Polariton condensation, beyond the weakly interacting Bose gas.

Jonathan Keeling[1]

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[1] *University of Cambridge*, [2] *University of Oxford*

July 24th 2007

Polariton condensation vs W.I.D.B.G

Non-interacting Bose gas

- Three dimensional
- Non-interacting bosons
- Structureless bosons
- Infinite lifetime
- Solved 80 years ago

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- Polariton internal structure

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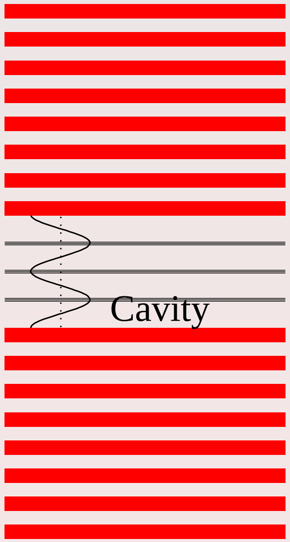
  - Lineshape, linewidth
  - Density profile

Excitons in a disordered Quantum well



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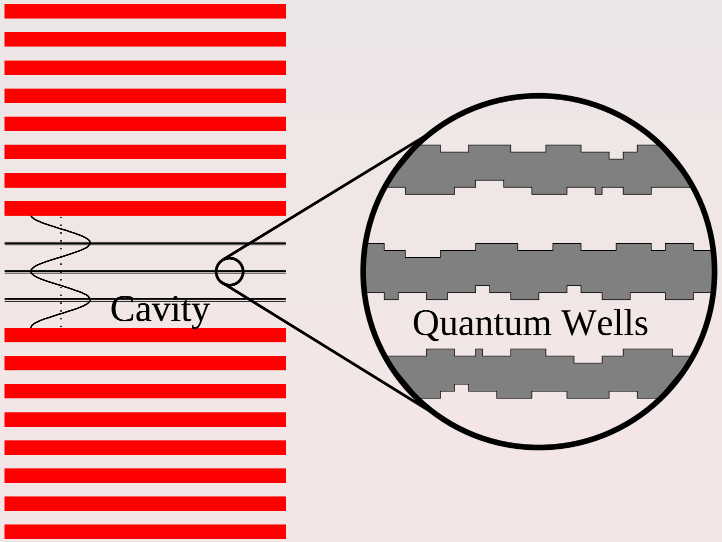

Centre-of-mass wavefunction satisfies:

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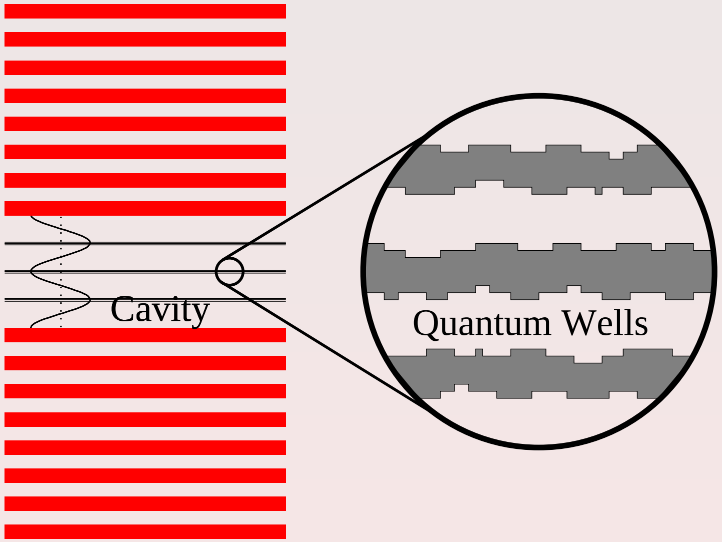

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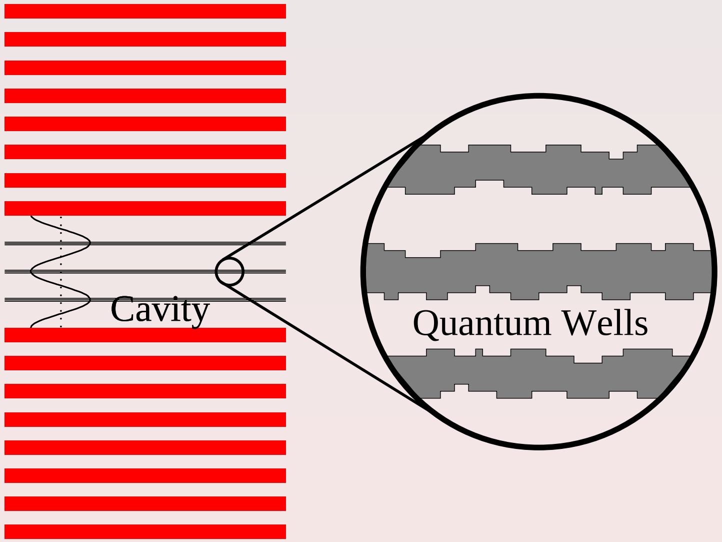

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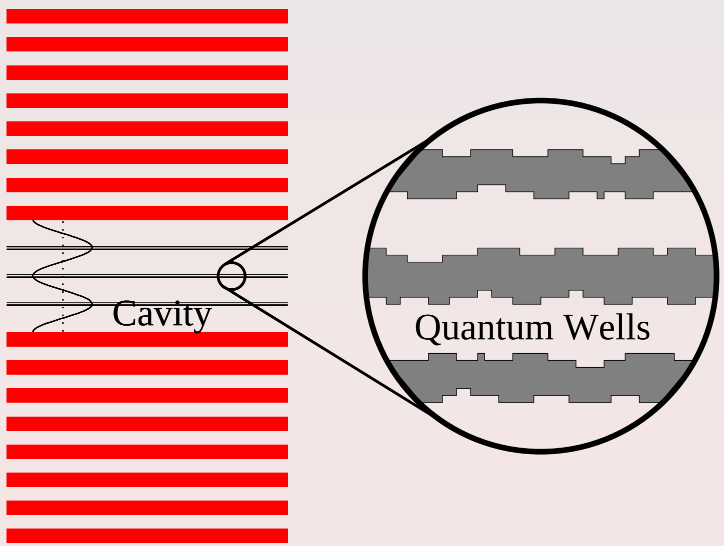

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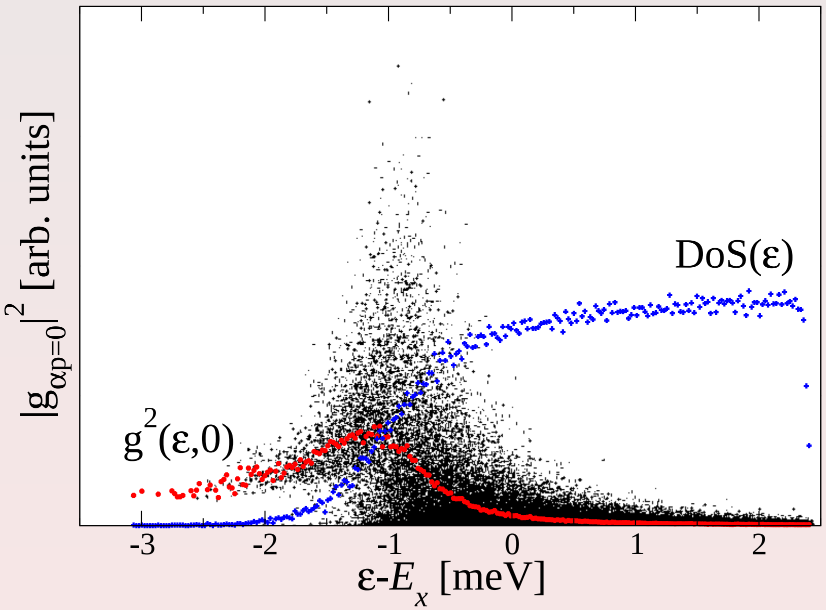
Exciton energies and oscillator strengths



[FMM, JK, MHS, PBL `cond-mat/0608096`]

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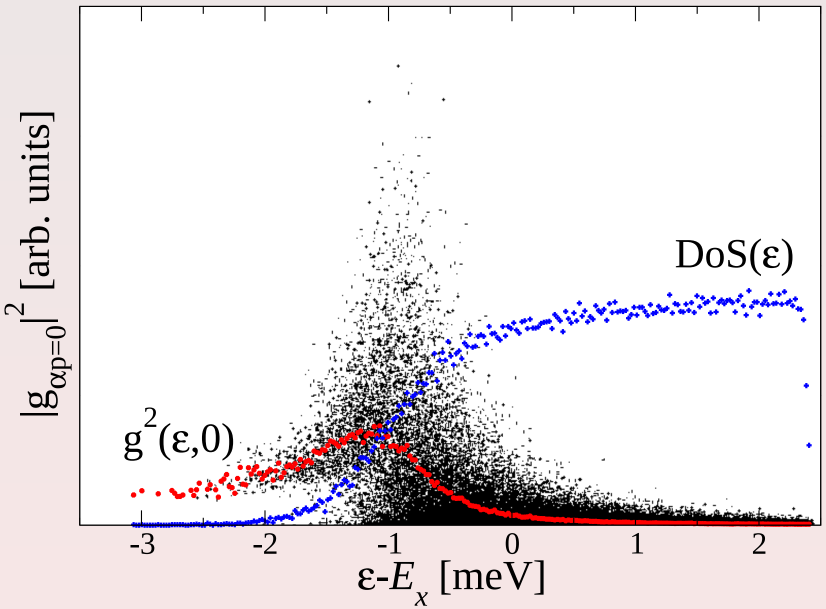

