BCS-BEC crossover in a system of microcavity polaritons

Jonathan Keeling, P. R. Eastham, M. H. Szymanska, P. B. Littlewood
Theory of Condensed Matter, Cambridge

October 19, 2005

Overview

- Microcavity polariton condensation: review of experiments.

Overview

- Microcavity polariton condensation: review of experiments.
- Dicke model for polaritons.

Overview

- Microcavity polariton condensation: review of experiments.
- Dicke model for polaritons.
- Summary of results from mean field theory.

Overview

- Microcavity polariton condensation: review of experiments.
- Dicke model for polaritons.
- Summary of results from mean field theory.
- Diversion: Atomic gases near Feshbach resonance – analogies

Overview

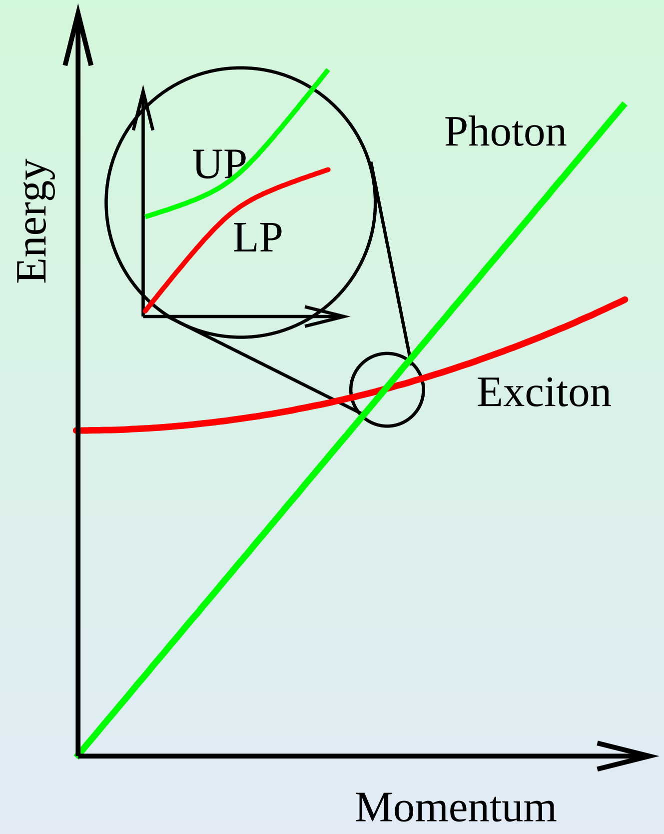
- Microcavity polariton condensation: review of experiments.
- Dicke model for polaritons.
- Summary of results from mean field theory.
- Diversion: Atomic gases near Feshbach resonance – analogies
- Fluctuations in two dimensions; fluctuations with condensate

Overview

- Microcavity polariton condensation: review of experiments.
- Dicke model for polaritons.
- Summary of results from mean field theory.
- Diversion: Atomic gases near Feshbach resonance – analogies
- Fluctuations in two dimensions; fluctuations with condensate
- Consider crossover between "B.E.C." and "B.C.S.-like" transition.

Exciton Polaritons

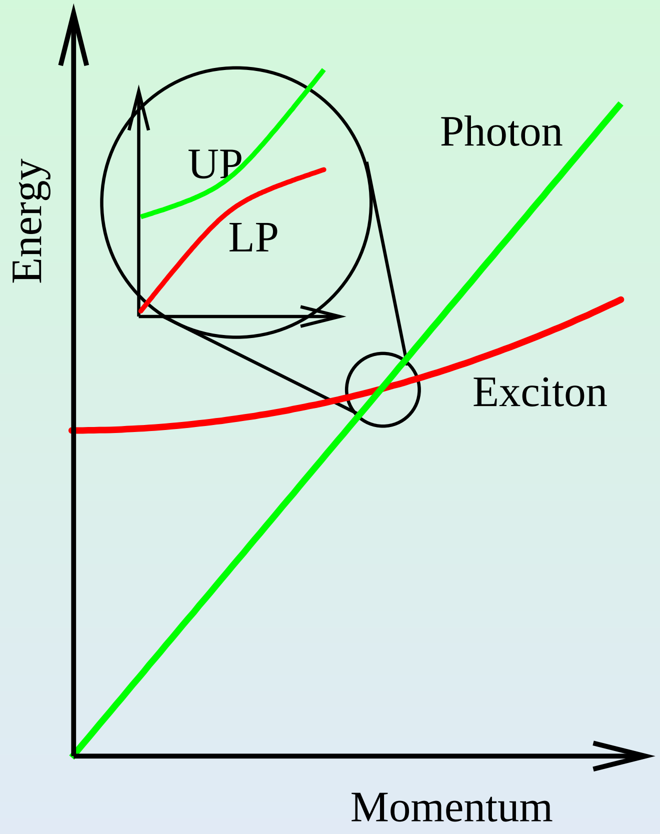
- Strong coupling of photons to excitons



[*Pekar, JETP* **6** 785 (1958)]
[*Hopfield, Phys. Rev.* **112** 1555 (1958)]

Exciton Polaritons

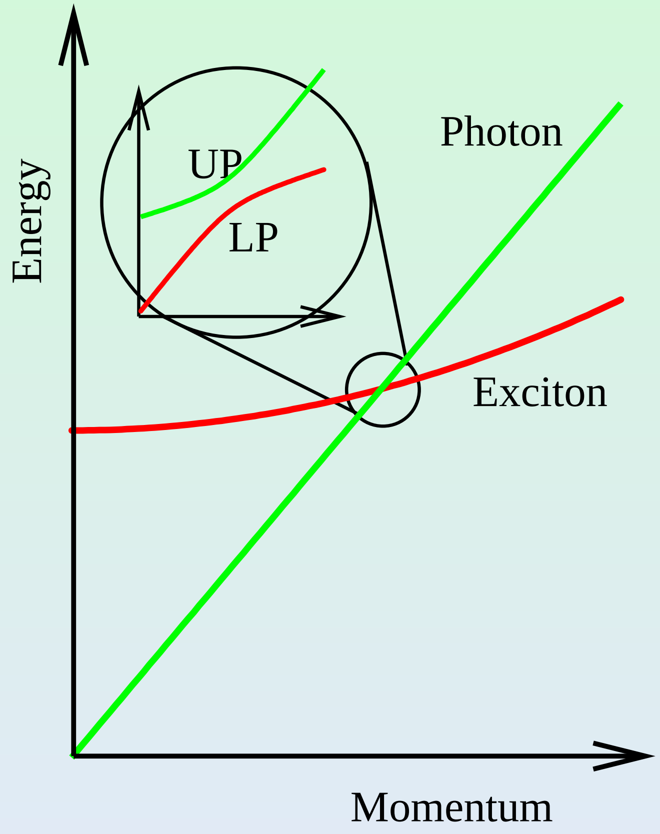
- Strong coupling of photons to excitons
- Anti-crossing – form two new modes



[*Pekar, JETP* **6** 785 (1958)]
[*Hopfield, Phys. Rev.* **112** 1555 (1958)]

Exciton Polaritons

- Strong coupling of photons to excitons
- Anti-crossing – form two new modes
- No condensation – can relax to photon mode.



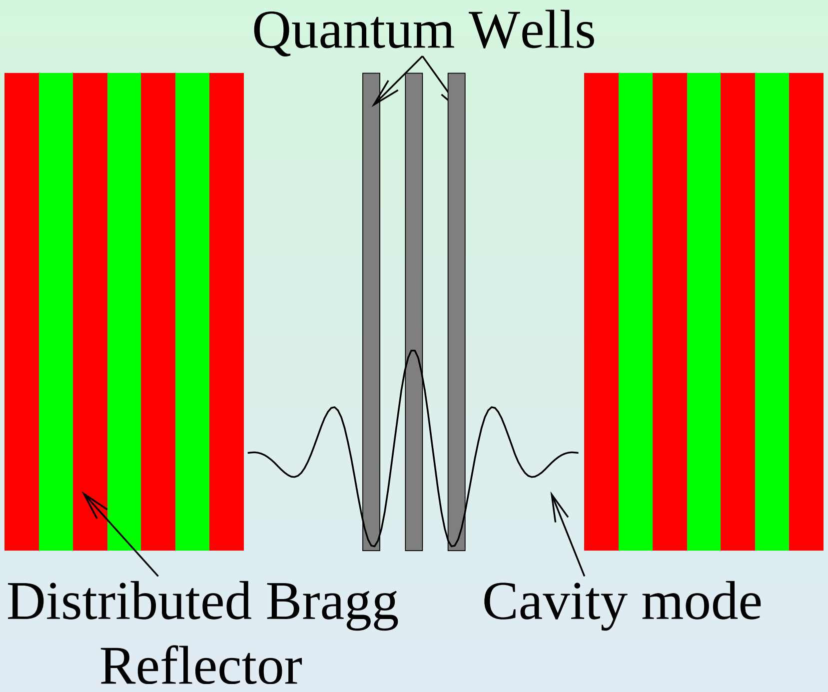
[*Pekar, JETP* **6** 785 (1958)]
[*Hopfield, Phys. Rev.* **112** 1555 (1958)]

Microcavity polaritons

Quantum well excitons coupled to photons confined in a microcavity.

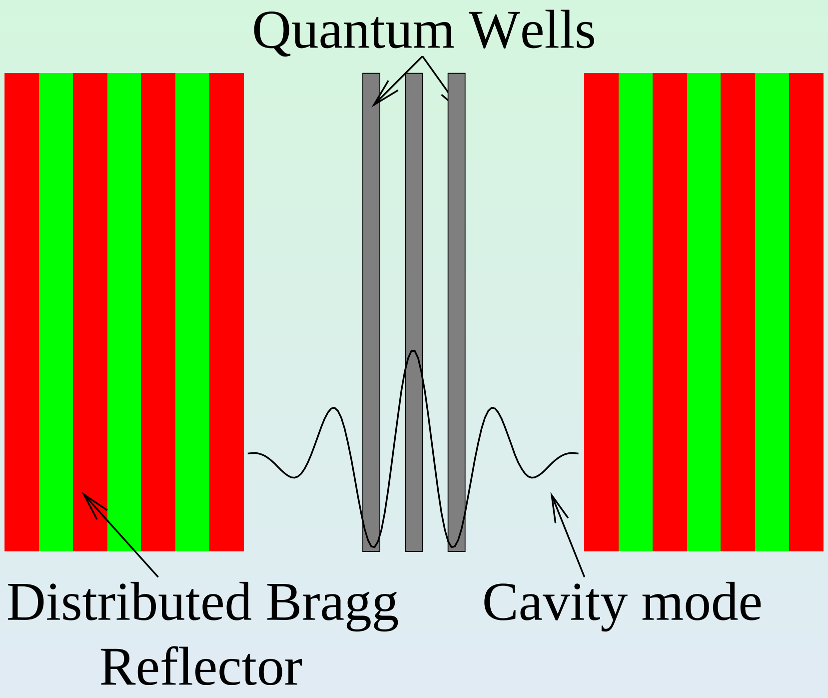
Microcavity polaritons

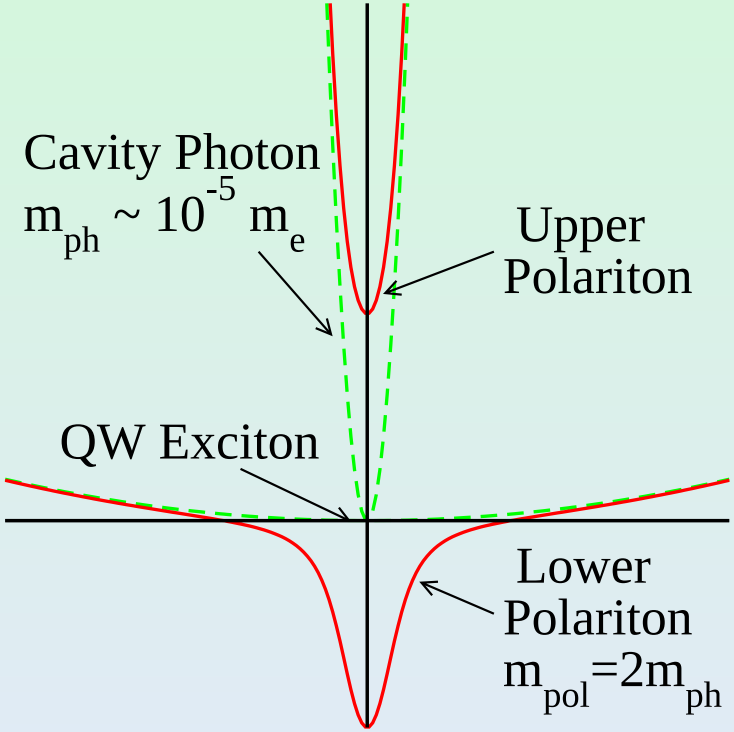
Quantum well excitons coupled to photons confined in a microcavity.



Microcavity polaritons

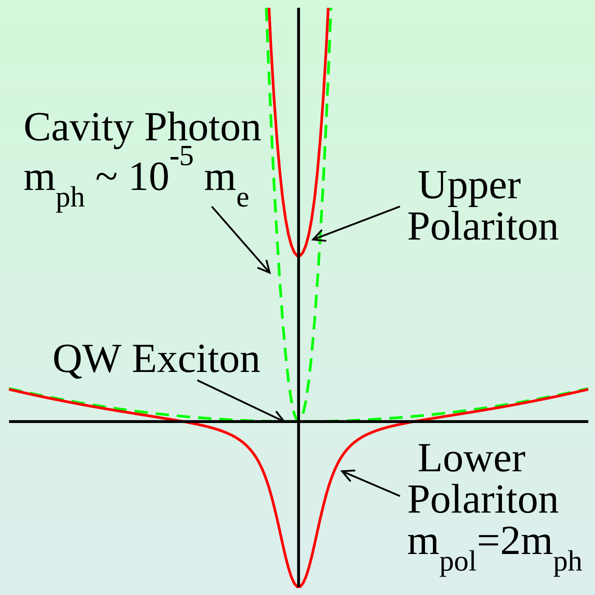
Quantum well excitons coupled to photons confined in a microcavity.





Why polariton condensation

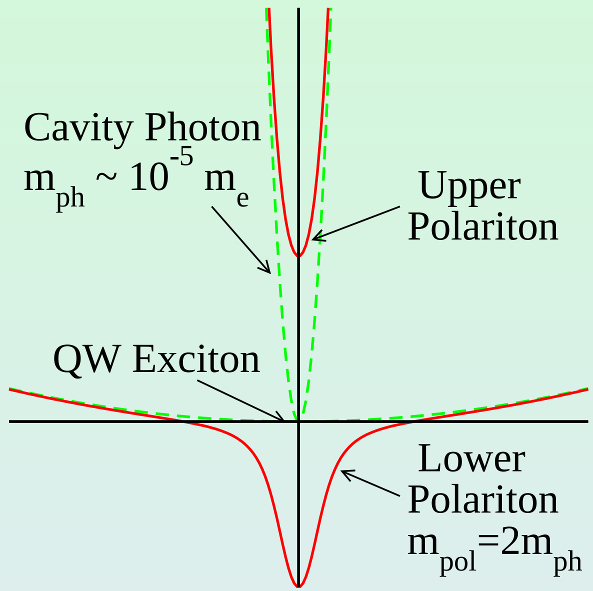
Why polariton condensation:



Why polariton condensation

Why polariton condensation:

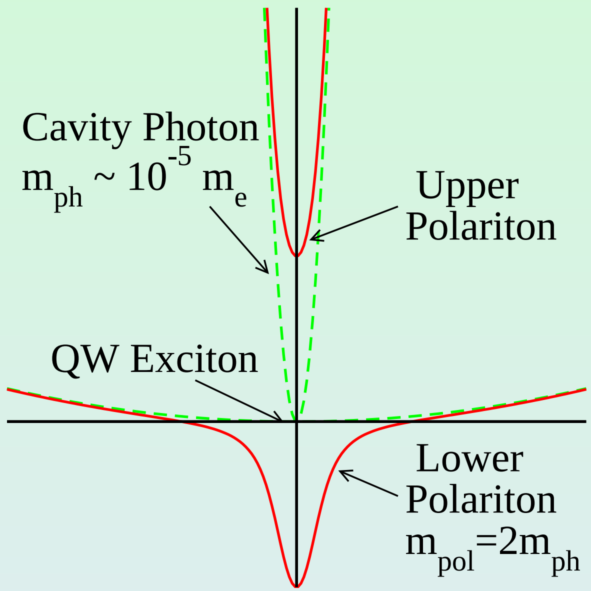
- Polariton mass $10^{-4} m_{\mathrm{electron}}$, high T_c.



