

Landau Zener processes in many body systems

Two-level system coupled to a photon field

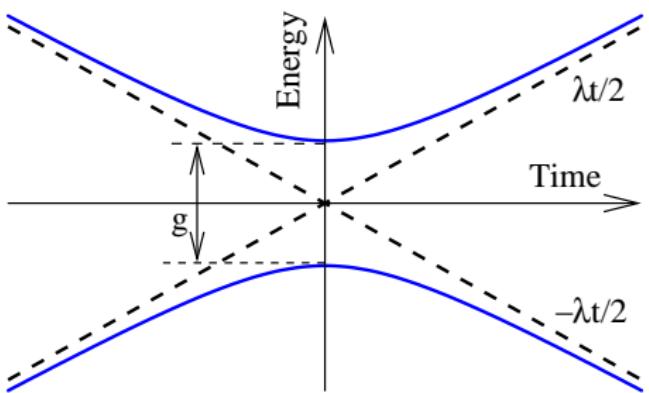
Jonathan Keeling¹ and V. Gurarie²

¹University of Cambridge

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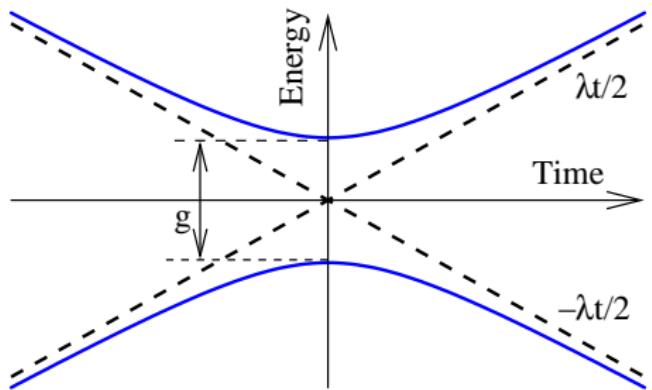
November 28, 2007

The Landau-Zener problem



$$i\partial_t \psi = \begin{pmatrix} \lambda t/2 & g \\ g & -\lambda t/2 \end{pmatrix} \psi$$

The Landau-Zener problem



Initially, $\psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

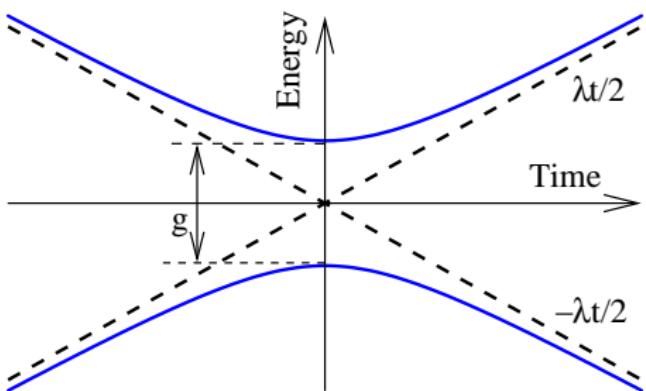
$$i\partial_t \psi = \begin{pmatrix} \lambda t/2 & g \\ g & -\lambda t/2 \end{pmatrix} \psi$$

Finally $\psi = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-g^2/\lambda}$

$$t = i\ln(\psi_1) / g = \sqrt{\lambda/2}$$

$$|\psi\rangle = \sqrt{1 - e^{-2g^2/\lambda}}$$

The Landau-Zener problem

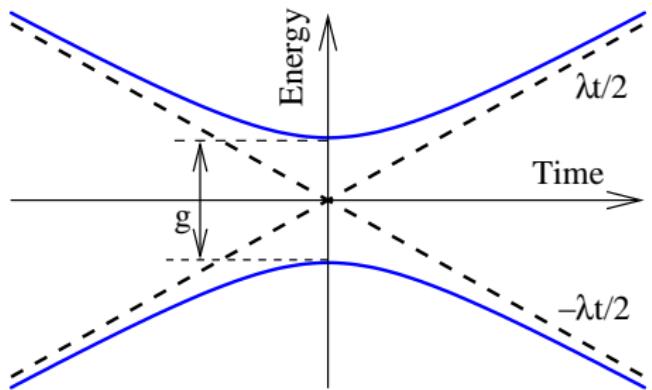


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Finally, $\psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$,

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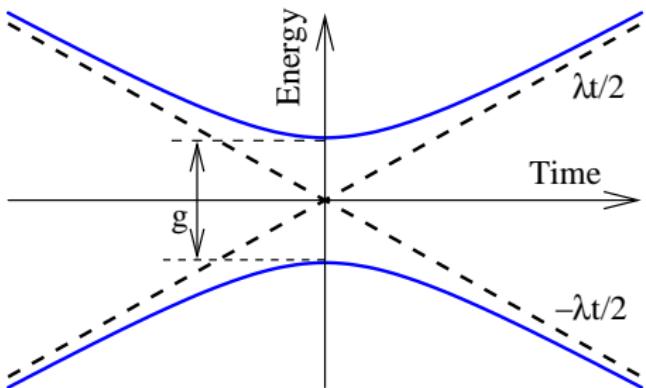


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The Landau-Zener problem



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Finally, $\psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \quad z = g^2/\lambda$

$$t = +\infty : \quad |\alpha| = e^{-\pi z}$$

$$|\beta| = \sqrt{1 - e^{-\pi z}}$$

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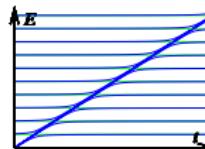
Many-body Landau-Zener generalisation

- One body, many-level generalisations:
 - Demkov-Osherov problem
 - Here, instead many-body
 - Present two simple but striking examples.

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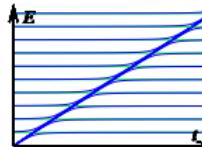
- Here, instead **many-body**

- ▶ Time varying energy of single-particle states
 - ▶ Automatic elevation to many-level problem
 - ▶ Interference
- ▶ Present two simple but striking examples.

Many-body Landau-Zener generalisation

- One body, many-level generalisations:

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- Here, instead **many-body**

- ▶ Time varying energy of single-particle states

→ Interference between different paths in momentum space

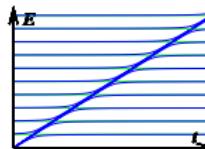
→ Interference

- ▶ Present two simple but striking examples.

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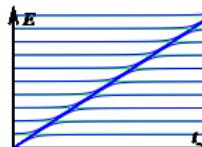
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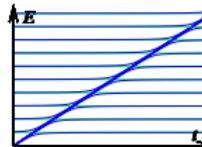
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Overview

1 Introduction and Landau-Zener

- Introduction

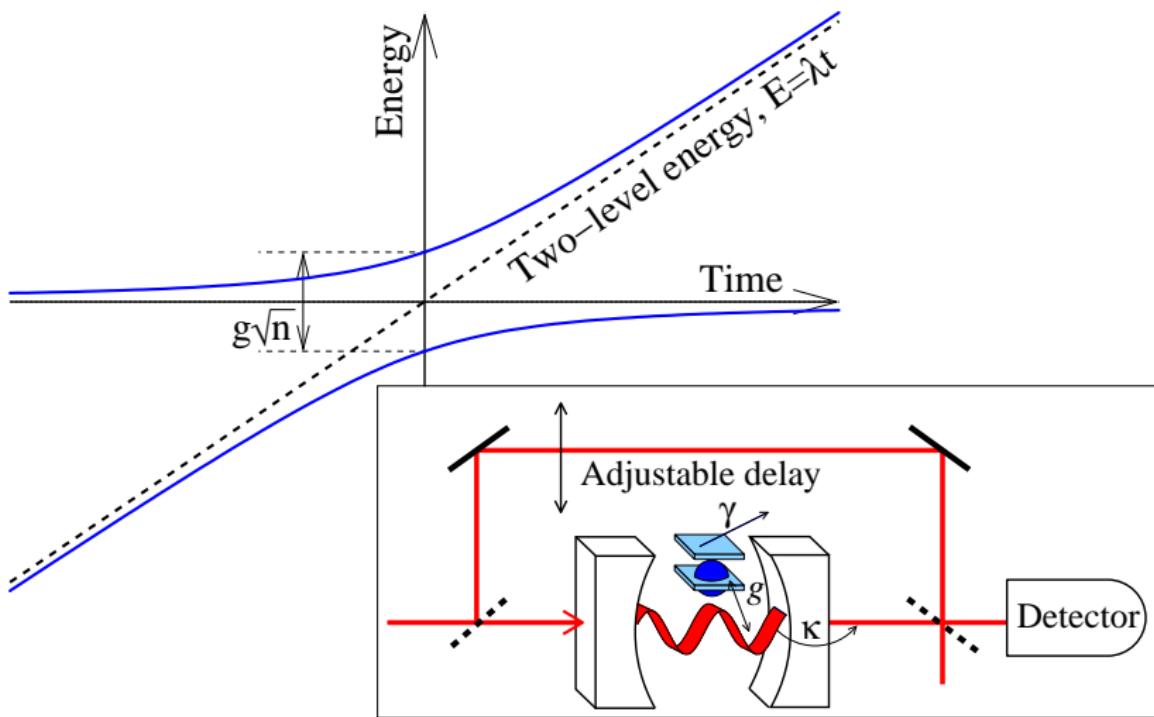
2 Two-level system/photon field

- Setting up the many body problem
- Collapse and revivals
- Effects of decay

3 Localised fermion coupled to continuum

- Relating one- and many-body problems
- Cancellation and “clean electrons”
- Arbitrary time dependence and noise

Physical system



Description

- Hamiltonian: $\hat{H} = \omega_0 \hat{\psi}^\dagger \hat{\psi} + \frac{\lambda t}{2} \hat{\sigma}_z + g (\hat{\psi}^\dagger \hat{\sigma}^- + \hat{\psi} \hat{\sigma}^+)$,

Initial coherent state $|n, \downarrow\rangle = |n, 0\rangle - \Delta n |n+1, 1\rangle$

- Each pair $|n, 1\rangle \rightarrow |n+1, 0\rangle$ undergoes LZ transition

$$H_{n,n+1} = \begin{pmatrix} \lambda t/2 & g\sqrt{n+1} \\ g\sqrt{n+1} & -\lambda t/2 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix}$$

- Final state:

$$|\Psi(t=\infty)\rangle = e^{-i\omega_0^2/2} \sum_{n=0}^{\infty} \frac{e^{i\omega_0 n}}{\sqrt{n!}} [A_{n+1}|n, 1\rangle + B_{n+1}|n+1, 0\rangle]$$

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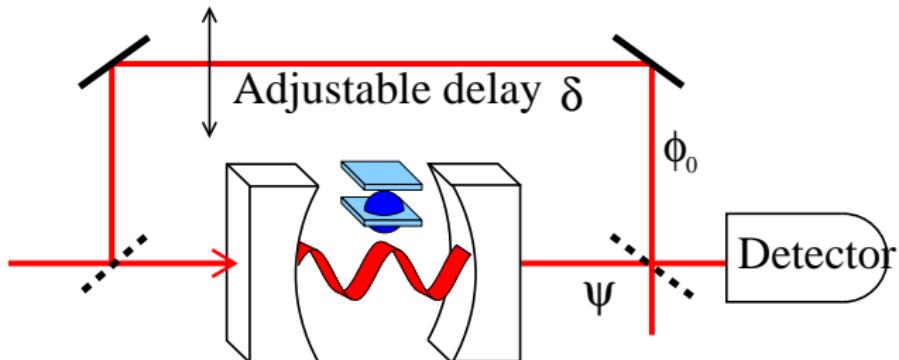
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What to measure?

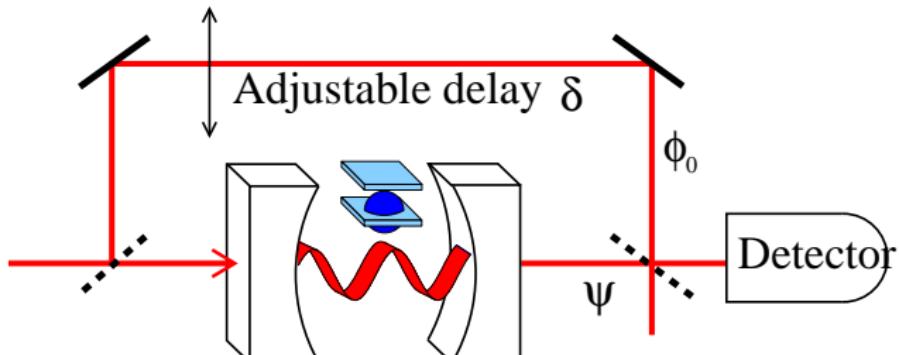


After mixing: $\hat{\phi} \rightarrow \phi_0 e^{i\delta}$ intensity

$$\begin{aligned} I &\propto \langle (\hat{\phi}) + \hat{\phi}_0 e^{-i\delta} | (\hat{\phi} + \hat{\phi}_0 e^{i\delta}) \rangle \\ &= \langle \hat{\phi}^{\dagger} \hat{\phi} \rangle + \phi_0^2 + \phi_0 (\langle \hat{\phi}^{\dagger} \rangle e^{i\delta} + \langle \hat{\phi} \rangle e^{-i\delta}) \end{aligned}$$

Measure $\langle \hat{\phi}^{\dagger} \hat{\phi} \rangle$

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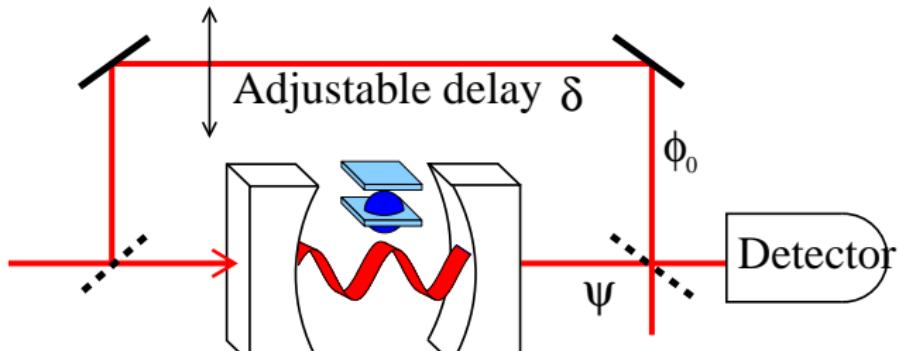


After mixing: $\hat{\psi} + \hat{\phi}_0 e^{i\delta}$ intensity

$$\begin{aligned} I &\propto \langle (\hat{\psi})^2 + \hat{\phi}_0^2 e^{-2i\delta} \rangle (\hat{\psi} + \hat{\phi}_0 e^{i\delta}) \\ &= \langle \hat{\psi}^2 \rangle + \phi_0^2 + \phi_0 \left(\langle \hat{\psi} \rangle e^{i\delta} + \langle \hat{\psi} \rangle e^{-i\delta} \right) \end{aligned}$$

Measure $\langle \hat{\psi}^2 \rangle$

What to measure?