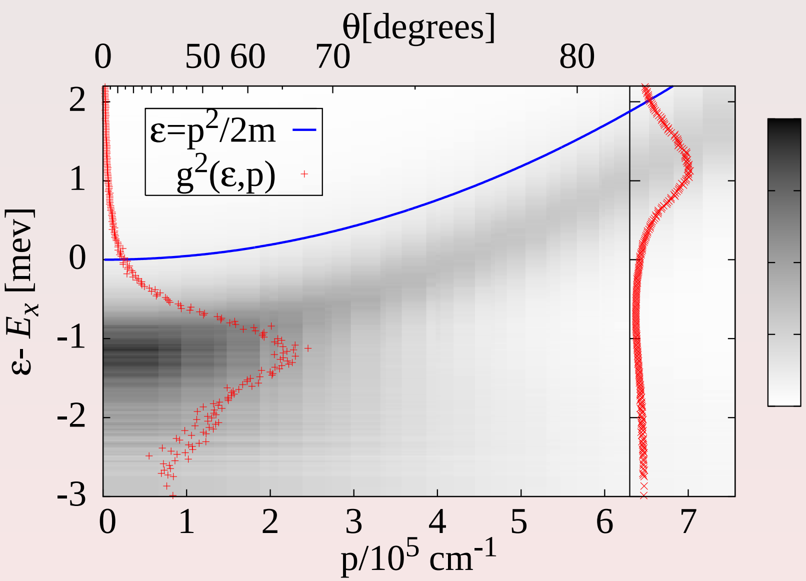

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Polariton model

- Couple excitons to photons

[JK, FMM, MHS, PBL *Semicond. Sci. Technol.* **22** R1 (2007)]

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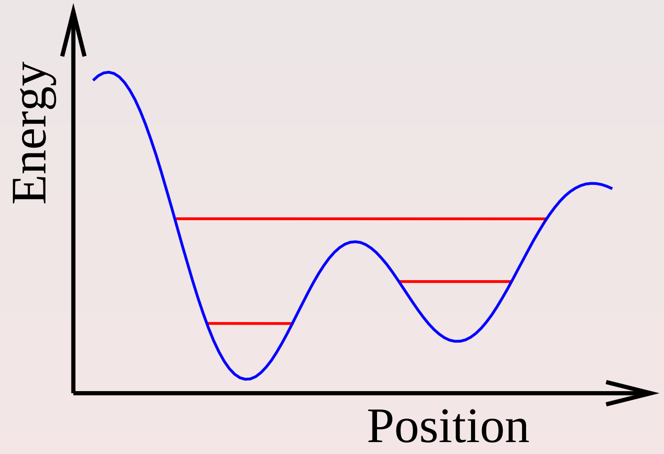

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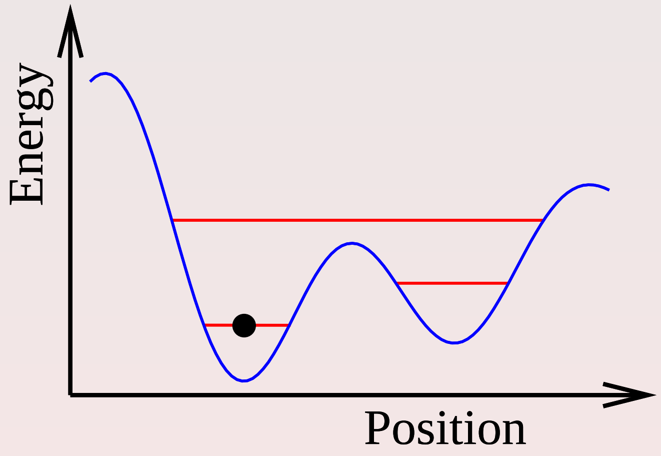

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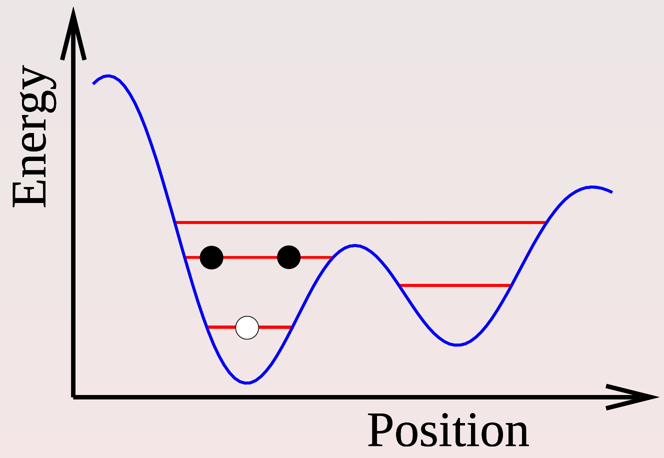

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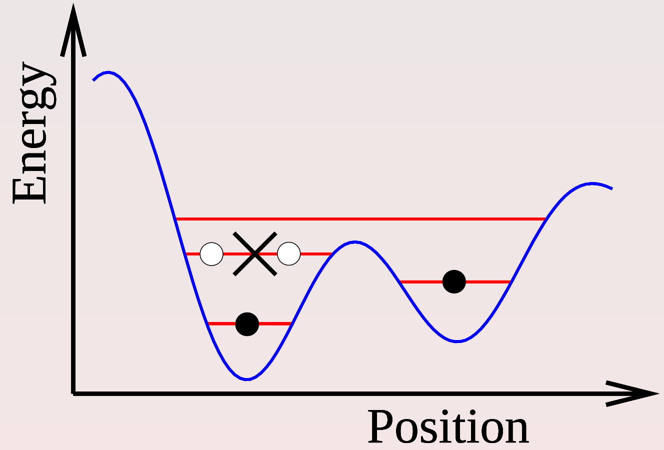

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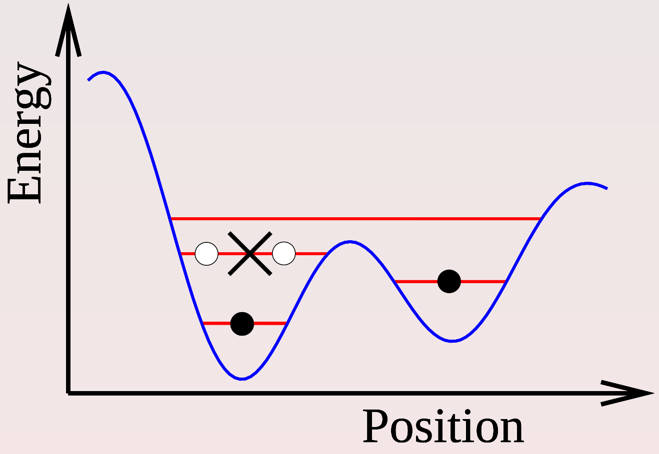

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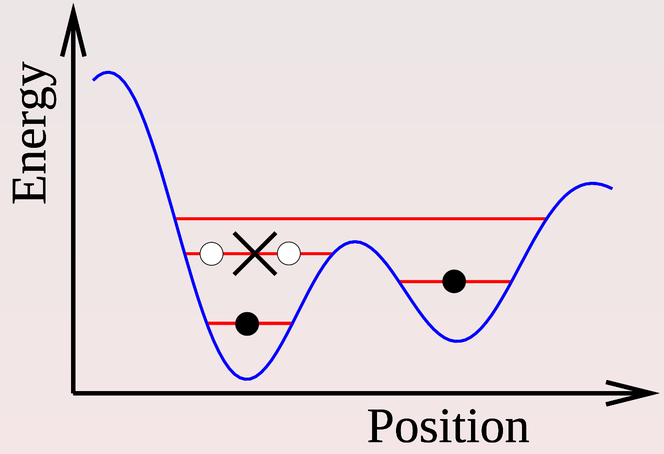

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$$H = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} \psi_{\mathbf{k}}$$