Why polariton condensation

Why polariton condensation:

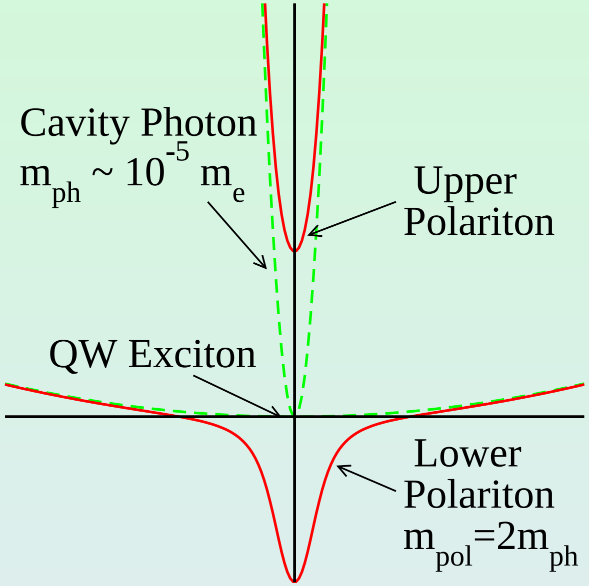
- Polariton mass $10^{-4} m_{\mathrm{electron}}$, high T_c.
- Photon component – Non-classical light.



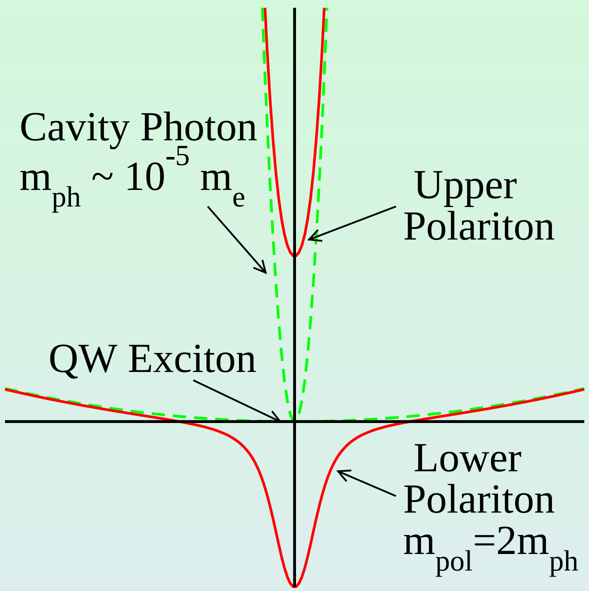
Why polariton condensation

Why polariton condensation:

- Polariton mass $10^{-4} m_{\mathrm{electron}}$, high T_c.
- Photon component – Non-classical light.
- Greater control.



Why polariton condensation



Why polariton condensation:

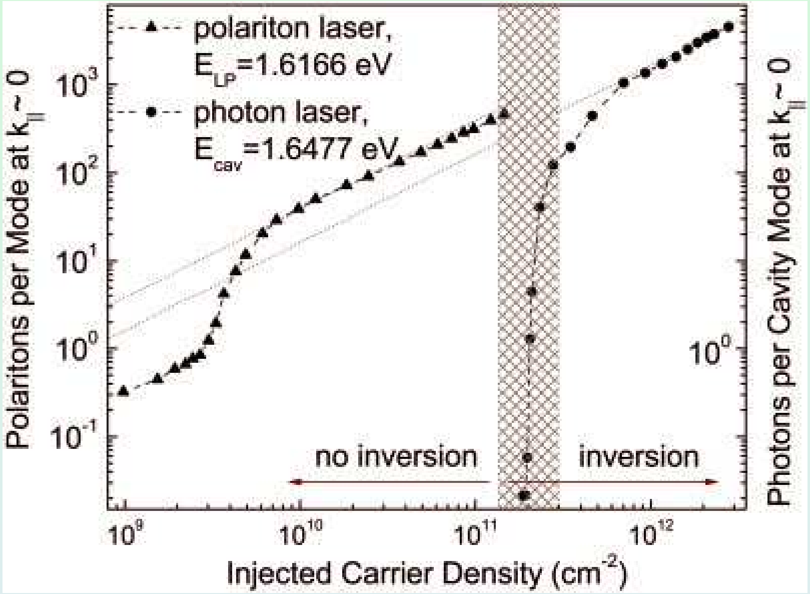
- Polariton mass $10^{-4} m_{\mathrm{electron}}$, high T_c.
- Photon component – Non-classical light.
- Greater control.

Problems?

- Cavity lifetime is short (ps), hard to thermalise.

Polariton Experiments

Non-linear ground state occupation.

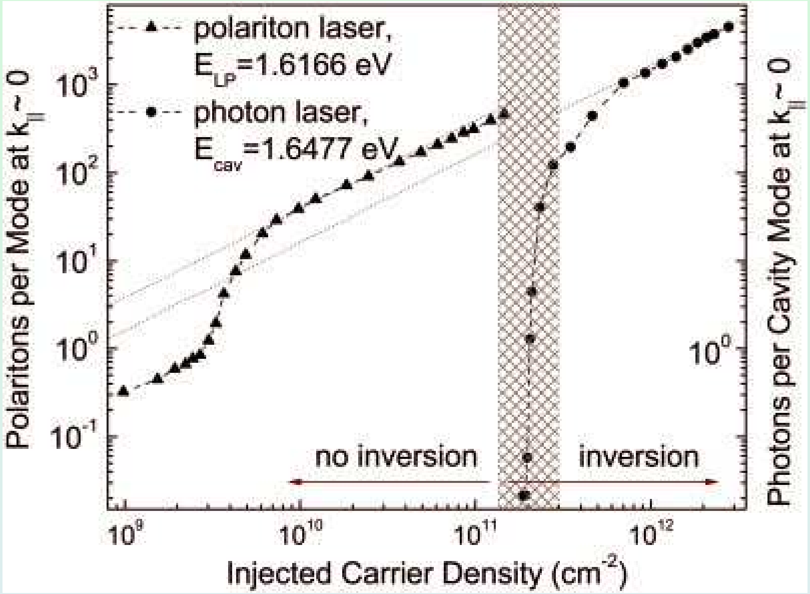


[*Deng et al.* Science **298** 199 (2002)]

(also [*Dang et al.* PRL. **81** 3920 (1998)])

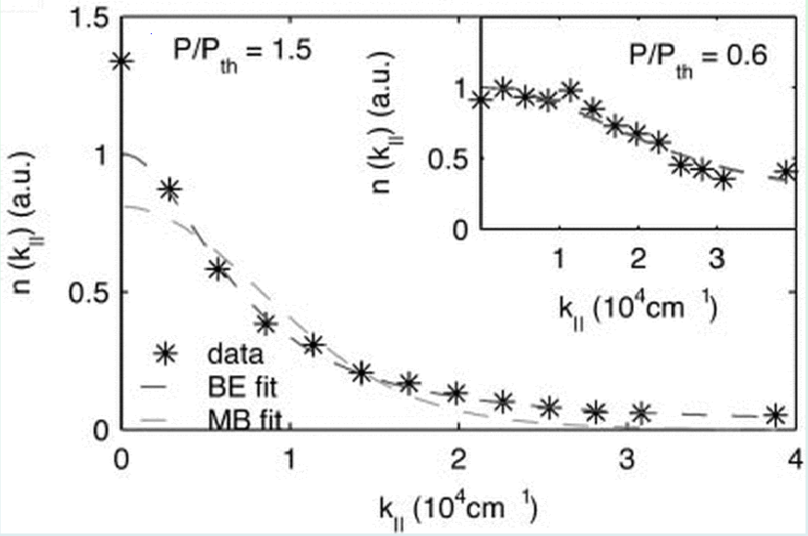
Polariton Experiments

Non-linear ground state occupation.



[*Deng et al.* Science **298** 199 (2002)]
(also [*Dang et al.* PRL. **81** 3920 (1998)])

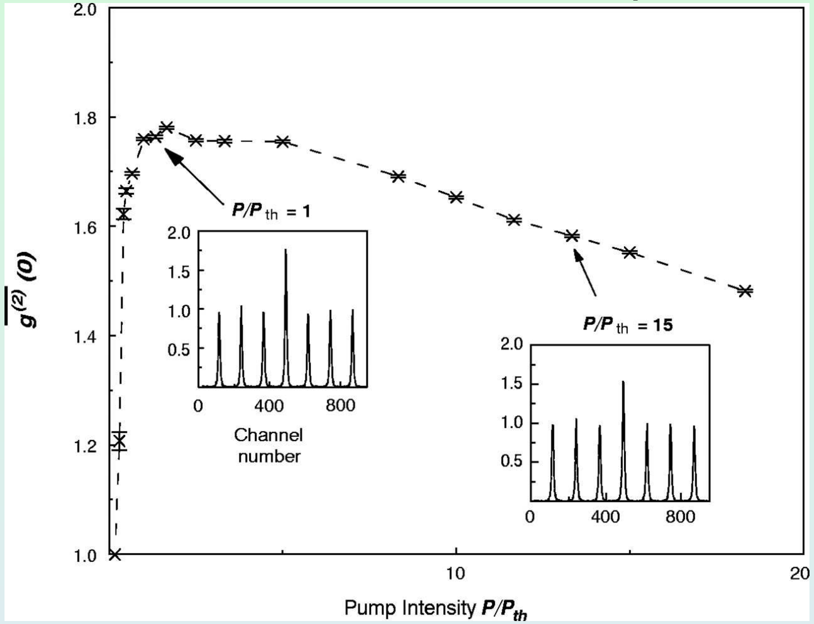
Peak in angular distribution.



[*Deng et al. PNAS* **100** 15318]

Polariton Experiments (2)

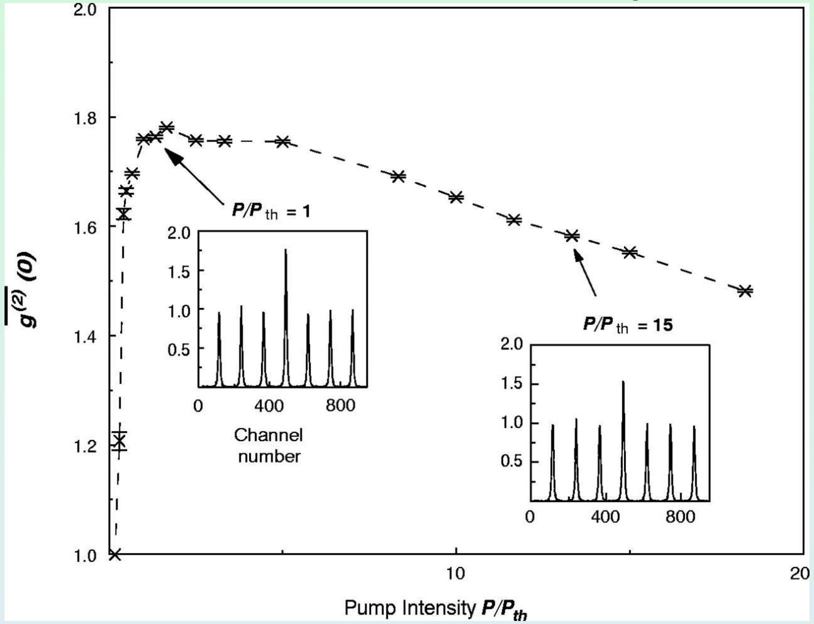
Second order coherence of photons.



[*Deng et al.* Science **298** 199 (2002)]

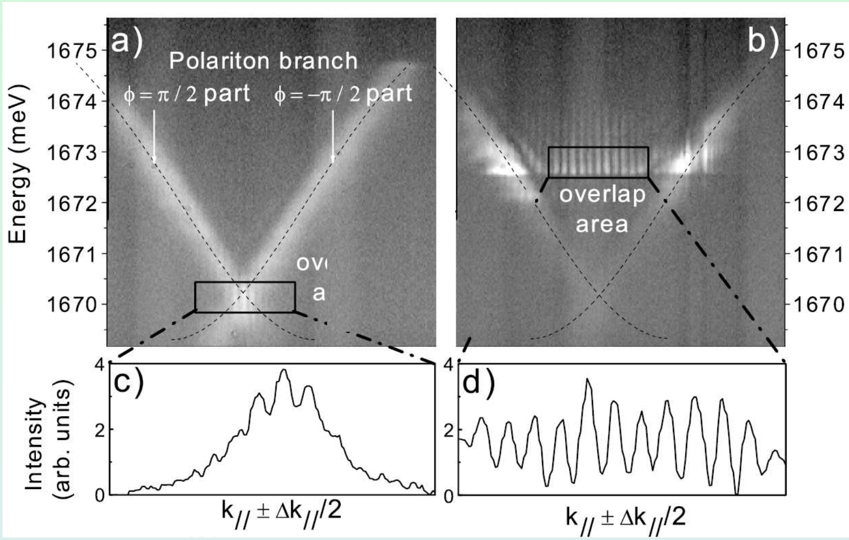
Polariton Experiments (2)

Second order coherence of photons.



[*Deng et al.* Science **298** 199 (2002)]

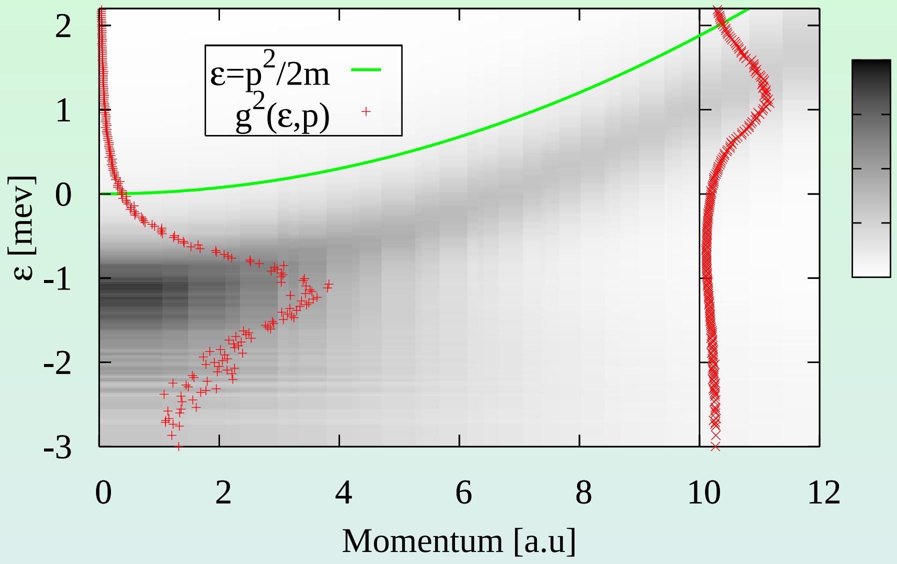
Interference fringes:



[*Richard et al.* Phys. Rev. Lett. **94** 187401 (2005)]

Localised two level systems

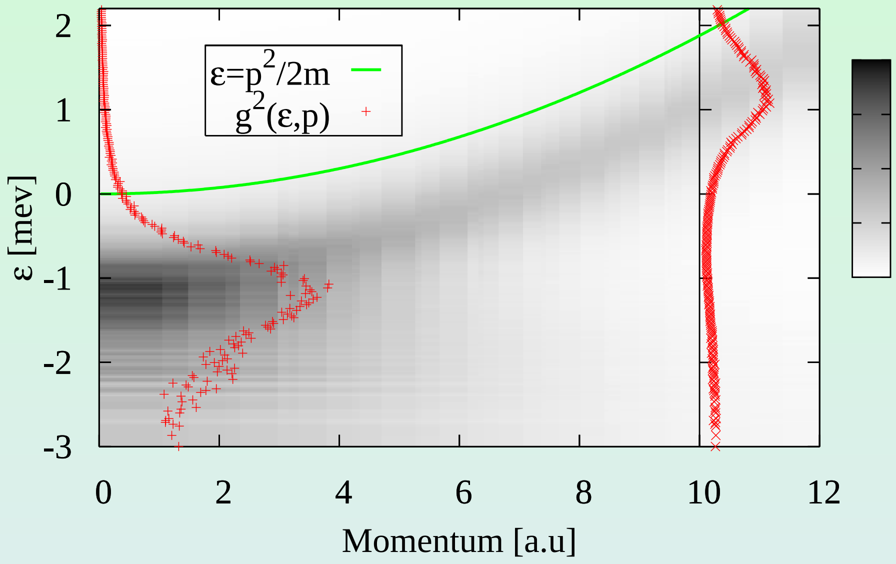
Coupling to light:

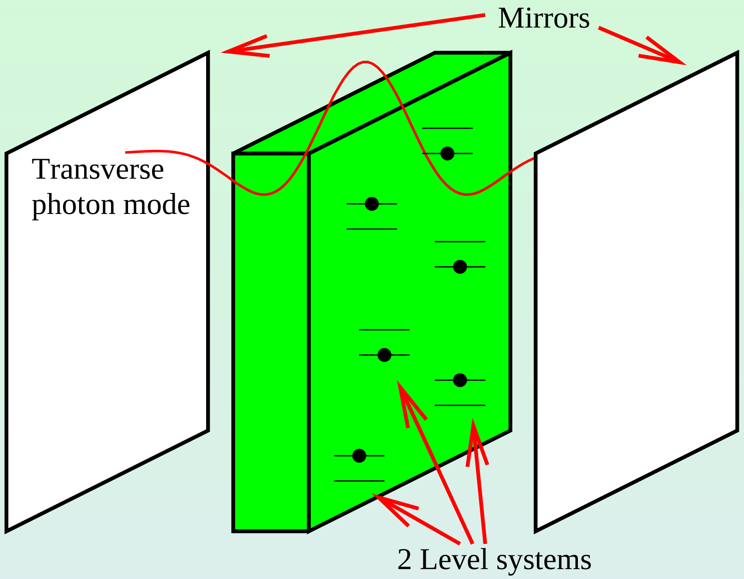


[*Marchetti et al.* cond-mat/0509438].

