

# Landau Zener processes in many body systems

## Two-level system coupled to a photon field

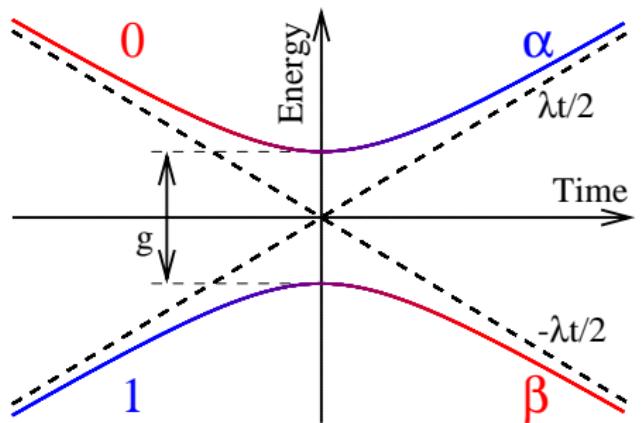
Jonathan Keeling<sup>1</sup> and V. Gurarie<sup>2</sup>

<sup>1</sup>University of Cambridge

<sup>2</sup>University of Colorado at Boulder

December 10, 2007

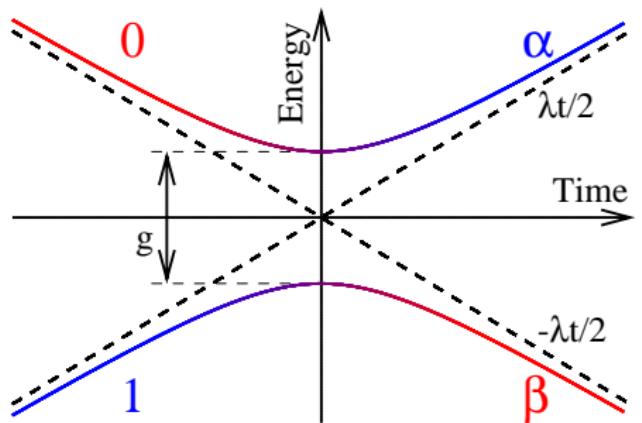
# The Landau-Zener problem



Initially,  $\psi =$   
Finally,  $\psi =$

$$i\partial_t \psi = \begin{pmatrix} \lambda t/2 & g \\ g & -\lambda t/2 \end{pmatrix} \psi$$

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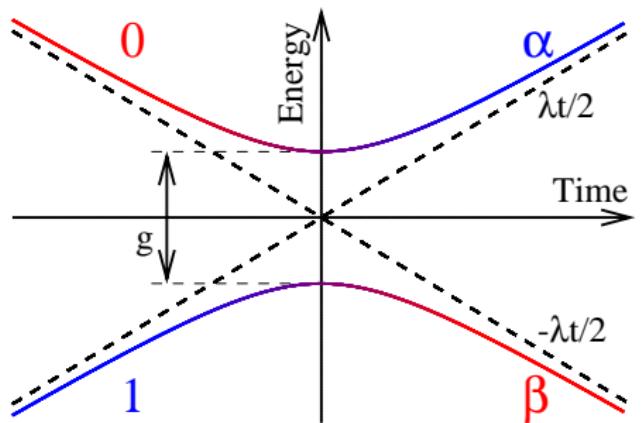


Initially,  $\psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ .

Finally,  $\psi = \begin{pmatrix} e^{-i\pi/4} \\ e^{i\pi/4} \end{pmatrix} = e^{i\pi/4} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

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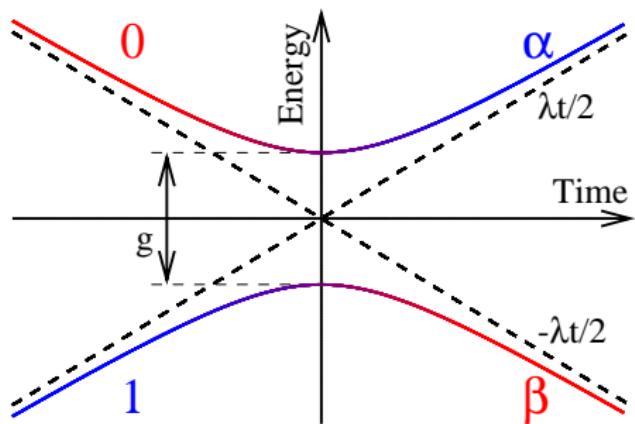
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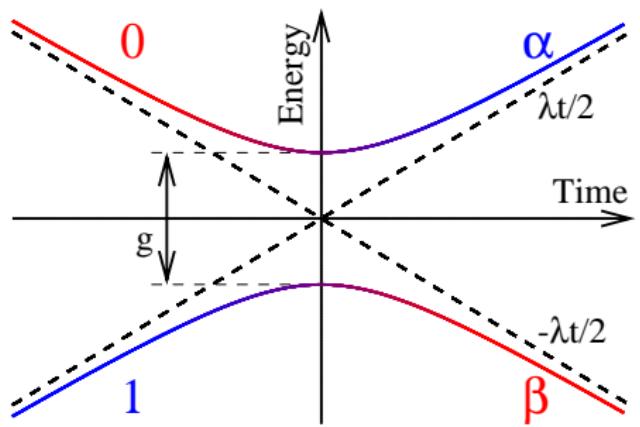


Initially,  $\psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ .

$$\text{Finally, } \psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \quad z = g^2/\lambda$$

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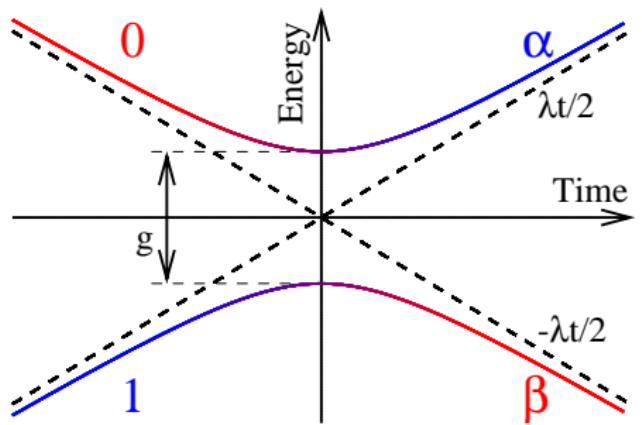
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$$|\beta| = \sqrt{1 - e^{-2\pi z}}$$

$$|\alpha| = e^{-\pi z}$$

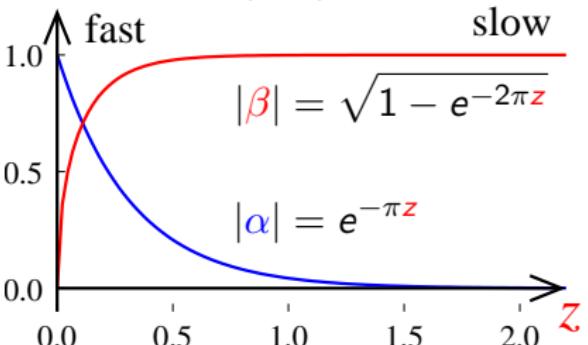
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# Many-body Landau-Zener generalisation

- One body, many-level generalisations:

- Exact solutions:

- Demkov-Osherov

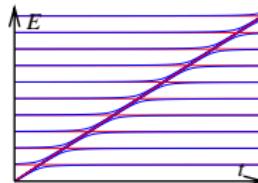
- Here, instead many-body

- Present two simple but striking examples.

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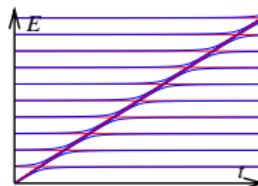


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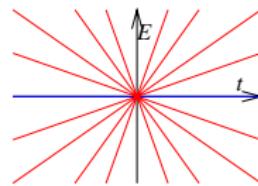
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Bowtie



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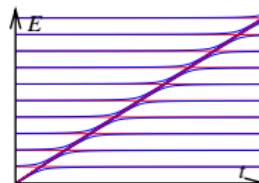
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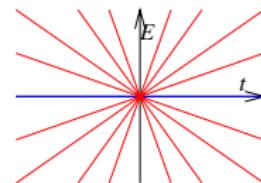
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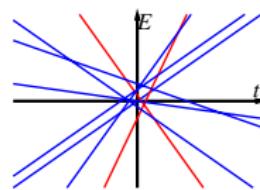
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Smallest/largest slope [Shytov]



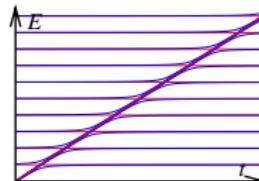
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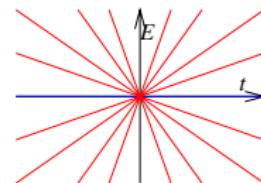
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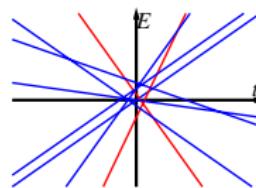
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- Here, instead **many-body**

↳ Many-body generalisation of single-particle states

↳ Automatic elevation to many-level problem

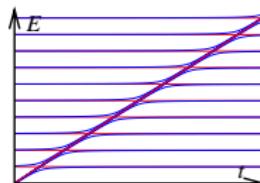
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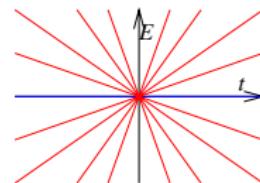
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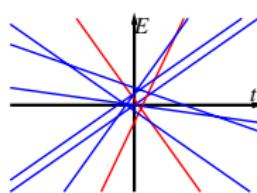
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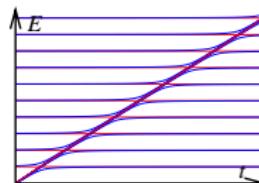
- ▶ Time varying energy of single-particle states
  - ▶ Automatic elevation to many-level problem
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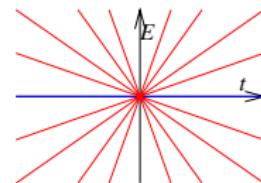
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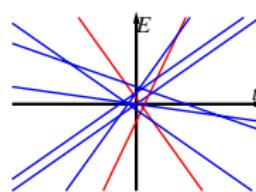
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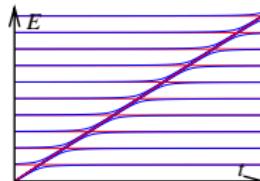
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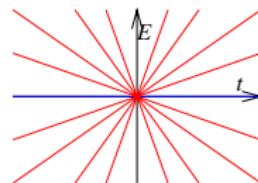
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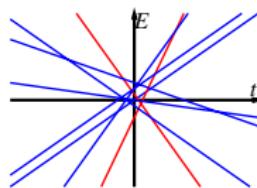
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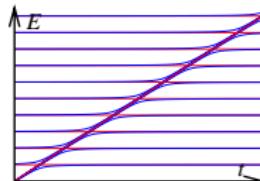
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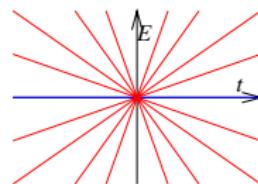
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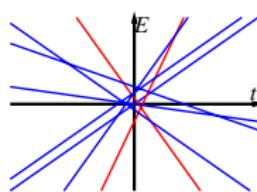
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# Overview

## 1 Introduction and Landau-Zener

- Introduction

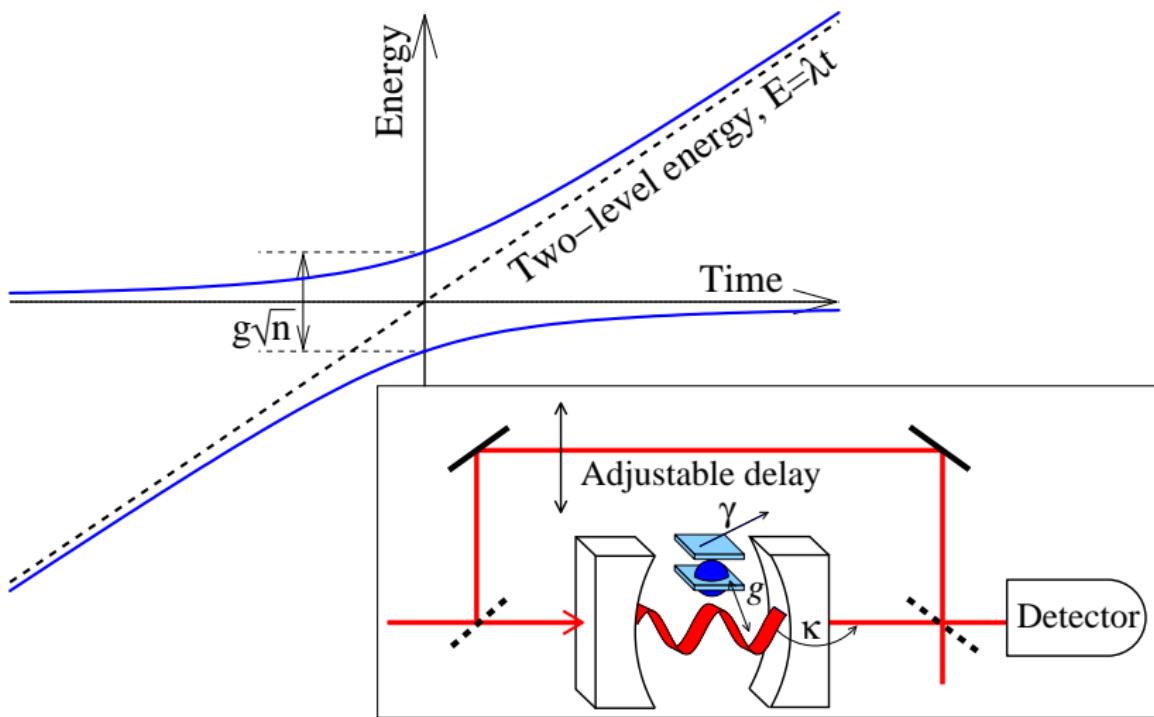
## 2 Two-level system/photon field

- Many body problem, Collapse and revivals
- Understanding collapse and revivals
- Effects of decay

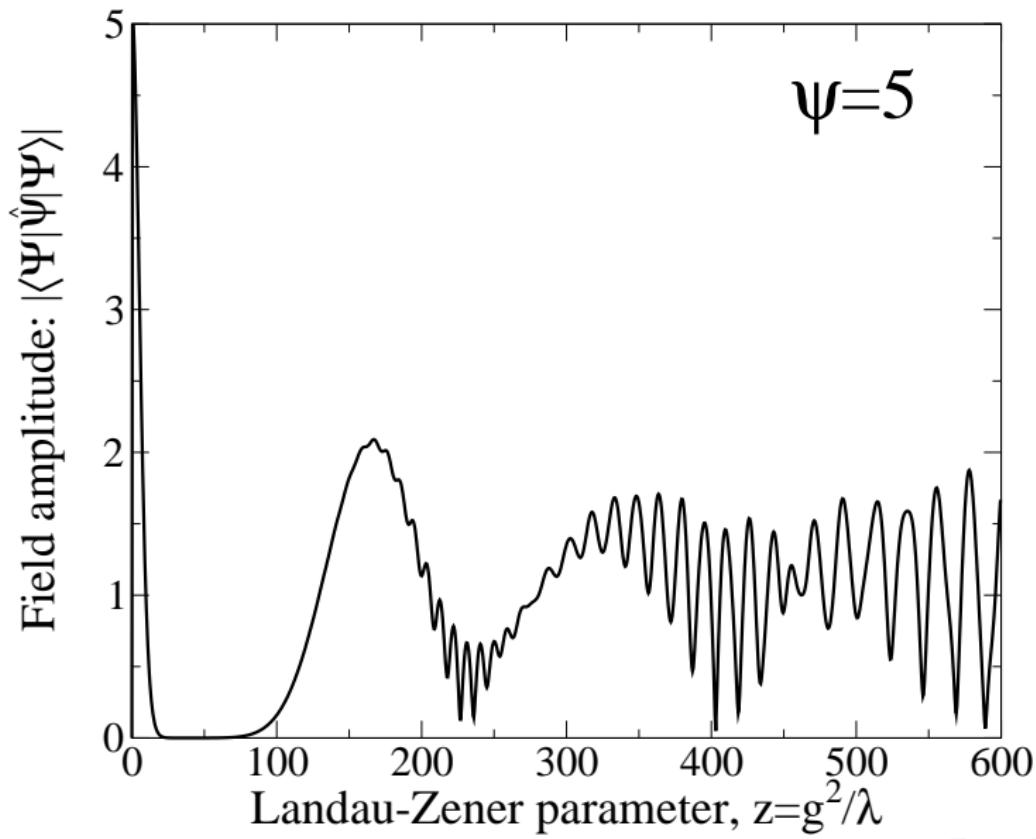
## 3 Localised fermion coupled to continuum