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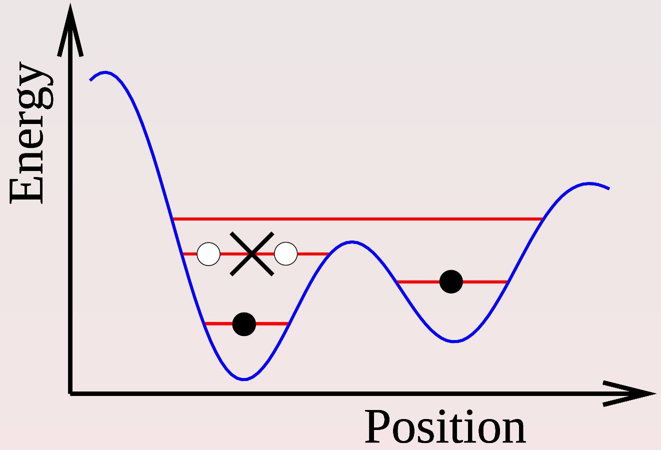

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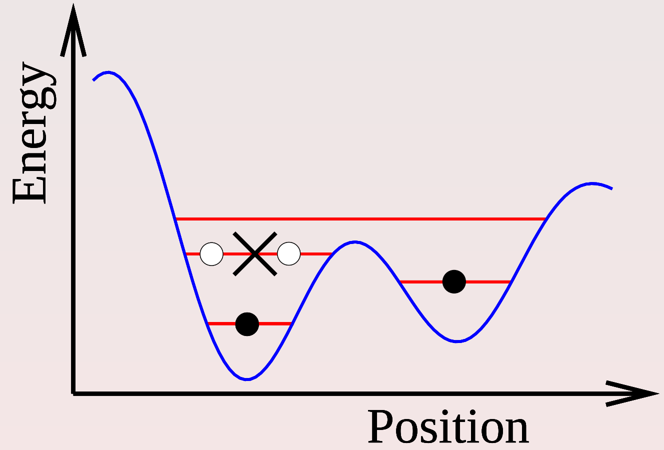

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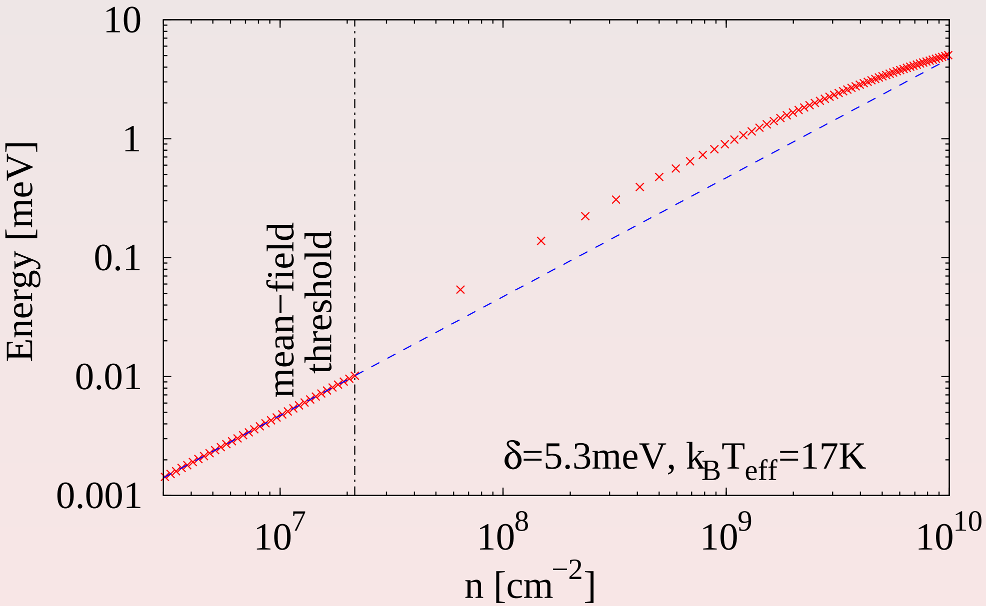
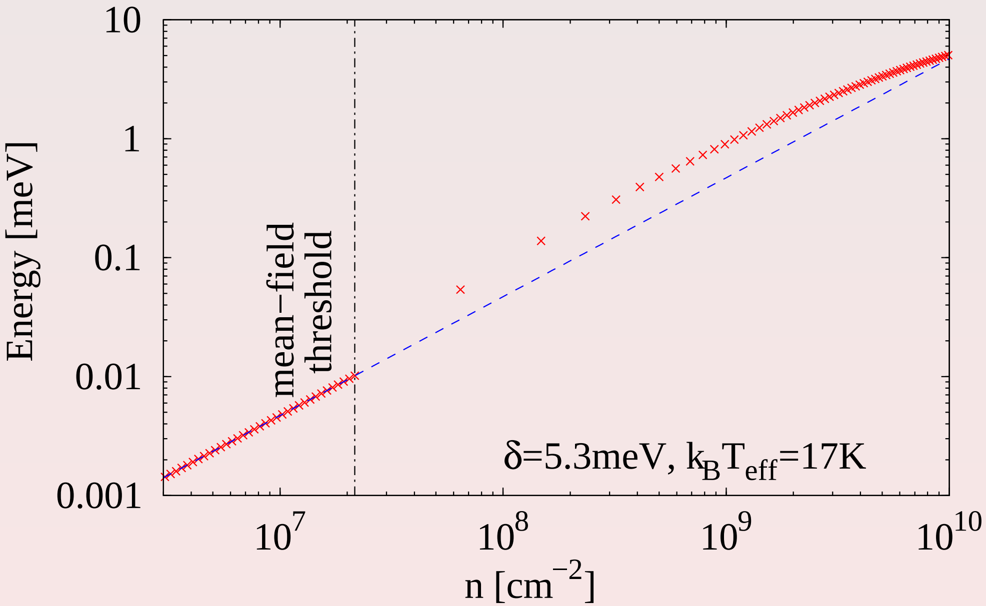
[JK, FMM, MHS, PBL *Semicond. Sci. Technol.* **22** R1 (2007)]

Blueshift


10
1
0.1
0.01
0.001
Energy [meV]
mean−field
threshold
δ=5.3meV, k_B T_eff=17K
10^7
10^8
10^9
10^10
n [cm^-2]

Blueshift



Clean limit:

$$\delta E_{\mathsf{LP}} \simeq \mathcal{R}y_{\mathsf{X}} a_{\mathsf{X}}^2 n + \Omega_{\mathsf{R}} a_{\mathsf{X}}^2 n$$

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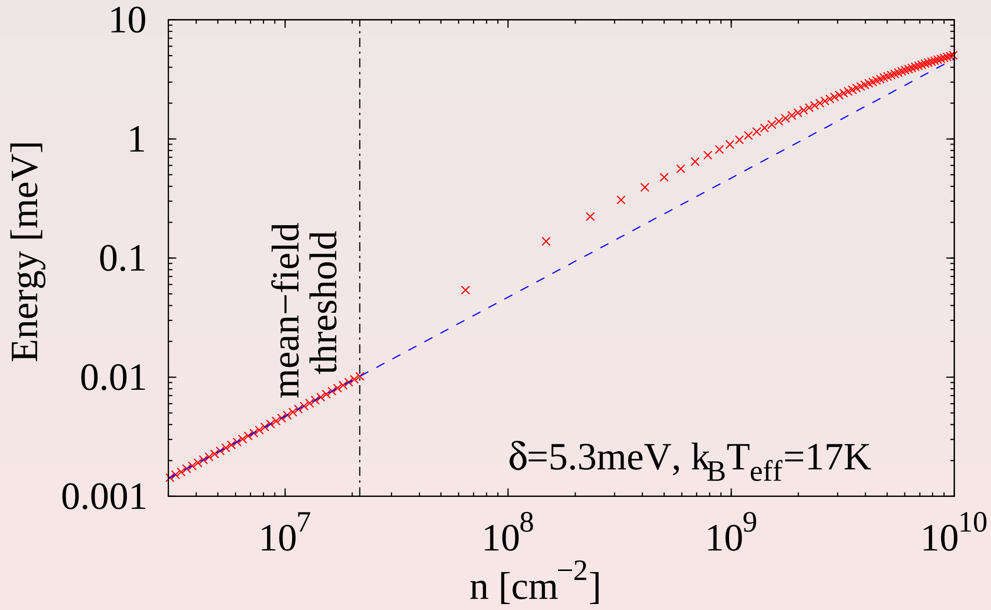

