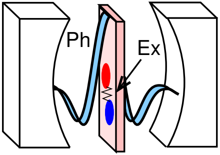
Microcavity polaritons: a non-equilibrium quantum fluid.

N. G. Berloff[1], J. Keeling[1], P. B. Littlewood[1], M. Szymańska[2]

[1]University of Cambridge, [2] University of Warwick

3rd July, 2008

Microcavity Polaritons



[**Review**: Keeling, et al., Semicond. Sci. Technol., 2007]

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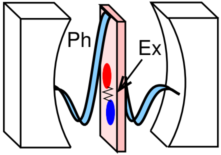

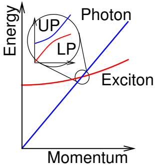

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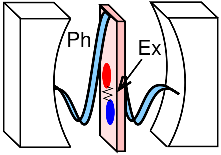

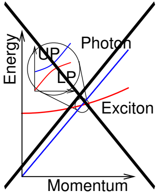

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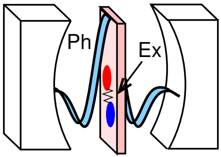

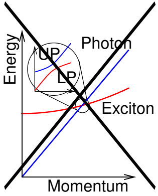

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Cavity photons:

$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$
$$\simeq \omega_0 + k^2/2m^*$$
$$m^* \sim 10^{-4} m_e$$

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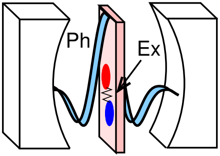

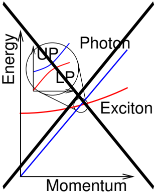

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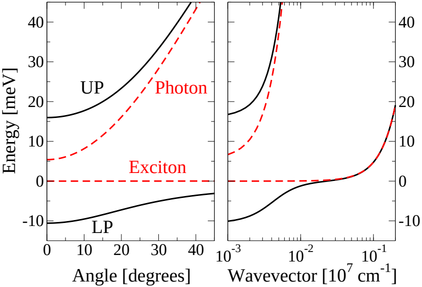

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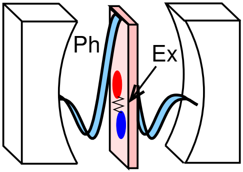
Nonequilibrium particle flux



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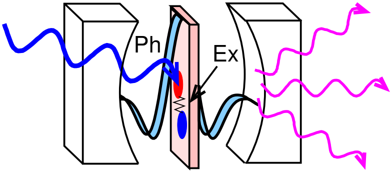

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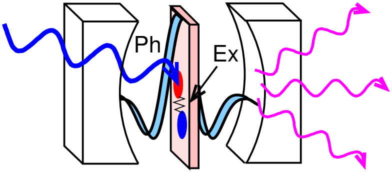

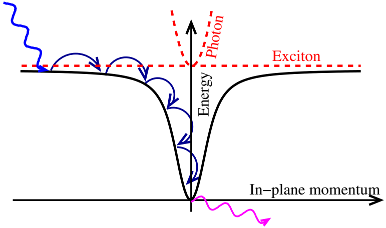

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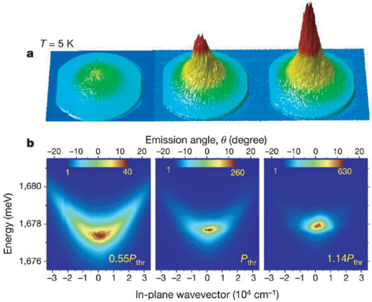
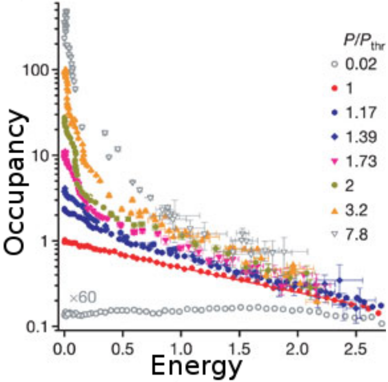
Polariton experiments: Momentum/Energy distribution