- Relating one- and many-body problems
- Cancellation and “clean electrons”

# Physical system



# Exact numerical result



# Description

- Hamiltonian:  $\hat{H} = \omega_0 \hat{\psi}^\dagger \hat{\psi} + \frac{\lambda t}{2} \hat{\sigma}_z + g (\hat{\psi}^\dagger \hat{\sigma}^- + \hat{\psi} \hat{\sigma}^+)$ ,

Initial coherent state  $|n, \downarrow\rangle = |n, 0\rangle - \Delta n |n+1, 1\rangle$

- Each pair  $|n, 1\rangle \rightarrow |n+1, 0\rangle$  undergoes LZ transition

$$H_{n,n+1} = \begin{pmatrix} \lambda t/2 & g\sqrt{n+1} \\ g\sqrt{n+1} & -\lambda t/2 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix}$$

- Final state:

$$|\Psi(t=\infty)\rangle = e^{-i\omega_0^2/2} \sum_{n=0}^{\infty} \frac{e^{i\omega_0 n}}{\sqrt{n!}} [A_{n+1}|n, 1\rangle + B_{n+1}|n+1, 0\rangle]$$

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$$|\Psi(+\infty)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} [A_{n,n}(n, \uparrow) + B_{n,n}(n+1, \downarrow)]$$

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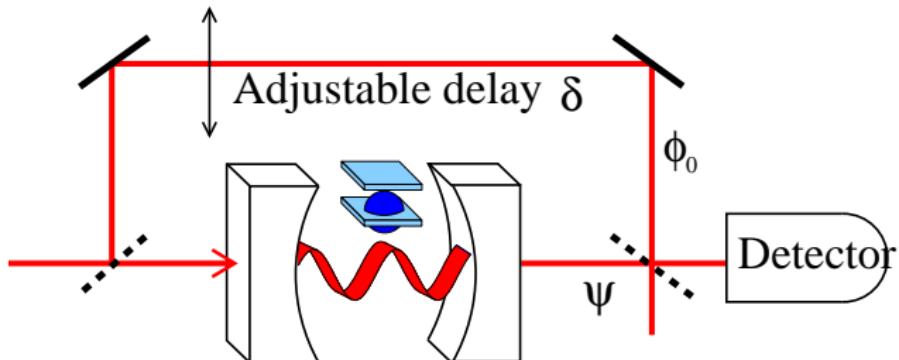
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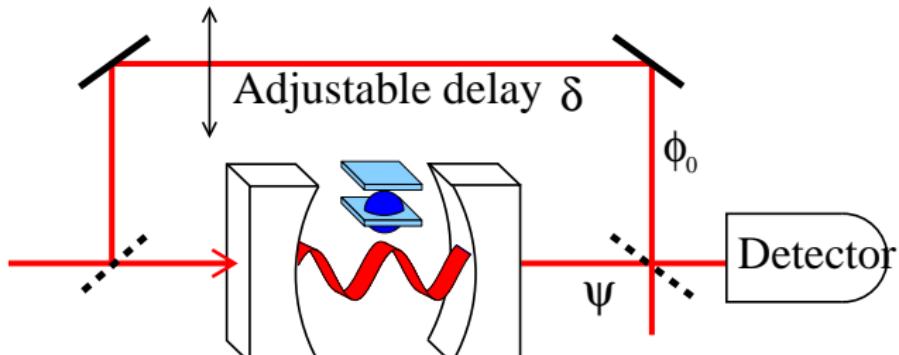


After mixing:  $\hat{\phi} \rightarrow \hat{\phi}_0 e^{i\delta}$  intensity

$$\begin{aligned} I &\propto \langle (\hat{\phi}) + \hat{\phi}_0 e^{-i\delta} | (\hat{\phi} + \hat{\phi}_0 e^{i\delta}) \rangle \\ &= \langle \hat{\phi}^{\dagger} \hat{\phi} \rangle + \phi_0^2 + \phi_0 (\langle \hat{\phi}^{\dagger} \rangle e^{i\delta} + \langle \hat{\phi} \rangle e^{-i\delta}) \end{aligned}$$

Measure  $\langle \hat{\phi}^{\dagger} \hat{\phi} \rangle$

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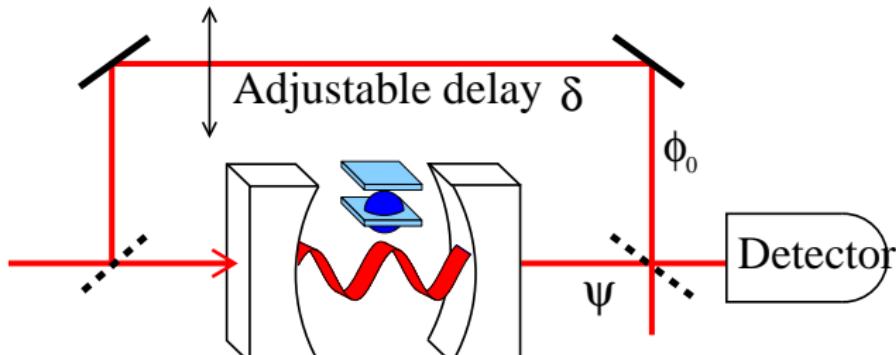


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$$\begin{aligned} I &\propto \langle (\hat{\psi})^2 + \hat{\phi}_0^2 e^{-2i\delta} \rangle (\hat{\psi} + \hat{\phi}_0 e^{i\delta}) \\ &= \langle \hat{\psi}^2 \rangle + \phi_0^2 + \phi_0 (\langle \hat{\psi} \rangle e^{i\delta} + \langle \hat{\psi} \rangle e^{-i\delta}) \end{aligned}$$

Measure  $\langle \hat{\psi}^2 \rangle$

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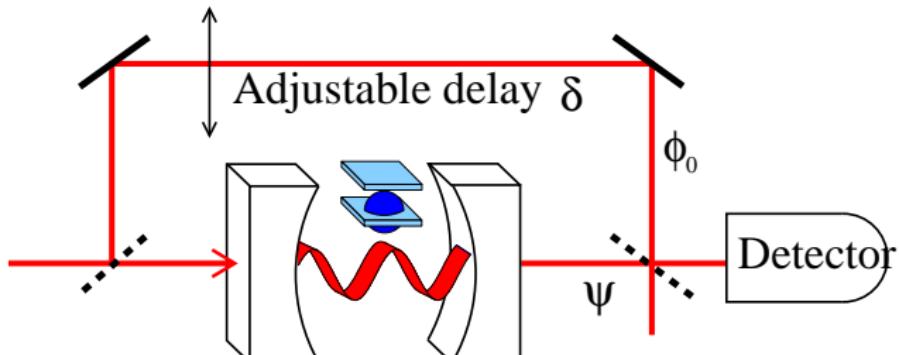
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$$I \propto \langle (\hat{\psi}^\dagger + \hat{\phi}_0^\dagger e^{-i\delta})(\hat{\psi} + \hat{\phi}_0 e^{i\delta}) \rangle$$

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Wigner function:

$$W(x, p) = \frac{1}{\pi} \int dy \Psi^*(x + y) \Psi(x - y) e^{2ipy}.$$

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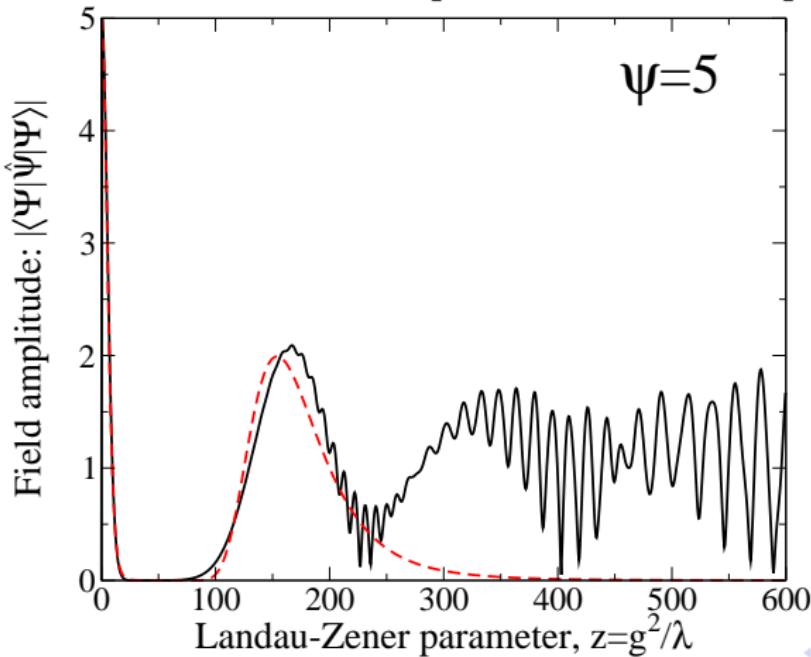
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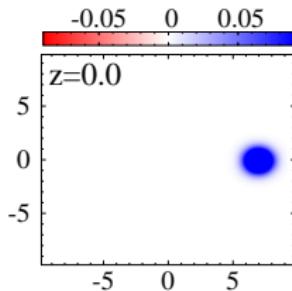
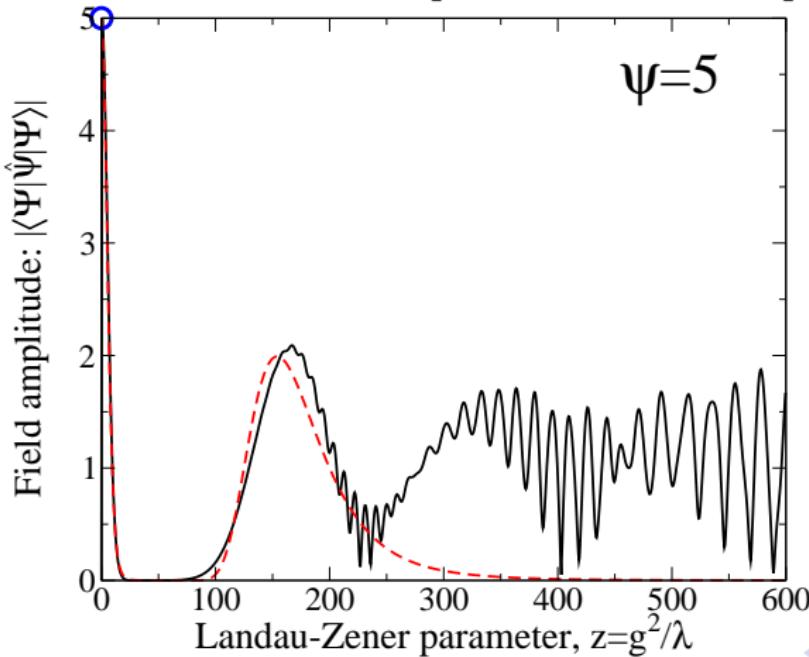


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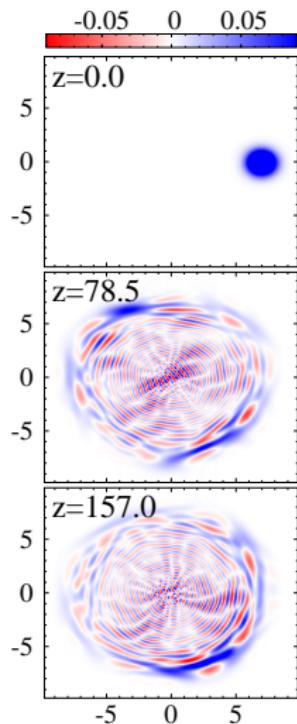
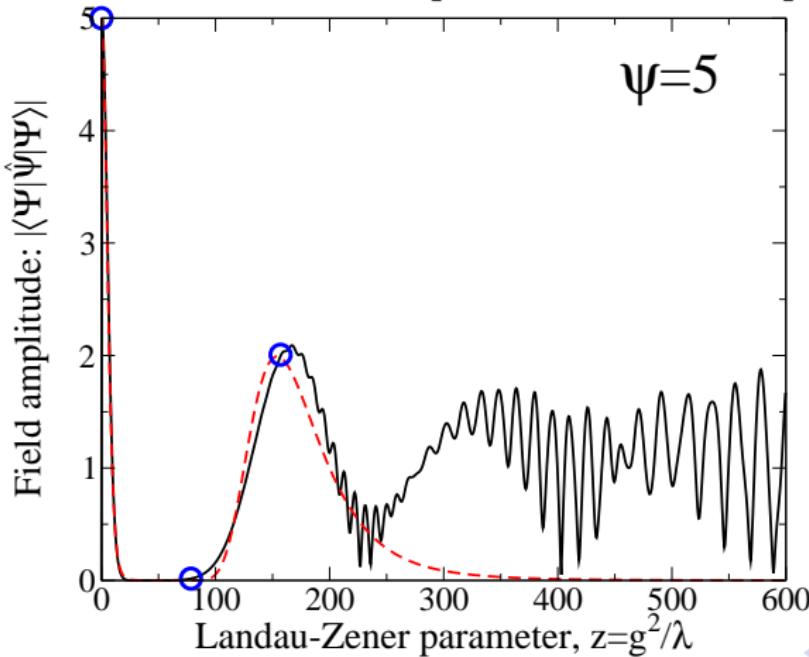


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# Explaining results

Adiabatic limit:  $\gamma = g^2/\lambda \gg 1$

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Adiabatic limit:  $z = g^2/\lambda \gg 1$

$$|\Psi(T)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[ A_{n+1}|n, \uparrow\rangle + B_{n+1}|n+1, \downarrow\rangle \right]$$

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# Explaining results

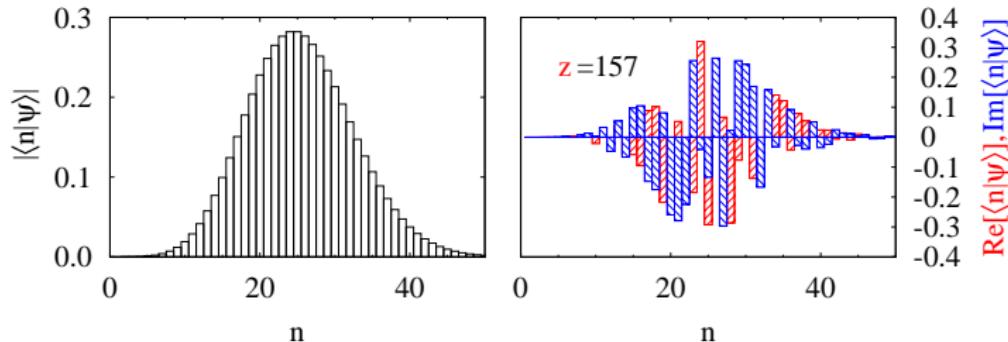
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# Understanding collapse and revival

$$\langle \Psi | \psi | \Psi \rangle = \psi e^{-|\psi|^2} \sum_n \frac{|\psi|^{2n}}{n!} \sqrt{\frac{n+2}{n+1}} e^{i(\phi_{n+2} - \phi_{n+1})}$$