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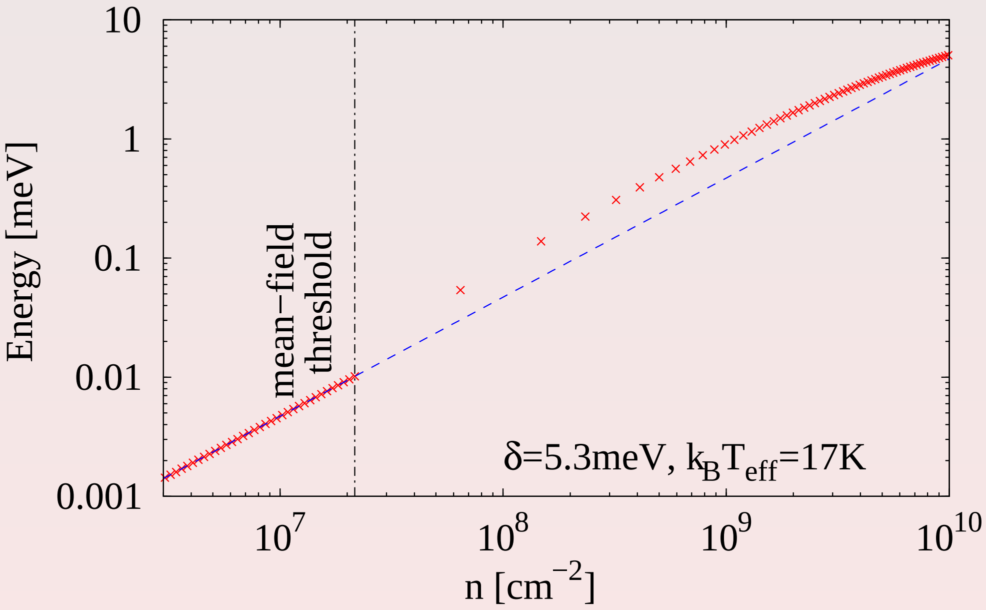

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For CdTe, $\times 100$ larger

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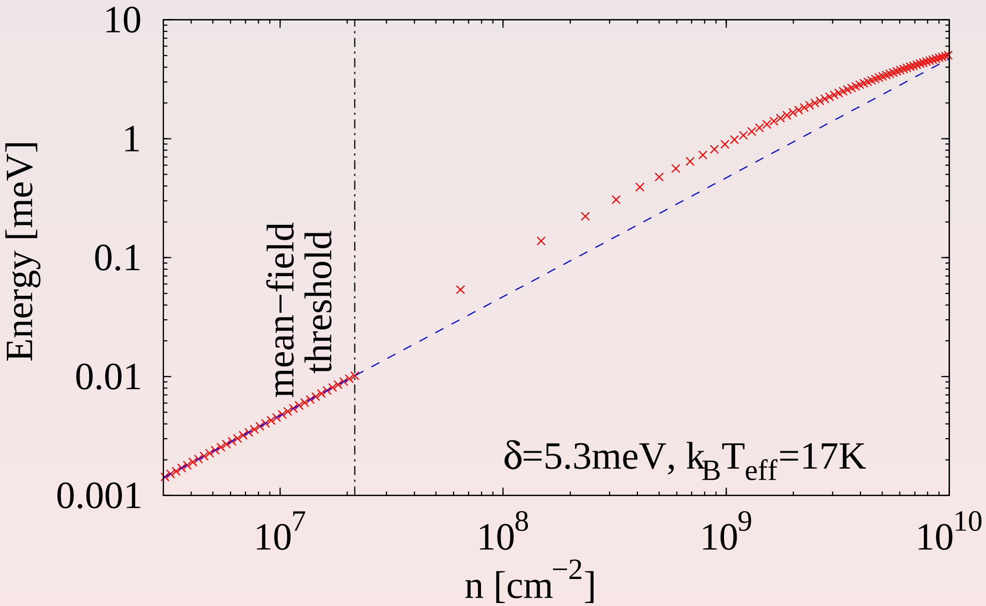

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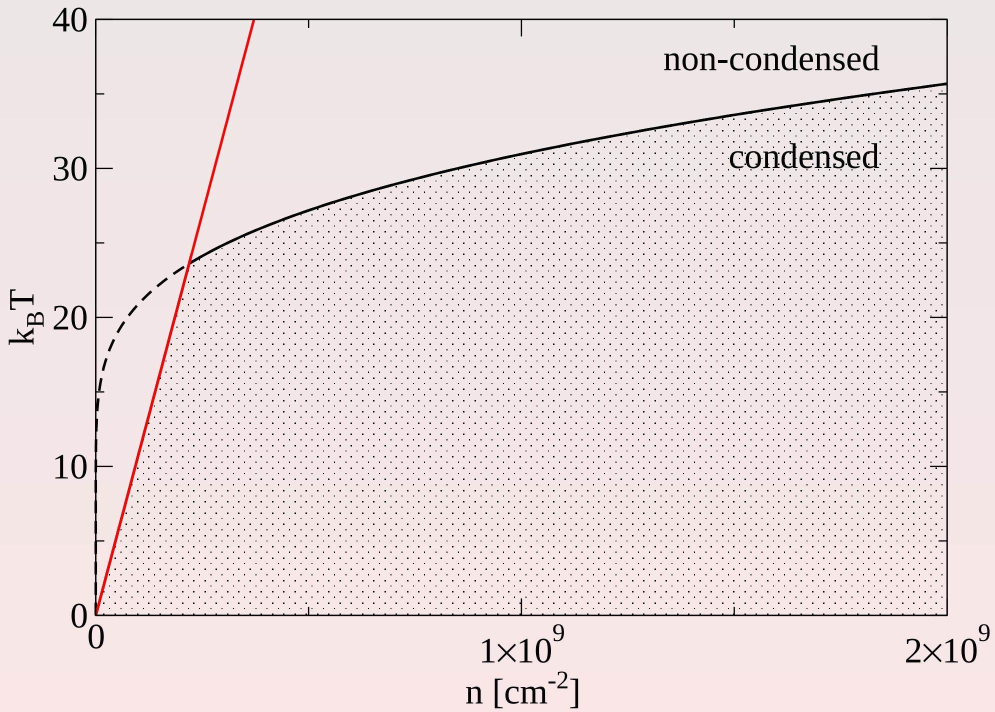
Upper polariton:

$$\delta E_{\mathsf{UP}} \simeq \mathcal{R}y_{\mathsf{X}} a_{\mathsf{X}}^2 n - \Omega_{\mathsf{R}} a_{\mathsf{X}}^2 n$$

Phase diagram



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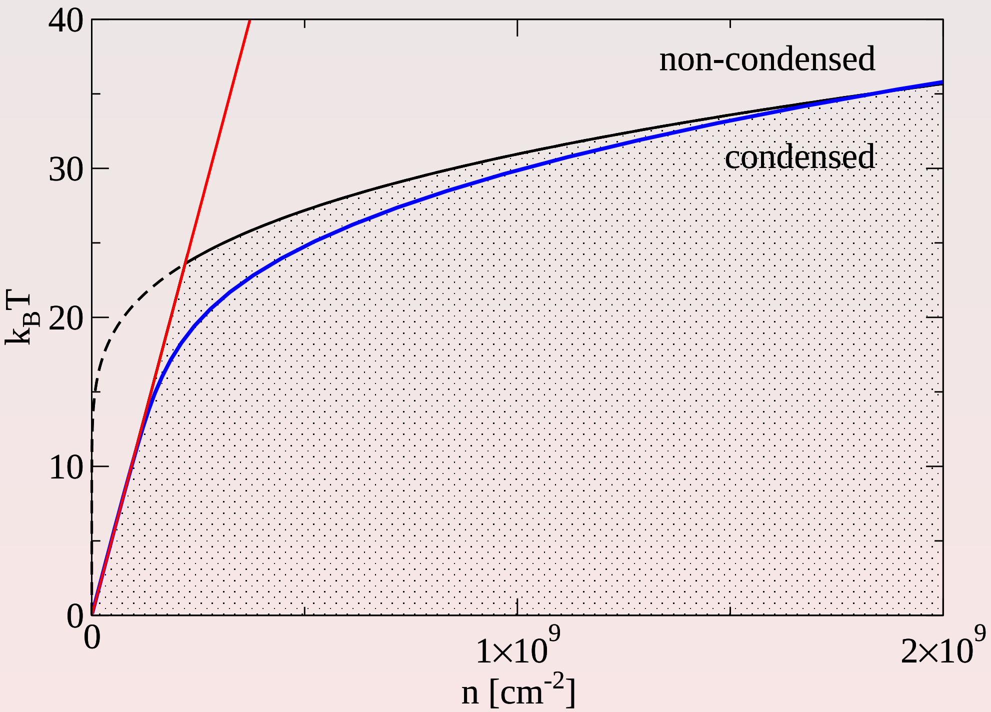