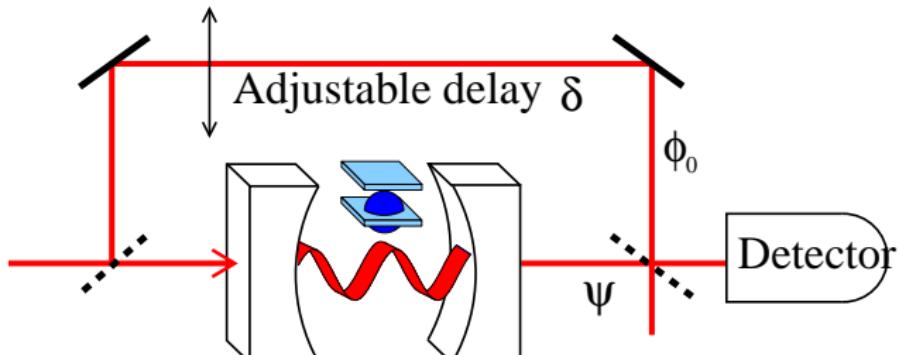
After mixing: $\hat{\psi} + \hat{\phi}_0 e^{i\delta}$ Intensity:

$$I \propto \langle (\hat{\psi}^\dagger + \hat{\phi}_0^\dagger e^{-i\delta})(\hat{\psi} + \hat{\phi}_0 e^{i\delta}) \rangle$$

$$= \langle \hat{\psi}^\dagger \hat{\psi} \rangle + \phi_0 + \phi_0 \langle (\hat{\psi}^\dagger)^2 e^{i\delta} + \langle \hat{\psi} \rangle e^{-i\delta} \rangle$$

Measure $\langle \hat{\psi}^\dagger \hat{\psi} \rangle$

What to measure?

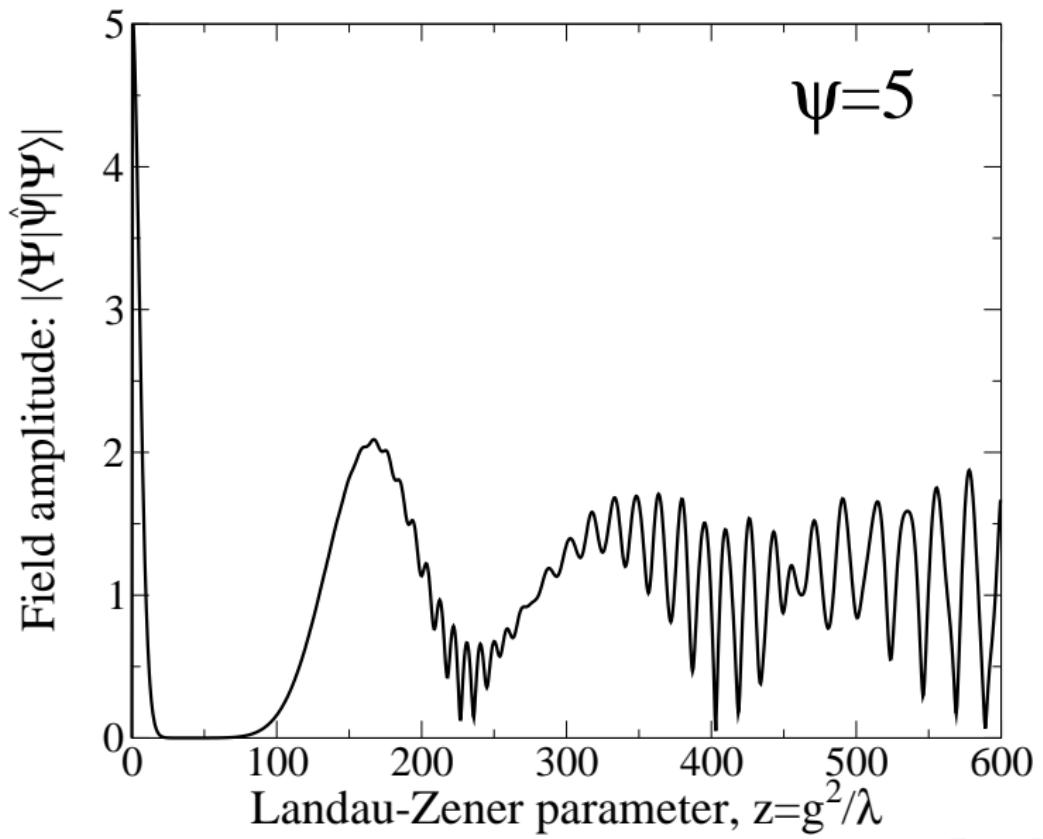


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Measure $\langle \Psi | \hat{\psi} | \Psi \rangle$

Exact numerical result



Explaining results

Adiabatic limit: $\lambda \rightarrow \infty$, $\omega \ll \lambda$

$$|\Psi(T)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1} |n, \uparrow\rangle + B_{n+1} |n+1, \downarrow\rangle \right]$$

$$\phi_n(z, T) = \phi_0 + nz \ln \left(\frac{e^{\lambda z}}{e^{\lambda T^2}} \right)$$

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Adiabatic limit: $z = g^2/\lambda \gg 1$

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Explaining results: field amplitude

$$\langle \Psi | \psi | \Psi \rangle = \psi e^{-|\psi|^2} \sum_n \frac{|\psi|^{2n}}{n!} \sqrt{\frac{n+2}{n+1}} e^{i(\phi_{n+2} - \phi_{n+1})}$$

$$\Delta\phi_n = z \left[(n+1) \ln(n+1) - n \ln n \right]$$

$$\Delta\phi_{10^2+m} \approx \Delta\phi_{10^2} + \frac{zm}{10^2} - \frac{zm^2}{2 \cdot 10^4}$$

Revival at $z = 2\pi N |\psi|^2$, fails at $z \gtrsim |\psi|^2$

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$$\Delta\phi_n = \pi \left[(n+1) \ln(n+1) - n \ln n \right]$$

$$\Delta\phi_{np+m} \approx \Delta\phi_{np} + \frac{\pi m}{4\pi^2} - \frac{\pi m^2}{2\pi^2}$$

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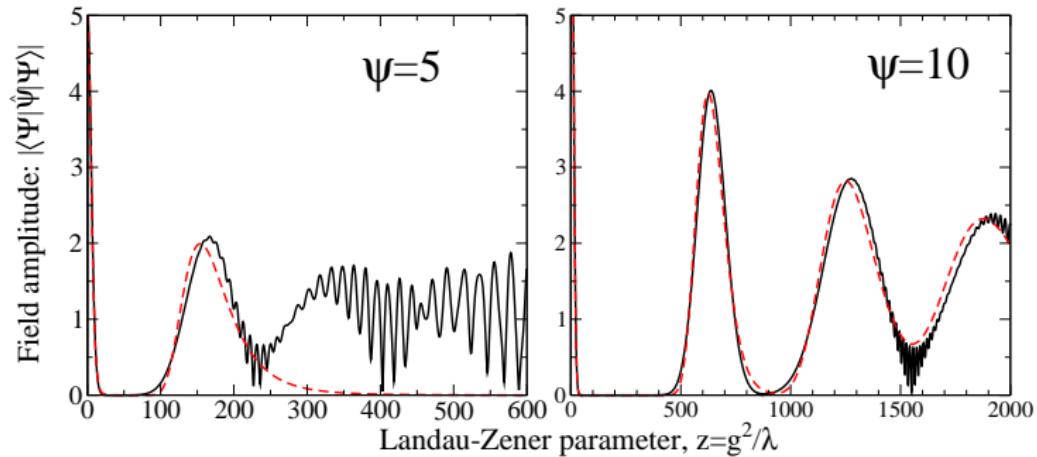
Revival at $z = 2\pi N |\psi|^2$, fails at $z \gtrsim |\psi|^3$

Explaining results: approximation

$$\langle \Psi | \psi | \Psi \rangle = \frac{|\psi|}{(1 + z^2/|\psi|^4)^{1/4}} \sum_{N=0}^{N_{\max}} \exp \left[-\frac{(z - 2\pi N |\psi|^2)^2}{2|\psi|^2(1 + z^2/|\psi|^4)} \right]$$

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Revivals are **not** coherent states

Wigner function $W(x, p) = \frac{1}{\pi} \int dy \Psi^*(x + y)\Psi(x - y)e^{2ipy}$.

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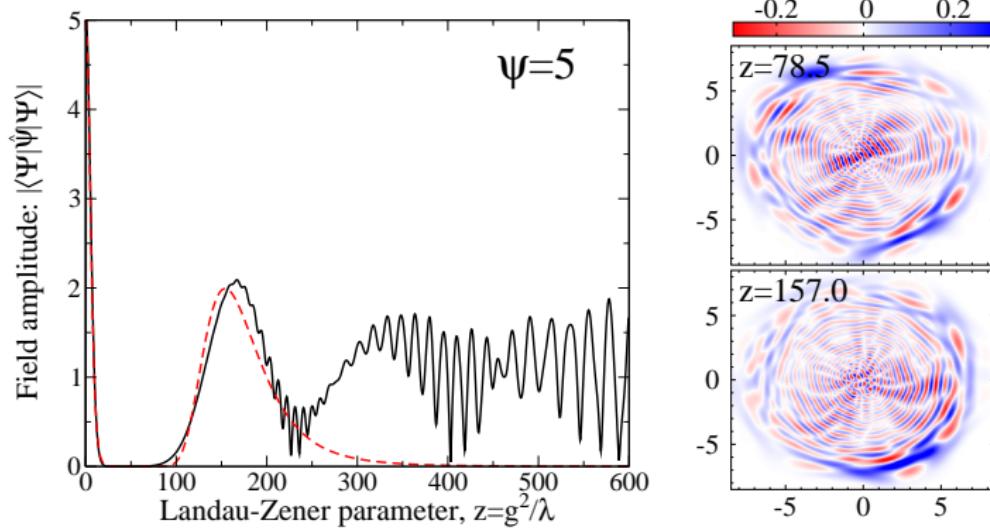
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For a coherent state, $W(x, p) = \frac{1}{\pi} \exp \left[- (x - \sqrt{2}\psi)^2 - p^2 \right]$

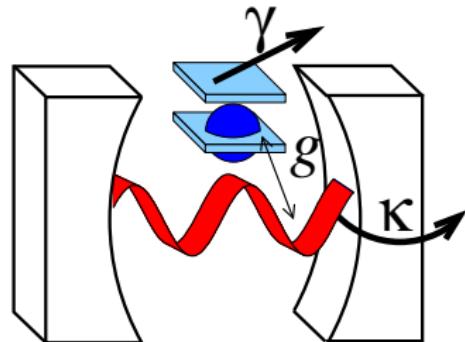
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Possible decay channels



$$\partial_t \hat{\rho} = -i [\hat{H}, \hat{\rho}] + L_\kappa[\hat{\rho}] + L_\gamma[\hat{\rho}]$$

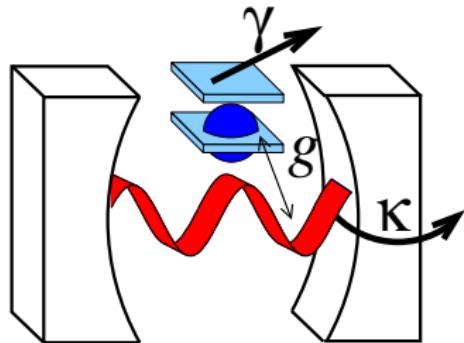
$$L_\kappa[\rho] = -\frac{i}{2} (\partial_x \partial_x \rho - \partial_y \partial_y \rho - \partial_z \partial_z \rho)$$

$$L_\gamma[\rho] = -\frac{1}{2} (\partial_x \partial_x \rho + \partial_y \partial_y \rho - 2 \partial_x \partial_y \rho)$$

Wave estimate: $(\psi^*(t')\psi^*(t)) < 1$, with $t' = g|t|/2$

$$\left(\frac{\psi^*(t')}{\psi^*(t)}\right) \leq \frac{1}{g^2(t')} \leq \frac{1}{2\pi|t'|^2} \approx 10^{-2}$$