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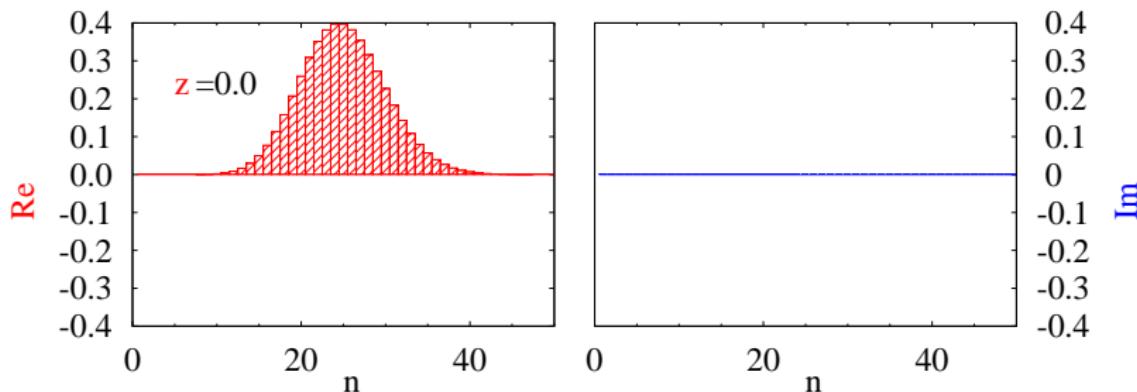
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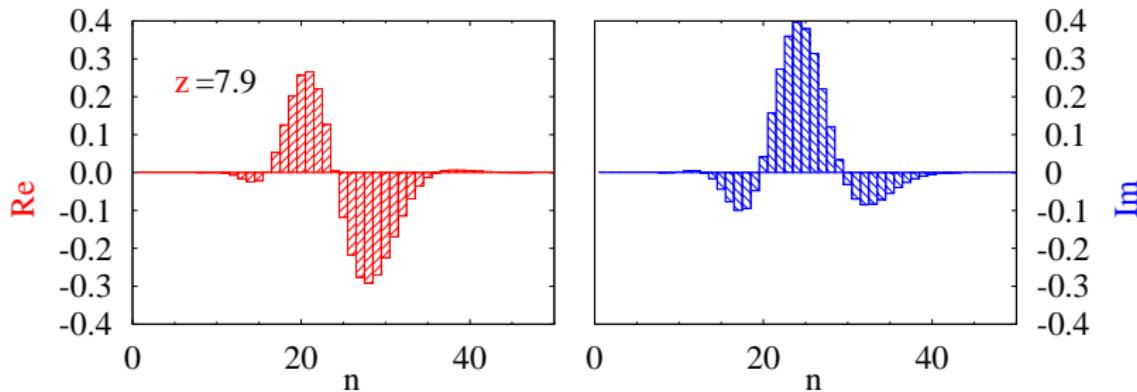
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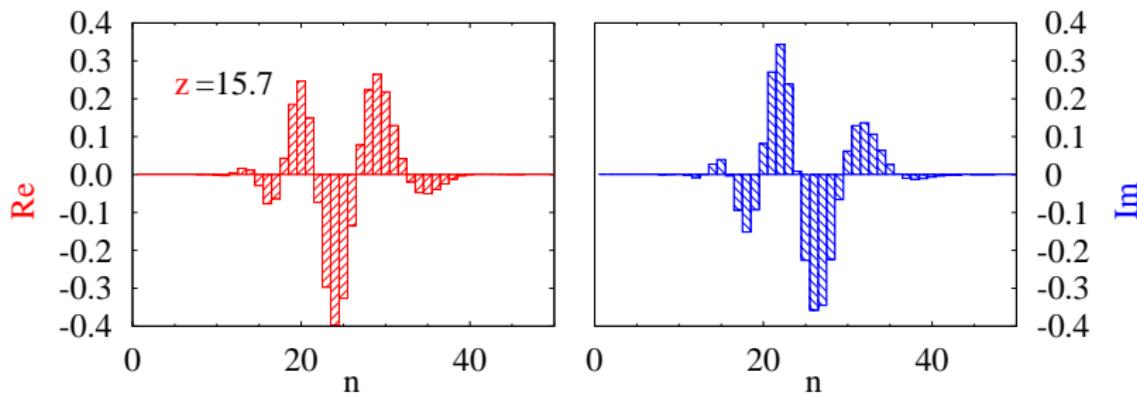
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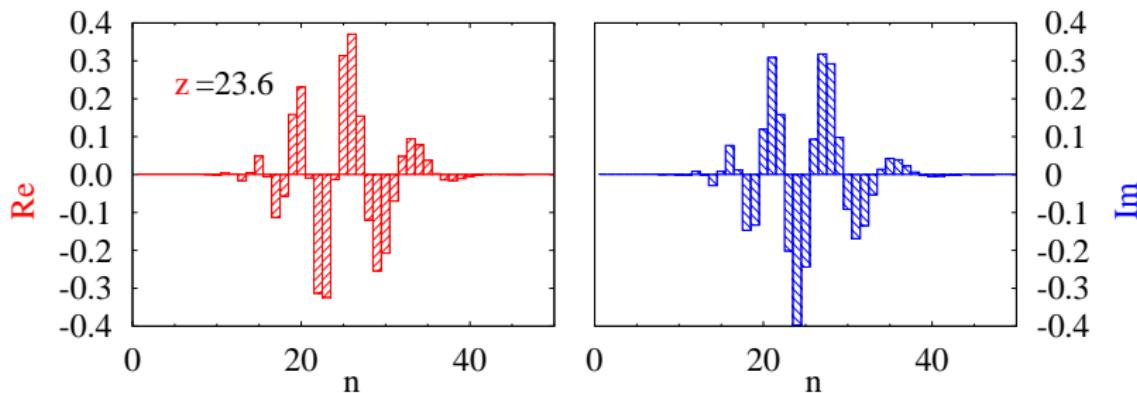
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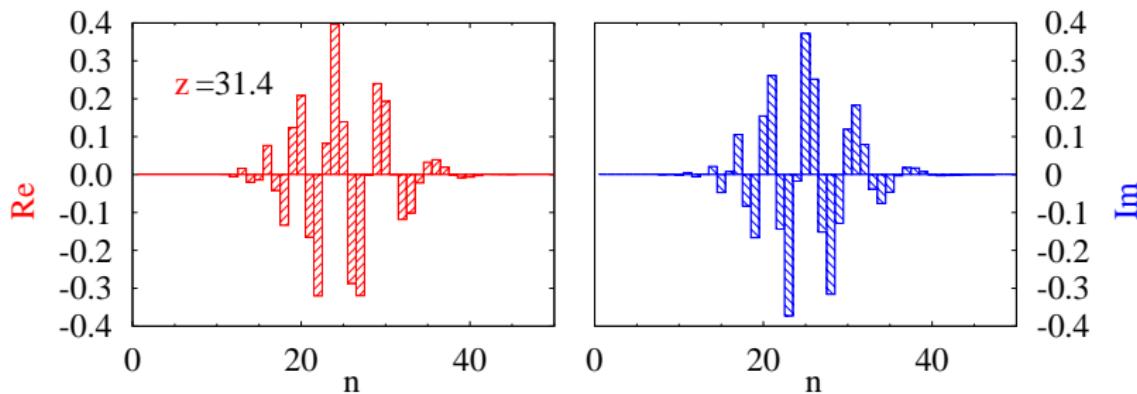
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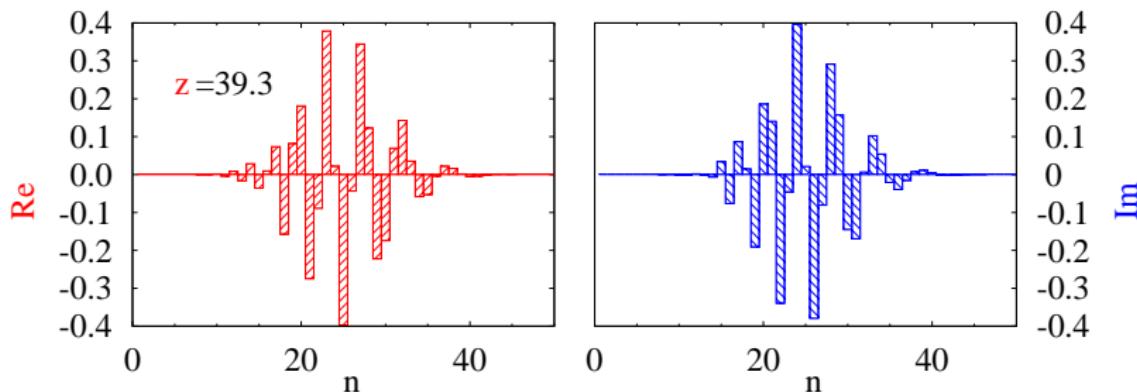
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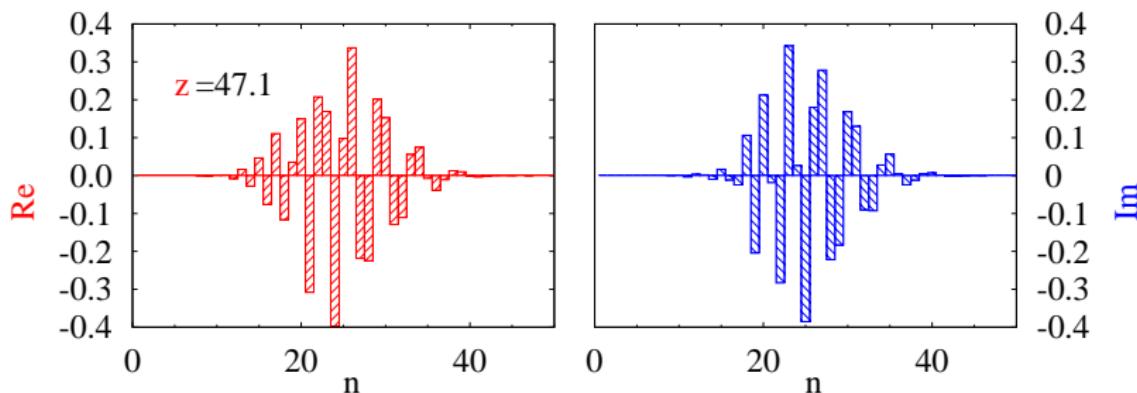
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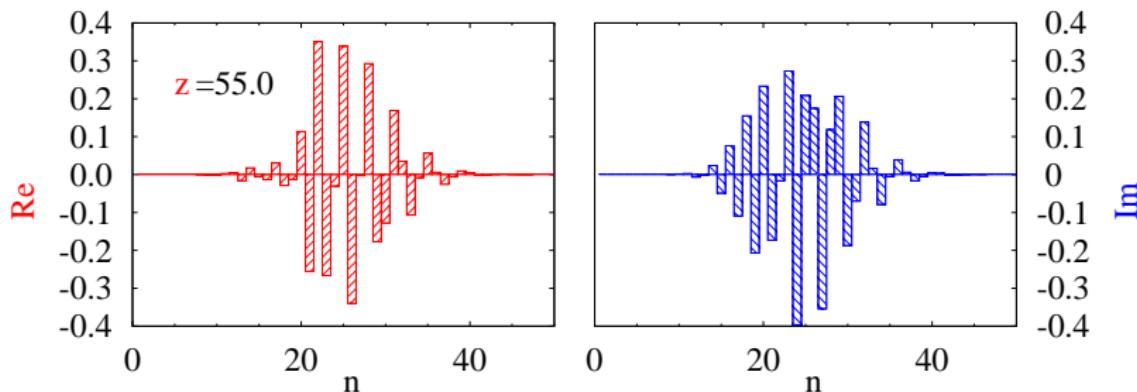
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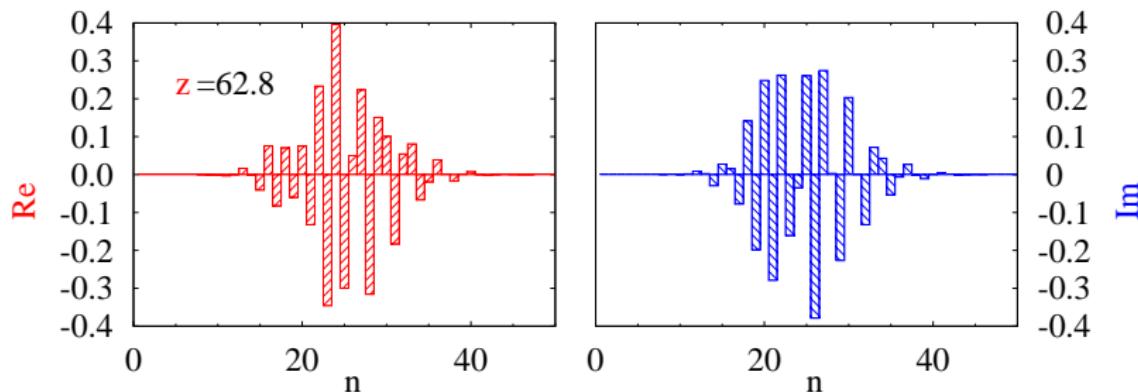
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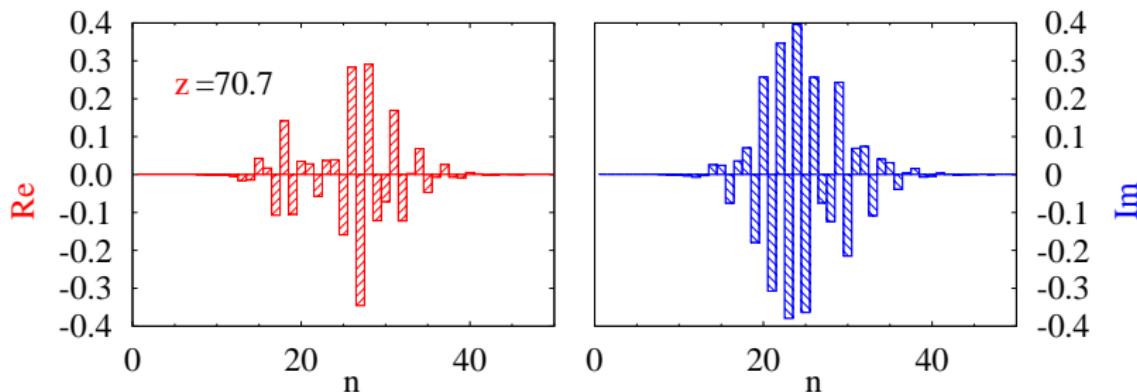
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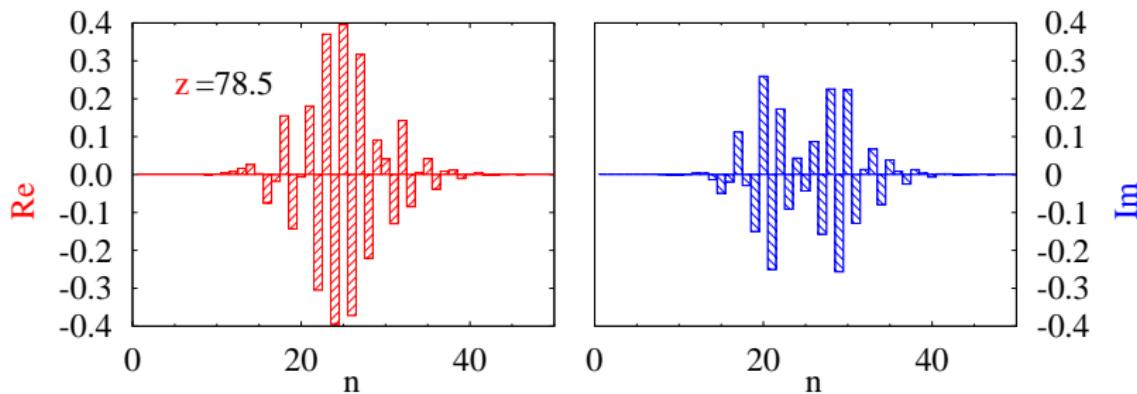
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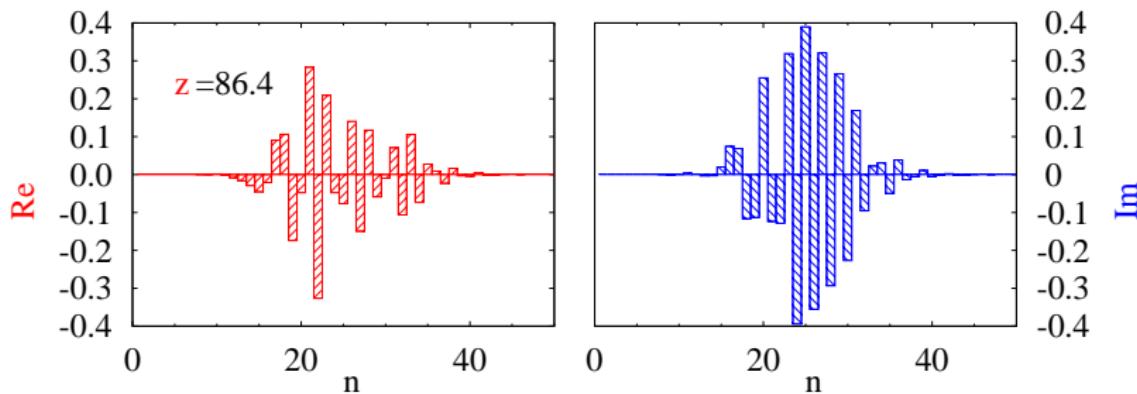
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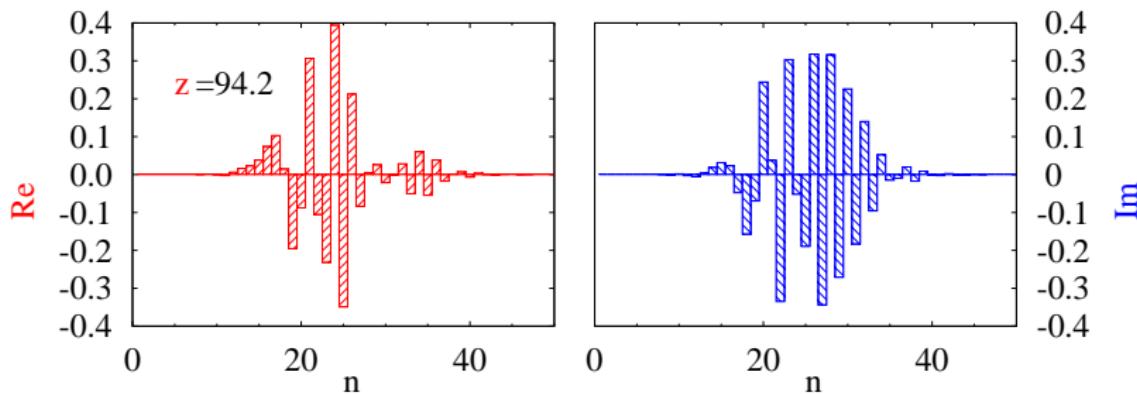
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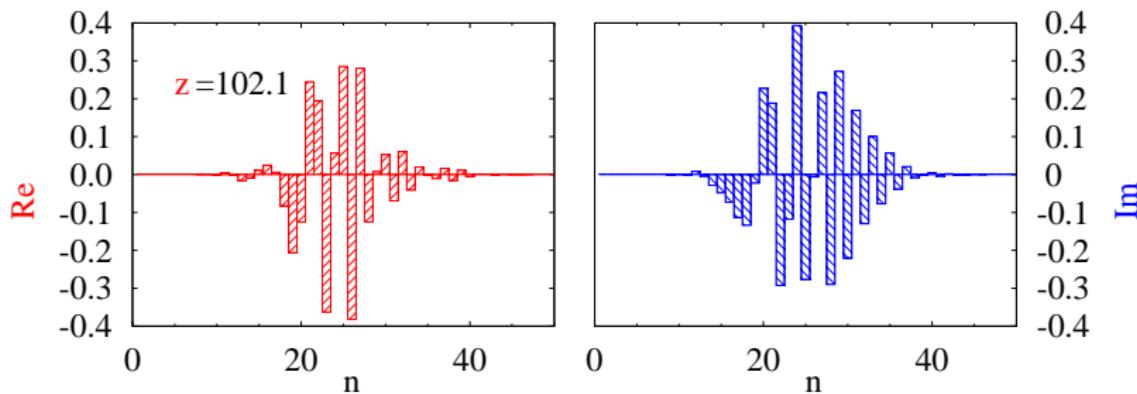
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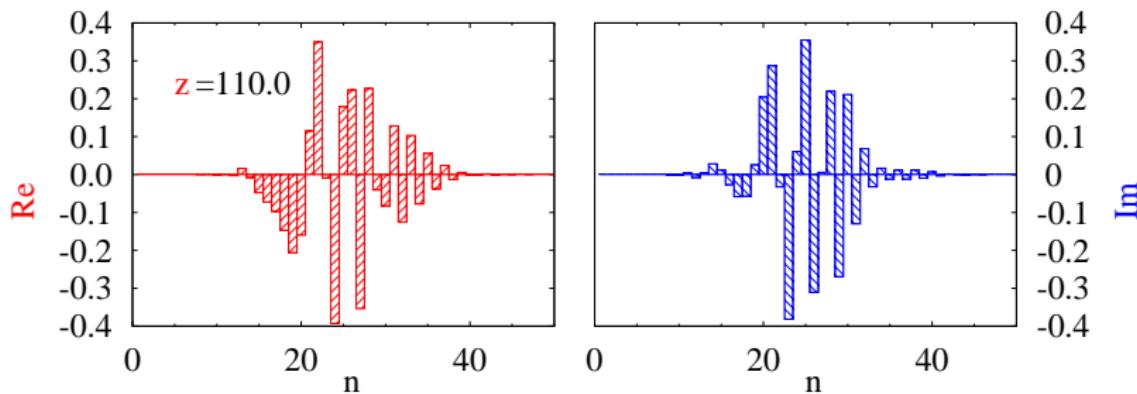
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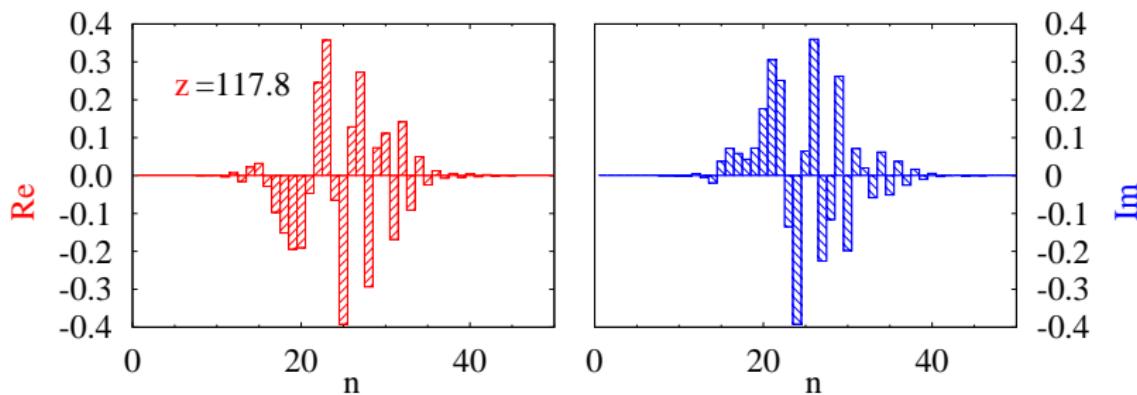
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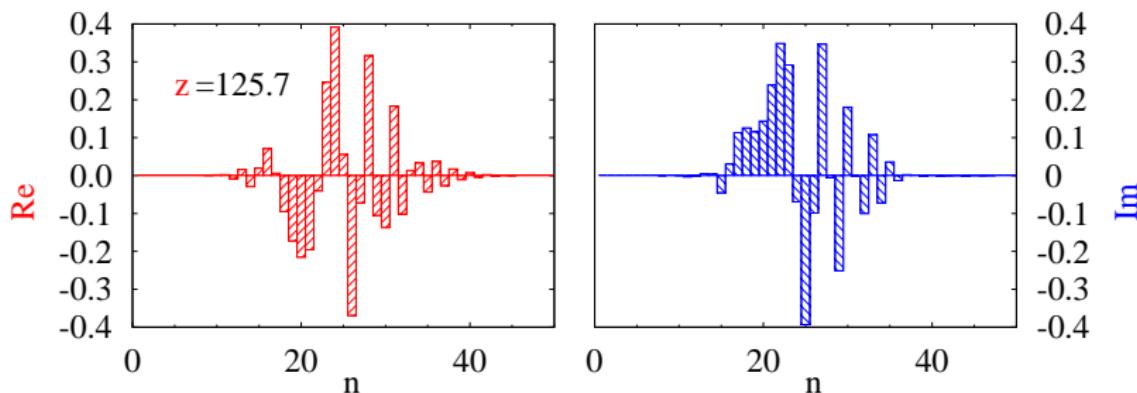
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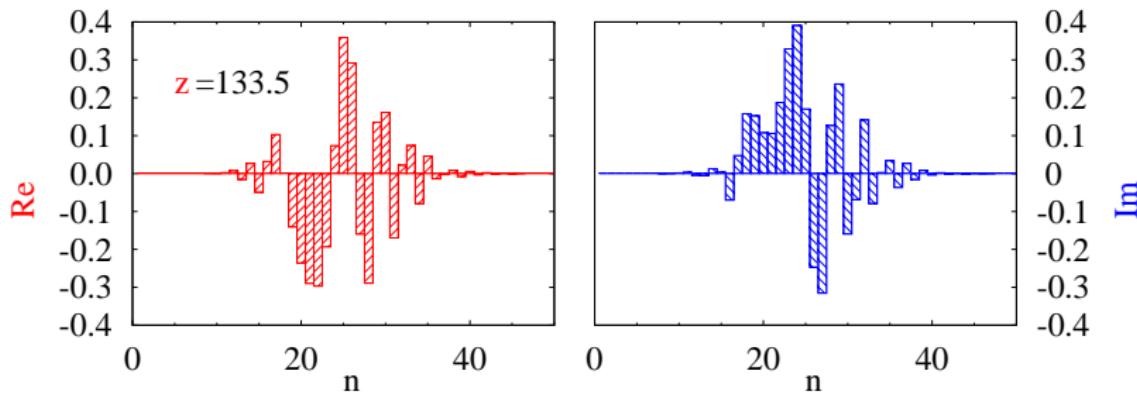
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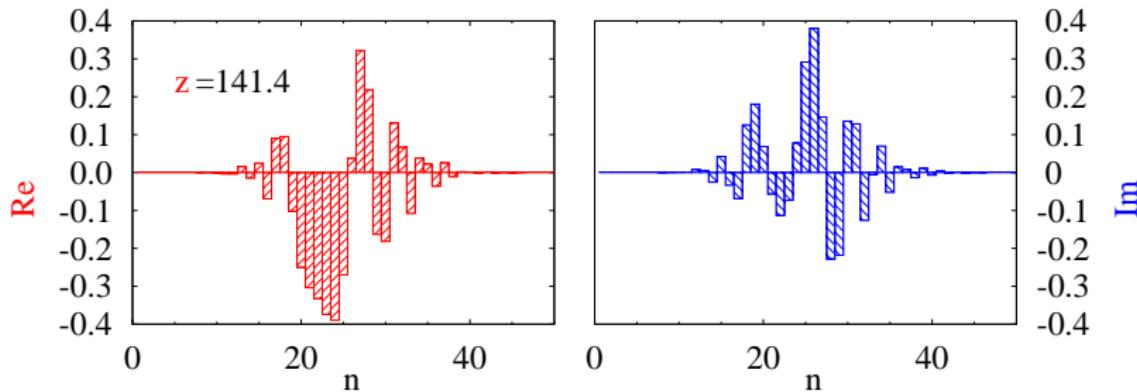
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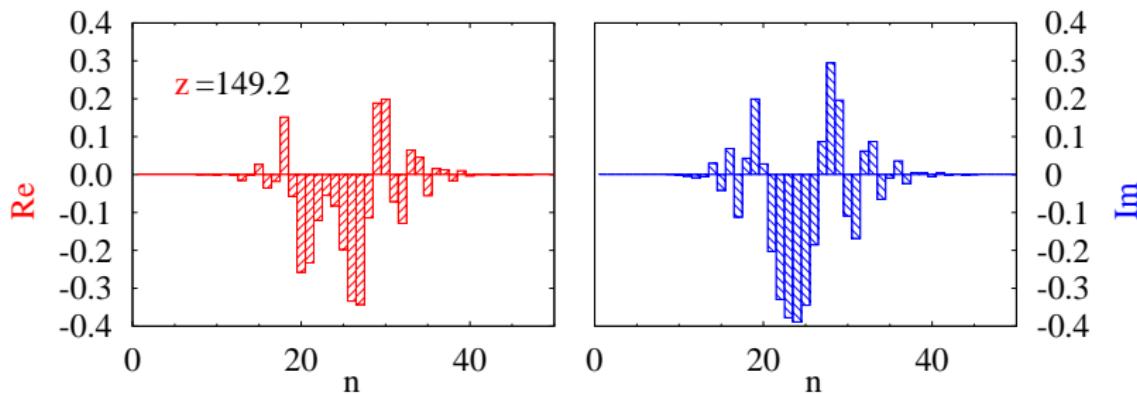
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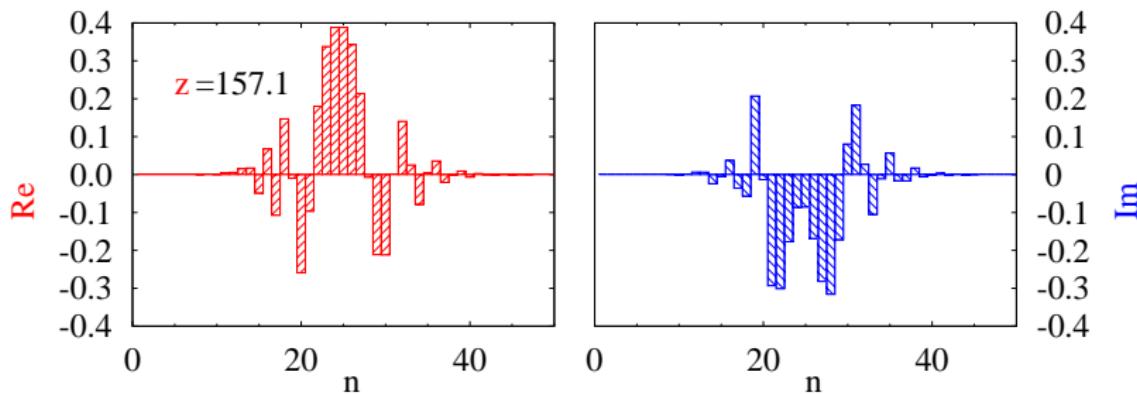
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# Explaining results: approximating phase

