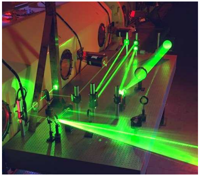
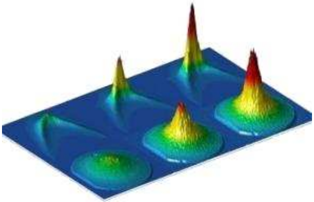
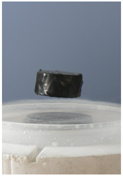
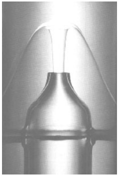
From Lasers to Bose-Einstein condensates

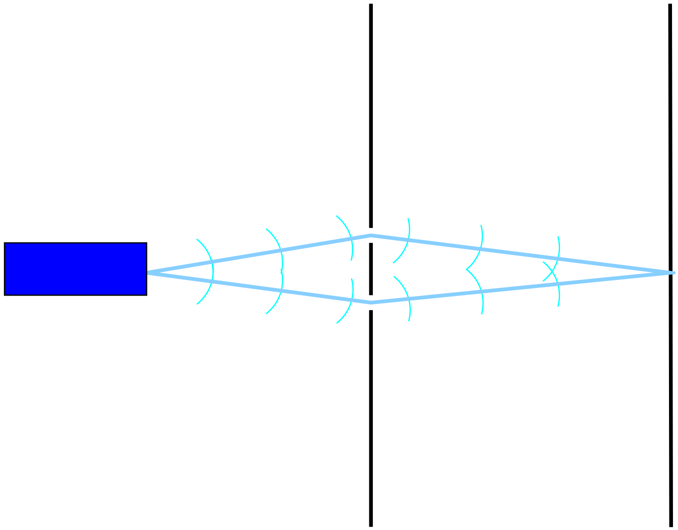
How superfluids, superconductors, polaritons and lasers fit together

Jonathan Keeling

Stokes Society, November 2008

The two-slit experiment



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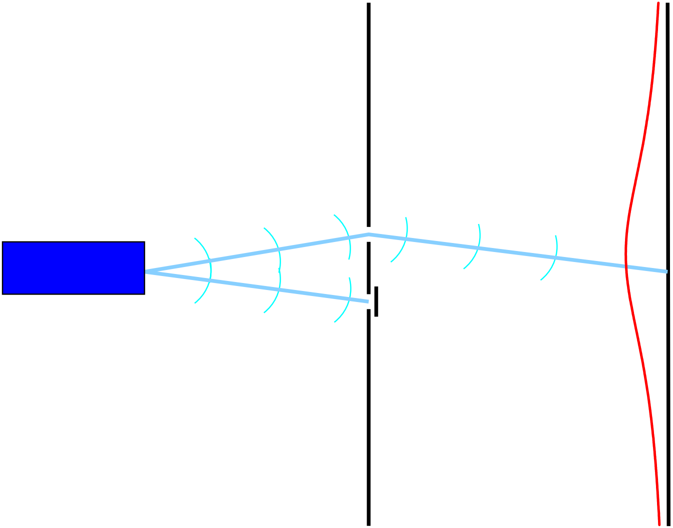

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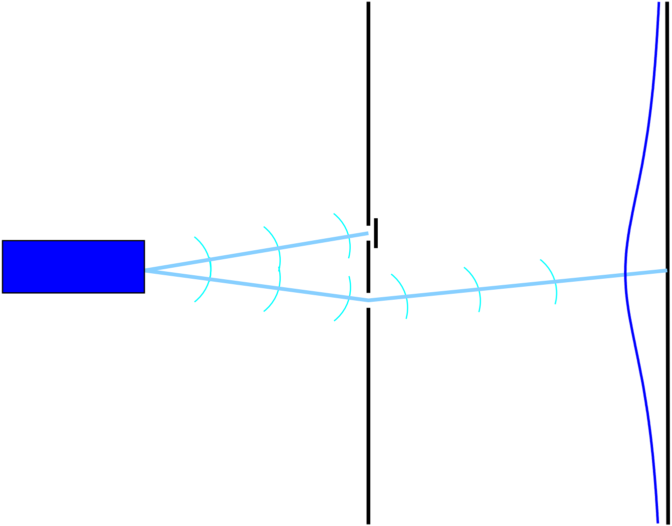

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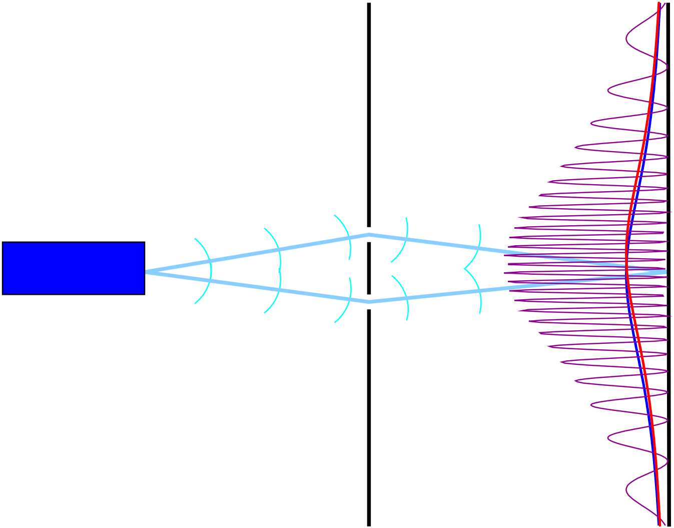

Macroscopic objects

Why no such interference for macroscopic objects?

- Wavelength would be very small $\lambda \sim 1/\sqrt{m}$
- Internal degrees of freedom remember "which way".
- Different initial conditions wash out path.

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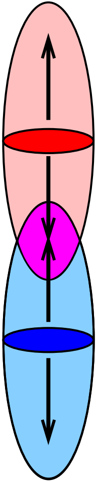

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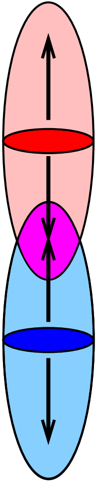