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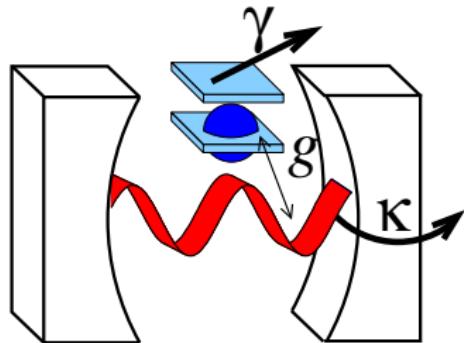
$$L_\kappa[\hat{\rho}] = -\frac{\kappa}{2} (\hat{\psi}^\dagger \hat{\psi} \hat{\rho} + \hat{\rho} \hat{\psi}^\dagger \hat{\psi} - 2 \hat{\psi} \hat{\rho} \hat{\psi}^\dagger)$$

$$L_\gamma[\hat{\rho}] = -\frac{i}{2} (\hat{\rho} \hat{\delta}_+ \hat{\delta}_- + \hat{\delta}_+ \hat{\delta}_- \hat{\rho} - 2 \hat{\delta}_+ \hat{\rho} \hat{\delta}_-)$$

Wave estimate: $(\delta \hat{\rho})/\delta \hat{\rho}^*$ < 1, with $\delta \hat{\rho}^* = g|\delta|/2$

$$\left(\frac{\delta \hat{\rho}}{\delta \hat{\rho}^*}\right) \leq \frac{\lambda}{\delta^2 |\delta|} \approx \frac{1}{2\pi |\delta|^2} \approx 10^{-3}$$

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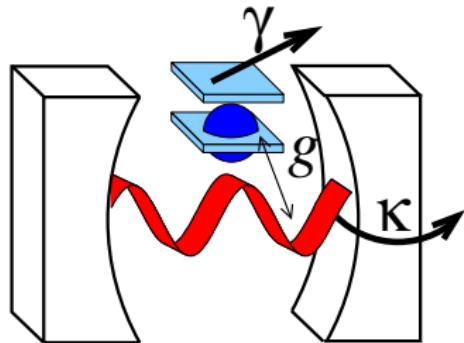
$$L_\kappa[\hat{\rho}] = -\frac{\kappa}{2} (\hat{\psi}^\dagger \hat{\psi} \hat{\rho} + \hat{\rho} \hat{\psi}^\dagger \hat{\psi} - 2 \hat{\psi} \hat{\rho} \hat{\psi}^\dagger)$$

$$L_\gamma[\hat{\rho}] = -\frac{\gamma}{2} (\hat{\sigma}_+ \hat{\sigma}_- \hat{\rho} + \hat{\rho} \hat{\sigma}_+ \hat{\sigma}_- - 2 \hat{\sigma}_- \hat{\rho} \hat{\sigma}_+)$$

Wave estimate: $(\delta t)^2 \ll 1$, with $\delta t = g|\delta|/2$

$$\left(\frac{\delta t}{g|\delta|}\right) \leq \frac{\lambda}{g^2 |\delta|} \approx \frac{1}{2\pi |\delta|^2} \approx 10^{-2}$$

Possible decay channels



$$\partial_t \hat{\rho} = -i [\hat{H}, \hat{\rho}] + L_\kappa[\hat{\rho}] + L_\gamma[\hat{\rho}]$$

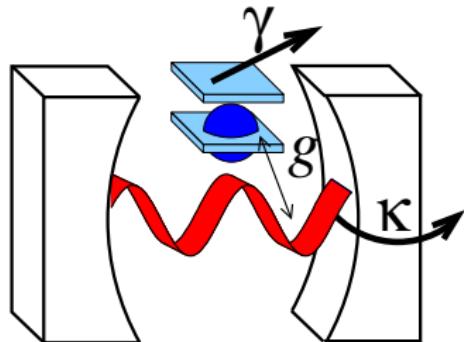
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Naive estimate: $(\gamma t^*, \kappa t^*) \ll 1$, with $t^* \simeq g|\psi|/\lambda$

$$\left(\frac{\gamma}{g}\right) \leq \frac{1}{g^2 |\psi|} = \frac{1}{2\pi |\psi|^2} \simeq 10^{-3}$$

Possible decay channels



$$\partial_t \hat{\rho} = -i [\hat{H}, \hat{\rho}] + L_\kappa[\hat{\rho}] + L_\gamma[\hat{\rho}]$$

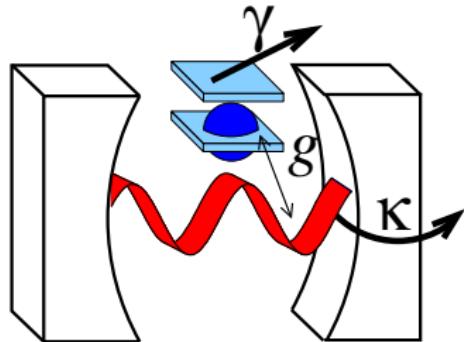
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$$\left(\frac{\gamma}{g}, \frac{\kappa}{g} \right) \ll \frac{\lambda}{g^2} \frac{1}{|\psi|}$$

Possible decay channels



$$\partial_t \hat{\rho} = -i [\hat{H}, \hat{\rho}] + L_\kappa[\hat{\rho}] + L_\gamma[\hat{\rho}]$$

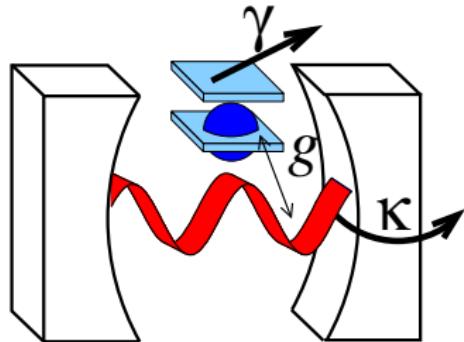
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Possible decay channels



$$\partial_t \hat{\rho} = -i [\hat{H}, \hat{\rho}] + L_\kappa[\hat{\rho}] + L_\gamma[\hat{\rho}]$$

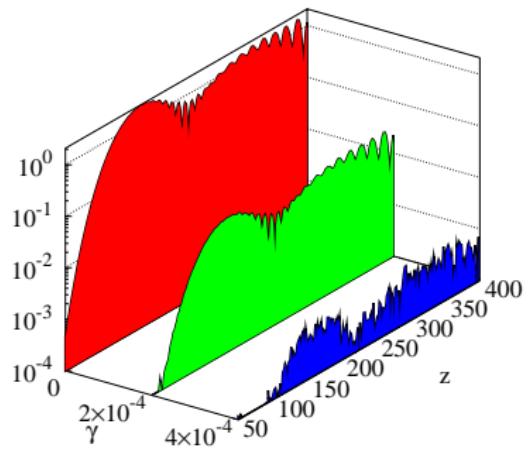
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Naive estimate: $(\gamma t^*, \kappa t^*) \ll 1$, with $t^* \simeq g|\psi|/\lambda$

$$\left(\frac{\gamma}{g}, \frac{\kappa}{g} \right) \ll \frac{\lambda}{g^2} \frac{1}{|\psi|} \simeq \frac{1}{2\pi |\psi|^3} \simeq 10^{-3}$$

Results including decay



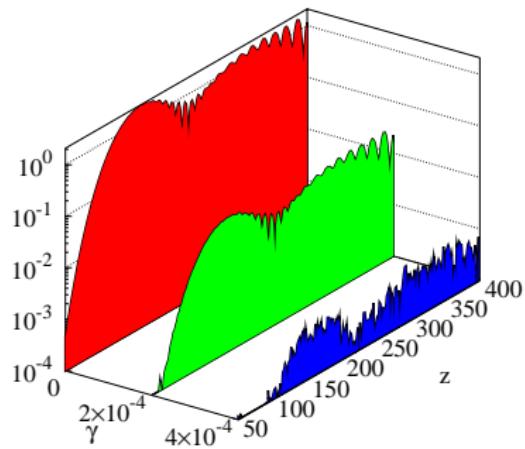
$$\gamma_{\max}/g \simeq 2 \times 10^{-4}$$

$$T_{\text{decay}}/E \simeq 10 \times 10^{-4}$$

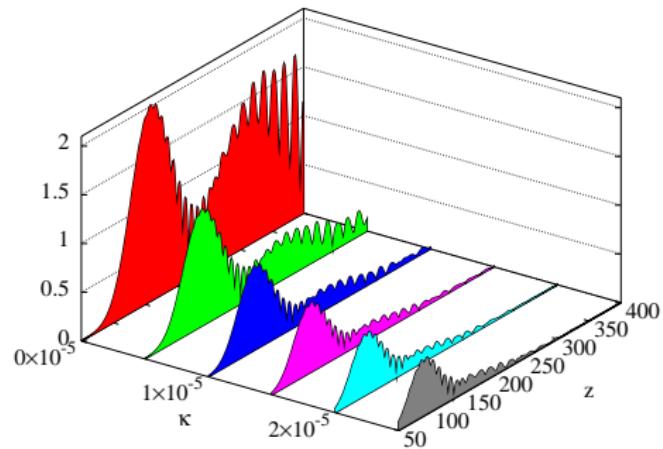
$$R_{\text{decay}}/E \simeq 1 \times 10^{-5}$$

$$R_{\text{decay}}/E < 100 \times 10^{-5}$$

Results including decay

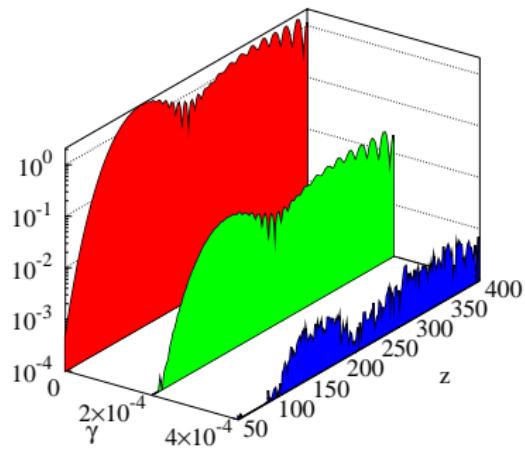


$$\gamma_{\max}/g \simeq 2 \times 10^{-4}$$



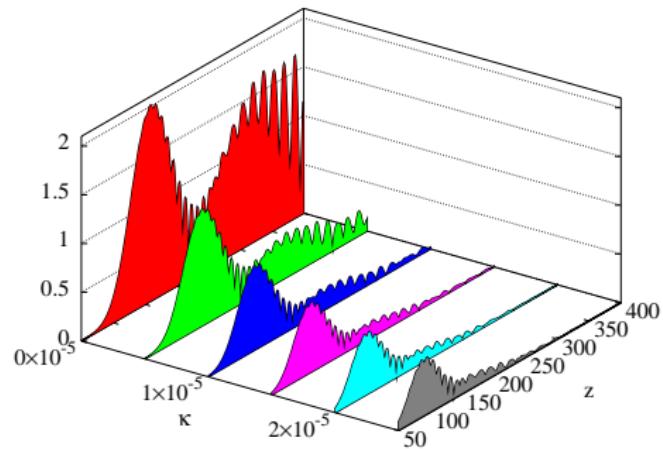
$$\kappa_{\max}/g \simeq 1 \times 10^{-5}$$

Results including decay



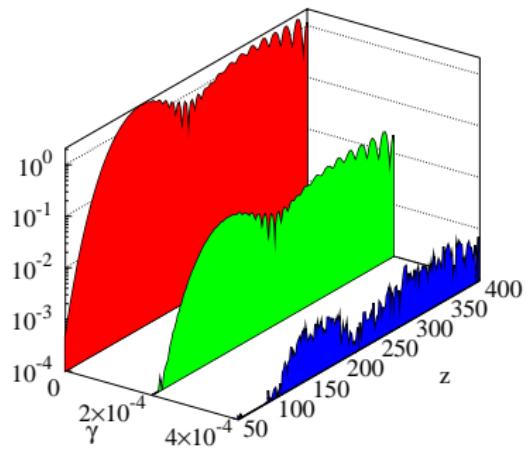
$$\gamma_{\max}/g \simeq 2 \times 10^{-4}$$

$$\gamma_{\text{theory}}/g \ll 10 \times 10^{-4}$$



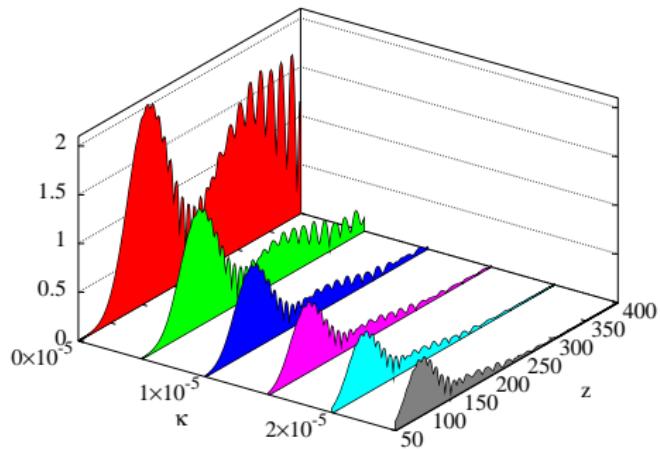
$$\kappa_{\max}/g \simeq 1 \times 10^{-5}$$

