Landau Zener processes in many body systems

Two-level system coupled to a photon field

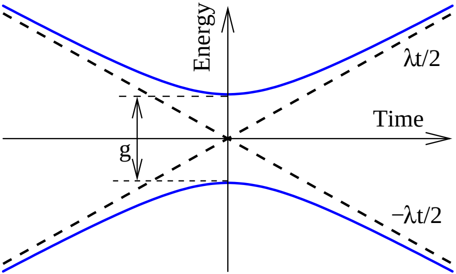
Jonathan Keeling[1] and V. Gurarie[2]

[1]University of Cambridge

[2]University of Colorado at Boulder

November 28, 2007

The Landau-Zener problem



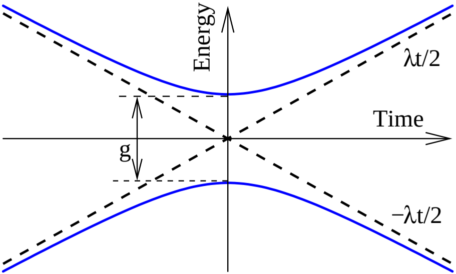
$$i\partial_t \psi = \begin{pmatrix} \lambda t/2 & g \\ g & -\lambda t/2 \end{pmatrix} \psi$$

Initially, $\psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Finally, $\psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$, $\quad z = g^2/\lambda$

$$t = +\infty: \quad |\alpha| = e^{-\pi z}$$
$$|\beta| = \sqrt{1 - e^{-\pi z}}$$

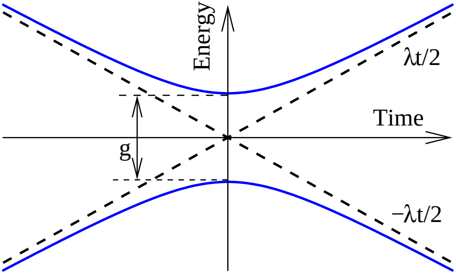
The Landau-Zener problem



Initially, $\psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$$i\partial_t \psi = \begin{pmatrix} \lambda t/2 & g \\ g & -\lambda t/2 \end{pmatrix} \psi$$

The Landau-Zener problem

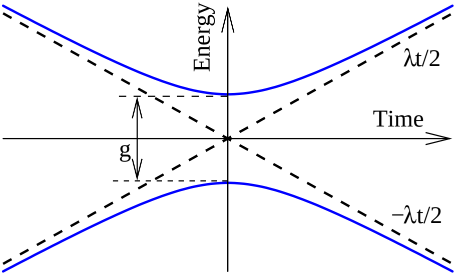


Initially, $\psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Finally, $\psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$,

$$i\partial_t \psi = \begin{pmatrix} \lambda t/2 & g \\ g & -\lambda t/2 \end{pmatrix} \psi$$

The Landau-Zener problem

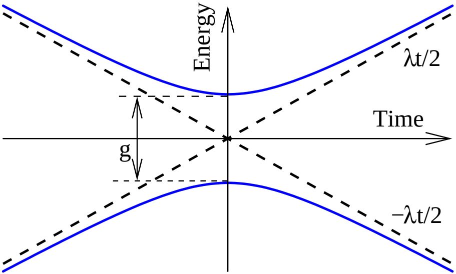


Initially, $\psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Finally, $\psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$, $\quad z = g^2/\lambda$

$$i\partial_t \psi = \begin{pmatrix} \lambda t/2 & g \\ g & -\lambda t/2 \end{pmatrix} \psi$$

The Landau-Zener problem



$$i\partial_t \psi = \begin{pmatrix} \lambda t/2 & g \\ g & -\lambda t/2 \end{pmatrix} \psi$$

Initially, $\psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Finally, $\psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \quad z = g^2/\lambda$

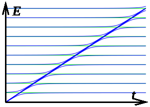
$$t = +\infty : \quad |\alpha| = e^{-\pi z}$$

$$|\beta| = \sqrt{1 - e^{-\pi z}}$$

Many-body Landau-Zener generalisation

- One body, many-level generalisations:
  - Demkov-Osherov problem
- Here, instead **many-body**
- Present two simple but striking examples.

Many-body Landau-Zener generalisation

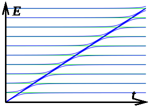
- One body, many-level generalisations:
  - Demkov-Osherov problem 

- Here, instead **many-body**

- Present two simple but striking examples.

Many-body Landau-Zener generalisation

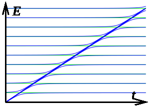
- One body, many-level generalisations:
  - Demkov-Osherov problem



- Here, instead **many-body**
  - Time varying energy of single-particle states
  - Automatic elevation to many-level problem
  - Interference
- Present two simple but striking examples.

Many-body Landau-Zener generalisation

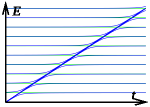
- One body, many-level generalisations:
  - Demkov-Osherov problem



- Here, instead **many-body**
  - Time varying energy of single-particle states
  - Automatic elevation to many-level problem
  - Interference
- Present two simple but striking examples.

Many-body Landau-Zener generalisation

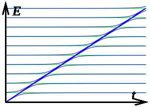
- One body, many-level generalisations:
  - Demkov-Osherov problem



- Here, instead **many-body**
  - Time varying energy of single-particle states
  - Automatic elevation to many-level problem
  - Interference
- Present two simple but striking examples.

Many-body Landau-Zener generalisation

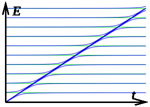
- One body, many-level generalisations:
    - Demkov-Osherov problem



- Here, instead **many-body**
    - Time varying energy of single-particle states
    - Automatic elevation to many-level problem
    - Interference
- Present two simple but striking examples.

Many-body Landau-Zener generalisation

- One body, many-level generalisations:
  - Demkov-Osherov problem

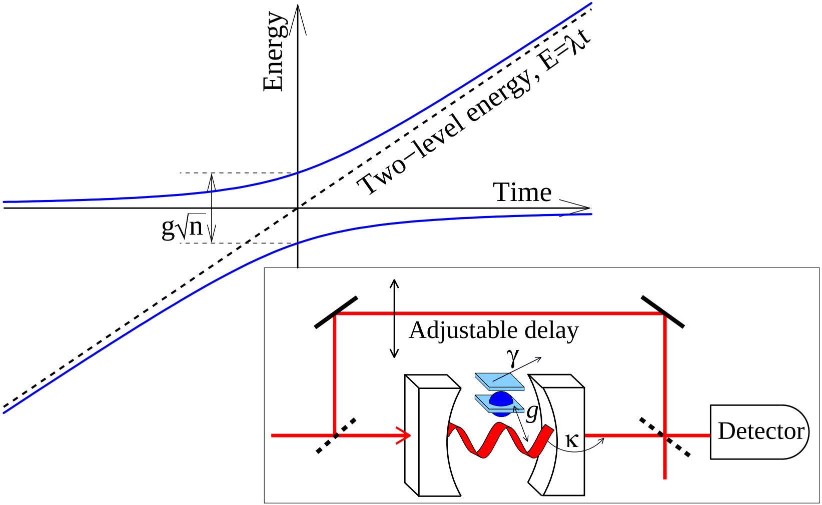


- Here, instead **many-body**
  - Time varying energy of single-particle states
  - Automatic elevation to many-level problem
  - Interference
- Present two simple but striking examples.

Overview

1. Introduction and Landau-Zener
   - Introduction
2. Two-level system/photon field
   - Setting up the many body problem
   - Collapse and revivals
   - Effects of decay
3. Localised fermion coupled to continuum
   - Relating one- and many-body problems
   - Cancellation and "clean electrons"
   - Arbitrary time dependence and noise

Physical system



Description

- Hamiltonian: $\hat{H} = \omega_0\, \hat{\psi}^\dagger \hat{\psi} + \frac{\lambda t}{2} \hat{\sigma}_z + g \left(\hat{\psi}^\dagger \hat{\sigma}^- + \hat{\psi}\, \hat{\sigma}^+ \right),$
- Initial coherent state: $|\Psi(-\infty)\rangle = e^{-|\psi|^2/2} \sum_n \frac{\psi^n}{\sqrt{n!}} |n, \uparrow\rangle$
- Each pair $|n, \uparrow\rangle \leftrightarrow |n+1, \downarrow\rangle$ undergoes LZ transition

$$H_{n,n+1} = \begin{pmatrix} \lambda t/2 & g\sqrt{n+1} \\ g\sqrt{n+1} & -\lambda t/2 \end{pmatrix}; \qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix}$$

- Final state:

$$|\Psi(+\infty)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1} |n, \uparrow\rangle + B_{n+1} |n+1, \downarrow\rangle \right]$$

Description

- Hamiltonian: $\hat{H} = \omega_0 \, \hat{\psi}^\dagger \hat{\psi} + \frac{\lambda t}{2} \hat{\sigma}_z + g \left(\hat{\psi}^\dagger \hat{\sigma}^- + \hat{\psi} \, \hat{\sigma}^+ \right),$
- Initial coherent state: $|\Psi(-\infty)\rangle = e^{-|\psi|^2/2} \sum_n \frac{\psi^n}{\sqrt{n!}} |n, \uparrow\rangle$
- Each pair $|n, \uparrow\rangle \leftrightarrow |n+1, \downarrow\rangle$ undergoes LZ transition

$$H_{n,n+1} = \begin{pmatrix} \lambda t/2 & g\sqrt{n+1} \\ g\sqrt{n+1} & -\lambda t/2 \end{pmatrix} : \qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix}$$

- Final state:

$$|\Psi(+\infty)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1} |n, \uparrow\rangle + B_{n+1} |n+1, \downarrow\rangle \right]$$

Description

- Hamiltonian: $\hat{H} = \omega_0 \hat{\psi}^\dagger \hat{\psi} + \frac{\lambda t}{2} \hat{\sigma}_z + g\left(\hat{\psi}^\dagger \hat{\sigma}^- + \hat{\psi}\,\hat{\sigma}^+\right)$,
- Initial coherent state: $|\Psi(-\infty)\rangle = e^{-|\psi|^2/2} \sum_n \frac{\psi^n}{\sqrt{n!}} |n, \uparrow\rangle$
- Each pair $|n, \uparrow\rangle \leftrightarrow |n+1, \downarrow\rangle$ undergoes LZ transition

$$H_{n,n+1} = \begin{pmatrix} \lambda t/2 & g\sqrt{n+1} \\ g\sqrt{n+1} & -\lambda t/2 \end{pmatrix} : \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix}$$

- Final state:

$$|\Psi(+\infty)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1} |n, \uparrow\rangle + B_{n+1} |n+1, \downarrow\rangle \right]$$

Description

- Hamiltonian: $\hat{H} = \omega_0 \hat{\psi}^\dagger \hat{\psi} + \frac{\lambda t}{2} \hat{\sigma}_z + g \left(\hat{\psi}^\dagger \hat{\sigma}^- + \hat{\psi}\, \hat{\sigma}^+ \right),$
- Initial coherent state: $|\Psi(-\infty)\rangle = e^{-|\psi|^2/2} \sum_n \frac{\psi^n}{\sqrt{n!}} |n, \uparrow\rangle$
- Each pair $|n, \uparrow\rangle \leftrightarrow |n+1, \downarrow\rangle$ undergoes LZ transition

$$H_{n,n+1} = \begin{pmatrix} \lambda t/2 & g\sqrt{n+1} \\ g\sqrt{n+1} & -\lambda t/2 \end{pmatrix}, \qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix}$$

- Final state:

$$|\Psi(+\infty)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1} |n, \uparrow\rangle + B_{n+1} |n+1, \downarrow\rangle \right]$$

Description

- Hamiltonian: $\hat{H} = \omega_0 \hat{\psi}^\dagger \hat{\psi} + \frac{\lambda t}{2} \hat{\sigma}_z + g \left(\hat{\psi}^\dagger \hat{\sigma}^- + \hat{\psi}\, \hat{\sigma}^+ \right)$,
- Initial coherent state: $|\Psi(-\infty)\rangle = e^{-|\psi|^2/2} \sum_n \frac{\psi^n}{\sqrt{n!}} |n, \uparrow\rangle$
- Each pair $|n, \uparrow\rangle \leftrightarrow |n+1, \downarrow\rangle$ undergoes LZ transition

$$H_{n,n+1} = \begin{pmatrix} \lambda t/2 & g\sqrt{n+1} \\ g\sqrt{n+1} & -\lambda t/2 \end{pmatrix}, \qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix}$$

- Final state:

$$|\Psi(+\infty)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1} |n, \uparrow\rangle + B_{n+1} |n+1, \downarrow\rangle \right]$$

Description

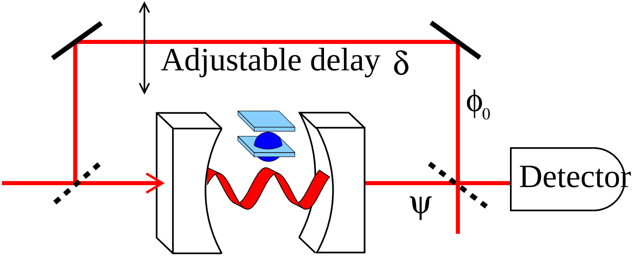
- Hamiltonian: $\hat{H} = \omega_0 \hat{\psi}^\dagger \hat{\psi} + \frac{\lambda t}{2} \hat{\sigma}_z + g \left(\hat{\psi}^\dagger \hat{\sigma}^- + \hat{\psi} \, \hat{\sigma}^+ \right)$,
- Initial coherent state: $|\Psi(-\infty)\rangle = e^{-|\psi|^2/2} \sum_n \frac{\psi^n}{\sqrt{n!}} |n, \uparrow\rangle$
- Each pair $|n, \uparrow\rangle \leftrightarrow |n+1, \downarrow\rangle$ undergoes LZ transition

$$H_{n,n+1} = \begin{pmatrix} \lambda t/2 & g\sqrt{n+1} \\ g\sqrt{n+1} & -\lambda t/2 \end{pmatrix}, \qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix}$$

- Final state:

$$|\Psi(+\infty)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1} |n, \uparrow\rangle + B_{n+1} |n+1, \downarrow\rangle \right]$$

What to measure?

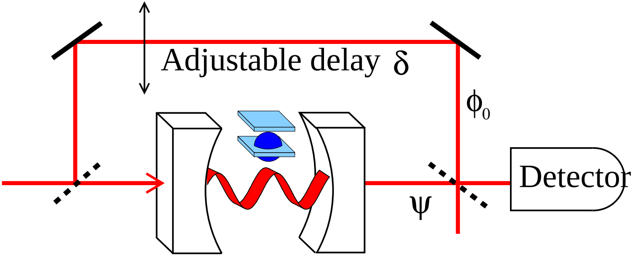


After mixing: $\hat{\psi} + \hat{\phi}_0 e^{i\delta}$ Intensity:

$$I \propto \left\langle (\hat{\psi}^\dagger + \hat{\phi}_0^\dagger e^{-i\delta})(\hat{\psi} + \hat{\phi}_0 e^{i\delta}) \right\rangle$$
$$= \left\langle \hat{\psi}^\dagger \hat{\psi} \right\rangle + \phi_0^2 + \phi_0 \left(\left\langle \psi^\dagger \right\rangle e^{i\delta} + \left\langle \psi \right\rangle e^{-i\delta} \right)$$

Measure $\langle \Psi | \hat{\psi} | \Psi \rangle$

What to measure?

