Landau Zener processes in many body systems

Two-level system coupled to a photon field

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[1]University of Cambridge

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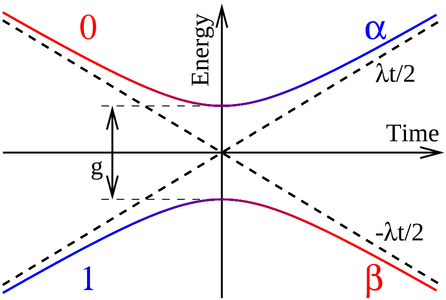
The Landau-Zener problem



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Finally, $\psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$, $\quad z = g^2/\lambda$

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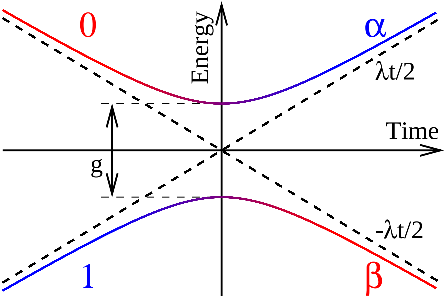

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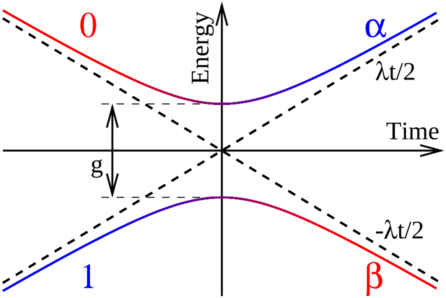

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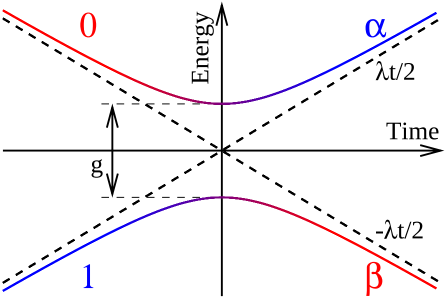

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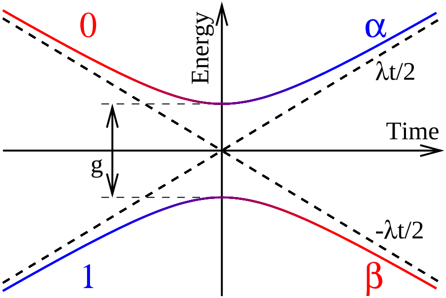

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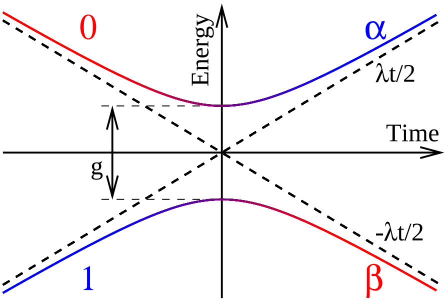

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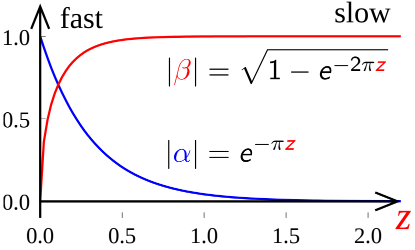

Many-body Landau-Zener generalisation

- One body, many-level generalisations:
  - Exact solutions:

    Demkov–Osherov

- Here, instead **many-body**

- Present two simple but striking examples.

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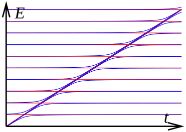

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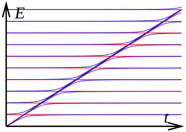

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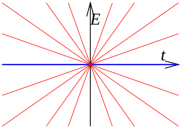

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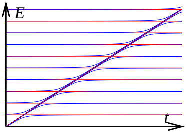

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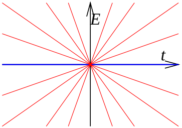

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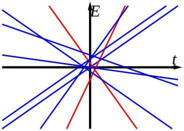

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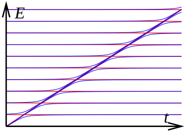

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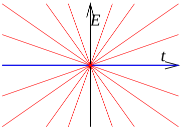

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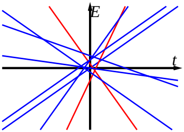

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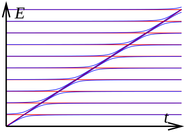

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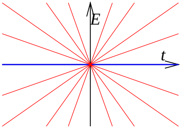

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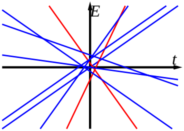

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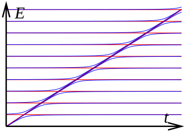

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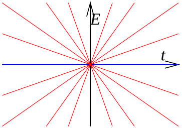

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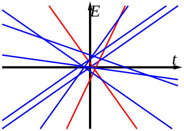

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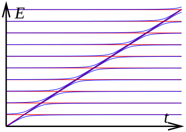

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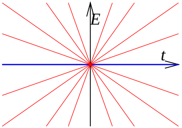

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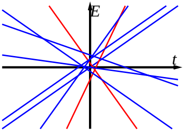

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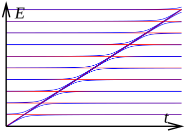

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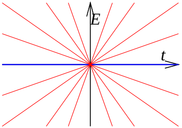

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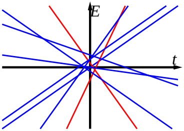

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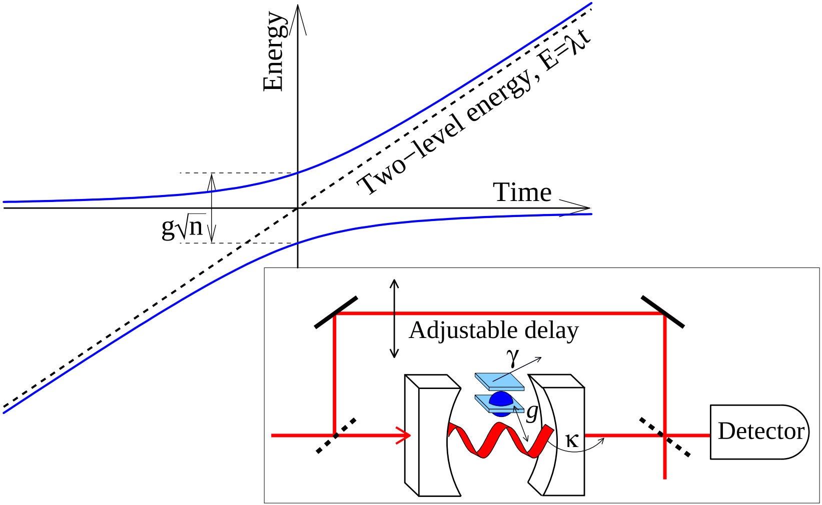
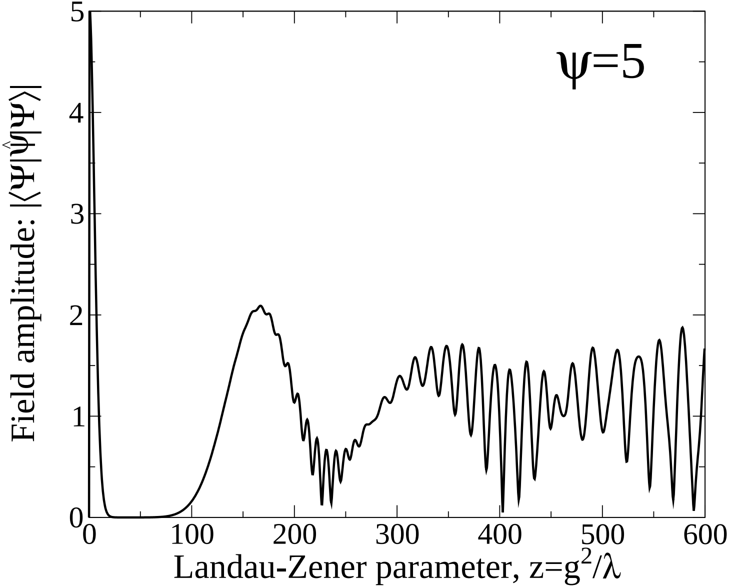
Overview