Want  $\Delta\phi_n$  for  $n \simeq |\psi|^2$ .

$$\Delta\phi_n = \pi [(n+1) \ln(n+1) - n \ln n + \text{const}]$$

$$\frac{d\Delta\phi_n}{dn} = \pi \ln \left( \frac{n+1}{n} \right) \simeq \frac{\pi}{n}$$

$$\Delta\phi_{n=|\psi|^2+m} \simeq \Delta\phi_{|\psi|^2} + \frac{cm}{|\psi|^2} - \frac{cm^2}{2|\psi|^4}$$

Revival at  $x = 2\pi N|\psi|^2$ , fails at  $x \gtrsim |\psi|^2$

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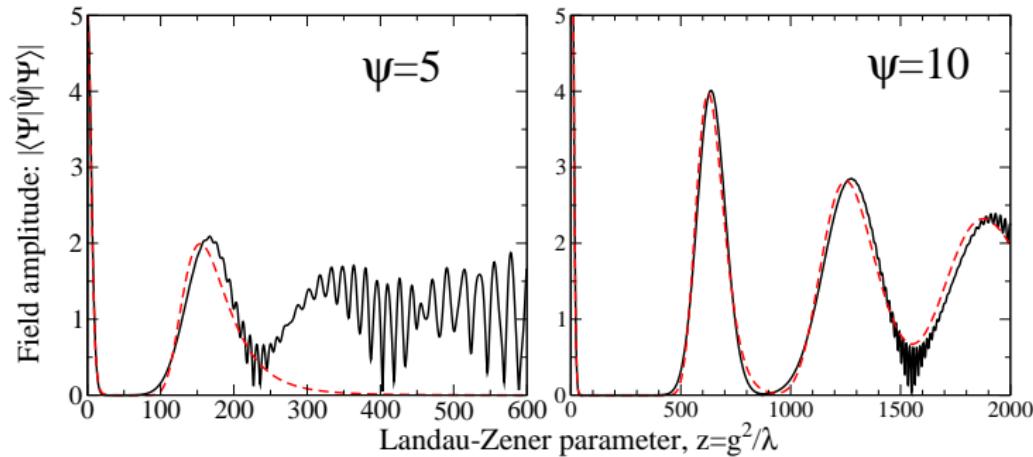
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# Explaining results: approximation