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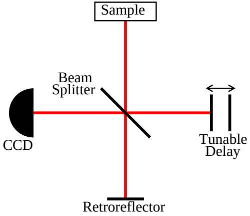
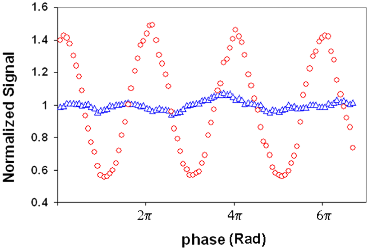
Polariton experiments: Coherence

Basic idea:





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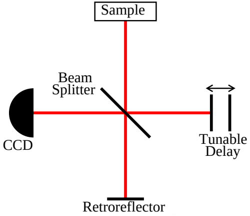
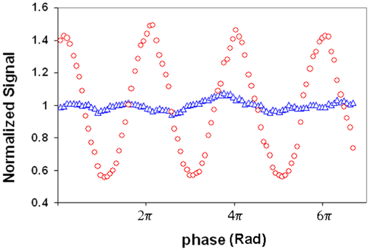
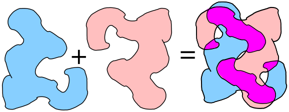
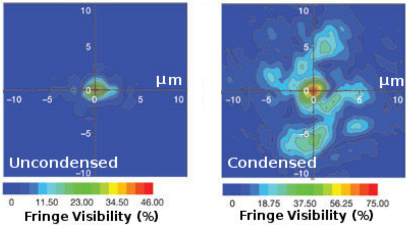




Coherence map:





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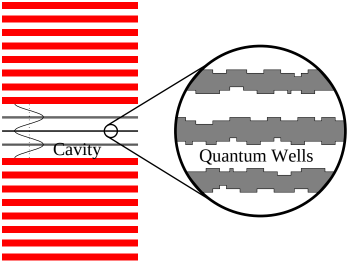
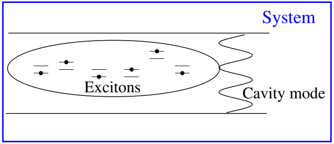
Non-equilibrium model

Polariton model

- Disorder-localised excitons
- Treat disorder sites as two-level (exciton/no-exciton)
- **Propagating** (2D) photons
- Exciton–photon coupling g.





[Szymańska et al., PRL '06; PRB '07]

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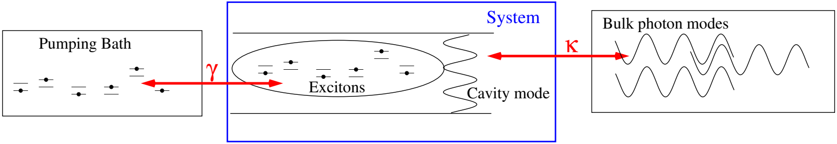
- Excitons coupling γ to high energy reservoir (μ_B, T_B)
- Photons decay κ into empty bulk modes.



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Non-equilibrium theory; mean-field

Look for mean-field solution, $\psi(\mathbf{r}, t) = \psi_0 e^{-i\mu_S t}$.

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$$(\mu_s + i\kappa)\,\psi_0 = \chi(\psi_0, \mu_s)\psi_0$$

Polarisation susceptibility,

$$\chi(\psi_0, \mu_s) = g^2\gamma \sum_{\text{excitons}} \int \frac{d\nu}{2\pi} \frac{(F_a + F_b)\nu + (F_b - F_a)(\tilde{\epsilon}_\alpha + i\gamma)}{[(\nu - E_\alpha)^2 + \gamma^2][(\nu + E_\alpha)^2 + \gamma^2]}.$$

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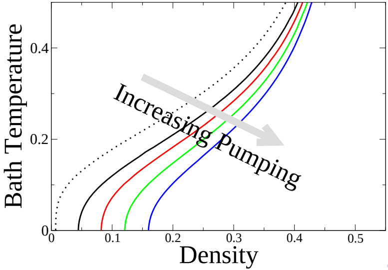
Shift of phase boundary:



Non-equilibrium theory; fluctuations

Approach transition, Gap Equation/Hugenholtz-Pines relation:

$$\mu_s + i\kappa = \chi(\psi_0 = 0, \mu_s) \Leftrightarrow \mathcal{G}^{-1}(\omega = \mu_S, k = 0) = 0$$

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$$\mathcal{G}^{-1}(\omega, k) = (\omega - \omega_k^*) + i\alpha$$