1. Introduction and Landau-Zener
   - Introduction
2. Two-level system/photon field
   - Many body problem, Collapse and revivals
   - Understanding collapse and revivals
   - Effects of decay
3. Localised fermion coupled to continuum
   - Relating one- and many-body problems
   - Cancellation and "clean electrons"

Physical system



Exact numerical result



Description

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- Initial coherent state: $|\Psi(-\infty)\rangle = e^{-|\psi|^2/2} \sum_n \frac{\psi^n}{\sqrt{n!}} |n, \uparrow\rangle$
- Each pair $|n, \uparrow\rangle \leftrightarrow |n+1, \downarrow\rangle$ undergoes LZ transition

$$H_{n,n+1} = \begin{pmatrix} \lambda t/2 & g\sqrt{n+1} \\ g\sqrt{n+1} & -\lambda t/2 \end{pmatrix}; \qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix}$$

- Final state:

$$|\Psi(+\infty)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1} |n, \uparrow\rangle + B_{n+1} |n+1, \downarrow\rangle \right]$$

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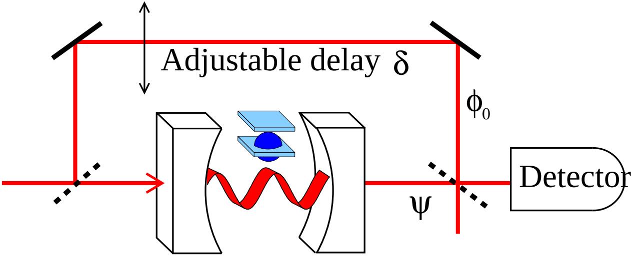

After mixing: $\hat{\psi} + \hat{\phi}_0 e^{i\delta}$ Intensity:

$$I \propto \left\langle (\hat{\psi}^\dagger + \hat{\phi}_0^\dagger e^{-i\delta})(\hat{\psi} + \hat{\phi}_0 e^{i\delta}) \right\rangle$$
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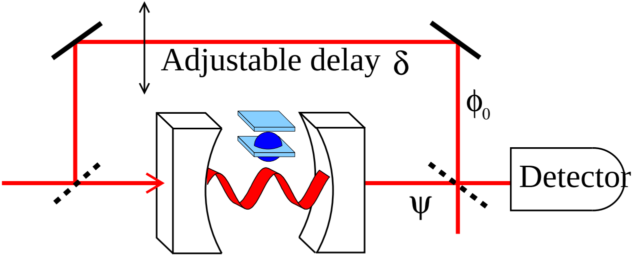

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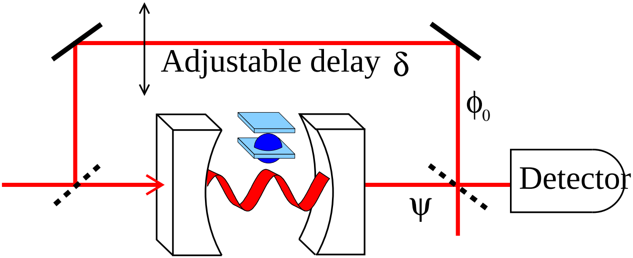

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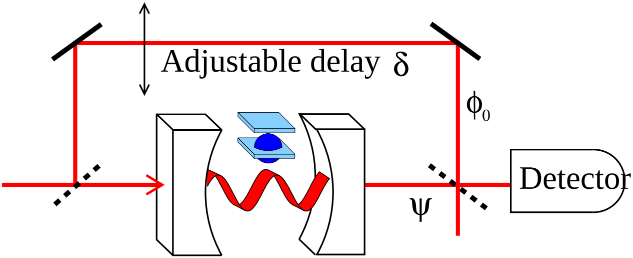

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Wigner function:
$W(x,p) = \frac{1}{\pi}\int dy\, \Psi^*(x+y)\Psi(x-y)e^{2ipy}.$

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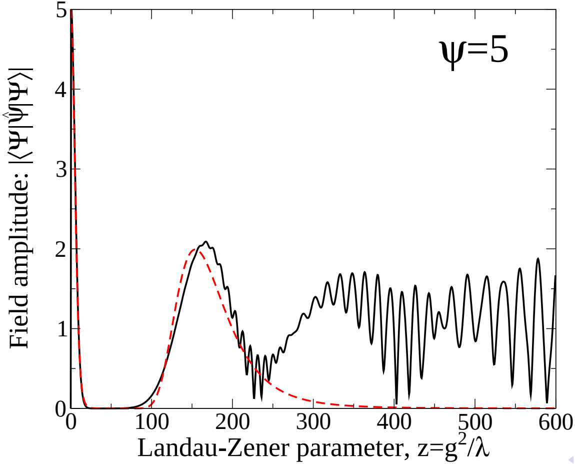

Revivals are **not** coherent states

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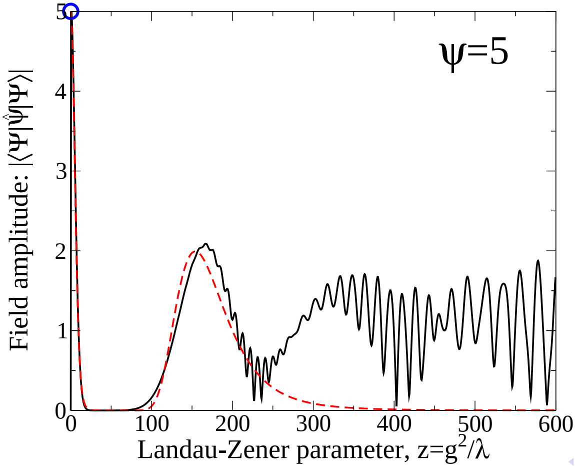

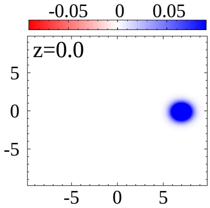

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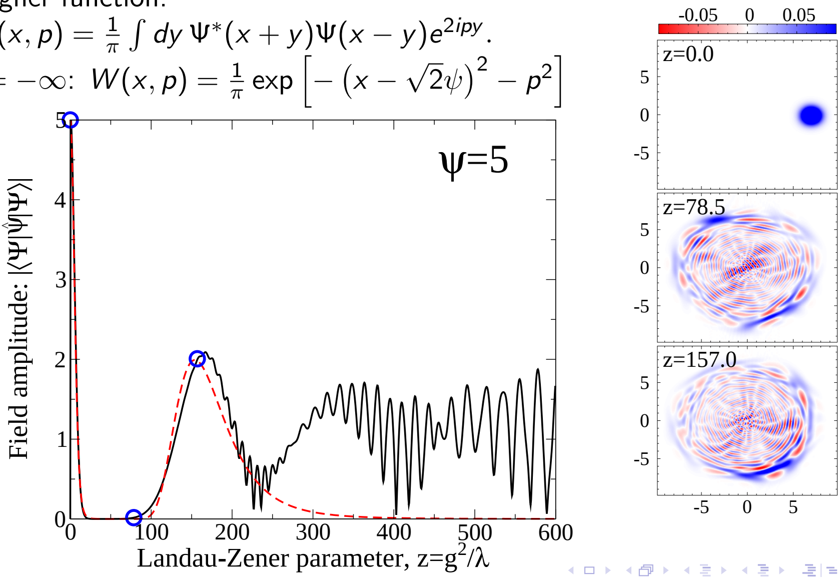

Explaining results

Adiabatic limit: $z = g^2/\lambda \gg 1$

$$|\Psi(T)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1} |n, \uparrow\rangle + \qquad B_{n+1} |n+1, \downarrow\rangle \right]$$