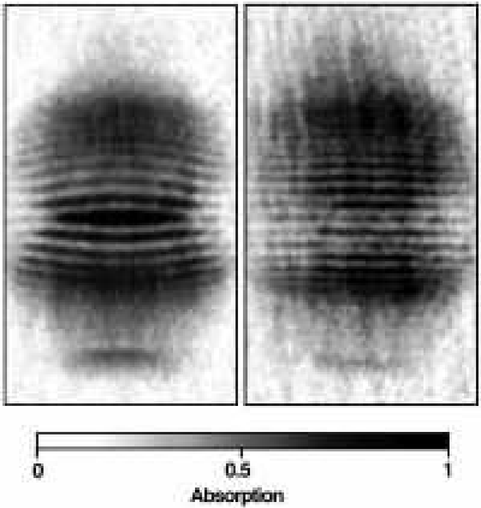

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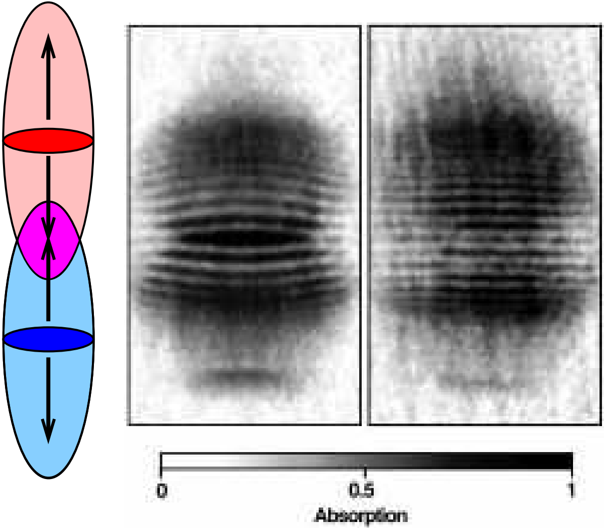
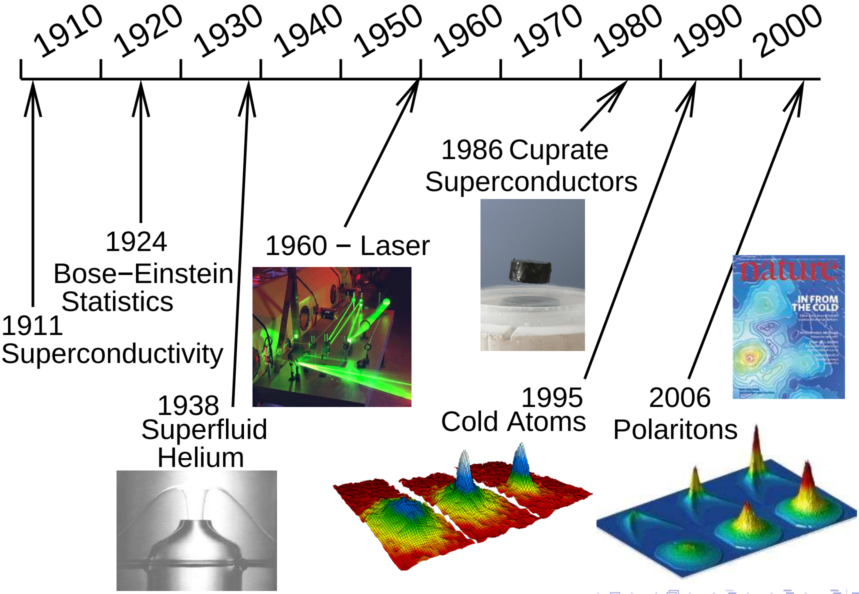


All atoms in single quantum state — like a classical wave.

History of Quantum Condensates

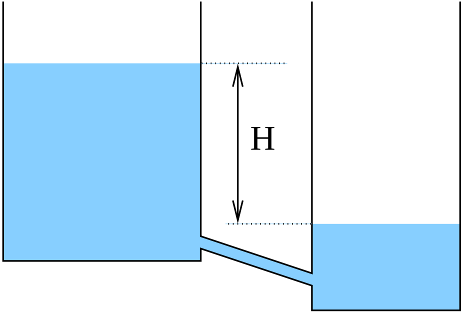


Overview

1. Particles and waves
   - The two-slit experiment with atoms
   - History of quantum condensates
2. Signatures of macroscopic occupation
   - Superfluidity
   - Superconductivity
3. Why low temperature
4. What about Lasers
5. Polaritons
   - What are excitons, polaritons,
   - What do they do
   - Why (else) are they interesting

Overview

What is superfluidity



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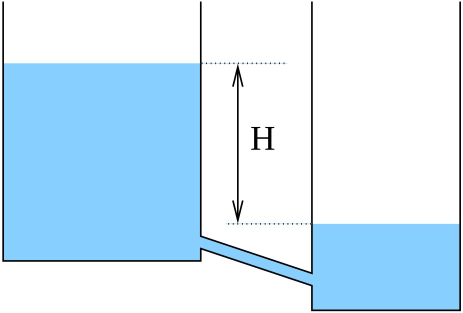

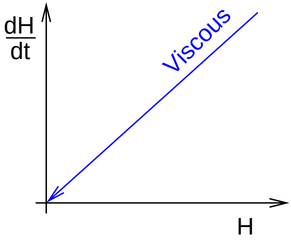

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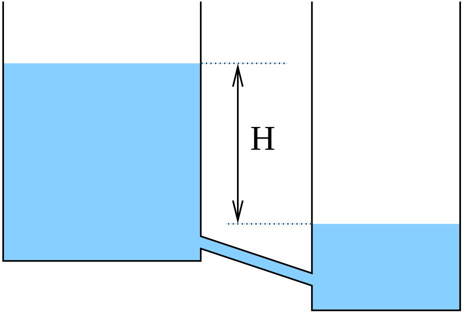

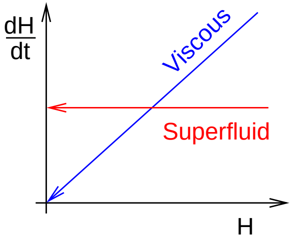

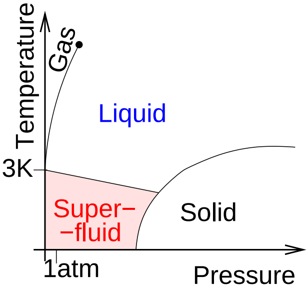

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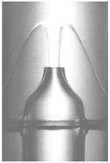

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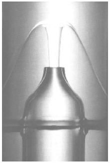

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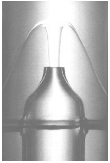

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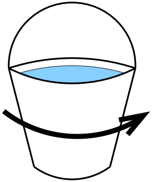

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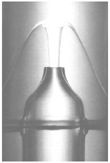

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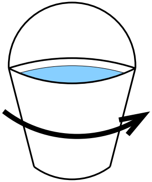

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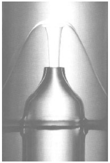

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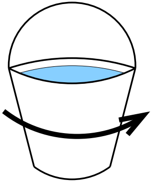

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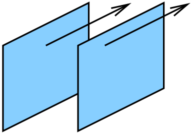

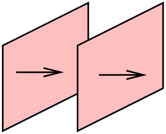

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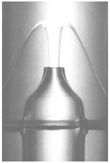

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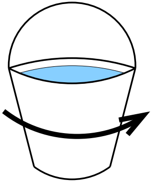

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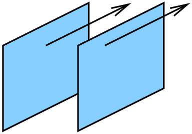