Results including decay



$$\gamma_{\text{max}}/g \simeq 2 \times 10^{-4}$$

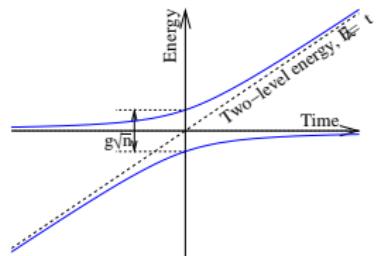
$$\gamma_{\text{theory}}/g \ll 10 \times 10^{-4}$$



$$\kappa_{\text{max}}/g \simeq 1 \times 10^{-5}$$

$$\kappa_{\text{theory}}/g \ll 100 \times 10^{-5}$$

Understanding additional decay: adiabatic approx



$$|n, +\rangle = \cos \theta_n |n, 1\rangle + \sin \theta_n |n+1, 1\rangle \quad \text{Adiabatic if } \kappa t \ll \hbar$$

$$\rho_{\text{trans}} = \frac{(n-1)(n+2)}{(n+1)(n+2)} e^{-\frac{\kappa^2 t^2}{2\hbar^2}} \leq \frac{27}{256P} < 1$$

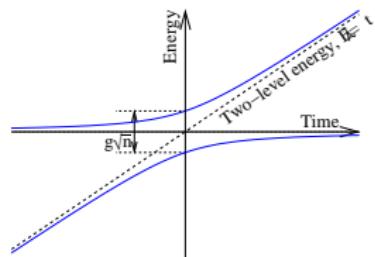
If adiabatic, $\Lambda_n = \rho_{n,n+1}$

$$\frac{d\Lambda_n}{dt} \rightarrow \left[\frac{d\Delta \phi_{n+1}}{dt} \right] \Lambda_n \rightarrow \kappa \left[\left(n - \frac{1}{2} \right) \Lambda_n - \sqrt{n(n+1)} \Lambda_{n+1} \right]$$

When $|\dot{\phi}| \leq g\sqrt{n}/\Lambda$, decay rate $\kappa \theta \approx \kappa |\dot{\phi}|^2 \gg \kappa$

$$\kappa/g \ll 1/(2\pi^2 c^2) \approx 5 \times 10^{-5}$$

Understanding additional decay: adiabatic approx



$$|n, +\rangle = \cos \theta_n |n, \downarrow\rangle + \sin \theta_n |n-1, \uparrow\rangle \quad \text{Adiabatic if } \kappa \ll g\sqrt{n}/\hbar$$

$$P_{\text{trans}} = \frac{(n-1)(n+1)}{(n+1)^2(n-1)} \leq \frac{27}{2500} < 1\%$$

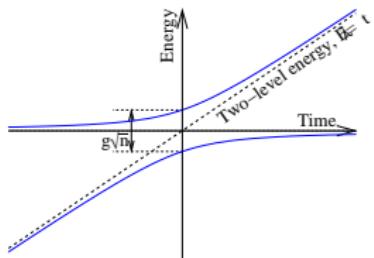
If adiabatic, $\Delta_n = p_{n,n+1}$

$$\frac{d\Delta_n}{dt} \rightarrow \left[\frac{d\Delta_{n+1}}{dt} \right] \Delta_n - \kappa \left[\left(n - \frac{1}{2} \right) \Delta_n - \sqrt{n(n+1)} \Delta_{n+1} \right]$$

When $|\dot{\epsilon}| \leq g\sqrt{n}/\lambda$, decay rate $\kappa n \approx n |\dot{\epsilon}|^2 \gg n$

$$\kappa/g \ll 1/(2\pi^2 c^3) \approx 5 \times 10^{-5}$$

Understanding additional decay: adiabatic approx



$$|n, +\rangle = \cos \theta_n |n, \downarrow\rangle + \sin \theta_n |n-1, \uparrow\rangle \quad \text{Adiabatic if:}$$

$$P_{\text{trans}} = \frac{|\langle n-1, -|\hat{\psi}|n, +\rangle|^2}{\langle n, +|\hat{\psi}^\dagger \hat{\psi}|n, +\rangle} \leq \frac{27}{256n^2} \ll 1$$

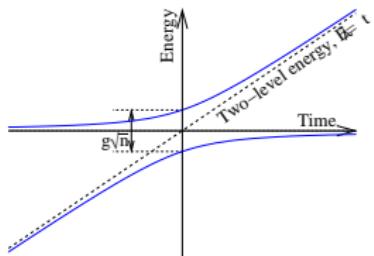
If adiabatic, $\Delta_n = \mu_{n,n+1}$

$$\frac{d\Delta_n}{dt} \rightarrow \left[\frac{d\Delta_{n-1}}{dt} \right] \Delta_n - \kappa \left[\left(n - \frac{1}{2} \right) \Delta_n - \sqrt{n(n+1)} \Delta_{n-1} \right]$$

When $|\dot{t}| \leq g\sqrt{n}/\lambda$, decay rate $\kappa \dot{t} \approx \kappa |\dot{t}|^2 \gg \kappa$

$$\kappa/g < 1/(2\pi^2 c^3) \approx 5 \times 10^{-5}$$

Understanding additional decay: adiabatic approx



$$|n, +\rangle = \cos \theta_n |n, \downarrow\rangle + \sin \theta_n |n-1, \uparrow\rangle \quad \text{Adiabatic if:}$$

$$P_{\text{trans}} = \frac{|\langle n-1, -|\hat{\psi}|n, +\rangle|^2}{\langle n, +|\hat{\psi}^\dagger \hat{\psi}|n, +\rangle} \leq \frac{27}{256n^2} \ll 1$$

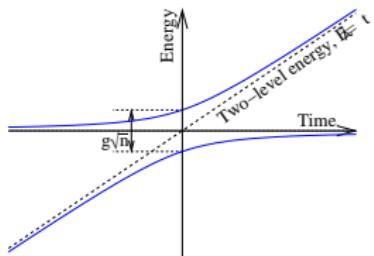
If adiabatic, $\Lambda_n = \rho_{n,n+1}$:

$$\frac{d\Lambda_n}{dt} = - \left[\frac{d\Lambda_{n+1}}{dt} \right] \Lambda_n - \kappa \left[\left(n - \frac{1}{2} \right) \Lambda_n - \sqrt{n(n+1)} \Lambda_{n+1} \right]$$

When $|\dot{\tau}| \leq g\sqrt{n}/\Lambda$, decay rate $\kappa\tau \approx \kappa|\dot{\tau}|^2 \gg \kappa$

$$\kappa/g < 1/(2\pi^2 c^3) \approx 5 \times 10^{-5}$$

Understanding additional decay: adiabatic approx



$$|n, +\rangle = \cos \theta_n |n, \downarrow\rangle + \sin \theta_n |n-1, \uparrow\rangle \quad \text{Adiabatic if:}$$

$$P_{\text{trans}} = \frac{|\langle n-1, -|\hat{\psi}|n, +\rangle|^2}{\langle n, +|\hat{\psi}^\dagger \hat{\psi}|n, +\rangle} \leq \frac{27}{256n^2} \ll 1$$

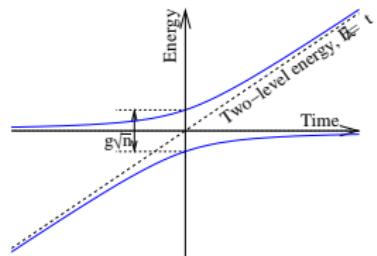
If adiabatic, $\Lambda_n = \rho_{n,n+1}$:

$$\frac{d\Lambda_n}{dt} = i \left[\frac{d\Delta\phi_{n-1}}{dt} \right] \Lambda_n - \left[\left(n - \frac{1}{2} \right) \Lambda_n - \sqrt{n(n+1)} \Lambda_{n+1} \right]$$

When $\delta \gg \hbar\omega$, the decay rate $\kappa\delta \approx \hbar|\delta|^2 > \kappa$

$$\kappa/g \ll 1/(2\pi\hbar\omega^2) \approx 5 \times 10^{-5}$$

Understanding additional decay: adiabatic approx



$$|n, +\rangle = \cos \theta_n |n, \downarrow\rangle + \sin \theta_n |n-1, \uparrow\rangle \quad \text{Adiabatic if:}$$

$$P_{\text{trans}} = \frac{|\langle n-1, -|\hat{\psi}|n, +\rangle|^2}{\langle n, +|\hat{\psi}^\dagger \hat{\psi}|n, +\rangle} \leq \frac{27}{256n^2} \ll 1$$

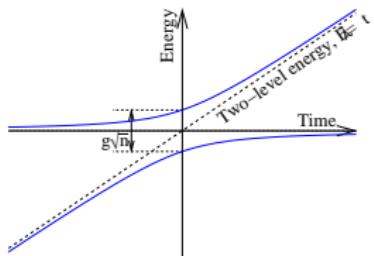
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What does it mean? Adiabatic approximation?

$$\kappa/g < 1/(2\pi^2 c^3) \approx 5 \times 10^{-5}$$

Understanding additional decay: adiabatic approx



$$|n, +\rangle = \cos \theta_n |n, \downarrow\rangle + \sin \theta_n |n-1, \uparrow\rangle \quad \text{Adiabatic if:}$$

$$P_{\text{trans}} = \frac{|\langle n-1, - | \hat{\psi} | n, + \rangle|^2}{\langle n, + | \hat{\psi}^\dagger \hat{\psi} | n, + \rangle} \leq \frac{27}{256 n^2} \ll 1$$

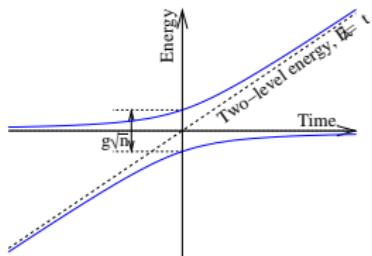
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When $|t| \leq g\sqrt{n}/\lambda$, decay rate κn

$$\kappa/g < 1/(2\pi c \ell^3) \approx 5 \times 10^{-5}$$

Understanding additional decay: adiabatic approx



$$|n, +\rangle = \cos \theta_n |n, \downarrow\rangle + \sin \theta_n |n-1, \uparrow\rangle \quad \text{Adiabatic if:}$$

$$P_{\text{trans}} = \frac{|\langle n-1, -|\hat{\psi}|n, +\rangle|^2}{\langle n, +|\hat{\psi}^\dagger \hat{\psi}|n, +\rangle} \leq \frac{27}{256n^2} \ll 1$$

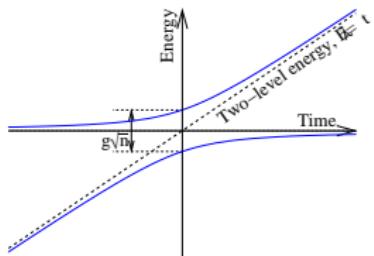
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When $|t| \leq g\sqrt{n}/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

$$\kappa/g < 1/(2\pi c \delta^2) \simeq 5 \times 10^{-5}$$

Understanding additional decay: adiabatic approx



$$|n, +\rangle = \cos \theta_n |n, \downarrow\rangle + \sin \theta_n |n-1, \uparrow\rangle \quad \text{Adiabatic if:}$$

$$P_{\text{trans}} = \frac{|\langle n-1, - | \hat{\psi} | n, + \rangle|^2}{\langle n, + | \hat{\psi}^\dagger \hat{\psi} | n, + \rangle} \leq \frac{27}{256n^2} \ll 1$$

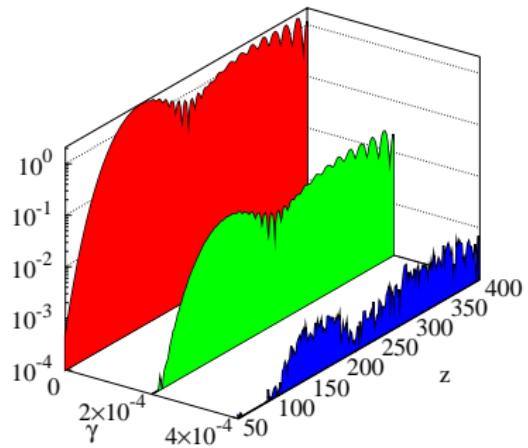
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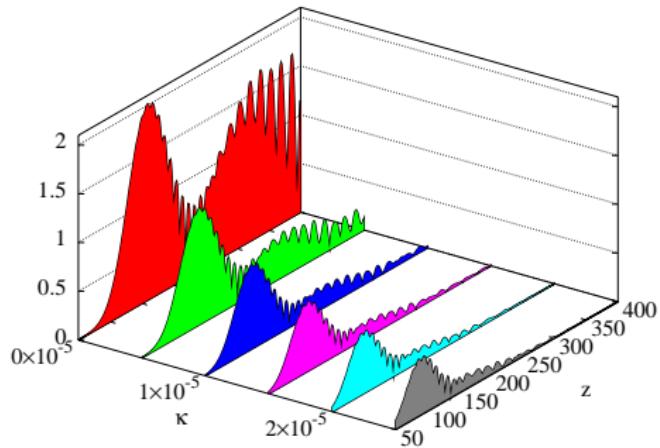
When $|t| \leq g\sqrt{n}/\lambda$, decay rate $\kappa n \simeq \kappa |\psi|^2 \gg \kappa$.