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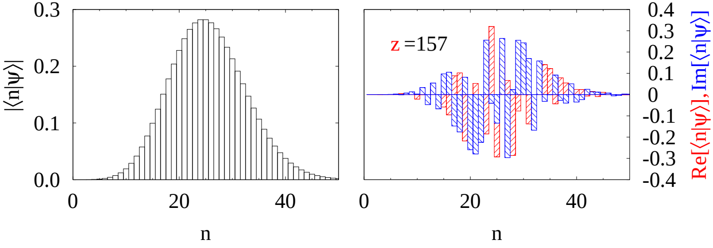

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$$\langle\Psi|\psi|\Psi\rangle = \psi e^{-|\psi|^2} \sum_n \frac{|\psi|^{2n}}{n!} \sqrt{\frac{n+2}{n+1}} e^{i(\phi_{n+2}-\phi_{n+1})}$$

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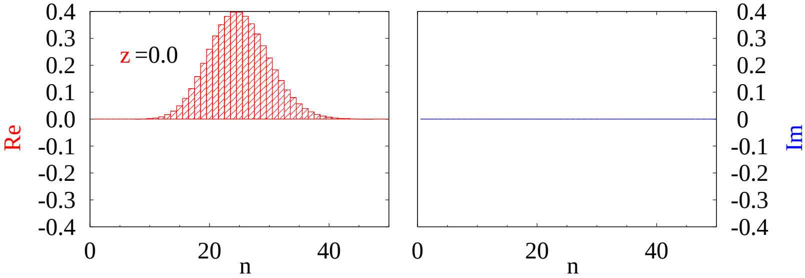

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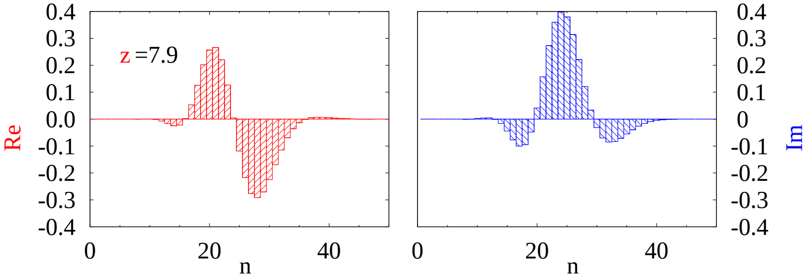

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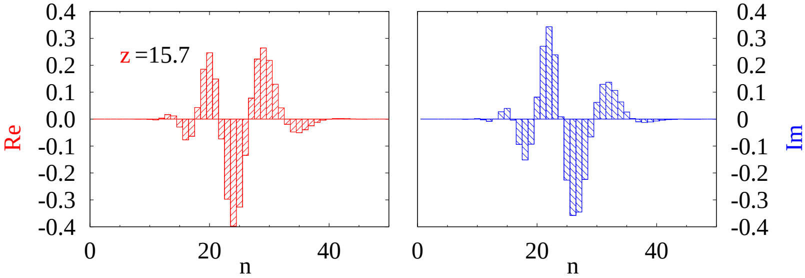

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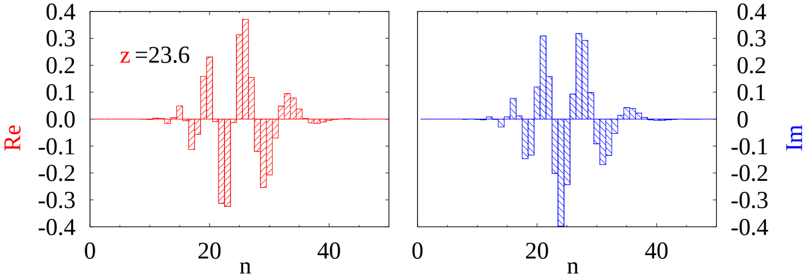

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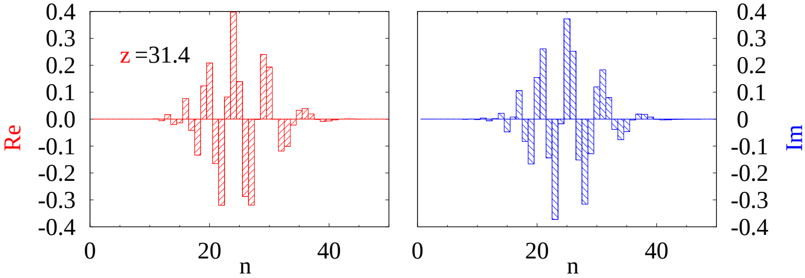

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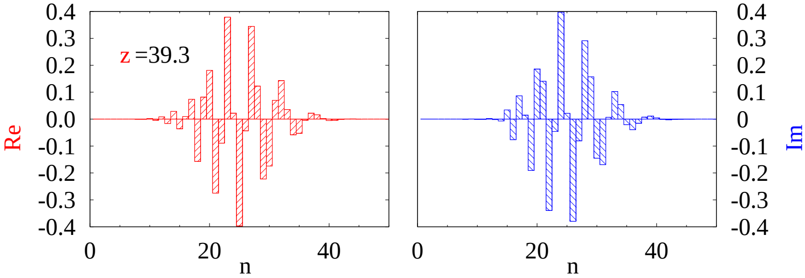

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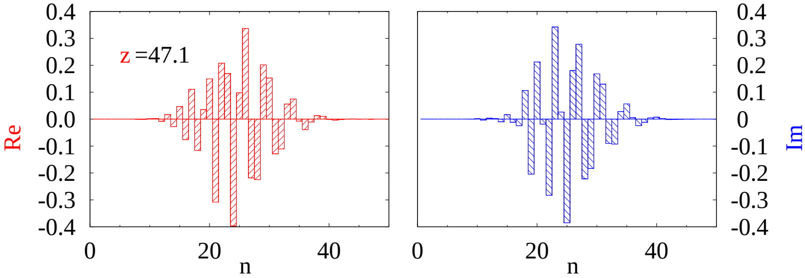

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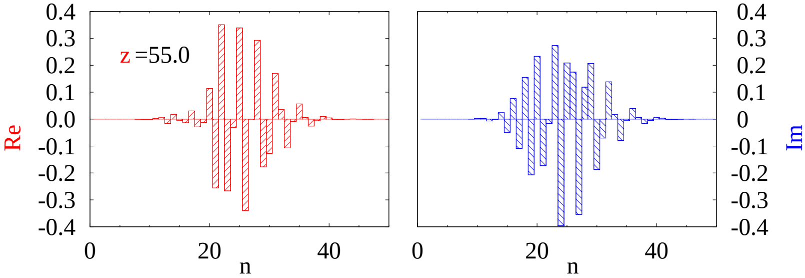

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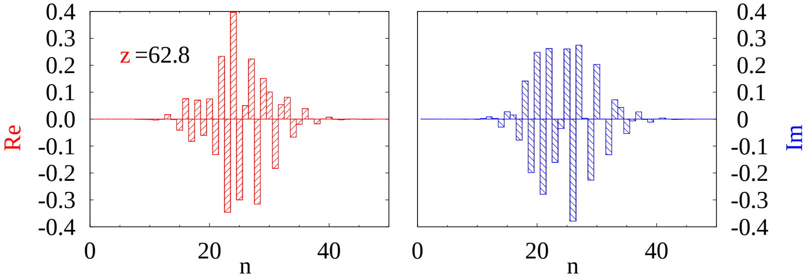

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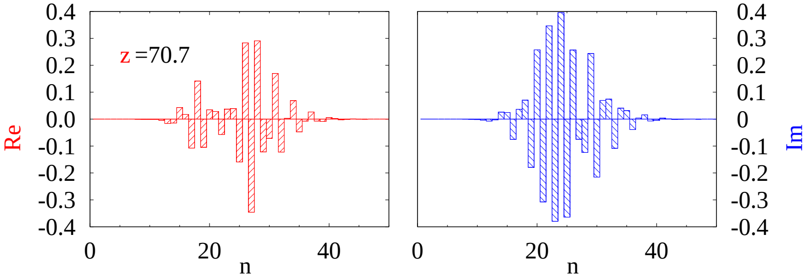