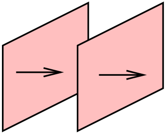

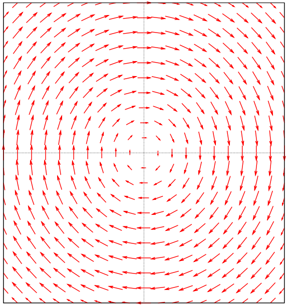

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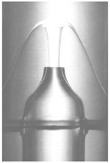

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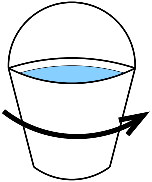

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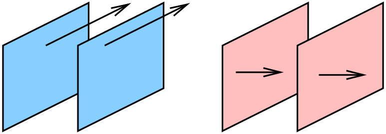

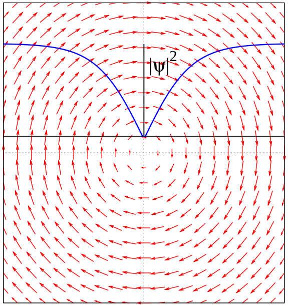

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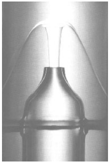

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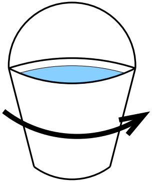

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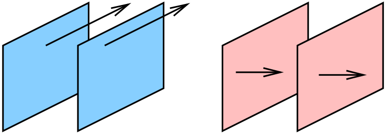

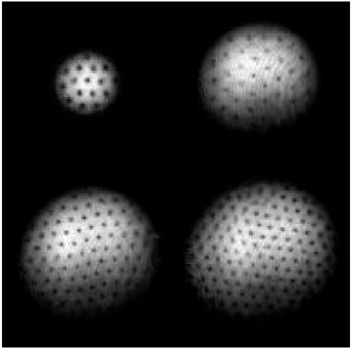

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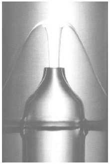

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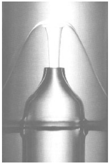

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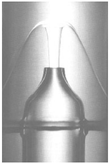

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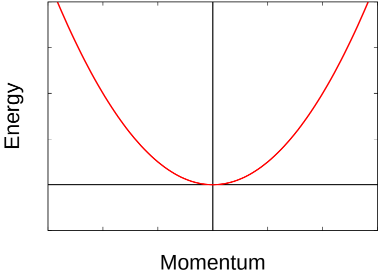

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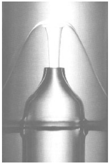

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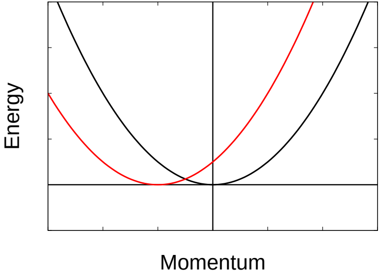

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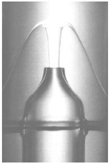

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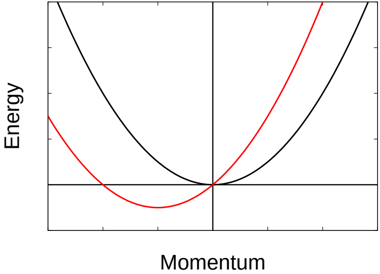

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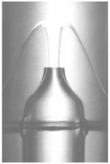

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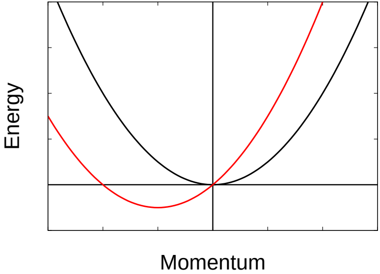

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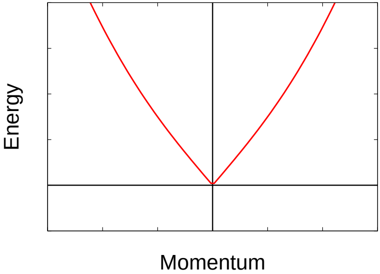

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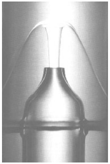

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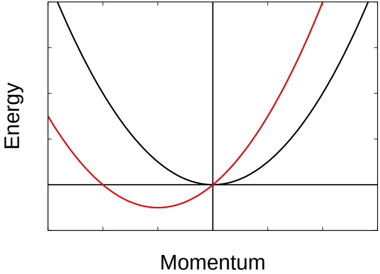

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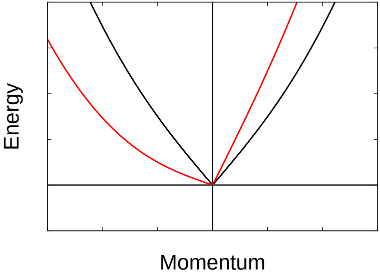

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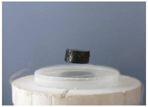

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- Fermions and macroscopic occupation
-

- Probabilities: $\Psi \to P = |\Psi|^2$

$$\Psi(r_1, r_2) = \pm\Psi(r_2, r_1)$$

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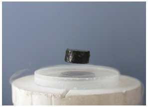

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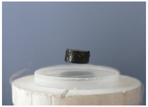

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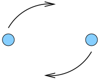
-  $\Psi(r_1, r_2) = \pm\Psi(r_2, r_1)$
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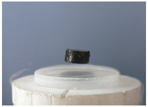

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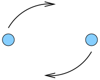
-  $\Psi(r_1, r_2) = \pm\Psi(r_2, r_1)$
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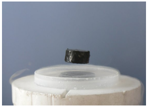

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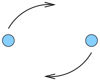
-  $\Psi(r_1, r_2) = \pm\Psi(r_2, r_1)$
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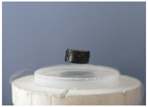

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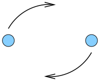
-  $\Psi(r_1, r_2) = \pm\Psi(r_2, r_1)$
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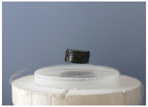

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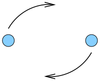
-  $\Psi(r_1, r_2) = \pm\Psi(r_2, r_1)$
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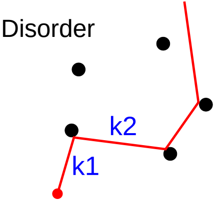
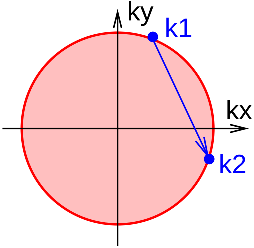
Macroscopically occupy pair state:

$$\prod_{i \neq j} \Phi\left(r_i - r_j\right)$$

Scattering and conductivity

Normally conductivity disrupted by disorder:





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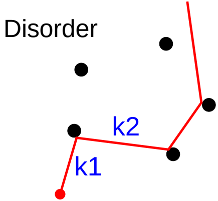