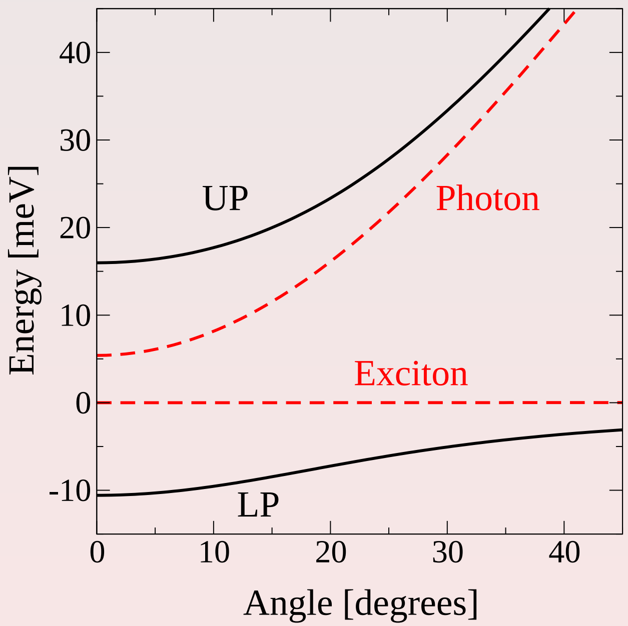

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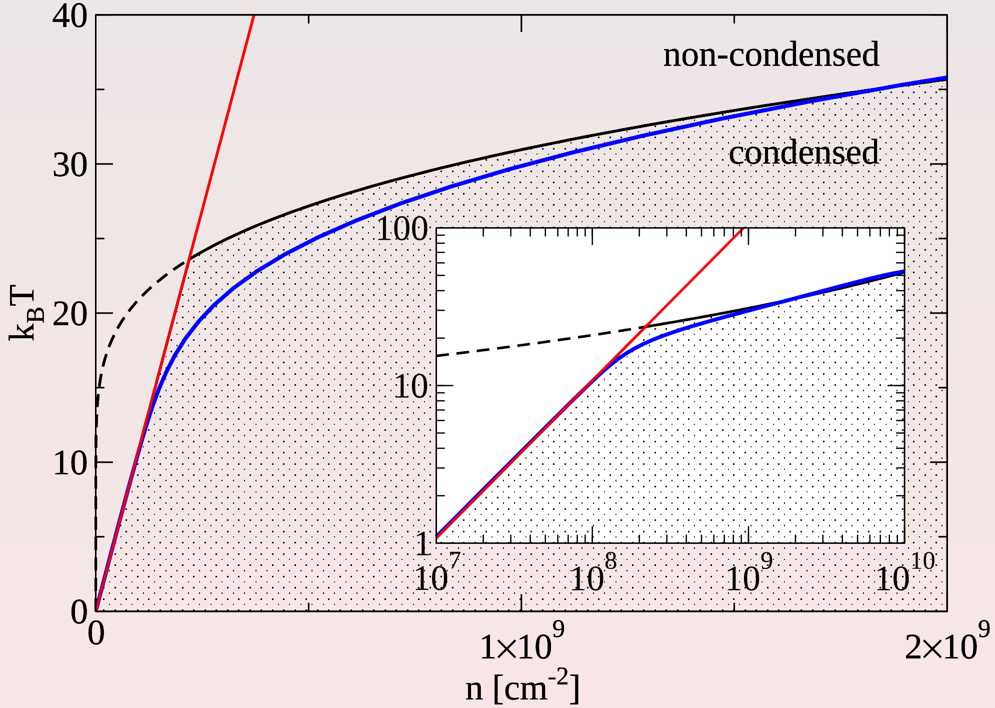

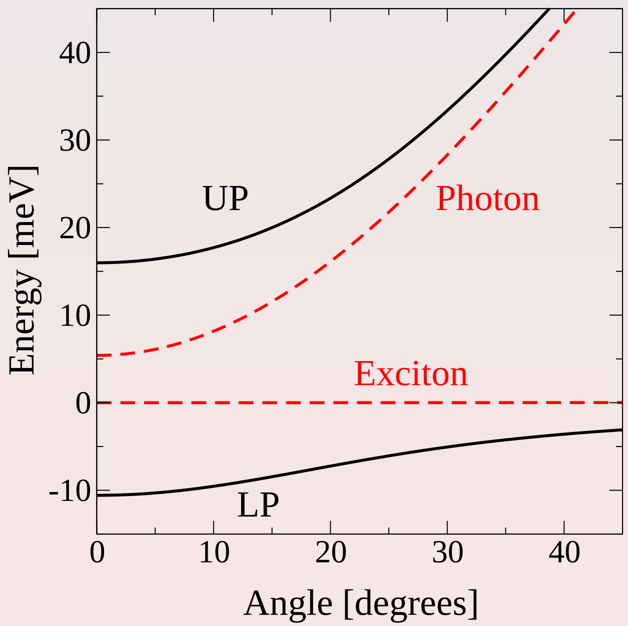

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Pumping and decay; energy scales

	Lifetime	Thermalisation
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Atoms		

Pumping and decay; energy scales

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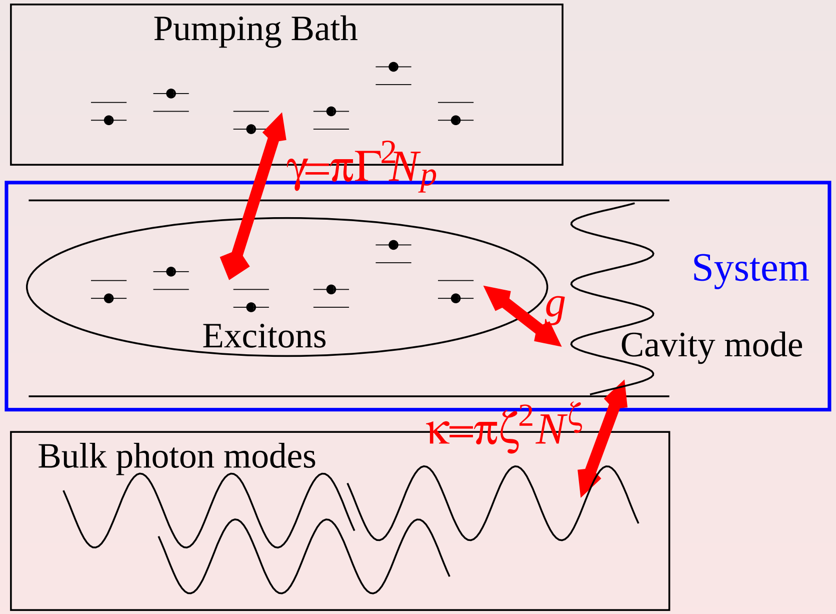

Linewidth on approaching transition

Approaching transition, susceptibility diverges

[MHS, JK, PBL *Phys. Rev. B* **75** 195331 (2007)]

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$$0 = \mathcal{G}^{-1}(\omega, k)$$

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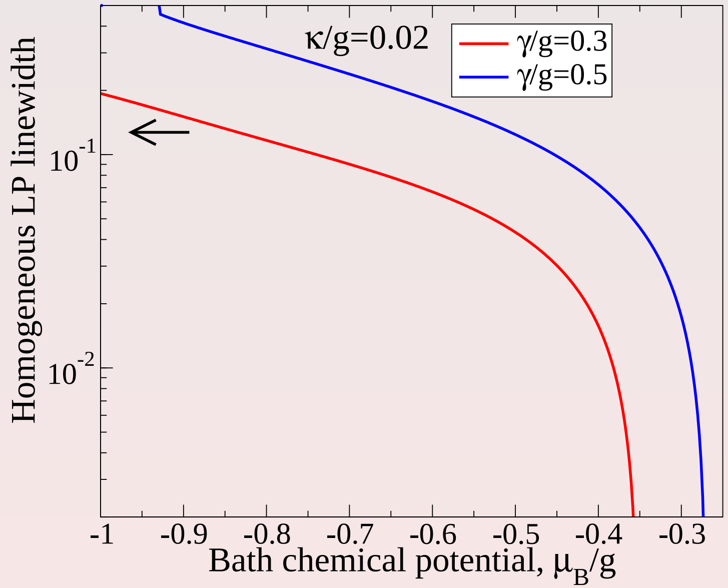

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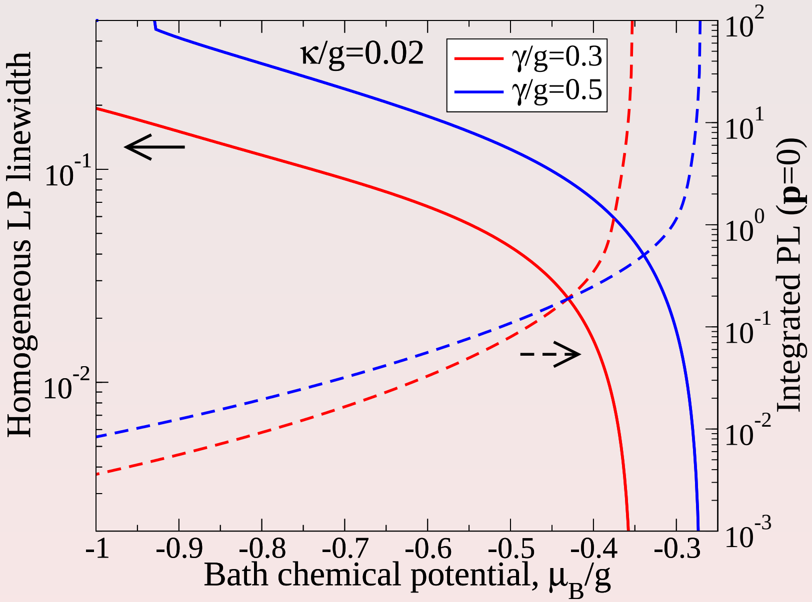