Localised two level systems

Coupling to light:

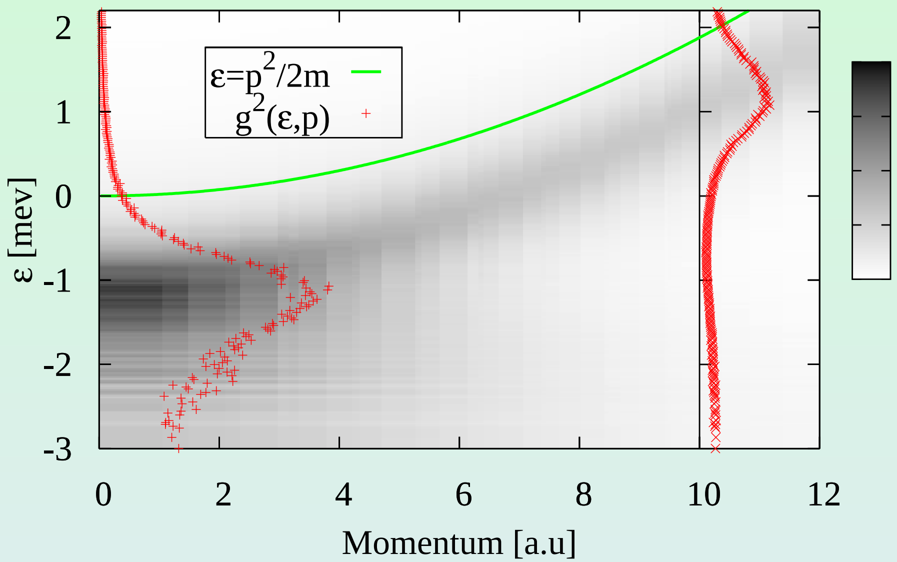


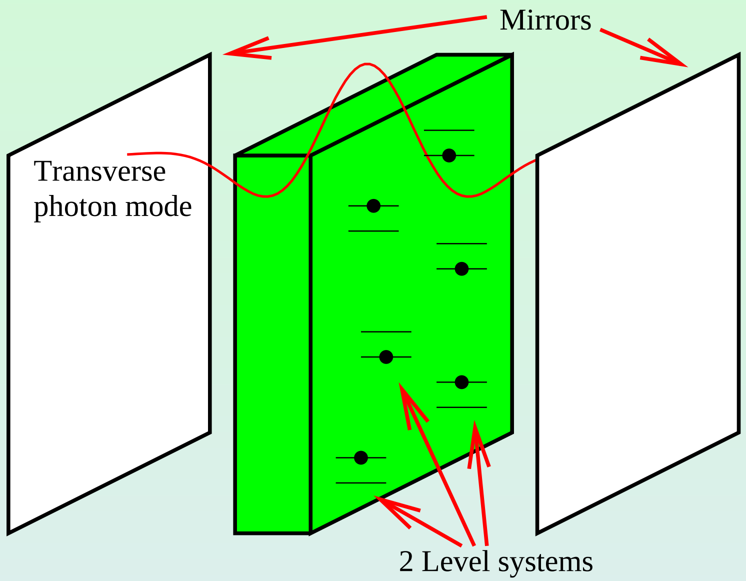


[*Marchetti et al.* cond-mat/0509438].

Localised two level systems

Coupling to light:



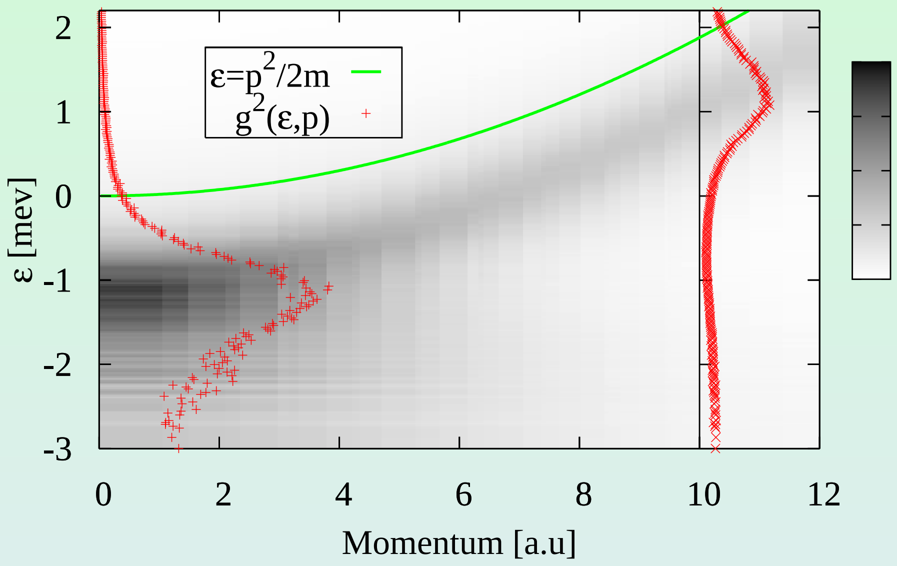


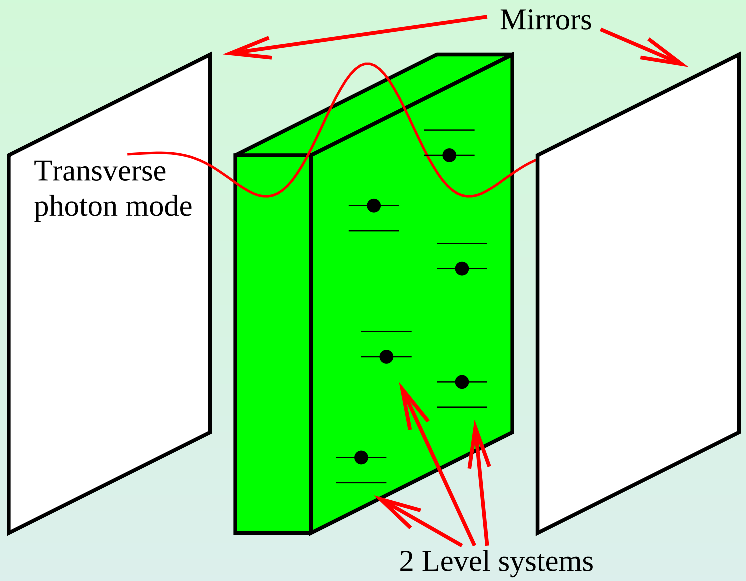
- Effective hard-core exciton-exciton interaction exists.

[*Marchetti et al.* cond-mat/0509438].

Localised two level systems

Coupling to light:





- Effective hard-core exciton-exciton interaction exists.
- Energy difference between levels represents energy of bound exciton state.

[*Marchetti et al.* cond-mat/0509438].

The Dicke Model Hamiltonian

$$H = \sum_{\alpha=1}^{\alpha=nA} \epsilon \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha\right)$$

The Dicke Model Hamiltonian

$$H = \sum_{\alpha=1}^{\alpha=nA} \epsilon \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \sum_{k=l/\sqrt{A}} \hbar \omega_k \psi^\dagger_k \psi_k$$

The Dicke Model Hamiltonian

$$
\begin{aligned}
H &= \sum_{\alpha=1}^{\alpha=nA} \epsilon \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \sum_{k=l/\sqrt{A}} \hbar\omega_k \psi^\dagger_k \psi_k \\
&+ \frac{g}{\sqrt{A}} \sum_{\alpha,k} \left(e^{2\pi i \mathbf{k}.\mathbf{r}_n} \psi_k b^\dagger_\alpha a_\alpha + e^{-2\pi i \mathbf{k}.\mathbf{r}_n} \psi^\dagger_k a^\dagger_\alpha b_\alpha \right).
\end{aligned}
$$

The Dicke Model Hamiltonian

$$
\begin{aligned}
H &= \sum_{\alpha=1}^{\alpha=nA} \epsilon \left(b^\dagger_\alpha b_\alpha - a^\dagger_\alpha a_\alpha \right) + \sum_{k=l/\sqrt{A}} \hbar\omega_k \psi^\dagger_k \psi_k \\
&+ \frac{g}{\sqrt{A}} \sum_{\alpha,k} \left(e^{2\pi i \mathbf{k}.\mathbf{r}_n} \psi_k b^\dagger_\alpha a_\alpha + e^{-2\pi i \mathbf{k}.\mathbf{r}_n} \psi^\dagger_k a^\dagger_\alpha b_\alpha \right).
\end{aligned}
$$

- T.L.S. areal density n

The Dicke Model Hamiltonian

$$H = \sum_{\alpha=1}^{\alpha=nA} \epsilon \left(b_\alpha^\dagger b_\alpha - a_\alpha^\dagger a_\alpha \right) + \sum_{k=l/\sqrt{A}} \hbar\omega_k \psi_k^\dagger \psi_k$$

$$+ \frac{g}{\sqrt{A}} \sum_{\alpha,k} \left(e^{2\pi i \mathbf{k}.\mathbf{r}_n} \psi_k b_\alpha^\dagger a_\alpha + e^{-2\pi i \mathbf{k}.\mathbf{r}_n} \psi_k^\dagger a_\alpha^\dagger b_\alpha \right).$$

- T.L.S. areal density n
- Photon dispersion in cavity: $\omega_k = \sqrt{\omega_0^2 + c^2k^2} \approx \omega_0 + \hbar k^2/2m$

The Dicke Model Hamiltonian

$$H = \sum_{\alpha=1}^{\alpha=nA} \epsilon \left(b_\alpha^\dagger b_\alpha - a_\alpha^\dagger a_\alpha\right) + \sum_{k=l/\sqrt{A}} \hbar\omega_k \psi_k^\dagger \psi_k$$
$$+ \frac{g}{\sqrt{A}} \sum_{\alpha,k} \left(e^{2\pi i \mathbf{k}.\mathbf{r}_n} \psi_k b_\alpha^\dagger a_\alpha + e^{-2\pi i \mathbf{k}.\mathbf{r}_n} \psi_k^\dagger a_\alpha^\dagger b_\alpha\right).$$

- T.L.S. areal density n
- Photon dispersion in cavity: $\omega_k = \sqrt{\omega_0^2 + c^2k^2} \approx \omega_0 + \hbar k^2/2m$

Assume thermal equilibrium with fixed number of excitations, $\tilde{H} = H - \mu N$

$$N = \sum_{\alpha=1}^{\alpha=nA} \frac{1}{2}\left(b_\alpha^\dagger b_\alpha - a_\alpha^\dagger a_\alpha + 1\right) + \sum_{k=l/\sqrt{A}} \psi_k^\dagger \psi_k.$$

Mean field theory

At zero temperature, BCS-like ansatz is exact minimum

$$|\Psi\rangle = e^{\lambda(\psi_0^\dagger + \sum_\alpha X_\alpha b_\alpha^\dagger a_\alpha)} \prod_\alpha a_\alpha^\dagger |0\rangle$$

[*Eastham & Littlewood. Phys. Rev. B* **64** 235101].

Mean field theory

At zero temperature, BCS-like ansatz is exact minimum

$$|\Psi\rangle = e^{\lambda(\psi_0^\dagger + \sum_\alpha X_\alpha b_\alpha^\dagger a_\alpha)} \prod_\alpha a_\alpha^\dagger |0\rangle = e^{\lambda\psi_0^\dagger} \prod_\alpha \left(v_\alpha b_\alpha^\dagger + u_\alpha a_\alpha^\dagger\right) |0\rangle$$

[*Eastham & Littlewood. Phys. Rev. B* **64** 235101].

Mean field theory

At zero temperature, BCS-like ansatz is exact minimum

$$|\Psi\rangle = e^{\lambda(\psi_0^\dagger + \sum_\alpha X_\alpha b_\alpha^\dagger a_\alpha)} \prod_\alpha a_\alpha^\dagger |0\rangle = e^{\lambda \psi_0^\dagger} \prod_\alpha \left(v_\alpha b_\alpha^\dagger + u_\alpha a_\alpha^\dagger \right) |0\rangle$$

At finite T, Integrate out TLS, and minimise w.r.t ψ.

[*Eastham & Littlewood. Phys. Rev. B* **64** 235101].

Mean field theory

At zero temperature, BCS-like ansatz is exact minimum

$$|\Psi\rangle = e^{\lambda(\psi_0^\dagger + \sum_\alpha X_\alpha b_\alpha^\dagger a_\alpha)} \prod_\alpha a_\alpha^\dagger |0\rangle = e^{\lambda\psi_0^\dagger} \prod_\alpha \left(v_\alpha b_\alpha^\dagger + u_\alpha a_\alpha^\dagger\right) |0\rangle$$

At finite T, Integrate out TLS, and minimise w.r.t ψ.
Gap equation:

$$\begin{aligned} \tilde{\omega}_0 \psi_0 &= g^2 n \frac{\tanh(\beta E)}{E} \psi_0 \\ E &= \sqrt{\tilde{\epsilon}^2 + g^2 n |\psi_0|^2} \end{aligned}$$

[*Eastham & Littlewood. Phys. Rev. B* **64** 235101].

Mean field theory

At zero temperature, BCS-like ansatz is exact minimum

$$|\Psi\rangle = e^{\lambda(\psi_0^\dagger + \sum_\alpha X_\alpha b_\alpha^\dagger a_\alpha)} \prod_\alpha a_\alpha^\dagger |0\rangle = e^{\lambda\psi_0^\dagger} \prod_\alpha \left(v_\alpha b_\alpha^\dagger + u_\alpha a_\alpha^\dagger\right)|0\rangle$$

At finite T, Integrate out TLS, and minimise w.r.t ψ.
Gap equation:

$$\tilde{\omega}_0 \psi_0 = g^2 n \frac{\tanh(\beta E)}{E} \psi_0$$

$$E = \sqrt{\tilde{\epsilon}^2 + g^2 n |\psi_0|^2}$$

Excitation density:

$$\frac{\rho_{ex}}{n} = -\frac{\tilde{\epsilon}}{2E} \tanh(\beta E) + |\psi_0|^2$$

[*Eastham & Littlewood. Phys. Rev. B* **64** 235101].