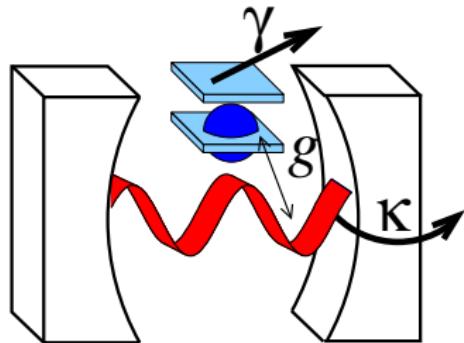
$$\langle \Psi | \psi | \Psi \rangle = \frac{|\psi|}{(1 + z^2/|\psi|^4)^{1/4}} \sum_{N=0}^{N_{\max}} \exp \left[ -\frac{(z - 2\pi N |\psi|^2)^2}{2|\psi|^2(1 + z^2/|\psi|^4)} \right]$$

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# Possible decay channels



$$\partial_t \hat{\rho} = -i [\hat{H}, \hat{\rho}] + L_\kappa[\hat{\rho}] + L_\gamma[\hat{\rho}]$$

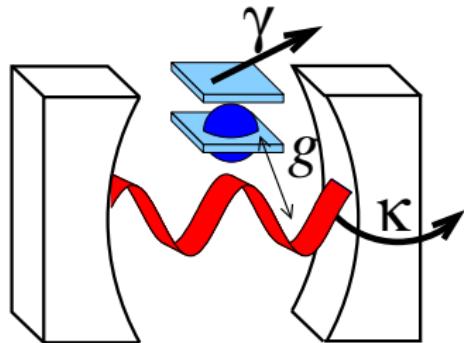
$$L_\kappa[\rho] = -\frac{i}{2} (\partial_x \partial_x \rho - \partial_y \partial_y \rho - \partial_z \partial_z \rho)$$

$$L_\gamma[\rho] = -\frac{1}{2} (\partial_x \partial_x \rho - \partial_y \partial_y \rho - 2\partial_x \partial_y \rho)$$

Wave estimate:  $(\psi^*(t')\psi(t)) < 1$ , with  $t' = g|\eta|/2$

$$\left(\frac{\psi^*(t')}{\psi(t)}\right) \leq \frac{1}{g^2 |\eta|^2} \leq \frac{1}{2\pi |\eta|^2} \simeq 10^{-3}$$

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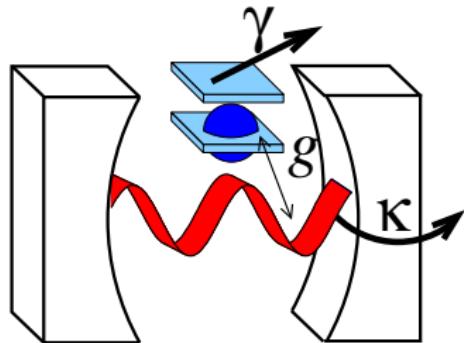
$$L_\kappa[\hat{\rho}] = -\frac{\kappa}{2} (\hat{\psi}^\dagger \hat{\psi} \hat{\rho} + \hat{\rho} \hat{\psi}^\dagger \hat{\psi} - 2 \hat{\psi} \hat{\rho} \hat{\psi}^\dagger)$$

$$L_\gamma[\hat{\rho}] = -\frac{i}{2} (\hat{\rho} \hat{\delta}_+ \hat{\delta}_- + \hat{\delta}_+ \hat{\delta}_- \hat{\rho} - 2 \hat{\delta}_+ \hat{\rho} \hat{\delta}_-)$$

Wave estimate:  $(\delta \hat{\rho})/\delta \hat{\rho}^*$  < 1, with  $\delta \hat{\rho}^* = g|\delta|/2$

$$\left(\frac{\delta \hat{\rho}}{\delta \hat{\rho}^*}\right) \leq \frac{\lambda}{g^2 |\delta|} \approx \frac{1}{2\pi |\delta|^2} \approx 10^{-2}$$

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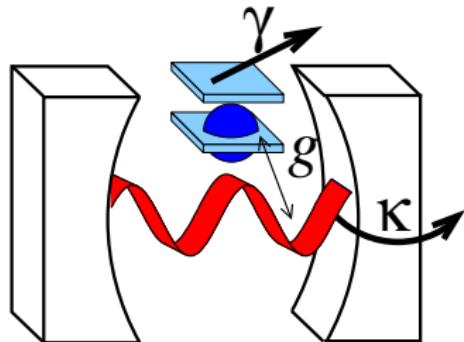
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$$L_\gamma[\hat{\rho}] = -\frac{\gamma}{2} (\hat{\sigma}_+ \hat{\sigma}_- \hat{\rho} + \hat{\rho} \hat{\sigma}_+ \hat{\sigma}_- - 2 \hat{\sigma}_- \hat{\rho} \hat{\sigma}_+)$$

Wave estimate:  $(\delta t)^2 \ll 1$ , with  $\delta t = g|\delta|/2$

$$\left(\frac{\delta t}{g|\delta|}\right) \leq \frac{\lambda}{g^2 |\delta|} \approx \frac{1}{2\pi |\delta|^2} \approx 10^{-3}$$

# Possible decay channels



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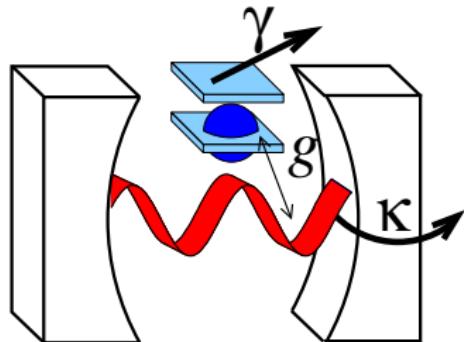
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Naive estimate:  $(\gamma t^*, \kappa t^*) \ll 1$ , with  $t^* \simeq g|\psi|/\lambda$

$$\left(\frac{\gamma}{g}\right) \leq \frac{1}{g^2 |\psi|} = \frac{1}{2\pi |\psi|^2} \simeq 10^{-3}$$

# Possible decay channels



$$\partial_t \hat{\rho} = -i [\hat{H}, \hat{\rho}] + L_\kappa[\hat{\rho}] + L_\gamma[\hat{\rho}]$$

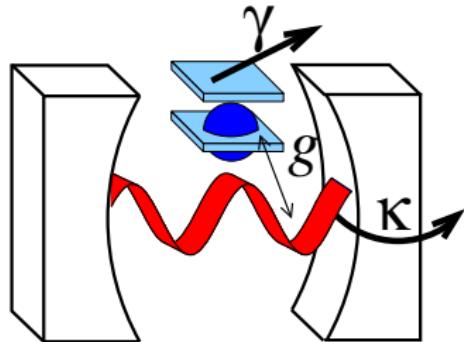
$$L_\kappa[\hat{\rho}] = -\frac{\kappa}{2} (\hat{\psi}^\dagger \hat{\psi} \hat{\rho} + \hat{\rho} \hat{\psi}^\dagger \hat{\psi} - 2 \hat{\psi} \hat{\rho} \hat{\psi}^\dagger)$$

$$L_\gamma[\hat{\rho}] = -\frac{\gamma}{2} (\hat{\sigma}_+ \hat{\sigma}_- \hat{\rho} + \hat{\rho} \hat{\sigma}_+ \hat{\sigma}_- - 2 \hat{\sigma}_- \hat{\rho} \hat{\sigma}_+)$$

Naive estimate:  $(\gamma t^*, \kappa t^*) \ll 1$ , with  $t^* \simeq g|\psi|/\lambda$

$$\left( \frac{\gamma}{g}, \frac{\kappa}{g} \right) \ll \frac{\lambda}{g^2} \frac{1}{|\psi|}$$

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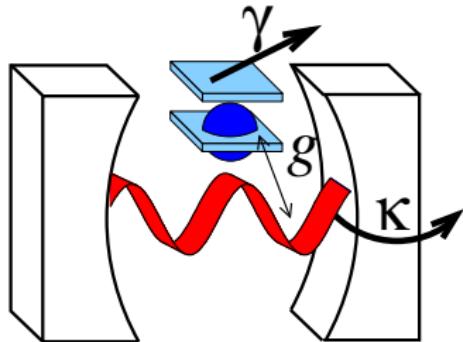
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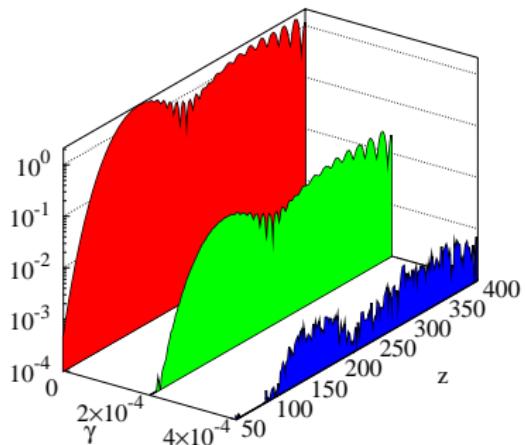
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# Results including decay



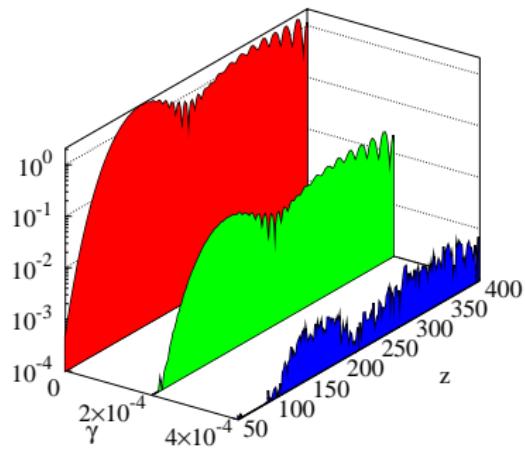
$$\gamma_{\max}/g \simeq 2 \times 10^{-4}$$

$$T_{\text{decay}}/t_0 \simeq 10 \times 10^{-4}$$

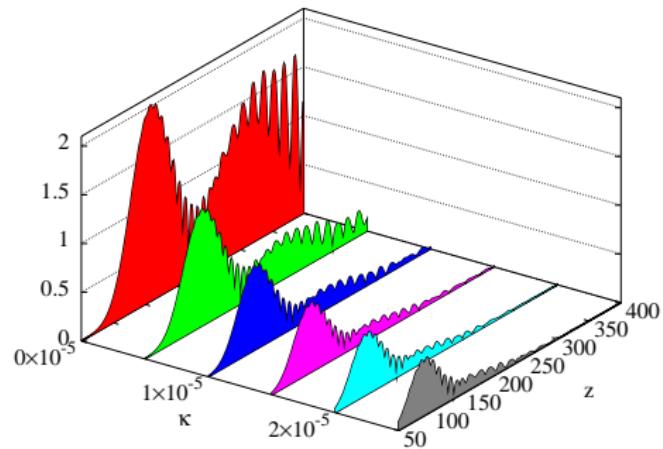
$$E_{\text{decay}}/E \simeq 1 \times 10^{-5}$$

$$R_{\text{decay}}/E < 100 \times 10^{-5}$$

# Results including decay

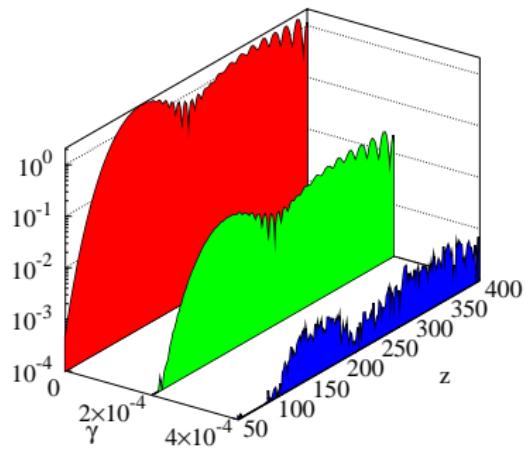


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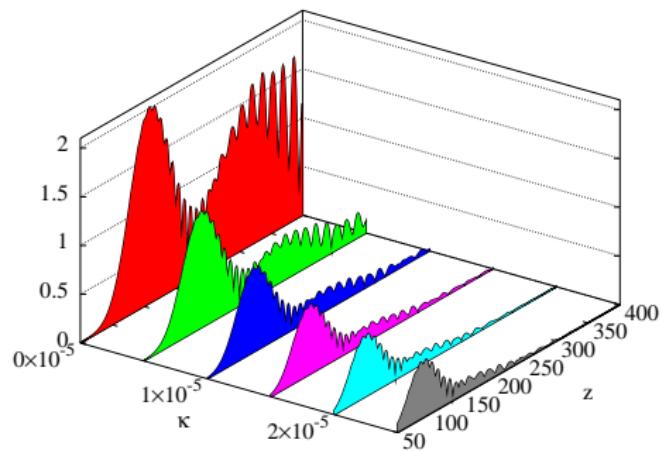


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# Results including decay

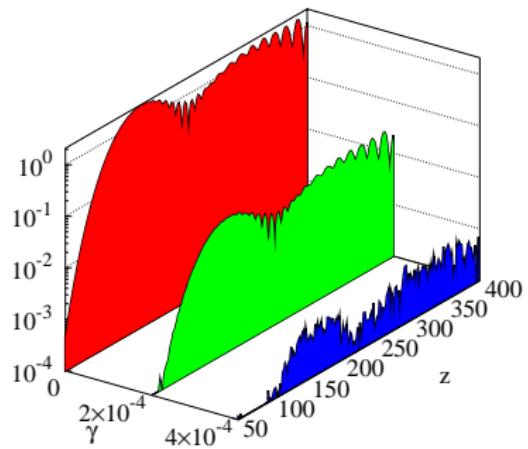


$$\gamma_{\max}/g \simeq 2 \times 10^{-4}$$
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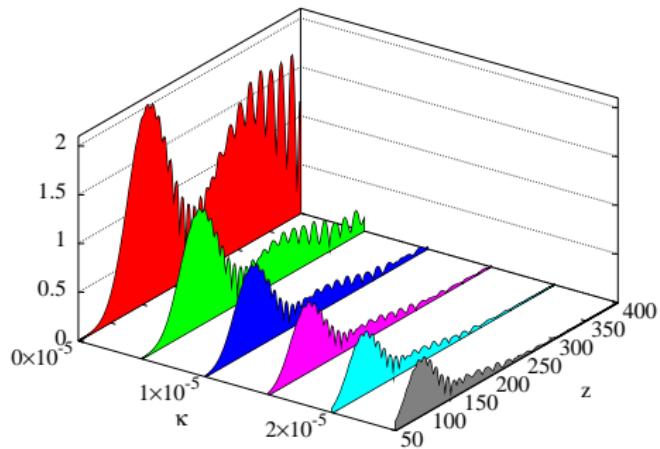
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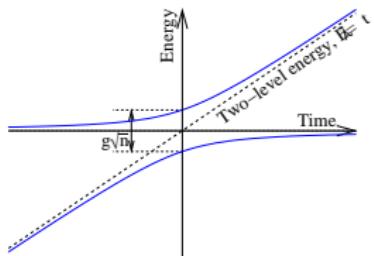
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# Understanding additional decay: adiabatic approx



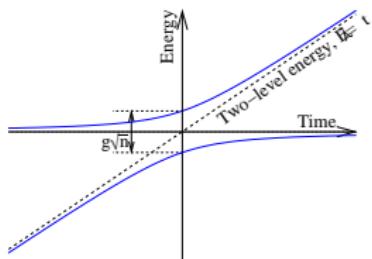
$$|\psi, t\rangle = \cos \theta_n |n, t\rangle + \sin \theta_n |n+1, t\rangle \quad \text{Adiabatic if } \frac{\partial \theta_n}{\partial t} \ll 1$$
$$P_{\text{trans}} = \frac{(n+1 - \theta_n(n+1))^2}{(n+1 + \theta_n(n+1))^2} \leq \frac{27}{2500} < 1$$

If adiabatic,  $\Lambda_n = P_{n,n+1}$

$$\frac{d\Lambda_n}{dt} = i \left[ \frac{d\Delta \phi_{n+1}}{dt} \right] \Lambda_n - \kappa \left[ \left( n - \frac{1}{2} \right) \Lambda_n - \sqrt{n(n+1)} \Lambda_{n+1} \right]$$

When  $|t| \leq g\sqrt{n}/\Lambda$ , decay rate  $\kappa \theta_n \approx \kappa |d|^2 \gg \kappa$