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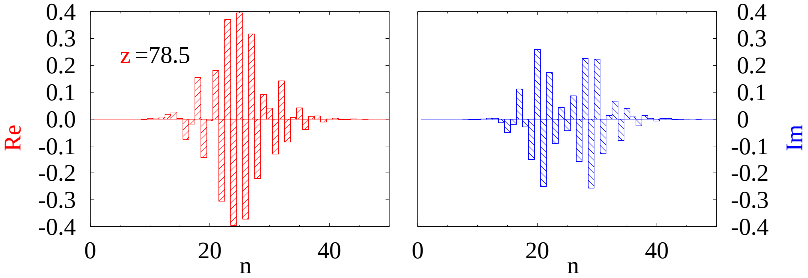

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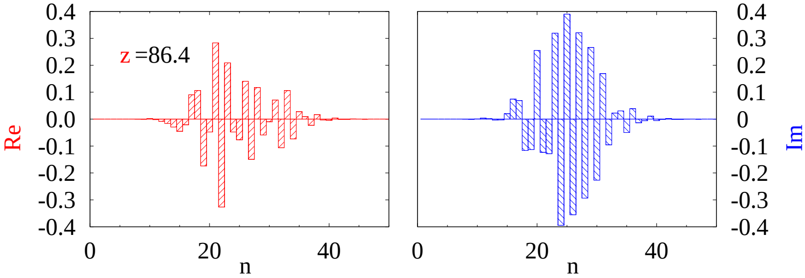

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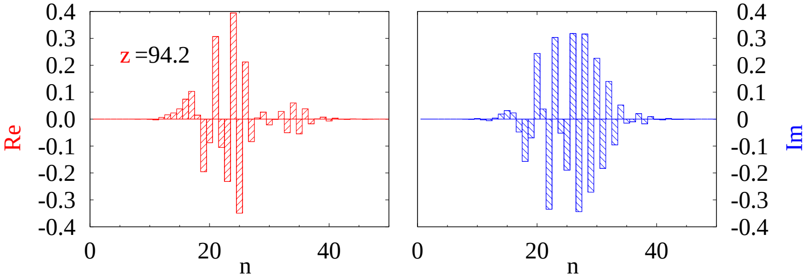

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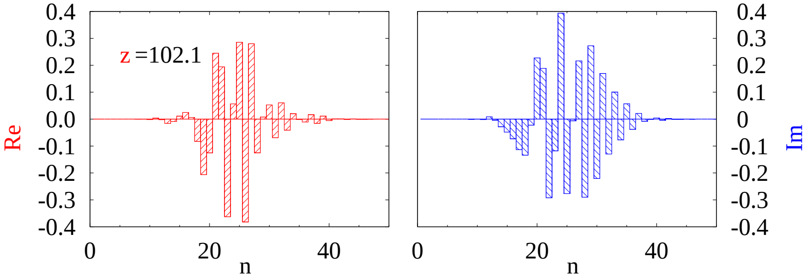

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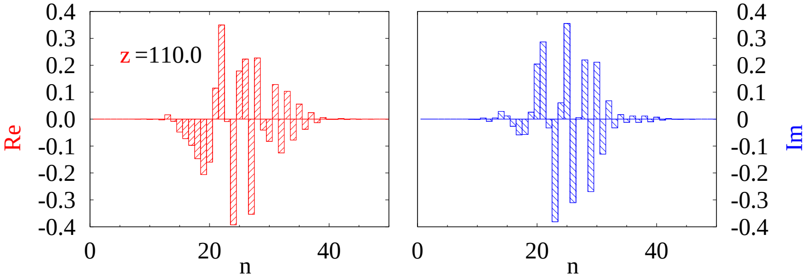

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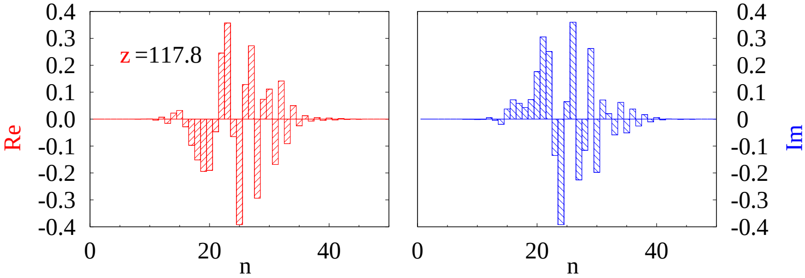

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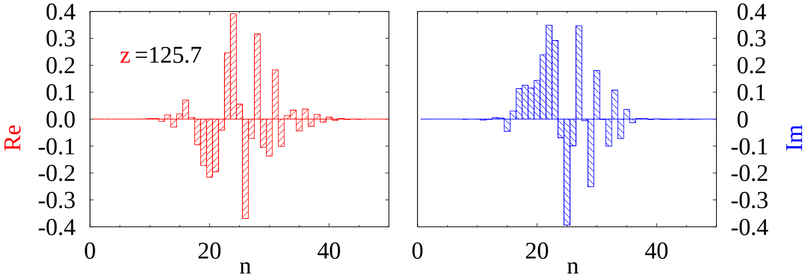

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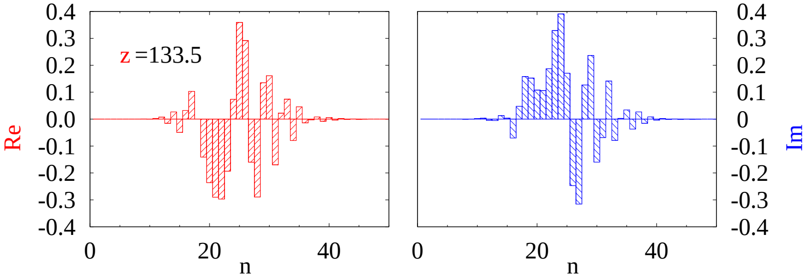

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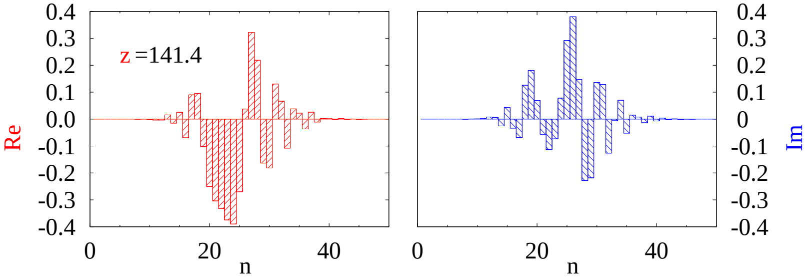

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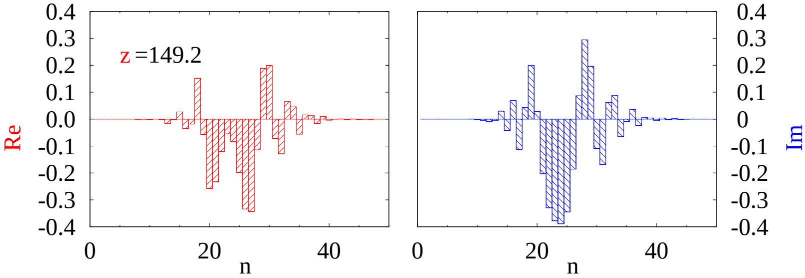

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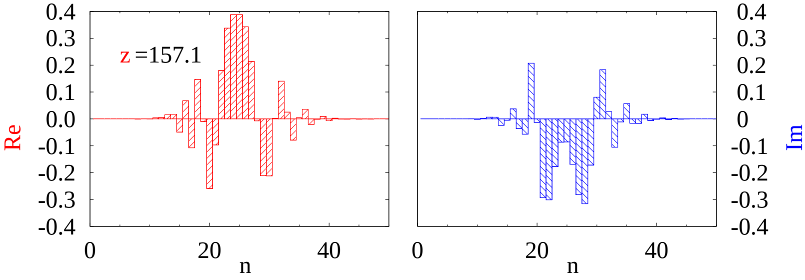