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Excitations of decaying condensate; power spectrum

When condensed, poles become:

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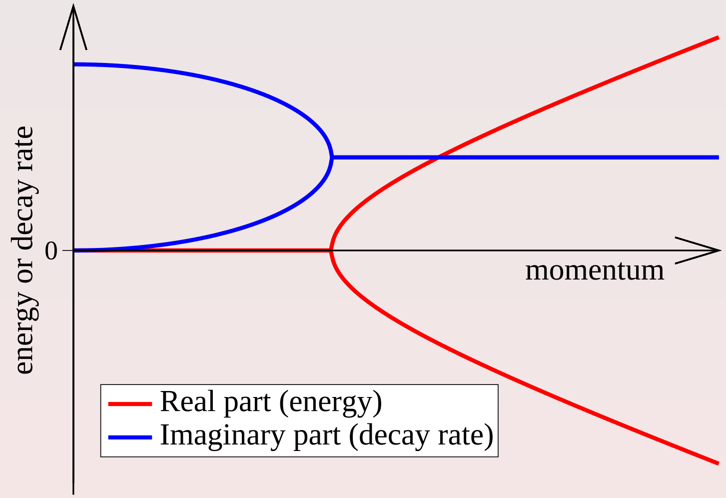

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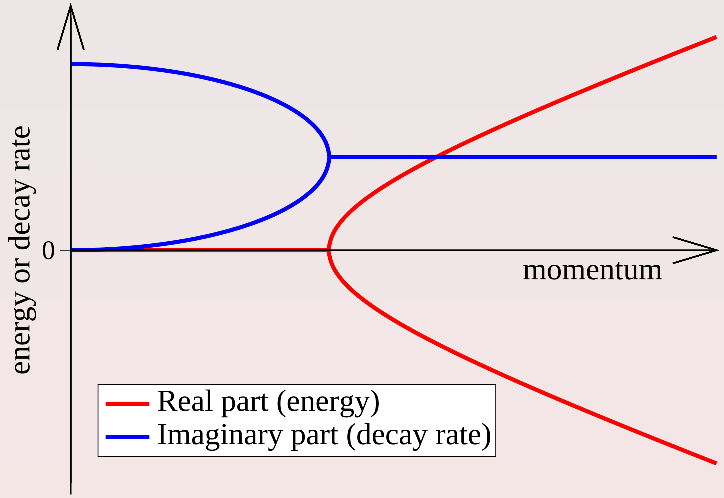

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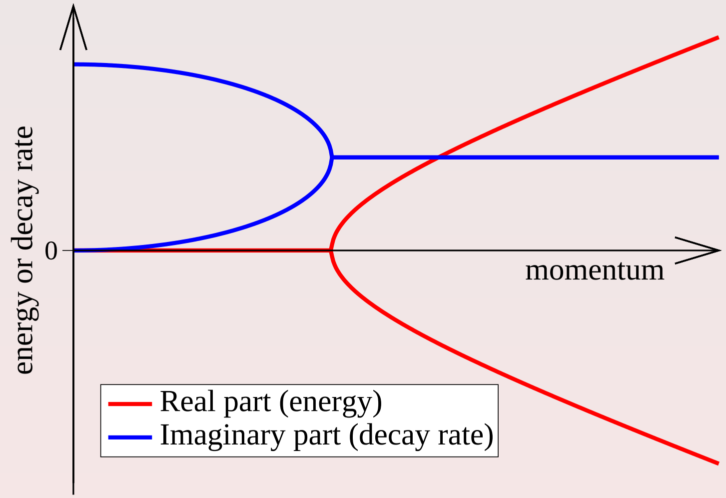
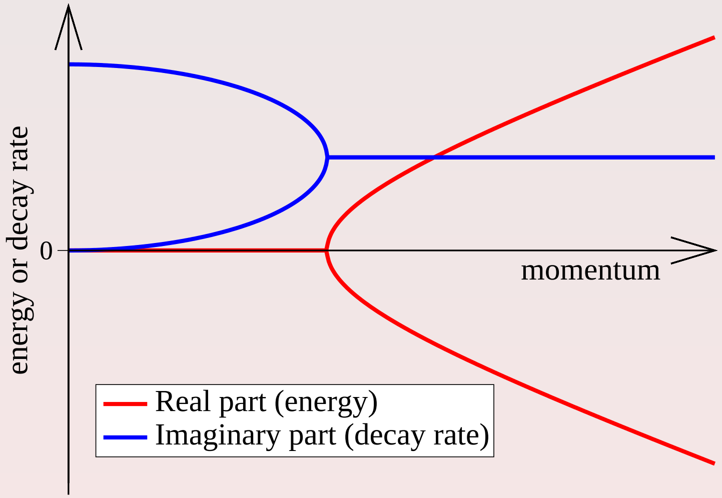


$$\langle \psi^\dagger(r,t)\psi(0,0)\rangle \simeq \rho_0 \exp\left[-\begin{cases} \eta \ln(r/\xi) & r \to \infty, t \simeq 0 \\ & \end{cases}\right]$$

Excitations of decaying condensate; power spectrum

When condensed, poles become:

$$\omega = -ix \pm \sqrt{c^2\mathbf{k}^2 - x^2}$$



$$\langle \psi^\dagger(r,t)\psi(0,0)\rangle \simeq \rho_0 \exp\left[-\begin{cases}\eta \ln(r/\xi) & r\to\infty, t\simeq 0\\ \frac{\eta}{2}\ln(c^2t/x\xi^2) & r\simeq 0, t\to\infty\end{cases}\right]$$

Phase boundary; effect of dephasing

Mean-field theory of pumped decaying system; self-consistent distribution:

[MHS, JK, PBL *Phys. Rev. B* **75** 195331 (2007)]

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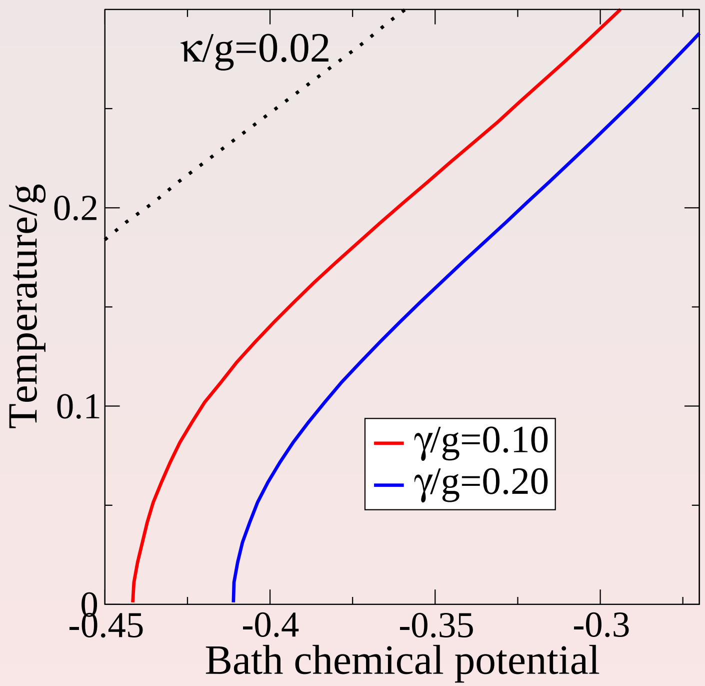

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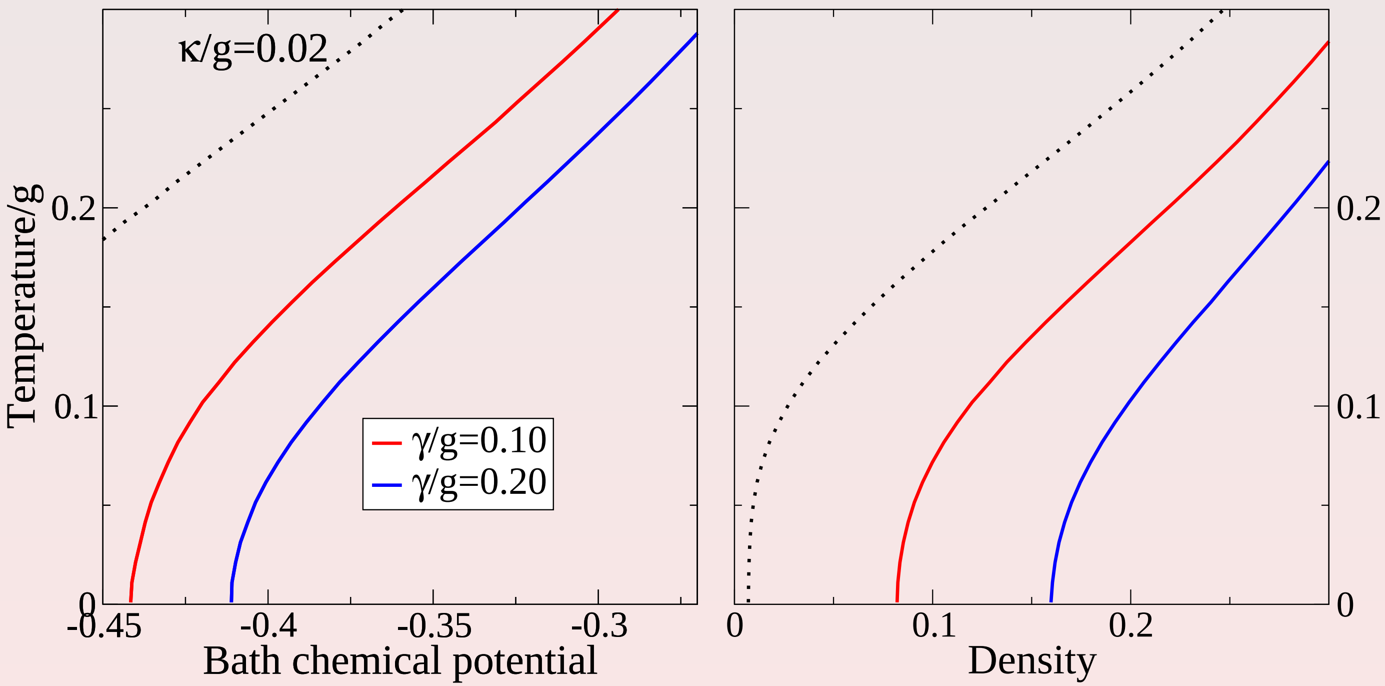