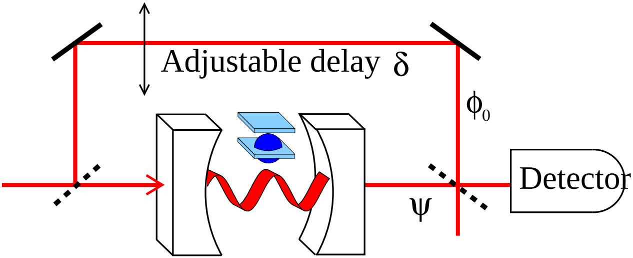


After mixing: $\hat{\psi} + \hat{\phi}_0 e^{i\delta}$ Intensity:

$$I \propto \left\langle (\hat{\psi}^\dagger + \hat{\phi}_0^\dagger e^{-i\delta})(\hat{\psi} + \hat{\phi}_0 e^{i\delta}) \right\rangle$$
$$= \left\langle \hat{\psi}^\dagger \hat{\psi} \right\rangle + \phi_0^2 + \phi_0 \left(\left\langle \psi^\dagger \right\rangle e^{i\delta} + \left\langle \psi \right\rangle e^{-i\delta} \right)$$

Measure $\langle \Psi | \hat{\psi} | \Psi \rangle$

What to measure?



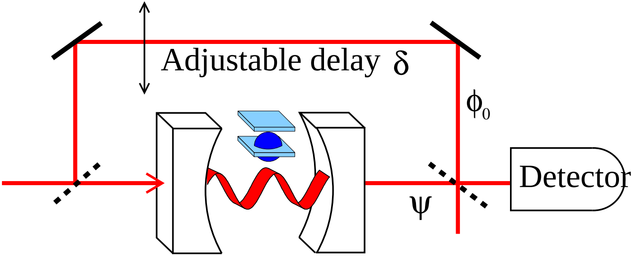
After mixing: $\hat{\psi} + \hat{\phi}_0 e^{i\delta}$ Intensity:

$$I \propto \left\langle (\hat{\psi}^\dagger + \hat{\phi}_0^\dagger e^{-i\delta})(\hat{\psi} + \hat{\phi}_0 e^{i\delta}) \right\rangle$$

$$= \left\langle \hat{\psi}^\dagger \hat{\psi} \right\rangle + \phi_0^2 + \phi_0 \left(\left\langle \hat{\psi}^\dagger \right\rangle e^{i\delta} + \left\langle \hat{\psi} \right\rangle e^{-i\delta} \right)$$

Measure $\langle \Psi | \hat{\psi} | \Psi \rangle$

What to measure?

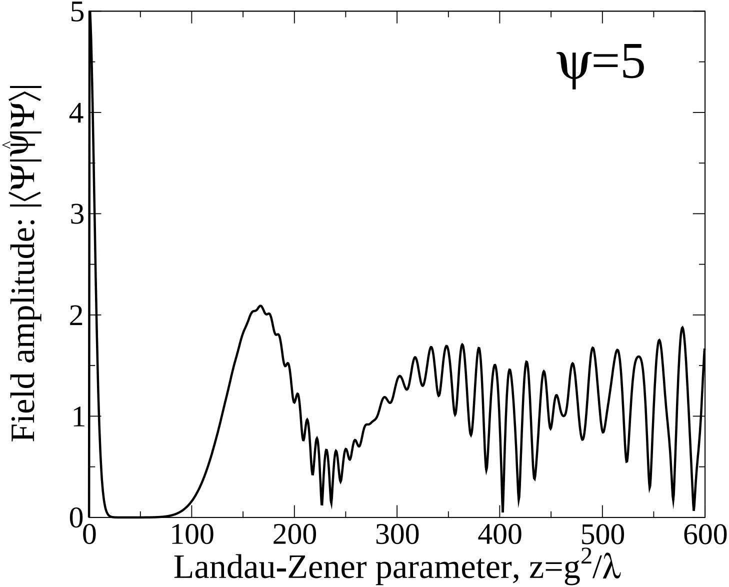


After mixing: $\hat{\psi} + \hat{\phi}_0 e^{i\delta}$ Intensity:

$$I \propto \left\langle (\hat{\psi}^\dagger + \hat{\phi}_0^\dagger e^{-i\delta})(\hat{\psi} + \hat{\phi}_0 e^{i\delta}) \right\rangle$$
$$= \left\langle \hat{\psi}^\dagger \hat{\psi} \right\rangle + \phi_0^2 + \phi_0 \left(\left\langle \psi^\dagger \right\rangle e^{i\delta} + \left\langle \psi \right\rangle e^{-i\delta} \right)$$

Measure $\langle \Psi | \hat{\psi} | \Psi \rangle$

Exact numerical result



Explaining results

Adiabatic limit: $z = g^2/\lambda \gg 1$

$$|\Psi(T)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1} |n, \uparrow\rangle + \qquad B_{n+1} |n+1, \downarrow\rangle \right]$$

$$\phi_n(z, T) = \phi_0 + nz \ln\left(\frac{nz}{e\lambda T^2}\right)$$

Explaining results

Adiabatic limit: $z = g^2/\lambda \gg 1$

$$|\Psi(T)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1} |n, \uparrow\rangle + \qquad B_{n+1} |n+1, \downarrow\rangle \right]$$

$$\phi_n(z, T) = \phi_0 + nz \ln\left(\frac{nz}{e\lambda T^2}\right)$$

Explaining results

Adiabatic limit: $z = g^2/\lambda \gg 1$

$$|\Psi(T)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1} |n, \uparrow\rangle + \qquad B_{n+1} |n+1, \downarrow\rangle \right]$$

$$\phi_n(z, T) = \phi_0 + nz \ln\left(\frac{nz}{e\lambda T^2}\right)$$

Explaining results

Adiabatic limit: $z = g^2/\lambda \gg 1$

$$|\Psi(T)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1}|n,\uparrow\rangle + e^{i\phi_{n+1}(T,z)}|n+1,\downarrow\rangle \right]$$

$$\phi_n(z,T) = \phi_0 + nz \ln\left(\frac{nz}{e\lambda T^2}\right)$$

Explaining results

Adiabatic limit: $z = g^2/\lambda \gg 1$

$$|\Psi(T)\rangle = e^{-|\psi|^2/2} \sum_{n=0}^{\infty} \frac{\psi^n}{\sqrt{n!}} \left[A_{n+1} |n, \uparrow\rangle + e^{i\phi_{n+1}(T,z)} |n+1, \downarrow\rangle \right]$$

$$\phi_n(z, T) = \phi_0 + nz \ln\left(\frac{nz}{e\lambda T^2}\right)$$

Explaining results: field amplitude

$$\langle \Psi | \psi | \Psi \rangle = \psi e^{-|\psi|^2} \sum_n \frac{|\psi|^{2n}}{n!} \sqrt{\frac{n+2}{n+1}} e^{i(\phi_{n+2} - \phi_{n+1})}$$

$$\Delta \phi_n = z \left[(n+1) \ln(n+1) - n \ln n \right]$$

$$\Delta \phi_{|\psi|^2 + m} \simeq \Delta \phi_{|\psi|^2} + \frac{zm}{|\psi|^2} - \frac{zm^2}{2|\psi|^4}$$

Revival at $z = 2\pi N |\psi|^2$, fails at $z \gtrsim |\psi|^3$

Explaining results: field amplitude

$$\langle \Psi | \psi | \Psi \rangle = \psi e^{-|\psi|^2} \sum_n \frac{|\psi|^{2n}}{n!} e^{i(\phi_{n+2} - \phi_{n+1})}$$

$$\Delta\phi_n = z\left[(n+1)\ln(n+1) - n\ln n \right]$$

$$\Delta\phi_{|\psi|^2+m} \simeq \Delta\phi_{|\psi|^2} + \frac{zm}{|\psi|^2} - \frac{zm^2}{2|\psi|^4}$$

Revival at $z = 2\pi N|\psi|^2$, fails at $z \gtrsim |\psi|^3$

Explaining results: field amplitude

$$\langle \Psi | \psi | \Psi \rangle = \psi e^{-|\psi|^2} \sum_n \frac{|\psi|^{2n}}{n!} e^{i\Delta\phi_{n+1}}$$

$$\Delta\phi_n = z \left[(n+1)\ln(n+1) - n \ln n + \ln\left(\frac{z}{e\lambda T^2}\right) \right]$$

$$\Delta\phi_{|\psi|^2+m} \simeq \Delta\phi_{|\psi|^2} + \frac{zm}{|\psi|^2} - \frac{zm^2}{2|\psi|^4}$$

Revival at $z = 2\pi N |\psi|^2$, fails at $z \gtrsim |\psi|^3$

Explaining results: field amplitude

$$\langle\Psi|\psi|\Psi\rangle = \psi e^{-|\psi|^2} \sum_n \frac{|\psi|^{2n}}{n!} e^{i\Delta\phi_{n+1}}$$

$$\Delta\phi_n = z\left[(n+1)\ln(n+1) - n\ln n + \ln\left(\frac{z}{e\lambda T^2}\right)\right]$$

$$\Delta\phi_{|\psi|^2+m} \simeq \Delta\phi_{|\psi|^2} + \frac{zm}{|\psi|^2} - \frac{zm^2}{2|\psi|^4}$$

Revival at $z = 2\pi N|\psi|^2$, fails at $z \gtrsim |\psi|^3$

Explaining results: field amplitude

$$\langle\Psi|\psi|\Psi\rangle = \psi e^{-|\psi|^2} \sum_n \frac{|\psi|^{2n}}{n!} e^{i\Delta\phi_{n+1}}$$

$$\Delta\phi_n = z\left[(n+1)\ln(n+1) - n\ln n + \ln\left(\frac{z}{e\lambda T^2}\right)\right]$$

$$\Delta\phi_{|\psi|^2+m} \simeq \Delta\phi_{|\psi|^2} + \frac{zm}{|\psi|^2} - \frac{zm^2}{2|\psi|^4}$$

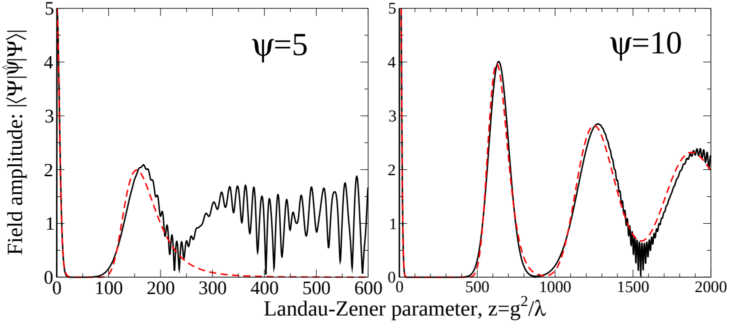
Revival at $z = 2\pi N|\psi|^2$, fails at $z \gtrsim |\psi|^3$

Explaining results: approximation

$$\langle \Psi | \psi | \Psi \rangle = \frac{|\psi|}{(1 + z^2/|\psi|^4)^{1/4}} \sum_{N=0}^{N_{\text{max}}} \exp\left[-\frac{(z - 2\pi N |\psi|^2)^2}{2|\psi|^2 (1 + z^2/|\psi|^4)} \right]$$

Explaining results: approximation

$$\langle\Psi|\psi|\Psi\rangle = \frac{|\psi|}{(1+z^2/|\psi|^4)^{1/4}} \sum_{N=0}^{N_{\max}} \exp\left[-\frac{(z-2\pi N|\psi|^2)^2}{2|\psi|^2(1+z^2/|\psi|^4)}\right]$$



Revivals are **not** coherent states

Wigner function $W(x, p) = \frac{1}{\pi} \int dy \, \Psi^*(x + y)\Psi(x - y)e^{2ipy}$.

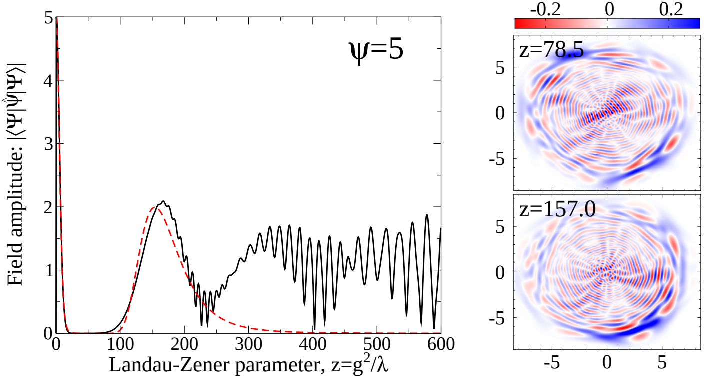
For a coherent state, $W(x, p) = \frac{1}{\pi} \exp\left[-\left(x - \sqrt{2}\psi\right)^2 - p^2\right]$

Revivals are **not** coherent states

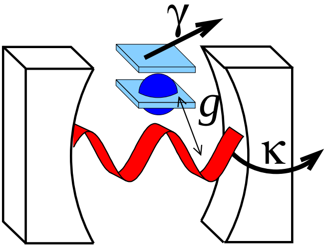
Wigner function $W(x,p) = \frac{1}{\pi}\int dy\, \Psi^*(x+y)\Psi(x-y)e^{2ipy}$.
For a coherent state, $W(x,p) = \frac{1}{\pi}\exp\left[-\left(x-\sqrt{2}\psi\right)^2 - p^2\right]$

Revivals are **not** coherent states

Wigner function $W(x,p) = \frac{1}{\pi}\int dy\, \Psi^*(x+y)\Psi(x-y)e^{2ipy}$.
For a coherent state, $W(x,p) = \frac{1}{\pi}\exp\left[-\left(x-\sqrt{2}\psi\right)^2 - p^2\right]$



Possible decay channels



$$\partial_t \hat{\rho} = -i\left[\hat{H}, \hat{\rho}\right] + L_\kappa[\hat{\rho}] + L_\gamma[\hat{\rho}]$$

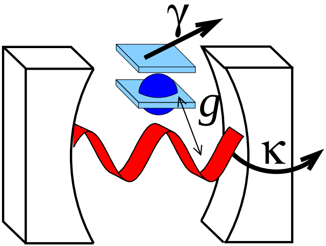
$$L_\kappa[\hat{\rho}] = -\frac{\kappa}{2}\left(\hat{\psi}^\dagger\hat{\psi}\hat{\rho} + \hat{\rho}\hat{\psi}^\dagger\hat{\psi} - 2\hat{\psi}\hat{\rho}\hat{\psi}^\dagger\right)$$

$$L_\gamma[\hat{\rho}] = -\frac{\gamma}{2}\left(\hat{\sigma}_+\hat{\sigma}_-\hat{\rho} + \hat{\rho}\hat{\sigma}_+\hat{\sigma}_- - 2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+\right)$$