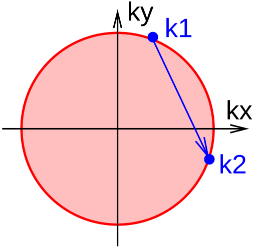

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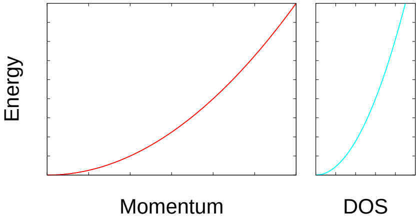

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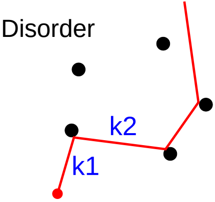

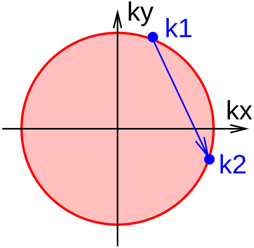

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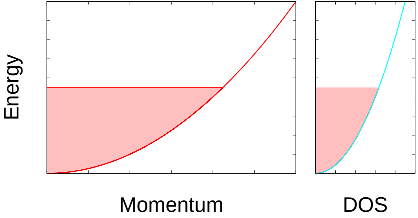

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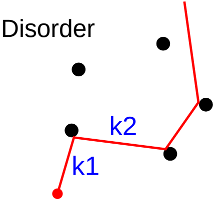

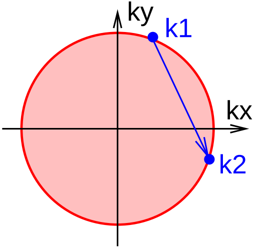

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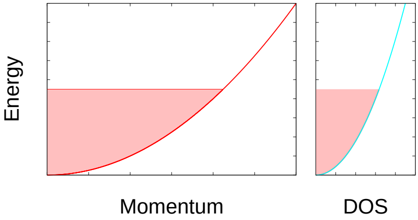

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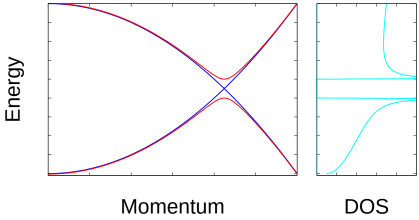

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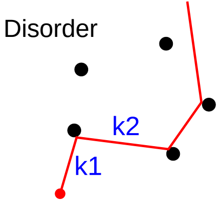

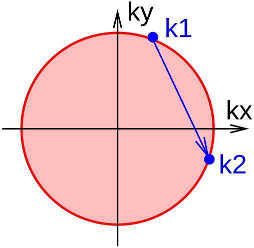

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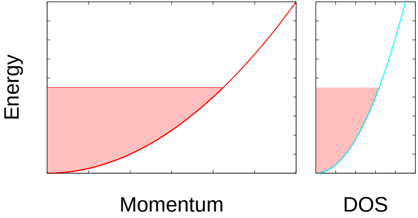

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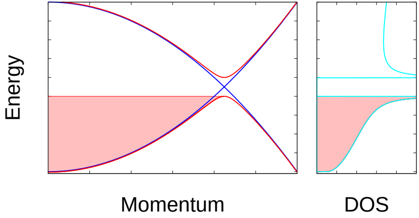

Overview

Why only at low temperatures

Temperature populates excitations – Depletes condensate

- Superconductors — electron pairs break apart
  - Mercury (first experiment) 4K.
  - Record (at $P = 1\text{atm}$) 138K.

- Helium, cold atoms — populate low momentum excitations

- Chemical potential at bottom of band:

$$k_B T_c = \frac{\hbar^2}{2m}\left(\frac{n}{2.612}\right)^{2/3}$$

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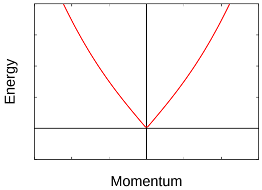

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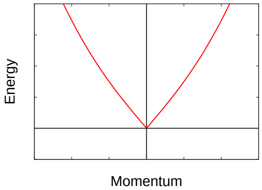

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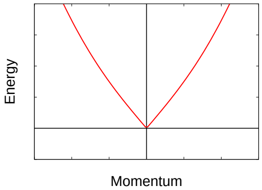

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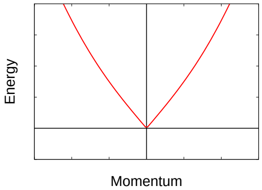

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- Other condensates show:
  Particle-like $\rightarrow$ wave-like
- Light is normally wave-like.

- **But** Thermal radiation — many frequecies; fringe patterns wash out.
- Laser provides light at single frequency
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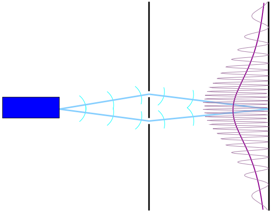

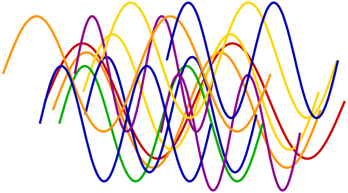

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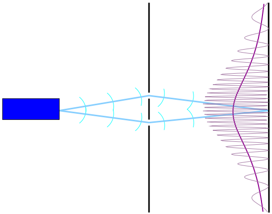

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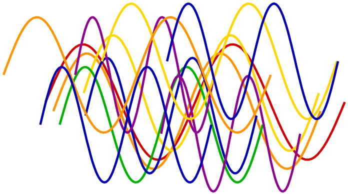

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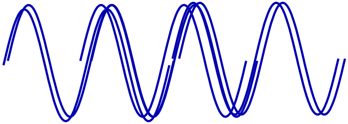

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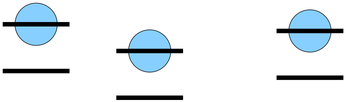

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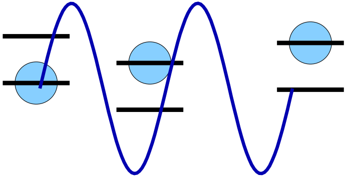

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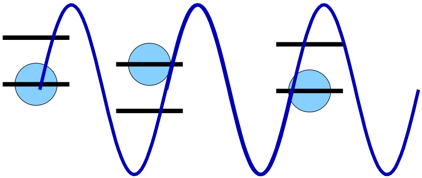

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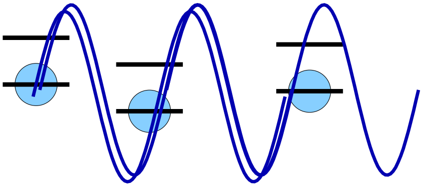