# Understanding additional decay: adiabatic approx



$$|n, +\rangle = \cos \theta_n |n, \downarrow\rangle + \sin \theta_n |n-1, \uparrow\rangle \quad \text{Adiabatic if } \Gamma t \ll \hbar \omega$$

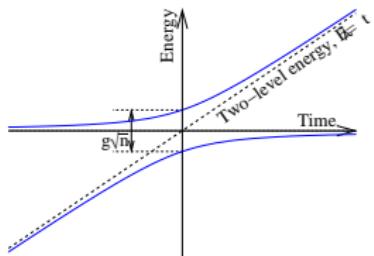
$$P_{\text{trans}} = \frac{(n-1)(n+1)}{(n+1)^2 + (n-1)^2} \leq \frac{27}{2500} < 1\%$$

If adiabatic,  $\Lambda_n = \rho_{n,n+1}$

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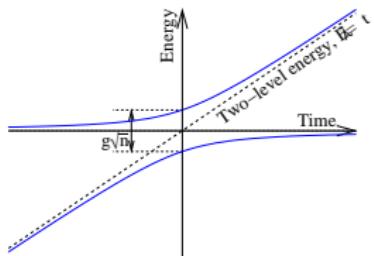
$$P_{\text{trans}} = \frac{|\langle n-1, -|\hat{\psi}|n, +\rangle|^2}{\langle n, +|\hat{\psi}^\dagger \hat{\psi}|n, +\rangle} \leq \frac{27}{256n^2} \ll 1$$

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When  $|t| \leq g\sqrt{n}/\lambda$ , decay rate  $\kappa n \approx n|d|^2 > n$

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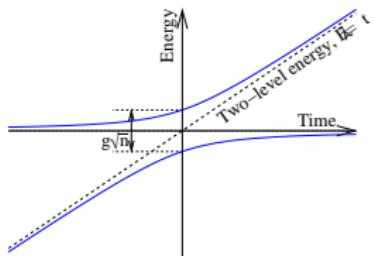
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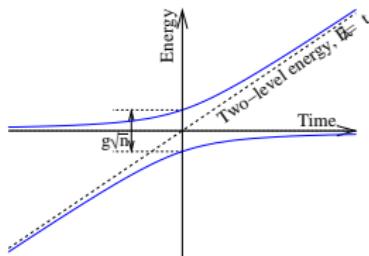
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When the initial decay rate  $\kappa_0 \approx n^{1/2} > n$

# Understanding additional decay: adiabatic approx



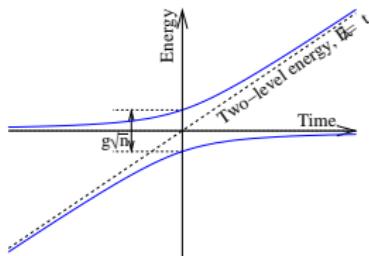
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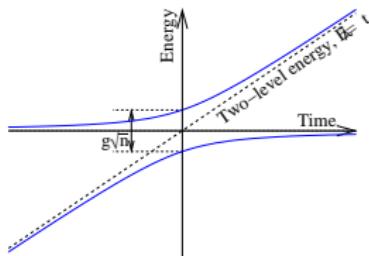
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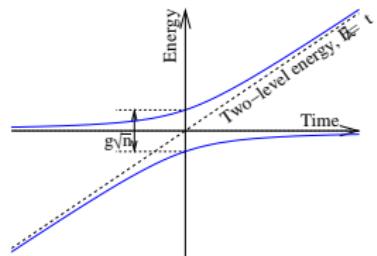
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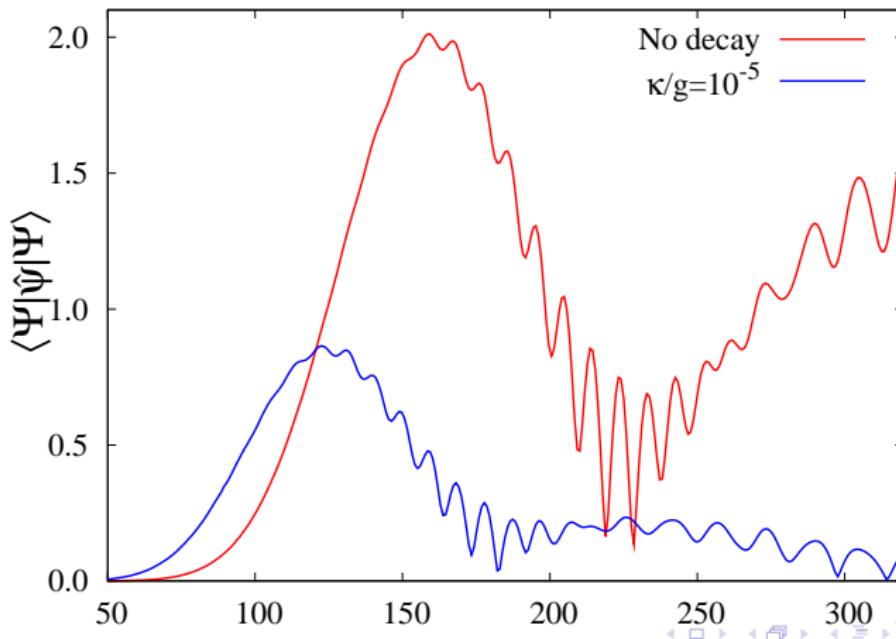
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When  $|t| \leq g\sqrt{n}/\lambda$ , decay rate  $\kappa n \simeq \kappa |\psi|^2 \gg \kappa$ .

$$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5 \times 10^{-5}$$

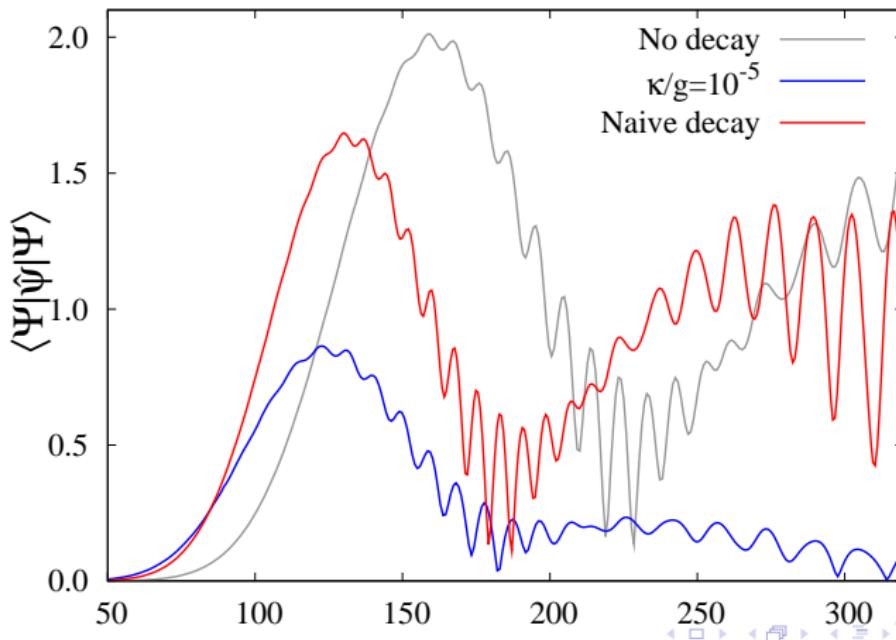
# Effect of extra decay

$$\langle \Psi | \hat{\psi}(\psi_0, \kappa) | \Psi \rangle$$



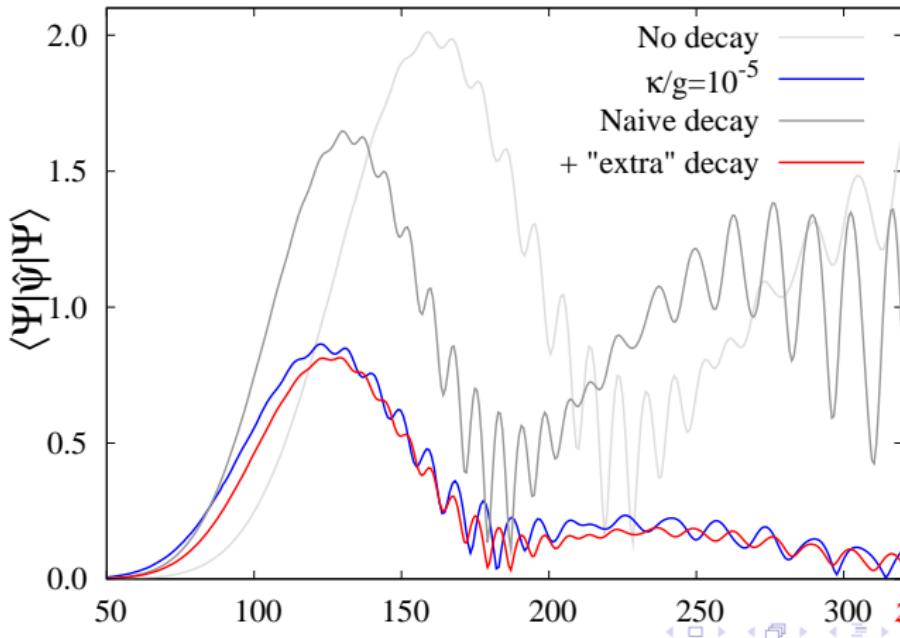
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$$\langle \Psi | \hat{\psi}(\psi_0, \kappa) | \Psi \rangle \simeq \langle \Psi | \hat{\psi}(\psi_0 e^{-\kappa T/2}, 0) | \Psi \rangle \exp \left[ -\frac{\kappa T}{2} \right]$$

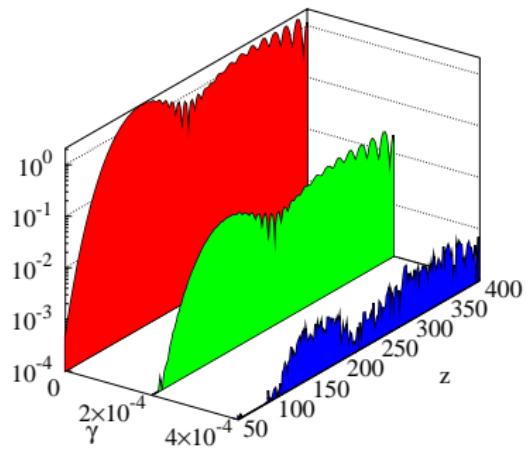


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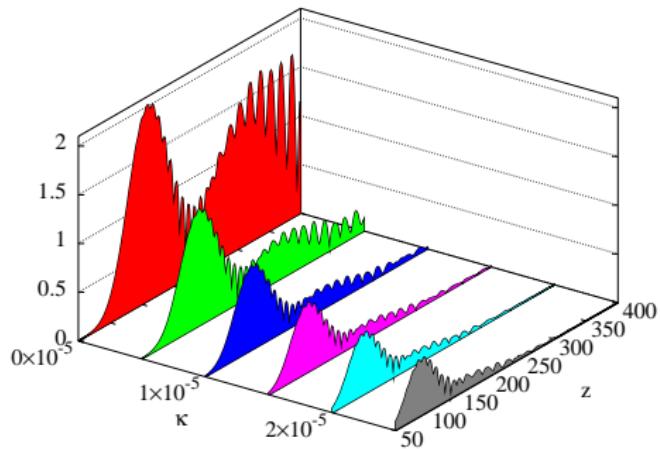
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# Results including decay



$$\begin{aligned}\gamma_{\text{max}}/g &\simeq 2 \times 10^{-4} \\ \gamma_{\text{theory}}/g &\ll 10 \times 10^{-4}\end{aligned}$$



$$\begin{aligned}\kappa_{\text{max}}/g &\simeq 1 \times 10^{-5} \\ \kappa_{\text{theory}}/g &\ll 5 \times 10^{-5}\end{aligned}$$

# Possible systems

Requirement:  $\kappa/g \lesssim 1 \times 10^{-5}$ ,  $\gamma/g \lesssim 2 \times 10^{-4}$

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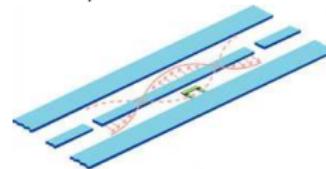
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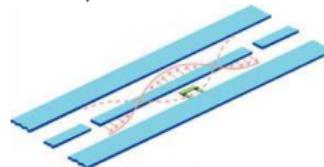
System	Source	$\kappa/g$	$\gamma/g$
Quantum dots/Microdisk	CNRS 2005	$2 \times 10^{-1}$	$3 \times 10^{-1}$
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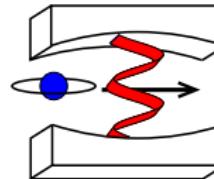
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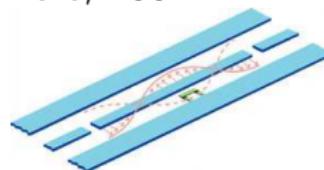
Atom/Microwave cavity	ENS, 2004	$7 \times 10^{-3}$	$2 \times 10^{-4}$
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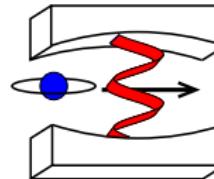
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# Landau Zener processes in many body systems

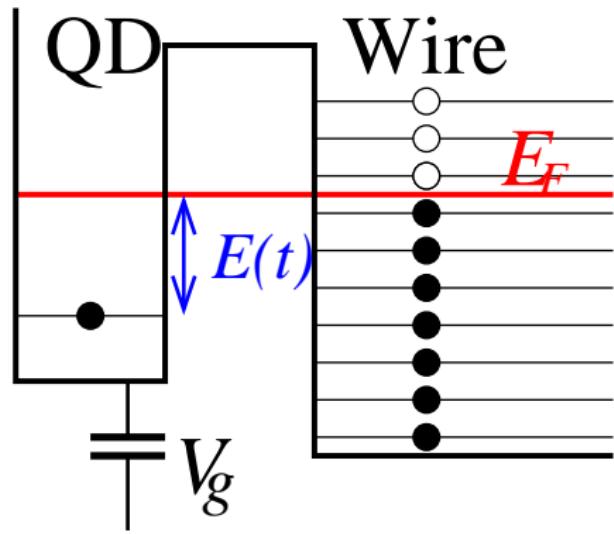
Localised fermion coupled to a continuum of states

Jonathan Keeling<sup>1</sup>, L. S. Levitov<sup>2</sup> and A. Shytov<sup>3</sup>

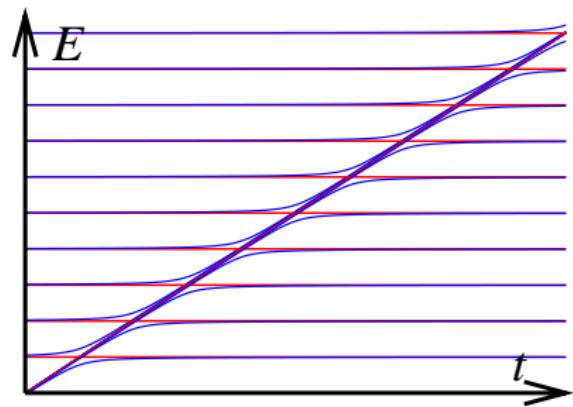
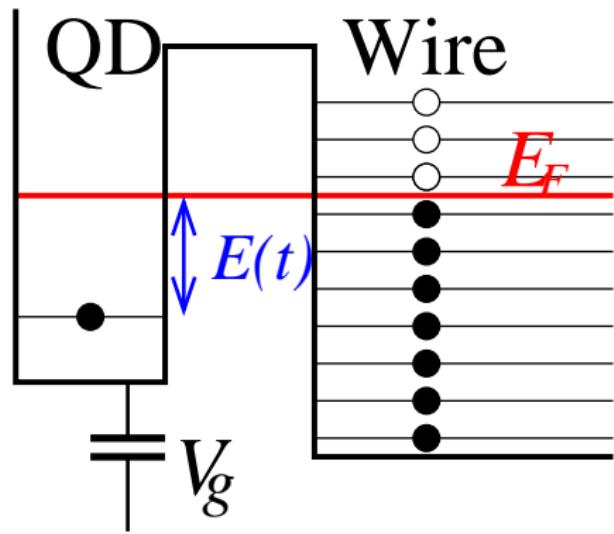
<sup>1</sup>University of Cambridge <sup>2</sup>Massachusetts Institute of Technology <sup>3</sup>Brookhaven National Lab