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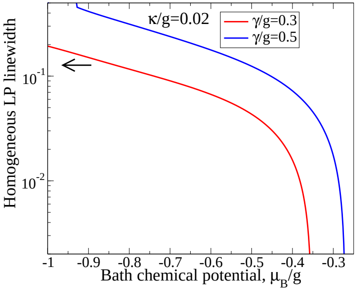

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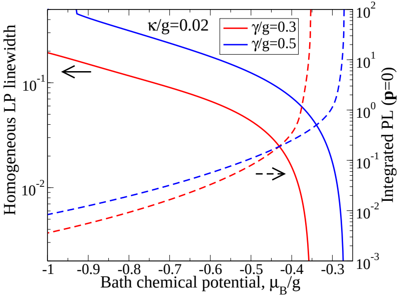

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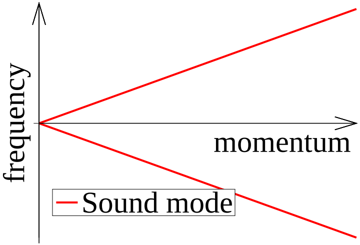
Fluctuations above transition

When condensed

$$\mathcal{G}^{-1}(\omega, k) = \omega^2 - c^2\mathbf{k}^2$$

Poles:

$$\omega^* = c|\mathbf{k}|$$



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Fluctuations above transition

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$$\mathcal{G}^{-1}(\omega, k) = (\omega + ix)^2 + x^2 - c^2\mathbf{k}^2$$

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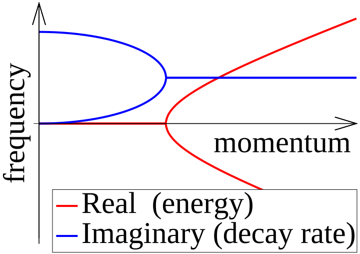
$$\omega^* = -ix \pm \sqrt{c^2\mathbf{k}^2 - x^2}$$



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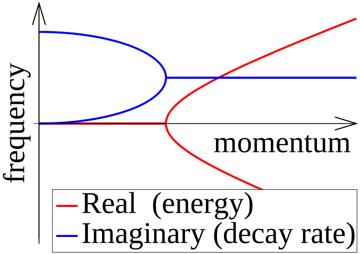

Correlations (in 2D):

$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0,0) \right\rangle \simeq |\psi_0|^2 \exp\left[-\eta \begin{cases} \ln(r/\xi) & r \to \infty, t \simeq 0 \\ \frac{1}{2}\ln(c^2 t/x\xi^2) & r \simeq, t \to \infty \end{cases}\right]$$

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Gross-Pitaevskii equation:

Gap equation:

$$\left(i\hbar\partial_t + i\kappa - \left[V(r) - \frac{\hbar^2\nabla^2}{2m} \right] \right) \psi(r) = \chi(\psi(r,t))\psi(r,t)$$

Nonlinear, complex susceptibility $\chi(\psi(r,t))$

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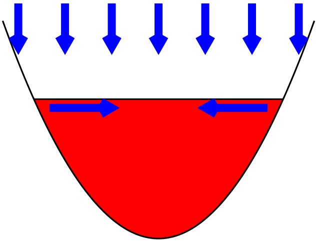

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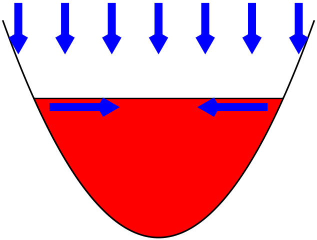

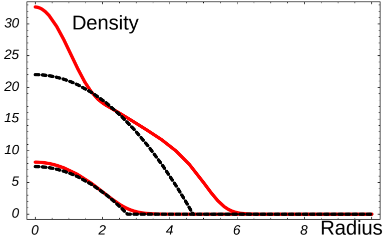

[Keeling & Berloff, PRL, '08]

Stability of profile?

$$\psi = \sqrt{\rho} e^{i\phi} \quad \mathbf{v} = \nabla\phi$$

$$\begin{aligned} \frac{1}{2}\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) &= (\alpha - \sigma\rho)\rho \\ \partial_t \mathbf{v} + \nabla(\rho + r^2 + |\mathbf{v}|^2) &= 0 \end{aligned}$$

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Consider $\rho \to \rho + \delta\rho, \mathbf{v} \to \mathbf{v} + \delta\mathbf{v}$.