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Effect of pumping on density profile

Gross-Pitaevskii equation:

[JK, NGB `arXiv:0706.3686v1 [cond-mat.other]`]

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Gross-Pitaevskii equation:

$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2\nabla^2}{2m} + V(r) + U|\psi|^2 \qquad\qquad\qquad\qquad\qquad\qquad \right]\psi$$

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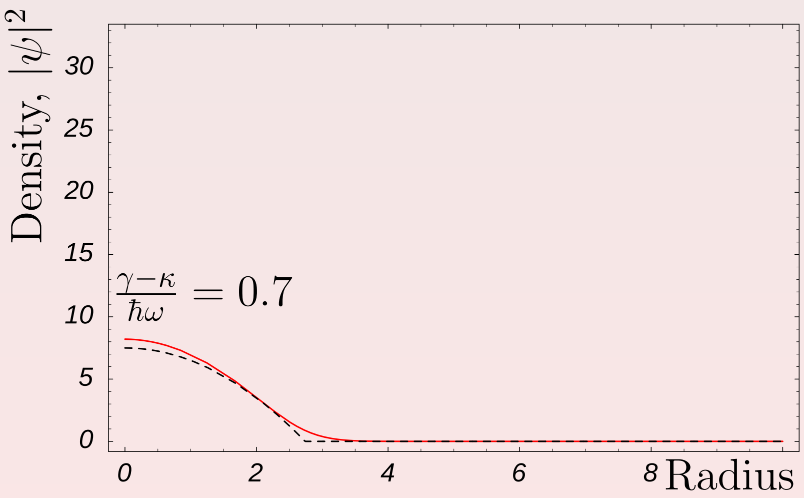
Consider $V(r) = m\omega^2 r^2/2$



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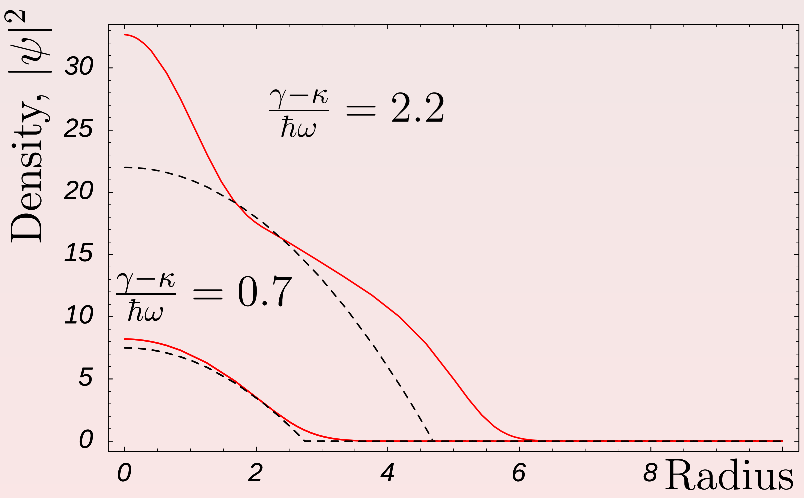

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Summary

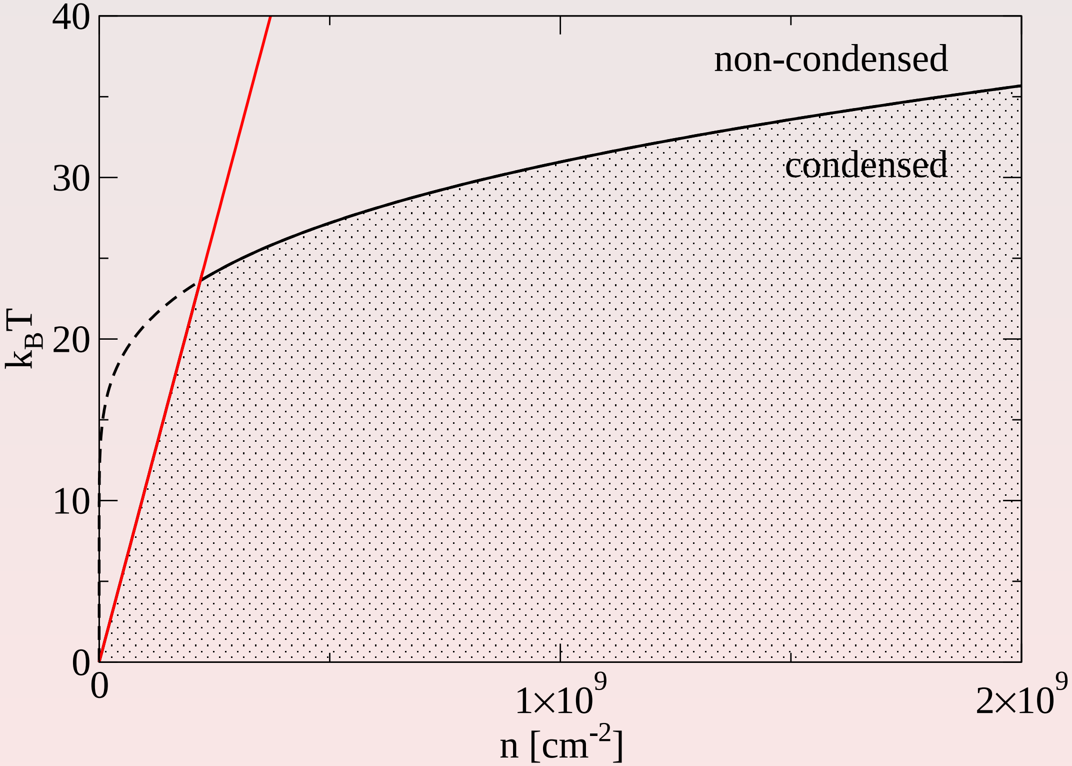
Differences between polariton condensate and non-interacting Bose gas allow investigation of **interesting physics**

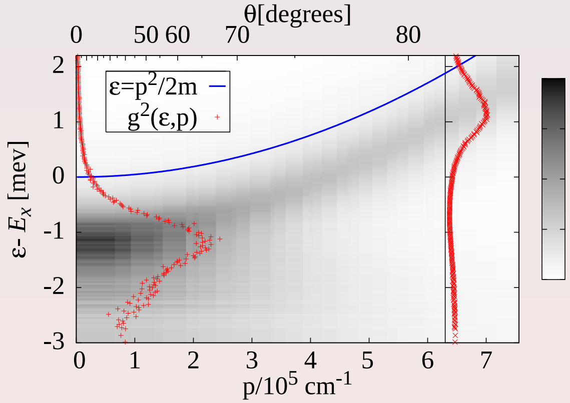
- Internal structure/disorder
  - Critical temperature comparable to Ω_R, $\mathcal{R}y$
  - Exciton disorder affects effective interactions

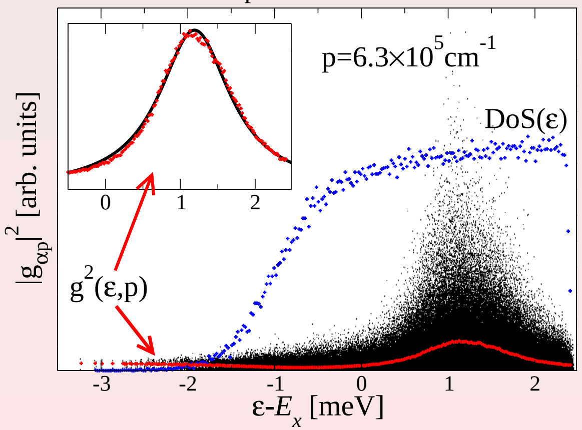
- Non-equilibrium
  - Diffusive spectrum / Lineshape
  - Phase boundary
  - Persistent supercurrents and density profile