Mean field theory

At zero temperature, BCS-like ansatz is exact minimum

$$|\Psi\rangle = e^{\lambda(\psi_0^\dagger + \sum_\alpha X_\alpha b_\alpha^\dagger a_\alpha)} \prod_\alpha a_\alpha^\dagger |0\rangle = e^{\lambda\psi_0^\dagger} \prod_\alpha \left(v_\alpha b_\alpha^\dagger + u_\alpha a_\alpha^\dagger\right) |0\rangle$$

At finite T, Integrate out TLS, and minimise w.r.t ψ.
Gap equation:

$$\begin{aligned} \tilde{\omega}_0 \psi_0 &= g^2 n \frac{\tanh(\beta E)}{E} \psi_0 \\ E &= \sqrt{\tilde{\epsilon}^2 + g^2 n |\psi_0|^2} \end{aligned}$$

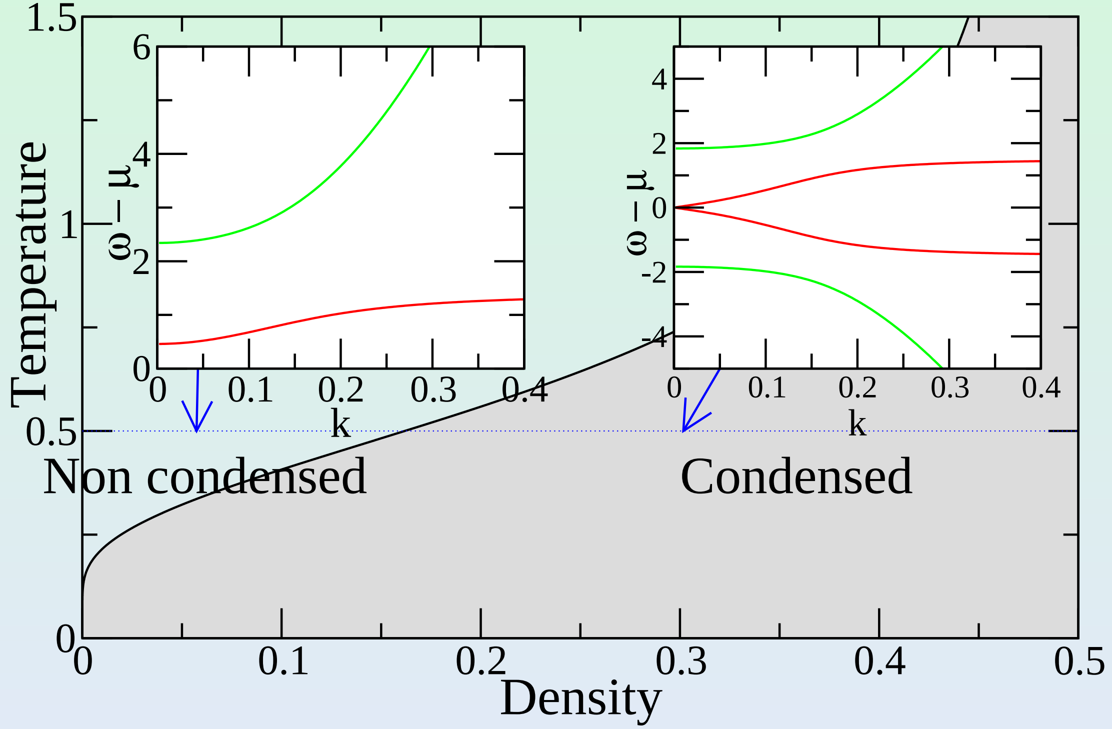
Excitation density:

$$\frac{\rho_{ex}}{n} = -\frac{\tilde{\epsilon}}{2E} \tanh(\beta E) + |\psi_0|^2$$

[*Eastham & Littlewood. Phys. Rev. B* **64** 235101].

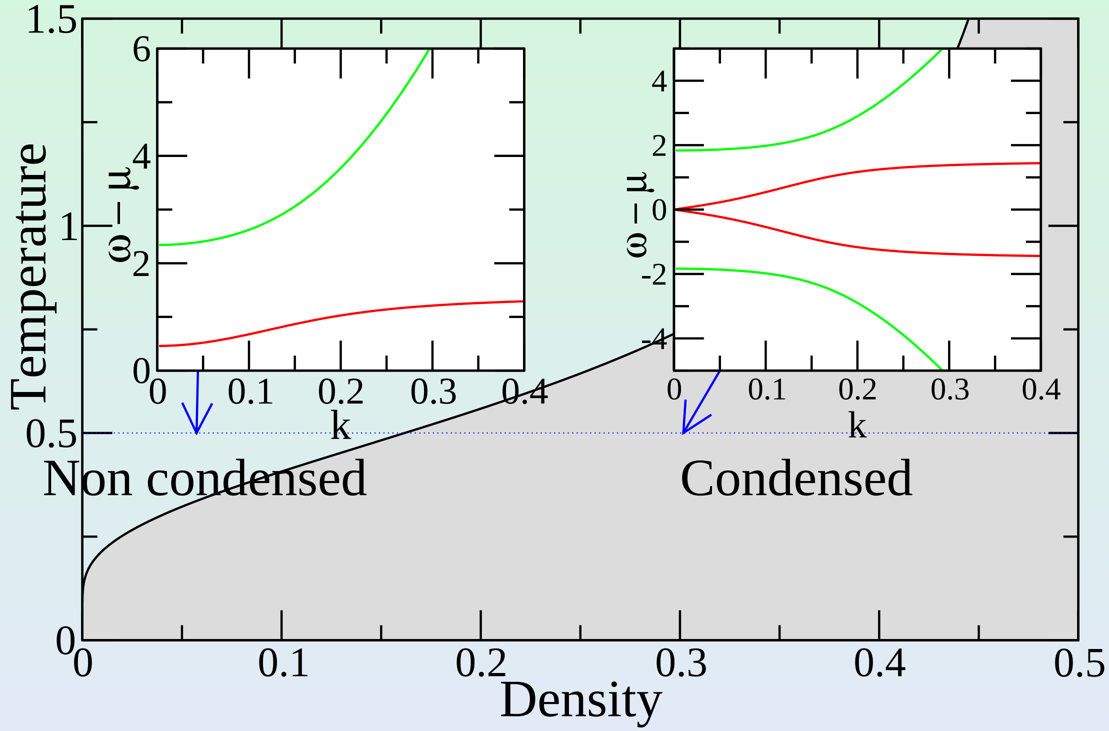
Fluctuation spectrum

Consider fluctuations about mean field — Poles of greens function for photon response.



Fluctuation spectrum

Consider fluctuations about mean field — Poles of greens function for photon response.

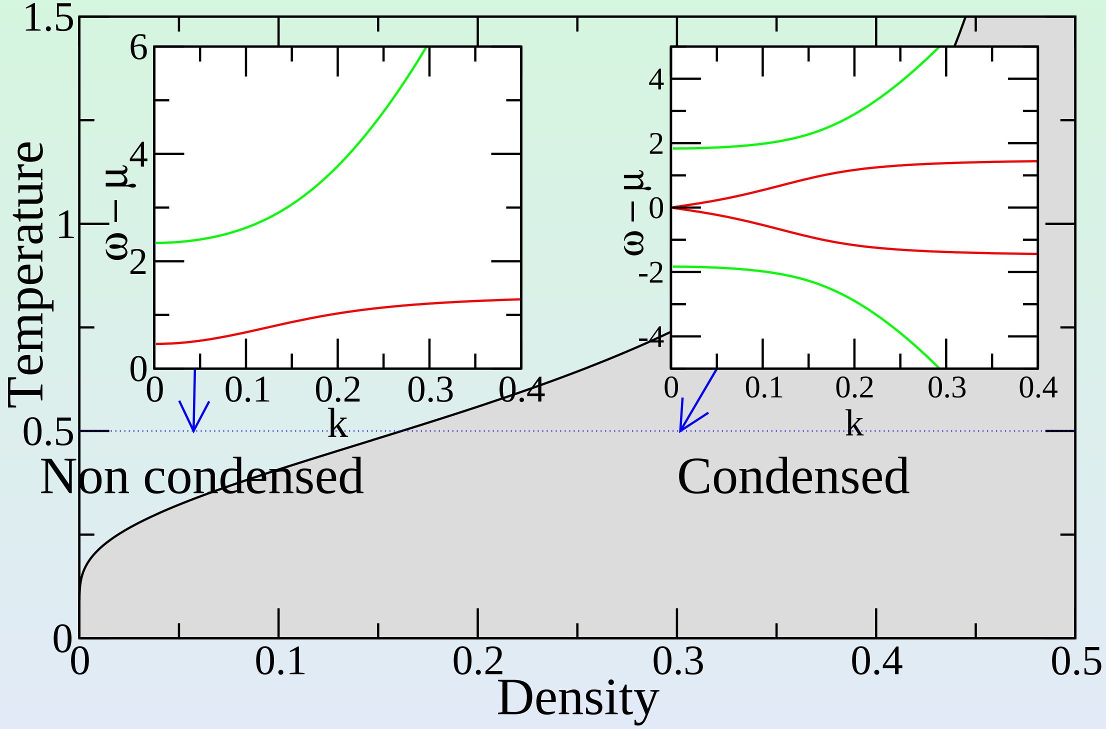


For small k, linear dispersion mode,

$$\xi_1 = \pm ck + O(k^2)$$

Fluctuation spectrum

Consider fluctuations about mean field — Poles of greens function for photon response.



For small k, linear dispersion mode,

$$\xi_1 = \pm ck + O(k^2)$$

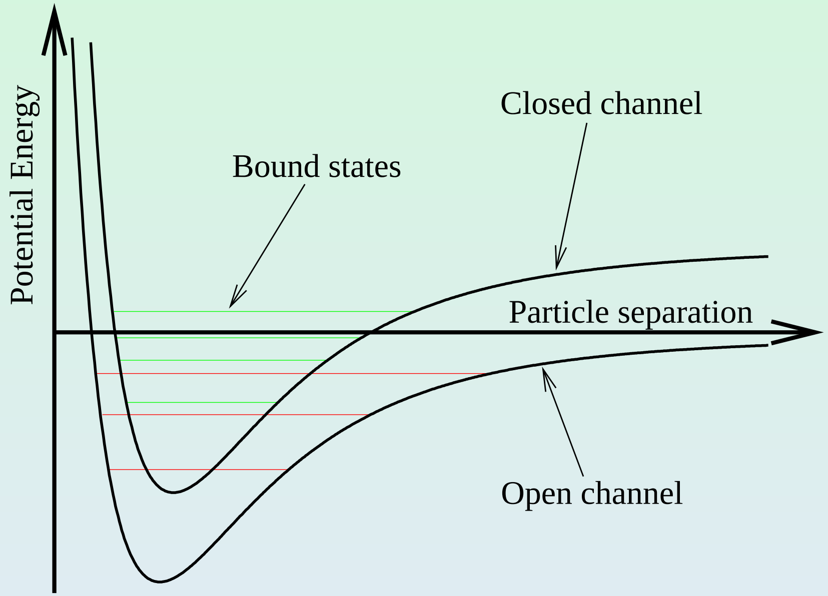
At large k, recover bare exciton/photon spectra.

Diversion: Feshbach resonance

Condensation in system of bosons coupled to fermion pairs — analogies to Feshbach resonance.

Diversion: Feshbach resonance

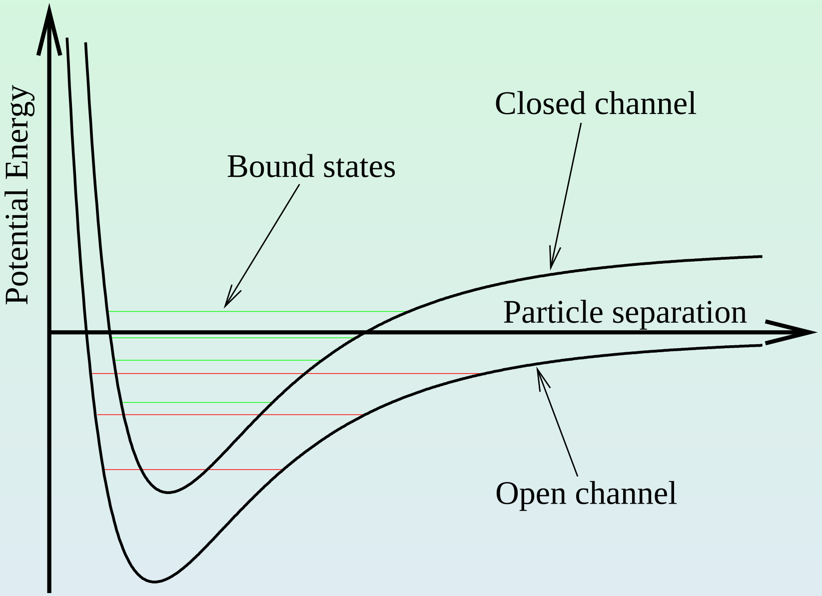
Condensation in system of bosons coupled to fermion pairs — analogies to Feshbach resonance.

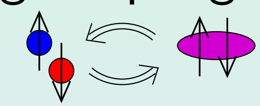


- Interaction between fermions depends on spin states.

Diversion: Feshbach resonance

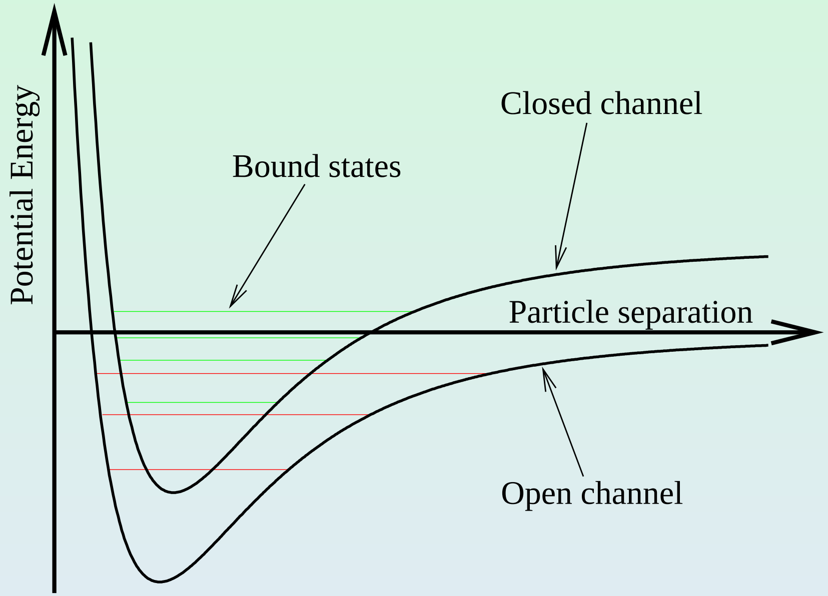
Condensation in system of bosons coupled to fermion pairs — analogies to Feshbach resonance.

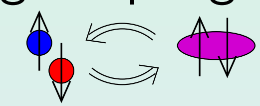


- Interaction between fermions depends on spin states.
- At resonance, “strong-coupling” of atoms and molecule: 

Diversion: Feshbach resonance

Condensation in system of bosons coupled to fermion pairs — analogies to Feshbach resonance.



- Interaction between fermions depends on spin states.
- At resonance, “strong-coupling” of atoms and molecule: 
- Detuning gives crossover from BCS of atoms to BEC of molecules.

Diversion: Analogies and differences

Comparison of physical systems:

Feshbach resonance $\Longleftrightarrow$ **Microcavity Polaritons**

Diversion: Analogies and differences

Comparison of physical systems:

Feshbach resonance	$\Longleftrightarrow$	**Microcavity Polaritons**
Closed channel molecules	$\Longleftrightarrow$	Microcavity Photons

Diversion: Analogies and differences

Comparison of physical systems:

Feshbach resonance	$\Longleftrightarrow$	**Microcavity Polaritons**
Closed channel molecules	$\Longleftrightarrow$	Microcavity Photons
Atoms	$\Longleftrightarrow$	Electron/Holes

Diversion: Analogies and differences

Comparison of physical systems:

Feshbach resonance	$\Longleftrightarrow$	**Microcavity Polaritons**
Closed channel molecules	$\Longleftrightarrow$	Microcavity Photons
Atoms	$\Longleftrightarrow$	Electron/Holes
Inter-channel coupling	$\Longleftrightarrow$	Electric dipole interaction

Diversion: Analogies and differences

Comparison of physical systems:

Feshbach resonance	$\Longleftrightarrow$	**Microcavity Polaritons**
Closed channel molecules	$\Longleftrightarrow$	Microcavity Photons
Atoms	$\Longleftrightarrow$	Electron/Holes
Inter-channel coupling	$\Longleftrightarrow$	Electric dipole interaction
Background potential	$\Longleftrightarrow$	Coulomb attraction

Diversion: Analogies and differences

Comparison of physical systems:

Feshbach resonance	$\Longleftrightarrow$	**Microcavity Polaritons**
Closed channel molecules	$\Longleftrightarrow$	Microcavity Photons
Atoms	$\Longleftrightarrow$	Electron/Holes
Inter-channel coupling	$\Longleftrightarrow$	Electric dipole interaction
Background potential	$\Longleftrightarrow$	Coulomb attraction

Important differences

- Polaritons: Measure only emitted photons.

