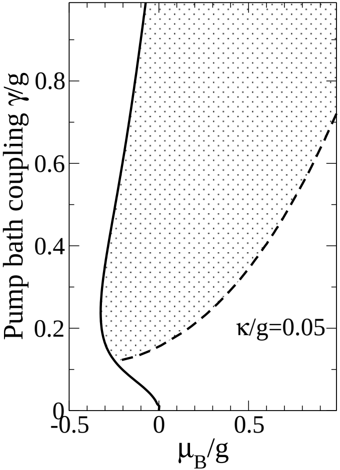
Zero temperature phase diagram

$$\tilde{\omega}_0 - i\kappa = g^2\gamma \sum_\alpha \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(\tilde{\epsilon}_\alpha + i\gamma)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}.$$



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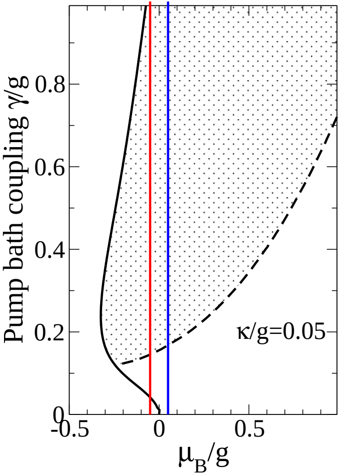

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