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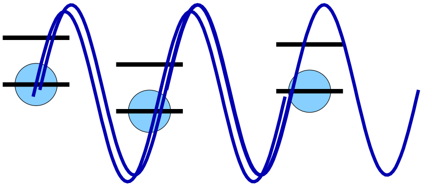

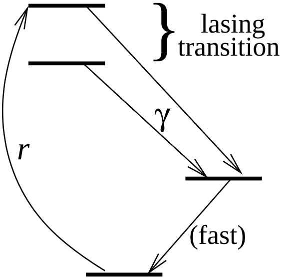

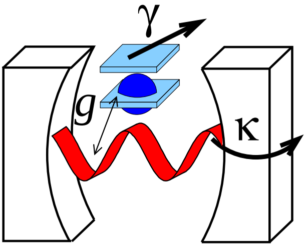

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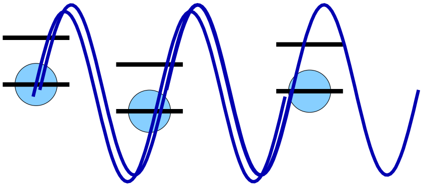
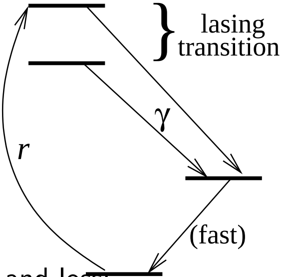
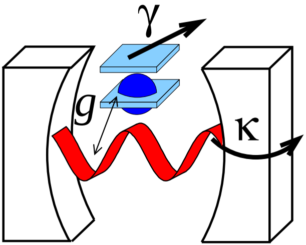


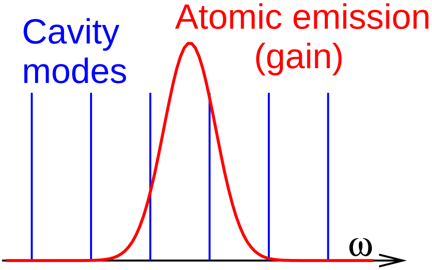




Balance of gain and loss:

$$\partial_t \langle n \rangle = \left[2r\frac{g^2}{\gamma^2} - \kappa \right] \langle n \rangle + 2r\frac{g^2}{\gamma^2} - 8r\left(\frac{g^2}{\gamma^2}\right)^2 \langle (n+1)^2 \rangle.$$

Laser spectrum



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- At threshold, all emission → single mode.
- Linewidth and threshold controlled by gain/loss
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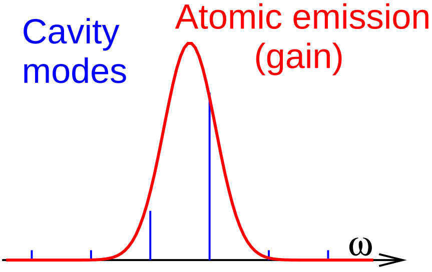

Below threshold
Spontaneous emission

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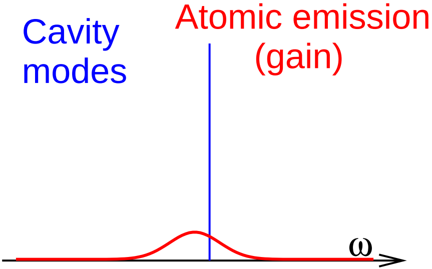

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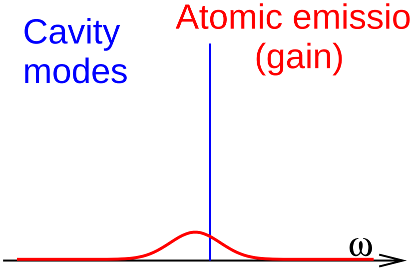

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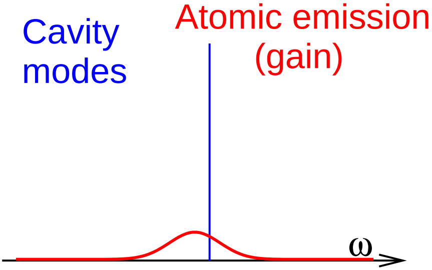

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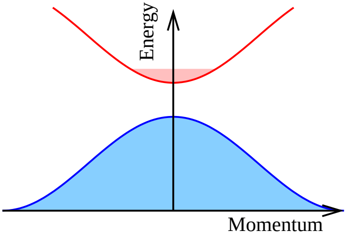

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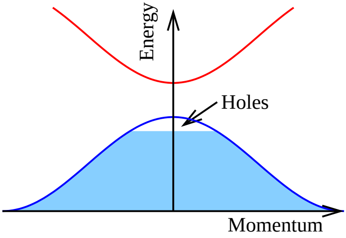

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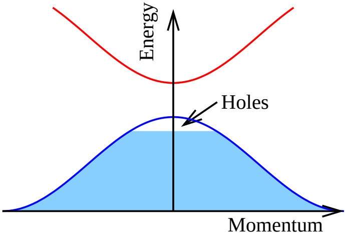

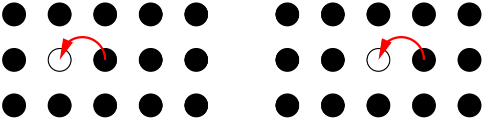

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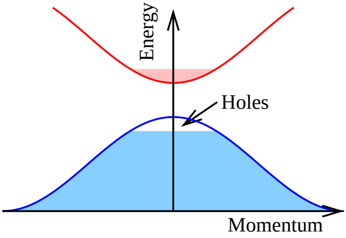

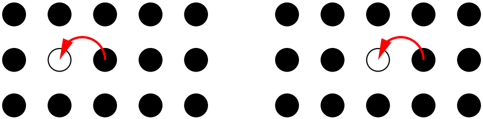

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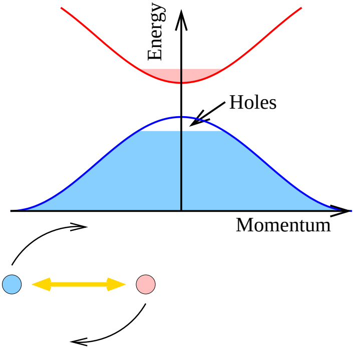
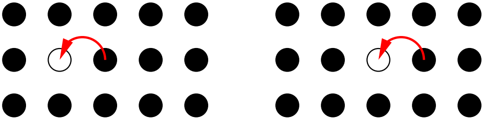
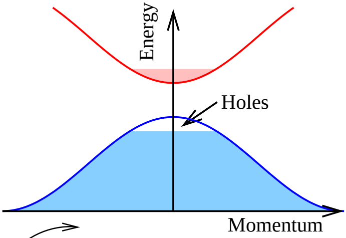
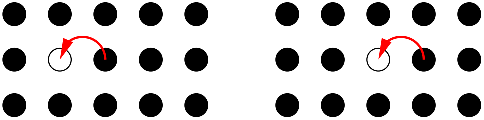


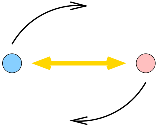


Electrostatic attraction:
Bound state.