Naive estimate: $(\gamma t^*, \kappa t^*) \ll 1$, with $t^* \simeq g|\psi|/\lambda$

$$\left(\frac{\gamma}{g}, \frac{\kappa}{g}\right) \ll \frac{\lambda}{g^2}\frac{1}{|\psi|} \simeq \frac{1}{2\pi|\psi|^3} \simeq 10^{-3}$$

Possible decay channels



$$\partial_t \hat{\rho} = -i \left[\hat{H}, \hat{\rho} \right] + L_\kappa[\hat{\rho}] + L_\gamma[\hat{\rho}]$$

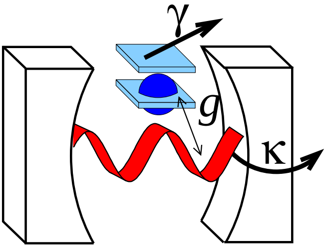
$$L_\kappa[\hat{\rho}] = -\frac{\kappa}{2} \left(\hat{\psi}^\dagger \hat{\psi} \hat{\rho} + \hat{\rho} \hat{\psi}^\dagger \hat{\psi} - 2 \hat{\psi} \hat{\rho} \hat{\psi}^\dagger \right)$$

$$L_\gamma[\hat{\rho}] = -\frac{\gamma}{2} \left(\hat{\sigma}_+ \hat{\sigma}_- \hat{\rho} + \hat{\rho} \hat{\sigma}_+ \hat{\sigma}_- - 2 \hat{\sigma}_- \hat{\rho} \hat{\sigma}_+ \right)$$

Naive estimate: $(\gamma t^*, \kappa t^*) \ll 1$, with $t^* \simeq g|\psi|/\lambda$

$$\left(\frac{\gamma}{g}, \frac{\kappa}{g} \right) \ll \frac{\lambda}{g^2} \frac{1}{|\psi|} \simeq \frac{1}{2\pi|\psi|^3} \simeq 10^{-3}$$

Possible decay channels



$$\partial_t \hat{\rho} = -i\left[\hat{H}, \hat{\rho}\right] + L_\kappa[\hat{\rho}] + L_\gamma[\hat{\rho}]$$

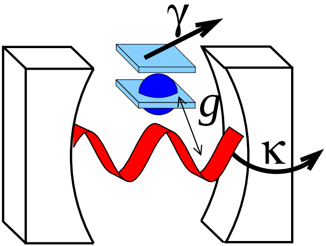
$$L_\kappa[\hat{\rho}] = -\frac{\kappa}{2}\left(\hat{\psi}^\dagger\hat{\psi}\hat{\rho} + \hat{\rho}\hat{\psi}^\dagger\hat{\psi} - 2\hat{\psi}\hat{\rho}\hat{\psi}^\dagger\right)$$

$$L_\gamma[\hat{\rho}] = -\frac{\gamma}{2}\left(\hat{\sigma}_+\hat{\sigma}_-\hat{\rho} + \hat{\rho}\hat{\sigma}_+\hat{\sigma}_- - 2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+\right)$$

Naive estimate: $(\gamma t^*, \kappa t^*) \ll 1$, with $t^* \simeq g|\psi|/\lambda$

$$\left(\frac{\gamma}{g}, \frac{\kappa}{g}\right) \ll \frac{\lambda}{g^2}\frac{1}{|\psi|} \simeq \frac{1}{2\pi|\psi|^3} \simeq 10^{-3}$$

Possible decay channels



$$\partial_t \hat{\rho} = -i\left[\hat{H}, \hat{\rho}\right] + L_\kappa[\hat{\rho}] + L_\gamma[\hat{\rho}]$$

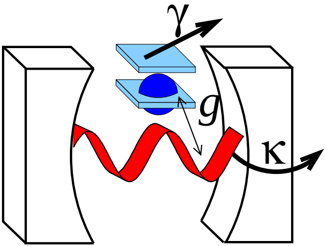
$$L_\kappa[\hat{\rho}] = -\frac{\kappa}{2}\left(\hat{\psi}^\dagger\hat{\psi}\hat{\rho} + \hat{\rho}\hat{\psi}^\dagger\hat{\psi} - 2\hat{\psi}\hat{\rho}\hat{\psi}^\dagger\right)$$

$$L_\gamma[\hat{\rho}] = -\frac{\gamma}{2}\left(\hat{\sigma}_+\hat{\sigma}_-\hat{\rho} + \hat{\rho}\hat{\sigma}_+\hat{\sigma}_- - 2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+\right)$$

Naive estimate: $(\gamma t^*, \kappa t^*) \ll 1$, with $t^* \simeq g|\psi|/\lambda$

$$\left(\frac{\gamma}{g}, \frac{\kappa}{g}\right) \ll \frac{\lambda}{g^2}\frac{1}{|\psi|} \simeq \frac{1}{2\pi|\psi|^3} \simeq 10^{-3}$$

Possible decay channels



$$\partial_t \hat\rho = -i\left[\hat H, \hat\rho\right] + L_\kappa[\hat\rho] + L_\gamma[\hat\rho]$$

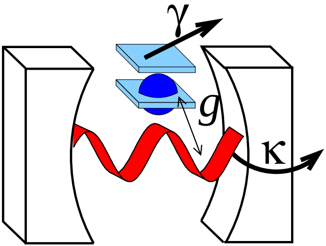
$$L_\kappa[\hat\rho] = -\frac{\kappa}{2}\left(\hat\psi^\dagger\hat\psi\hat\rho + \hat\rho\hat\psi^\dagger\hat\psi - 2\hat\psi\hat\rho\hat\psi^\dagger\right)$$

$$L_\gamma[\hat\rho] = -\frac{\gamma}{2}\left(\hat\sigma_+\hat\sigma_-\hat\rho + \hat\rho\hat\sigma_+\hat\sigma_- - 2\hat\sigma_-\hat\rho\hat\sigma_+\right)$$

Naive estimate: $(\gamma t^*, \kappa t^*) \ll 1$, with $t^* \simeq g|\psi|/\lambda$

$$\left(\frac{\gamma}{g}, \frac{\kappa}{g}\right) \ll \frac{\lambda}{g^2}\frac{1}{|\psi|} \simeq \frac{1}{2\pi|\psi|^3} \simeq 10^{-3}$$

Possible decay channels



$$\partial_t \hat{\rho} = -i\left[\hat{H}, \hat{\rho}\right] + L_\kappa[\hat{\rho}] + L_\gamma[\hat{\rho}]$$

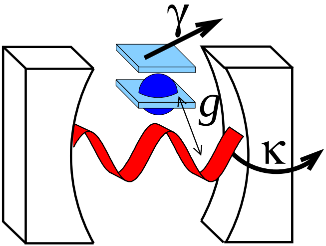
$$L_\kappa[\hat{\rho}] = -\frac{\kappa}{2}\left(\hat{\psi}^\dagger\hat{\psi}\hat{\rho} + \hat{\rho}\hat{\psi}^\dagger\hat{\psi} - 2\hat{\psi}\hat{\rho}\hat{\psi}^\dagger\right)$$

$$L_\gamma[\hat{\rho}] = -\frac{\gamma}{2}\left(\hat{\sigma}_+\hat{\sigma}_-\hat{\rho} + \hat{\rho}\hat{\sigma}_+\hat{\sigma}_- - 2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+\right)$$

Naive estimate: $(\gamma t^*, \kappa t^*) \ll 1$, with $t^* \simeq g|\psi|/\lambda$

$$\left(\frac{\gamma}{g}, \frac{\kappa}{g}\right) \ll \frac{\lambda}{g^2}\frac{1}{|\psi|} \simeq \frac{1}{2\pi|\psi|^3}$$

Possible decay channels



$$\partial_t \hat{\rho} = -i\left[\hat{H}, \hat{\rho}\right] + L_\kappa[\hat{\rho}] + L_\gamma[\hat{\rho}]$$

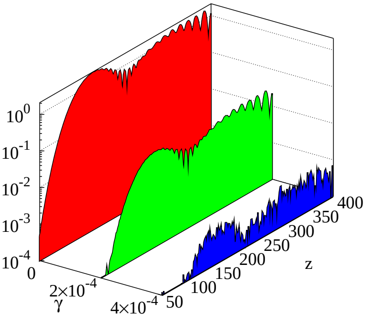
$$L_\kappa[\hat{\rho}] = -\frac{\kappa}{2}\left(\hat{\psi}^\dagger\hat{\psi}\hat{\rho} + \hat{\rho}\hat{\psi}^\dagger\hat{\psi} - 2\hat{\psi}\hat{\rho}\hat{\psi}^\dagger\right)$$

$$L_\gamma[\hat{\rho}] = -\frac{\gamma}{2}\left(\hat{\sigma}_+\hat{\sigma}_-\hat{\rho} + \hat{\rho}\hat{\sigma}_+\hat{\sigma}_- - 2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+\right)$$

Naive estimate: $(\gamma t^*, \kappa t^*) \ll 1$, with $t^* \simeq g|\psi|/\lambda$

$$\left(\frac{\gamma}{g}, \frac{\kappa}{g}\right) \ll \frac{\lambda}{g^2}\frac{1}{|\psi|} \simeq \frac{1}{2\pi|\psi|^3} \simeq 10^{-3}$$

Results including decay



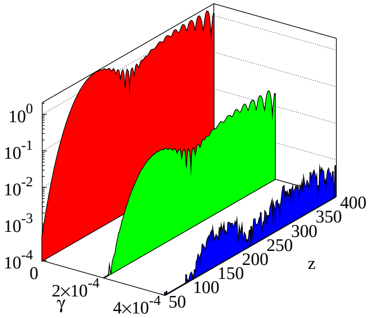
$\gamma_{\mathrm{max}}/g \simeq 2 \times 10^{-4}$

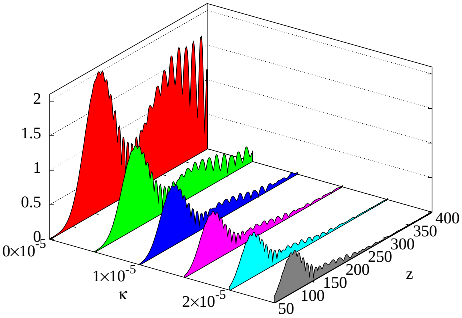
$\gamma_{\mathrm{theory}}/g \ll 10 \times 10^{-4}$

$\kappa_{\mathrm{max}}/g \simeq 1 \times 10^{-5}$

$\kappa_{\mathrm{theory}}/g \ll 100 \times 10^{-5}$

Results including decay





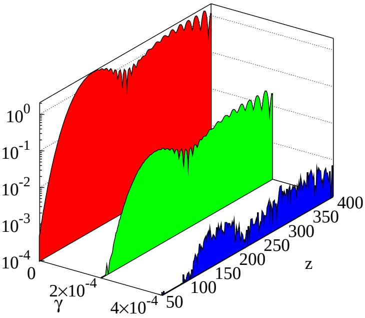
$\gamma_{\text{max}}/g \simeq 2 \times 10^{-4}$

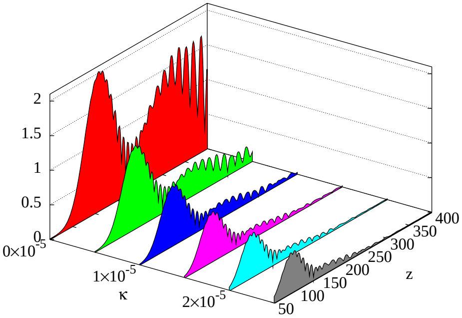
$\kappa_{\text{max}}/g \simeq 1 \times 10^{-5}$

$\gamma_{\text{theory}}/g \ll 10 \times 10^{-4}$

$\kappa_{\text{theory}}/g \ll 100 \times 10^{-5}$

Results including decay

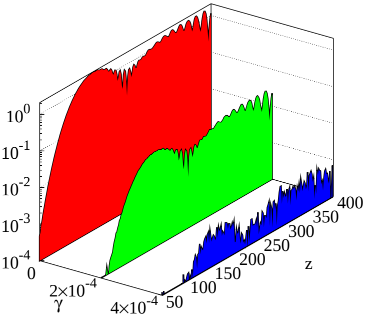


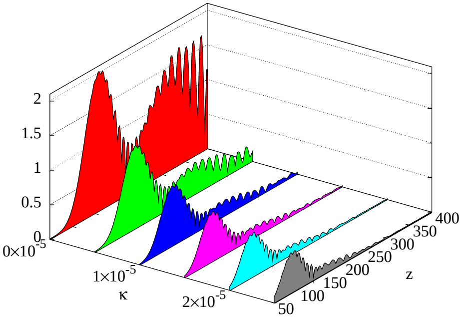


$\gamma_{\text{max}}/g \simeq 2 \times 10^{-4}$
$\gamma_{\text{theory}}/g \ll 10 \times 10^{-4}$

$\kappa_{\text{max}}/g \simeq 1 \times 10^{-5}$
$\kappa_{\text{theory}}/g \ll 100 \times 10^{-5}$

Results including decay





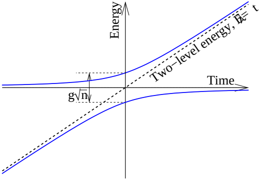
$\gamma_{\mathrm{max}}/g \simeq 2 \times 10^{-4}$

$\gamma_{\mathrm{theory}}/g \ll 10 \times 10^{-4}$

$\kappa_{\mathrm{max}}/g \simeq 1 \times 10^{-5}$

$\kappa_{\mathrm{theory}}/g \ll 100 \times 10^{-5}$

Understanding additional decay: adiabatic approx



$|n,+\rangle = \cos\theta_n|n,\downarrow\rangle + \sin\theta_n|n-1,\uparrow\rangle$ Adiabatic if:

$$P_{\text{trans}} = \frac{\left|\langle n-1,-|\hat{\psi}|n,+\rangle\right|^2}{\langle n,+|\hat{\psi}^\dagger\hat{\psi}|n,+\rangle} \le \frac{27}{256n^2} \ll 1$$

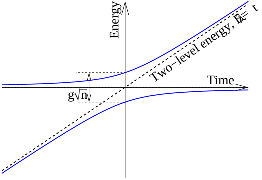
If adiabatic, $\Lambda_n = \rho_{n,n+1}$:

$$\frac{d\Lambda_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]\Lambda_n - \kappa\left[\left(n-\frac{1}{2}\right)\Lambda_n - \sqrt{n(n+1)}A_{n+1}\right]$$

When $|t| \le g\sqrt{n}/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5\times10^{-5}$