Explaining results: approximating phase

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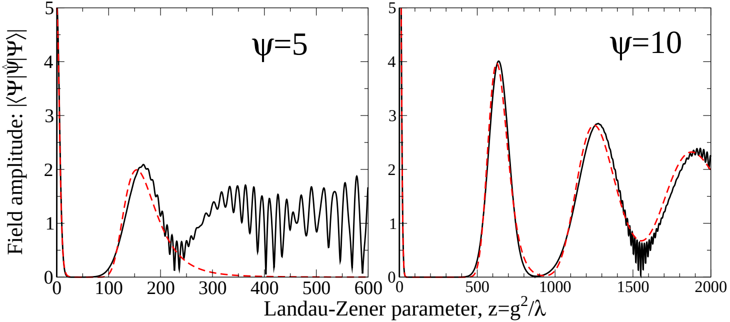
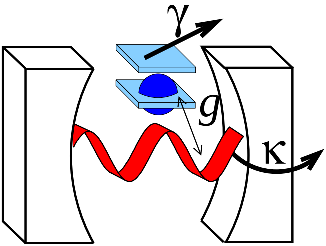


Possible decay channels



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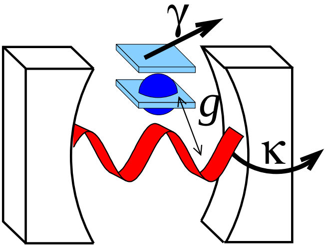

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$$L_\gamma[\hat{\rho}] = -\frac{\gamma}{2}\left(\hat{\sigma}_+\hat{\sigma}_-\hat{\rho} + \hat{\rho}\hat{\sigma}_+\hat{\sigma}_- - 2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+\right)$$

Naive estimate: $(\gamma t^*, \kappa t^*) \ll 1$, with $t^* \simeq g|\psi|/\lambda$

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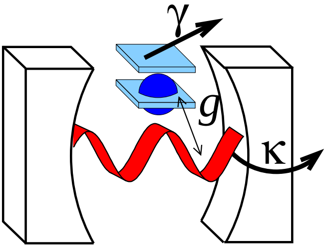
Possible decay channels



$$\partial_t \hat{\rho} = -i\left[\hat{H}, \hat{\rho}\right] + L_\kappa[\hat{\rho}] + L_\gamma[\hat{\rho}]$$

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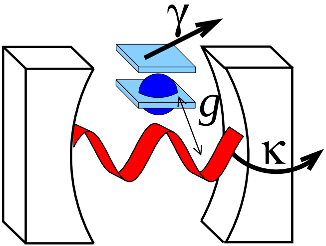

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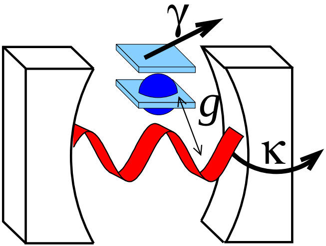

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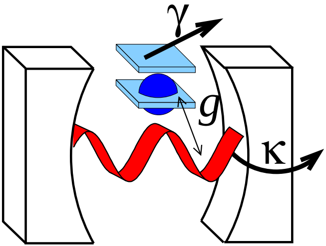

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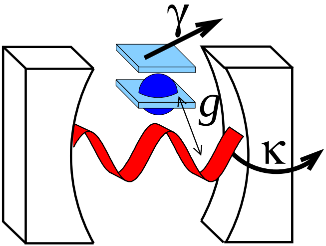

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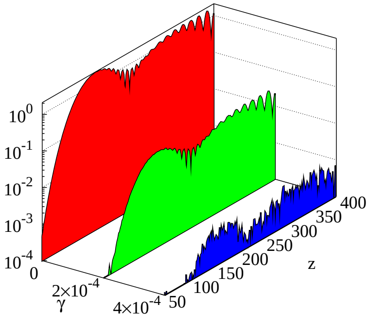
Results including decay



$\gamma_{\max}/g \simeq 2 \times 10^{-4}$

$\gamma_{\text{theory}}/g \ll 10 \times 10^{-4}$

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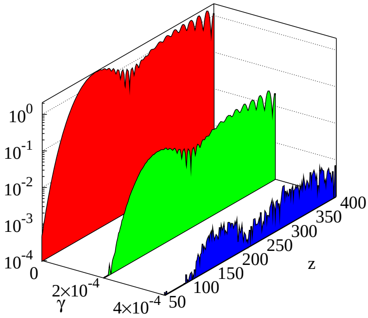

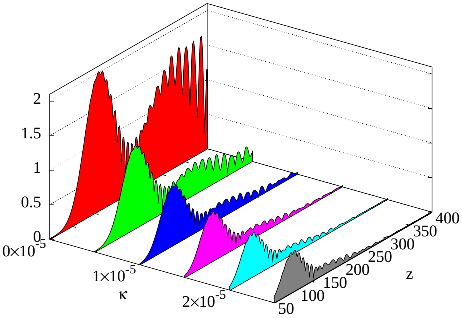

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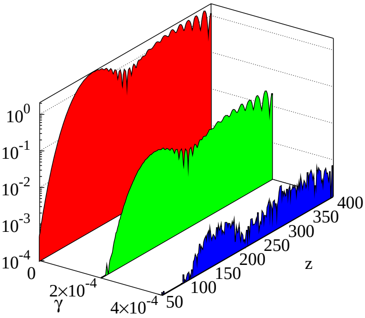

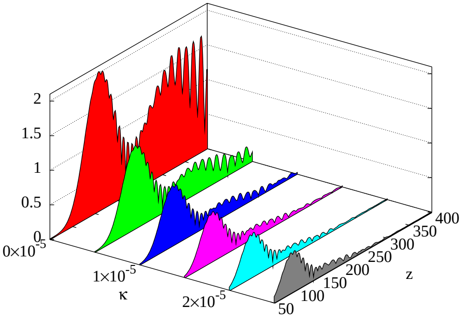

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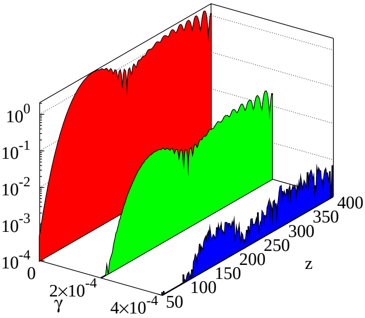

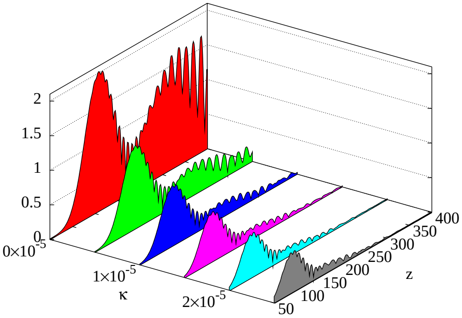

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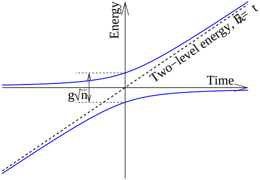
Understanding additional decay: adiabatic approx



$|n,+\rangle = \cos\theta_n|n,\downarrow\rangle + \sin\theta_n|n-1,\uparrow\rangle$ Adiabatic if:

$$P_{\text{trans}} = \frac{\left|\langle n-1,-|\hat{\psi}|n,+\rangle\right|^2}{\langle n,+|\hat{\psi}^\dagger\hat{\psi}|n,+\rangle} \leq \frac{27}{256n^2} \ll 1$$

If adiabatic, $\Lambda_n = \rho_{n,n+1}$:

$$\frac{d\Lambda_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]\Lambda_n - \kappa\left[\left(n-\frac{1}{2}\right)\Lambda_n - \sqrt{n(n+1)}\Lambda_{n+1}\right]$$