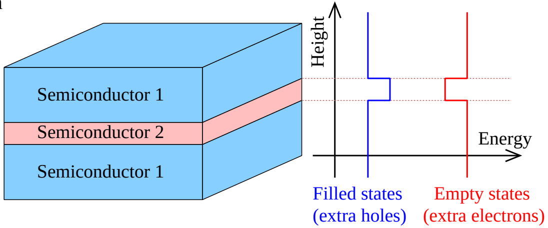
Excitons: quasiparticles in semiconductors



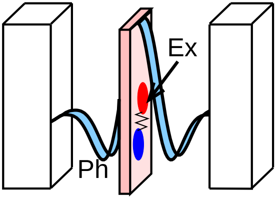




Electrostatic attraction:
Bound state.



Microcavity Polaritons



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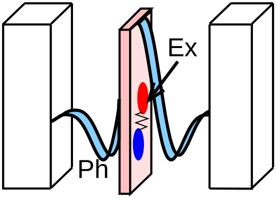

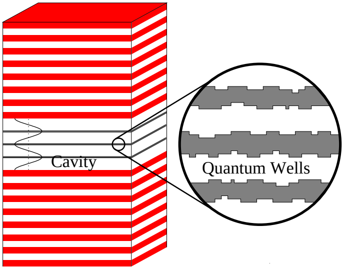

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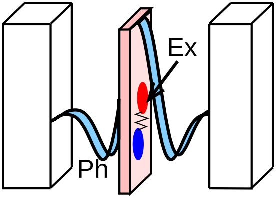
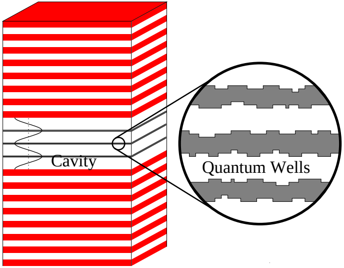
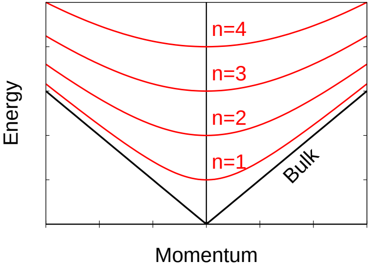




Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$



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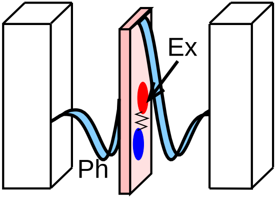

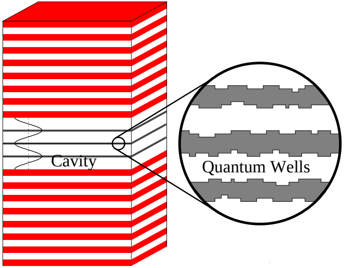

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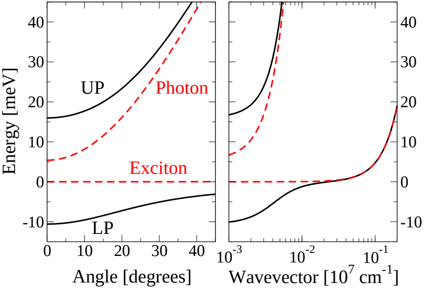
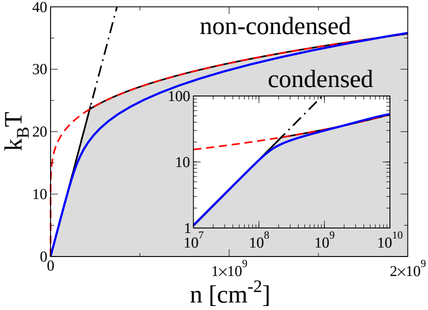


Why polaritons



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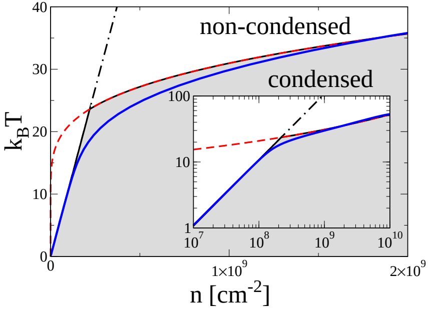

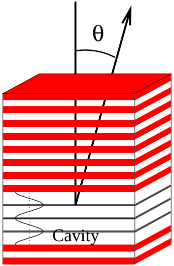

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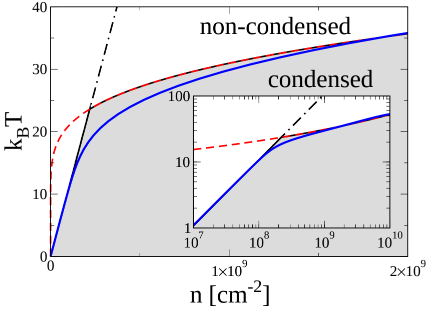
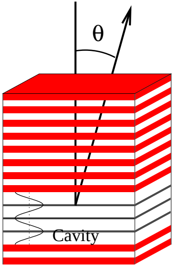
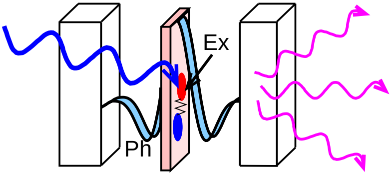
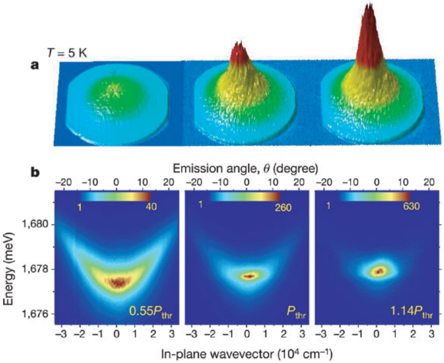
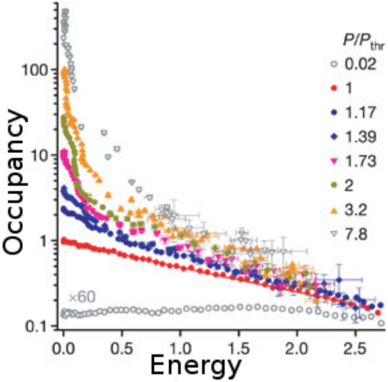






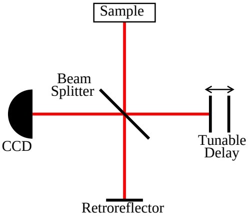
Polariton experiments: Momentum/Energy distribution