Understanding additional decay: adiabatic approx



$$|n,+\rangle = \cos\theta_n |n,\downarrow\rangle + \sin\theta_n |n-1,\uparrow\rangle$$

Adiabatic if:

$$P_{\text{trans}} = \frac{\left|\langle n-1,-|\hat{\psi}|n,+\rangle\right|^2}{\langle n,+|\hat{\psi}^\dagger\hat{\psi}|n,+\rangle} \le \frac{27}{256n^2} \ll 1$$

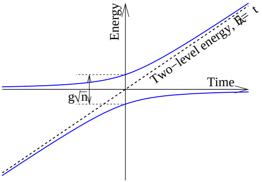
If adiabatic, $\Lambda_n = \rho_{n,n+1}$:

$$\frac{d\Lambda_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]\Lambda_n - \kappa\left[\left(n-\frac{1}{2}\right)\Lambda_n - \sqrt{n(n+1)}\Lambda_{n+1}\right]$$

When $|t| \le g\sqrt{n}/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5\times10^{-5}$

Understanding additional decay: adiabatic approx



$|n,+\rangle = \cos\theta_n |n,\downarrow\rangle + \sin\theta_n |n-1,\uparrow\rangle$ Adiabatic if:

$$P_{\text{trans}} = \frac{\left|\langle n-1,-|\hat{\psi}|n,+\rangle\right|^2}{\langle n,+|\hat{\psi}^\dagger\hat{\psi}|n,+\rangle} \leq \frac{27}{256n^2} \ll 1$$

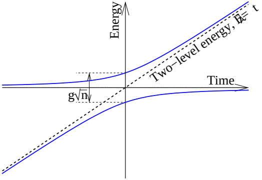
If adiabatic, $\Lambda_n = \rho_{n,n+1}$:

$$\frac{d\Lambda_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]\Lambda_n - \kappa\left[\left(n-\frac{1}{2}\right)\Lambda_n - \sqrt{n(n+1)}A_{n+1}\right]$$

When $|t| \leq g\sqrt{n}/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5\times 10^{-5}$

Understanding additional decay: adiabatic approx



$|n,+\rangle = \cos\theta_n |n,\downarrow\rangle + \sin\theta_n |n-1,\uparrow\rangle$ Adiabatic if:

$$P_{\text{trans}} = \frac{\left|\langle n-1,-|\hat{\psi}|n,+\rangle\right|^2}{\langle n,+|\hat{\psi}^\dagger\hat{\psi}|n,+\rangle} \leq \frac{27}{256 n^2} \ll 1$$

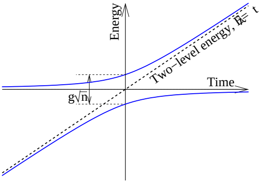
If adiabatic, $\Lambda_n = \rho_{n,n+1}$:

$$\frac{d\Lambda_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]\Lambda_n - \kappa\left[\left(n-\frac{1}{2}\right)\Lambda_n - \sqrt{n(n+1)}\Lambda_{n+1}\right]$$

When $|t| \leq g\sqrt{n}/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5\times 10^{-5}$

Understanding additional decay: adiabatic approx



$|n,+\rangle = \cos\theta_n |n,\downarrow\rangle + \sin\theta_n |n-1,\uparrow\rangle$ Adiabatic if:

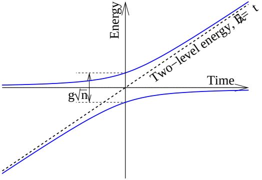
$$P_{\text{trans}} = \frac{\left|\langle n-1,-|\hat{\psi}|n,+\rangle\right|^2}{\langle n,+|\hat{\psi}^\dagger\hat{\psi}|n,+\rangle} \le \frac{27}{256 n^2} \ll 1$$

If adiabatic, $\Lambda_n = \rho_{n,n+1}$:

$$\frac{d\Lambda_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]\Lambda_n - \kappa\left[\left(n-\frac{1}{2}\right)\Lambda_n - \sqrt{n(n+1)}A_{n+1}\right]$$

When $|t| \le g\sqrt{n}/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5\times 10^{-5}$



$|n,+\rangle = \cos\theta_n |n,\downarrow\rangle + \sin\theta_n |n-1,\uparrow\rangle$ Adiabatic if:

$$P_{\text{trans}} = \frac{\left| \langle n-1,-|\hat{\psi}|n,+\rangle \right|^2}{\langle n,+|\hat{\psi}^\dagger \hat{\psi}|n,+\rangle} \leq \frac{27}{256 n^2} \ll 1$$

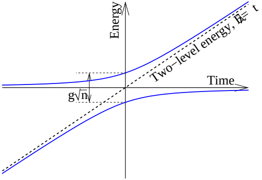
If adiabatic, $\Lambda_n = \rho_{n,n+1}$:

$$\frac{d\Lambda_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]\Lambda_n - \kappa\left[\left(n-\frac{1}{2}\right)\Lambda_n - \sqrt{n(n+1)}A_{n+1}\right]$$

When $|t| \leq g\sqrt{n}/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5\times 10^{-5}$

Understanding additional decay: adiabatic approx



$|n,+\rangle = \cos\theta_n |n,\downarrow\rangle + \sin\theta_n |n-1,\uparrow\rangle$ Adiabatic if:

$$P_{\text{trans}} = \frac{\left|\langle n-1,-|\hat{\psi}|n,+\rangle\right|^2}{\langle n,+|\hat{\psi}^\dagger\hat{\psi}|n,+\rangle} \le \frac{27}{256 n^2} \ll 1$$

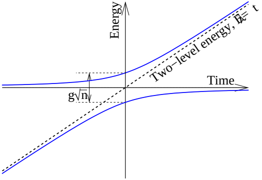
If adiabatic, $\Lambda_n = \rho_{n,n+1}$:

$$\frac{d\Lambda_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]\Lambda_n - \kappa\left[\left(n-\frac{1}{2}\right)\Lambda_n - \sqrt{n(n+1)}A_{n+1}\right]$$

When $|t| \le g\sqrt{n}/\lambda$, decay rate κn $\simeq \kappa|\psi|^2 \gg \kappa$.

$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5 \times 10^{-5}$

Understanding additional decay: adiabatic approx



$|n,+\rangle = \cos\theta_n |n,\downarrow\rangle + \sin\theta_n |n-1,\uparrow\rangle$ Adiabatic if:

$$P_{\text{trans}} = \frac{\left|\langle n-1,-|\hat{\psi}|n,+\rangle\right|^2}{\langle n,+|\hat{\psi}^\dagger\hat{\psi}|n,+\rangle} \leq \frac{27}{256n^2} \ll 1$$

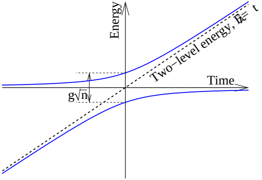
If adiabatic, $\Lambda_n = \rho_{n,n+1}$:

$$\frac{d\Lambda_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]\Lambda_n - \kappa\left[\left(n-\frac{1}{2}\right)\Lambda_n - \sqrt{n(n+1)}A_{n+1}\right]$$

When $|t| \leq g\sqrt{n}/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5 \times 10^{-5}$

Understanding additional decay: adiabatic approx



$|n,+\rangle = \cos\theta_n |n,\downarrow\rangle + \sin\theta_n |n-1,\uparrow\rangle$ Adiabatic if:

$$P_{\text{trans}} = \frac{\left|\langle n-1,-|\hat{\psi}|n,+\rangle\right|^2}{\langle n,+|\hat{\psi}^\dagger\hat{\psi}|n,+\rangle} \leq \frac{27}{256n^2} \ll 1$$

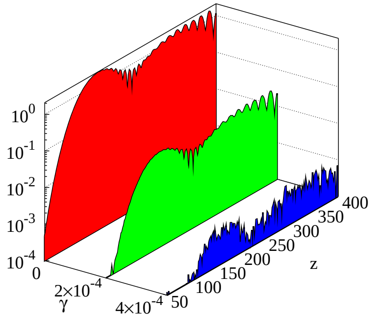
If adiabatic, $\Lambda_n = \rho_{n,n+1}$:

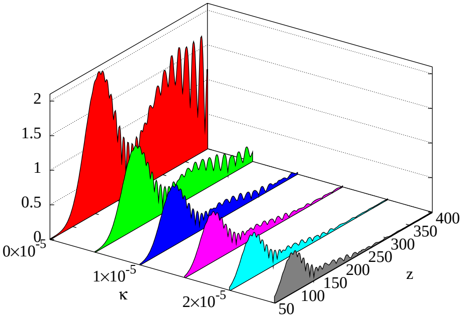
$$\frac{d\Lambda_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]\Lambda_n - \kappa\left[\left(n-\frac{1}{2}\right)\Lambda_n - \sqrt{n(n+1)}A_{n+1}\right]$$

When $|t| \leq g\sqrt{n}/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5\times 10^{-5}$

Results including decay

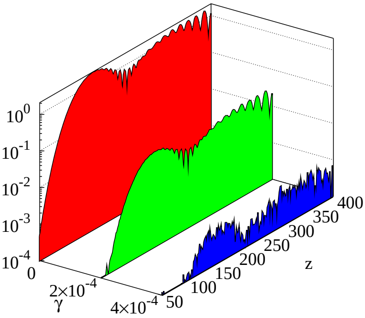


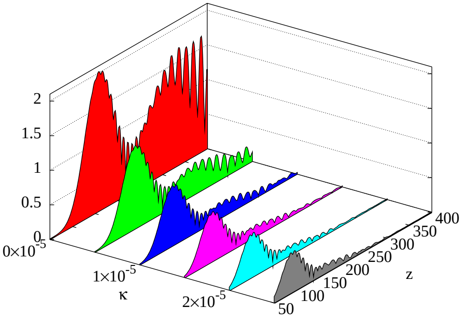


$\gamma_{\text{max}}/g \simeq 2 \times 10^{-4}$
$\gamma_{\text{theory}}/g \ll 10 \times 10^{-4}$

$\kappa_{\text{max}}/g \simeq 1 \times 10^{-5}$
$\kappa_{\text{theory}}/g \ll 100 \times 10^{-5}$

Results including decay





$\gamma_{\text{max}}/g \simeq 2 \times 10^{-4}$

$\gamma_{\text{theory}}/g \ll 10 \times 10^{-4}$

$\kappa_{\text{max}}/g \simeq 1 \times 10^{-5}$

$\kappa_{\text{theory}}/g \ll 5 \times 10^{-5}$

Landau Zener processes in many body systems

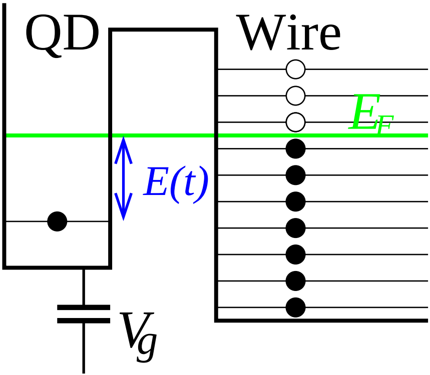
Localised fermion coupled to a continuum of states

Jonathan Keeling[1], L. S. Levitov[2] and A. Shytov[3]

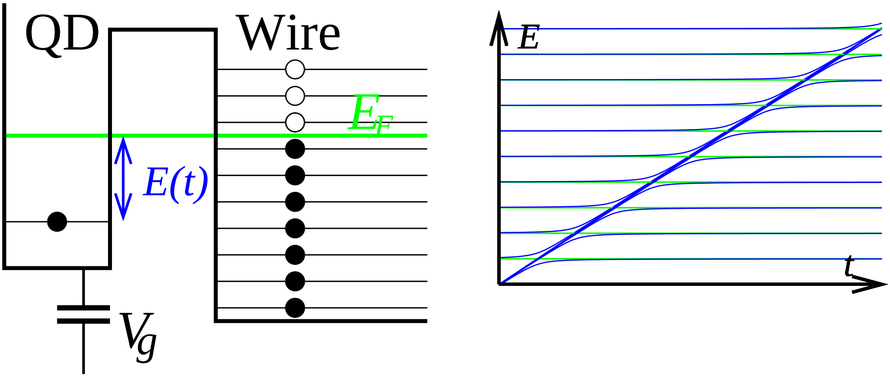
[1]University of Cambridge [2]Massachusetts Institute of Technology [3]Brookhaven National Lab

November 28, 2007

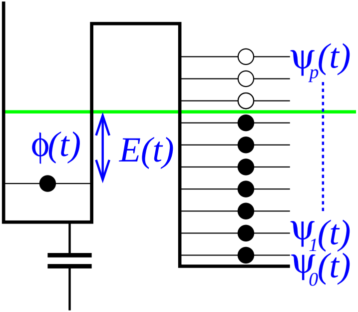
Physical problem



Physical problem



Single particle problem



Regularly spaced continuum:

$$\psi(x,t) = \sum \psi_p(t) e^{ipx}$$

Thus, continuum equations:

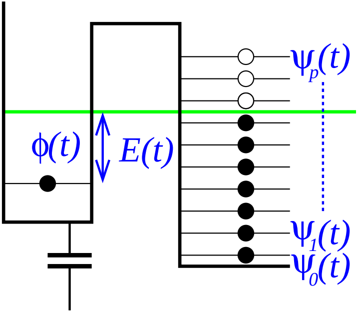
$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x,t)\delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x,t) = g\delta(x)\phi(t)$$

Given: $\psi(x<0,t) = \frac{e^{-i\epsilon' t}}{\sqrt{2\pi}}$, find:

$$U(\epsilon,\epsilon') = \int dt \psi(x>0,t) \frac{e^{i\epsilon t}}{\sqrt{2\pi}}$$

Single particle problem



Regularly spaced continuum:

$$\psi(x, t) = \sum \psi_p(t) e^{ipx}$$

Thus, continuum equations:

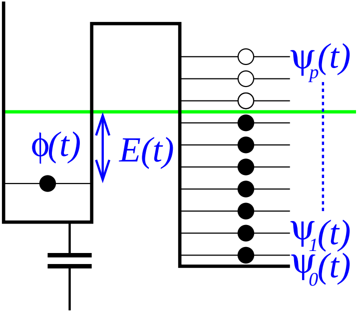
$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x, t)\delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x, t) = g\delta(x)\phi(t)$$

Given: $\psi(x < 0, t) = \frac{e^{-i\epsilon' t}}{\sqrt{2\pi}}$, find:

$$U(\epsilon, \epsilon') = \int dt \psi(x > 0, t) \frac{e^{i\epsilon t}}{\sqrt{2\pi}}$$

Single particle problem



Regularly spaced continuum:

$$\psi(x, t) = \sum \psi_p(t) e^{ipx}$$

Thus, continuum equations:

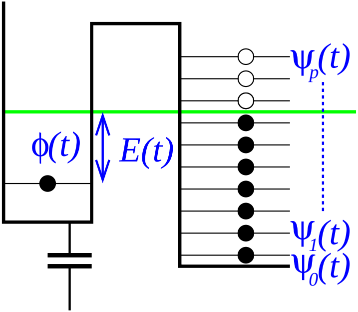
$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x, t)\delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x, t) = g\delta(x)\phi(t)$$

Given: $\psi(x < 0, t) = \frac{e^{-i\epsilon' t}}{\sqrt{2\pi}}$, find:

$$U(\epsilon, \epsilon') = \int dt \psi(x > 0, t) \frac{e^{i\epsilon t}}{\sqrt{2\pi}}$$

Single particle problem



Regularly spaced continuum:

$$\psi(x,t) = \sum \psi_p(t) e^{ipx}$$

Thus, continuum equations:

$$[i\partial_t - E(t)]\phi(t) = g\int dx \psi(x,t)\delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x,t) = g\delta(x)\phi(t)$$

Given: $\psi(x<0,t) = \frac{e^{-i\epsilon' t}}{\sqrt{2\pi}}$, find:

$$U(\epsilon,\epsilon') = \int dt \psi\left(x>0,t\right)\frac{e^{i\epsilon t}}{\sqrt{2\pi}}$$

Solving Schrodinger equations

Equations:

$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x,t)\delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x,t) = g\delta(x)\phi(t)$$

Can solve for general $E(t)$, writing:

$$\psi(x,t) = \psi_0\left(t - \frac{x}{v}\right) - i\frac{g}{v}\phi\left(t - \frac{x}{v}\right)\Theta(x)$$

$$\text{so: } \left[i\partial_t - E(t) + i\frac{g^2}{2v}\right]\phi(t) = g\psi_0(t)$$

Introduce decay rate $\Gamma = g^2/v$

Solving Schrodinger equations

Equations:

$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x,t)\delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x,t) = g\delta(x)\phi(t)$$

Can solve for general $E(t)$, writing:

$$\psi(x,t) = \psi_0\left(t - \frac{x}{v}\right) - i\frac{g}{v}\phi\left(t - \frac{x}{v}\right)\Theta(x)$$

$$\text{so: } \left[i\partial_t - E(t) + i\frac{g^2}{2v}\right]\phi(t) = g\psi_0(t)$$

Introduce decay rate $\Gamma = g^2/v$

Solving Schrodinger equations

Equations:

$$[i\partial_t - E(t)]\phi(t) = g \int dx \psi(x,t)\delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x,t) = g\delta(x)\phi(t)$$

Can solve for general $E(t)$, writing:

$$\psi(x,t) = \psi_0\left(t - \frac{x}{v}\right) - i\frac{g}{v}\phi\left(t - \frac{x}{v}\right)\Theta(x)$$

$$\text{so: } \left[i\partial_t - E(t) + i\frac{g^2}{2v}\right]\phi(t) = g\psi_0(t)$$

Introduce decay rate $\Gamma = g^2/v$

Solving Schrodinger equations

Equations:

$$[i\partial_t - E(t)]\phi(t) = g\int dx\psi(x,t)\delta(x)$$

$$(i\partial_t + iv\partial_x)\psi(x,t) = g\delta(x)\phi(t)$$

Can solve for general $E(t)$, writing:

$$\psi(x,t) = \psi_0\left(t - \frac{x}{v}\right) - i\frac{g}{v}\phi\left(t - \frac{x}{v}\right)\Theta(x)$$

$$\text{so: } \left[i\partial_t - E(t) + i\frac{g^2}{2v}\right]\phi(t) = g\psi_0(t)$$

Introduce decay rate $\Gamma = g^2/v$

Scattering matrix for arbitrary times

Solution for $\phi(t) = g \int_{-\infty}^{t} dt' \psi_0(t') \exp\left[-\frac{\Gamma}{2}(t-t') + i\int_{t'}^{t} E(\tau) d\tau\right]$

Then: $\psi_0(t') = \frac{1}{\sqrt{2\pi}} e^{-i\epsilon' t'} \to \phi(t)$

Scattering matrix for arbitrary times

Solution for $\phi(t) = g \int_{-\infty}^{t} dt' \psi_0(t') \exp\left[-\frac{\Gamma}{2}(t - t') + i \int_{t'}^{t} E(\tau) d\tau \right]$

Then: $\psi_0(t') = \frac{1}{\sqrt{2\pi}} e^{-i\epsilon' t'} \to \phi(t) \to \psi(x, t)$

$$U(\epsilon, \epsilon') = \frac{\Gamma}{2\pi} \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \exp\left[i(\epsilon t - \epsilon' t') - \frac{\Gamma}{2}(t - t') + i \int_{t'}^{t} E(\tau) d\tau \right]$$

If $E(t) = \lambda t$, elementary form:

$$U(\epsilon, \epsilon') \propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)}$$

Scattering matrix for arbitrary times

$$\text{Solution for } \phi(t) = g \int_{-\infty}^{t} dt' \psi_0(t') \exp\left[-\frac{\Gamma}{2}(t-t') + i \int_{t'}^{t} E(\tau) d\tau\right]$$

Then: $\psi_0(t') = \frac{1}{\sqrt{2\pi}} e^{-i\epsilon' t'} \rightarrow \phi(t) \rightarrow \psi(x,t)$

$$U(\epsilon,\epsilon') = \frac{\Gamma}{2\pi} \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \exp\left[i(\epsilon t - \epsilon' t') - \frac{\Gamma}{2}(t-t') + i \int_{t'}^{t} E(\tau) d\tau\right]$$

If $E(t) = \lambda t$, elementary form:

$$U(\epsilon,\epsilon') \propto e^{-(\Gamma/\lambda)(\epsilon-\epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)}$$

Scattering matrix for arbitrary times

Solution for $\phi(t) = g \int_{-\infty}^{t} dt' \psi_0(t') \exp\left[-\frac{\Gamma}{2}(t - t') + i \int_{t'}^{t} E(\tau) d\tau\right]$

Then: $\psi_0(t') = \frac{1}{\sqrt{2\pi}} e^{-i\epsilon' t'} \rightarrow \phi(t) \rightarrow \psi(x, t)$

$$U(\epsilon, \epsilon') = \frac{\Gamma}{2\pi} \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \exp\left[i(\epsilon t - \epsilon' t') - \frac{\Gamma}{2}(t - t') + i \int_{t'}^{t} E(\tau) d\tau\right]$$

If $E(t) = \lambda t$, elementary form:

$$U(\epsilon, \epsilon') \propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)}$$

Scattering matrix for arbitrary times

Solution for $\phi(t) = g \int_{-\infty}^{t} dt' \psi_0(t') \exp\left[-\frac{\Gamma}{2}(t - t') + i \int_{t'}^{t} E(\tau) d\tau\right]$

Then: $\psi_0(t') = \frac{1}{\sqrt{2\pi}} e^{-i\epsilon' t'} \to \phi(t) \to \psi(x, t)$

$$U(\epsilon, \epsilon') = \frac{\Gamma}{2\pi} \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \exp\left[i(\epsilon t - \epsilon' t') - \frac{\Gamma}{2}(t - t') + i \int_{t'}^{t} E(\tau) d\tau\right]$$

If $E(t) = \lambda t$, elementary form:

$$U(\epsilon, \epsilon') \propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)}$$

Scattering matrix for arbitrary times

$$\text{Solution for } \phi(t) = g \int_{-\infty}^{t} dt' \psi_0(t') \exp\left[-\frac{\Gamma}{2}(t-t') + i \int_{t'}^{t} E(\tau) d\tau\right]$$

Then: $\psi_0(t') = \frac{1}{\sqrt{2\pi}} e^{-i\epsilon' t'} \rightarrow \phi(t) \rightarrow \psi(x,t)$

$$U(\epsilon, \epsilon') = \frac{\Gamma}{2\pi} \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \exp\left[i(\epsilon t - \epsilon' t') - \frac{\Gamma}{2}(t-t') + i \int_{t'}^{t} E(\tau) d\tau\right]$$

If $E(t) = \lambda t$, elementary form:

$$U(\epsilon, \epsilon') \propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)}$$

Scattering matrix for arbitrary times

Solution for $\phi(t) = g \int_{-\infty}^{t} dt' \psi_0(t') \exp\left[-\frac{\Gamma}{2}(t-t') + i\int_{t'}^{t} E(\tau)d\tau\right]$

Then: $\psi_0(t') = \frac{1}{\sqrt{2\pi}} e^{-i\epsilon' t'} \to \phi(t) \to \psi(x,t)$

$$U(\epsilon,\epsilon') = \frac{\Gamma}{2\pi}\int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \exp\left[i(\epsilon t - \epsilon' t') - \frac{\Gamma}{2}(t-t') + i\int_{t'}^{t} E(\tau)d\tau\right]$$

If $E(t) = \lambda t$, elementary form:

$$U(\epsilon,\epsilon') \propto e^{-(\Gamma/\lambda)(\epsilon-\epsilon')-(i/2\lambda)(\epsilon^2-\epsilon'^2)}$$

Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$

For $E = \lambda t$,

$$\langle \epsilon | U | \epsilon' \rangle \propto e^{-(\Gamma/\lambda)(\epsilon-\epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)}$$
$$= U_1(\epsilon) U_2(\epsilon')$$
$$= c |\phi_+\rangle\langle\phi_-|$$

$$P_2 = U_{a\to a'} U_{b\to b'} - U_{a\to b'} U_{b\to a'}$$
$$= \langle a'|\phi_+\rangle\langle\phi_-|a\rangle\langle b'|\phi_+\rangle\langle\phi_-|b\rangle$$
$$- \langle a'|\phi_+\rangle\langle\phi_-|b\rangle\langle b'|\phi_+\rangle\langle\phi_-|a\rangle = 0.$$

Max number of particles transfered = rank of U.

Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$

For $E = \lambda t$,

$$\langle \epsilon | U | \epsilon' \rangle \propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)}$$
$$= U_1(\epsilon) U_2(\epsilon')$$
$$= c |\phi_+\rangle\langle\phi_-|$$

$$\begin{aligned} P_2 &= U_{a\to a'} U_{b\to b'} - U_{a\to b'} U_{b\to a'} \\ &= \langle a'|\phi_+\rangle\langle\phi_-|a\rangle\langle b'|\phi_+\rangle\langle\phi_-|b\rangle \\ &- \langle a'|\phi_+\rangle\langle\phi_-|b\rangle\langle b'|\phi_+\rangle\langle\phi_-|a\rangle = 0. \end{aligned}$$

Max number of particles transfered = rank of U.

Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$

For $E = \lambda t$,

$$\begin{aligned} \langle \epsilon | U | \epsilon' \rangle &\propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)} \\ &= U_1(\epsilon) U_2(\epsilon') \\ &= c|\phi_+\rangle\langle\phi_-| \end{aligned}$$

$$\begin{aligned} P_2 &= U_{a\to a'}U_{b\to b'} - U_{a\to b'}U_{b\to a'} \\ &= \langle a'|\phi_+\rangle\langle\phi_-|a\rangle\langle b'|\phi_+\rangle\langle\phi_-|b\rangle \\ &- \langle a'|\phi_+\rangle\langle\phi_-|b\rangle\langle b'|\phi_+\rangle\langle\phi_-|a\rangle = 0. \end{aligned}$$

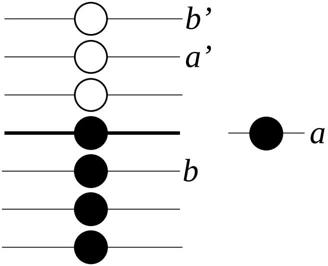
Max number of particles transfered = rank of U.

Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$

For $E = \lambda t$,

$$\langle \epsilon | U | \epsilon' \rangle \propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)}$$
$$= U_1(\epsilon) U_2(\epsilon')$$
$$= c|\phi_+\rangle\langle\phi_-|$$



$$P_2 = U_{a\to a'} U_{b \to b'} - U_{a \to b'} U_{b \to a'}$$
$$= \langle a'|\phi_+\rangle\langle\phi_-|a\rangle\langle b'|\phi_+\rangle\langle\phi_-|b\rangle$$
$$- \langle a'|\phi_+\rangle\langle\phi_-|b\rangle\langle b'|\phi_+\rangle\langle\phi_-|a\rangle = 0.$$

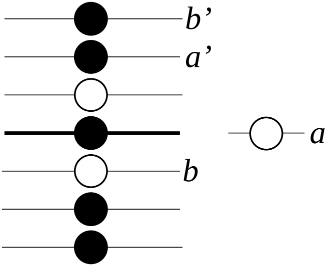
Max number of particles transfered = rank of U.

Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$

For $E = \lambda t$,

$$\begin{aligned}\langle \epsilon | U | \epsilon' \rangle &\propto e^{-(\Gamma/\lambda)(\epsilon-\epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)} \\ &= U_1(\epsilon) U_2(\epsilon') \\ &= c|\phi_+\rangle\langle\phi_-|\end{aligned}$$

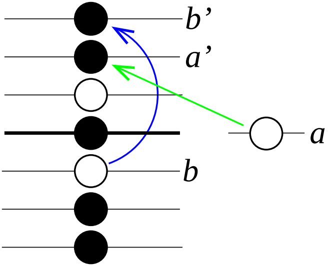


$$\begin{aligned}P_2 &= U_{a\to a'} U_{b\to b'} - U_{a\to b'} U_{b\to a'} \\ &= \langle a'|\phi_+\rangle\langle\phi_-|a\rangle\langle b'|\phi_+\rangle\langle\phi_-|b\rangle \\ &- \langle a'|\phi_+\rangle\langle\phi_-|b\rangle\langle b'|\phi_+\rangle\langle\phi_-|a\rangle = 0.\end{aligned}$$

Max number of particles transfered = rank of U.

Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$



For $E = \lambda t$,

$$\langle \epsilon | U | \epsilon' \rangle \propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)}$$
$$= U_1(\epsilon) U_2(\epsilon')$$
$$= c |\phi_+\rangle\langle\phi_-|$$

$$P_2 = U_{a\to a'} U_{b \to b'} - U_{a \to b'} U_{b \to a'}$$
$$= \langle a' | \phi_+ \rangle \langle \phi_- | a \rangle \langle b' | \phi_+ \rangle \langle \phi_- | b \rangle$$
$$- \langle a' | \phi_+ \rangle \langle \phi_- | b \rangle \langle b' | \phi_+ \rangle \langle \phi_- | a \rangle = 0.$$

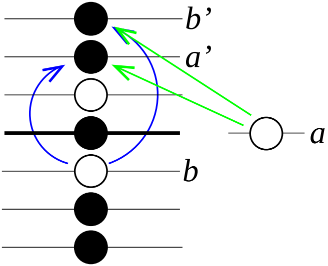
Max number of particles transfered = rank of U.

Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$

For $E = \lambda t$,

$$\begin{aligned} \langle \epsilon | U | \epsilon' \rangle &\propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)} \\ &= U_1(\epsilon) U_2(\epsilon') \\ &= c |\phi_+\rangle\langle\phi_-| \end{aligned}$$



$$\begin{aligned} P_2 &= U_{a\to a'} U_{b\to b'} - U_{a\to b'} U_{b\to a'} \\ &= \langle a'|\phi_+\rangle\langle\phi_-|a\rangle\langle b'|\phi_+\rangle\langle\phi_-|b\rangle \\ &- \langle a'|\phi_+\rangle\langle\phi_-|b\rangle\langle b'|\phi_+\rangle\langle\phi_-|a\rangle = 0. \end{aligned}$$

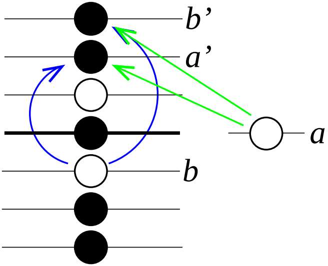
Max number of particles transfered = rank of U.

Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$

For $E = \lambda t$,

$$\begin{aligned} \langle \epsilon | U | \epsilon' \rangle &\propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)} \\ &= U_1(\epsilon) U_2(\epsilon') \\ &= c |\phi_+\rangle \langle \phi_-| \end{aligned}$$



$$\begin{aligned} P_2 &= U_{a \to a'} U_{b \to b'} - U_{a \to b'} U_{b \to a'} \\ &= \langle a' | \phi_+ \rangle \langle \phi_- | a \rangle \langle b' | \phi_+ \rangle \langle \phi_- | b \rangle \\ &- \langle a' | \phi_+ \rangle \langle \phi_- | b \rangle \langle b' | \phi_+ \rangle \langle \phi_- | a \rangle = 0. \end{aligned}$$

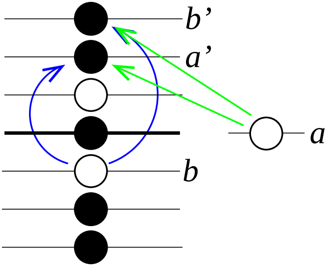
Max number of particles transfered = rank of U.

Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$

For $E = \lambda t$,

$$\langle \epsilon | U | \epsilon' \rangle \propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)}$$
$$= U_1(\epsilon) U_2(\epsilon')$$
$$= c |\phi_+\rangle\langle\phi_-|$$



$$P_2 = U_{a\to a'} U_{b\to b'} - U_{a\to b'} U_{b\to a'}$$
$$= \langle a'|\phi_+\rangle\langle\phi_-|a\rangle\langle b'|\phi_+\rangle\langle\phi_-|b\rangle$$
$$- \langle a'|\phi_+\rangle\langle\phi_-|b\rangle\langle b'|\phi_+\rangle\langle\phi_-|a\rangle = 0.$$

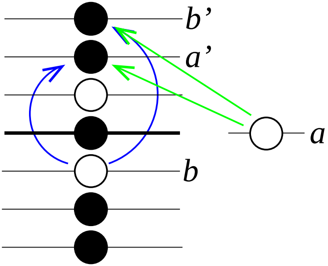
Max number of particles transfered = rank of U.

Elevating to many particle problem: linear time

$$\tilde{a}_\epsilon = \sum_{\epsilon'} \langle \epsilon | U | \epsilon' \rangle a_{\epsilon'}$$

For $E = \lambda t$,

$$\langle \epsilon | U | \epsilon' \rangle \propto e^{-(\Gamma/\lambda)(\epsilon - \epsilon') - (i/2\lambda)(\epsilon^2 - \epsilon'^2)}$$
$$= U_1(\epsilon) U_2(\epsilon')$$
$$= c|\phi_+\rangle\langle\phi_-|$$



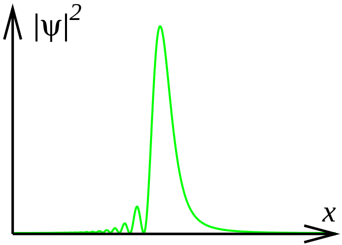
$$P_2 = U_{a\to a'} U_{b\to b'} - U_{a\to b'} U_{b\to a'}$$
$$= \langle a'|\phi_+\rangle\langle\phi_-|a\rangle\langle b'|\phi_+\rangle\langle\phi_-|b\rangle$$
$$- \langle a'|\phi_+\rangle\langle\phi_-|b\rangle\langle b'|\phi_+\rangle\langle\phi_-|a\rangle = 0.$$

Max number of particles transfered = rank of U.

Linear timed dependence, exact state

$$\Psi_p \propto e^{-(\Gamma/c)vp-(i/2\lambda)(vp)^2}$$

Spatial profile:



General time dependence: measuring noise

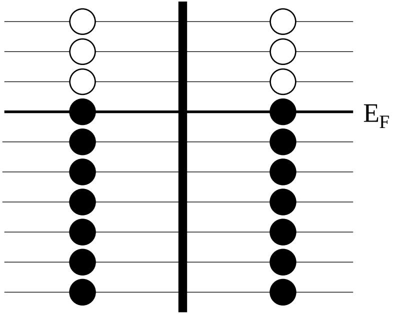
Number of excitations:

$$N^{\mathrm{ex}} = \sum_{\epsilon > \epsilon_{\mathrm{F}} > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_{\mathrm{F}} < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$

General time dependence: measuring noise

Number of excitations:

$$N^{\rm ex} = \sum_{\epsilon > \epsilon_{\rm F} > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_{\rm F} < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



Define: $N^{\rm ex}_{r,l} = N^e_{r,l} + N^h_{r,l}$

$q_{r,l} = e(N^e_{r,l} - N^h_{r,l})$

Then

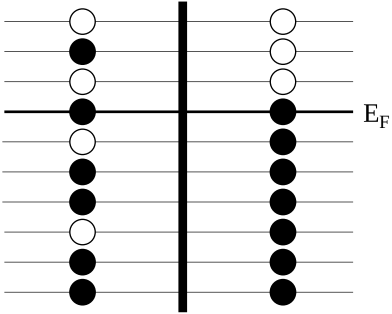
$$\langle q_r \rangle = T \langle q_l \rangle$$

$$\langle \Delta q_r^2 \rangle = T^2 \langle \Delta q_l^2 \rangle + e^2 T(1-T) \langle N_l^{\rm ex} \rangle$$

General time dependence: measuring noise

Number of excitations:

$$N^{\mathrm{ex}} = \sum_{\epsilon > \epsilon_{\mathrm{F}} > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_{\mathrm{F}} < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



Define: $N^{\mathrm{ex}}_{r,l} = N^{e}_{r,l} + N^{h}_{r,l}$

$q_{r,l} = e(N^{e}_{r,l} - N^{h}_{r,l})$

Then

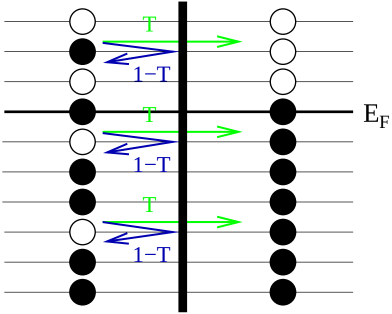
$$\langle q_r \rangle = T \langle q_l \rangle$$

$$\langle \Delta q_r^2 \rangle = T^2 \langle \Delta q_l^2 \rangle + e^2 T(1-T) \langle N_l^{\mathrm{ex}} \rangle$$

General time dependence: measuring noise

Number of excitations:

$$N^{\mathrm{ex}} = \sum_{\epsilon > \epsilon_F > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_F < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



Define: $N^{\mathrm{ex}}_{r,l} = N^{e}_{r,l} + N^{h}_{r,l}$

$q_{r,l} = e(N^{e}_{r,l} - N^{h}_{r,l})$

Then

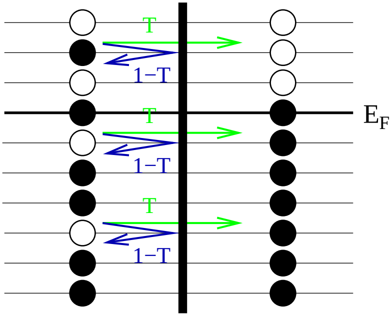
$$\langle q_r \rangle = T \langle q_l \rangle$$

$$\langle \Delta q_r^2 \rangle = T^2 \langle \Delta q_l^2 \rangle + e^2 T(1-T) \langle N_l^{\mathrm{ex}} \rangle$$

General time dependence: measuring noise

Number of excitations:

$$N^{\text{ex}} = \sum_{\epsilon > \epsilon_F > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_F < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



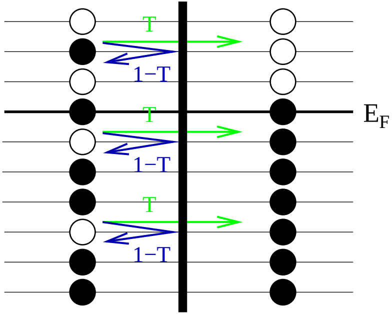
Define: $N^{\text{ex}}_{r,l} = N^{e}_{r,l} + N^{h}_{r,l}$

$$q_{r,l} = e(N^{e}_{r,l} - N^{h}_{r,l})$$

General time dependence: measuring noise

Number of excitations:

$$N^{\mathrm{ex}} = \sum_{\epsilon > \epsilon_{\mathrm{F}} > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_{\mathrm{F}} < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



Define: $N^{\mathrm{ex}}_{r,l} = N^e_{r,l} + N^h_{r,l}$

$$q_{r,l} = e(N^e_{r,l} - N^h_{r,l})$$

Then

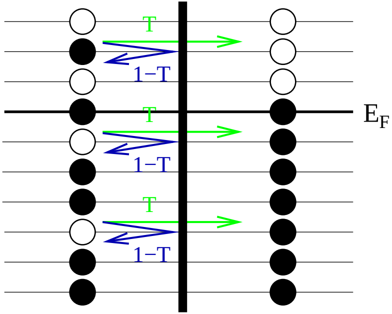
$$\langle q_r \rangle = T \langle q_l \rangle$$

$$\langle \Delta q_r^2 \rangle = T^2 \langle \Delta q_l^2 \rangle + e^2 T(1-T) \langle N_l^{\mathrm{ex}} \rangle$$

General time dependence: measuring noise

Number of excitations:

$$N^{\rm ex} = \sum_{\epsilon > \epsilon_{\rm F} > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_{\rm F} < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



Define: $N^{\rm ex}_{r,l} = N^e_{r,l} + N^h_{r,l}$

$$q_{r,l} = e(N^e_{r,l} - N^h_{r,l})$$

Then

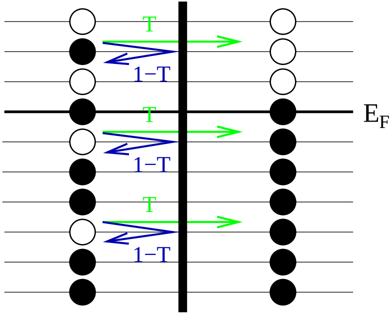
$$\langle q_r \rangle = T \langle q_l \rangle$$

$$\langle \Delta q_r^2 \rangle = T^2 \langle \Delta q_l^2 \rangle + e^2 T(1-T) \langle N_l^{\rm ex} \rangle$$

General time dependence: measuring noise

Number of excitations:

$$N^{\mathrm{ex}} = \sum_{\epsilon > \epsilon_{\mathrm{F}} > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_{\mathrm{F}} < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



Define: $N^{\mathrm{ex}}_{r,l} = N^e_{r,l} + N^h_{r,l}$

$$q_{r,l} = e(N^e_{r,l} - N^h_{r,l})$$

Then

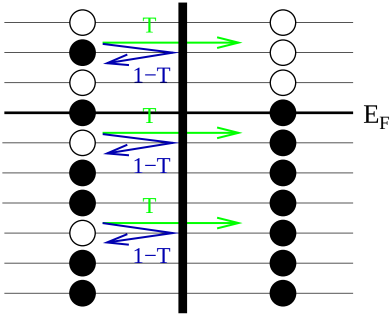
$$\langle q_r \rangle = T \langle q_l \rangle$$

$$\langle \Delta q_r^2 \rangle = T^2 \langle \Delta q_l^2 \rangle + e^2 T(1-T) \langle N_l^{\mathrm{ex}} \rangle$$

General time dependence: measuring noise

Number of excitations:

$$N^{\rm ex} = \sum_{\epsilon > \epsilon_{\rm F} > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_{\rm F} < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



Define: $N^{\rm ex}_{r,l} = N^e_{r,l} + N^h_{r,l}$

$$q_{r,l} = e(N^e_{r,l} - N^h_{r,l})$$

Then

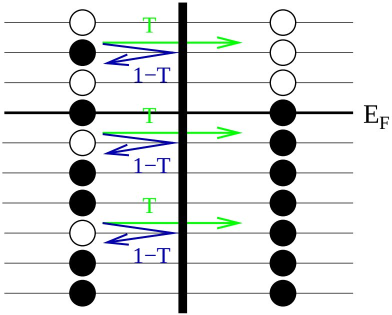
$$\langle q_r \rangle = T \langle q_l \rangle$$

$$\langle \Delta q_r^2 \rangle = T^2 \langle \Delta q_l^2 \rangle + e^2 T(1-T)\langle N_l^{\rm ex} \rangle$$

General time dependence: measuring noise

Number of excitations:

$$N^{\mathrm{ex}} = \sum_{\epsilon > \epsilon_{\mathrm{F}} > \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2 + \sum_{\epsilon < \epsilon_{\mathrm{F}} < \epsilon'} |\langle \epsilon | U | \epsilon' \rangle|^2$$



Define: $N^{\mathrm{ex}}_{r,l} = N^e_{r,l} + N^h_{r,l}$

$$q_{r,l} = e(N^e_{r,l} - N^h_{r,l})$$

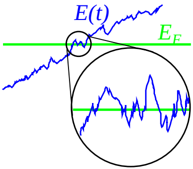
Then

$$\langle q_r \rangle = T \langle q_l \rangle$$

$$\langle \Delta q_r^2 \rangle = T^2 \langle \Delta q_l^2 \rangle + e^2 T(1-T) \langle N_l^{\mathrm{ex}} \rangle$$

Noisy driving

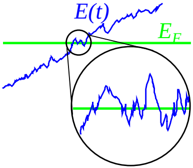
- Suppose $E(t) = \lambda t + \eta(t)$
  $\langle \eta(t) \rangle = 0, \ \langle \eta(t)\eta(t') \rangle = \Gamma_2 \delta(t - t')$.



- To find N^{ex}, need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$
- Integral log divergent: white noise limit
- Infinite no. crossings of Fermi surface

Noisy driving

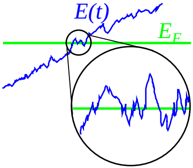
- Suppose $E(t) = \lambda t + \eta(t)$
  $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \Gamma_2 \delta(t - t')$.