Diversion: Analogies and differences

Comparison of physical systems:

Feshbach resonance	$\Longleftrightarrow$	**Microcavity Polaritons**
Closed channel molecules	$\Longleftrightarrow$	Microcavity Photons
Atoms	$\Longleftrightarrow$	Electron/Holes
Inter-channel coupling	$\Longleftrightarrow$	Electric dipole interaction
Background potential	$\Longleftrightarrow$	Coulomb attraction

Important differences

- Polaritons: Measure only emitted photons.
- Cannot dynamically change exciton-photon detuning.

Diversion: Holland-Timmermans model

One model of Feshbach resonance, very similar to Dicke model:

Diversion: Holland-Timmermans model

One model of Feshbach resonance, very similar to Dicke model:

$$H - \mu N = \sum_{k,\sigma} (\epsilon_k - \mu) c^\dagger_{k,\sigma} c_{k,\sigma}$$

Diversion: Holland-Timmermans model

One model of Feshbach resonance, very similar to Dicke model:

$$H - \mu N = \sum_{k,\sigma} (\epsilon_k - \mu) c^\dagger_{k,\sigma} c_{k,\sigma} + \sum_k (\epsilon_k + 2\Delta - 2\mu) b^\dagger_k b_k$$

Diversion: Holland-Timmermans model

One model of Feshbach resonance, very similar to Dicke model:

$$
\begin{aligned}
H - \mu N &= \sum_{k,\sigma} (\epsilon_k - \mu) c^\dagger_{k,\sigma} c_{k,\sigma} + \sum_k (\epsilon_k + 2\Delta - 2\mu) b^\dagger_k b_k \\
&+ g \sum_{k,q} \left(b^\dagger_q c_{-k+q/2,\downarrow} c_{k+q/2,\uparrow} + c^\dagger_{k+q/2,\uparrow} c^\dagger_{-k+q/2,\downarrow} b_q \right)
\end{aligned}
$$

Diversion: Holland-Timmermans model

One model of Feshbach resonance, very similar to Dicke model:

$$\begin{aligned} H - \mu N &= \sum_{k,\sigma} (\epsilon_k - \mu) c^\dagger_{k,\sigma} c_{k,\sigma} + \sum_k (\epsilon_k + 2\Delta - 2\mu) b^\dagger_k b_k \\ &+ g \sum_{k,q} \left(b^\dagger_q c_{-k+q/2,\downarrow} c_{k+q/2,\uparrow} + c^\dagger_{k+q/2,\uparrow} c^\dagger_{-k+q/2,\downarrow} b_q \right) \\ &- \frac{U}{2} \sum_{k,k',q} c^\dagger_{k+q,\uparrow} c^\dagger_{k'-q,\downarrow} c_{k,\downarrow} c_{k',\uparrow}. \end{aligned}$$

Diversion: Holland-Timmermans model

One model of Feshbach resonance, very similar to Dicke model:

$$
\begin{aligned}
H - \mu N &= \sum_{k,\sigma} (\epsilon_k - \mu) c^\dagger_{k,\sigma} c_{k,\sigma} + \sum_k (\epsilon_k + 2\Delta - 2\mu) b^\dagger_k b_k \\
&+ g \sum_{k,q} \left(b^\dagger_q c_{-k+q/2,\downarrow} c_{k+q/2,\uparrow} + c^\dagger_{k+q/2,\uparrow} c^\dagger_{-k+q/2,\downarrow} b_q \right) \\
&- \frac{U}{2} \sum_{k,k',q} c^\dagger_{k+q,\uparrow} c^\dagger_{k'-q,\downarrow} c_{k,\downarrow} c_{k',\uparrow}.
\end{aligned}
$$

Diversion: Holland-Timmermans model

One model of Feshbach resonance, very similar to Dicke model:

$$
\begin{aligned}
H - \mu N &= \sum_{k,\sigma} (\epsilon_k - \mu) c^\dagger_{k,\sigma} c_{k,\sigma} + \sum_k (\epsilon_k + 2\Delta - 2\mu) b^\dagger_k b_k \\
&+ g \sum_{k,q} \left(b^\dagger_q c_{-k+q/2,\downarrow} c_{k+q/2,\uparrow} + c^\dagger_{k+q/2,\uparrow} c^\dagger_{-k+q/2,\downarrow} b_q \right) \\
&- \frac{U}{2} \sum_{k,k',q} c^\dagger_{k+q,\uparrow} c^\dagger_{k'-q,\downarrow} c_{k,\downarrow} c_{k',\uparrow}.
\end{aligned}
$$

Gives energy dependant fermion-fermion scattering.

Diversion: Holland-Timmermans model

One model of Feshbach resonance, very similar to Dicke model:

$$\begin{aligned} H - \mu N &= \sum_{k,\sigma} (\epsilon_k - \mu) c^\dagger_{k,\sigma} c_{k,\sigma} + \sum_k (\epsilon_k + 2\Delta - 2\mu) b^\dagger_k b_k \\ &+ g \sum_{k,q} \left(b^\dagger_q c_{-k+q/2,\downarrow} c_{k+q/2,\uparrow} + c^\dagger_{k+q/2,\uparrow} c^\dagger_{-k+q/2,\downarrow} b_q \right) \\ &- \frac{U}{2} \sum_{k,k',q} c^\dagger_{k+q,\uparrow} c^\dagger_{k'-q,\downarrow} c_{k,\downarrow} c_{k',\uparrow}. \end{aligned}$$

Gives energy dependant fermion-fermion scattering. Unnecessary for current experiments. e.g. [*Simonucci et al. Europhys. Lett.* **69** 713 (2005)]

Comparing mean field theories

General form

$$\frac{1}{U_{\text{eff}}} = \int \nu_s(\epsilon) \frac{\tanh(\beta(\epsilon - \mu))}{\epsilon - \mu} d\epsilon$$

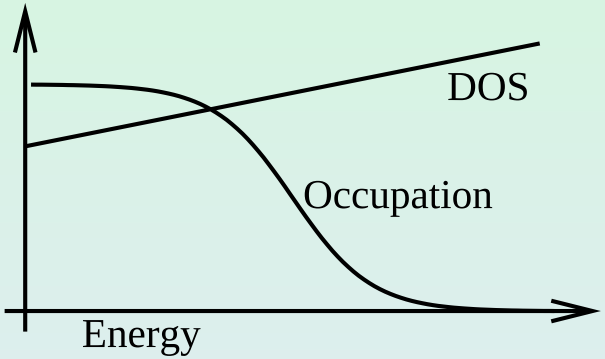
BCS superconductor **Holland-Timmermans** **Dicke model**

Comparing mean field theories

General form

$$\frac{1}{U_{\text{eff}}} = \int \nu_s(\epsilon) \frac{\tanh(\beta(\epsilon - \mu))}{\epsilon - \mu} d\epsilon$$

BCS superconductor **Holland-Timmermans** **Dicke model**

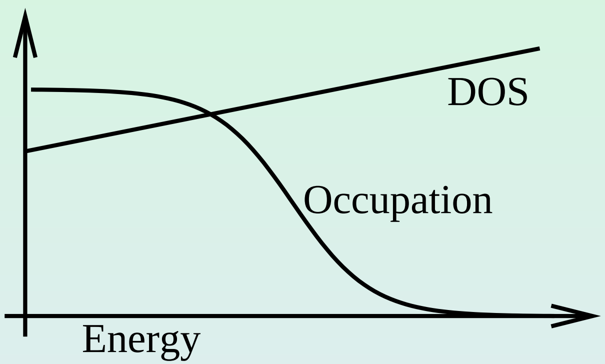


Comparing mean field theories

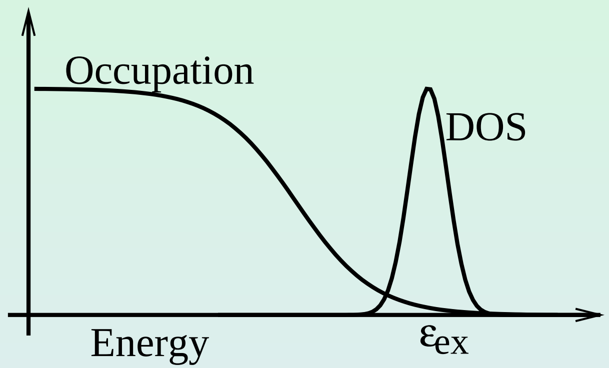
General form

$$\frac{1}{U_{\text{eff}}} = \int \nu_s(\epsilon) \frac{\tanh(\beta(\epsilon - \mu))}{\epsilon - \mu} d\epsilon$$

BCS superconductor **Holland-Timmermans**



Dicke model

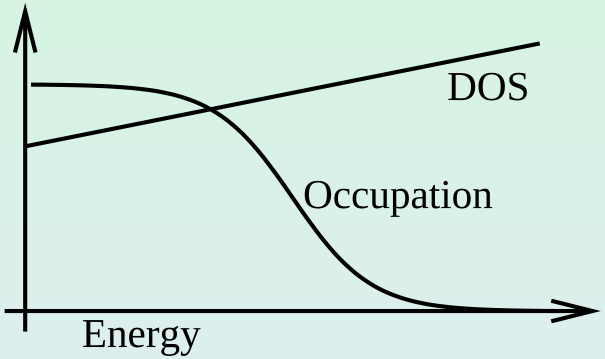


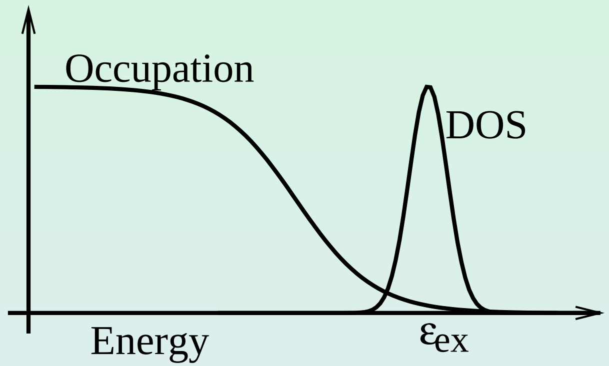
Comparing mean field theories

General form

$$\frac{1}{U_{\text{eff}}} = \int \nu_s(\epsilon) \frac{\tanh(\beta(\epsilon - \mu))}{\epsilon - \mu} d\epsilon$$

BCS superconductor **Holland-Timmermans** **Dicke model**





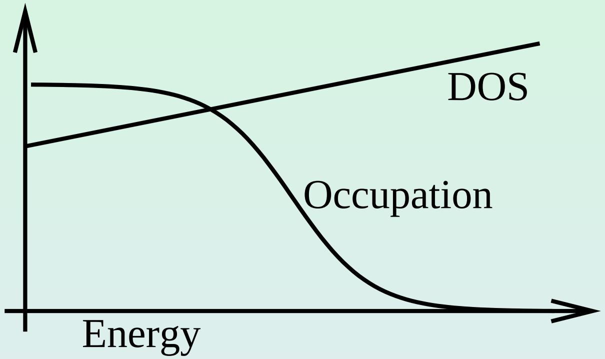
$$\frac{1}{U} = \nu(\mu) \ln\left(\frac{\Omega}{T}\right),$$

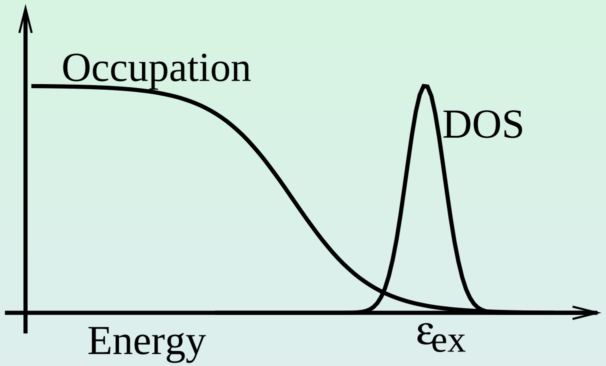
Comparing mean field theories

General form

$$\frac{1}{U_{\text{eff}}} = \int \nu_s(\epsilon) \frac{\tanh(\beta(\epsilon - \mu))}{\epsilon - \mu} d\epsilon$$

BCS superconductor **Holland-Timmermans** **Dicke model**





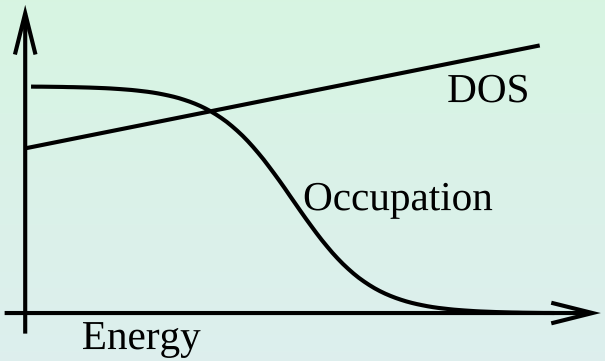
$$\frac{1}{U} = \nu(\mu) \ln\left(\frac{\Omega}{T}\right), \qquad \frac{1}{U_{\text{bg}} + \frac{g^2}{\Delta - \mu}} = \nu(\mu) \ln\left(\frac{\Omega}{T}\right),$$

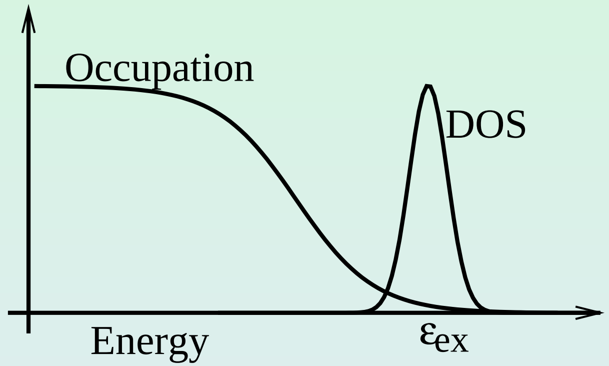
Comparing mean field theories

General form

$$\frac{1}{U_{\text{eff}}} = \int \nu_s(\epsilon) \frac{\tanh(\beta(\epsilon-\mu))}{\epsilon-\mu} d\epsilon$$

BCS superconductor **Holland-Timmermans** **Dicke model**





$$\frac{1}{U} = \nu(\mu) \ln\left(\frac{\Omega}{T}\right), \qquad \frac{1}{U_{\text{bg}} + \frac{g^2}{\Delta-\mu}} = \nu(\mu) \ln\left(\frac{\Omega}{T}\right), \quad \frac{\omega-\mu}{g^2} = \frac{\tanh(\beta(\epsilon_{ex}-\mu))}{\epsilon_{ex}-\mu}$$

Fluctuation corrections

- Consider crossover to BEC with changing density.