$$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5 \times 10^{-5}$$

Results including decay

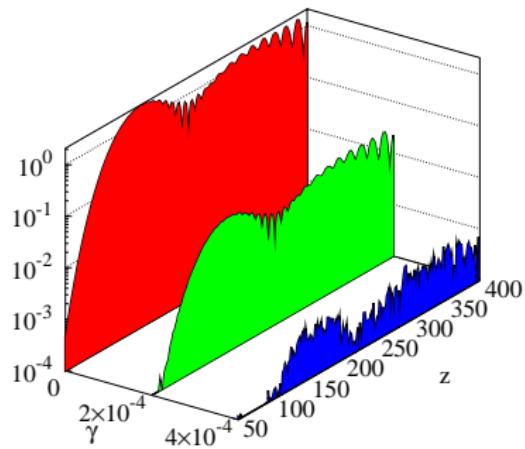


$$\begin{aligned}\gamma_{\text{max}}/g &\simeq 2 \times 10^{-4} \\ \gamma_{\text{theory}}/g &\ll 10 \times 10^{-4}\end{aligned}$$



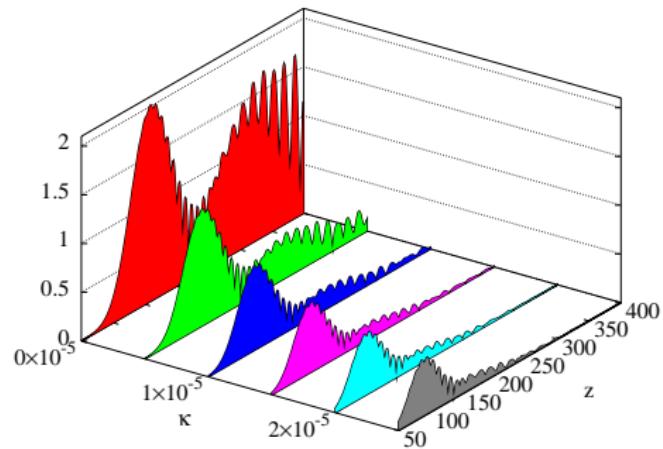
$$\begin{aligned}\kappa_{\text{max}}/g &\simeq 1 \times 10^{-5} \\ \kappa_{\text{theory}}/g &\ll 100 \times 10^{-5}\end{aligned}$$

Results including decay



$$\gamma_{\text{max}}/g \simeq 2 \times 10^{-4}$$

$$\gamma_{\text{theory}}/g \ll 10 \times 10^{-4}$$



$$\kappa_{\text{max}}/g \simeq 1 \times 10^{-5}$$

$$\kappa_{\text{theory}}/g \ll 5 \times 10^{-5}$$

Landau Zener processes in many body systems

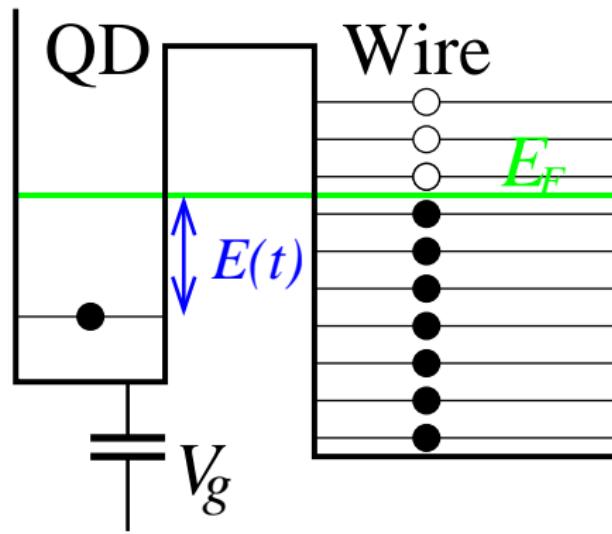
Localised fermion coupled to a continuum of states

Jonathan Keeling¹, L. S. Levitov² and A. Shytov³

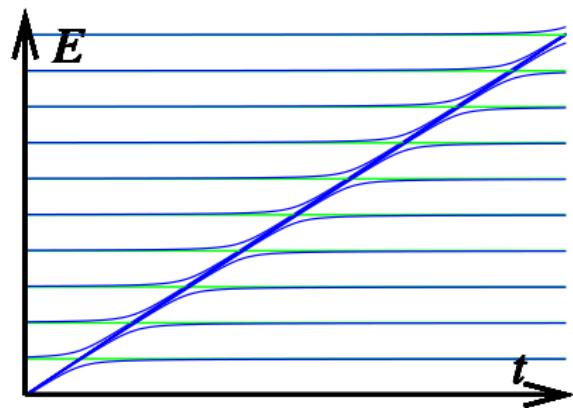
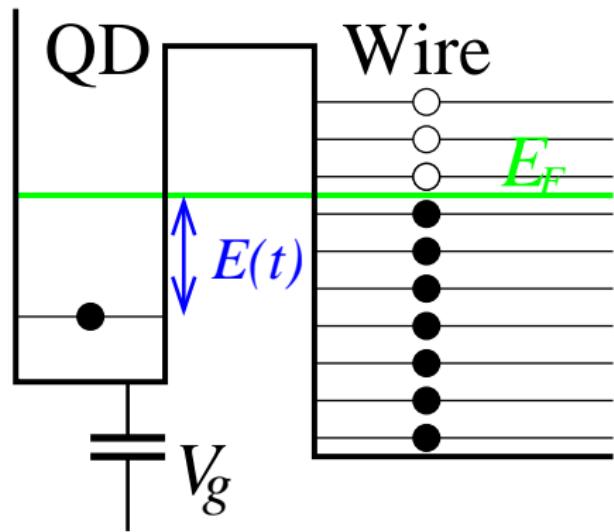
¹University of Cambridge ²Massachusetts Institute of Technology ³Brookhaven National Lab

November 28, 2007

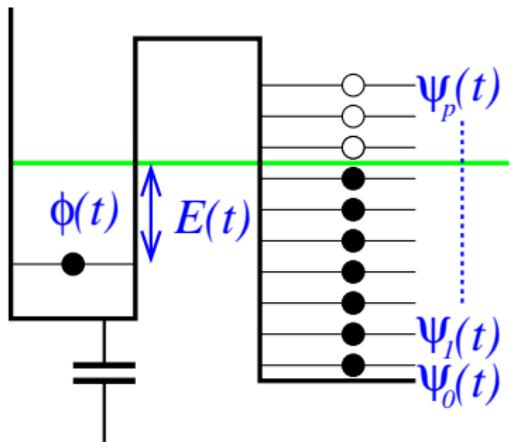
Physical problem



Physical problem



Single particle problem



Regularly spaced continuum:

$$\psi(x, t) = \sum \psi_p(t) e^{ikx}$$

Thus, continuum equations:

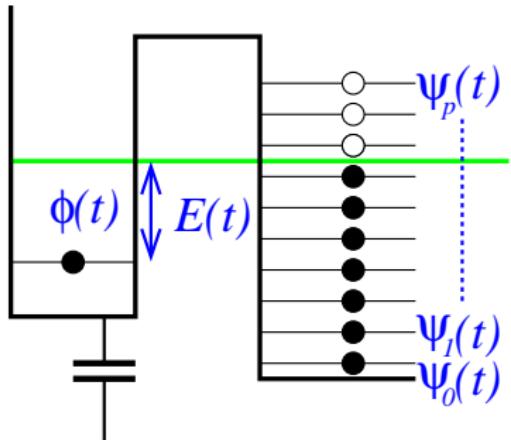
$$[i\partial_t - E(t)]\psi(t) = g \int dx \psi(x, t)\delta(x)$$

$$(i\partial_t + i\nu\partial_x)\psi(x, t) = g\delta(x)\psi(t)$$

Given: $\psi(x < 0, t) = \frac{e^{-i\theta}}{\sqrt{2\pi}}$, find

$$U(\nu, d) = \int dt \psi(x > 0, t) \frac{e^{i\theta}}{\sqrt{2\pi}}$$

Single particle problem



Regularly spaced continuum:

$$\psi(x, t) = \sum \psi_p(t) e^{ipx}$$

Thus, continuum equations:

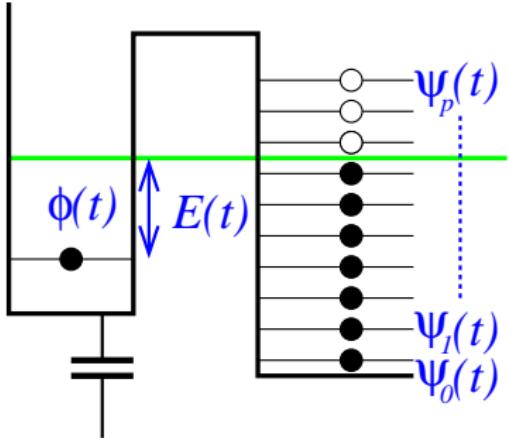
$$[i\partial_t - E(t)]\psi(t) = g \int dx \psi(x, t) \delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x, t) = g\delta(x)\psi(t)$$

Given: $\psi(x < 0, t) = \frac{e^{-i\frac{pt}{m}}}{\sqrt{2\pi}}$, find

$$U(e, d) = \int dt \psi(x > 0, t) \frac{e^{i\frac{pt}{m}}}{\sqrt{2\pi}}$$

Single particle problem



Regularly spaced continuum:

$$\psi(x, t) = \sum \psi_p(t) e^{ipx}$$

Thus, continuum equations:

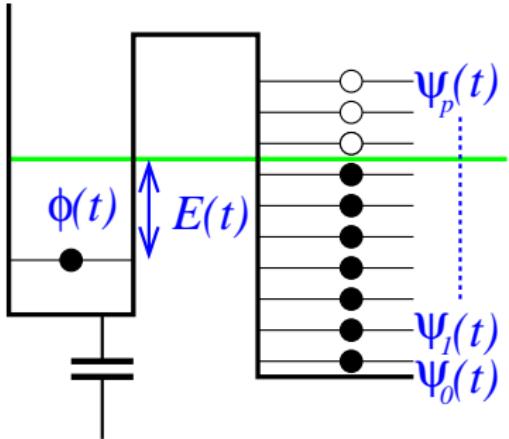
$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x, t) \delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x, t) = g\delta(x)\phi(t)$$

Given: $\psi(x < 0, t) = \frac{e^{i\theta}}{\sqrt{2\pi}}$, find

$$U(\epsilon, d) = \int dt \psi(x > 0, t) \frac{d}{dx} \frac{\phi(t)}{\phi(0)}$$

Single particle problem



Regularly spaced continuum:

$$\psi(x, t) = \sum \psi_p(t) e^{ipx}$$

Thus, continuum equations:

$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x, t) \delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x, t) = g\delta(x)\phi(t)$$

Given: $\psi(x < 0, t) = \frac{e^{-i\epsilon' t}}{\sqrt{2\pi}}$, find:

$$U(\epsilon, \epsilon') = \int dt \psi(x > 0, t) \frac{e^{i\epsilon t}}{\sqrt{2\pi}}$$

Solving Schrodinger equations

Equations:

$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x, t) \delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x, t) = g\delta(x)\phi(t)$$

Can solve for general $E(t)$, writing

$$\psi(x, t) = \phi_0 \left(t - \frac{x}{v} \right) e^{-i\frac{E}{\hbar}t} + \frac{g}{\hbar v} \phi \left(t - \frac{x}{v} \right) e(x)$$

$$\text{so: } [i\partial_t - E(t) + i\frac{E^2}{2\hbar}] \phi(t) = g\phi_0(t)$$

Introduce decay rate $\Gamma = g^2/v$

Solving Schrodinger equations

Equations:

$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x, t) \delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x, t) = g\delta(x)\phi(t)$$

Can solve for general $E(t)$, writing:

$$\psi(x, t) = \psi_0 \left(t - \frac{x}{v} \right) - i \frac{g}{v} \phi \left(t - \frac{x}{v} \right) \Theta(x)$$

$$\text{so: } [i\partial_t - E(t) + i\frac{g^2}{2v}] \phi(t) = g\psi_0(t)$$

Introduce decay rate $\Gamma = g^2/v$

Solving Schrodinger equations

Equations:

$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x, t) \delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x, t) = g\delta(x)\phi(t)$$

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Introduce decay rate $\Gamma = g^2/v$

Scattering matrix for arbitrary times

Solution for $\phi(t)$

$$\phi(t) = \phi_0 \int_{-\infty}^t dt' \phi_0(t') \exp\left[-\frac{i}{\hbar}(t-t') + i \int_{t'}^t E(\tau) d\tau\right]$$

Then: $\phi_0(t') = \frac{1}{\sqrt{2\pi}} e^{-iEt'} \rightarrow \phi(t)$

Scattering matrix for arbitrary times

Solution for $\phi(t) = g \int_{-\infty}^t dt' \psi_0(t') \exp \left[-\frac{\Gamma}{2}(t-t') + i \int_{t'}^t E(\tau) d\tau \right]$

Then: $\psi_0(t') = \frac{1}{\sqrt{2\pi}} e^{-\Gamma t'/2} \rightarrow \phi(t) = g \psi_0(t)$

$$U(t,t') = \frac{g}{2\pi} \int_{-\infty}^t dt \int_{-\infty}^t dt' \exp \left[i(t-t') - \frac{\Gamma}{2}(t-t') + i \int_{t'}^t E(\tau) d\tau \right]$$

If $E(t) = \lambda t$, elementary form:

$$U(t,t') \propto e^{-(\Gamma/\lambda)(t-t') - (i/2\lambda)(t^2-t'^2)}$$

Scattering matrix for arbitrary times

Solution for $\phi(t) = g \int_{-\infty}^t dt' \psi_0(t') \exp \left[-\frac{\Gamma}{2}(t-t') + i \int_{t'}^t E(\tau) d\tau \right]$