If $\alpha, \sigma \to 0$, can find normal modes in 2D trap:

$$\delta\rho_{n,m}(r,\theta,t) = e^{im\theta} h_{n,m}(r) e^{i\omega_{n,m} t}$$
$$\omega_{n,m} = 2\sqrt{m(1+2n) + 2n(n+1)}$$

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Consider $\rho \to \rho + \delta\rho, \mathbf{v} \to \mathbf{v} + \delta\mathbf{v}$.

Add weak pumping/decay:

$$\omega_{n,n} \to \omega_{n,m} + i\alpha \left[\frac{m(1+2n) + 2n(n+1) - m^2}{2m(1+2n) + 4n(n+1) + m^2} \right]$$

Instability!

Restoring stability

Instability

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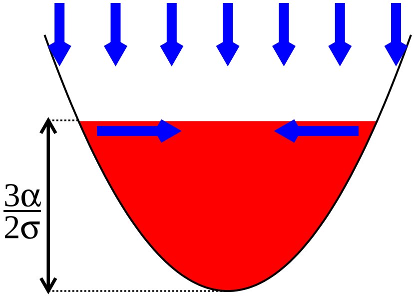
High m modes: $\delta\rho_{n,m} \simeq e^{im\theta} r^m \ldots$

Restoring stability

Instability

$$\omega_{n,n} \rightarrow \omega_{n,m} + i\alpha \left[\frac{m(1+2n) + 2n(n+1) - m^2}{2m(1+2n) + 4n(n+1) + m^2} \right]$$

High m modes: $\delta\rho_{n,m} \simeq e^{im\theta} r^m \ldots$



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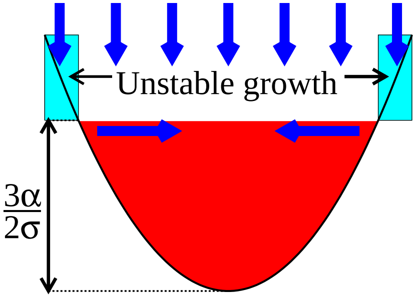

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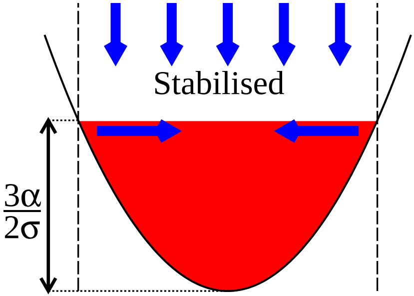

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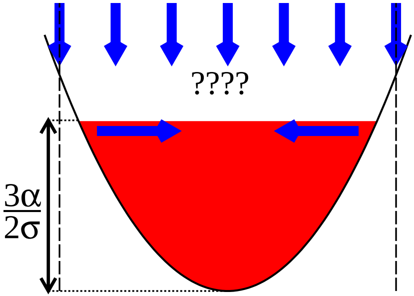
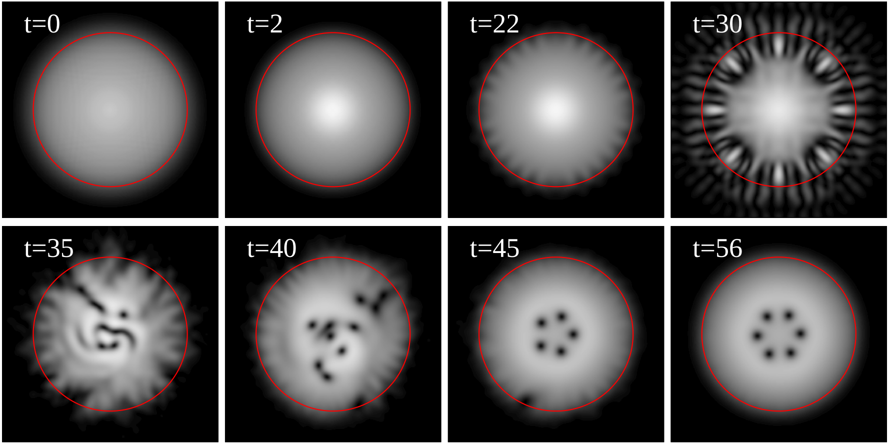
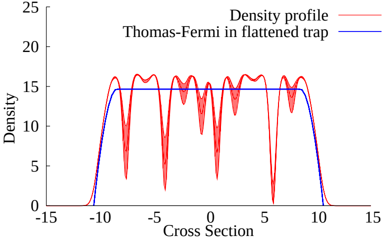