Fluctuation corrections

- Consider crossover to BEC with changing density.
- Treatment similar to [*Nozières & Schmitt-Rink J.L.T.P* **59** 195 (1985)]

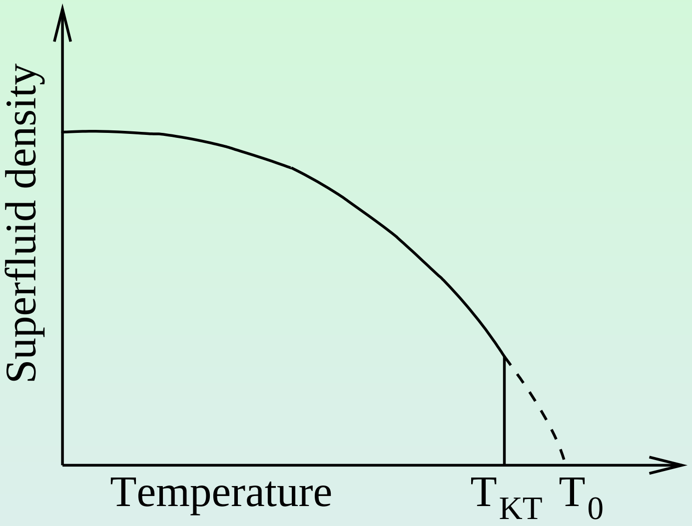
Fluctuation corrections

- Consider crossover to BEC with changing density.
- Treatment similar to [*Nozières & Schmitt-Rink J.L.T.P* **59** 195 (1985)]
- However:
  - Two dimensional system — consider Kosterlitz-Thouless

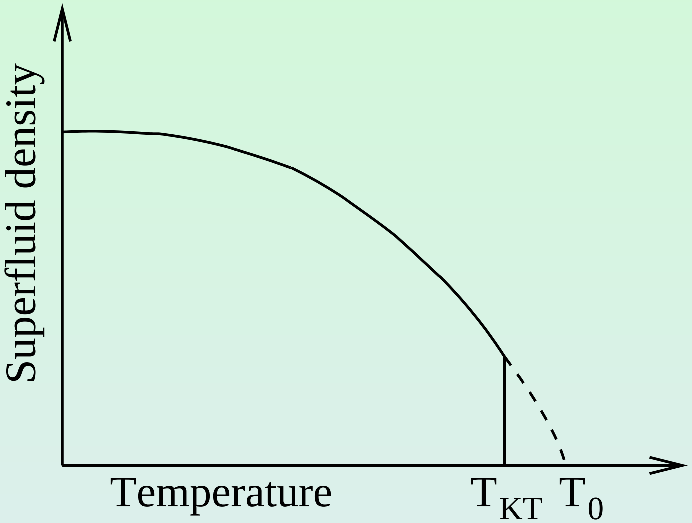
Fluctuation corrections

- Consider crossover to BEC with changing density.
- Treatment similar to [*Nozières & Schmitt-Rink J.L.T.P* **59** 195 (1985)]
- However:
  - Two dimensional system — consider Kosterlitz-Thouless
  - Boson field dynamic, with chemical potential — similar to Holland-Timmermans model, e.g. [*Ohashi & Griffin, PRA.* **67** 063612 (2003)]

Fluctuations in 2d

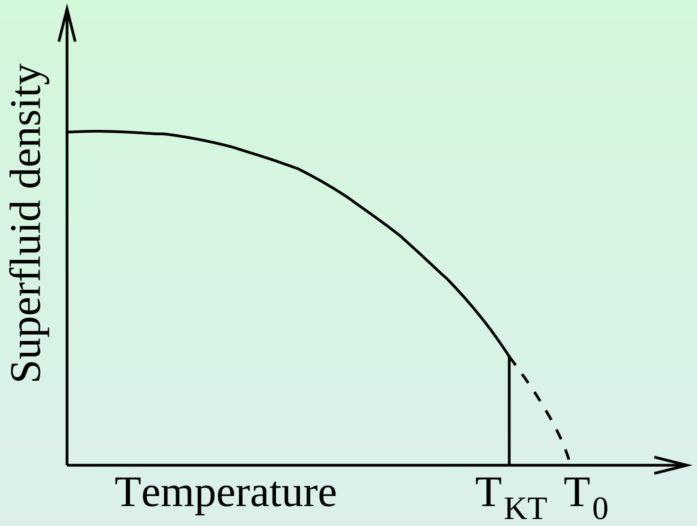


Fluctuations in 2d



$$\rho_s = \# \frac{2 m k_B T}{\hbar^2}$$

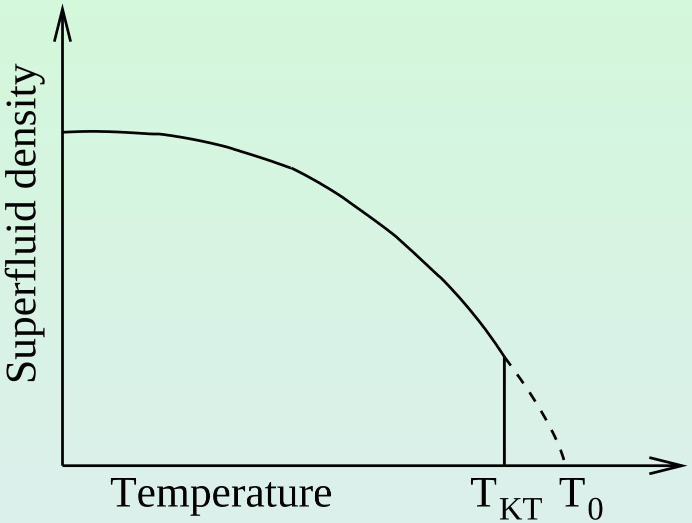
Fluctuations in 2d



$$\rho_s = \# \frac{2mk_BT}{\hbar^2}$$

Need $\rho_{\text{sf}} = \rho_{\text{total}} - \rho_{\text{normal}}$.

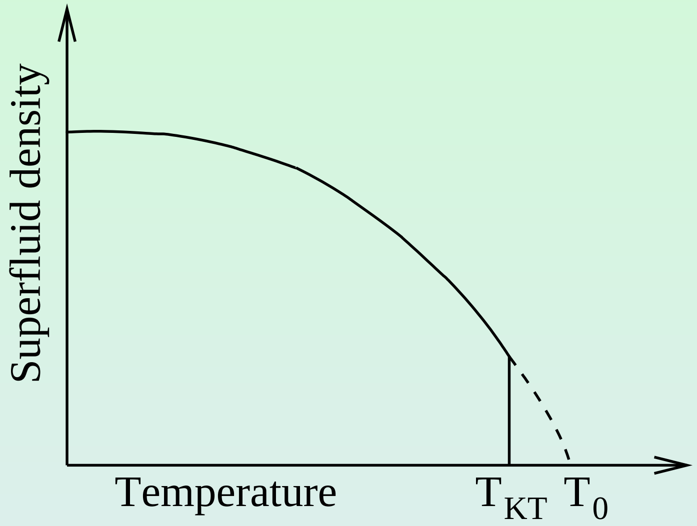
Fluctuations in 2d



$$\rho_s = \# \frac{2mk_BT}{\hbar^2}$$

Need $\rho_{\text{sf}} = \rho_{\text{total}} - \rho_{\text{normal}}$.
ρ_{normal} found by current response:
$J_i(\mathbf{q}) = \chi_{ij}(\mathbf{q})F_j(\mathbf{q})$.

Fluctuations in 2d



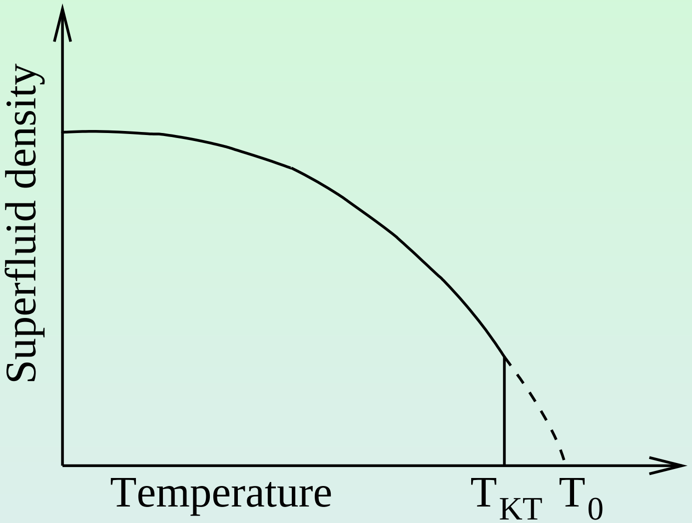
$$\rho_s = \# \frac{2mk_BT}{\hbar^2}$$

Need $\rho_{\rm sf} = \rho_{\rm total} - \rho_{\rm normal}$.
$\rho_{\rm normal}$ found by current response:
$J_i(\mathbf{q}) = \chi_{ij}(\mathbf{q}) F_j(\mathbf{q})$.

$$\chi_{ij} = \chi_L \frac{q_i q_j}{q^2} + \chi_T \left(\delta_{ij} - \frac{q_i q_j}{q^2} \right)$$

Thus $\rho_{\rm normal} = m\chi_T(\mathbf{q} \to 0)$

Fluctuations in 2d



$$\rho_s = \# \frac{2mk_BT}{\hbar^2}$$

Need $\rho_{\rm sf} = \rho_{\rm total} - \rho_{\rm normal}$.
$\rho_{\rm normal}$ found by current response:
$J_i(\mathbf{q}) = \chi_{ij}(\mathbf{q}) F_j(\mathbf{q})$.

$$\chi_{ij} = \chi_L \frac{q_i q_j}{q^2} + \chi_T \left(\delta_{ij} - \frac{q_i q_j}{q^2} \right)$$

Thus $\rho_{\rm normal} = m\chi_T(\mathbf{q} \to 0)$

Thus, need to find: $\rho_{\rm total}$ in presence of condensate.

Fluctuations in presence of condensate

Density is total derivative of free energy:

$$\rho = -\frac{\partial F}{\partial \mu} - \frac{d\psi_0}{d\mu}\frac{\partial F}{\partial \psi_0}$$

Fluctuations in presence of condensate

Density is total derivative of free energy:

$$\rho = -\frac{\partial F}{\partial \mu} - \frac{d\psi_0}{d\mu}\frac{\partial F}{\partial \psi_0}$$

Write $F = F_{\mathrm{m.f.}} + F_{\mathrm{fluct}}$.

Fluctuations in presence of condensate

Density is total derivative of free energy:

$$\rho = -\frac{\partial F}{\partial \mu} - \frac{d\psi_0}{d\mu}\frac{\partial F}{\partial \psi_0}$$

Write $F = F_{\mathrm{m.f.}} + F_{\mathrm{fluct}}$.

Schematically,

$$F_{\mathrm{fluct}} = -k_B T \ln \left\langle e^{-\beta(H_{\mathrm{fluct}}[\psi_0] - \mu\rho_{\mathrm{uncondensed}})} \right\rangle$$

Fluctuations in presence of condensate

Density is total derivative of free energy:

$$\rho = -\frac{\partial F}{\partial \mu} - \frac{d\psi_0}{d\mu}\frac{\partial F}{\partial \psi_0}$$

Write $F = F_{\mathrm{m.f.}} + F_{\mathrm{fluct}}$.

Schematically,

$$F_{\mathrm{fluct}} = -k_B T \ln \left\langle e^{-\beta(H_{\mathrm{fluct}}[\psi_0] - \mu\rho_{\mathrm{uncondensed}})} \right\rangle$$

By definition, $\partial F_{\mathrm{m.f.}}/\partial \psi_0 = 0$, so:

Fluctuations in presence of condensate

Density is total derivative of free energy:

$$\rho = -\frac{\partial F}{\partial \mu} - \frac{d\psi_0}{d\mu}\frac{\partial F}{\partial \psi_0}$$

Write $F = F_{\mathrm{m.f.}} + F_{\mathrm{fluct}}$.

Schematically,

$$F_{\mathrm{fluct}} = -k_B T \ln \left\langle e^{-\beta(H_{\mathrm{fluct}}[\psi_0] - \mu\rho_{\mathrm{uncondensed}})} \right\rangle$$

By definition, $\partial F_{\mathrm{m.f.}}/\partial\psi_0 = 0$, so:

$$\rho = \left(\rho_{\mathrm{m.f.}} - \frac{d\psi_0}{d\mu}\frac{\partial F_{\mathrm{fluct}}}{\partial \psi_0}\right) - \frac{\partial F_{\mathrm{fluct}}}{\partial \mu}$$

Fluctuations in presence of condensate

Density is total derivative of free energy:

$$\rho = -\frac{\partial F}{\partial \mu} - \frac{d\psi_0}{d\mu}\frac{\partial F}{\partial \psi_0}$$

Write $F = F_{\text{m.f.}} + F_{\text{fluct}}$.

Schematically,

$$F_{\text{fluct}} = -k_B T \ln \left\langle e^{-\beta(H_{\text{fluct}}[\psi_0] - \mu\rho_{\text{uncondensed}})} \right\rangle$$

By definition, $\partial F_{\text{m.f.}}/\partial \psi_0 = 0$, so:

$$\rho = \left(\rho_{\text{m.f.}} - \frac{d\psi_0}{d\mu}\frac{\partial F_{\text{fluct}}}{\partial \psi_0}\right) - \frac{\partial F_{\text{fluct}}}{\partial \mu}$$

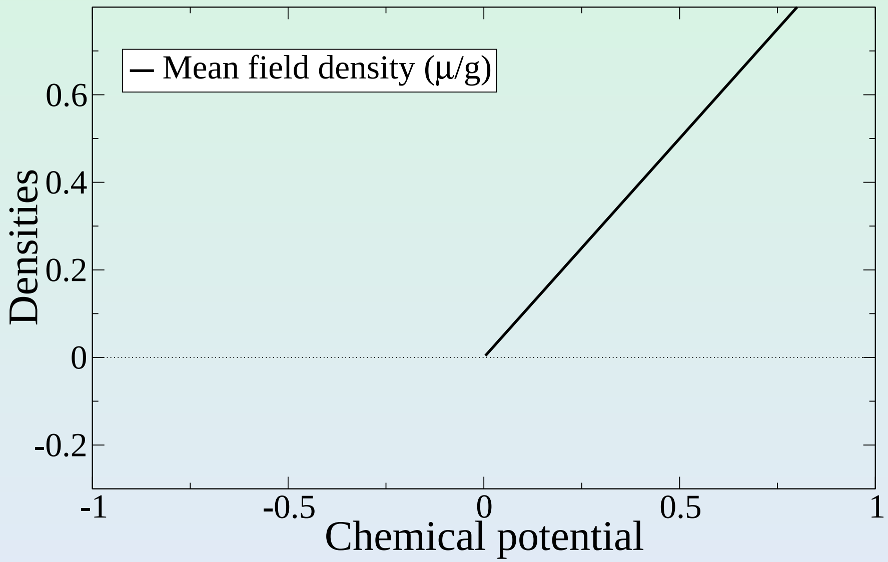
Condensate depletion changes critical chemical potential.