Supplementary Slides

Density scales/exciton localisation





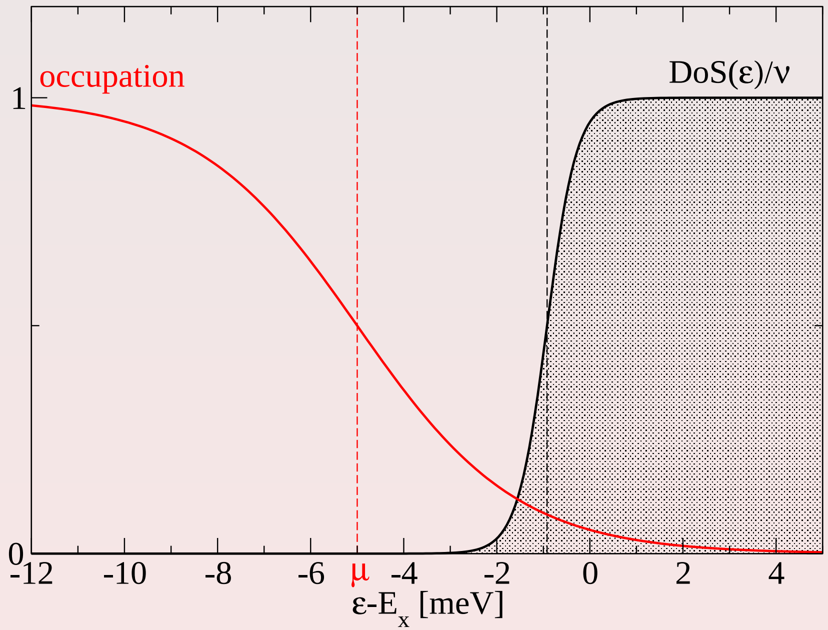


Limit of validity

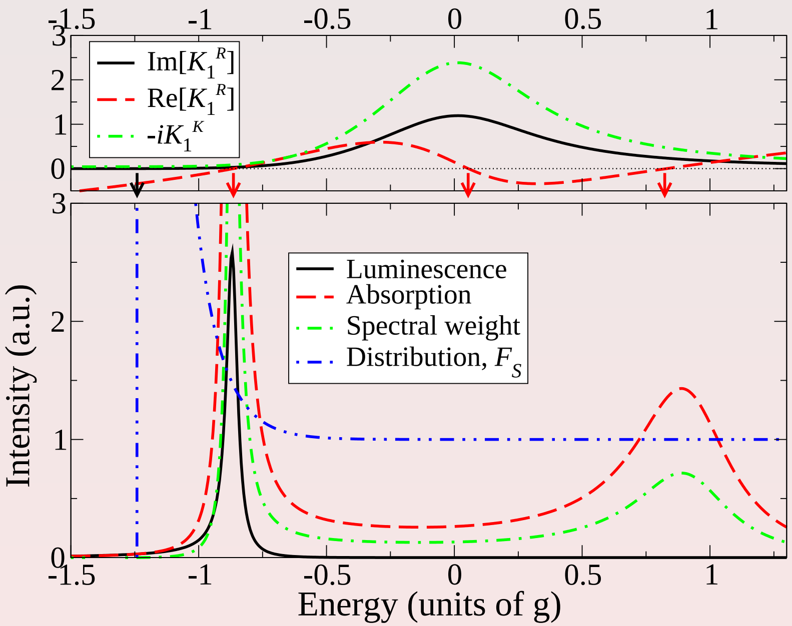
Model neglects:

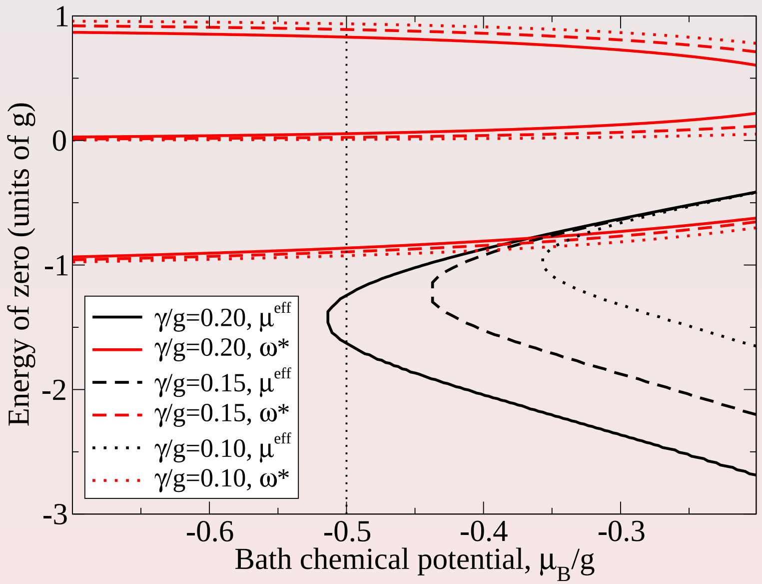
- Inter-site Coulomb
- (High energy) multiple occupancy

Limits to low density: How low?
Limited to states below band-edge



Lineshape in the normal state



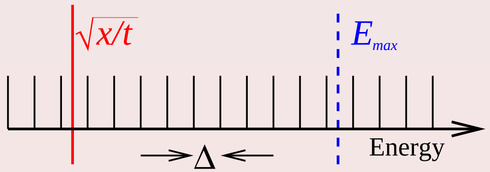


Finite size, condensate linewidth

Correlations, $\langle \psi^\dagger(t,r)\psi(0,r)\rangle \propto \exp(-f(t,r,r)$, with:

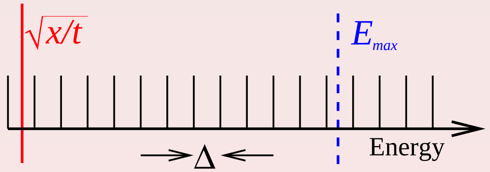
$$f(t,r,r) = -\sum_{n}^{n_{max}} \int \frac{d\omega}{2\pi} \frac{C|\varphi_n(r)|^2(1-e^{i\omega t})}{[\omega^2-(n\Delta)^2]^2+4\omega^2x^2},$$

$\Delta \ll \sqrt{x/t} \ll E_{\mathsf{max}}$



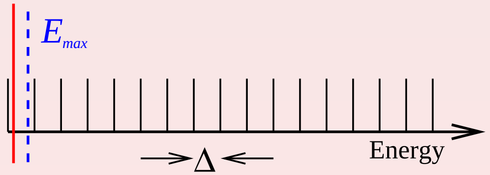
$f(t,r,r) \sim 1 + \ln(E_{\mathsf{max}}\sqrt{t/x})$

$\sqrt{x/t} \ll \Delta \ll E_{\mathsf{max}}$



$f(t,r,r) \sim \left(\frac{\pi C}{2x}\right)\left(\frac{t}{2x}\right)$

$\sqrt{x/t} \ll E_{\mathsf{max}} \ll \Delta$



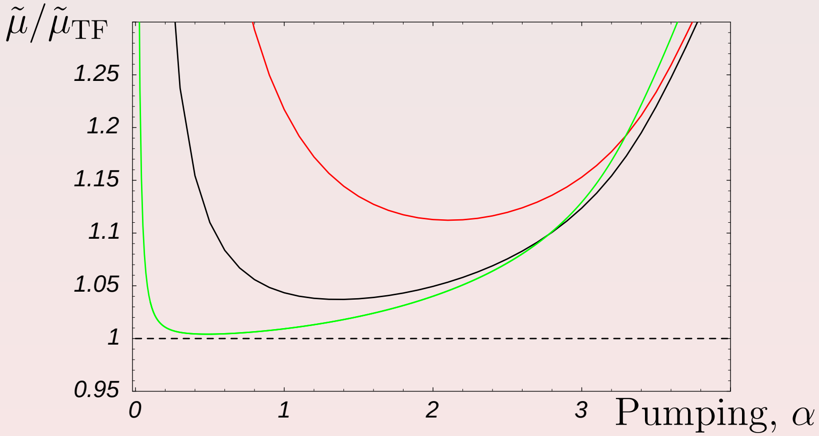
$f(t,r,r) \sim 1$

Pumped, Decaying GPE

Rescaling:

$$\tilde{\mu}\psi = \left[-\nabla^2 + r^2 + |\psi|^2 + i(\alpha - \sigma|\psi|^2)\right]\psi$$

Then, writing: $\psi = \sqrt{\rho}e^{i\phi}$



$$\nabla \cdot [\rho \nabla \phi] = (\alpha - \sigma\rho)$$

$$\mu = |\nabla\phi|^2 + r^2 + \rho - \frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}$$