Then: $\psi_0(t') = \frac{1}{\sqrt{2\pi}} e^{-i\epsilon' t'}$

$$U(t, t') = \frac{1}{2\pi} \int_{-\infty}^t dt \int_{-\infty}^t dt' \exp \left[i(\epsilon t - \epsilon t') - \frac{\Gamma}{2}(t-t') + i \int_{t'}^t E(\tau) d\tau \right]$$

If $E(t) = \lambda t$, elementary form

$$U(t, t') \propto e^{-(\Gamma/\lambda)(t-t') - (\Gamma/2\lambda)(t^2 - t'^2)}$$

Scattering matrix for arbitrary times

Solution for $\phi(t) = g \int_{-\infty}^t dt' \psi_0(t') \exp \left[-\frac{\Gamma}{2}(t-t') + i \int_{t'}^t E(\tau) d\tau \right]$

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$$U(x,t) = \frac{1}{2\pi} \int_{-\infty}^t dt \int_{-\infty}^t dt' \exp \left[i(xt - ct) - \frac{\Gamma}{2}(t-t') + i \int_{t'}^t E(\tau) d\tau \right]$$

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Scattering matrix for arbitrary times

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$U(\epsilon, \epsilon') = \frac{\Gamma}{2\pi} \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' \exp \left[i(\epsilon t - \epsilon' t') - \frac{\Gamma}{2}(t-t') + i \int_{t'}^t E(\tau) d\tau \right]$

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$$U(\epsilon, \epsilon') \propto e^{-(\Gamma/\lambda)(\epsilon-\epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)}$$

Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$

For $E = \lambda I$,

$$\langle \epsilon | U | \epsilon' \rangle \propto e^{-(E/2)(\epsilon - \epsilon') - i(2\pi/\hbar)^2(\epsilon^2 - \epsilon'^2)}$$

$$= U_1(\epsilon) U_2(\epsilon')$$

$$= c|\phi_+\rangle\langle\phi_-|$$

$$P_2 = U_{a \rightarrow b} U_{b \rightarrow a} - U_{a \rightarrow b} U_{b \rightarrow a}$$

$$= \langle a' |\phi_+\rangle\langle\phi_-|a\rangle \langle b' |\phi_+\rangle\langle\phi_-|b\rangle$$

$$- \langle a' |\phi_+\rangle\langle\phi_-|b\rangle \langle b' |\phi_+\rangle\langle\phi_-|a\rangle = 0$$

Max number of particles transferred = rank of U .

Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$

For $E = \lambda t$,

$$\langle \epsilon | U | \epsilon' \rangle \propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)}$$

$$= c(\phi_+)(\phi_-)$$

$$= c(\phi_+)(\phi_-)c(\phi_+)(\phi_-)$$

$$P_2 = U_{\phi_+\phi_+} U_{\phi_-\phi_-} - U_{\phi_+\phi_-} U_{\phi_-\phi_+}$$

$$= (c(\phi_+)(\phi_-))c(\phi_+)(\phi_-)c(\phi_+)(\phi_-)$$

$$= (c(\phi_+)(\phi_-)b)(U(\phi_+)U(\phi_-))c(\phi_-) = 0$$

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Elevating to many particle problem: linear time

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$$= U_1(\epsilon) U_2(\epsilon')$$

$$= c |\phi_+\rangle \langle \phi_-|$$

$$\begin{aligned} P_2 &= U_{a\rightarrow b} U_{b\rightarrow a} - U_{a\rightarrow b} U_{b\rightarrow a} \\ &= (\delta|\phi_+\rangle \langle \phi_-| \delta) (\delta'|\phi_+\rangle \langle \phi_-| \delta') \\ &= (\delta|\phi_+\rangle \langle \phi_-| b) (\delta'|\phi_+\rangle \langle \phi_-| a) = 0 \end{aligned}$$

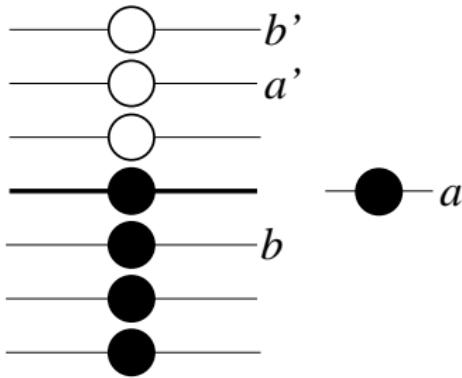
Max number of particles transferred = rank of U .

Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$

For $E = \lambda t$,

$$\begin{aligned}\langle \epsilon | U | \epsilon' \rangle &\propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)} \\ &= U_1(\epsilon) U_2(\epsilon') \\ &= c |\phi_+\rangle \langle \phi_-|\end{aligned}$$



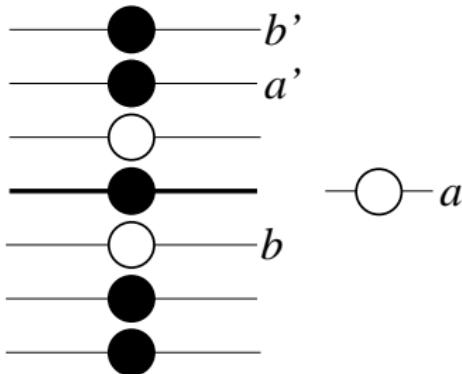
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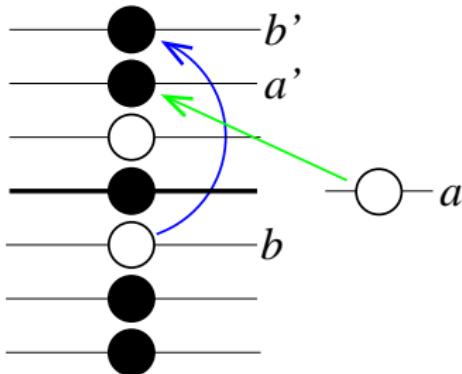
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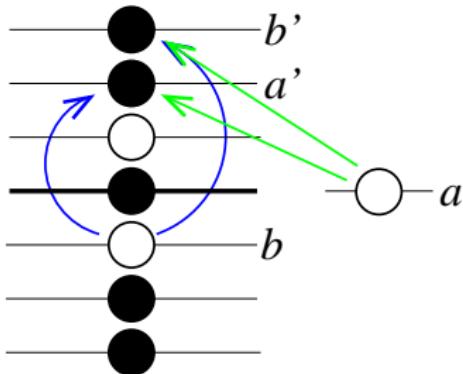
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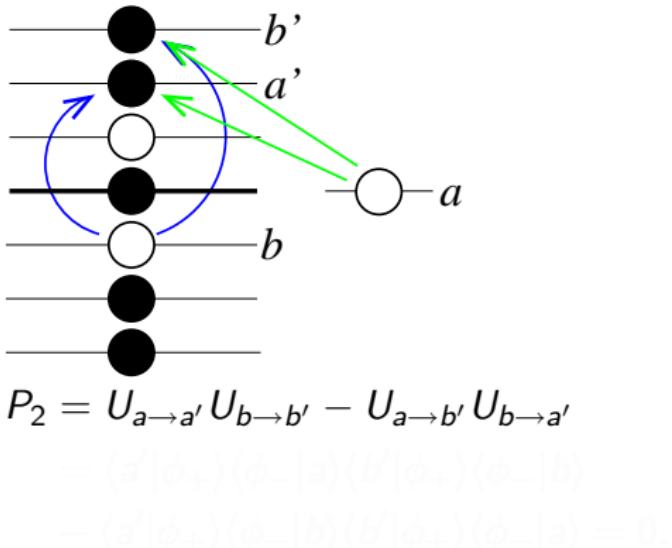
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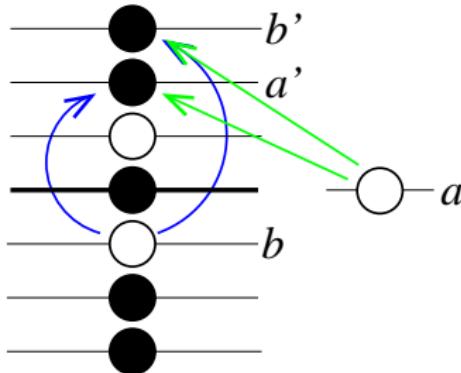
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$$\begin{aligned}P_2 &= U_{a \rightarrow a'} U_{b \rightarrow b'} - U_{a \rightarrow b'} U_{b \rightarrow a'} \\ &= \langle a' | \phi_+ \rangle \langle \phi_- | a \rangle \langle b' | \phi_+ \rangle \langle \phi_- | b \rangle \\ &\quad - \langle a' | \phi_+ \rangle \langle \phi_- | b \rangle \langle b' | \phi_+ \rangle \langle \phi_- | a \rangle = 0\end{aligned}$$

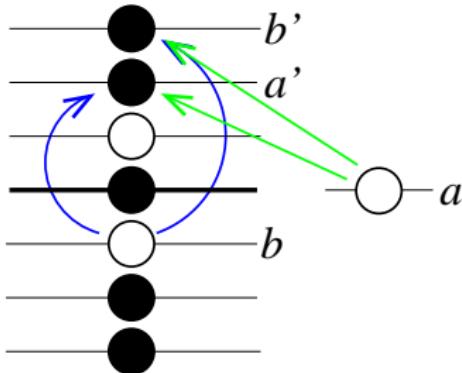
Max number of particles transferred = rank of P_2

Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$

For $E = \lambda t$,

$$\begin{aligned}\langle \epsilon | U | \epsilon' \rangle &\propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)} \\ &= U_1(\epsilon) U_2(\epsilon') \\ &= c |\phi_+\rangle \langle \phi_-|\end{aligned}$$



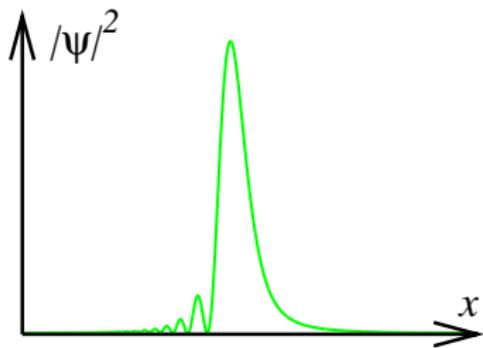
$$\begin{aligned}P_2 &= U_{a \rightarrow a'} U_{b \rightarrow b'} - U_{a \rightarrow b'} U_{b \rightarrow a'} \\ &= \langle a' | \phi_+ \rangle \langle \phi_- | a \rangle \langle b' | \phi_+ \rangle \langle \phi_- | b \rangle \\ &\quad - \langle a' | \phi_+ \rangle \langle \phi_- | b \rangle \langle b' | \phi_+ \rangle \langle \phi_- | a \rangle = 0.\end{aligned}$$

Max number of particles transferred = rank of U .

Linear timed dependence, exact state

$$\Psi_p \propto e^{-(\Gamma/c)vp - (i/2\lambda)(vp)^2}$$

Spatial profile:



General time dependence: measuring noise

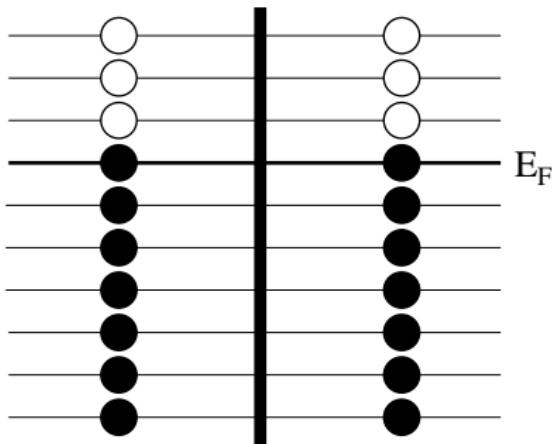
Number of excitations:

$$N^{\text{ex}} = \sum_{\epsilon > \epsilon_F > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_F < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$

General time dependence: measuring noise

Number of excitations:

$$N^{\text{ex}} = \sum_{\epsilon > \epsilon_F > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_F < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



$$\text{Define: } N_{ij} = N_j - N_i \\ g_{ij} = e(N_j - N_i)$$

then

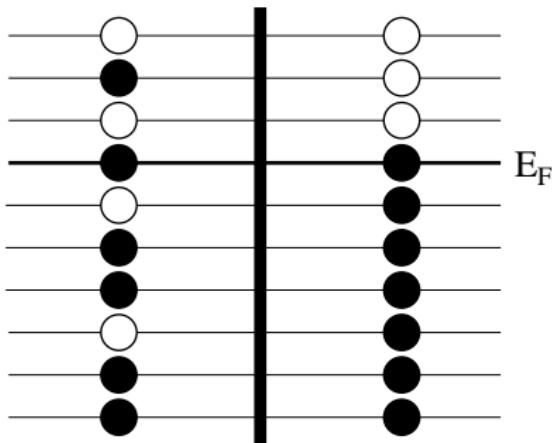
$$\langle g \rangle = T(g)$$

$$\langle \Delta g^2 \rangle = T^2(\Delta g)^2 + \delta^2 T(1-T)(N)^2$$

General time dependence: measuring noise

Number of excitations:

$$N^{\text{ex}} = \sum_{\epsilon > \epsilon_F > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_F < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



$$\text{Define: } N_j = N_j - N_j^0$$
$$g_{j,j} = \delta(N_j^0 - N_j)$$

then

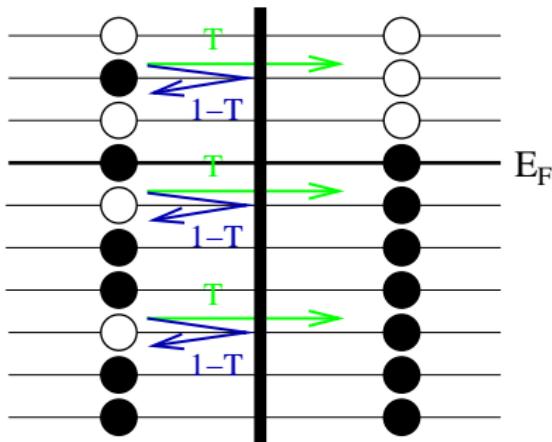
$$\langle \eta \rangle = T(\eta)$$

$$\langle \Delta \eta^2 \rangle = T^2 \langle \Delta \eta^2 \rangle + \delta^2 T(1-T) \langle N \rangle^2$$