Simple example: Weakly interacting Bose gas

$$H - \mu N = \sum_k (\epsilon_k - \mu) a^\dagger_k a_k + \frac{g}{2} \sum_{k,k',q} a^\dagger_{k+q} a^\dagger_{k'-q} a_k a_{k'}.$$

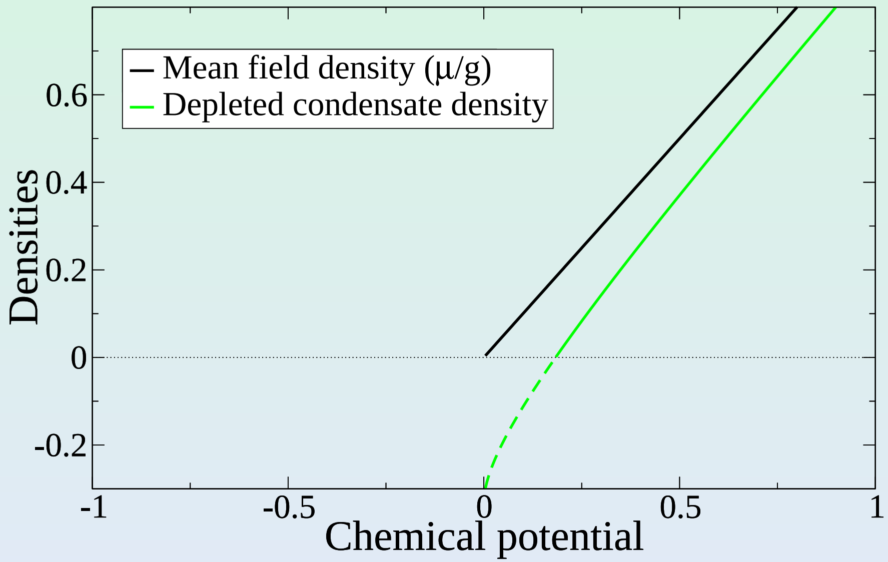
Simple example: Weakly interacting Bose gas

$$H - \mu N = \sum_k (\epsilon_k - \mu) a_k^\dagger a_k + \frac{g}{2} \sum_{k,k',q} a_{k+q}^\dagger a_{k'-q}^\dagger a_k a_{k'}.$$



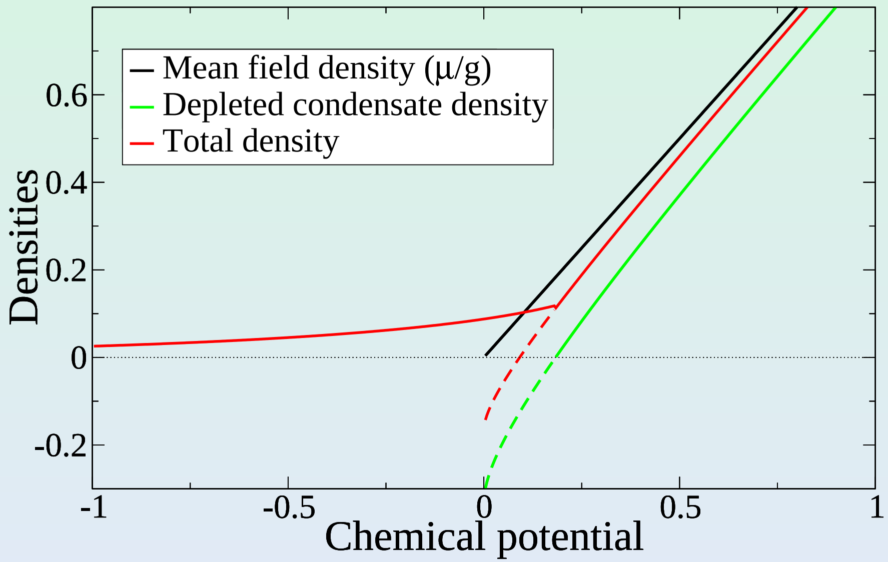
Simple example: Weakly interacting Bose gas

$$H - \mu N = \sum_k (\epsilon_k - \mu) a_k^\dagger a_k + \frac{g}{2} \sum_{k,k',q} a_{k+q}^\dagger a_{k'-q}^\dagger a_k a_{k'}.$$



Simple example: Weakly interacting Bose gas

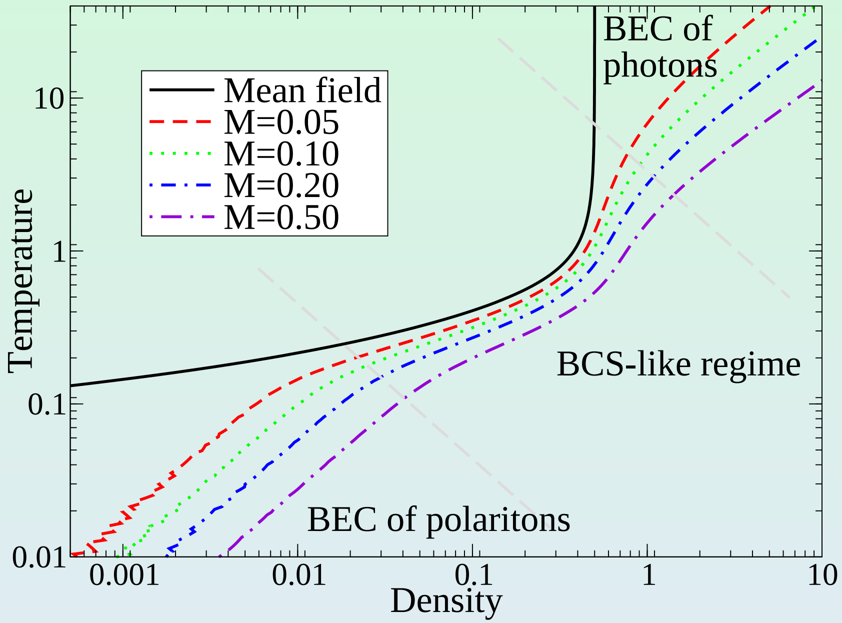
$$H - \mu N = \sum_k (\epsilon_k - \mu) a_k^\dagger a_k + \frac{g}{2} \sum_{k,k',q} a_{k+q}^\dagger a_{k'-q}^\dagger a_k a_{k'}.$$



Normal state exists for $\mu > 0$: Need self energy.

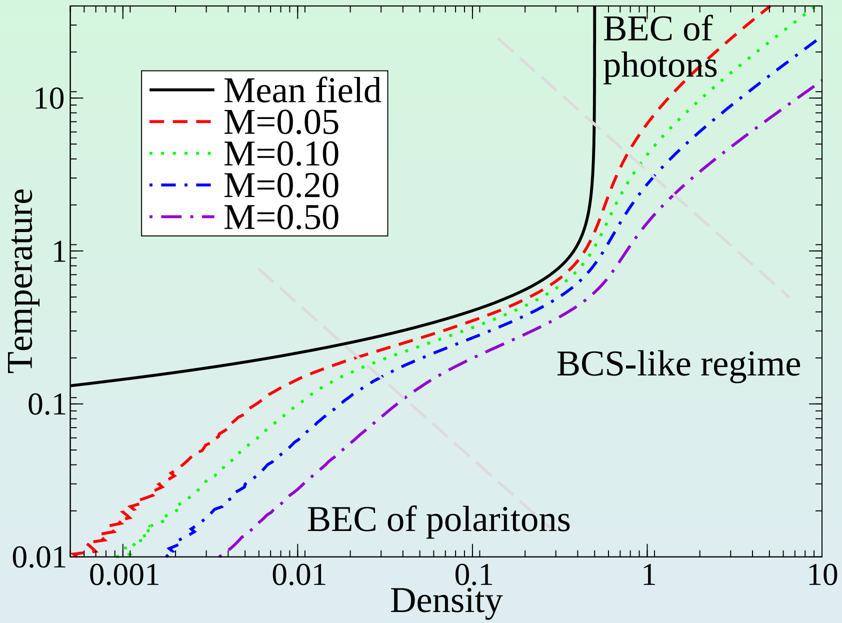
The phase diagram

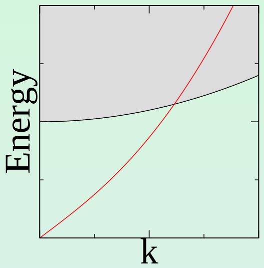
Calculate density where $\rho_{\text{superfluid}} = 0$.



The phase diagram

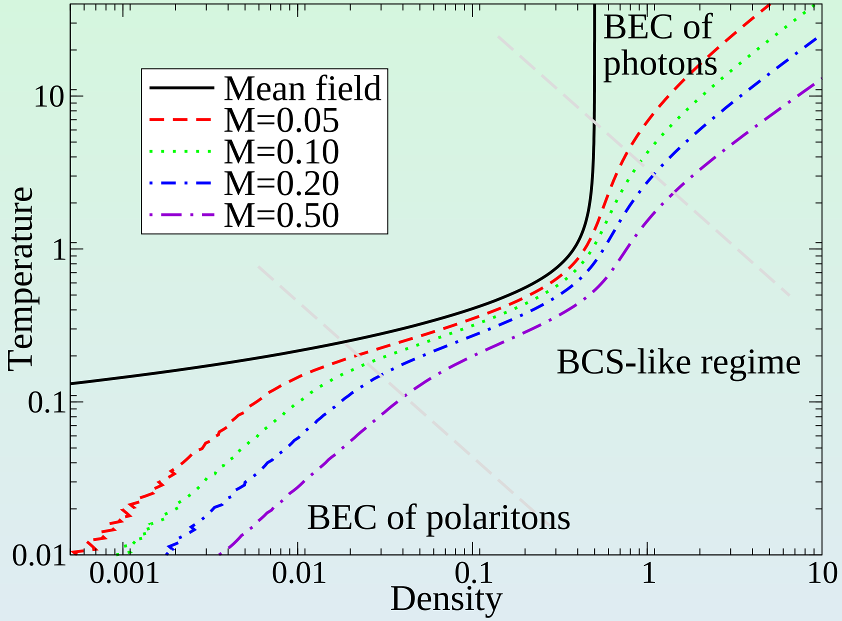
Calculate density where $\rho_{\text{superfluid}} = 0$.

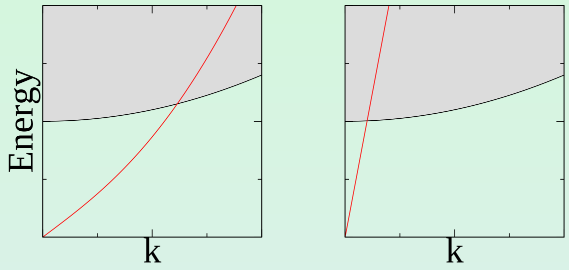




The phase diagram

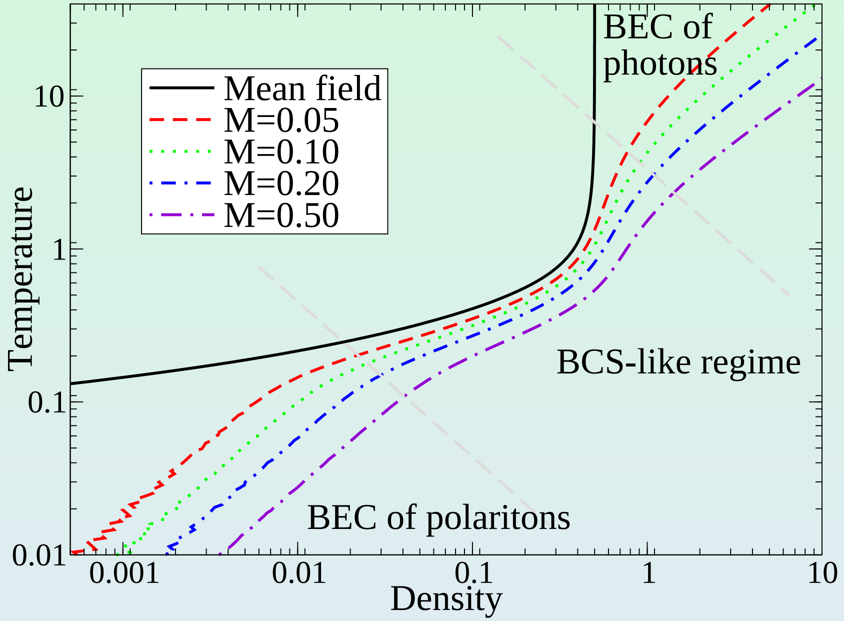
Calculate density where $\rho_{\text{superfluid}} = 0$.

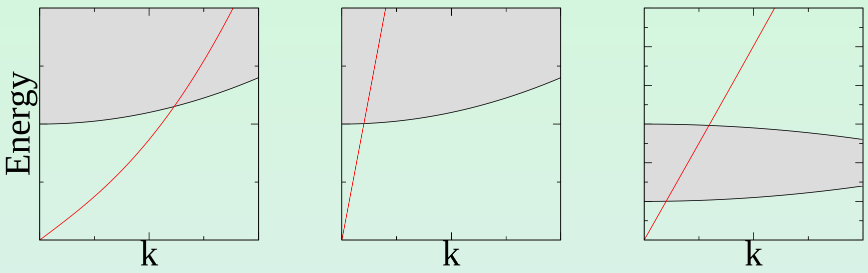




The phase diagram

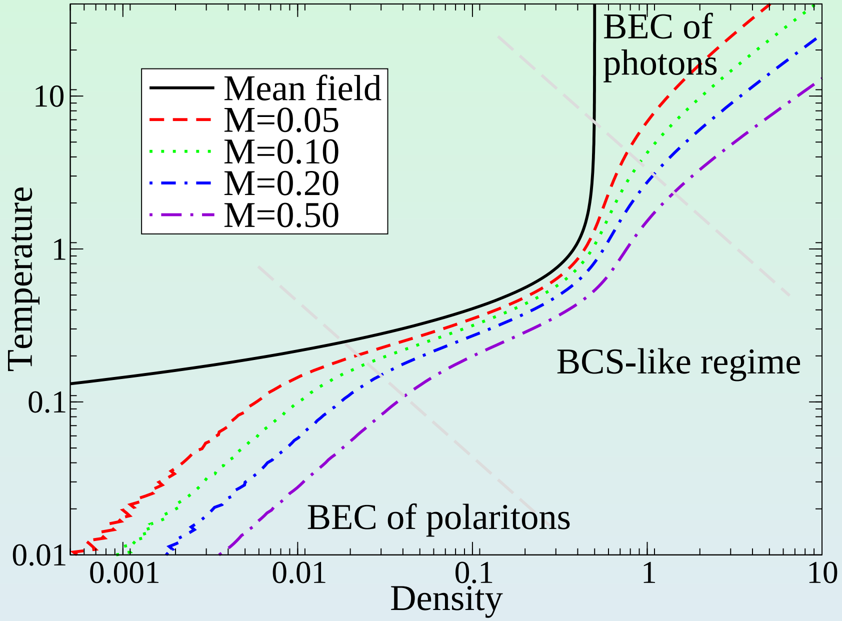
Calculate density where $\rho_{\text{superfluid}} = 0$.

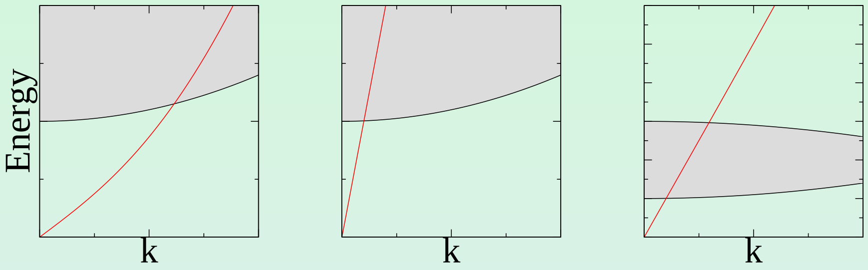




The phase diagram

Calculate density where $\rho_{\text{superfluid}} = 0$.



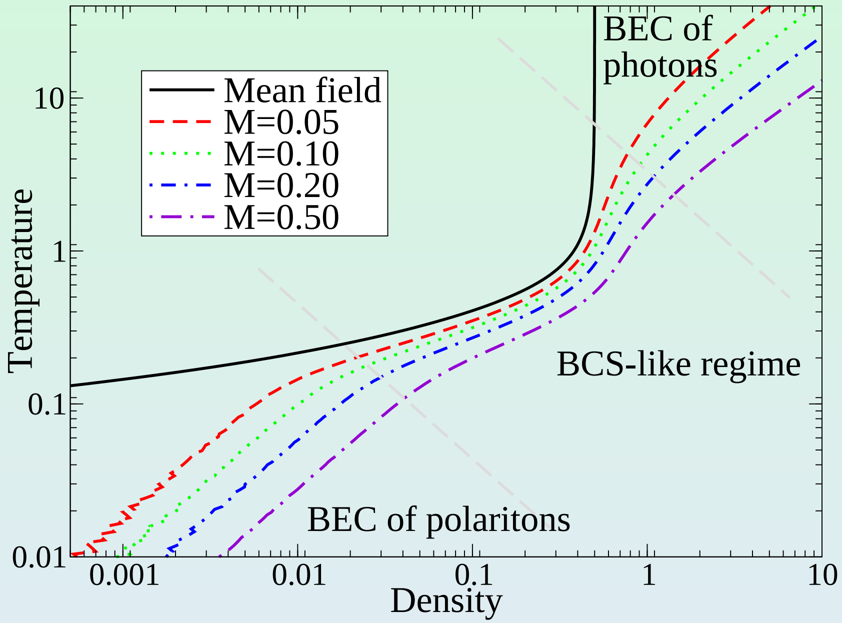


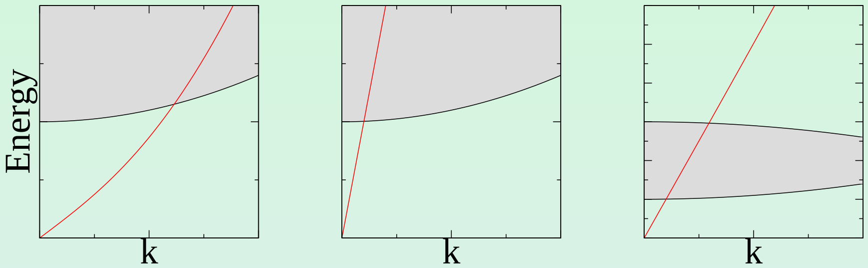
Crossover when:

$$T_{deg} = \frac{\rho}{m}$$

The phase diagram

Calculate density where $\rho_{\text{superfluid}} = 0$.