- To find N^{ex}, need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$
- Integral log divergent: white noise limit
- Infinite no. crossings of Fermi surface

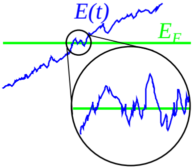
Noisy driving

- Suppose $E(t) = \lambda t + \eta(t)$
  $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \Gamma_2 \delta(t - t')$.



- To find N^{ex}, need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$
- Integral log divergent: white noise limit
- Infinite no. crossings of Fermi surface

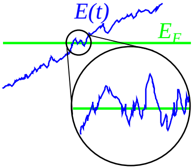
Noisy driving



- Suppose $E(t) = \lambda t + \eta(t)$
  $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \Gamma_2 \delta(t - t')$.
- To find N^{ex}, need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$
- Integral log divergent: white noise limit
- Infinite no. crossings of Fermi surface

Noisy driving

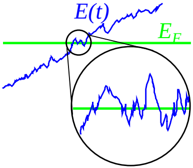
- Suppose $E(t) = \lambda t + \eta(t)$
  $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \Gamma_2 \delta(t - t')$.



- To find N^{ex}, need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$
- Integral log divergent: white noise limit
- Infinite no. crossings of Fermi surface

Noisy driving

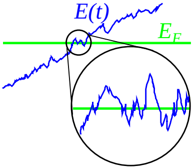
- Suppose $E(t) = \lambda t + \eta(t)$
  $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \Gamma_2 \delta(t - t')$.



- To find N^{ex}, need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$
- Integral log divergent: white noise limit
- Infinite no. crossings of Fermi surface

$$N^{\mathrm{ex}} = \begin{cases} 1 & \lambda \gg \Gamma\Gamma_2 \\ \frac{\Gamma^2}{(\Gamma+\Gamma_2)^2} + \frac{2\Gamma\Gamma_2}{\pi\lambda} \ln \frac{\omega}{\Gamma+\Gamma_2} & \lambda \ll \Gamma\Gamma_2 \end{cases}$$

Noisy driving



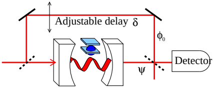
- Suppose $E(t) = \lambda t + \eta(t)$
  $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \Gamma_2 \delta(t - t')$.
- To find N^{ex}, need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$
- Integral log divergent: white noise limit
- Infinite no. crossings of Fermi surface

$$N^{\mathrm{ex}} = \begin{cases} 1 & \lambda \gg \Gamma\Gamma_2 \\ \frac{\Gamma^2}{(\Gamma+\Gamma_2)^2} + \frac{2\Gamma\Gamma_2}{\pi\lambda} \ln \frac{\omega_*}{\Gamma+\Gamma_2} & \lambda \ll \Gamma\Gamma_2 \end{cases}$$

Conclusions

- Exactly solvable many-body generalisations of LZ.
- Photons/Spin:
- Localised/continuum fermions

Conclusions

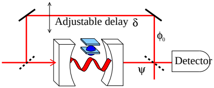
- Exactly solvable many-body generalisations of LZ.
- Photons/Spin: 

  - Coherent state; interference
  - Decay: larger than expected, but feasible.

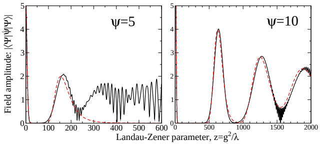
- Localised/continuum fermions

Conclusions

- Exactly solvable many-body generalisations of LZ.
- Photons/Spin:



  - Coherent state; interference

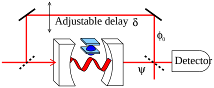


  - Decay: larger than expected, but feasible.

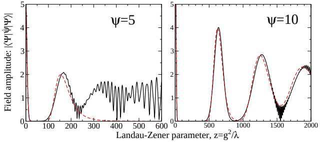
- Localised/continuum fermions

Conclusions

- Exactly solvable many-body generalisations of LZ.
- Photons/Spin:



  - Coherent state; interference

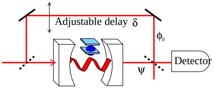


  - Decay: larger than expected, but feasible.

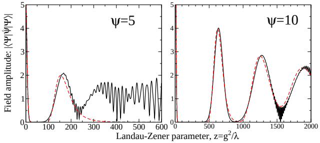
- Localised/continuum fermions

Conclusions

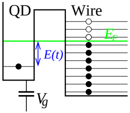
- Exactly solvable many-body generalisations of LZ.
- Photons/Spin:



  - Coherent state; interference



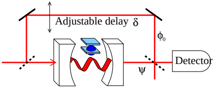
  - Decay: larger than expected, but feasible.
- Localised/continuum fermions



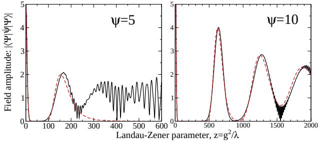
  - Filled fermi sea; interference → cancellations

Conclusions

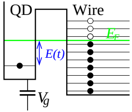
- Exactly solvable many-body generalisations of LZ.
- Photons/Spin:



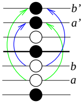
  - Coherent state; interference



  - Decay: larger than expected, but feasible.
- Localised/continuum fermions

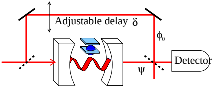


  - Filled fermi sea; interference → cancellations

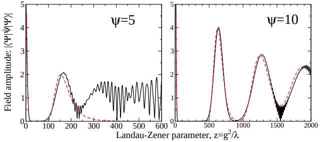


Conclusions

- Exactly solvable many-body generalisations of LZ.
- Photons/Spin:

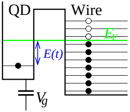


  - Coherent state; interference

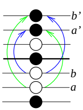


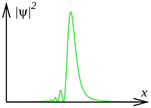
  - Decay: larger than expected, but feasible.

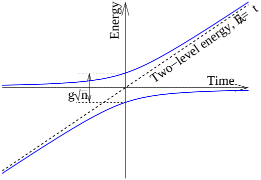
- Localised/continuum fermions



  - Filled fermi sea; interference → cancellations







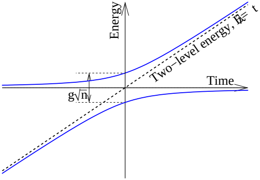
$|n,+\rangle = [\cos(\theta_n)\,|n,\downarrow\rangle + \sin(\theta_n)\,|n-1,\uparrow\rangle]$.
Adiabatic if:

$$P_{\text{trans}}(\theta_n(t)) = \frac{|\langle n-1,-|a|n,+\rangle|^2}{\langle n,+|a^\dagger a|n,+\rangle} \ll 1$$

At $t \to \pm\infty$, $P_{\text{trans}} \to 0$.
At $t \simeq 0$,

$$P_{\text{trans}} \simeq \frac{|\sqrt{n}-\sqrt{n-1}|^2}{2(2n+1)} \simeq \frac{1}{16n^2} \simeq \frac{1}{16|\psi|^4}$$

Understanding additional decay: adiabatic approx



$$|n,+\rangle = [\cos(\theta_n)\,|n,\downarrow\rangle + \sin(\theta_n)\,|n-1,\uparrow\rangle]$$

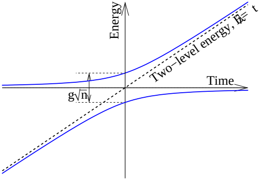
Adiabatic if:

$$P_{\text{trans}}(\theta_n(t)) = \frac{|\langle n-1,-|a|n,+\rangle|^2}{\langle n,+|a^\dagger a|n,+\rangle} \ll 1$$

At $t \to \pm\infty$, $P_{\text{trans}} \to 0$.
At $t \simeq 0$,

$$P_{\text{trans}} \simeq \frac{|\sqrt{n}-\sqrt{n-1}|^2}{2(2n+1)} \simeq \frac{1}{16n^2} \simeq \frac{1}{16|\psi|^4}$$

Understanding additional decay: adiabatic approx



$$|n,+\rangle = [\cos(\theta_n)\,|n,\downarrow\rangle + \sin(\theta_n)\,|n-1,\uparrow\rangle]$$

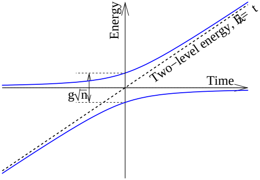
Adiabatic if:

$$P_{\mathrm{trans}}(\theta_n(t)) = \frac{|\langle n-1,-|a|n,+\rangle|^2}{\langle n,+|a^\dagger a|n,+\rangle} \ll 1$$

At $t \to \pm\infty$, $P_{\mathrm{trans}} \to 0$.
At $t \simeq 0$,

$$P_{\mathrm{trans}} \simeq \frac{|\sqrt{n}-\sqrt{n-1}|^2}{2(2n+1)} \simeq \frac{1}{16n^2} \simeq \frac{1}{16|\psi|^4}$$

Understanding additional decay: adiabatic approx



$$|n,+\rangle = [\cos(\theta_n)\,|n,\downarrow\rangle + \sin(\theta_n)\,|n-1,\uparrow\rangle]$$

Adiabatic if:

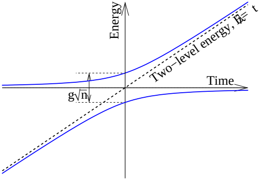
$$P_{\text{trans}}(\theta_n(t)) = \frac{|\langle n-1,-|a|n,+\rangle|^2}{\langle n,+|a^\dagger a|n,+\rangle} \ll 1$$

At $t \to \pm\infty$, $P_{\text{trans}} \to 0$.

At $t \simeq 0$,

$$P_{\text{trans}} \simeq \frac{|\sqrt{n}-\sqrt{n-1}|^2}{2(2n+1)} \simeq \frac{1}{16n^2} \simeq \frac{1}{16|\psi|^4}$$

Understanding additional decay: adiabatic approx



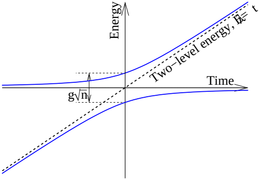
$|n,+\rangle = [\cos(\theta_n)\,|n,\downarrow\rangle + \sin(\theta_n)\,|n-1,\uparrow\rangle]$
Adiabatic if:

$$P_{\text{trans}}(\theta_n(t)) = \frac{|\langle n-1,-|a|n,+\rangle|^2}{\langle n,+|a^\dagger a|n,+\rangle} \ll 1$$

At $t \to \pm\infty$, $P_{\text{trans}} \to 0$.
At $t \simeq 0$,

$$P_{\text{trans}} \simeq \frac{|\sqrt{n} - \sqrt{n-1}|^2}{2(2n+1)} \simeq \frac{1}{16n^2} \simeq \frac{1}{16|\psi|^4}$$

Understanding additional decay: adiabatic approx



$$|n,+\rangle = [\cos(\theta_n)\,|n,\downarrow\rangle + \sin(\theta_n)\,|n-1,\uparrow\rangle]$$

Adiabatic if:

$$P_{\text{trans}}(\theta_n(t)) = \frac{|\langle n-1,-|a|n,+\rangle|^2}{\langle n,+|a^\dagger a|n,+\rangle} \ll 1$$

At $t \to \pm\infty$, $P_{\text{trans}} \to 0$.

At $t \simeq 0$,

$$P_{\text{trans}} \simeq \frac{|\sqrt{n}-\sqrt{n-1}|^2}{2(2n+1)} \simeq \frac{1}{16n^2} \simeq \frac{1}{16|\psi|^4}$$

Understanding additional decay: Density matrix equation

If adiabatic, $A_n = \rho_{n,n+1}$:

$$\frac{dA_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]A_n - \kappa\left[\left(n-\frac{1}{2}\right)A_n - \sqrt{n(n+1)}A_{n+1}\right]$$

When $|t| \leq g/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5\times10^{-5}$

Understanding additional decay: Density matrix equation

If adiabatic, $A_n = \rho_{n,n+1}$:

$$\frac{dA_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]A_n - \kappa\left[\left(n - \frac{1}{2}\right)A_n - \sqrt{n(n+1)}A_{n+1}\right]$$

When $|t| \leq g/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5 \times 10^{-5}$

If adiabatic, $A_n = \rho_{n,n+1}$:

$$\frac{dA_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]A_n - \kappa\left[\left(n-\frac{1}{2}\right)A_n - \sqrt{n(n+1)}A_{n+1}\right]$$

When $|t| \leq g/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5 \times 10^{-5}$

Understanding additional decay: Density matrix equation

If adiabatic, $A_n = \rho_{n,n+1}$:

$$\frac{dA_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]A_n - \kappa\left[\left(n - \frac{1}{2}\right)A_n - \sqrt{n(n+1)}A_{n+1}\right]$$

When $|t| \leq g/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5 \times 10^{-5}$

Understanding additional decay: Density matrix equation

If adiabatic, $A_n = \rho_{n,n+1}$:

$$\frac{dA_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]A_n - \kappa\left[\left(n - \frac{1}{2}\right)A_n - \sqrt{n(n+1)}A_{n+1}\right]$$

When $|t| \leq g/\lambda$, decay rate $\kappa n \simeq \kappa|\psi|^2 \gg \kappa$.