General time dependence: measuring noise

Number of excitations:

$$N^{\text{ex}} = \sum_{\epsilon > \epsilon_F > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_F < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



$$\text{Define: } N_T = N_U - N_D$$
$$g_{eff} = e(N_T - N_D)$$

then

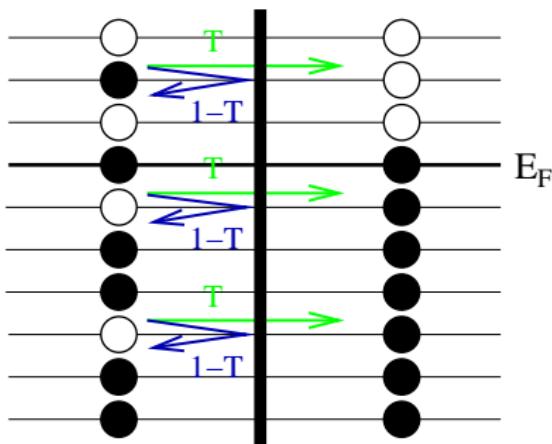
$$\langle g \rangle = T \langle g \rangle$$

$$\langle \Delta g^2 \rangle = T^2 \langle \Delta g^2 \rangle + \delta^2 T(1-T)(N)^2$$

General time dependence: measuring noise

Number of excitations:

$$N^{\text{ex}} = \sum_{\epsilon > \epsilon_F > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_F < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$

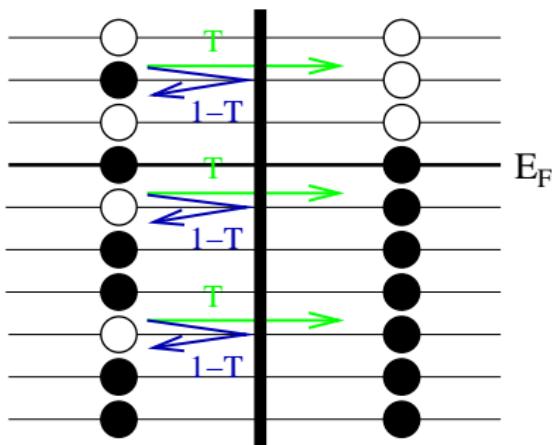


Define: $N_{r,I}^{\text{ex}} = N_{r,I}^e + N_{r,I}^h$
 $q_{r,I} = e(N_{r,I}^e - N_{r,I}^h)$

General time dependence: measuring noise

Number of excitations:

$$N^{\text{ex}} = \sum_{\epsilon > \epsilon_F > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_F < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



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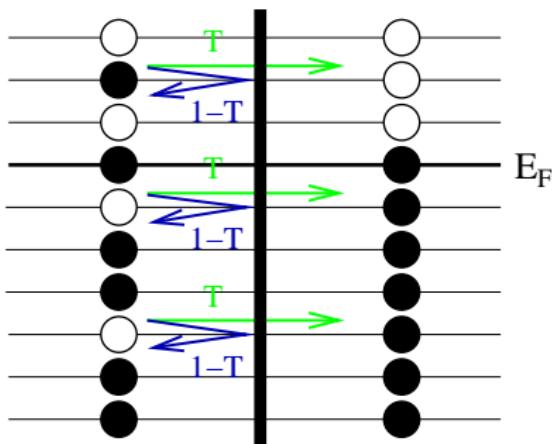
$$\langle q_r \rangle = T \langle q_I \rangle$$

$$(N^e - N^h) + 2T(1-T)(N^h)$$

General time dependence: measuring noise

Number of excitations:

$$N^{\text{ex}} = \sum_{\epsilon > \epsilon_F > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_F < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



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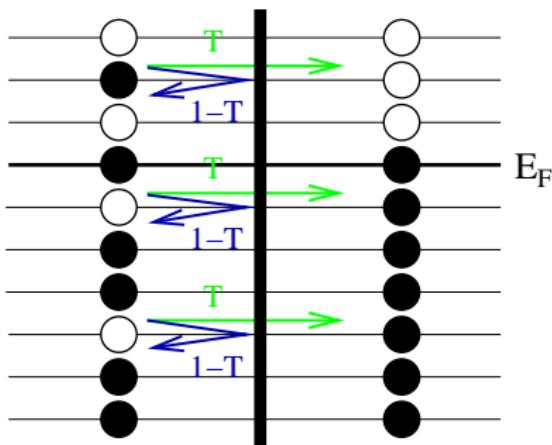
Then

$$\langle q_r \rangle = T \langle q_I \rangle$$
$$\langle \Delta q_r^2 \rangle$$

General time dependence: measuring noise

Number of excitations:

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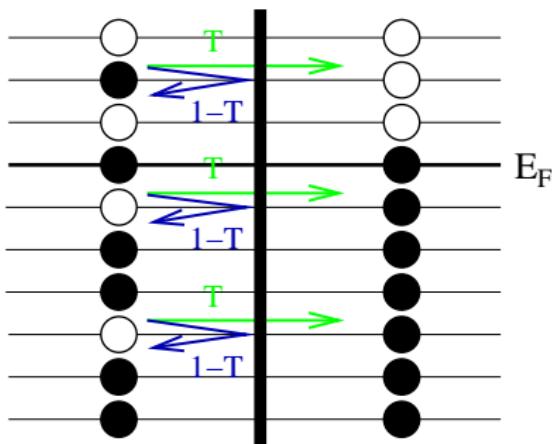
$$\langle q_r \rangle = T \langle q_I \rangle$$

$$\langle \Delta q_r^2 \rangle =$$

General time dependence: measuring noise

Number of excitations:

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Define: $N_{r,I}^{\text{ex}} = N_{r,I}^e + N_{r,I}^h$
 $q_{r,I} = e(N_{r,I}^e - N_{r,I}^h)$

Then

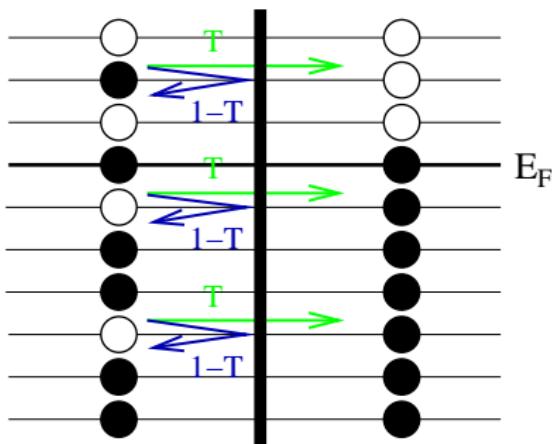
$$\langle q_r \rangle = T \langle q_I \rangle$$

$$\langle \Delta q_r^2 \rangle = T^2 \langle \Delta q_I^2 \rangle$$

General time dependence: measuring noise

Number of excitations:

$$N^{\text{ex}} = \sum_{\epsilon > \epsilon_F > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_F < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



Define: $N_{r,I}^{\text{ex}} = N_{r,I}^e + N_{r,I}^h$
 $q_{r,I} = e(N_{r,I}^e - N_{r,I}^h)$

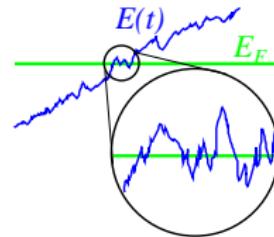
Then

$$\langle q_r \rangle = T \langle q_I \rangle$$

$$\langle \Delta q_r^2 \rangle = T^2 \langle \Delta q_I^2 \rangle + e^2 T (1 - T) \langle N_I^{\text{ex}} \rangle$$

Noisy driving

- Suppose $E(t) = \lambda t + \eta(t)$
- $\langle \eta(t) \rangle = 0, \langle \eta(t)\eta(t') \rangle = T_2\delta(t-t')$.

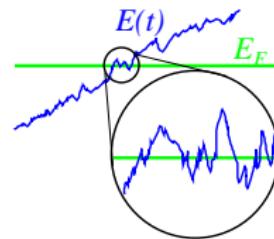


- To find N^∞ , need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$
- Integral log divergent: white noise limit
- Infinite no. crossings of Fermi surface

{

Noisy driving

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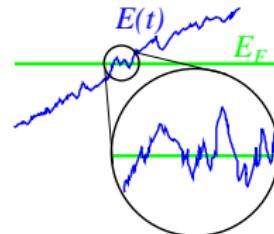


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{

Noisy driving

- Suppose $E(t) = \lambda t + \eta(t)$
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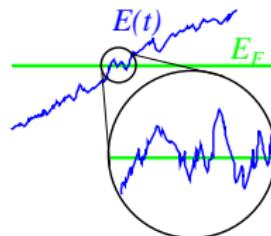
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Noisy driving

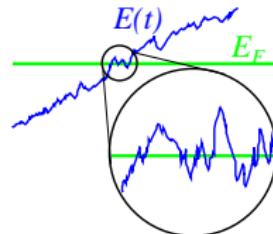
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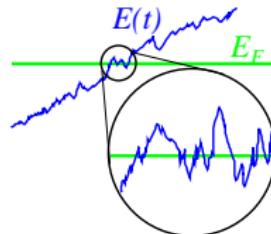


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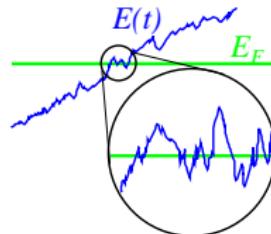


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$$N^{\text{ex}} = \begin{cases} 1 & \lambda \gg \Gamma\Gamma_2 \\ \dots & \dots \end{cases}$$

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Conclusions

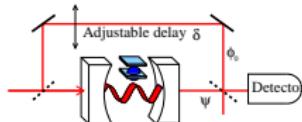
- Exactly solvable many-body generalisations of LZ.

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- Localised/continuum fermions

Conclusions

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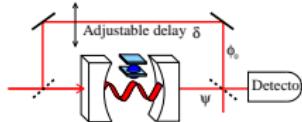


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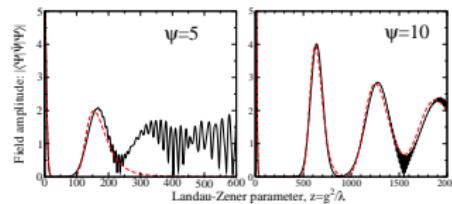
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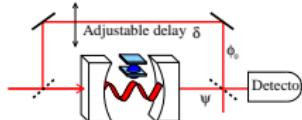


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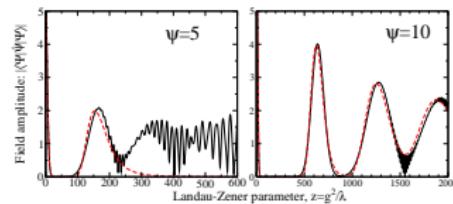
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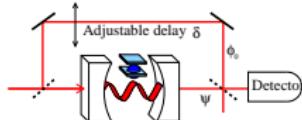


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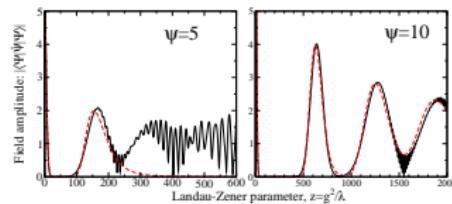
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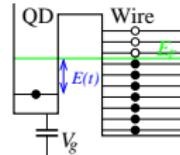
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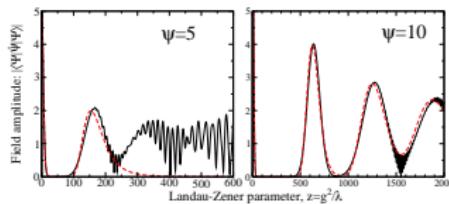
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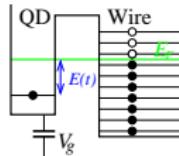
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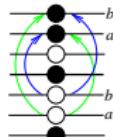
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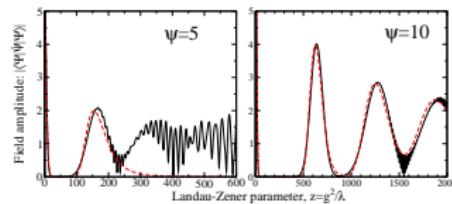
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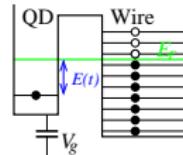
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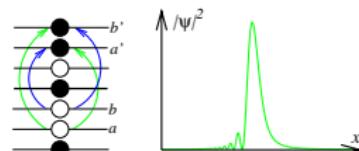
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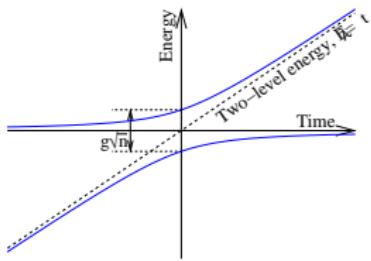
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Understanding additional decay: adiabatic approx



$$|\psi_{\pm}\rangle = [\cos(\theta_0)|n,1\rangle + \sin(\theta_0)|n-1,0\rangle]$$