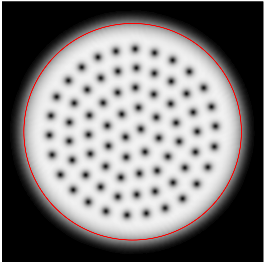
Increase pump spot size



[Keeling & Berloff, PRL, '08]

Why vortices





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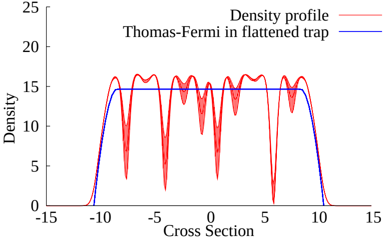

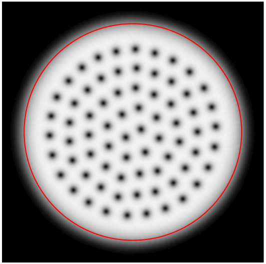

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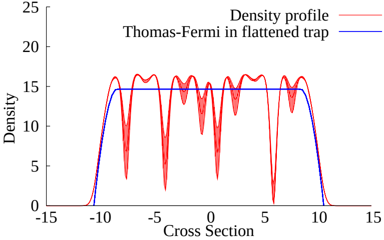

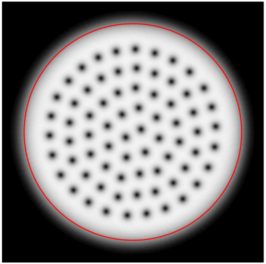

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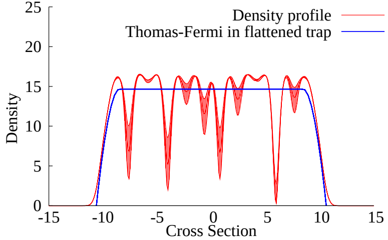

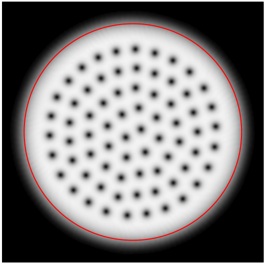

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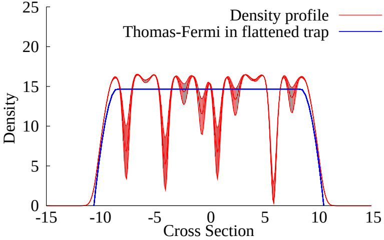

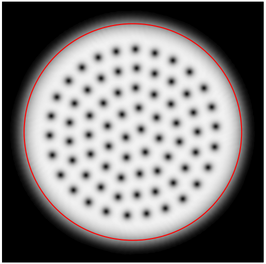

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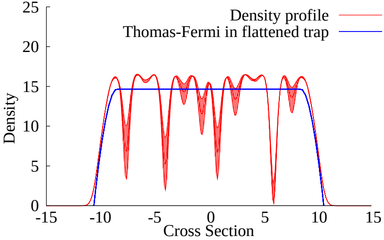

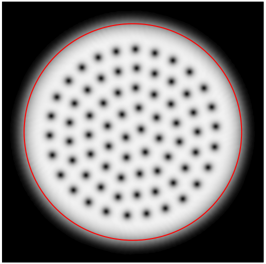

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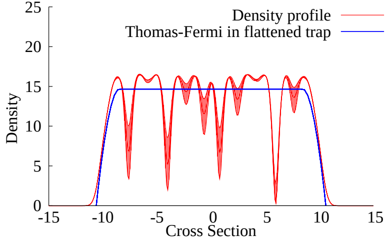

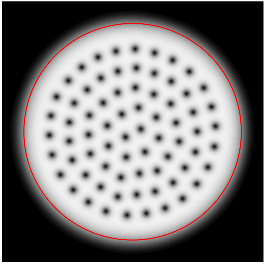

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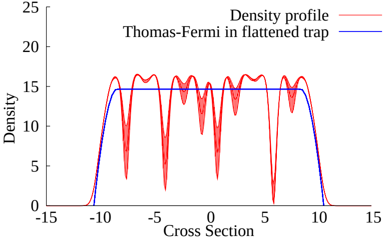