When $|t| \leq g\sqrt{n}/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

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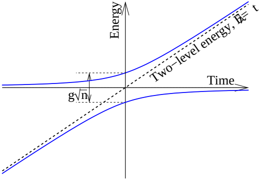

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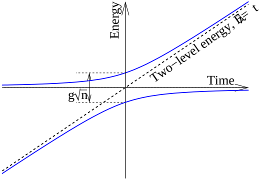

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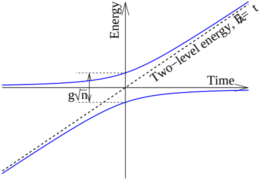

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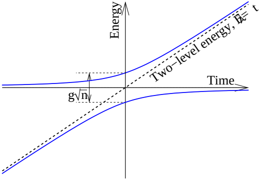

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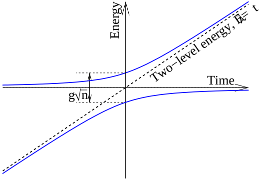

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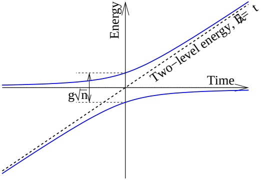

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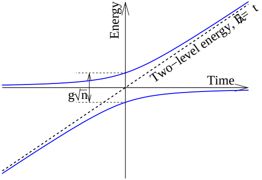

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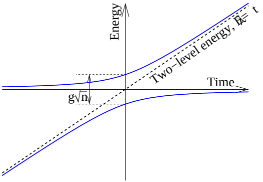

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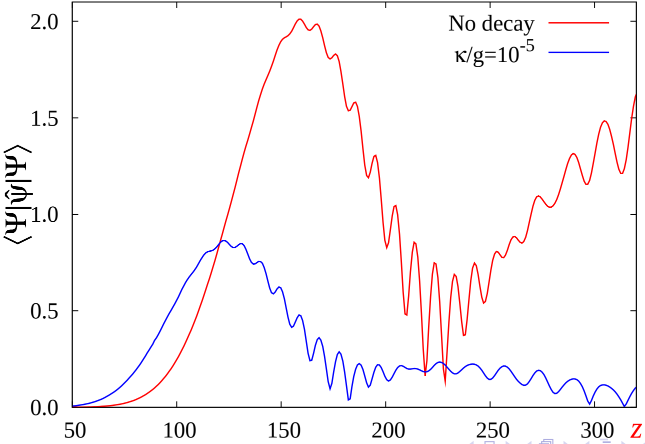
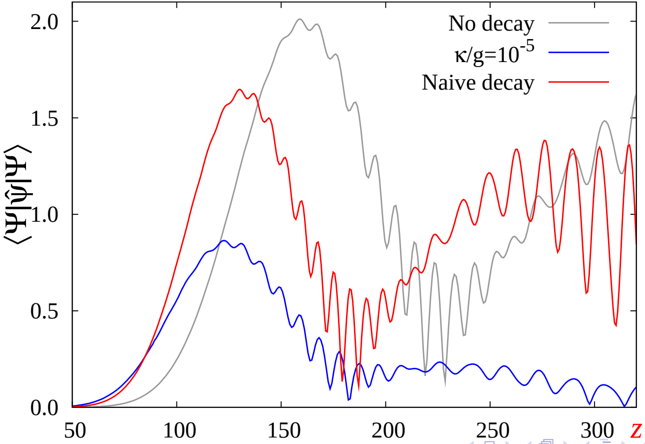
Effect of extra decay

$\langle\Psi|\, \hat{\psi}(\psi_0, \kappa)\, |\Psi\rangle$



Effect of extra decay

$$\langle\Psi|\,\hat{\psi}(\psi_0,\kappa)\,|\Psi\rangle \simeq \langle\Psi|\,\hat{\psi}(\psi_0 e^{-\kappa T/2},0)\,|\Psi\rangle \exp\left[-\frac{\kappa T}{2}\right]$$



Effect of extra decay

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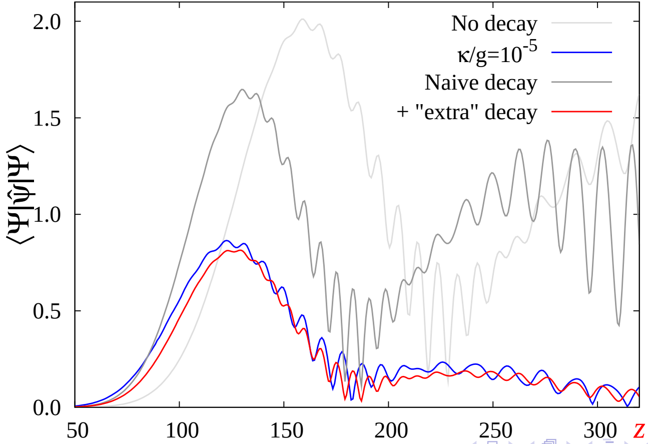
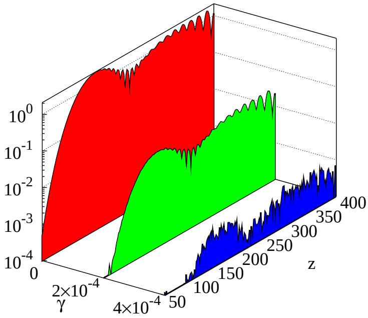
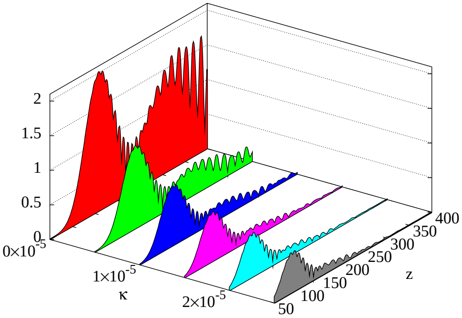


Results including decay





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Possible systems

Requirement: $\kappa/g \lesssim 1 \times 10^{-5}$, $\gamma/g \lesssim 2 \times 10^{-4}$

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System	**Source**	κ/g	γ/g

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Possible systems

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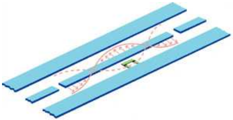

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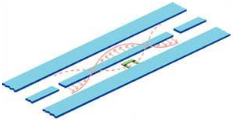
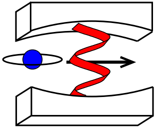
Atom/Microwave cavity	ENS, 2004	7×10^{-3}	2×10^{-4}





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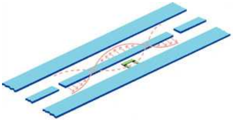
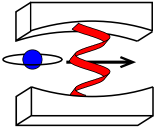
Josephson junction/stripline	Yale, 2004	2×10^{-2}	2×10^{-3}
Atom/Microwave cavity	ENS, 2004	7×10^{-3}	2×10^{-4}
	ENS, 2007	2×10^{-4}	2×10^{-4}





Landau Zener processes in many body systems

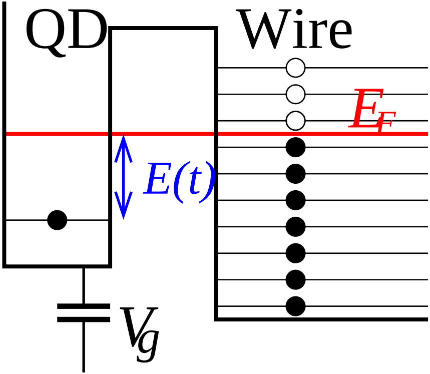
Localised fermion coupled to a continuum of states

Jonathan Keeling[1], L. S. Levitov[2] and A. Shytov[3]

[1]University of Cambridge [2]Massachusetts Institute of Technology [3]Brookhaven National Lab