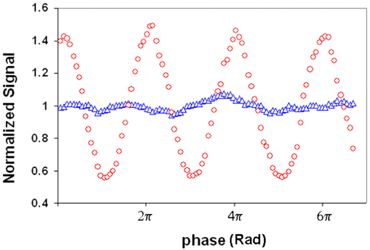




Polariton experiments: Coherence

Basic idea:





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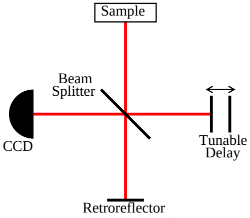
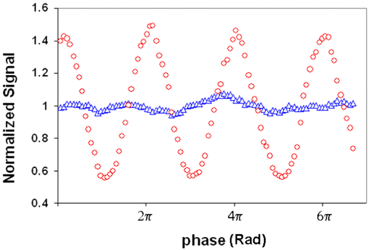
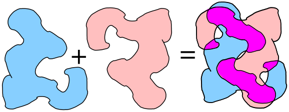
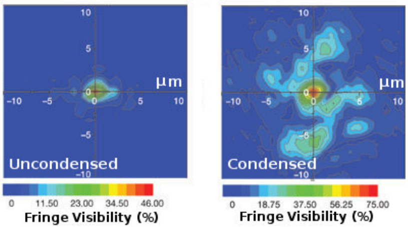




Coherence map:





Non-equilbrium condensation

- Condensate $\leftrightarrow$ Laser.

- What kinds of coherence out of equilibrium.
- What happens to superfluidity?

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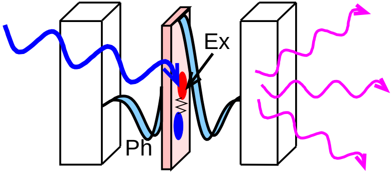

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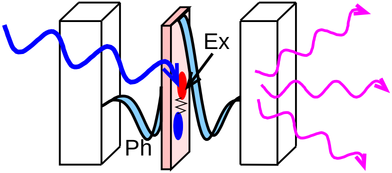

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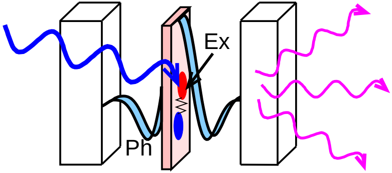

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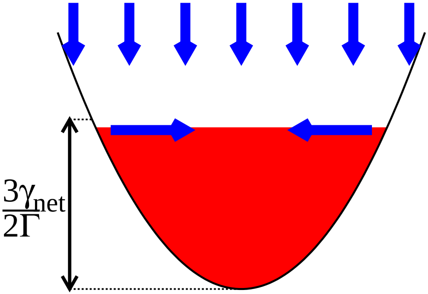
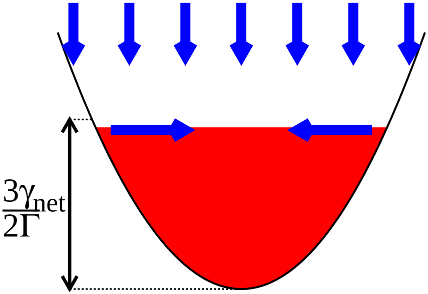
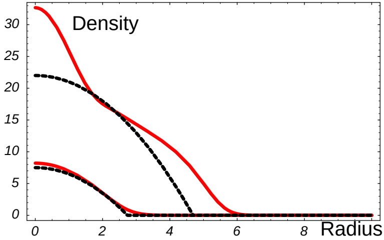

$\frac{3\gamma_{net}}{2\Gamma}$

Non-equilibrium condensate in a trap





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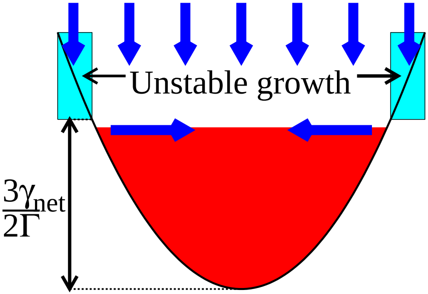
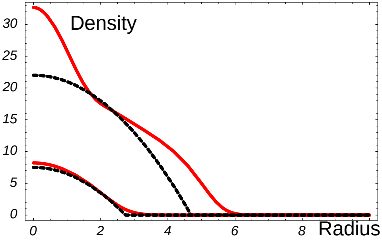
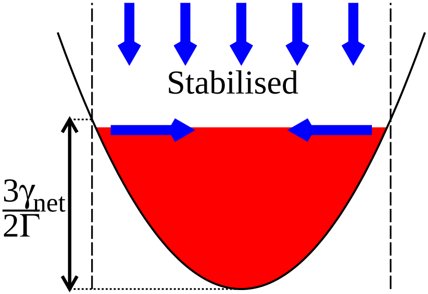
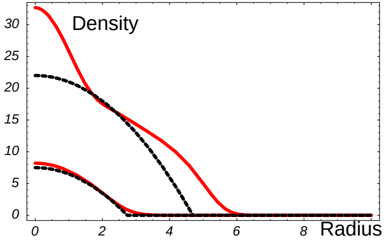
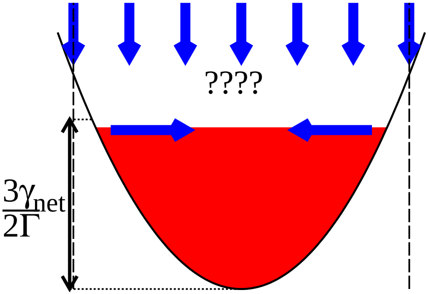