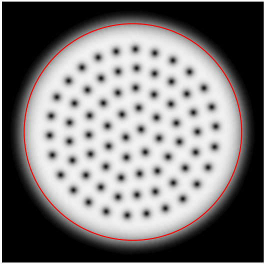

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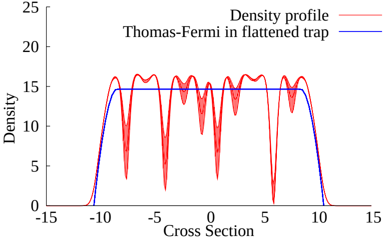

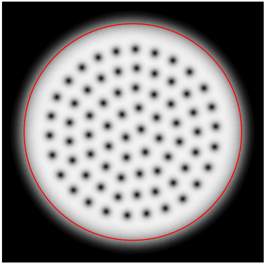

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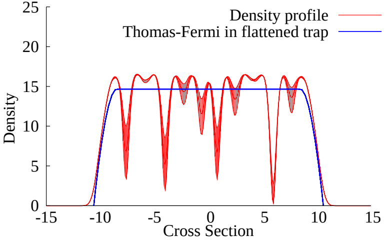

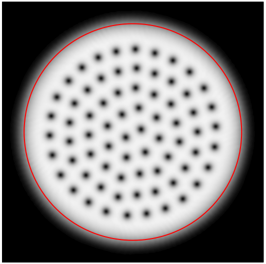

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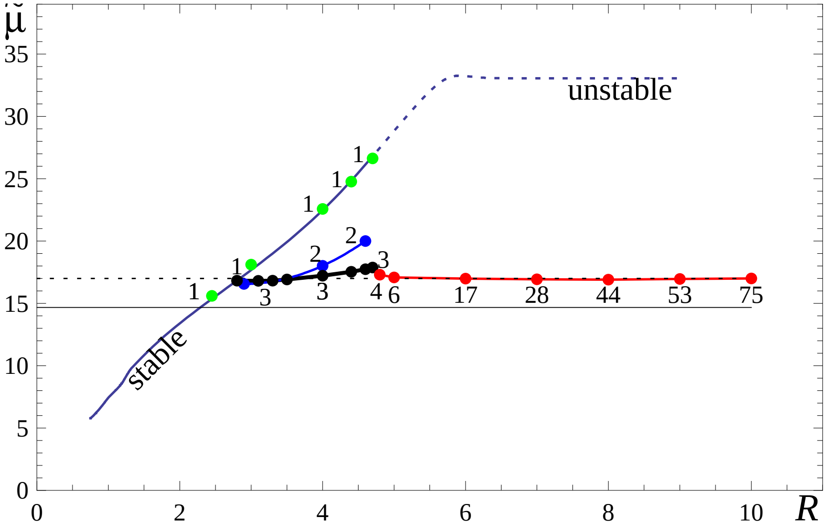
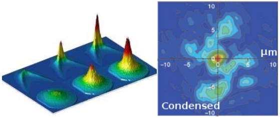
μ vs R, and hysteresis



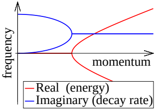
[Keeling & Berloff, PRL, '08]

Conclusions

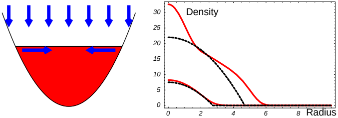
- Polariton condensates



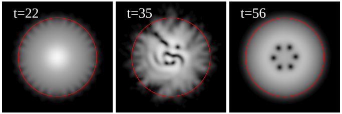
- Change to spectrum and correlations



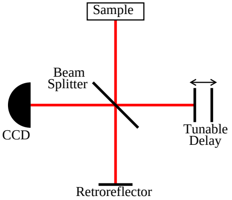
- Modification to Thomas-Fermi profile

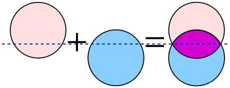


- Spontaneous rotating vortex lattice



Observing vortices: fringe pattern





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