December 10, 2007

Physical problem



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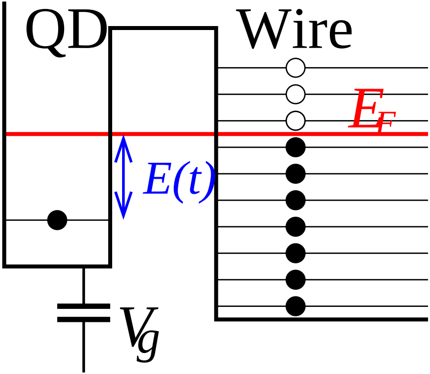
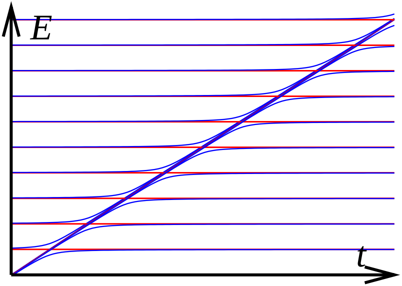
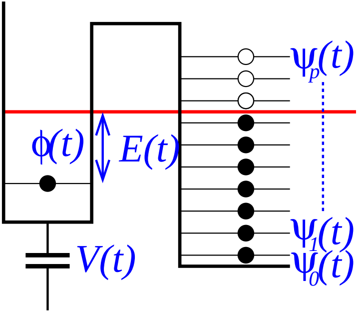




Single particle problem



Regularly spaced continuum:

$$\psi(x,t) = \sum \psi_p(t) e^{ipx}$$

Thus, continuum equations:

$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x,t)\delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x,t) = g\delta(x)\phi(t)$$

Given: $\psi(x<0,t) = \frac{e^{-i\epsilon' t}}{\sqrt{2\pi}}$, find:

$$U(\epsilon,\epsilon') = \int dt \psi(x>0,t) \frac{e^{i\epsilon t}}{\sqrt{2\pi}}$$

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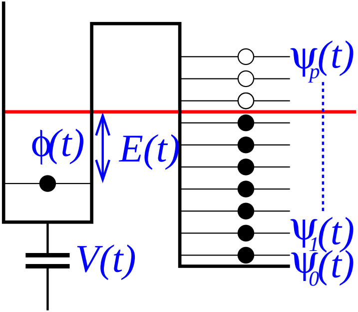

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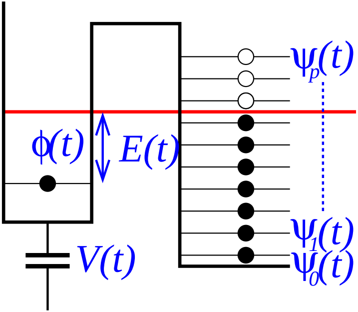

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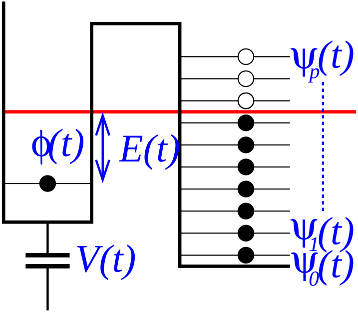

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Solution for $\phi(t) = g \int_{-\infty}^{t} dt' \psi_0(t') \exp\left[-\frac{\Gamma}{2}(t - t') + i \int_{t'}^{t} E(\tau) d\tau \right]$

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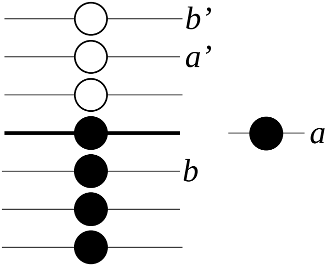

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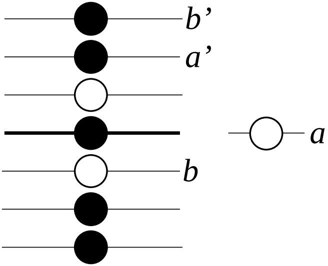

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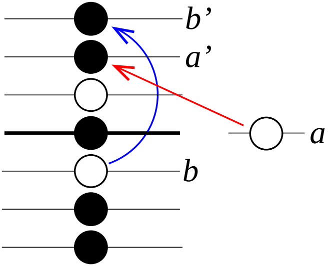

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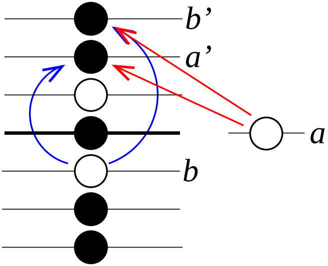

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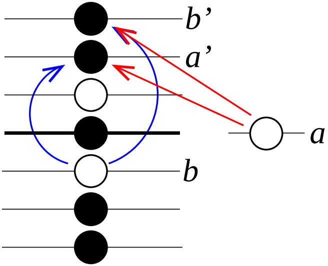

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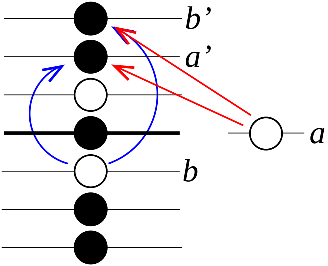

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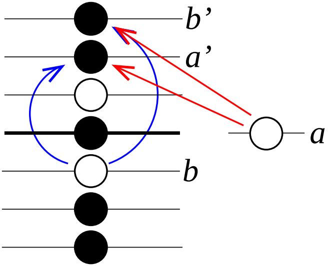

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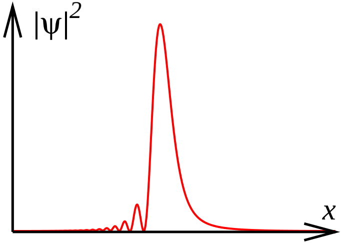
Linear timed dependence, exact state

$$\Psi_p \propto e^{-(\Gamma/\lambda)vp-(i/2\lambda)(vp)^2}$$

Spatial profile:



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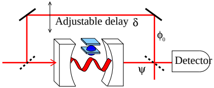

  - Coherent state; interference
  - Decay: larger than expected, but feasible.
- Localised/continuum fermions

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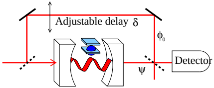

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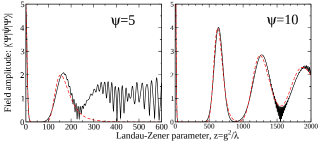

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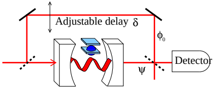

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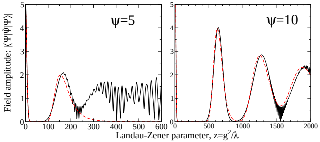

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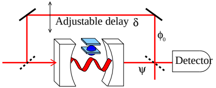

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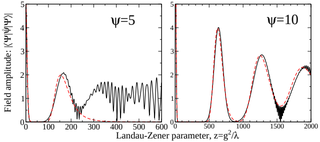

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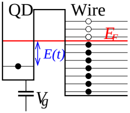

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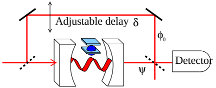

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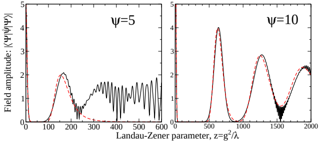

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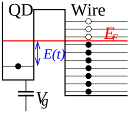

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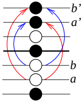

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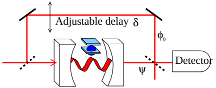

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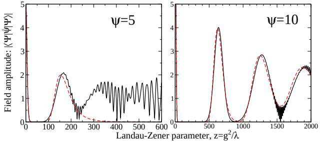

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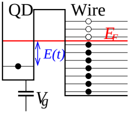

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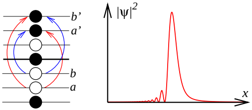
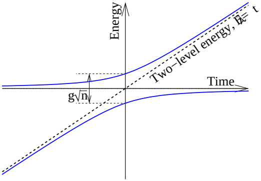