$\kappa/g \ll 1/(2\pi|\psi|^5) \simeq 5 \times 10^{-5}$

Additional decay perturbatively

$$\frac{dA_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]A_n - \kappa\left[\left(n-\frac{1}{2}\right)A_n - \sqrt{n(n+1)}A_{n+1}\right]$$

Gauge transform: $A_n = \tilde{A}_n e^{-i\Delta\phi_{n-1}}$

$$\frac{d\tilde{A}_n}{dt} = -\kappa\left[\left(n-\frac{1}{2}\right)\tilde{A}_n - \sqrt{n(n+1)}\tilde{A}_{n+1}e^{i[\Delta\phi_{n-1}-\Delta\phi_n]}\right]$$

$$\tilde{A}_n(T) = \tilde{A}_n(-T)$$

$$-\kappa\int_{-T}^{T}dt\left[\left(n-\frac{1}{2}\right)\tilde{A}_n - \sqrt{n(n+1)}\tilde{A}_{n+1}e^{i[\Delta\phi_{n-1}-\Delta\phi_n]}\right]$$

Additional decay perturbatively

$$\frac{dA_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]A_n - \kappa\left[\left(n-\frac{1}{2}\right)A_n - \sqrt{n(n+1)}A_{n+1}\right]$$

Gauge transform: $A_n = \tilde{A}_n e^{-i\Delta\phi_{n-1}}$

$$\frac{d\tilde{A}_n}{dt} = -\kappa\left[\left(n-\frac{1}{2}\right)\tilde{A}_n - \sqrt{n(n+1)}\tilde{A}_{n+1}e^{i[\Delta\phi_{n-1}-\Delta\phi_n]}\right]$$

$$\tilde{A}_n(T) = \tilde{A}_n(-T) - \kappa\int_{-T}^{T}dt\left[\left(n-\frac{1}{2}\right)\tilde{A}_n - \sqrt{n(n+1)}\tilde{A}_{n+1}e^{i[\Delta\phi_{n-1}-\Delta\phi_n]}\right]$$

Additional decay perturbatively

$$\frac{dA_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right]A_n - \kappa\left[\left(n-\frac{1}{2}\right)A_n - \sqrt{n(n+1)}A_{n+1}\right]$$

Gauge transform: $A_n = \tilde{A}_n e^{-i\Delta\phi_{n-1}}$

$$\frac{d\tilde{A}_n}{dt} = -\kappa\left[\left(n-\frac{1}{2}\right)\tilde{A}_n - \sqrt{n(n+1)}\tilde{A}_{n+1}e^{i[\Delta\phi_{n-1}-\Delta\phi_n]}\right]$$

$$\tilde{A}_n(T) = \tilde{A}_n(-T) - \kappa\int_{-T}^{T} dt\left[\left(n-\frac{1}{2}\right)\tilde{A}_n - \sqrt{n(n+1)}\tilde{A}_{n+1}e^{i[\Delta\phi_{n-1}-\Delta\phi_n]}\right]$$

Additional decay perturbatively

$$\frac{dA_n}{dt} = i\left[\frac{d\Delta\phi_{n-1}}{dt}\right] A_n - \kappa\left[\left(n-\frac{1}{2}\right)A_n - \sqrt{n(n+1)}A_{n+1}\right]$$

Gauge transform: $A_n = \tilde{A}_n e^{-i\Delta\phi_{n-1}}$

$$\frac{d\tilde{A}_n}{dt} = -\kappa\left[\left(n-\frac{1}{2}\right)\tilde{A}_n - \sqrt{n(n+1)}\tilde{A}_{n+1}e^{i[\Delta\phi_{n-1}-\Delta\phi_n]}\right]$$

$$\tilde{A}_n(T) = \tilde{A}_n(-T) - \kappa\int_{-T}^{T} dt\left[\left(n-\frac{1}{2}\right)\tilde{A}_n - \sqrt{n(n+1)}\tilde{A}_{n+1}e^{i[\Delta\phi_{n-1}-\Delta\phi_n]}\right]$$

Results of perturbative decay

Remove "naive decay",

$$\langle \Psi | \psi(\kappa, \psi_0) | \Psi \rangle_{\text{naive}} = \left\langle \Psi | \psi(0, \psi_0 e^{-\kappa T/2}) | \Psi \right\rangle e^{-\kappa T/2}$$

$$\delta \langle \Psi | \psi | \Psi \rangle = \psi(\kappa |\psi|^2) \sum_n P_n \times \\ \left[T e^{i[\Delta\phi_{n-1}(-T) - \Delta\phi_{n-1}(T)]} + T e^{i[\Delta\phi_n(-T) - \Delta\phi_n(T)]} \right. \\ \left. - e^{i[\Delta\phi_n(-T) - \Delta\phi_{n-1}(T)]} \int_{-T}^{T} e^{i[\Delta\phi_{n-1} - \Delta\phi_n]} dt \right]$$

Results of perturbative decay

Remove "naive decay",

$$\langle \Psi | \psi(\kappa, \psi_0) | \Psi \rangle_{\text{naive}} = \left\langle \Psi | \psi(0, \psi_0 e^{-\kappa T/2}) | \Psi \right\rangle e^{-\kappa T/2}$$

$$\delta \langle \Psi | \psi | \Psi \rangle = \psi(\kappa |\psi|^2) \sum_n P_n \times \left[T e^{i[\Delta\phi_{n-1}(-T) - \Delta\phi_{n-1}(T)]} + T e^{i[\Delta\phi_n(-T) - \Delta\phi_n(T)]} - e^{i[\Delta\phi_n(-T) - \Delta\phi_{n-1}(T)]} \int_{-T}^{T} e^{i[\Delta\phi_{n-1} - \Delta\phi_n]} dt \right]$$

Results of perturbative decay

Explicit form in large n approx:

$$\delta \langle \Psi | \psi | \Psi \rangle = -\psi(\kappa|\psi|^2) \sum_n P_n e^{-iz \ln(T^2/zn) + iz/2n} \sqrt{\frac{zn}{\lambda}} \frac{\pi z}{2n} J_1\left(\frac{z}{2n}\right)$$

At $g^2/\lambda = z = 2\pi N |\psi|^2$

$$|\delta \langle \Psi | \psi | \Psi \rangle| \propto \psi \left[\frac{\kappa}{g} (2\pi N)^{3/2} |\psi|^5 \right]$$

Results of perturbative decay

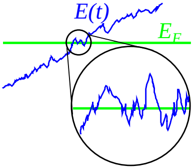
Explicit form in large n approx:

$$\delta \langle \Psi|\psi|\Psi\rangle = -\psi(\kappa|\psi|^2) \sum_n P_n e^{-iz \ln(T^2/zn) + iz/2n} \sqrt{\frac{zn}{\lambda}} \frac{\pi z}{2n} J_1\left(\frac{z}{2n}\right)$$

At $g^2/\lambda = z = 2\pi N|\psi|^2$

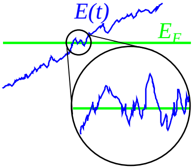
$$|\delta\langle \Psi|\psi|\Psi\rangle| \propto \psi \left[\frac{\kappa}{g}(2\pi N)^{3/2}|\psi|^5\right]$$

Noisy driving



- Suppose $E(t) = ct + \eta(t)$
  $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \Gamma_2 \delta(t - t')$.
- To find N^{ex}, need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$thus:

Noisy driving

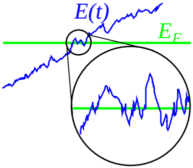


- Suppose $E(t) = ct + \eta(t)$
  $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \Gamma_2 \delta(t - t')$.

- To find N^{ex}, need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$thus:

Noisy driving

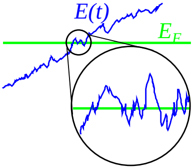
- Suppose $E(t) = ct + \eta(t)$
  $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \Gamma_2 \delta(t - t')$.



- To find N^{ex}, need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$thus:

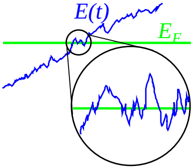
Noisy driving

- Suppose $E(t) = ct + \eta(t)$
  $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \Gamma_2 \delta(t - t')$.



- To find $N^{\rm ex}$, need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$

Noisy driving

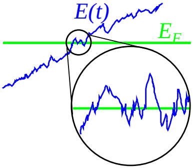


- Suppose $E(t) = ct + \eta(t)$
  $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \Gamma_2 \delta(t - t')$.
- To find N^{ex}, need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$thus:

$$N^{\mathrm{ex}} \propto \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \int_{-\infty}^{\infty} ds \int_{-\infty}^{t} ds' \ldots \times F_{\mathrm{noise}}(t, t', s, s')$$

$$F_{\mathrm{noise}} = \left\langle \exp\left[i \int_{t'}^{t} \eta(\tau) d\tau - i \int_{s'}^{s} \eta(\sigma) d\sigma \right] \right\rangle$$

Noisy driving



- Suppose $E(t) = ct + \eta(t)$
  $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \Gamma_2 \delta(t - t')$.
- To find N^{ex}, need: $\langle |U(\epsilon, \epsilon')|^2 \rangle$thus:

$$N^{\mathrm{ex}} \propto \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \int_{-\infty}^{\infty} ds \int_{-\infty}^{t} ds' \ldots \times F_{\mathrm{noise}}(t, t', s, s')$$

$$F_{\mathrm{noise}} = \left\langle \exp\left[i \int_{t'}^{t} \eta(\tau) d\tau - i \int_{s'}^{s} \eta(\sigma) d\sigma \right] \right\rangle$$

Noisy driving

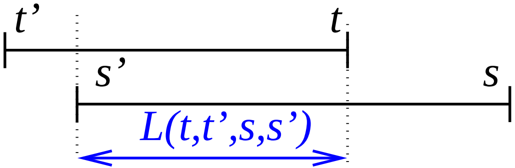
- Find $F_{\text{noise}} = \exp\left[-\Gamma_2\left(|t - t'| + |s - s'| - L(t, t', s, s')\right)\right]$

- Can simplify to $\Delta = t - s$ and $\Lambda = t - t' = s - s'$

$$N^{ex} = \frac{-\Gamma^2}{2\pi c}\int_{-\infty}^{\infty}\frac{d\Delta}{(\Delta - i0)^2}\int_0^{\infty} d\Lambda e^{(ic\Delta - \Gamma - \Gamma_2)\Lambda + \Gamma_2 L(\Delta,\Lambda)}$$

Noisy driving

- Find $F_{\text{noise}} = \exp\left[-\Gamma_2\left(|t-t'| + |s-s'| - L(t,t',s,s')\right)\right]$

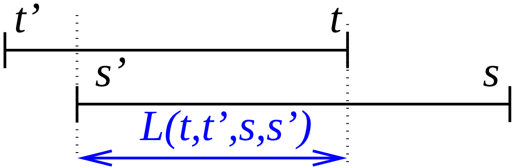


- Can simplify to $\Delta = t - s$ and $\Lambda = t - t' = s - s'$

$$N^{ex} = \frac{-\Gamma^2}{2\pi c} \int_{-\infty}^{\infty} \frac{d\Delta}{(\Delta - i0)^2} \int_0^{\infty} d\Lambda e^{(ic\Delta - \Gamma - \Gamma_2)\Lambda + \Gamma_2 L(\Delta,\Lambda)}$$

Noisy driving

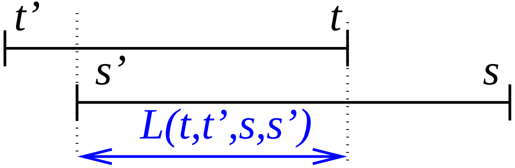
- Find $F_{\mathrm{noise}} = \exp\left[-\Gamma_2\left(|t-t'| + |s-s'| - L(t,t',s,s')\right)\right]$



- Can simplify to $\Delta = t - s$ and $\Lambda = t - t' = s - s'$

$$N^{\mathrm{ex}} = \frac{-\Gamma^2}{2\pi c}\int_{-\infty}^{\infty}\frac{d\Delta}{(\Delta - i0)^2}\int_0^{\infty} d\Lambda e^{(ic\Delta-\Gamma-\Gamma_2)\Lambda+\Gamma_2 L(\Delta,\Lambda)}$$

- Find $F_{\text{noise}} = \exp\left[-\Gamma_2\left(|t-t'| + |s-s'| - L(t,t',s,s')\right)\right]$

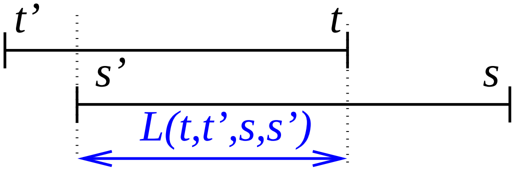


- Can simplify to $\Delta = t - s$ and $\Lambda = t - t' = s - s'$

$$N^{ex} = \frac{-\Gamma^2}{2\pi c}\int_{-\infty}^{\infty}\frac{d\Delta}{(\Delta - i0)^2}\int_0^{\infty} d\Lambda e^{(ic\Delta - \Gamma - \Gamma_2)\Lambda + \Gamma_2 L(\Delta,\Lambda)}$$

Noisy driving

- Find $F_{\text{noise}} = \exp\left[-\Gamma_2\left(|t-t'| + |s-s'| - L(t,t',s,s')\right)\right]$



- Can simplify to $\Delta = t - s$ and $\Lambda = t - t' = s - s'$

$$N^{\text{ex}} = \frac{-\Gamma^2}{2\pi c}\int_{-\infty}^{\infty}\frac{d\Delta}{(\Delta - i0)^2}\int_0^{\infty} d\Lambda e^{(ic\Delta - \Gamma - \Gamma_2)\Lambda + \Gamma_2 L(\Delta,\Lambda)}$$

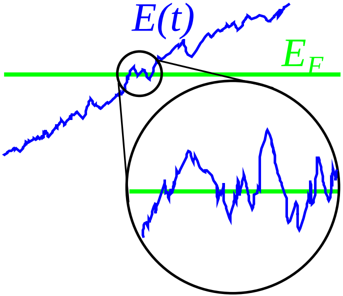
Noisy driving: results

- Integral log divergent: white noise limit
- Infinite no. crossings of Fermi surface
- Can extract logarithmic contribution

$$N^{\mathrm{ex}} = \begin{cases} 1 & c \gg \Gamma\Gamma_2 \\ \frac{\Gamma^2}{(\Gamma+\Gamma_2)^2} + \frac{2\Gamma\Gamma_2}{\pi c} \ln \frac{\omega_*}{\Gamma+\Gamma_2} & c \ll \Gamma\Gamma_2 \end{cases}$$

Noisy driving: results

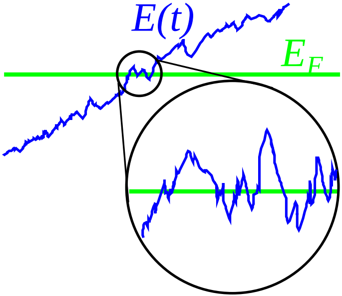
- Integral log divergent: white noise limit
- Infinite no. crossings of Fermi surface
- Can extract logarithmic contribution



$$N^{\mathrm{ex}} = \begin{cases} 1 & c \gg \Gamma\Gamma_2 \\ \frac{\Gamma^2}{(\Gamma+\Gamma_2)^2} + \frac{2\Gamma\Gamma_2}{\pi c} \ln \frac{\omega_*}{\Gamma+\Gamma_2} & c \ll \Gamma\Gamma_2 \end{cases}$$

Noisy driving: results

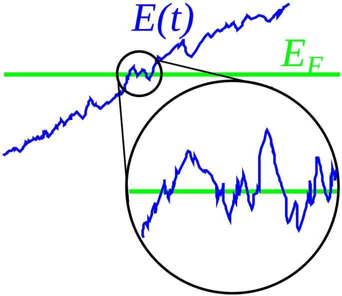
- Integral log divergent: white noise limit
- Infinite no. crossings of Fermi surface
- Can extract logarithmic contribution



$$N^{\mathrm{ex}} = \begin{cases} 1 & c \gg \Gamma\Gamma_2 \\ \frac{\Gamma^2}{(\Gamma+\Gamma_2)^2} + \frac{2\Gamma\Gamma_2}{\pi c} \ln \frac{\omega_*}{\Gamma+\Gamma_2} & c \ll \Gamma\Gamma_2 \end{cases}$$

Noisy driving: results

- Integral log divergent: white noise limit
- Infinite no. crossings of Fermi surface
- Can extract logarithmic contribution



$$N^{\mathrm{ex}} = \begin{cases} 1 & c \gg \Gamma\Gamma_2 \\ \frac{\Gamma^2}{(\Gamma+\Gamma_2)^2} + \frac{2\Gamma\Gamma_2}{\pi c} \ln \frac{\omega_*}{\Gamma+\Gamma_2} & c \ll \Gamma\Gamma_2 \end{cases}$$