Adiabatic fit

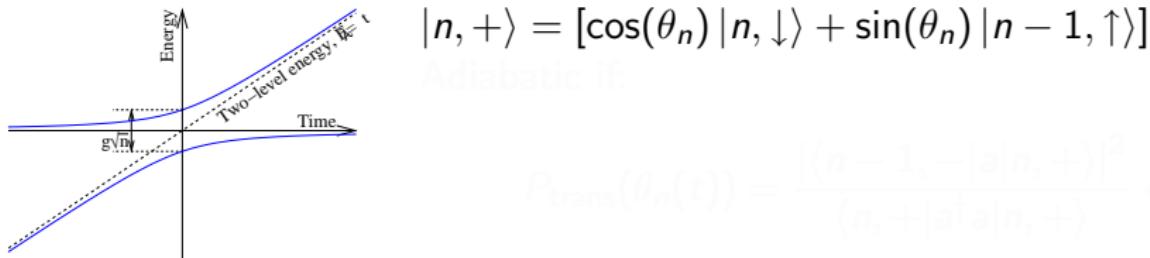
$$P_{trans}(\theta_0(t)) = \frac{(\alpha - 1 - i\beta n_{\perp})^2}{(\alpha + i\beta n_{\perp})|\psi_{\pm}\rangle} < 1$$

At $t \rightarrow \infty, P_{trans} \rightarrow 0$

At $t=0$,

$$P_{trans} \approx \frac{(\sqrt{n} - \sqrt{n-1})^2}{2(2n+1)} \approx \frac{1}{16n^2} \approx \frac{1}{16|g|^2}$$

Understanding additional decay: adiabatic approx

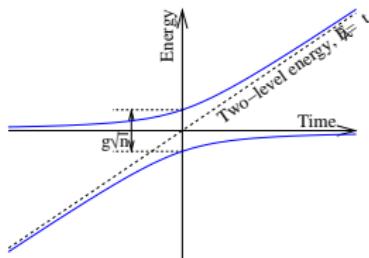


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Understanding additional decay: adiabatic approx



$$|n, +\rangle = [\cos(\theta_n) |n, \downarrow\rangle + \sin(\theta_n) |n-1, \uparrow\rangle]$$

Adiabatic if:

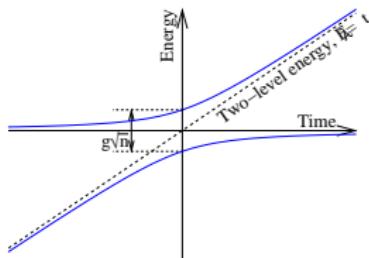
$$P_{\text{trans}}(\theta_n(t)) = \frac{|\langle n-1, -|a|n, +\rangle|^2}{\langle n, +|a^\dagger a|n, +\rangle} \ll 1$$

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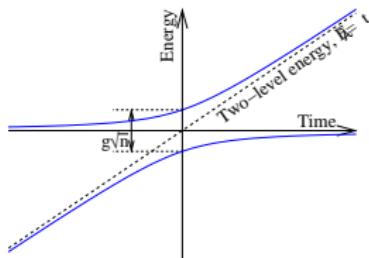
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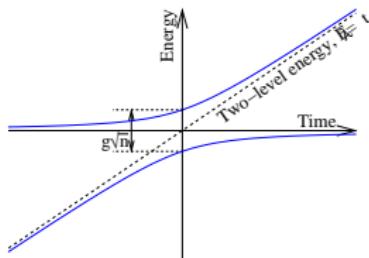
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At $t \simeq 0$,

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1
16n²

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Understanding additional decay: Density matrix equation

If adiabatic, $A_n = \rho_{n,n+1}$:

$$\frac{dA_n}{dt} = i \left[\frac{d\Delta\phi_{n-1}}{dt} \right] A_n \rightarrow \left(n - \frac{1}{2} \right) A_n - \sqrt{n(n+1)} A_{n+1}$$

When $|t| \leq g/\lambda$, decay rate $\kappa_n \approx \pi v^2 \gg \kappa$

$$v/g \ll 1/(2\pi v^2) \approx 5 \times 10^{-3}$$

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When $|t| \leq g/\lambda$, decay rate κn

$$\kappa/8 \ll 1/(2\pi\omega_0) \sim 2 \times 10^{-10}$$

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Additional decay perturbatively

$$\frac{dA_n}{dt} = i \left[\frac{d\Delta\phi_{n-1}}{dt} \right] A_n - \kappa \left[\left(n - \frac{1}{2} \right) A_n - \sqrt{n(n+1)} A_{n+1} \right]$$

Gauge transform: $A_n = \tilde{A}_n e^{-i\Delta\phi_{n-1}}$

$$\frac{d\tilde{A}_n}{dt} = -\kappa \left[\left(n - \frac{1}{2} \right) \tilde{A}_n - \sqrt{n(n+1)} \tilde{A}_{n+1} e^{i(\Delta\phi_{n-1} - \tilde{\phi}_n)} \right]$$

$$\tilde{A}_n(T) = \tilde{A}_n(-T)$$

$$-\kappa \int_{-T}^T dt \left[\left(n - \frac{1}{2} \right) \tilde{A}_n - \sqrt{n(n+1)} \tilde{A}_{n+1} e^{i(\Delta\phi_{n-1} - \tilde{\phi}_n)} \right]$$

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Results of perturbative decay

Remove “naive decay”,

$$\langle \Psi | \psi(\kappa, \psi_0) | \Psi \rangle_{\text{naive}} = \left\langle \Psi | \psi(0, \psi_0 e^{-\kappa T/2}) | \Psi \right\rangle e^{-\kappa T/2}$$

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$$At \quad g^2/\lambda = z = 2\pi n/kT^2$$

$$\langle \delta \langle \Psi | \psi | \Psi \rangle \rangle \propto \psi \left[\frac{E}{\hbar} (2\pi n)^{3/2} / kT^2 \right]$$

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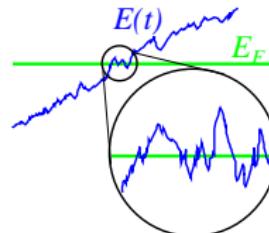
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Noisy driving

- Suppose $E(t) = ct + \eta(t)$
- $\langle \eta(t) \rangle = 0, \langle \eta(t)\eta(t') \rangle = T_2\delta(t-t')$.

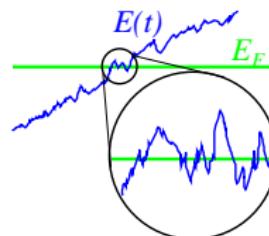
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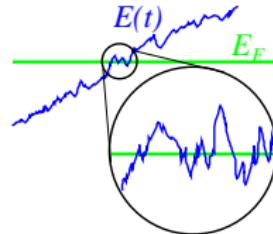
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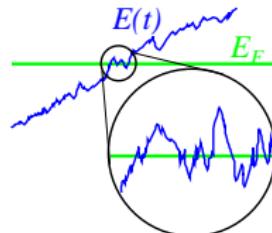
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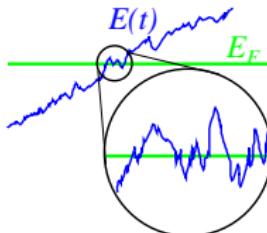
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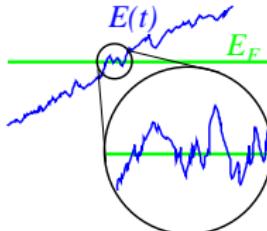
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$$N^{\text{ex}} \propto \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' \int_{-\infty}^{\infty} ds \int_{-\infty}^t ds' \dots \times F_{\text{noise}}(t, t', s, s')$$

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$$F_{\text{noise}} = \left\langle \exp \left[i \int_{t'}^t \eta(\tau) d\tau - i \int_{s'}^s \eta(\sigma) d\sigma \right] \right\rangle$$

Noisy driving

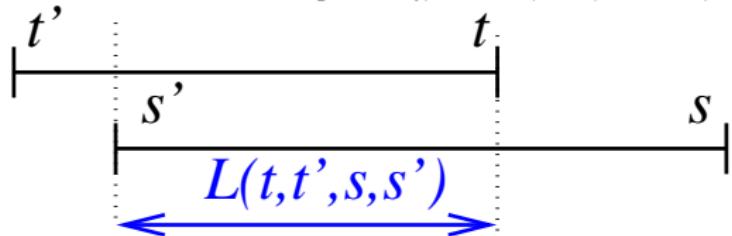
- Find $F_{\text{noise}} = \exp[-\Gamma_2(|t - t'| + |s - s'| - L(t, t', s, s'))]$

• Can simplify to $\Delta = t - s$ and $\lambda = t' - t = s' - s'$

$$N^{\infty} = \frac{-\Gamma^2}{2\pi c} \int_{-\infty}^{\infty} \frac{d\Delta}{(\Delta - i0)^2} \int_0^{\infty} d\lambda e^{(i\omega\Delta - \Gamma - \Gamma_2)\lambda + \Gamma_2(\Delta)\lambda}$$

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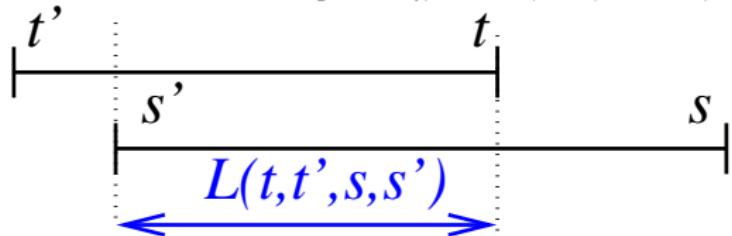


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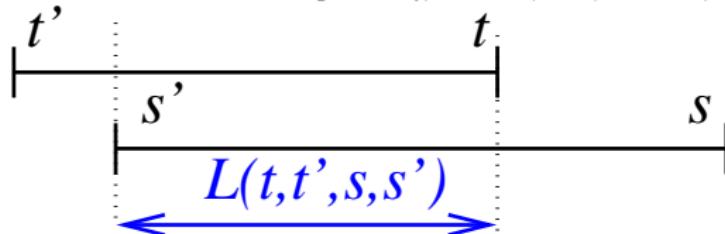


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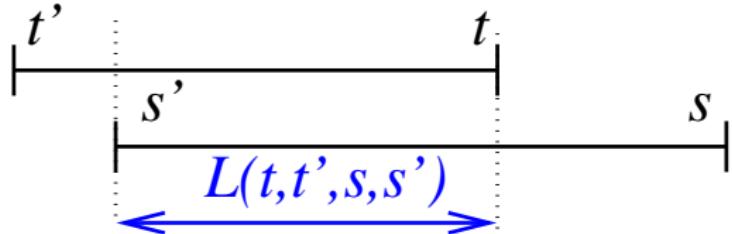


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Noisy driving: results

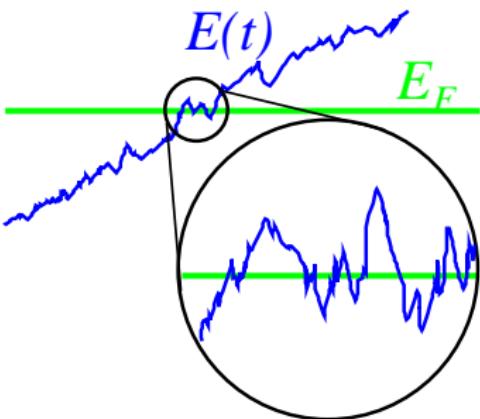
- Integral log divergent: white noise limit
 - infinite no. crossings of Fermi surface
 - Can extract logarithmic contribution

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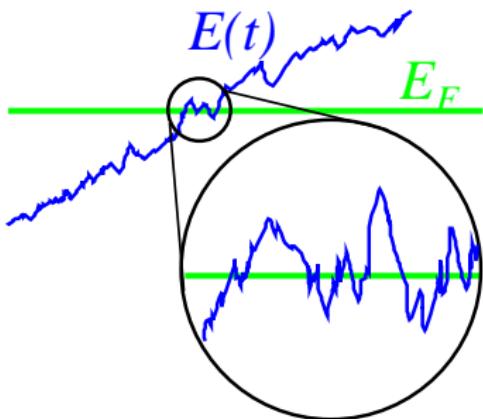
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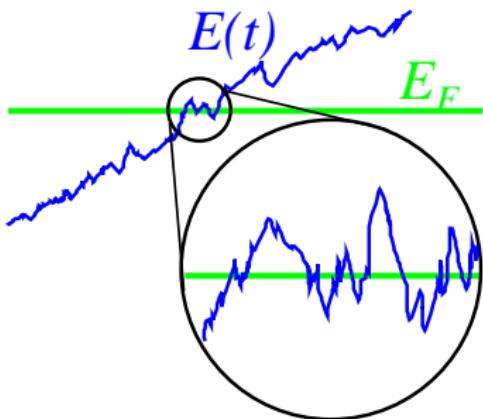
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