December 10, 2007

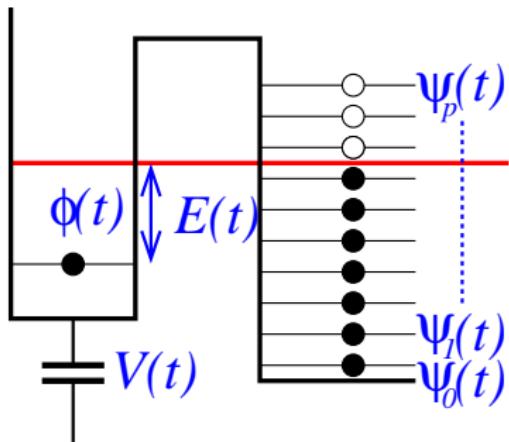
# Physical problem



# Physical problem



# Single particle problem



Regularly spaced continuum:

$$\psi(x, t) = \sum \psi_p(t) e^{ipx}$$

Thus, continuum equations:

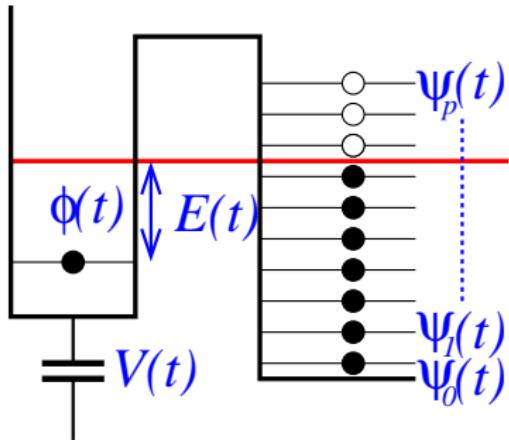
$$(i\partial_t - E(t))\psi(t) = g \int dx \psi(x, t) \delta(x)$$

$$(i\partial_t + i\nu\partial_x)\psi(x, t) = g\delta(x)\psi(t)$$

Given:  $\psi(x < 0, t) = \frac{e^{-iEt}}{\sqrt{2\pi}}$ , find

$$U(x, t) = \int dt \psi(x > 0, t) \frac{e^{iEt}}{\sqrt{2\pi}}$$

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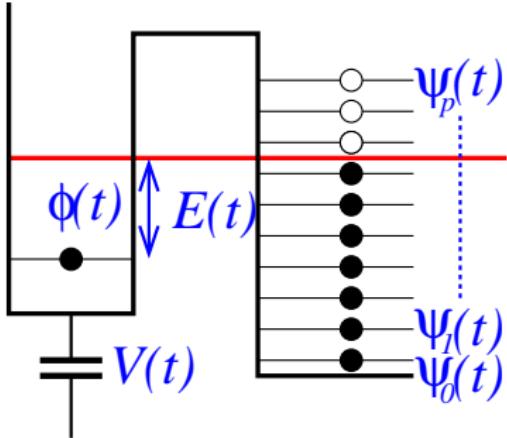
$$(i\partial_t - E(t))\psi(t) = g \int dx \delta(x-t)\psi(x)$$

$$(i\partial_t + iv\partial_x)\psi(x, t) = g\delta(x)\psi(t)$$

Given:  $\psi(x < 0, t) = \frac{e^{-ikx}}{\sqrt{2\pi}}$ , find

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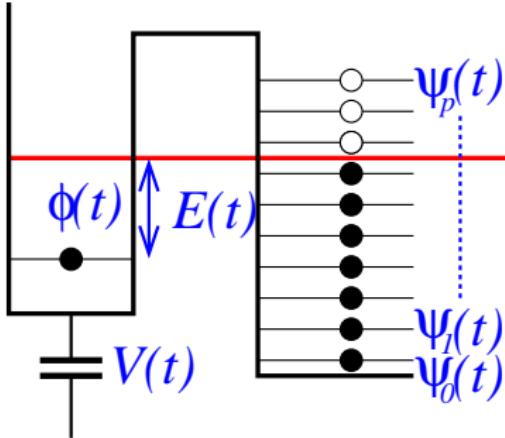
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$$(i\partial_t + iv\partial_x)\psi(x, t) = g\delta(x)\phi(t)$$

Given:  $\psi(x < 0, t) = \frac{e^{i\theta}}{\sqrt{2}}$ , find

$$U(e, d) = \int dt \psi(x > 0, t) \frac{d\phi}{dx}$$

# Single particle problem



Regularly spaced continuum:

$$\psi(x, t) = \sum \psi_p(t) e^{ipx}$$

Thus, continuum equations:

$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x, t) \delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x, t) = g\delta(x)\phi(t)$$

Given:  $\psi(x < 0, t) = \frac{e^{-i\epsilon' t}}{\sqrt{2\pi}}$ , find:

$$U(\epsilon, \epsilon') = \int dt \psi(x > 0, t) \frac{e^{i\epsilon t}}{\sqrt{2\pi}}$$

# Solving Schrodinger equations

Equations:

$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x, t) \delta(x)$$
$$(i\partial_t + iv\partial_x)\psi(x, t) = g\delta(x)\phi(t)$$

Can solve for general  $E(t)$ , writing

$$\psi(x, t) = \phi_0\left(t - \frac{x}{v}\right) e^{-i\int_{t_0}^t \frac{E}{2v} dt} e^{i\int_{t_0}^t \frac{\partial E}{2v} dt}$$

$$\text{so: } [i\partial_t - E(t) + i\frac{E^2}{2v}] \phi(t) = g\phi_0(t)$$

Introduce decay rate  $\Gamma = g^2/v$

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Can solve for general  $E(t)$ , writing:

$$\psi(x, t) = \psi_0 \left( t - \frac{x}{v} \right) - i \frac{g}{v} \phi \left( t - \frac{x}{v} \right) \Theta(x)$$

$$\text{so: } [i\partial_t - E(t) + i\frac{g^2}{2v}] \phi(t) = g\psi_0(t)$$

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Introduce decay rate  $\Gamma = g^2/v$

# Scattering matrix for arbitrary times

Solution for  $\phi(t)$

$$\phi(t) = \phi_0 \int_{-\infty}^t dt' \phi_0(t') \exp\left[-\frac{i}{\hbar}(t-t') + i \int_{t'}^t E(\tau) d\tau\right]$$

Then:  $\phi_0(t') = \frac{1}{\sqrt{2\pi}} e^{-ik't'} \rightarrow \phi(t)$

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Solution for  $\phi(t) = g \int_{-\infty}^t dt' \psi_0(t') \exp \left[ -\frac{\Gamma}{2}(t-t') + i \int_{t'}^t E(\tau) d\tau \right]$

Then:  $\psi_0(t') = \frac{1}{\sqrt{2\pi}} e^{-\Gamma t'/2} \rightarrow \phi(t) = g \psi_0(t)$

$$U(c, c') = \frac{g}{2\pi} \int_{-\infty}^c dt \int_{-\infty}^{c'} dt' \exp \left[ i(c t - c' t') - \frac{\Gamma}{2}(t-t') + i \int_t^{t'} E(\tau) d\tau \right]$$

If  $E(t) = \lambda t$ , elementary form:

$$U(c, c') \propto e^{-(\Gamma/\lambda)(c-c') - (\Gamma/2\lambda)(c^2 - c'^2)}$$

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# Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$

For  $E = \lambda I$ ,

$$\langle \epsilon | U | \epsilon' \rangle \propto e^{-(E/2)(\epsilon - \epsilon') - i(2\pi/\hbar)^2(\epsilon^2 - \epsilon'^2)}$$

$$= U_1(\epsilon) U_2(\epsilon')$$

$$= c|\phi_+\rangle\langle\phi_-|$$

$$P_2 = U_{a \rightarrow b} U_{b \rightarrow a} - U_{a \rightarrow b} U_{b \rightarrow a}$$

$$= \langle a' |\phi_+\rangle\langle\phi_-|a\rangle \langle b' |\phi_+\rangle\langle\phi_-|b\rangle$$

$$= \langle a' |\phi_+\rangle\langle\phi_-|b\rangle \langle b' |\phi_+\rangle\langle\phi_-|a\rangle = 0$$

Max number of particles transferred = rank of  $U$ .

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$$= c(\phi_+)(\phi_-)$$

$$\begin{aligned} P_2 &= U_{+-+}U_{+-+} - U_{+-+}U_{+-+} \\ &= (\phi'_+|\phi_+)(\phi_-|c)(\phi'_+|\phi_+)(\phi_-|b) \\ &\quad - (\phi'_+|\phi_+)(\phi_-|b)(\phi'_+|\phi_+)(\phi_-|c) = 0 \end{aligned}$$

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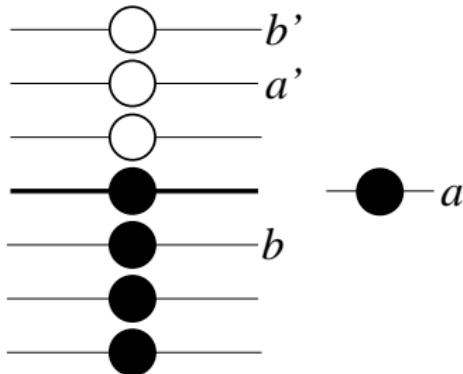
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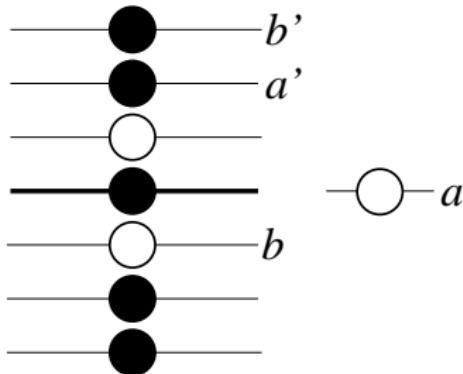
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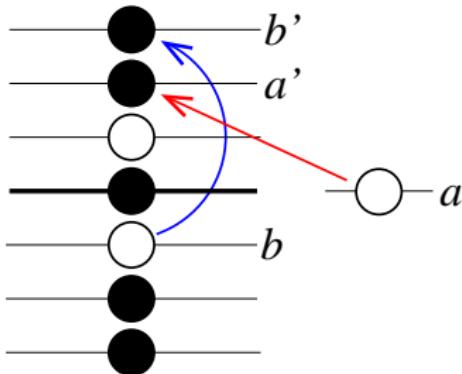
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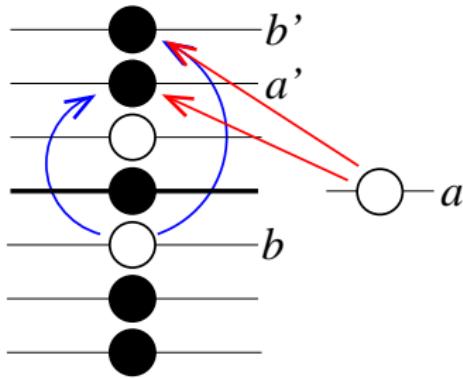
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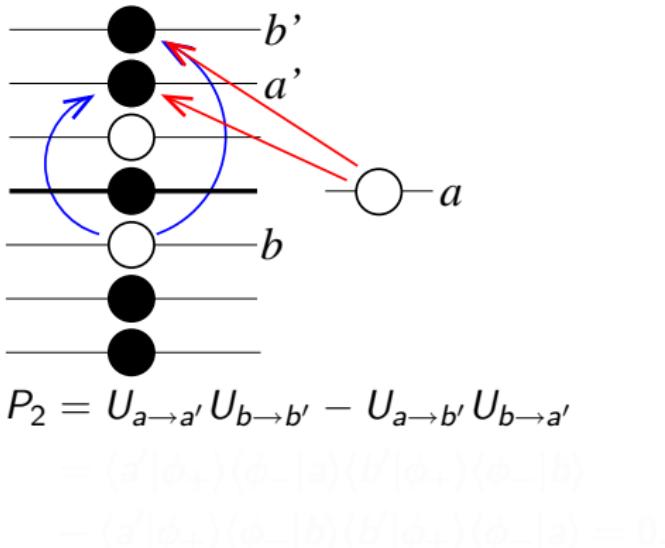
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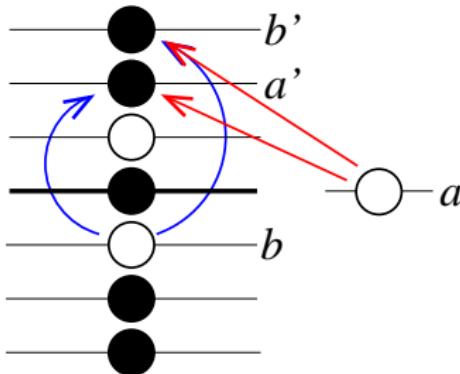
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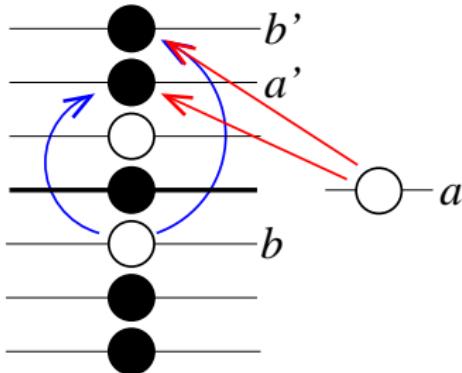
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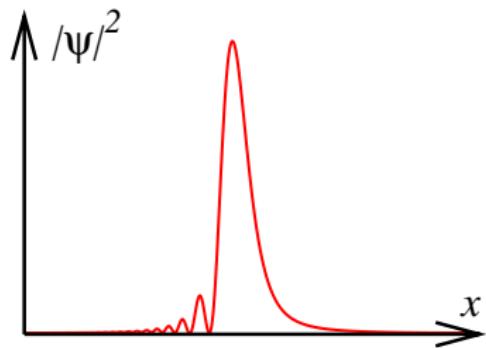
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# Linear timed dependence, exact state

$$\Psi_p \propto e^{-(\Gamma/\lambda)vp - (i/2\lambda)(vp)^2}$$

Spatial profile:



# Conclusions

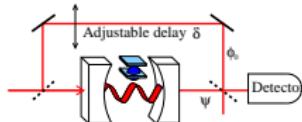
- Exactly solvable many-body generalisations of LZ.