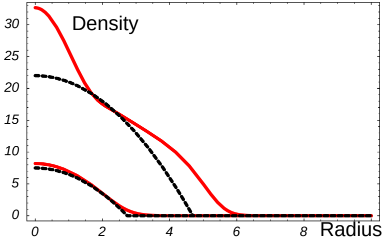



Stabilised
$\frac{3\gamma_{net}}{2\Gamma}$

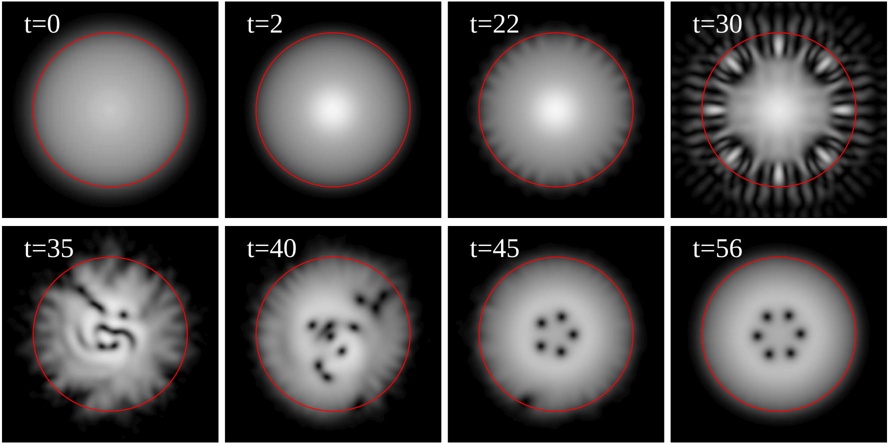

Density
Radius
0
5
10
15
20
25
30
0
2
4
6
8

Non-equilibrium condensate in a trap

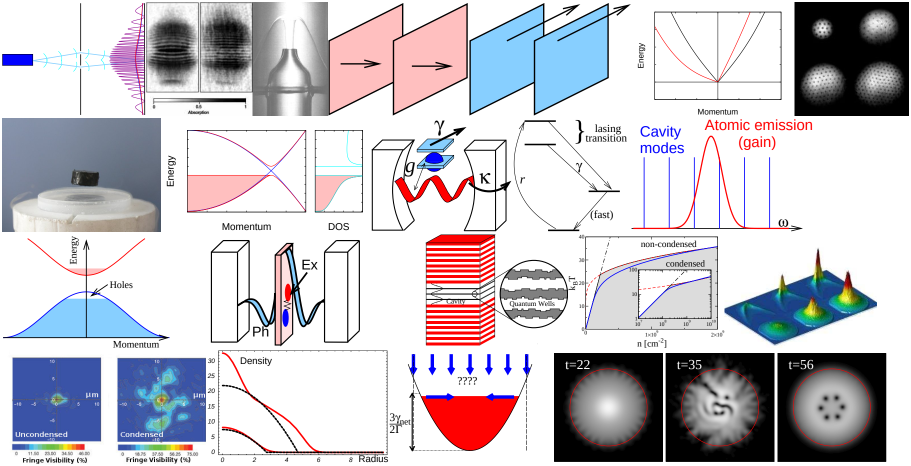




Time evolution:



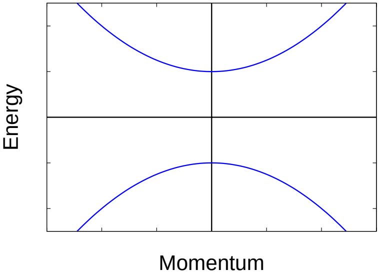
The End


Energy
Momentum
lasing
transition
Cavity
modes
Atomic emission
(gain)
γ
g
κ
r
γ
(fast)
ω
Energy
Momentum
DOS
Energy
Holes
Momentum
Ex
Ph
Cavity
Quantum Wells
non-condensed
condensed
k_BT
n [cm^{-2}]
Uncondensed
Condensed
Fringe Visibility (%)
Fringe Visibility (%)
μm
μm
Density
Radius
????
$\frac{3\gamma_{net}}{2\Gamma}$
t=22
t=35
t=56

Why change of excitations?

Macroscopic occupation of Ψ:

$\{N \text{ in } \Psi\} \rightarrow \{(N-2) \text{ in } \Psi, +\vec{k}, -\vec{k}\}$



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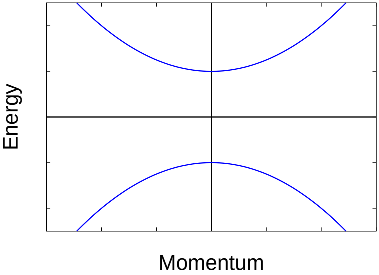
Number of **excitations** not fixed



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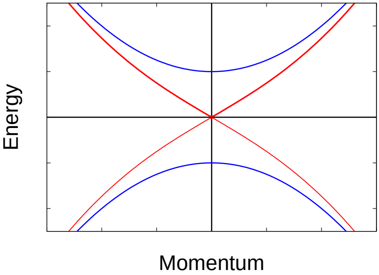

Non-equilibrium theory; fluctuations

Approach transition, Gap Equation/Hugenholtz-Pines relation:

$$\mu_s + i\kappa = \chi(\psi_0 = 0, \mu_s) \Leftrightarrow \mathcal{G}^{-1}(\omega = \mu_S, k = 0) = 0$$

[Szymańska et al., PRL '06; PRB '07]

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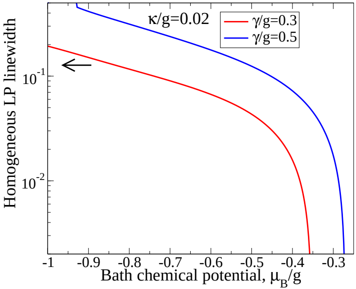

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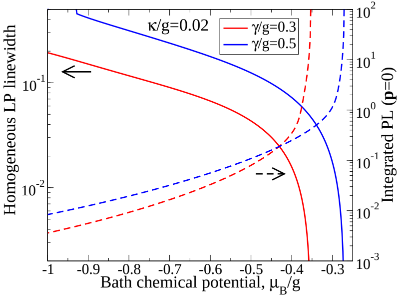


[Szymańska et al., PRL '06; PRB '07]

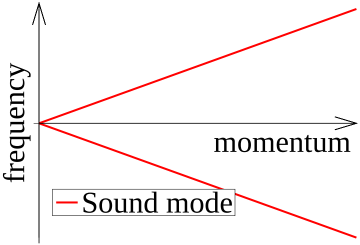
Fluctuations above transition

When condensed

$$\mathcal{G}^{-1}(\omega, k) = \omega^2 - c^2\mathbf{k}^2$$

Poles:

$$\omega^* = c|\mathbf{k}|$$



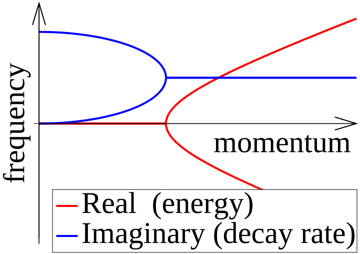
[Szymańska et al., PRL '06; PRB '07]

When condensed

$$\mathcal{G}^{-1}(\omega, k) = (\omega + ix)^2 + x^2 - c^2\mathbf{k}^2$$

Poles:

$$\omega^* = -ix \pm \sqrt{c^2\mathbf{k}^2 - x^2}$$



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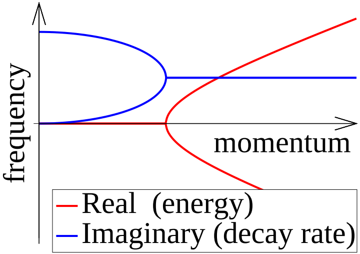

Correlations (in 2D):

$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0,0) \right\rangle \simeq |\psi_0|^2 \exp\left[-\eta \begin{cases} \ln(r/\xi) & r \to \infty, t \simeq 0 \\ \frac{1}{2}\ln(c^2 t / x\xi^2) & r \simeq, t \to \infty \end{cases} \right]$$

[Szymańska et al., PRL '06; PRB '07]