Understanding additional decay: adiabatic approx



$|n,+\rangle = [\cos(\theta_n)\,|n,\downarrow\rangle + \sin(\theta_n)\,|n-1,\uparrow\rangle]$.
Adiabatic if:

$$P_{\text{trans}}(\theta_n(t)) = \frac{|\langle n-1,-|a|n,+\rangle|^2}{\langle n,+|a^\dagger a|n,+\rangle} \ll 1$$

At $t \to \pm\infty$, $P_{\text{trans}} \to 0$.
At $t \simeq 0$,

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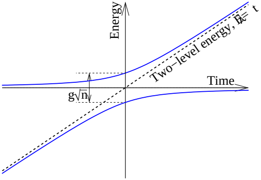

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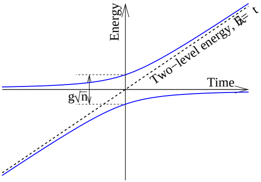

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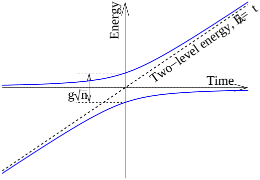

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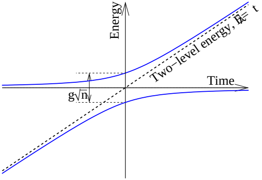

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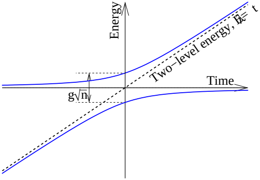

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If adiabatic, $A_n = \rho_{n,n+1}$:

$$\frac{dA_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]A_n - \kappa\left[\left(n-\frac{1}{2}\right)A_n - \sqrt{n(n+1)}A_{n+1}\right]$$

When $|t| \leq g/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

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Results of perturbative decay

Remove "naive decay",

$$\langle\Psi|\psi(\kappa,\psi_0)|\Psi\rangle_{\text{naive}} = \left\langle \Psi|\psi(0,\psi_0 e^{-\kappa T/2})|\Psi\right\rangle e^{-\kappa T/2}$$

$$\delta\langle\Psi|\psi|\Psi\rangle = \psi(\kappa|\psi|^2)\sum_n P_n \times$$
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Explicit form in large n approx:

$$\delta \langle \Psi | \psi | \Psi \rangle = -\psi(\kappa|\psi|^2) \sum_n P_n e^{-iz \ln(T^2/zn) + iz/2n} \sqrt{\frac{zn}{\lambda}} \frac{\pi z}{2n} J_1 \left(\frac{z}{2n} \right)$$

At $g^2/\lambda = z = 2\pi N |\psi|^2$

$$|\delta \langle \Psi | \psi | \Psi \rangle| \propto \psi \left[\frac{\kappa}{g} (2\pi N)^{3/2} |\psi|^5 \right]$$

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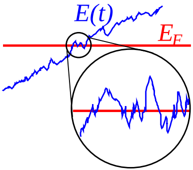
- Suppose $E(t) = ct + \eta(t)$
  $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \Gamma_2 \delta(t - t')$.



- To find N^{ex}, need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$thus:

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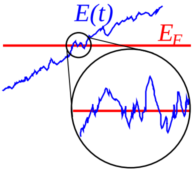

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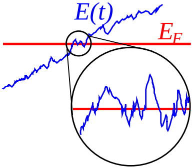

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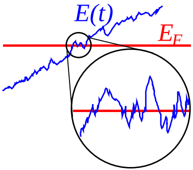

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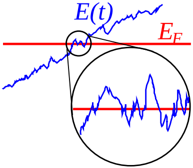

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$$N^{\mathrm{ex}} \propto \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \int_{-\infty}^{\infty} ds \int_{-\infty}^{t} ds' \ldots \times F_{\mathrm{noise}}(t, t', s, s')$$

$$F_{\mathrm{noise}} = \left\langle \exp\left[i \int_{t'}^{t} \eta(\tau) d\tau - i \int_{s'}^{s} \eta(\sigma) d\sigma \right] \right\rangle$$

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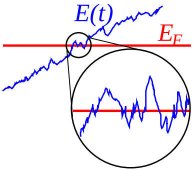

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- Find $F_{\mathrm{noise}} = \exp\left[-\Gamma_2\left(|t-t'| + |s-s'| - L(t,t',s,s')\right)\right]$

- Can simplify to $\Delta = t - s$ and $\Lambda = t - t' = s - s'$

$$N^{\mathrm{ex}} = \frac{-\Gamma^2}{2\pi c}\int_{-\infty}^{\infty}\frac{d\Delta}{(\Delta - i0)^2}\int_0^{\infty} d\Lambda e^{(ic\Delta - \Gamma - \Gamma_2)\Lambda + \Gamma_2 L(\Delta,\Lambda)}$$

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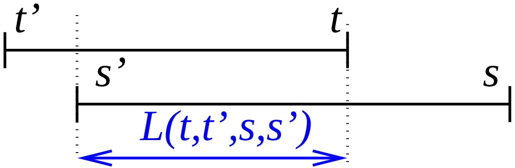

- Can simplify to $\Delta = t - s$ and $\Lambda = t - t' = s - s'$

$$N^{\text{ex}} = \frac{-\Gamma^2}{2\pi c}\int_{-\infty}^{\infty}\frac{d\Delta}{(\Delta - i0)^2}\int_0^{\infty} d\Lambda e^{(ic\Delta-\Gamma-\Gamma_2)\Lambda+\Gamma_2 L(\Delta,\Lambda)}$$

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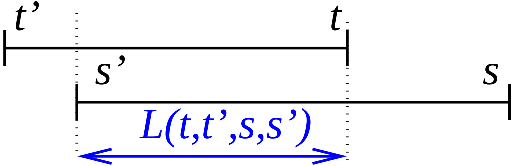

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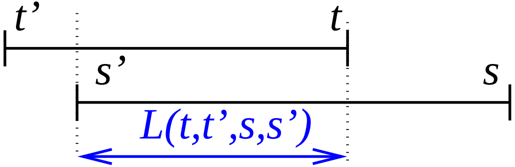

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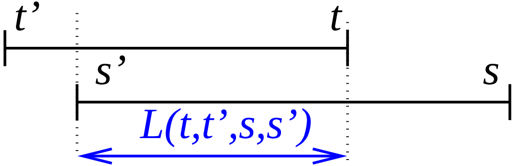

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Noisy driving: results

- Integral log divergent: white noise limit
- Infinite no. crossings of Fermi surface
- Can extract logarithmic contribution

$$N^{\mathrm{ex}} = \begin{cases} 1 & c \gg \Gamma\Gamma_2 \\ \frac{\Gamma^2}{(\Gamma+\Gamma_2)^2} + \frac{2\Gamma\Gamma_2}{\pi c} \ln \frac{\omega_*}{\Gamma+\Gamma_2} & c \ll \Gamma\Gamma_2 \end{cases}$$

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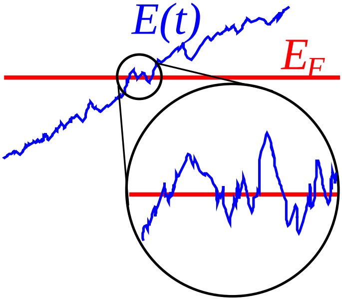

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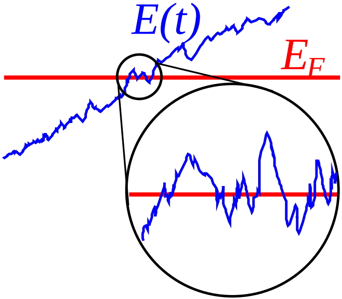

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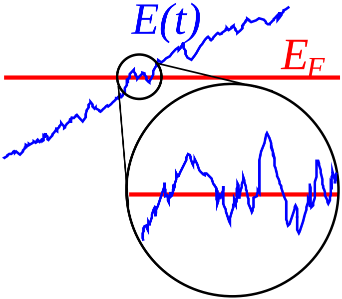

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