- Photons/Spins

- Localised/continuum fermions

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- Decay: larger than expected, but feasible.
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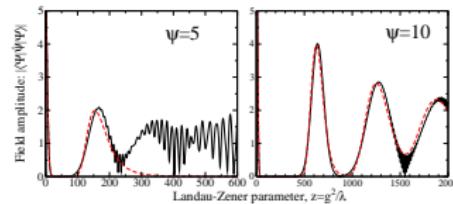
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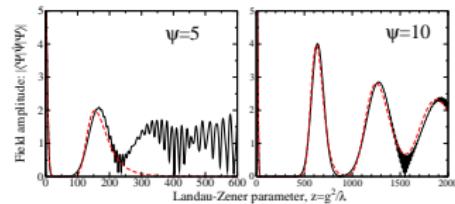
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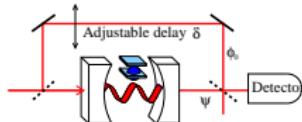
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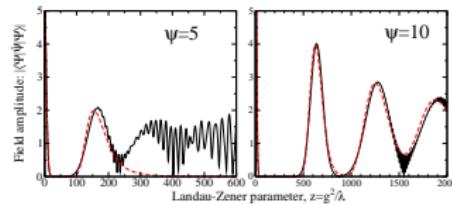
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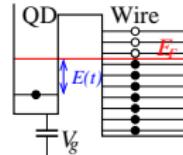
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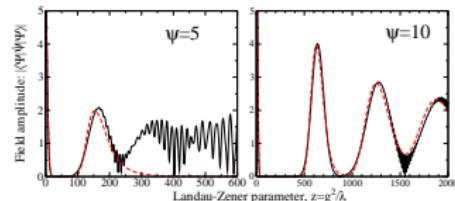
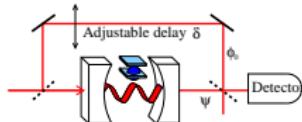


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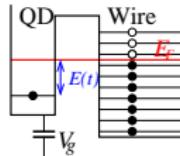
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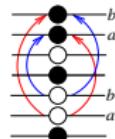


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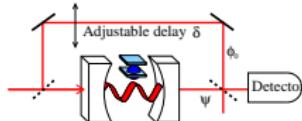
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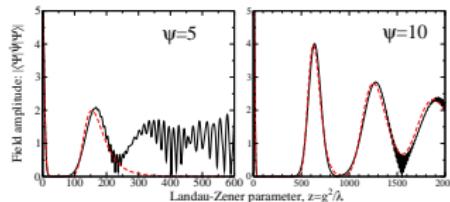
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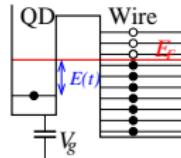
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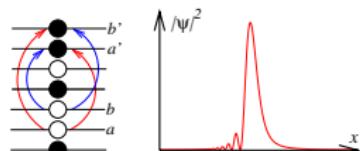
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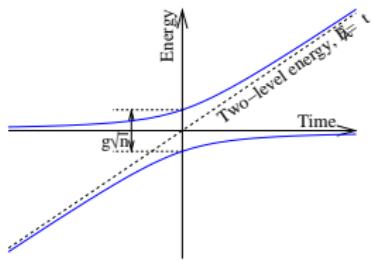
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# Understanding additional decay: adiabatic approx



$$|\psi_{\pm}\rangle = [\cos(\theta_0)|n,1\rangle + \sin(\theta_0)|n-1,0\rangle]$$

Adiabatic fit

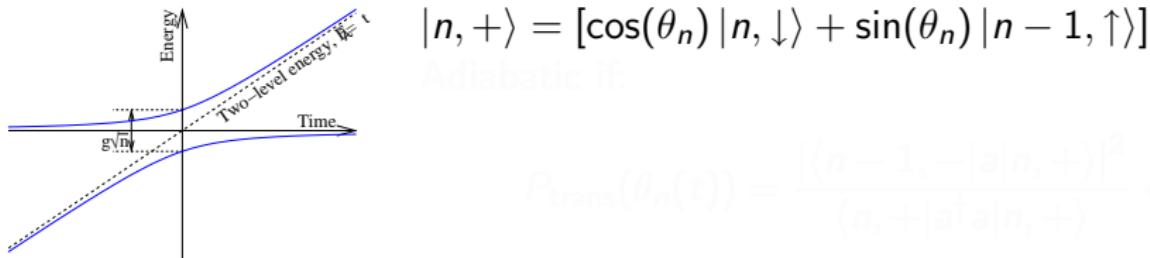
$$P_{trans}(\theta_0(t)) = \frac{(\alpha - 1 - \beta n_{\pm})^2}{(\alpha + \beta)^2 n_{\pm}} < 1$$

At  $t \rightarrow \infty, P_{trans} \rightarrow 0$

At  $t=0$ ,

$$P_{trans} \approx \frac{(\sqrt{n} - \sqrt{n-1})^2}{2(2n+1)} \approx \frac{1}{16n^2} \approx \frac{1}{16|g|^2}$$

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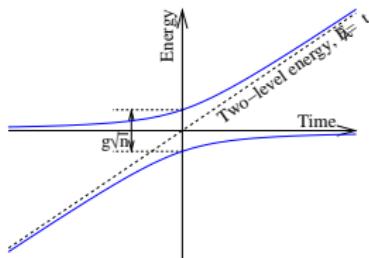


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# Understanding additional decay: adiabatic approx



$$|n, +\rangle = [\cos(\theta_n) |n, \downarrow\rangle + \sin(\theta_n) |n - 1, \uparrow\rangle]$$

Adiabatic if:

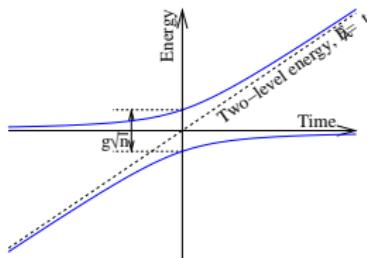
$$P_{\text{trans}}(\theta_n(t)) = \frac{|\langle n - 1, - | a | n, + \rangle|^2}{\langle n, + | a^\dagger a | n, + \rangle} \ll 1$$

At  $t \rightarrow \infty, P_{\text{trans}} \rightarrow 0$

At  $t = 0$

$$P_{\text{trans}} \approx \frac{(\sqrt{n} - \sqrt{n-1})^2}{2(2n+1)} \approx \frac{1}{16n^2} \approx \frac{1}{16m^2}$$

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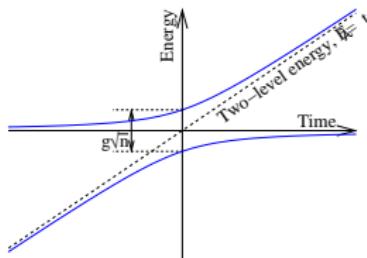
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At  $t \rightarrow \pm\infty$ ,  $P_{\text{trans}} \rightarrow 0$ .

$$P_{\text{trans}} \approx \frac{[\sqrt{n} - \sqrt{n-1}]^2}{2(2n+1)} \approx \frac{1}{16n^2} \approx \frac{1}{16m^2}$$

# Understanding additional decay: adiabatic approx



$$|n, +\rangle = [\cos(\theta_n) |n, \downarrow\rangle + \sin(\theta_n) |n - 1, \uparrow\rangle]$$

Adiabatic if:

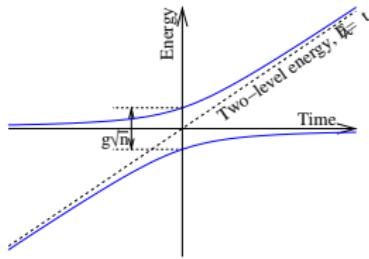
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# Understanding additional decay: Density matrix equation

If adiabatic,  $A_n = \rho_{n,n+1}$ :

$$\frac{dA_n}{dt} = i \left[ \frac{d\Delta\phi_{n-1}}{dt} \right] A_n \rightarrow \left( n - \frac{1}{2} \right) A_n - \sqrt{n(n+1)} A_{n+1}$$

When  $|t| \leq g/\lambda$ , decay rate  $\kappa_n \approx \pi v^2 \gg \kappa$

$$v/g \ll 1/(2\pi v^2) \approx 5 \times 10^{-3}$$

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When  $|t| \leq g/\lambda$ , decay rate  $\kappa n$

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## Additional decay perturbatively

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Gauge transform:  $A_n = \tilde{A}_n e^{-i\Delta\phi_{n-1}}$

$$\frac{d\tilde{A}_n}{dt} = -\kappa \left[ \left( n - \frac{1}{2} \right) \tilde{A}_n - \sqrt{n(n+1)} \tilde{A}_{n+1} e^{i(\Delta\phi_{n-1} - \tilde{\phi}_n)} \right]$$

$$\tilde{A}_n(T) = \tilde{A}_n(-T)$$

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# Results of perturbative decay

Remove “naive decay”,

$$\langle \Psi | \psi(\kappa, \psi_0) | \Psi \rangle_{\text{naive}} = \left\langle \Psi | \psi(0, \psi_0 e^{-\kappa T/2}) | \Psi \right\rangle e^{-\kappa T/2}$$

$$\begin{aligned} \delta \langle \Psi | \psi | \Psi \rangle &= \psi(\kappa |\psi|^2) \sum_n P_n \times \\ &\left[ T e^{i[\Delta\phi_{n-1}(-T) - \Delta\phi_{n-1}(T)]} + T e^{i[\Delta\phi_n(-T) - \Delta\phi_n(T)]} \right. \\ &\quad \left. - e^{i[\Delta\phi_n(-T) - \Delta\phi_{n-1}(T)]} \int_{-T}^T e^{i[\Delta\phi_{n-1} - \Delta\phi_n] dt} \right] \end{aligned}$$

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# Results of perturbative decay

Explicit form in large  $n$  approx:

$$\delta \langle \Psi | \psi | \Psi \rangle = -\psi(\kappa|\psi|^2) \sum_n P_n e^{-iz \ln(T^2/zn) + iz/2n} \sqrt{\frac{zn}{\lambda}} \frac{\pi z}{2n} J_1\left(\frac{z}{2n}\right)$$

$$At \quad g^2/\lambda = z = 2\pi n/kT^2$$

$$\langle \delta \langle \Psi | \psi | \Psi \rangle \rangle \propto \psi \left[ \frac{E}{\hbar} (2\pi n)^{3/2} / kT^2 \right]$$

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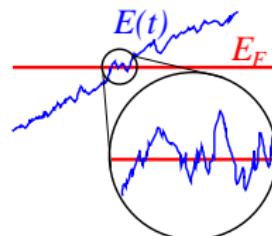
At  $g^2/\lambda = z = 2\pi N |\psi|^2$

$$|\delta \langle \Psi | \psi | \Psi \rangle| \propto \psi \left[ \frac{\kappa}{g} (2\pi N)^{3/2} |\psi|^5 \right]$$

# Noisy driving

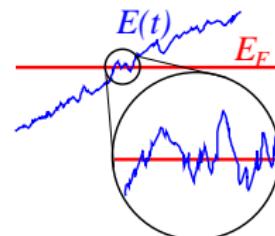
- Suppose  $E(t) = ct + \eta(t)$
- $\langle \eta(t) \rangle = 0, \langle \eta(t)\eta(t') \rangle = T_2\delta(t-t')$ .

To find  $N^\infty$ , need:  $\langle |U(z,z')|^2 \rangle$  thus:



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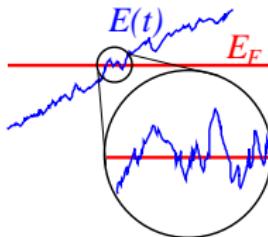


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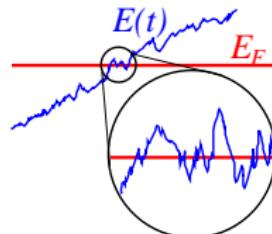
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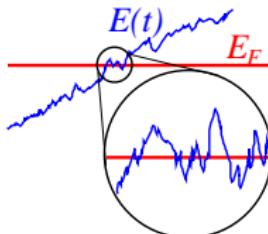
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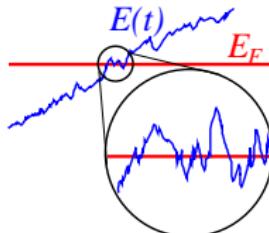
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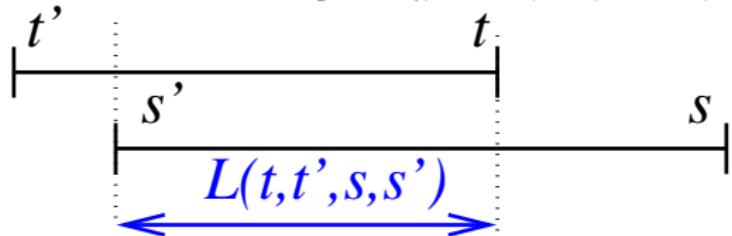
- Find  $F_{\text{noise}} = \exp[-\Gamma_2(|t - t'| + |s - s'| - L(t, t', s, s'))]$

- Can simplify to  $\Delta = t - s$  and  $\lambda = t' - t = s' - s'$

$$N^{\infty} = \frac{-\Gamma^2}{2\pi c} \int_{-\infty}^{\infty} \frac{d\Delta}{(\Delta - i0)^2} \int_0^{\infty} d\lambda e^{(i\omega\Delta - \Gamma - \Gamma_2)\lambda + \Gamma_2(\Delta)\lambda}$$

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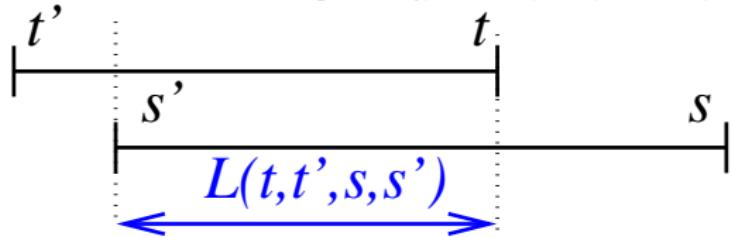


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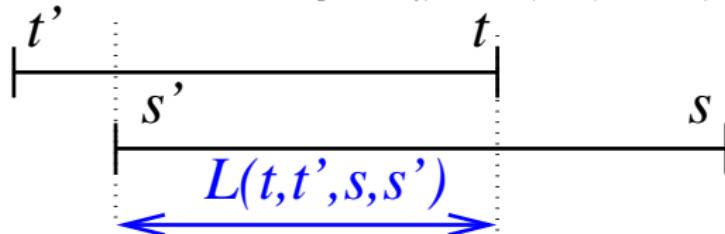


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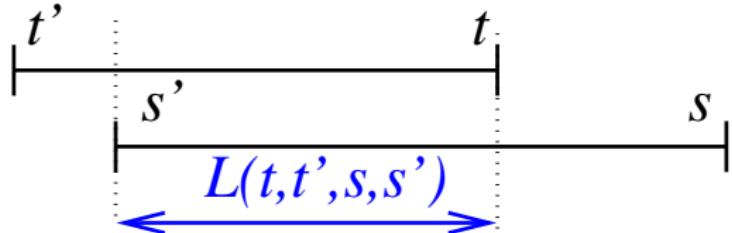


- Can simplify to  $\Delta = t - s$  and  $\Lambda = t - t' = s - s'$

$$N^2 = \frac{-\Gamma^2}{2\pi c} \int_{-\infty}^{\infty} d\Delta \int_{-\infty}^{\infty} d\Lambda e^{i(\Delta - \Gamma - \Gamma_2)\Lambda + \Gamma_2(\Delta)\Lambda}$$

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# Noisy driving: results

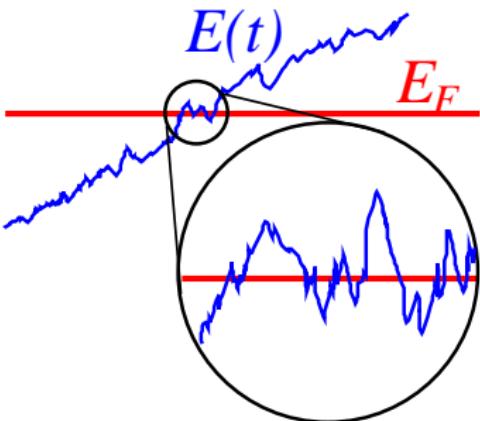
- Integral log divergent: white noise limit
  - infinite no. crossings of Fermi surface
  - Can extract logarithmic contribution

$$N^{\text{ex}} = \begin{cases} 1 & c \gg \Gamma\Gamma_2 \\ \frac{\Gamma^2}{(\Gamma+\Gamma_2)^2} + \frac{2\Gamma\Gamma_2}{\pi c} \ln \frac{\omega_*}{\Gamma+\Gamma_2} & c \ll \Gamma\Gamma_2 \end{cases}$$

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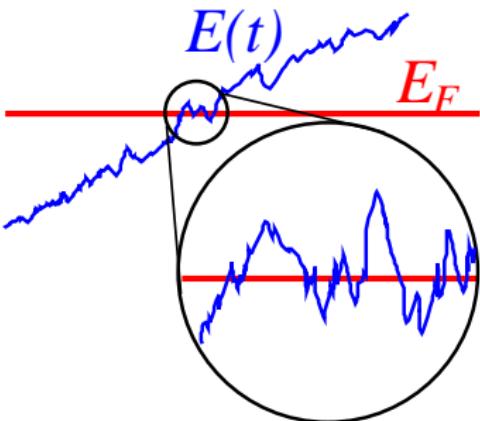
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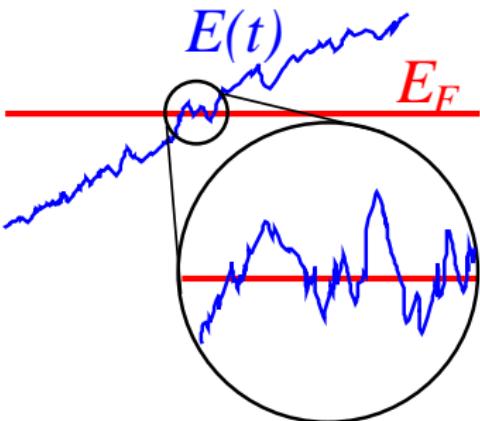
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