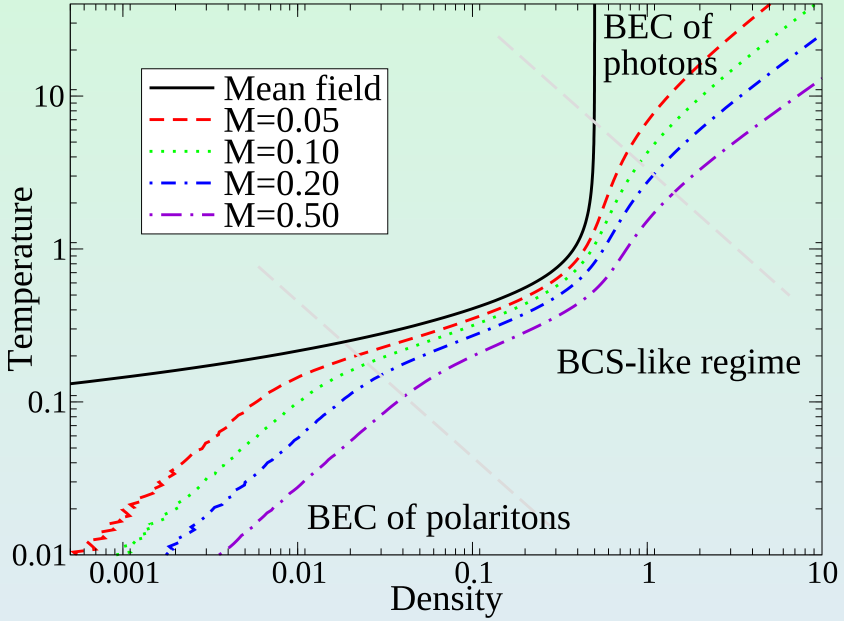


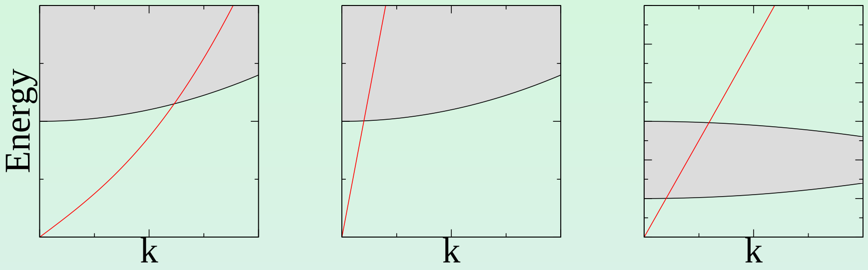
Crossover when:

$$T_{deg} = \frac{\rho}{m} \approx T_c \approx \frac{g\sqrt{n}}{-\ln(\rho/n)}$$

The phase diagram

Calculate density where $\rho_{\text{superfluid}} = 0$.





Crossover when:

$$T_{deg} = \frac{\rho}{m} \approx T_c \approx \frac{g\sqrt{n}}{-\ln(\rho/n)}$$

Current experiments in BCS-like regime: $\rho_{\text{crossover}}/n \approx mg/\sqrt{n} \approx 10^{-3}$, experiments around $\rho/n \approx 0.01$.

Conclusions

- Including fluctuations, B.E.C. transition at low density, internal structure matters at higher densities.

[*Keeling et al., Phys. Rev. Lett.* **93** 226403 (2004)]
[*Keeling et al., Phys. Rev. B* **72** 115320 (2005)]

Conclusions

- Including fluctuations, B.E.C. transition at low density, internal structure matters at higher densities.
- Fluctuations in 2D require considering fluctuations in condensate

[*Keeling et al., Phys. Rev. Lett.* **93** 226403 (2004)]
[*Keeling et al., Phys. Rev. B* **72** 115320 (2005)]

Conclusions

- Including fluctuations, B.E.C. transition at low density, internal structure matters at higher densities.
- Fluctuations in 2D require considering fluctuations in condensate
- Fluctuations in condensate deplete order parameter.

[*Keeling et al., Phys. Rev. Lett.* **93** 226403 (2004)]
[*Keeling et al., Phys. Rev. B* **72** 115320 (2005)]

Conclusions

- Including fluctuations, B.E.C. transition at low density, internal structure matters at higher densities.
- Fluctuations in 2D require considering fluctuations in condensate
- Fluctuations in condensate deplete order parameter.
- Density crossover set by wavelength, so experiments in B.C.S. regime.

[*Keeling et al., Phys. Rev. Lett.* **93** 226403 (2004)]
[*Keeling et al., Phys. Rev. B* **72** 115320 (2005)]

Conclusions

- Including fluctuations, B.E.C. transition at low density, internal structure matters at higher densities.
- Fluctuations in 2D require considering fluctuations in condensate
- Fluctuations in condensate deplete order parameter.
- Density crossover set by wavelength, so experiments in B.C.S. regime.

[*Keeling et al., Phys. Rev. Lett.* **93** 226403 (2004)]
[*Keeling et al., Phys. Rev. B* **72** 115320 (2005)]

Supplementary material

Experimental signatures: N(k)

From spectrum find:

$$N(k) = \left\langle \psi_k^\dagger(\tau + \eta)\psi_k(\tau) \right\rangle$$

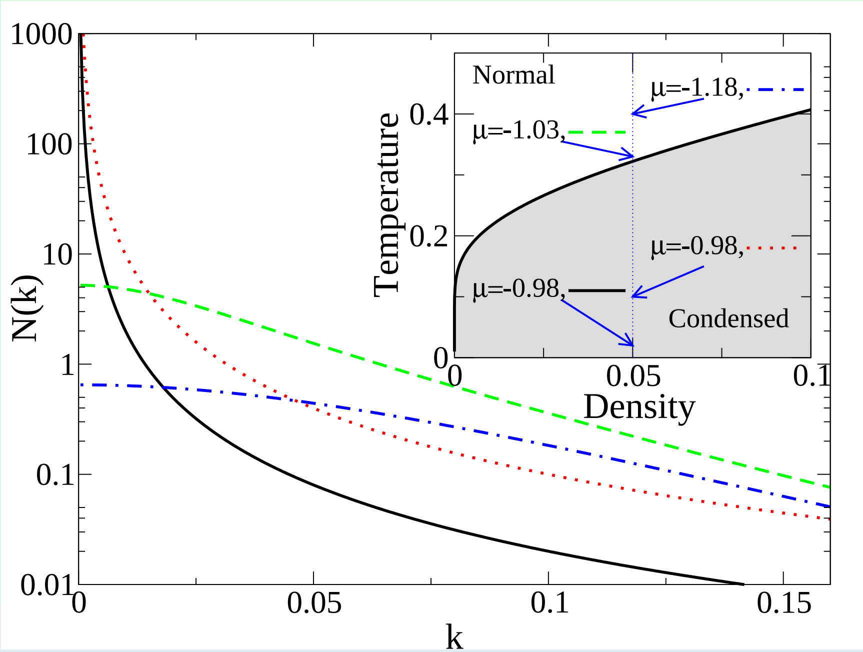
Experimental signatures: N(k)

From spectrum find:

$$N(k) = \left\langle \psi_k^\dagger(\tau + \eta)\psi_k(\tau) \right\rangle$$

Can calculate if:

- Uncondensed,
- **or** low T, phase fluctuations.



Experimental signatures: N(k)

From spectrum find:

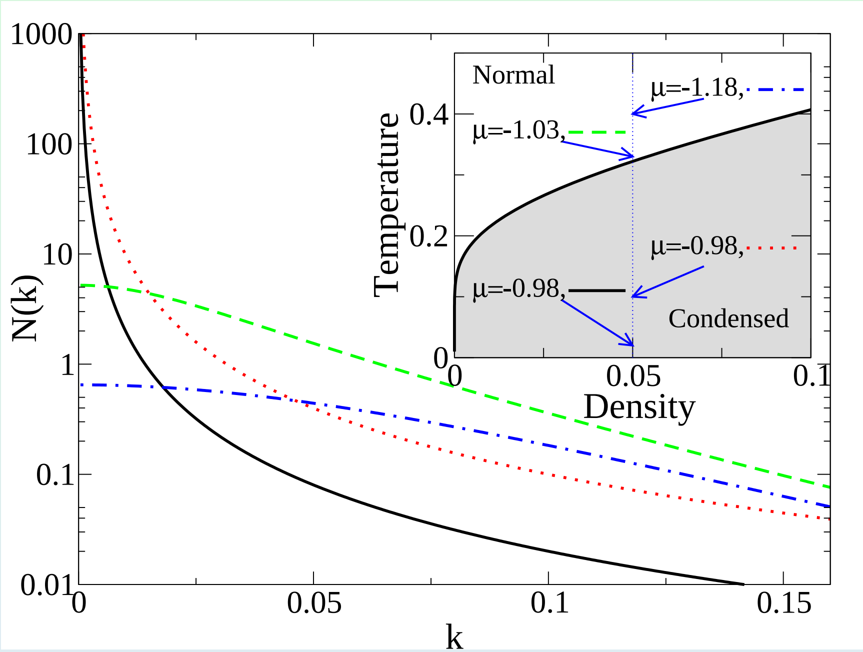
$$N(k) = \left\langle \psi^\dagger_k(\tau + \eta)\psi_k(\tau) \right\rangle$$

Can calculate if:

- Uncondensed,
- **or** low T, phase fluctuations.

Universal form:

$$N(p) \propto \rho_0 \frac{\xi_T^\eta}{p^{2-\eta}}, \qquad \eta = \frac{m}{2\pi\beta\rho_0\hbar^2}$$



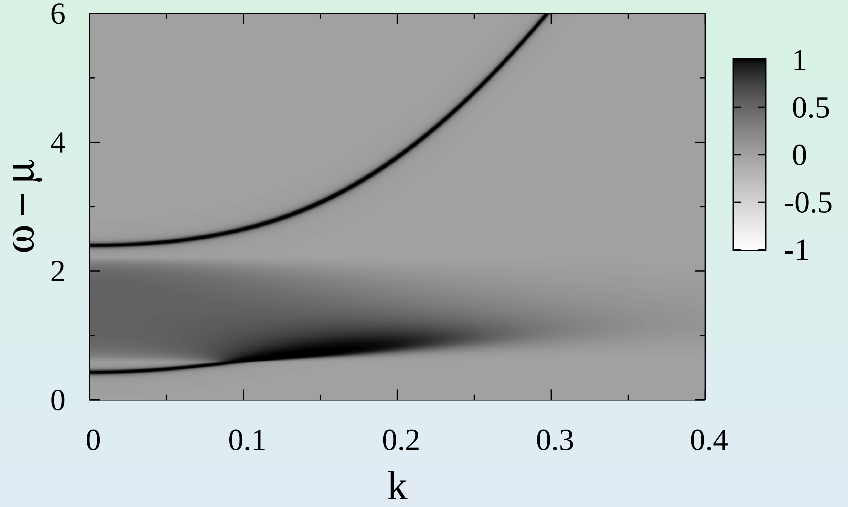
Inhomogeneous broadening — spectral weight

With inhomogeneous broadening of exciton energies, lines become broadened.
Can plot $\Im\mathcal{G}(i\omega = z + i\eta)$, absorption coefficient.

Inhomogeneous broadening — spectral weight

With inhomogeneous broadening of exciton energies, lines become broadened.

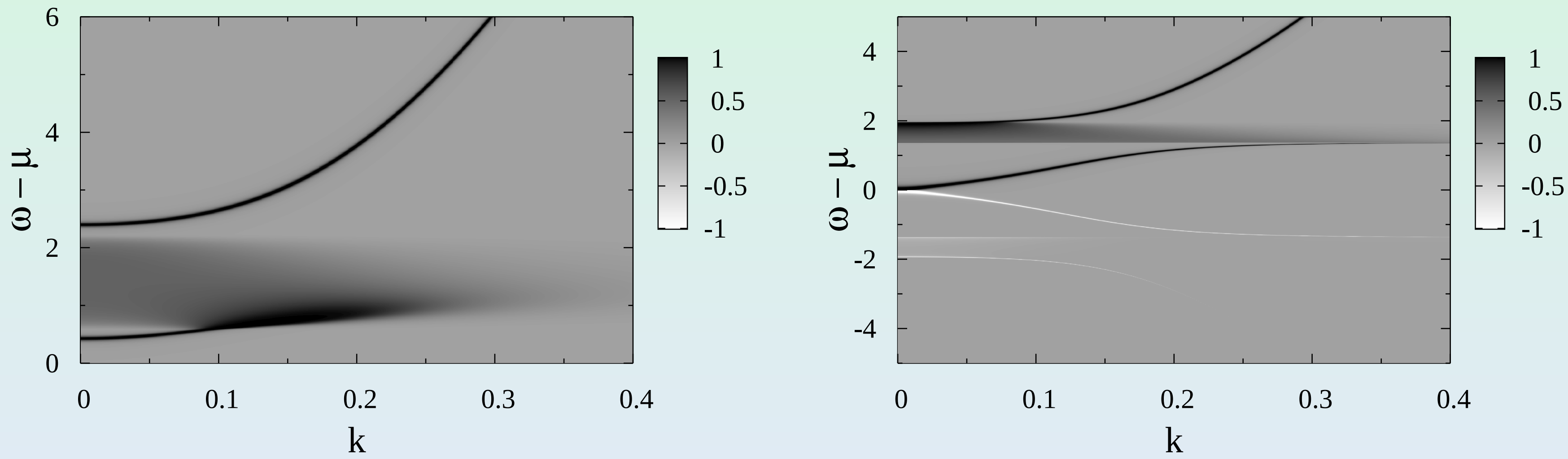
Can plot $\Im\mathcal{G}(i\omega = z + i\eta)$, absorption coefficient. Figures for broadening, $0.3g\sqrt{n}$ other parameters as previously.



Inhomogeneous broadening — spectral weight

With inhomogeneous broadening of exciton energies, lines become broadened.

Can plot $\Im\mathcal{G}(i\omega = z + i\eta)$, absorption coefficient. Figures for broadening, $0.3g\sqrt{n}$ other parameters as previously.



Note Goldstone mode is not broadened.

Inhomogeneous broadening: What the spectrum means

Absorption probability is:

$$P_{\text{absorb}}(x) = \sum_{n,m} \left| \langle m \left| \psi^\dagger \right| n \rangle \right|^2 e^{\beta(F-E_n)} \delta(x - E_{mn}) = (1 + n_{\text{B}}(x))\rho_{\text{L}}(x).$$

Inhomogeneous broadening: What the spectrum means

Absorption probability is:

$$P_{\mathrm{absorb}}(x) = \sum_{n,m} \left| \langle m \left| \psi^\dagger \right| n \rangle \right|^2 e^{\beta(F-E_n)} \delta(x - E_{mn}) = (1 + n_{\mathrm{B}}(x))\rho_{\mathrm{L}}(x).$$

Emission probability is:

$$P_{\mathrm{emit}}(x) = \sum_{n,m} \left| \langle m \left| \psi \right| n \rangle \right|^2 e^{\beta(F-E_n)} \delta(x + E_{mn}) = n_{\mathrm{B}}(x)\rho_{\mathrm{L}}(x).$$

Inhomogeneous broadening: What the spectrum means

Absorption probability is:

$$P_{\mathrm{absorb}}(x) = \sum_{n,m} \left| \langle m \left| \psi^\dagger \right| n \rangle \right|^2 e^{\beta(F-E_n)} \delta(x - E_{mn}) = (1 + n_{\mathrm{B}}(x)) \rho_{\mathrm{L}}(x).$$

Emission probability is:

$$P_{\mathrm{emit}}(x) = \sum_{n,m} \left| \langle m \left| \psi \right| n \rangle \right|^2 e^{\beta(F-E_n)} \delta(x + E_{mn}) = n_{\mathrm{B}}(x) \rho_{\mathrm{L}}(x).$$

Where ρ_{L}, the difference is given by:

$$\rho_{\mathrm{L}}(x) = \lim_{\eta \to 0} \Im \mathcal{G}(i\omega = x + i\eta) = P_{\mathrm{absorb}}(x) - P_{\mathrm{emit}}(x).$$

Inhomogeneous broadening — Emission probability

Alternative plots: P_{emit} Figures for broadening, $0.1 g\sqrt{n}$ other parameters as previously.